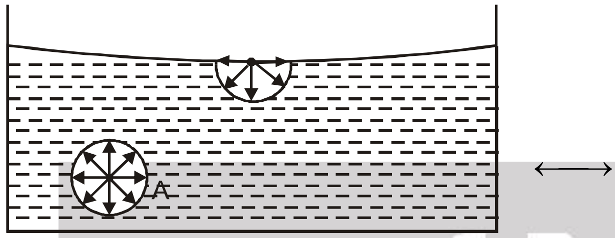
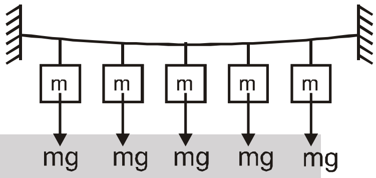
SURFACE TENSION

"Tension force generated in (applied by) the liquid surface is called surface tension force". In the fluid mechanics we have studied about the inner part of liquid, but in this chapter we will concentrate only on the surface of the liquid. The forces on the surface molecules are slightly different than the forces on inner molecules. Lets see how !

Explanation of surface tension on the basis of intermolecular forces :



Due to net downward force, a tension is generated in the surface



Due to suspended weight, tension is generated in rope

Actually surface tension is created due to cohesive forces, which is attractive force between the molecules of same substance.

Figure shows a container filled with a liquid. Consider a molecule 'A' which is inside the liquid. Equal cohesive force from all the direction acts on it. So net cohesive force on it is zero.

So cohesive force is meaningless for the liquid inside. That 's why we didn't used it in fluid mechanics.

Now lets consider a molecule 'B' on the surface. Water molecules are only below it, but there is no water molecule above it. So only the water molecules below it applies cohesive forces, and the resulting cohesive force is downwards.

Due to this downward force, a tension is generated in the surface, just like due to suspended weight, tension is generated in the rope.

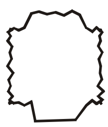
The tension generated in the surface is called surface tension force. Due to surface tension, the liquid surface behaves like a stretched membrane (rubber sheet) and try to minimize its area.

Explanation of surface tension on the basis of energy :

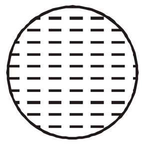
As we have seen, the molecule inside the liquid is attracted by the surrounding liquid molecules from all the directions. So it will has more negative energy (say –10). But the molecule on the surface is surrounded by liquid molecules only in lower half. So it will have less negative energy (say –5)

Less negative means more energy. So the molecules of surface have more energy than the molecule inside. For stability, the energy should be minimum possible. For minimum energy, the surface molecules should be minimum and hence surface area should be minimum. So the surface tries to minimize its area and due to this a tension is generated in the surface.

Some simple evidence of surface tension :



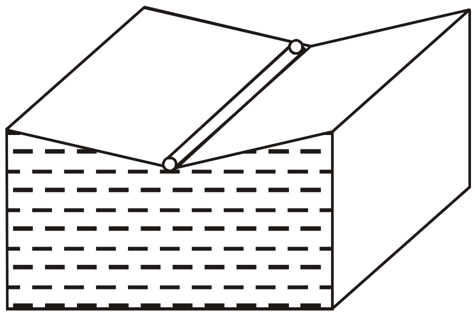
(i) Piece of stone



Piece of water (water drop)

A piece of stone can be of random shape because solids don't have surface tension. But a piece of water (water drop) is in spherical shape. Since there is tension in surface of water. So the water surface act like a tight membrane (tight bag). To minimize its surface area, the water drop takes spherical shape. For small drop gravitational pressure (ρgh) is negligible so the small drop is almost spherical. But in big drop gravitational pressure (ρgh) is considerable so the big drop has oval shape.

(ii) If we put a needle very slowly on the water surface, it will float on the surface as if it were put on a tight membrane. This also proves that there is a tension in the liquid surface due to which it act like a tight membrane.



(iii)

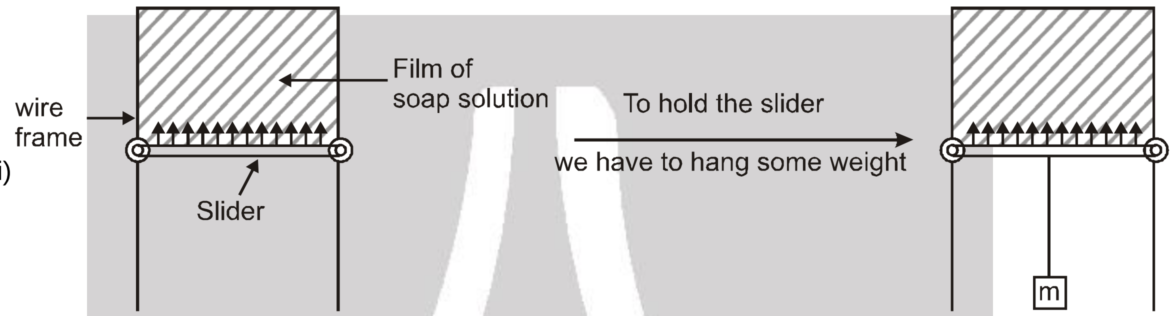
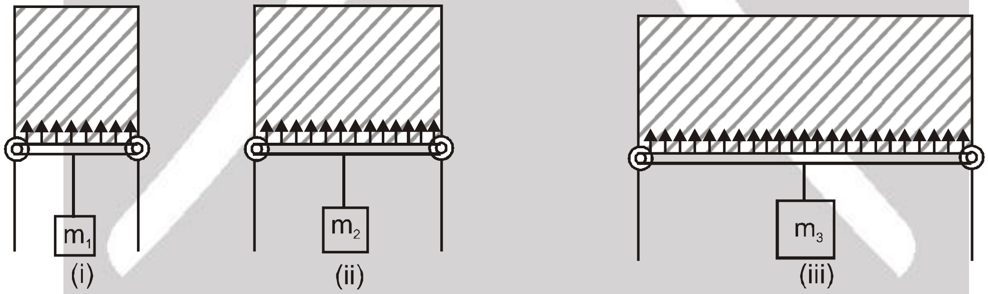
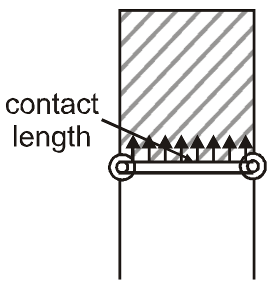


Figure shows a U shaped fixed wire frame, on which very light slider can slide. Dip the frame in soap solution and take it out. A thin film of soap solution is formed between the frame and slider, which is purely a surface. Now if we release the slider, it will move upwards, this shows that there is a tension in the liquid surface. The liquid surface applied tension force (pulling force) on the slider in contact, due to which the slider try to move upward. To keep the slider in equilibrium, we have to hang some weight. This is very close example. From this, we can also measure surface tension force.



Consider three cases (i), (ii) and (iii). In which case, the surface tension force on the slider is more? Practically it is observed that in case (i) surface tension force on slider is least, it is more in case (ii) and most in case (iii). In case (iii), we have to hang more weight to keep the slider in equilibrium. From this example it is clear that surface tension force depends on contact length which is greatest in case (iii)



Surface tension force (F) $\propto$ contact length (ℓ)

$$F = (T)\ \ell$$

Here T is a constant which is called surface tension constant. T depends on the properties of liquid and also on the medium which is on the other side of liquid.

- If we increase the temperature, surface tension constant (T) decreases.
- If we add highly soluble substances like NaCl, $ZnSO_4$ etc. then surface tension constant (T) increases.
- If we add sparingly soluble substances like soap, phenol, then surface tension (T) decreases.

