ELASTICITY & VISCOSITY

ELASTICITY AND PLASTICITY

The property of a material body by virtue of which it regains its original configuration (i.e. shape and size) when the external deforming force is removed is called elasticity. The property of the material body by virtue of which it does not regain its original configuration when the external force is removed is called plasticity.

Deforming force : An external force applied to a body which changes its size or shape or both is called deforming force.

Perfectly Elastic body : A body is said to be perfectly elastic if it completely regains its original form when the deforming force is removed. Since no material can regain completely its original form so the concept of perfectly elastic body is only an ideal concept. A quartz fiber is the nearest approach to the perfectly elastic body.

Perfectly Plastic body : A body is said to be perfectly plastic if it does not regain its original form even slightly when the deforming force is removed. Since every material partially regain its original form on the removal of deforming force, so the concept of perfectly plastic body is also only an ideal concept. Paraffin wax, wet clay are the nearest approach to a perfectly plastic bodies.

Cause of Elasticity : In a solid, atoms and molecules are arranged in such a way that each molecule is acted upon by the forces due to the neighboring molecules. These forces are known as intermolecular forces. When no deforming force is applied on the body, each molecule of the solid (i.e. body) is in its equilibrium position and the inter molecular forces between the molecules of the solid are minimum.
On applying the deforming force on the body, the molecules either come closer or go far apart from each other. As a result of this, the molecules are displaced from their equilibrium position. In other words, intermolecular forces get changed and restoring forces are developed on the molecules. When the deforming force is removed, these restoring forces bring the molecules of the solid to their respective equilibrium positions and hence the solid (or the body) regains its original form.

STRESS

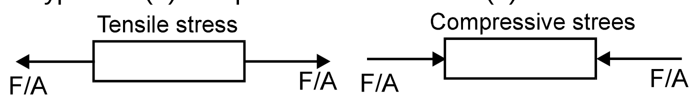
When deforming force is applied on the body then the equal restoring force in opposite direction is developed inside the body. The restoring forces per unit area of the body is called stress.

$$\text{stress} = \frac{\text{restoring force}}{\text{Area of the body}} = \frac{F}{A}$$

The unit of stress is N/m^2. There are three types of stress

1. Longitudinal or Normal stress

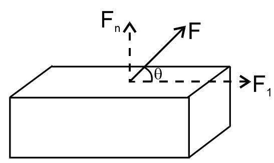
When object is one dimensional then force acting per unit area is called longitudinal stress.
It is of two types : (a) compressive stress (b) tensile stress



Examples :

(i) Consider a block of solid as shown in figure. Let a force F be applied to the face which has area A. Resolve $\vec{F}$ into two components :
$F_n = F \sin\theta$ called normal force and $F_t = F \cos\theta$ called tangential force.

$\therefore$ Normal (tensile) stress $= \frac{F_n}{A} = \frac{F\sin\theta}{A}$



(b)

2. Tangential or shear stress

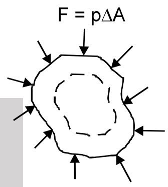
It is defined as the restoring force acting per unit area tangential to the surface of the body. Refer to shown in figure above.

Tangential (shear) stress = $\frac{F_t}{A} = \frac{F\cos\theta}{A}$

The effect of stress is to produce distortion or a change in size, volume and shape (i.e., configuration of the body).

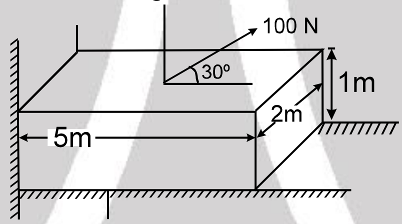
3. Bulk stress

When force is acting all along the surface normal to the area, then force acting per unit area is known as pressure. The effect of pressure is to produce volume change. The shape of the body may or may not change depending upon the homogeneity of body.



Solved Example

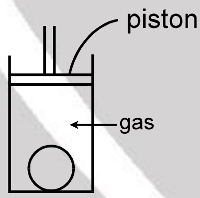
Example 1. Find out longitudinal stress and tangential stress on a fixed block.

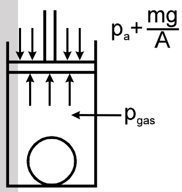


Solution : Longitudinal or normal stress $\Rightarrow$ $\sigma_l = \frac{100\sin 30^o}{5\times 2}$ = 5 N/m^2

Tangential stress $\Rightarrow$ $\sigma_t = \frac{100\ \cos\ 30^o}{5\times 2} = 5\sqrt{3}\,\text{N/m}^2$

Example 2. Find out Bulk stress on the spherical object of radius $\frac{10}{\pi}$ cm if area and mass of piston is 50 cm^2 and 50 kg respectively for a cylinder filled with gas.





Solution : $p_{gas} = \frac{mg}{A} + p_a = \frac{50\times 10}{50\times 10^{-4}}$ + 1 × 10^5 = 2 × 10^5 N/m^2

Bulk stress = p_{gas} = 2 × 10^5 N/m^2

STRAIN

The ratio of the change in configuration (i.e. shape, length or volume) to the original configuration of the body is called strain,

i.e. Strain, $\in = \frac{\text{change in configuration}}{\text{original configuration}}$

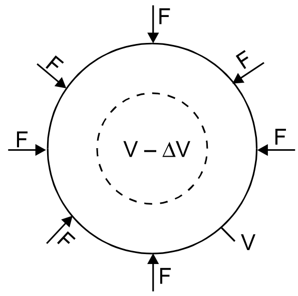
It has no unit

Types of strain : There are three types of strain

(i) Longitudinal strain : This type of strain is produced when the deforming force causes a change in length of the body. It is defined as the ratio of the change in length to the original length of the body. Consider a wire of length L **:** When the wire is stretched by a force F, then let the change in length of the wire is ΔL.

$\therefore$ Longitudinal strain, $\in_\ell = \frac{\text{change in length}}{\text{original length}}$ or Longitudinal strain = $\frac{\Delta L}{L}$

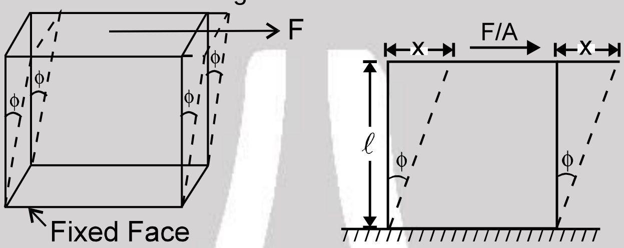
(ii) Volume strain : This type of strain is produced when the deforming force produces a change in volume of the body as shown in the figure. It is defined as the ratio of the change in volume to the original volume of the body.



