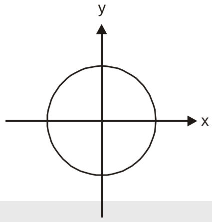
Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Kinematics of circular motion

A-1. Figure shows a circular path taken by a particle. If the instantaneous velocity of the particle is $\vec{v}$ = (2m /s) $\hat{i}$ – (2 m/s) $\hat{j}$. Through which quadrant is the particle moving when it is travelling (a) clockwise and (b) counter clockwise around the circle?



A-2.✍ Find the ratio of angular speeds of minute hand and hour hand of a watch and also find the angular speed of the second's hand in a watch.

A-3. A wheel is subjected to uniform angular acceleration about its axis. Initially its angular velocity is zero. In the first 2 seconds, it rotates through an angle θ_1. In the next 2 seconds, it rotates through an additional angle θ_2. Find the ratio of θ_2/θ_1.

A-4. If the equation for the angular displacement of a particle moving on a circular path is given by $(\theta) = 2t^3 + 0.5$, where θ is in radians and t in seconds, then find the angular velocity of the particle after 2 seconds from its start.

A-5. The length of second's hand in a watch is 1 cm. Find the magnitude of change in velocity of its tip in 15 seconds. Also find out the magnitude of average acceleration during this interval.

Section (B) : Radial and Tangential acceleration

B-1. A particle moves uniformly in a circle of radius 25 cm at two revolution per second. Find the acceleration of the particle in m/s^2.

B-2. A car is moving with speed 30 m/sec on a circular path of radius 500 m. Its speed is increasing at the rate of 2 m/sec^2. What is the acceleration of the car at that moment?

B-3.✍ A particle moves in a circle of radius 1.0 cm at a speed given by v = 2.0 t where v is in cm/s and t in seconds.
(a) Find the radial acceleration of the particle at t = 1s.
(b) Find the tangential acceleration at t = 1s
(c) Find the magnitude of the acceleration at t = 1s.

Section (C) : Circular Motion in Horizontal plane

C-1. A small sphere of mass 200 gm is attached to an inextensible string of length 130 cm whose upper end is fixed to the ceiling. The sphere is made to describe a horizontal circle of radius 50 cm. Calculate the time period of this conical pendulum and the tension in the string. ($\pi^2 = 10$)

C-2.✍ A motorcyclist wants to drive on the vertical surface of wooden 'well' of radius 5 m, in horizontal plane with speed of $5\sqrt{5}$ m/s. Find the minimum value of coefficient of friction between the tyres and the wall of the well . (Take g = 10 m/s^2)

C-3. A mass is kept on a horizontal frictionless surface. It is attached to a string and rotates about a fixed centre at an angular velocity ω_0. If the length of the string and angular velocity are doubled, find the tension in the string which was initially T_0.

C-4. A ceiling fan has a diameter (of the circle through the outer edges of the three blades) of 120 cm and rpm 1500 at full speed. Consider a particle of mass 1g sticking at the outer end of a blade. What is the net force on it, when the fan runs at full speed ? Who exerts this force on the particle ? How much force does the particle exert on the blade in the plane of motion ?

Section (D) : Radius of curvature

D-1. A ball is projected with initial speed u and making an angle θ with the vertical. Consider a small part of the trajectory near the highest position and take it approximately to be a circular arc. What is the radius of this circle? This radius is called the radius of curvature of the curve at the point.

D-2 A particle is projected with initial speed u and at an angle θ with horizontal. What is the radius of curvature of the parabola traced out by the projectile at a point where the particle velocity makes an angle $\theta/2$ with the horizontal?

Section (E) : Circular motion in vertical plane

E-1. A weightless thread can support tension upto 30 N. A stone of mass 0.5 kg is tied to it and is revolved in a circular path of radius 2 m in a vertical plane. If g = 10 m/s^2, find the maximum angular velocity of the stone.

E-2. A simple pendulum oscillates in a vertical plane. When it passes through the mean position, the tension in the string is 3 times the weight of the pendulum bob. What is the maximum angular displacement of the pendulum of the string with respect to the downward vertical?

E-3. A small body of mass m hangs at one end of a string of length a, the other end of which is fixed. It is given a horizontal velocity u at its lowest position so that the string would just becomes slack, when it makes an angle of 60° with the upward drawn vertical line. Find the tension in the string at point of projection.