Result :

Surface applies tension force (pulling force) on the other part of surface and also on any object (like slider) which is in contact.
Surface tension force
F = (T) (ℓ) where ℓ = contact length = length of Boundary line between the two surfaces

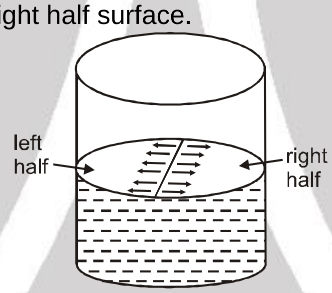
also T = so the definition of surface tension (T) can be written as

$T = F/\ell$

The surface tension of a liquid can be measured as the force per unit length on an imaginary line drawn on the liquid surface, which acts perpendicular to the line on its either side at every point and tangentially to the liquid surface.

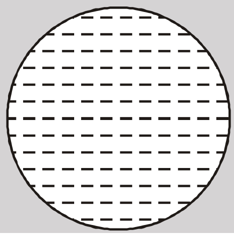
Solved Example

Example 1. Figure shows the container of radius R filled with water. Consider an imaginary diametric line dividing the surface in two parts: Left half and right half. Find surface tension force between the left half surface and the right half surface.

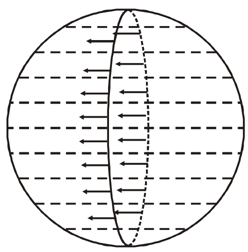


Solution : Both left half and right half surface will pull each other with a force F = (T) (ℓ)
where ℓ is the length of boundary lines between the two surfaces which is equal to 2R
So F = (T) (2R)

Example 2. Consider a water drop of radius R. Find surface tension force between the left half surface and right half surface ?

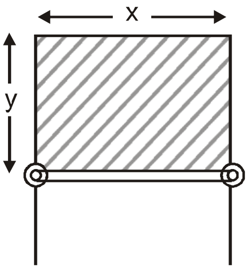


Solution : Surface tension force

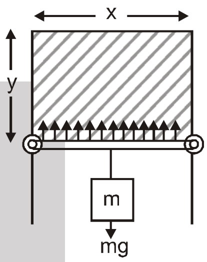


F = (T) (ℓ)
here ℓ = length of boundary line between left half and right half surface = $2\pi R$
So F = (T)(2πR)

Example 3. Between a frame and a light slider, a thin film of soap solution is made. Whose length is x and width is y. Find surface tension force on the slider. To keep the slider in equilibrium, how much weight should be suspended ?



Solution : The surface will act like a tight membrane and pull the slider with a force F = (T) (ℓ). Since this a film, it will have two surfaces: the front surface and the back surface. On the front surface, contact length is x, and also on the back surface contact length is x. So total contact length will be $\ell = x + x = 2x$



So surface tension force on slider.

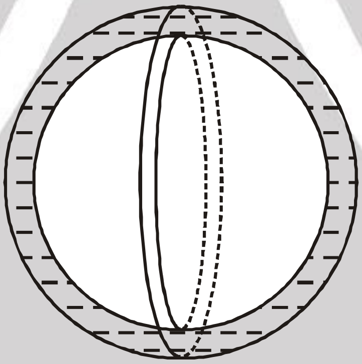
F = (T)(2x)

For equilibrium, this force will be balanced by weight of suspended block.

(T)(2x) = mg

$$m = \frac{2Tx}{g}$$

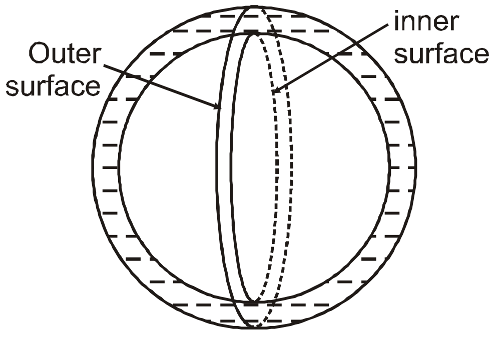
Example 4. Consider a bubble of soap solution. Find the surface tension force between the left half surface and right half surface



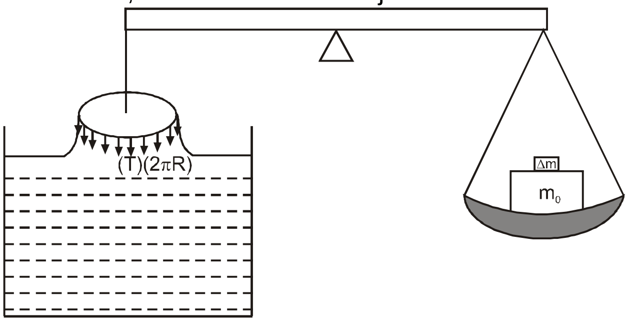
Solution : The bubble also have two surfaces: the inner surface and the outer surface. And in the small thickness between them, there is some liquid. So the surface tension force will be applied by inner surface as well as the outer surface [$T(2\pi R)$].

So total surface tension force between left half and right half surface is

$F = (T)(2\pi R) + (T)(2\pi R) = T(4\pi R)$



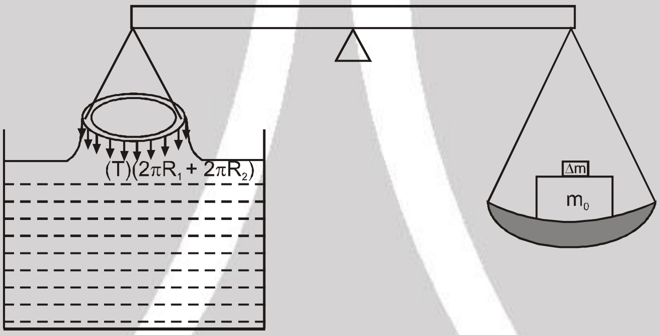
Example 5. A thin disc of radius R, just touching the liquid surface, forms one arm of a balance. The plate is balanced by some weight on the other side of the balance. How much extra weight should be added on the other side, so that the disc can just come out of water ?



Solution: Surface tension force on the disc is $(T)(2\pi R)$

For balance $(T)(2\pi R) = (\Delta m)g \Rightarrow \Delta m = \frac{(T)(2\pi R)}{g}$

Example 6. In the previous question, in place of disc a ring is used whose inner radius is R_1 and outer radius is R_2. Now how much extra weight should be added on the other side, so that the ring can just come out of water ?

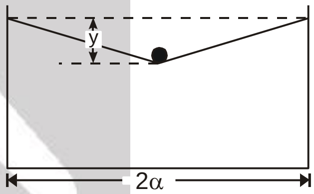


Solution : Surface tension force on the disc is $(T)\ 2\pi\ (R_1 + R_2)$

For balance $(T)\ 2\pi(R_1 + R_2) = (\Delta m)g \Rightarrow \Delta m = \frac{(T)2\pi(R_1 + R_2)}{g}$

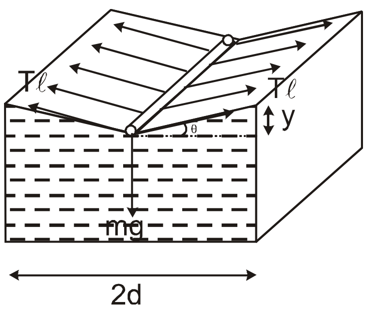
Example 7. **(Only for JEE Advanced)**

A long thin straight uniform wire of negligible radius is supported on the surface of a liquid. The width of the container is 2d and the wire is kept at its centre, parallel to its length (as shown in figure). The surface of the liquid is depressed by a vertical distance y(y << d) at the centre as shown in figure. If the wire has mass λ per unit length, what is the surface tension of the liquid? Ignore end effects.



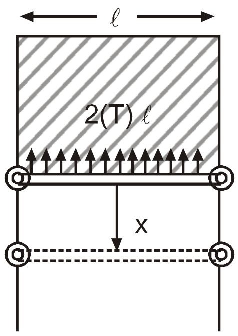
Solution : Balancing upward an downward forces $2(T\ell \sin\theta) = (\lambda\ell)g$

as angle is very small $\sin \approx \tan\theta = \frac{y}{d}$



$2(T\ell)\frac{y}{d} = (\lambda\ell)g \quad T = \frac{\lambda g d}{2y}$

Surface energy :



Potential energy stored due to surface tension force is called surface energy. To understand this, suppose a thin film of soap solution is formed between the fix frame and the slider. Both front and the back surface will pull the slider with a force of $F = 2(T\ell)$

Now we move the slider forward by a distance x.