If ΔV = change in volume $\quad$ V = original volume

$$\in_v = \text{volume strain} = \frac{\Delta V}{V}$$

(iii) Shear Strain : This type of strain is produced when the deforming force causes a change in the shape of the body. It is defined as the angle ϕ through which a face originally perpendicular to the fixed face is turned as shown in the figure.



$\tan\phi$ or $\phi = \frac{x}{\ell}$

HOOKE’S LAW AND MODULUS OF ELASTICITY

According to this law, within the elastic limit, stress is proportional to the strain.

i.e. stress $\propto$ strain

or $\quad$ stress = constant × strain or $\frac{\text{stress}}{\text{strain}}$ = Modulus of Elasticity.

This constant is called modulus of elasticity.

Thus, modulus of elasticity is defined as the ratio of the stress to the strain.

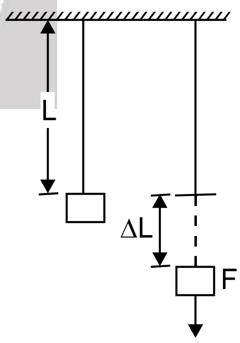
Modulus of elasticity depends on the nature of the material of the body and is independent of its dimensions (i.e. length, volume etc.).

Unit : The Sl unit of modulus of elasticity is Nm^{-2} or Pascal (Pa).

TYPES OF MODULUS OF ELASTICITY

Corresponding to the three types of strain there are three types of modulus of elasticity.

1. Young's modulus of elasticity (Y)
2. Bulk modulus of elasticity (K)
3. Modulus of rigidity (η).



1. Young's modulus of elasticity

It is defined as the ratio of the normal stress to the longitudinal strain.

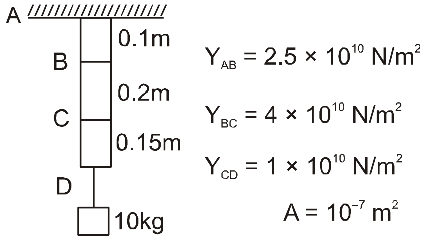
i.e. Young's modulus (Y) = $\frac{\text{Longitudinal stress}}{\text{Longitudinal strain}}$

Normal stress = F/A,

Longitudinal strain = $\Delta L/L$

$$Y = \frac{F/A}{\Delta L/L} = \frac{FL}{A\Delta L}$$

Example 3. Find out the shift in point B, C and D



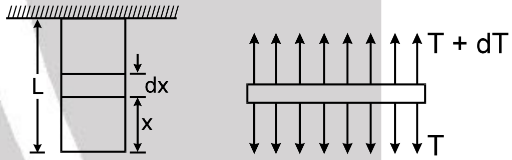
Solution : $\Delta L_B = \Delta L_{AB} = \dfrac{FL}{AY} = \dfrac{MgL}{AY} = \dfrac{10 \times 10 \times 0.1}{10^{-7} \times 2.5 \times 10^{10}} = 4 \times 10^{-3}$ m = 4mm

$\Delta L_C = \Delta L_B + \Delta L_{BC} = 4 \times 10^{-3} + \dfrac{100 \times 0.2}{10^{-7} \times 4 \times 10^{10}} = 4 \times 10^{-3} + 5 \times 10^{-3}$ = 9mm

$\Delta L_D = \Delta L_C + \Delta L_{CD} = 9 \times 10^{-3} + \dfrac{100 \times 0.15}{10^{-7} \times 1 \times 10^{10}} = 9 \times 10^{-3} + 15 \times 10^{-3}$ = 24 mm

ELONGATION OF ROD UNDER IT'S SELF WEIGHT

Let rod is having self weight 'W', area of cross-section 'A" and length 'L'. Considering on element at a distance 'x' from bottom.



then $T = \dfrac{W}{L}x$

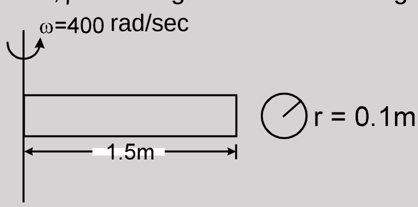
elongation in 'dx' element = $\dfrac{Tdx}{AY}$

Total elongation $s = \displaystyle\int_0^L \frac{Tdx}{AY} = \int_0^L \frac{Wx \;\; dx}{LAY} = \frac{WL}{2AY}$

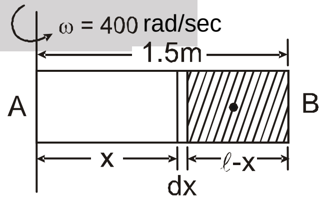
Note : One can do directly by considering total weight at C.M. and using effective length $\ell/2$.

Solved Example

Example 4. Given Y = 2 × 10^{11} N/m^2 , ρ = 10^4 kg/m^3. Find out elongation in rod.



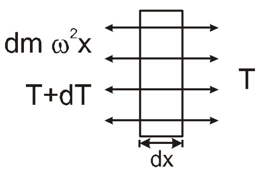
Solution : mass of shaded portion

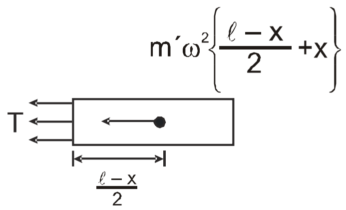


$m' = \dfrac{m}{\ell}(\ell - x)$ [where m = total mass = ρ Aλ]

T = $m'\omega^2\left[\dfrac{\ell - x}{2} + x\right]$

$\Rightarrow$ $T = \dfrac{m}{\ell}(\ell - x)\omega^2\left(\dfrac{\ell + x}{2}\right)$ T = $\dfrac{m\omega^2}{2\ell}(\ell^2 - x^2)$





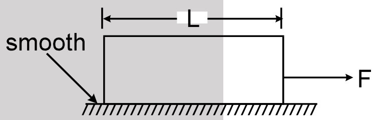
this tension will be maximum at A $\left(\frac{m\omega^2\ell}{2}\right)$ and minimum at 'B' (zero), elongation in element of width 'dx' = $\frac{Tdx}{AY}$