E-4. A body attached to a string of length ℓ describes a vertical circle such that it is just able to cross the highest point. Find the minimum velocity at the bottom of the circle.

Section (F) : Motion of a vehicle, Centrifugal force and rotation of earth

F-1. When the road is dry and coefficient of friction is μ, the maximum speed of a car in a circular path is 10 ms^{-1}. If the road becomes wet and coefficient of friction become $\mu/2$, what is the maximum speed permitted?

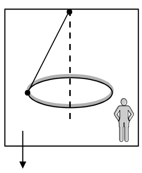
F-2 Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [g = 10 m/s^2]

F-3. A train has to negotiate a curve of radius 400 m. By how much height should the outer rail be raised with respect to inner rail for a speed of 48 km/hr ? The distance between the rails is 1 m :

F-4. A road surrounds a circular playing field having radius of 10 m. If a vehical goes around it at an average speed of 18 km/hr, find proper angle of banking for the road. If the road is horizontal (no banking), what should be the minimum friction coefficient so that a scooter going at 18 km/hr does not skid.

F-5. A circular road of radius 1000 m has banking angle 45°. Find the maximum safe speed of a car having mass 2000 kg, if the coefficient of friction between tyre and road is 0.5.

F-6. In the figure shown a lift goes downwards with a constant retardation. An observer in the lift observers a conical pendulum in the lift, revolving in a horizontal circle with time period 2 seconds. The distance between the centre of the circle and the point of suspension is 2.0 m. Find the retardation of the lift in m/s^2. Use π^2 = 10 and g = 10 m/s^2



F-7. A turn of radius 20 m is banked for the vehicles going at a speed of 36 km/h. If the coefficient of static friction between the road and the tyre is 0.4, what are the possible speeds of a vehicle so that it neither slips down nor skids up ?

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Kinematics of circular motion

A-1. Two racing cars of masses m_1 and m_2 are moving in circles of radii r and 2r respectively and their angular speeds are equal. The ratio of the time taken by cars to complete one revolution is :
(A) $m_1 : m_2$ (B) 1 : 2 (C) 1 : 1 (D) $m_1 : 2m_2$

A-2. A wheel is at rest. Its angular velocity increases uniformly with time and becomes 80 radian per second after 5 second. The total angular displacement is :
(A) 800 rad (B) 400 rad (C) 200 rad (D) 100 rad

A-3. A particle moves along a circle of radius $\left(\frac{20}{\pi}\right)$ m with tangential acceleration of constant magnitude. If the speed of the particle is 80 m/s at the end of the second revolution after motion has begun, the tangential acceleration is:
(A) $160\,\pi$ m/s^2 (B) $40\,\pi$ m/s^2 (C) 40 m/s^2 (D) $640\,\pi$ m/s^2

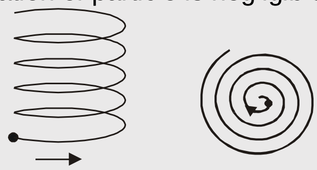
A-4. During the circular motion with constant speed :
(A) Both velocity and acceleration are constant
(B) velocity is constant but the acceleration changes
(C) acceleration is constant but the velocity changes
(D) velocity and acceleration both change

Section (B) : Radial and Tangential acceleration

B-1. Two particles P and Q are located at distances r_P and r_Q respectively from the axis of a rotating disc such that $r_P > r_Q$:
(A) Both P and Q have the same acceleration (B) Both P and Q do not have any acceleration
(C) P has greater acceleration than Q (D) Q has greater acceleration than P

B-2. Let a_r and a_t represent radial and tangential acceleration. The motion of a particle may be circular if :
(A) $a_r = 0, a_t = 0$ (B) $a_r = 0, a_t \neq 0$ (C) $a_r \neq 0, a_t = 0$ (D) none of these

B-3. A particle is going with constant speed along a uniform helical and spiral path separately as shown in figure (in case (a), verticle acceleration of particle is negligible)



(a) (b)

(A) The velocity of the particle is constant in both cases
(B) The magnitude of acceleration of the particle is constant in both cases
(C) The magnitude of accleration is constant in (a) and decreasing in (b)
(D) The magnitude of accleration is decreasing continuously in both the cases