During this :

Work done by surface tension force = $-(2T\ell)(x)$

(As surface tension force is opposite of displacement)

⇒ Work done against surface tension force = $+(2T\ell)x$

⇒ Increase in surface potential energy = $+(2T\ell)x$

where $2\ell x$ = increasing surface area (increase in front area = ℓx, increase in back area = ℓx)

⇒ Increase in surface potential energy $\Delta U = (T)(\Delta A)$ = (T) (increase in surface area)

or generally, we can say that

Surface energy U = (T)(A) = (T) (surface area)

also $T = \frac{U}{A}$ and previously we have seen that $T = \frac{F}{\ell}$

So Surface tension is surface energy per unit surface area

Surface tension is also tension force generated on the surface per unit length.

Solved Example

Example 8. 1000 small water drops, each of radius r, combine and form a big drop. In this process, find decrease in surface energy.

Solution : Suppose radius of big drop is R. During this process, mass will be conserved, so volume will also be conserved.

$(\text{Volume})_{\text{initial}} = (\text{Volume})_{\text{final}}$; $\left(\frac{4}{3}\pi r^3\right)\times 1000 = \left(\frac{4}{3}\pi R^3\right)$ ⇒ $R = 10r$

loss in surface energy $\Delta U_{loss} = T\Delta A_{loss} = T(\,(4\pi r^2)\times 1000 - 4\pi(10r)^2\,)$

⇒ $\Delta U_{loss} = (T)\,(900 \times 4\pi r^2)$

this energy loss will be converted into heat. So increase in temperature of the drop can be found from

$T(900 \times 4\pi r^2) = ms\Delta T$, From this get the increase in temperature ΔT.

Example 9. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given be $\frac{3T}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ where T is the surface tension of water and J is the mechanical equivalent of heat. Here r, R and T are in CGS system.

Solution : suppose n small water drop combine and form a big drop. During this process so volume will also be conserved

$(\text{Volume})_{\text{initial}} = (\text{Volume})_{\text{final}}$

$\left(\frac{4}{3}\pi r^3\right)\times n = \frac{4}{3}\pi R^3$ ⇒ $n = \frac{R^3}{r^3}$

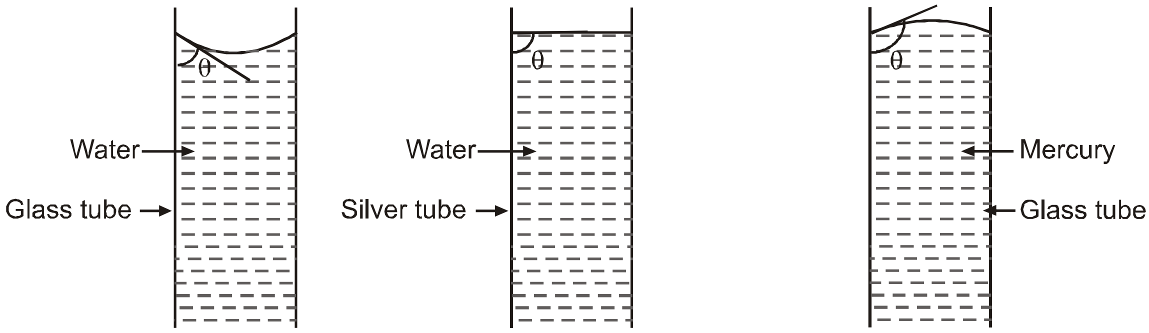
Loss in surface energy $\Delta U_{loss} = T\Delta A_{loss} = T(4\pi r^3 \times n - 4\pi R^2)$

Put $n = \frac{R^3}{r^3}$ get $\Delta U_{loss} = T\left(4\pi r^2 \times \frac{R^3}{r^3} - 4\pi R^2\right)$; $\Delta U_{loss} = T\,4\pi R^3\left(\frac{1}{r}-\frac{1}{R}\right)$

$T(4\pi R^3)\left(\frac{1}{r}-\frac{1}{R}\right) = ms\,\Delta\theta$ when $m = \rho(\text{vol})$

$T(4\pi R^3)\left(\frac{1}{r}-\frac{1}{R}\right) = (1\ \text{gm/cm}^3)\left(\frac{4}{3}\pi R^3\right)(1\ \text{cal/gm}\ {}^{o}\text{C})\Delta T$ get $\Delta\theta = \frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$

SHAPE OF LIQUID SURFACE :

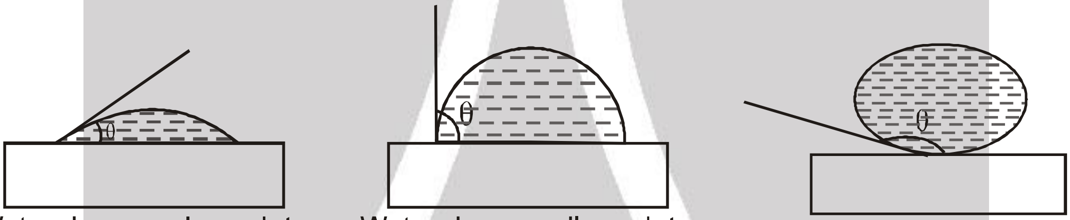


If we fill water in glass tube, the surface becomes concave in shape, if water is filled in silver tube, the surface becomes horizontal and if Hg is filled in glass tube, the surface becomes convex.

Shape of liquid surface is called meniscus. At point of contact, angle between the tangent to the liquid surface and solid surface submerged in liquid is called angle of contact (θ).

In figure (i) angle of contact is acute.

In figure (ii), angle of contact is 90° and in figure (iii) angle of contact is obtuse. Angle of contact can also be observed when a liquid drop is put on a plate as shown below :



Water drop on glass plate
$\theta < 90°$

Water drop on silver plate
$\theta = 90°$

Hg drop on glass plate
$\theta > 90°$

The shape of liquid surface depends on cohesive and adhesive forces.

Cohesive force : The force of attraction between the molecules of the same substance is called cohesive force. The cohesive forces is effective if distance between molecules is less than 10^{-9} m. If distance between molecule is greater than 10^{-9} m then cohesive force is negligible. The sphere drawn around a particular molecule as centre and range of cohesive forces (10^{-9} m) as radius is called **sphere of influence (sphere of molecular activity)**. The centre of molecule is attracted by only the molecules lying inside the sphere of influence.

Example : cohesive force between water molecules.

On the corner molecule (see the figure (i) (a) bellow), all the neighbouring water molecules will apply cohesive force, so net cohesive force (F_c) on it can be assumed to be centered at 45° angle with vertical.

Other examples of Cohesive force :

(i) Two drops of a liquid coalesce into one when brought in mutual contact because of the cohe sive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

Adhesive force : The force of attraction between different substances is called adhesive force.

Example Adhesive force between water and glass tube.

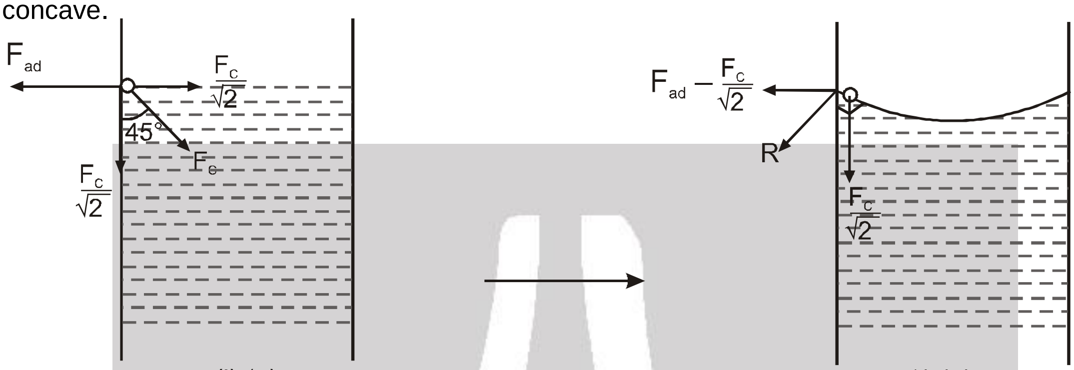
On the corner molecule, adhesive force will be towards the glass wall as shown in figure (i) (a) bellow.