Total elongation $\delta = \int \frac{Tdx}{AY} = \int_0^{\ell} \frac{m\omega^2(\ell^2 - x^2)}{2\ell AY} dx$

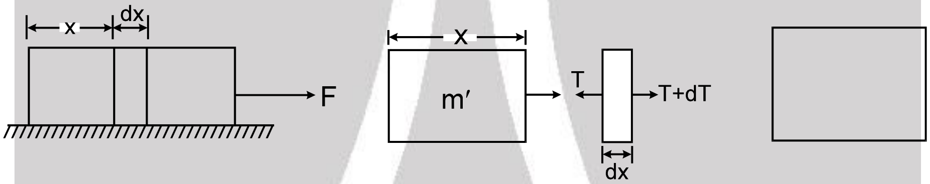
$$\delta = \frac{m\omega^2}{2\ell \quad AY}\left[\ell^2 x - \frac{x^3}{3}\right]_0^{\ell} = \frac{m\omega^2 \times 2\ell^3}{2\ell AY \times 3} = \frac{m\omega^2\ell^2}{3AY} = \frac{\rho A\ell\omega^2\ell^2}{3AY}$$

$$\delta = \frac{\rho\omega^2\ell^3}{3Y} = \frac{10^4 \times (400) \times (1.5)^3}{3 \times 2 \times 10^{11}} = 9 \times 10^{-3} \text{ m} = 9\text{mm}$$

Example 5. Find out the elongation in block. If mass, area of cross-section and young modulus of block are m, A and Y respectively.



Solution :



Acceleration, $a = \frac{F}{m}$ then $T = m'a$ where $\Rightarrow$ $m' = \frac{m}{\ell}x$

$T = \frac{m}{\ell} x \frac{F}{m} = \frac{F\ x}{\ell}$

Elongation in element 'dx' = $\frac{Tdx}{AY}$

total elongation, $\delta = \int_0^{\ell} \frac{Tdx}{AY}$ $\quad d = \int_0^{\ell} \frac{Fxdx}{A\ell Y} = \frac{F\ell}{2AY}$

Note : Try this problem, if friction is given between block and surface (μ = friction coefficient), and
Case : (I) F < μmg (II) F > μmg

Ans. In both cases answer will be $\frac{F\ell}{2AY}$

2. Bulk modulus :

It is defined as the ratio of the normal stress to the volume strain

i.e. $B = \frac{\text{Pressure}}{\text{Volume strain}}$

The stress being the normal force applied per unit area and is equal to the pressure applied (p).

$$B = \frac{p}{\frac{-\Delta V}{V}} = -\frac{pV}{\Delta V}$$

Negative sign shows that increase in pressure (p) causes decrease in volume (ΔV).

Compressibility : The reciprocal of bulk modulus of elasticity is called compressibility. Unit of compressibility in SI is $N^{-1}\ m^2$ or pascal^{-1} (Pa^{-1}).

Bulk modulus of solids is about fifty times that of liquids, and for gases it is 10^{-8} times of solids.

$B_{solids} > B_{liquids} > B_{gases}$

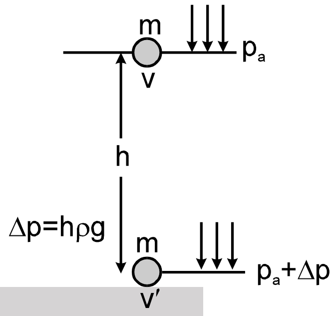
Isothermal bulk modulus of elasticity of gas B = P (pressure of gas)

Adiabatic bulk modulus of elasticity of gas $B = \gamma \times P$ where $\gamma = \frac{C_p}{C_v}$.

Solved Example

Example 6. Find the depth of lake at which density of water is 1% greater than at the surface. Given compressibility K = 50 × 10^{-6} /atm.

Solution : $B = \dfrac{\Delta p}{-\dfrac{\Delta V}{V}}$ $\quad \dfrac{\Delta V}{V} = -\dfrac{\Delta p}{B}$



m = ρV = const.

$d\rho\, V + dV \,.\, \rho = 0 \quad \Rightarrow \quad \dfrac{d\rho}{\rho} = -\dfrac{dV}{V}$

i.e. $\dfrac{\Delta\rho}{\rho} = \dfrac{\Delta p}{B} \quad \Rightarrow \quad \dfrac{\Delta\rho}{\rho} = \dfrac{1}{100}$

$\dfrac{1}{100} = \dfrac{h\rho g}{B}$ $\quad$ [assuming ρ = const.]

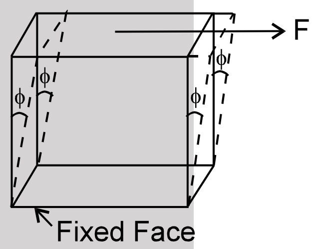
$h\,\rho\,g = \dfrac{B}{100} = \dfrac{1}{100K} \quad \Rightarrow \quad h\,\rho\,g = \dfrac{1\times10^5}{100\times50\times10^{-6}}$

$h = \dfrac{10^5}{5000\times10^{-6}\times1000\times10} = \dfrac{100\times10^3}{50} = 2\text{km}$ **Ans.**

---

3. **Modulus of Rigidity :**

It is defined as the ratio of the tangential stress to the shear strain. Let us consider a cube whose lower face is fixed and a tangential force F acts on the upper face whose area is A.



∴ Tangential stress = F/A.

Let the vertical sides of the cube shifts through an angle θ, called shear strain

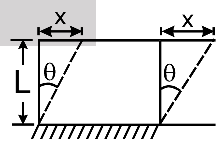
∴ Modulus of rigidity is given by

$\eta = \dfrac{\text{Tangential stress}}{\text{Shear strain}}$ or $\eta = \dfrac{F/A}{\phi} = \dfrac{F}{A\phi}$

Solved Example

Example 7. A rubber cube of side 5 cm has one side fixed while a tangential force equal to 1800 N is applied to opposite face find the shearing strain and the lateral displacement of the strained face. Modulus of rigidity for rubber is 2.4 × 10^6 N/m^2.