B-4. If the radii of circular paths of two particles of same masses are in the ratio of 1 : 2, then in order to have same centripetal force, their speeds should be in the ratio of :
(A) 1 : 4 (B) 4 : 1 (C) $1 : \sqrt{2}$ (D) $\sqrt{2} : 1$

Section (C) : Circular Motion in Horizontal plane

C-1. A stone of mass of 16 kg is attached to a string 144 m long and is whirled in a horizontal smooth surface. The maximum tension the string can withstand is 16 N. The maximum speed of revolution of the stone without breaking it, will be :
(A) 20 ms^{-1} (B) 16 ms^{-1} (C) 14 ms^{-1} (D) 12 ms^{-1}

C-2. On horizontal smooth surface a mass of 2 kg is whirled in a horizontal circle by means of a string at an initial angular speed of 5 revolutions per minute. Keeping the radius constant the tension in the string is doubled. The new angular speed is nearly:
(A) 14 rpm (B) 10 rpm (C) 2.25 rpm (D) 7 rpm

C-3. A particle is kept fixed on a uniformly rotating turn-table As seen from the ground , the particle goes in a circle, its speed is 10 cm/s and acceleration is 10 cm/s^2.The particle is now shifted to a new position to make the radius half of the original value.The new values of the speed and acceleration will be
(A) 20 cm/s, 20 cm/s^2 (B) 5 cm/s, 5 cm/s^2 (C) 40 cm/s, 10 cm/s^2 (D) 40 cm/s, 40 cm/s^2

C-4. A coin placed on a rotating turntable just slips if it is placed at a distance of 16 cm from the centre. If the angular velocity of the turntable is doubled, it will just slip at a distance of
(A) 1 cm (B) 2 cm (C) 4 cm (D) 8 cm

C-5. A rod of length L is hinged at one end and it is rotated with a constant angular velocity in a horizontal plane. Let T_1 and T_2 be the tensions at the points L/4 and 3L/4 away from the hinged end.
(A) $T_1 > T_2$ (B) $T_2 > T_1$ (C) $T_1 = T_2$
(D) The relation between T_1 and T_2 depends on whether the rod rotates clockwise or anticlockwise

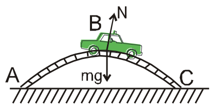
Section (D) : Radius of curvature

D-1. A stone is projected with speed u and angle of projection is θ. Find radius of curvature at t = 0.
(A) $\frac{u^2\cos^2\theta}{g}$ (B) $\frac{u^2}{g\sin\theta}$ (C) $\frac{u^2}{g\cos\theta}$ (D) $\frac{u^2\sin^2\theta}{g}$

D-2. A particle of mass m is moving with constant velocity $\vec{v}$ on smooth horizontal surface. A constant force $\vec{F}$ starts acting on particle perpendicular to velocity v. Radius of curvature after force F start acting is :
(A) $\frac{mv^2}{F}$ (B) $\frac{mv^2}{F\cos\theta}$ (C) $\frac{mv^2}{F\sin\theta}$ (D) none of these

Section (E) : Circular motion in vertical plane

E-1. A car is going on an overbridge of radius R, maintaining a constant speed. As the car is descending on the overbridge from point B to C, the normal force on it :



(A) increase (B) decreases
(C) remains constant (D) first increases then decreases.

E-2. In a circus, stuntman rides a motorbike in a circular track of radius R in the vertical plane. The minimum speed at highest point of track will be :
(A) $\sqrt{2gR}$ (B) 2gR (C) $\sqrt{3gR}$ (D) $\sqrt{gR}$

E-3. A particle is moving in a vertical circle. The tensions in the string when passing through two positions at angles 30° and 60° from downward vertical are T_1 and T_2 respectively. Then
(A) $T_1 = T_2$ (B) $T_2 > T_1$
(C) $T_1 > T_2$ (D) Tension in the string always remains the same

E-4. A bucket is whirled in a vertical circle with a string attached to it.The water in bucket does not fall down even when the bucket is inverted at the top of its path. In this position choose most appropiate option if v is the speed at the top.
(A) mg = $\frac{mv^2}{r}$ (B) mg is greater than $\frac{mv^2}{r}$
(C) mg is not greater than $\frac{mv^2}{r}$ (D) mg is not less than $\frac{mv^2}{r}$