Other examples of adhesive force :

Examples.

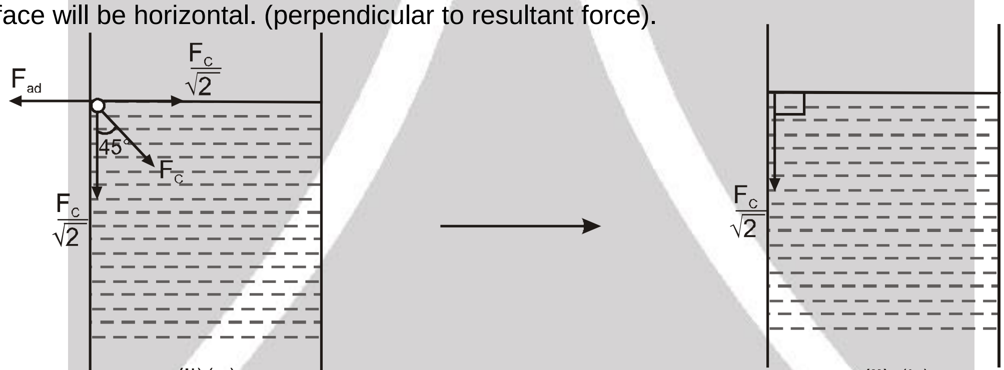
(i) Adhesive force enables us to write on the black board with a chalk.
(ii) Adhesive force helps us to write on the paper with ink.
(iii) Large force of adhesion between cement and bricks helps us in construction work.
(iv) Due to force of adhesive, water wets the glass plate.
(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

Case- I : If water is filled in a glass tube, $F_{ad} > \frac{F_c}{\sqrt{2}}$ then the resultant force will be as shown in (i) (b). As the water surface always adjusts itself perpendicular to the resultant force. So the surface will be concave.



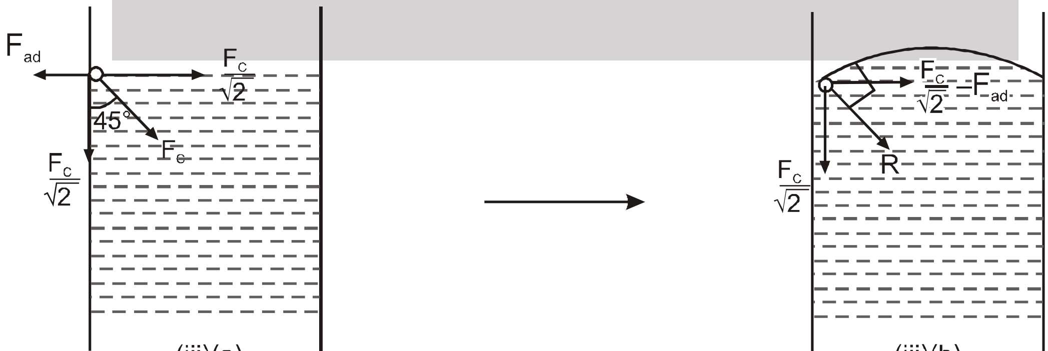
(i) (a) (i) (b)

Case -II : If water is filled in silver tube, $F_{ad} = \frac{F_c}{\sqrt{2}}$ so resultant will be vertically downwards. So liquid surface will be horizontal. (perpendicular to resultant force).



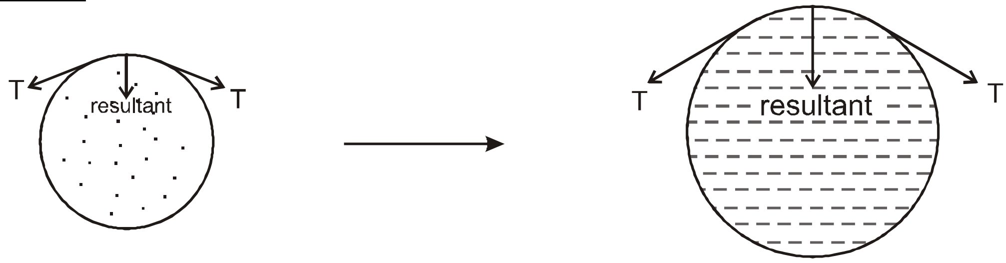
(ii)(a) (ii) (b)

Case -III : If Hg is filled in glass tube, $F_{ad} < \frac{F_c}{\sqrt{2}}$ so resultant force will be as shown in (iii) (b). As the surface adjusts itself perpendicular to the resultant force so surface will be convex.



(iii)(a) (iii)(b)

PRESSURE EXCESS INSIDE A LIQUID DROP :

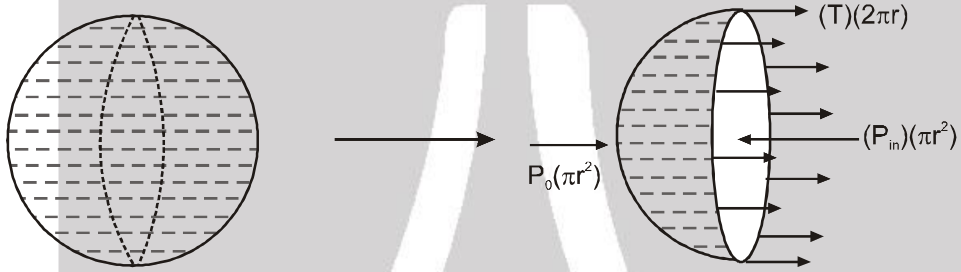


Due to stretched rubber, the air inside gets compressed. So pressure of air inside will be greater than pressure of air outside

The water surface also acts like a stretched rubber. So due to tension in the surface the water inside get compressed

So the pressure of water inside will be greater than the outside atmospheric pressure. This extra pressure is called pressure excess.

To find pressure excess, make free body diagram of the half part. The forces on this hemisphere are :



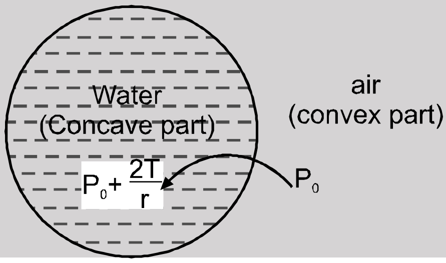
(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$

(ii) Pushing force due to atmospheric pressure will be $(P_0) \times$ (facing area) $= P_0(\pi r^2)$

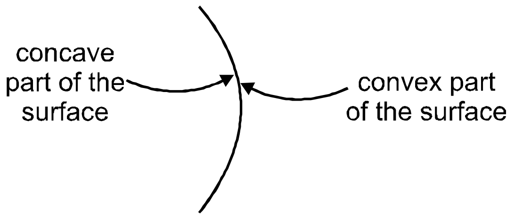
(iii) Surface tension force on left half surface due to right half surface will be $(T)(2\pi r)$

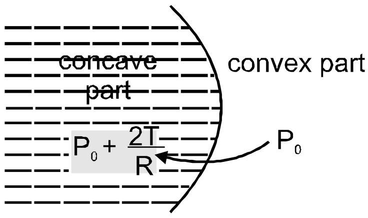
Applying force balance : $(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r)$

$\Rightarrow$ $P_{in} = P_0 + \frac{2T}{r}$, here $\frac{2T}{r}$ is called pressure excess. So pressure inside the drop will be greater than pressure outside the drop by $\frac{2T}{r}$



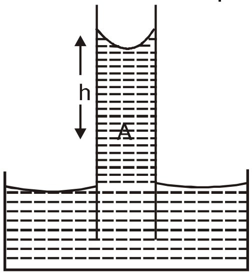
Generally we can say that pressure at concave part will be greater than pressure at convex part by $\frac{2T}{r}$ where r is radius of curvature of the surface between them.