Solution : $L = 5 \times 10^{-2}$ m $\Rightarrow \dfrac{F}{A} = \eta\ \dfrac{x}{L}$



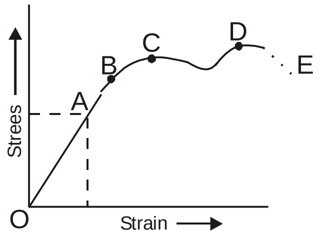
strain $\theta = \dfrac{F}{A\eta} = \dfrac{1800}{25\times10^{-4}\times2.4\times10^6} = \dfrac{180}{25\times24} = \dfrac{3}{10} = 0.3$ radian

$\dfrac{x}{L} = 0.3 \Rightarrow x = 0.3 \times 5 \times 10^{-2} = 1.5 \times 10^{-2}$ m = 1.5 cm **Ans.**

VARIATION OF STRAIN WITH STRESS

When a wire is stretched by a load, it is seen that for small value of load, the extension produced in the wire is proportional to the load. On removing the load, the wire returns to its original length. The wire regains its original dimensions only when load applied is less or equal to a certain limit. This limit is called elastic limit. Thus, elastic limit is the maximum stress on whose removal, the bodies regain their original dimensions. In shown figure, this type of behavior is represented by OB portion of the graph. Till

A the stress is proportional to strain and from A to B if deforming forces are removed then the wire comes to its original length but here stress is not proportional to strain.



OA → Limit of Proportionality
OB → Elastic limit
C → Yield Point
CD → Plastic behaviour
D → Ultimate point
DE → Fracture

As we go beyond the point B, then even for a very small increase in stress, the strain produced is very large. This type of behavior is observed around point C and at this stage the wire begins to flow like a viscous fluid. The point C is called yield point. If the stress is further increased, then the wire breaks off at a point D called the breaking point. The stress corresponding to this point is called breaking stress or tensile strength of the material of the wire. A material for which the plastic range CD is relatively high is called ductile material. These materials get permanently deformed before breaking. The materials for which plastic range is relatively small are called brittle materials. These materials break as soon as elastic limit is crossed.

Important points

- Breaking stress = Breaking force/area of cross section.
- Breaking stress is constant for a material.
- Breaking force depends upon the area of the section of the wire of a given material.
- The working stress is always kept lower than that of a breaking stress so that safety factor = breaking stress/working stress may have a large value.
- Breaking strain = elongation or compression/original dimension.
- Breaking strain is constant for a material.

Elastic after effect

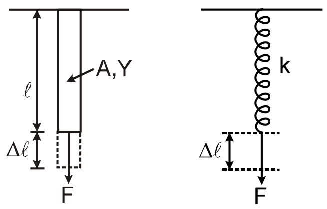
We know that some material bodies take some time to regain their original configuration when the deforming force is removed. The delay in regaining the original configuration by the bodies on the removal of deforming force is called elastic after effect. The elastic after effect is negligibly small for quartz fiber and phosphor bronze. For this reason, the suspensions made from quartz and phosphor-bronze are used in galvanometers and electrometers.

For glass fiber elastic after effect is very large. It takes hours for glass fiber to return to its original state on removal of deforming force.

Elastic Fatigue

The, the loss of strength of the material due to repeated strains on the material is called elastic fatigue. That is why bridges are declared unsafe after a longtime of their use.

Analogy of Rod as a spring

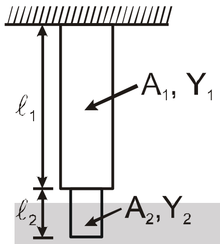


$$Y = \frac{\text{stress}}{\text{strain}} \quad \Rightarrow \quad Y = \frac{F\ell}{A\Delta\ell}$$

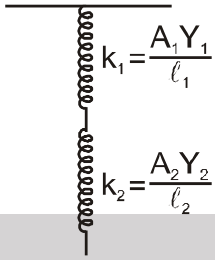
or $F = \frac{AY}{\ell}\Delta\ell$

$\frac{AY}{\ell}$ = constant, depends on type of material and geometry of rod. $\quad$ $F = k\Delta\ell$

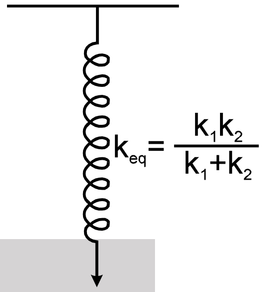
where $k = \frac{AY}{\ell}$ = equivalent spring constant.



(a)



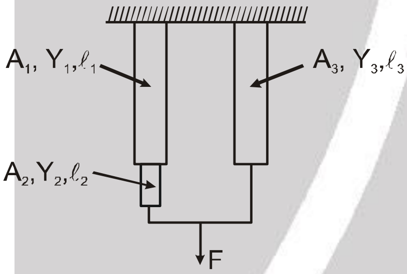
(b)



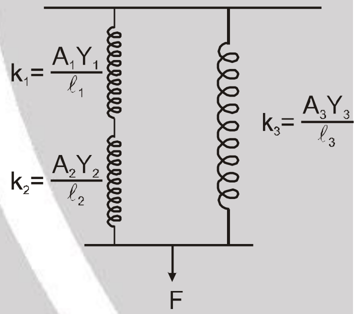
(c)

for the system of rods shown in figure (a), the replaced spring system is shown in figure (b) two spring in series]. Figure (c) represents equivalent spring system.

Figure (d) represents another combination of rods and their replaced spring system.

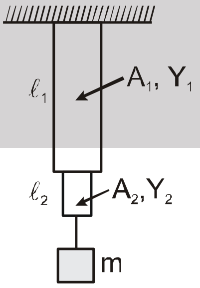


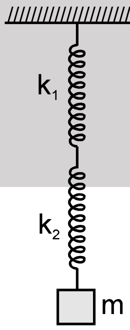
(d)



Solved Example

Example 8. A mass 'm' is attached with rods as shown in figure. This mass is slightly stretched and released whether the motion of mass is S.H.M., if yes then find out the time period.