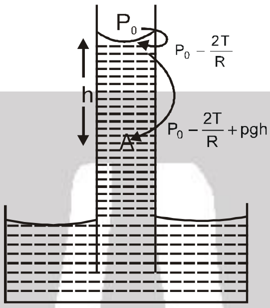


Solved Example

Example 10. Water is filled in a capillary tube of radius R. If the surface of water is hemispherical ($\theta = 0$), then find pressure at a point 'A' which is at h depth below the surface.

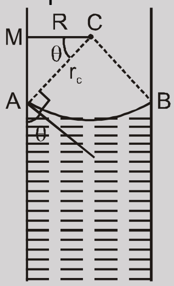


Solution :



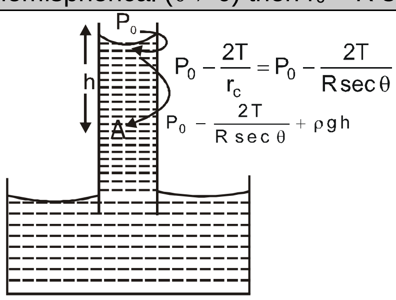
Water is on convex part. So pressure of water just below the surface will be less by $\frac{2T}{R}$. So pressure at point A is $P_0 - \frac{2T}{R} + \rho gh$. Here surface of water was hemispherical (contact angle $\theta = 0$) so radius of curvature of the surface = radius of the tube = R.

Example 11. In the previous question, suppose contact angle is not zero, but it is θ (the surface not hemispherical) now find pressure at point 'A'



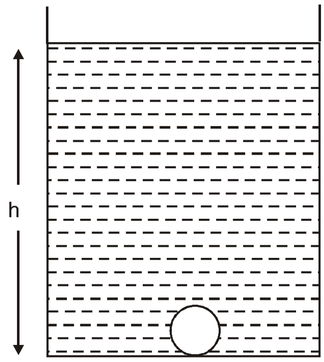
Solution : Draw normal (radial lines) at point A and B of periphery. The point (C) where radial lines meet is called centre of curvature. If contact angle is θ, from Δ ACM, $r_C = R\sec\theta$
So radius of curvature of the surface $r_C = R\sec\theta$.

Point to remember :
If the liquid surface is hemispherical ($\theta = 0$) then $r_c = R$
If liquid surface is not hemispherical ($\theta \neq 0$) then $r_c = R\sec\theta$

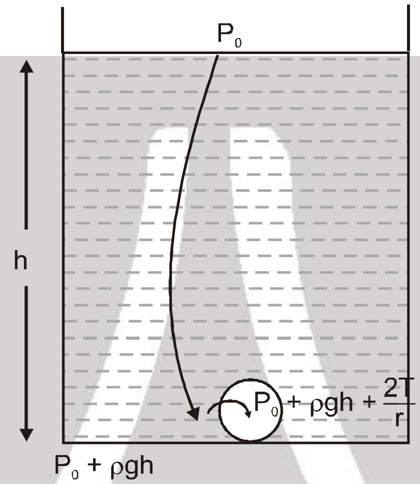


So pressure at A is $P_0 - \frac{2T}{R\sec\theta} + \rho gh$

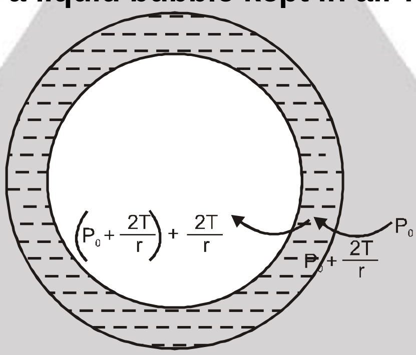
Example 12. A small air bubble (cavity of air) of radius r is at depth 'h'. Find the pressure inside the bubble.



Solution : Out of water and air (inside the bubble) air is on concave part.



Pressure excess inside a liquid bubble kept in air :-



So pressure inside the liquid bubble = $P_0 + \frac{4T}{r}$

So pressure excess inside the liquid bubble = $\frac{4T}{r}$

Alternative method :

Draw free body diagram of half part of bubble. The force on this hemisphere are :

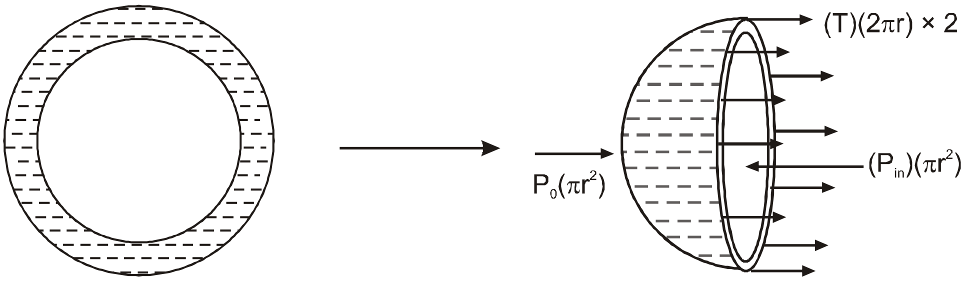
(i) Pushing force on the left half liquid due to right half liquid will be $(P_{in})(\pi r^2)$

(ii) Pushing force due to atmospheric pressure will be (P_0) × (facing area) = $P_0(\pi r^2)$

(iii) Surface tension force on both inner and outer surface will be $(T)(2\pi r) \times 2$

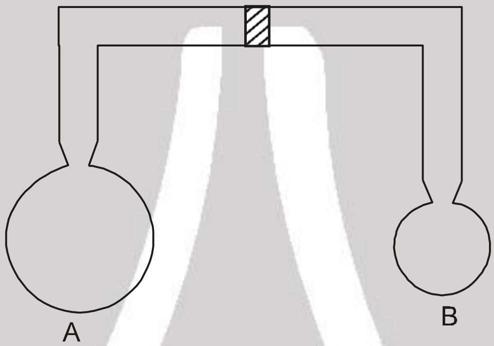
Applying force balance :

$(P_{in})(\pi r^2) = P_0(\pi r^2) + (T)(2\pi r) \times 2 \Rightarrow \quad P_{in} = P_0 + \frac{4T}{r}$, So pressure excess inside a liquid bubble = $\frac{4T}{r}$

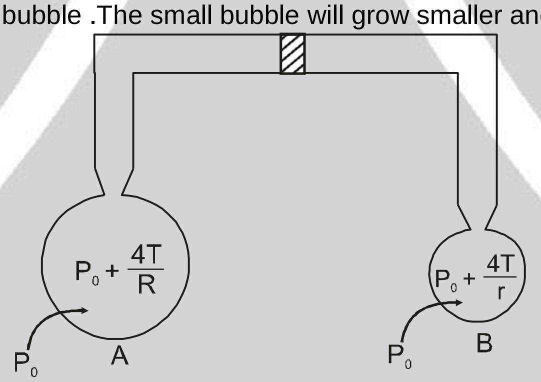


Solved Example

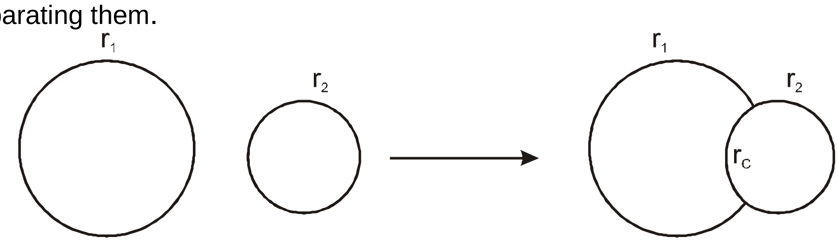
Example 13. Two soap bubbles are formed on the ends of the tube as shown. If valve is opened, in which direction will the air flow ?