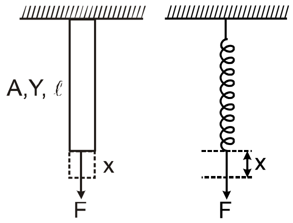


Solution : $k_{eq} = \frac{k_1k_2}{k_1+k_2}$

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}} = 2\pi\sqrt{\frac{m(k_1+k_2)}{k_1k_2}}$$

where $k_1 = \frac{A_1Y_1}{\ell_1}$ and $k_2 = \frac{A_2Y_2}{\ell_2}$

ELASTIC POTENTIAL ENERGY STORED IN A STRETCHED WIRE OR IN A ROD



Strain energy stored in equivalent spring

$$U = \frac{1}{2}kx^2$$

where $x = \frac{F\ell}{AY}$, $k = \frac{AY}{\ell}$

$$U = \frac{1}{2}\frac{AY}{\ell}\frac{F^2\ell^2}{A^2Y^2} = \frac{1}{2}\frac{F^2\ell}{AY}$$

equation can be re-arranged

$$U = \frac{1}{2}\frac{F^2}{A^2}\times\frac{\ell A}{Y} \quad [\ell A = \text{volume of rod, } F/A = \text{stress}]$$

$$U = \frac{1}{2}\frac{(\text{stress})^2}{Y}\times \text{volume}$$

again, $U = \frac{1}{2}\frac{F}{A}\times\frac{F}{AY}\times A\,\ell$ $\quad$ [Strain = $\frac{F}{AY}$]

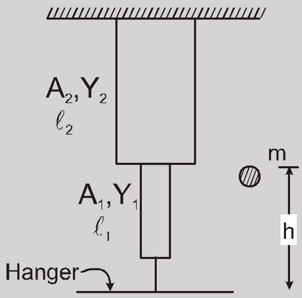
$$U = \frac{1}{2}\text{stress} \times \text{strain} \times \text{volume}$$

again, $U = \frac{1}{2}\frac{F^2}{A^2Y^2}A\ell Y \Rightarrow$ $\quad U = \frac{1}{2}Y(\text{strain})^2 \times \text{volume}$

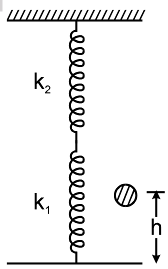
strain energy density $= \frac{\text{strain energy}}{\text{volume}} = \frac{1}{2}\frac{(\text{stress})^2}{Y} = \frac{1}{2}Y(\text{strain})^2 = \frac{1}{2}\text{stress} \times \text{strain}$

Solved Example

Example 9. A ball of mass 'm' drops from a height 'h', which sticks tomass-less hanger after striking. Neglect over turning, find out the maximum extension in rod. Assuming rod is massless.



Solution : Applying energy conservation mg (h + x) = $\frac{1}{2}\frac{k_1k_2}{k_1+k_2}x^2$



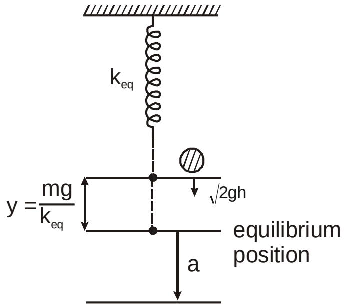
where $k_1 = \frac{A_1Y_1}{\ell_1}$ $\quad k_2 = \frac{A_2Y_2}{\ell_2}$

& $k_{eq} = \frac{A_1A_2Y_1Y_2}{A_1Y_1\ell_2 + A_2Y_2\ell_1}$

$k_{eq}x^2 - 2mgx - 2mgh = 0$

$$x = \frac{2mg \pm \sqrt{4m^2g^2 + 8mghk_{eq}}}{2k_{eq}}, \quad x_{max} = \frac{mg}{k_{eq}} + \sqrt{\frac{m^2g^2}{k_{eq}^2} + \frac{2mgh}{k_{eq}}}$$

OTHERWAY BY S.H.M.



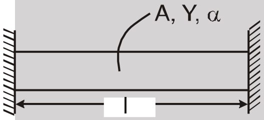
$\omega = \sqrt{\dfrac{k_{eq}}{m}}$ $\qquad$ $v = \omega\sqrt{a^2 - y^2}$

$\sqrt{2gh} = \sqrt{\dfrac{k_{eq}}{m}}\sqrt{a^2 - y^2} \quad \Rightarrow \quad \sqrt{\dfrac{2mgh}{k_{eq}} + \dfrac{m^2g^2}{k_{eq}^2}} = a$

max^m extension

$= a + y = \dfrac{mg}{k_{eq}} + \sqrt{\dfrac{m^2g^2}{k_{eq}} + \dfrac{2mgh}{k_{eq}}}$

THERMAL STRESS :



If temp of rod is increased by ΔT, then change in length $\Delta\ell = \ell\ \alpha\ \Delta T$

strain $= \dfrac{\Delta\ell}{\ell} = \alpha\ \ \Delta T$

But due to rigid support, there is no strain. Supports provide force on stresses to keep the length of rod same

$Y = \dfrac{\text{stress}}{\text{strain}}$

If ΔT = positive

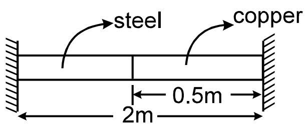
thermal stress = Y strain = $Y\ \alpha\ \Delta T$

If ΔT = negative

$\dfrac{F}{A} = Y\alpha\Delta T \qquad F = AY\ \alpha\ \Delta T$

Solved Example

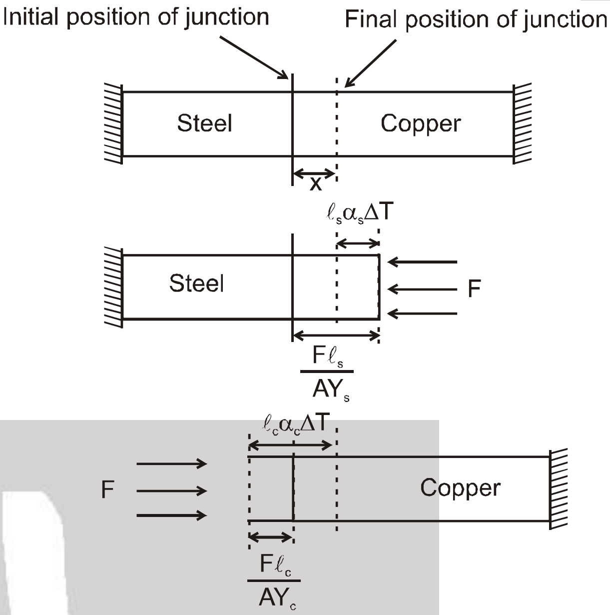
Example 10. When composite rod is free, then composite length increases to 2.002 m for temperature rise from 20ºC to 120ºC. When composite rod is fixed between the support, there is no change in component length find Y and α of steel, if $Y_{cu} = 1.5 \times 10^{13}$ N/m^2 $\alpha_{cu} = 1.6 \times 10^{-5}$/ºC.



Solution : $\Delta\ell = \ell_s\ \alpha_s\ \Delta T + \ell_c\ \alpha_c\Delta T$

$.002 = [1.5\ \alpha_s + 0.5 \times 1.6 \times 10^{-5}] \times 100$

$\alpha_s = \dfrac{1.2 \times 10^{-5}}{1.5} = 8 \times 10^{-6}$/ºC



there is no change in component length

For steel

$$x = \ell_s \alpha_s \Delta T - \frac{F\ell_s}{AY_s} = 0$$

$$\frac{F}{AY_s} = \alpha_s \Delta T \qquad \text{....(A)}$$

for copper

$$x = \frac{F\ell_c}{AY_c} - \ell_c \alpha_c \Delta T = 0$$

$$\frac{F}{AY_c} = \alpha_c \Delta T \qquad \text{...(B)}$$

$$B/A \Rightarrow \frac{Y_s}{Y_c} = \frac{\alpha_c}{\alpha_s}$$

$$Y_s = Y_c \frac{\alpha_c}{\alpha_s} = \frac{1.5\times10^{13}\times16\times10^{6}}{8\times10^{-6}}$$

$= 3\times10^{13}$ N/m^2

APPLICATIONS OF ELASTICITY

Some of the important applications of the elasticity of the materials are discussed as follows :

1. The material used in bridges lose its elastic strength with time bridges are declared unsafe after long use.

2. **To estimate the maximum height of a mountain :**

The pressure at the base of the mountain = hρg = stress. The elastic limit of a typical rock is 3×10^8 N m^{-2}

The stress must be less than the elastic limits, otherwise the rock begins to flow.

$h < \dfrac{3\times10^8}{\rho g} \Rightarrow h < 10^4$ m ($\because \rho = 3 \times 10^3$ kg m^{-3} ; g = 10 ms^{-2}) or h = 10 km

It may be noted that the height of Mount Everest is nearly 9 km.

TORSION CONSTANT OF A WIRE

$C = \dfrac{\pi\eta r^4}{2\ell}$ Where η is modulus of rigidity r and ℓ is radius and length of wire respectively.

(a) Toque required for twisting by an angle θ, $\tau = C\theta$.

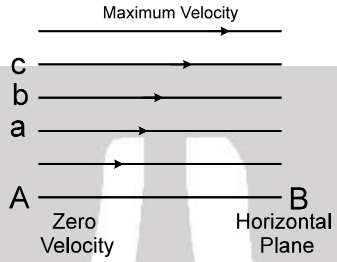
(b) Work done in twisting by an angle θ, $W = \dfrac{1}{2} C\theta^2$.

VISCOSITY

When a solid body slides over another solid body, a frictional-force begins to act between them. This force opposes the relative motion of the bodies. Similarly, when a layer of a liquid slides over another layer of the same liquid, a frictional-force acts between them which opposes the relative motion between the layers. This force is called 'internal frictional-force'.

Suppose a liquid is flowing in streamlined motion on a fixed horizontal surface AB (Fig.). The layer of the liquid which is in contact with the surface is at rest, while the velocity of other layers increases with distance from the fixed surface. In the Fig., the lengths of the arrows represent the increasing velocity of the layers. Thus there is a relative motion between adjacent layers of the liquid. Let us consider three parallel layers a, b and c. Their velocities are in the increasing order. The layer a tends to retard the layer b, while b tends to retard c. Thus each layer tends to decrease the velocity of the layer above it. Similarly, each layer tends to increase the velocity of the layer below it. This means that in between any two layers of the liquid, internal tangential forces act which try to destroy the relative motion between the layers. These forces are called 'viscous forces'. If the flow of the liquid is to be maintained, an external force must be applied to overcome the dragging viscous forces. In the absence of the external force, the viscous forces would soon bring the liquid to rest. **The property of the liquid by virtue of which it opposes the relative motion between its adjacent layers is known as 'viscosity'.**

The property of viscosity is seen in the following examples :



(i) A stirred liquid, when left, comes to rest on account of viscosity. Thicker liquids like honey, coaltar, glycerin, etc. have a larger viscosity than thinner ones like water. If we pour coaltar and water on a table, the coaltar will stop soon while the water will flow upto quite a large distance.

(ii) If we pour water and honey in separate funnels, water comes out readily from the hole in the funnel while honey takes enough time to do so. This is because honey is much more viscous than water. As honey tends to flow down under gravity, the relative motion between its layers is opposed strongly.

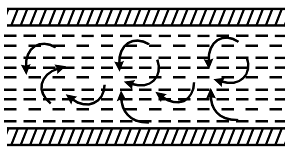
(iii) We can walk fast in air, but not in water. The reason is again viscosity which is very small for air but comparatively much larger for water.

(iv) The cloud particles fall down very slowly because of the viscosity of air and hence appear floating in the sky.

Viscosity comes into play only when there is a relative motion between the layers of the same material. This is why it does not act in solids.

FLOW OF LIQUID IN A TUBE: CRITICAL VELOCITY

When a liquid flows in a tube, the viscous forces oppose the flow of the liquid, Hence a pressure difference is applied between the ends of the tube which maintains the flow of the liquid. If all particles of the liquid passing through a particular point in the tube move along the same path, the flow of the liquid is called 'stream-lined flow'. This occurs only when the velocity of flow of the liquid is below a certain limiting value called 'critical velocity'. When the velocity of flow exceeds the critical velocity, the flow is no longer stream-lined but becomes turbulent. In this type of flow, the motion of the liquid becomes zig-zag and eddy-currents are developed in it.