Solution : Radius of curvature of smaller bubble 'B' is less so pressure inside the smaller bubble will be more $\left(P_0 + \frac{4T}{r}\right)$. Air will flow from high pressure to low pressure, so it will flow from smaller bubble to bigger bubble .The small bubble will grow smaller and the big bubble will grow bigger.

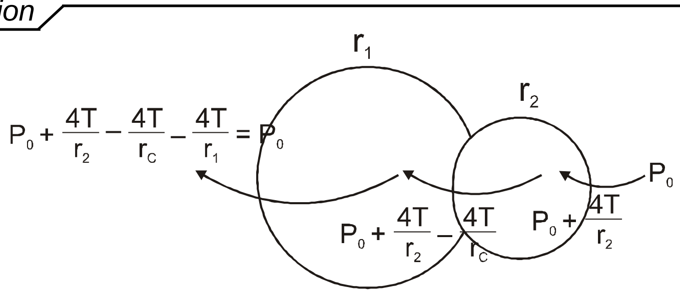


Example 14. Two soap bubbles of radius r_1 and r_2 combine .Find radius of curvature of the common surface separating them.



Solution : $P_0 + \frac{4T}{r_2} - \frac{4T}{r_c} - \frac{4T}{r_1} = P_0$

$\frac{1}{r_c} = \frac{1}{r_2} - \frac{1}{r_1}$



$$r_c = \frac{r_1 \; r_2}{r_1 - r_2}$$

Example 15. **(Only for JEE Advance)**

A soap bubble of radius r and surface tension constant T is given a charge, so that its surface charge density is σ. Due to charge, the radius of the soap bubble becomes double then find 'σ'. (atmospheric pressure = P_0)

Solution : Initial pressure inside the bubble $P_i = P_0 + \frac{4T}{r}$

Now a uniform surface charge in given to the bubble

The surface tension is a pulling force, which increases pressure inside the bubble (by $\frac{4T}{r}$)

But the charges given to the surface will repel each other. So due to the charge given, pressure inside the bubble will decrease (by $\frac{\sigma^2}{2\varepsilon_0}$)

So, final pressure inside the bubble $P_f = P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}$

As the temperature of the gas inside the bubble if constant so, $P_iV_i = P_fV_f$

$$\left(P_0 + \frac{4T}{r}\right)\left(\frac{4}{3}\pi r^3\right) = \left(P_0 + \frac{4T}{r_f} - \frac{\sigma^2}{2\varepsilon_0}\right)\left(\frac{4}{3}\pi r_f^3\right)$$

Here Put $r_f = 2r$ So, get $\sigma = \sqrt{\left(7P_0 - \frac{12T}{r}\right)2\varepsilon_0}$.

Example 16. **(Only for JEE Advance)**

A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100 cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76 cm of mercury.

Solution : The total pressure inside the bubble at depth h_1 is (P is atmospheric pressure)

$$= (P + h_1 \rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is $= (P + h_2 \rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyle's Law ; $P_1V_1 = P_2V_2$ where $V_1 = \frac{4}{3}\pi r_1^3$, and $V_2 = \frac{4}{3}\pi r_2^3$

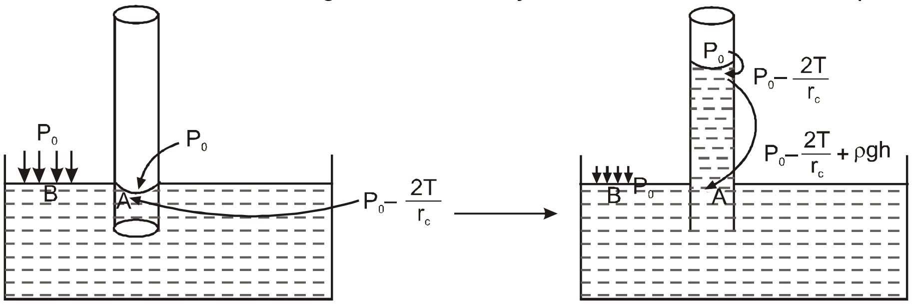
Hence we get $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right]\pi r_2^3$

or, $\left[(P + h_1\rho g) + \frac{2T}{r_1}\right] r_1^3 = \left[(P + h_2\rho g) + \frac{2T}{r_2}\right] r_2^3$

Given that : h_1 = 100 cm, r_1 = 0.1 mm = 0.01 cm, r_2 = 0.126 mm = 0.0126 cm, T = 567 dyne/cm, P = 76 cm of mercury. Substituting all the values, we get

h_2 = 9.48 cm.

CAPILLARY ACTION : A glass tube of very small diameter is called capillary



If we dip the capillary tube in water, due to the concave surface, pressure just below the surface becomes $P_0 - \frac{2T}{r_c}$, while on the other points at the same horizontal level, pressure is P_0. Due to this less pressure water level in the tube rises up, till pressure becomes equal at the same horizontal level (At point A and B)

$$P_0 - \frac{2T}{r_c} + \rho gh = P_0 \Rightarrow h = \frac{2T}{\rho g r_c}$$

where r_c = radius of curvature of the water surface. If the water surface is hemispherical ($\theta = 0$), then $r_c = R$ but if water surface is not hemispherical ($\theta \neq 0$), then $r_c = R\sec\theta \Rightarrow h = \frac{2T\cos\theta}{\rho g R}$

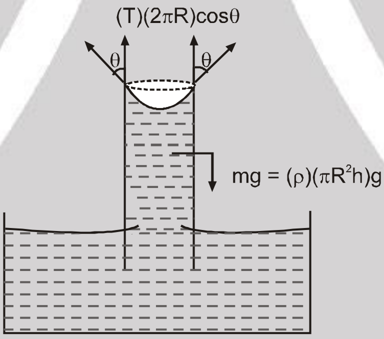
From this formula, we can say that

If $\theta < 90°$ then h = ⊕ ve, so water in capillary will rise up (Ex. water in glass tube).

If $\theta = 0$ so h = 0, so water in the capillary will not rise. (Ex. water in silver tube).

If $\theta > 90°$, h = –ve, so liquid in capillary will go down. (Ex. mercury in glass tube).

Deriving capillary rise from force balance :



As we dip the capillary in water, the surface pulls the capillary walls in downwards direction, so the capillary walls pulls the surface in upward direction as shown in figure, due to which water will rise up till the forces get balanced.

Lets draw free body diagram of the water raised up. Forces on it are :

(i) The surface pulls the capillary in downward direction, so as a reaction, the capillary pulls the surface in upward direction. Their horizontal components will be cancelled out and their vertical components will be added up. So net surface tension force will be vertically upwards and will be $(T)(2\pi R)\cos\theta$.

(ii) The weight of raised water; we can neglect the weight of meniscus. So the weight of raised water = $(\rho)(\pi R^2 h)g$

For equilibrium, forces should be balanced.

$$(T)(2\pi R)\cos\theta = (\rho)(\pi R^2 h)g \Rightarrow h = \frac{2T}{\rho g R}\cos\theta$$

From this equation we can say that $h \propto 1/R$. So if the capillary is thin, water will raise to more height.

If pure water is inside a glass tube, then $\theta \to 0$ so $h = \dfrac{2T}{\rho g R}$

Although in the previous derivation the volume of meniscus is negligeable, but if we have to consider the volume of meniscus then the volume of water raised will be $\pi r^2(h + r) - \dfrac{2}{3}\pi r^3$ so applying force balance

$(T)(2\pi R)\cos\theta = (\rho)(\pi r^2 (h + r) - \dfrac{2}{3}\pi r^3)g$ solving $\left(h + \dfrac{r}{3}\right) = \dfrac{2T}{\rho g R}\cos\theta$

Practical Applications of Capillarity

1. The oil in a lamp rises in the wick by capillary action.

2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tip of nib continuously.

3. Sap and water rise upto the top of the leaves of the tree by capillary action.

4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.

5. Ink is absorbed by the blotter due to capillary action.

6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.

7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture in the soil, capillaries must be. broken up. This is done by ploughing and leveling the fields

8. Bricks are porous and behave like capillaries.

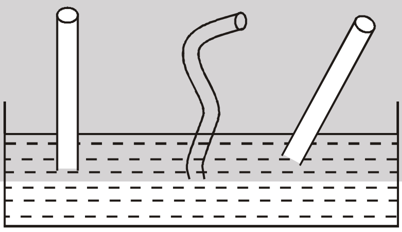
Solved Example

Example 17. A capillary of internal radius 4 mm, is dipped in water. To how much height, will the water rise in the capillary. ($T_{water} = 70 \times 10^{-3}$ N/m, g = 10 m/sec^2, ρ_{water} 10^3 kg/m^3, contact angle $\theta \to 0$)

Solution : Capillary rise

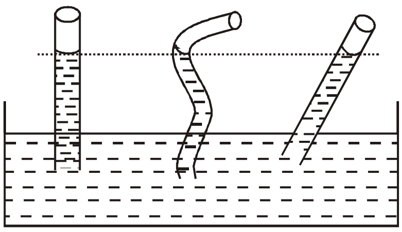
$h = \dfrac{2T}{\rho g R}\cos\theta = \dfrac{2 \times 70 \times 10^{-3}}{10^3 \times 10 \times 4 \times 10^{-3}}$ .....(1) h = 3.5 mm

Example 18. If all the glass capillaries have same internal radius, then in which of the capillary, water will rise to move height ?



Solution : The height of water in the capillary $\left(h = \dfrac{2T}{\rho g r}\cos\theta\right)$ doesn't depend on shape of the capillary.

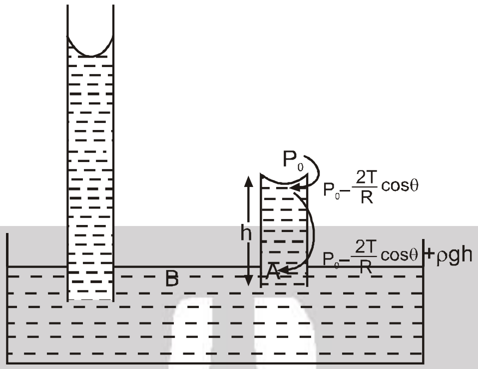
So water will raise to same height in all the tubes. (However the length of water column in the tubes can be different)



If capillary tube of insufficient length is used :

Suppose a thin capillary tube of radius 0.35 mm is dipped in water. $T_{water} = 70 \times 10^{-3}$ N/m, $\theta \to 0$.
In this case water will rise up to a height

$$h = \frac{2T}{\rho g R}\cos\theta = \frac{2\times70\times10^{-3}}{10^{3}\times10\times0.35\times10^{-3}} = 4\text{cm}$$



Now suppose we use shorter capillary of same radius, but its length is only 2 cm. It is slightly dipped in the water.
To balance the pressure, water level will rise up in the capillary, it will reach upto the upper end of the tube, and now the contact angle will change till the pressure at same horizontal level is balanced. Balancing pressure at point A (inside the capillary) and point B (outside)

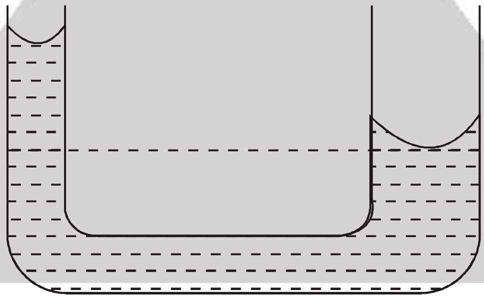
$$P_0 - \frac{2T}{R}\cos\theta + \rho g h = P_0 \quad \Rightarrow \quad h = \frac{2T}{\rho g R}\cos\theta$$

$$2\times10^{-2} = \frac{2\times70\times10^{-3}}{10^{3}\times10\times0.35\times10^{-3}}\cos\theta \quad \Rightarrow \quad \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

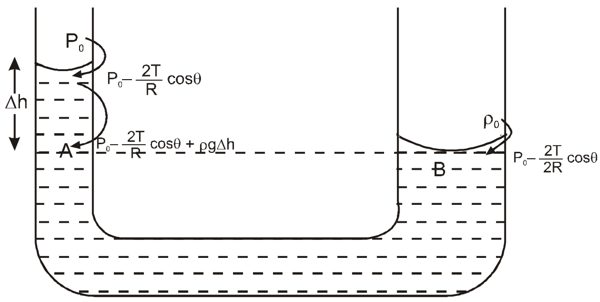
So water level will reach to the topmost point of the capillary (= 2cm) and now contact angle will change to 60°. Water will not overflow out of upper end in the form of fountain.

Solved Example

Example 19. In the U-tube, radius of one arm is R and the other arm is 2R. Find the difference in water level if contact angle is $\theta = 60°$ and surface tension of water is T.



Solution :

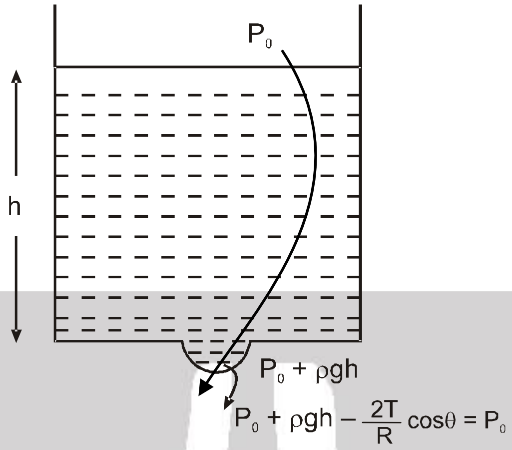


Balancing pressure at points A and B situated in same horizontal level.

$$P_0 - \frac{2T}{R}\cos\theta + \rho g\Delta h = P_0 - \frac{2T}{2R}\cos\theta \quad \text{here } \theta = 60°\text{, solving we get } \Delta h = \frac{T}{2\rho g R}$$

Example 20. There is a small hole of diameter 0.1 mm at the bottom of a large container. To what minimum height we can fill water in it, so that water doesn't come out of hole. ($T_{water} = 75 \times 10^{-3}$ N/m) $\rho_{water} = 10^3$ kg/m^3, g = 10m/sec^2

Solution :



The lower surface of water, which will try to come out will be spherical. Pressure just outside the spherical surface is :

$P_0 + \rho gh - \frac{2T}{R}\cos\theta = P_0$; $h = \frac{2T}{\rho gR}\cos\theta$

$(h)_{max} = \frac{2T}{\rho gR}(\cos\theta)_{max}$ and $(\cos\theta)_{max} = 1$

So $(h)_{max} = \frac{2T}{\rho gR} = \frac{2 \times 75 \times 10^{-3}}{10^3 \times 10 \times 0.05 \times 10^{-3}}$ $(h)_{max}$ = 30 cm

SOME OTHER APPLICATIONS OF SURFACE TENSION

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

Solved Example

Example 21. **(Only for JEE Advanced)**

A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. If the height of the liquid in the narrow tube is 0.2 m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Solution : Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the meniscus in the wide and narrow tubes respectively are

$\left(P_1 - \frac{2T}{r_1}\right)$ and $\left(P_2 - \frac{2T}{r_2}\right)$ [excess pressure = $\frac{2T}{r}$].

Difference in these pressures = $\left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference = $P_1 - P_2$

$= h\rho g + 2T \left(\frac{1}{r_1} - \frac{1}{r_2}\right) = 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3} \left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$

$= 1.86 \times 10^3 =$ **1860 N/m²**

Example 22. **(Only for JEE Advanced)**
A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate the excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0º.