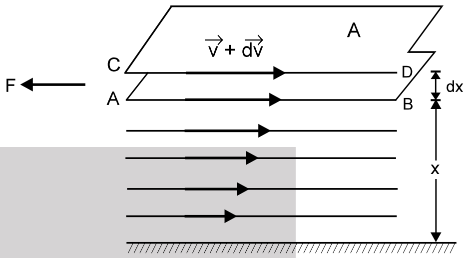
Reynold's proved that the critical velocity for a liquid flowing in a tube is $v_c = k\eta/\rho r$. where ρ is density and η is viscosity of the liquid, r is radius of the tube and k is 'Reynold's number' (whose value for a narrow tube and for water is about 1000). When the velocity of flow of the liquid is less than the critical velocity, then the flow of the liquid is controlled by the viscosity, the density having no effect on it. But when the velocity of flow is larger than the critical velocity, then the flow is mainly governed by the density, the effect of viscosity becoming less important. It is because of this reason that when a volcano erupts, then the lava coming out of it flows speedily inspite of being very thick (of large viscosity).

VELOCITY GRADIENT AND COEFFICIENT OF VISCOSITY

The property of a liquid by virtue of which an opposing force (internal friction) comes into play when ever there is a relative motion between the different layers of the liquid is called viscosity. Consider a flow of a liquid over the horizontal solid surface as shown in fig. Let us consider two layers AB and CD moving with velocities $\vec{v}$ and $\vec{v} + d\vec{v}$ at a distance x and (x + dx) respectively from the fixed solid surface. According to Newton, the viscous drag or back ward force (F) between these layers depends.

(i) directly proportional to the area (A) of the layer and

(ii) directly proportional to the velocity gradient $\left(\frac{dv}{dx}\right)$ between the layers.



i.e. $F \propto A\frac{dv}{dx}$ or $F = -\eta A\frac{dv}{dx}$ ...(1)

η is called Coefficient of viscosity. Negative sign shows that the direction of viscous drag (F) is just opposite to the direction of the motion of the liquid.

SIMILARITIES AND DIFFERENCES BETWEEN VISCOSITY AND SOLID FRICTION

Similarities

Viscosity and solid friction are similar as

1. Both oppose relative motion. Whereas viscosity opposes the relative motion between two adjacent liquid layers, solid friction opposes the relative motion between two solid layers.

2. Both come into play, whenever there is relative motion between layers of liquid or solid surfaces as the case may be.

3. Both are due to molecular attractions.

Differences between them →

Viscosity	Solid Friction
(i) Viscosity (or viscous drag) between layers of liquid is directly proportional to the area of the liquid layers.	(i) Friction between two solids is independent of the area of solid surfaces in contact.
(ii) Viscous drag is proportional to the relative velocity between two layers of liquid.	(ii) Friction is independent of the relative velocity between two surfaces.
(iii) Viscous drag is independent of normal reaction between two layers of liquid.	(iii) Friction is directly proportional to the normal reaction between two surfaces in contact.

SOME APPLICATIONS OF VISCOSITY

Knowledge of viscosity of various liquids and gases have been put to use in daily life. Some applications of its knowledge are discussed as under →

1. As the viscosity of liquids vary with temperature, proper choice of lubricant is made depending upon season.

2. Liquids of high viscosity are used in shock absorbers and buffers at railway stations.

3. The phenomenon of viscosity of air and liquid is used to damp the motion of some instruments.

4. The knowledge of the coefficient of viscosity of organic liquids is used in determining the molecular weight and shape of the organic molecules.

5. It finds an important use in the circulation of blood through arteries and veins of human body.

UNITS OF COEFFICIENT OF VISCOSITY

From the above formula, we have $\eta = \dfrac{F}{A(\Delta v_x / \Delta z)}$

$\therefore$ dimensions of $\eta = \dfrac{[MLT^{-2}]}{[L^2][LT^{-1}/L]} = \dfrac{[MLT^{-2}]}{[L^2T^{-1}]} = [ML^{-1}T^{-1}]$

Its unit is kg/(meter-second)*

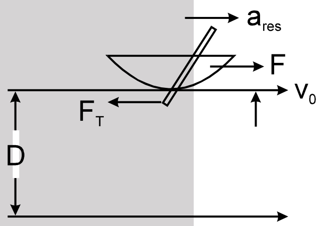
In C.G.S. system, the unit of coefficient of viscosity is dyne s cm^{-2} and is called poise. In SI the unit of coefficient of viscosity is N sm^{-2} and is called decapoise.

1 decapoise = 1 N sm^{-2} = (10^5 dyne) × s × (10^2 cm)$^{-2}$ = 10 dyne s cm^{-2} = 10 poise

Solved Example

Example 11. A man is rowing a boat with a constant velocity 'v_0' in a river the contact area of boat is 'A' and coefficient of viscosity is η. The depth of river is 'D'. Find the force required to row the boat.

Solution : $F - F_T = m\, a_{res}$



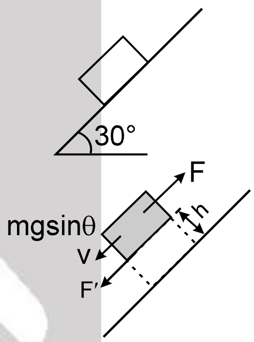
As boat moves with constant velocity $a_{res} = 0$

$F = F_T$

But $F_T = \eta A \dfrac{dv}{dz}$, but $\dfrac{dv}{dz} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$

then $F = F_T = \dfrac{\eta A v_0}{D}$

Example 12. A cubical block (of side 2m) of mass20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/sec. (g = 10 m/sec^2) find out the thickness of layer of liquid.

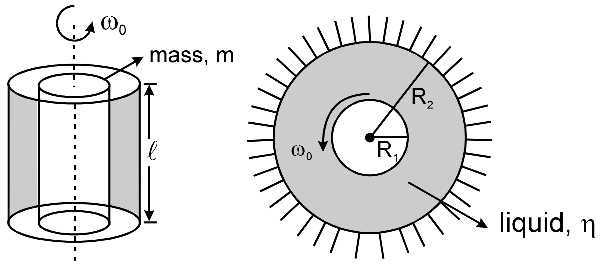


Solution : $F = \eta A \dfrac{dv}{dz} = mg \sin\theta \quad \dfrac{dv}{dz} = \dfrac{v}{h}$

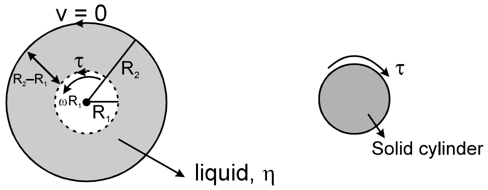
$20 \times 10 \times \sin 30° = \eta \times 4 \times \dfrac{10}{h}$

$h = \dfrac{40 \times 10^{-2}}{100}$ [$\eta = 10^{-1}$ poise $= 10^{-2}$ N-sec-m^{-2}] $= 4 \times 10^{-3}$ m = 4 mm

Example 13. As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.