Solution : The surface tension of the liquid is $T = \frac{rh\rho g}{2}$

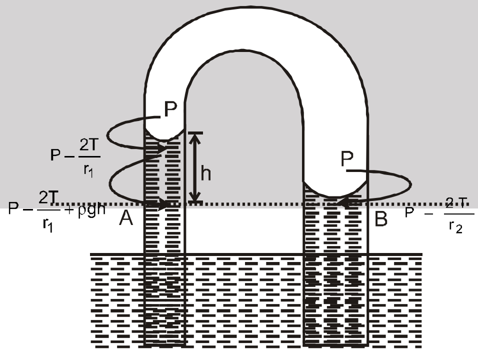
$= \frac{(0.025\text{cm})(3.0\text{cm})(1.5\text{gm/cm}^3)(980\text{cm/sec}^2)}{2}$ = 55 dyne/cm.

Hence excess pressure inside a spherical bubble

$p = \frac{4T}{R} = \frac{4 \times 55\,\text{dyne/cm}}{(0.5\,\text{cm})}$ = **440 dyne/cm²** .

Example 23. **(Only for JEE Advanced)**
A glass U-tube is such that the diameter of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm⁻¹. Assume that the angle of contact between water and glass is 0º.

Solution :



Equating pressure at point A and B which are in same horizontal level

$P - \frac{2T}{r_1} + \rho g h = P - \frac{2T}{r_2} \Rightarrow h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$

Given that T = 0.07 Nm⁻¹ , ρ = 1000 kgm⁻³

$r_1 = \frac{3}{2}$ mm $= \frac{3}{20}$ cm $= \frac{3}{20 \times 100}$ m $= 1.5 \times 10^{-3}$ m, $r_2 = 3 \times 10^{-3}$ m

$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right)$ m $= 4.76 \times 10^{-3}$ m = 4.76 mm

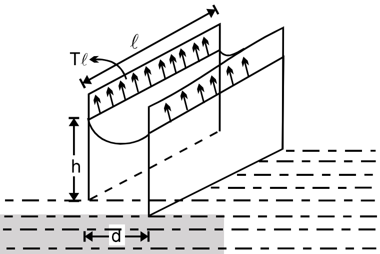
Example 24. **(Only for JEE Advanced)**

Two parallel plates which are separated by a very small distance d, are dipped in water. To how much height will the water raise between the plates (Assume contact angle $\theta \to 0$)

Solution : Lets draw free body diagram of the water raised up . Forces on it are :

(i) The plates pull the surface in upward direction with a force $2T\ell$

(ii) The weight of raised water = $(\rho)(\ell hd)g$ For equilibrium, forces should be balanced.



$$2T\ell = (\rho)(\ell hd)g \quad \Rightarrow \quad h = \frac{2T}{\rho gd}$$

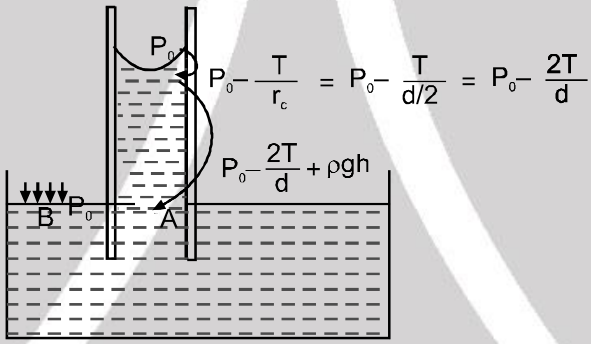
Also $= \frac{T}{d/2}$ ρgh ; so we can say that pressure

excess due to cylindrical surface = $\frac{T}{d/2} = \frac{T}{r_c}$

pressure excess due to spherical surface= $\frac{2T}{r_c}$

pressure excess due to cylindrical surface = $\frac{T}{r_c}$

Alternative method :



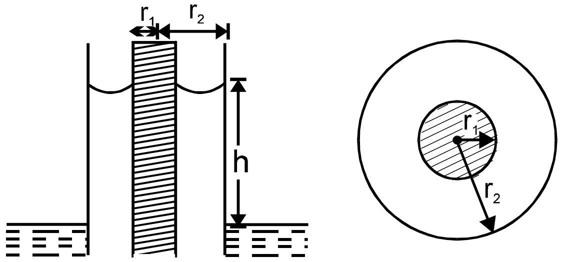
$$\Rightarrow P_0 - \frac{2T}{d} + \rho gh = P_0 \Rightarrow h = \frac{2T}{\rho gd}$$

Example 25. **(Only for JEE Advanced)**

A thin capillary of inner radius r_1 and outer radius r_2 (The inner tube is solid) is dipped in water. To how much height will the water raise in the tube ? (Assume contact angle $\theta \to 0$)

Solution : Applying force balance $T[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\rho g$

$$h = \frac{2T}{(r_2 - r_1)\rho g}$$

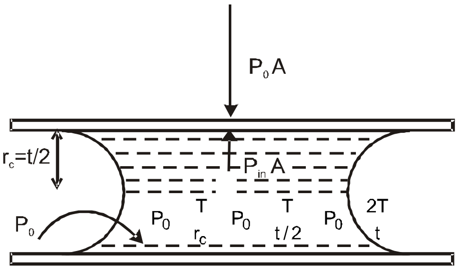


Example 26. **(Only for JEE Advanced)**

A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in Newton.

Solution : Pressure inside the surface

$$P_{in} = P_0 - \frac{T}{r_C} = P_0 - \frac{T}{t/2} = P_0 - \frac{2T}{t},$$



So, net inwards force = $P_0A - P_{in}A = \left(P_0 - \frac{2T}{t}\right)A - P_0A$

$$= \frac{2TA}{t}$$

Here volume between the plates V = A × t

$\Rightarrow \quad t = \frac{V}{A}$ Putting the value of t

$$F = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45 \text{ N};$$

So this much force is required to separate the plates

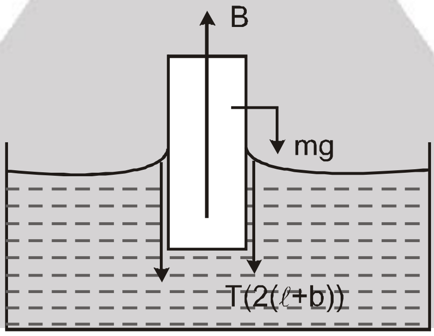
Example 27 **(Only for JEE Advanced)**

A glass plate of length 10 cm, breadth 1.54 cm and thickness 0.20 cm weighs 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water = 73 dyne per cm, g = 980 cm/sec^2.

Solution : The forces acting on the plate are

(i) buoyancive force of water acting upward

$$B = \rho_l V_{sub} g = 1 \times \frac{1.54\times10\times0.2}{2} \times 980 = 1509.2 \text{ dyne.}$$



(ii) Weight of the system acting downward = (8.2) × 980 dyne

(iii) Force of surface tension acting downward = 2 (ℓ + b)T = 2 (10 + 0.2) 73 = 1489.2

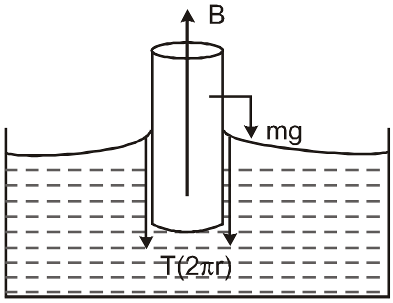
So net downward force = mg + (surface tension force) – B

= (8.2) × 980 + 1489.2 – 1509.2 = 8016.008 dyne = 8.1796 gm force

Example 28. **(Only for JEE Advanced)**

A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Given : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension of water 73 dyne/cm and g = 980 cm/sec^2.

Solution : Let ℓ be the length of the tube inside water. The forces acting on the tube are :



(i) buoyancive force of water acting upward

$B = \pi r^2 \ell \times 1 \times 980 = \frac{22}{7} \times (0.14)^2\, \ell \times 980 = 60.368\, \ell$ dyne.

(ii) Weight of the system acting downward = mg = 0.2 × 980 = 196 dyne.

(iii) Force of surface tension acting downward = $2\pi rT$

$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24$ dyne.

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,
$60.368\, \ell = 196 + 64.24 = 260.24$.

$\therefore \quad \ell = \frac{260.24}{60.368} =$ **4.31 cm.**