Solution : $F = \eta A \dfrac{dv}{dz}$, where $\dfrac{dv}{dz} = \dfrac{\omega R_1 - 0}{R_2 - R_1}$



$$F = \eta \frac{2\pi \quad R_1 \quad \ell \quad \omega \quad R_1}{R_2 - R_1}$$

and $\quad \tau = FR_1 = \dfrac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$

$$I\,\alpha = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

$$\Rightarrow \frac{mR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1} \qquad \text{or} \qquad -\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} \int_0^t dt$$

$$\Rightarrow t = \frac{m(R_2 - R_1)\ell n 2}{4\pi\eta \ell R_1}$$

EFFECT OF TEMPERATURE ON THE VISCOSITY

The viscosity of liquids decrease with increase in temperature and increase with the decrease in temperature. That is, $\eta \propto \dfrac{1}{\sqrt{T}}$ On the other hand, the value of viscosity of gases increases with the increase in temperature and vice-versa. That is, $\eta \propto \sqrt{T}$

STOKE’S LAW

Stokes proved that the viscous drag (F) on a spherical body of radius r moving with velocity v in a fluid of viscosity η is given by $F = 6\,\pi\,\eta\,r\,v$. This is called Stokes’ law.

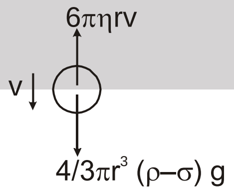
TERMINAL VELOCITY

When a body is dropped in a viscous fluid, it is first accelerated and then its acceleration becomes zero and it attains a constant velocity called terminal velocity.

Calculation of Terminal Velocity

Let us consider a small ball, whose radius is r and density is ρ, falling freely in a liquid (or gas), whose density is σ and coefficient of viscosity η. When it attains a terminal velocity v. It is subjected to two forces :

(i) effective force acting downward = $V(\rho - \sigma)\,g = \dfrac{4}{3}\pi r^3 (\rho - \sigma) g$,



(ii) viscous force acting upward = $6\,\pi\,\eta\,rv$.
Since the ball is moving with a constant velocity v i.e., there is no acceleration in it, the net force acting on it must be zero. That is

$$6\pi rv = p\, r^3 (\rho - \sigma)\, g \qquad \text{or} \quad v = \frac{2}{9}\,\frac{r^2(\rho - \sigma) g}{\eta}$$

Thus, terminal velocity of the ball is directly proportional to the square of its radius

Important point

Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is Negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

Solved Example

Example 14. A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Solution : Rate of heat loss = power = F × v = 6 π η r v × v = 6 π η r v^2 = 6p η r $\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$

Rate of heat loss α r^5

Example 15. A drop of water of radius 0.0015 mm is falling in air. If the coefficient of viscosity of air is 1.8 × 10^{-5} kg /(m-s), what will be the terminal velocity of the drop? (density of water = 1.0 × 10^3 kg/m^2 and g = 9.8 N/kg.) Density of air can be neglected.

Solution : By Stoke's law , the terminal velocity of a water drop of radius r is given by $\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$

where ρ is the density of water, σ is the density of air and η the coefficient of viscosity of air. Here σ is negligible and r = 0.0015 mm = 1.5 × 10^{-3} mm = 1.5 × 10^{-6} m. Substituting the values :

$$\nu = \frac{2}{9} \times \frac{(1.5\times10^{-6})^2\times(1.0\times10^3)\times9.8}{1.8\times10^{-5}} = \mathbf{2.72 \times 10^{-4}\ m/s}$$

Example 16. A metallic sphere of radius 1.0 × 10^{-3} m and density 1.0 × 10^4 kg/m^3 enters a tank of water, after a free fall through a distance of h in the earth's gravitational field. If its velocity remains unchanged after entering water, determine the value of h. Given : coefficient of viscosity of water = 1.0 × 10^{-3} N-s/m^2, g = 10 m/s^2 and density of water = 1.0 × 10^3 kg/m^3.

Solution : The velocity attained by the sphere in falling freely from a height h is

$\nu = \sqrt{2gh}$ ....(i)

This is the terminal velocity of the sphere in water. Hence by Stoke's law, we have

$$\nu = \frac{2}{9}\frac{r^2(\rho-\sigma)g}{\eta}$$

where r is the radius of the sphere, ρ is the density of the material of the sphere σ (= 1.0 × 10^3 kg/m^3) is the density of water and η is coefficient of viscosity of water.

$$\therefore \quad \nu = \frac{2\times(1.0\times10^{-3})^2(1.0\times10^4 - 1.0\times10^3)\times10}{9\times1.0\times10^{-3}} = 20\ m/s$$

from equation (i), we have $h = \frac{\nu^2}{2g} = \frac{20\times20}{2\times10}$ = 20 m

Applications of Stokes' Formula

(i) In determining the Electronic Charge by Millikan's Experiment : Stokes' formula is used in Millikan's method for determining the electronic charge. In this method the formula is applied for finding out the radii of small oil-drops by measuring their terminal velocity in air.

(ii) Velocity of Rain Drops : Rain drops are formed by the condensation of water vapour on dust particles. When they fall under gravity, their motion is opposed by the viscous drag in air. As the velocity of their fall increases, the viscous drag also increases and finally becomes equal to the effective force of gravity. The drops then attain a (constant) terminal velocity which is directly proportional to the square of the radius of the drops. In the beginning the raindrops are very small in size and so they fall with such a small velocity that they appear floating in the sky as cloud. As they grow in size by further condensation, then they reach the earth with appreciable velocity,

(iii) Parachute : When a soldier with a parachute jumps from a flying aeroplane, he descends very slowly in air. In the beginning the soldier falls with gravity acceleration g, but soon the acceleration goes on decreasing rapidly until in parachute is fully opened. Therefore, in the beginning the speed of the falling soldier increases somewhat rapidly but then very slowly. Due to the viscosity of air the acceleration of the soldier becomes ultimately zero and the soldier then falls with a constant terminal speed. In Fig graph is shown between the speed of the falling soldier and time.