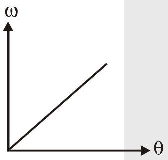
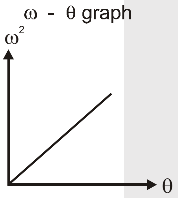
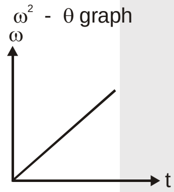
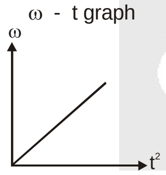
Section (F) : Motion of a vehicle, Centrifugal force and rotation of earth

F-1. A train A runs from east to west and another train B of the same mass runs from west to east at the same speed with respect to earth along the equator. Normal force by the track on train A is N_1 and that on train B is N_2:
(A) $N_1 > N_2$ (B) $N_1 < N_2$ (C) $N_1 = N_2$
(D) the information is insufficient to find the relation between N_1 and N_2.

F-2. If the apparent weight of the bodies at the equator is to be zero, then the earth should rotate with angular velocity
(A) $\sqrt{\frac{g}{R}}$ rad/sec (B) $\sqrt{\frac{2g}{R}}$ rad/sec (C) $\sqrt{\frac{g}{2R}}$ rad/sec (D) $\sqrt{\frac{3g}{2R}}$ rad/sec

PART - III : MATCH THE COLUMN

1. Each situation in column I gives graph of a particle moving in circular path. The variables ω, θ and t represent angular speed (at any time t), angular displacement (in time t) and time respectively. Column II gives certain resulting interpretation. Match the graphs in column I with statements in column II and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the OMR.

Column-I	**Column-II**
(A)  ω - θ graph	(p) Angular acceleration of particle is uniform
(B)  ω² - θ graph	(q) Angular acceleration of particle is non-uniform
(C)  ω - t graph	(r) Angular acceleration of particle is directly proportional to t.
(D)  ω - t² graph	(s) Angular acceleration of particle is directly proportional to θ.

2. A particle is moving with speed $v = 2t^2$ on the circumference of circle of radius R. Match the quantities given in column-I with corresponding results in column-II

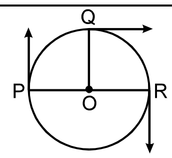
Column-I	**Column-II**
(A) Magnitude of tangential acceleration of particle	(p) decreases with time.
(B) Magnitude of Centripetal acceleration of particle	(q) increases with time
(C) Magnitude of angular speed of particle with respect to centre of circle	(r) remains constant
(D) Angle between the total acceleration vector and centripetal acceleration vector of particle	(s) depends on the value of radius R

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1.✍ Three point particles P, Q, R move in a circle of radius ' r ' with different but constant speeds. They start moving at t = 0 from their initial positions as shown in the figure. The angular velocities (in rad/sec) of P, Q and R are 5π, 2π & 3π respectively, in the same sense. The time at which they all meet is:

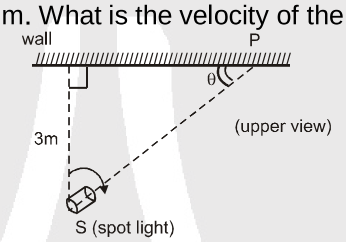


(A) 2/3 sec (B) 1/6 sec (C) 1/2 sec (D) 3/2 sec

2.✍ The kinetic energy K of a particle moving along a circle of radius R depends on the distance covered s as $K = as^2$ where a is a positive constant. The total force acting on the particle is :

(A) $2a\dfrac{s^2}{R}$ (B) $2as\left(1+\dfrac{s^2}{R^2}\right)^{1/2}$ (C) $2as$ (D) $2a\dfrac{R^2}{s}$

3. A spot light S rotates in a horizontal plane with a constant angular velocity of 0.1 rad/s. The spot of light P moves along the wall at a distance 3 m. What is the velocity of the spot P when $\theta = 45°$?



(A) 0.6 m/s (B) 0.5 m/s (C) 0.4 m/s (D) 0.3 m/s

4. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2\hat{i}$ m/s and $\vec{a} = 2\hat{i} + 4\hat{j}$ m/s^2 respectively at an instant of time. The radius of the circle is

(A) 1m (B) 2m (C) 3m (D) 4m

5.✍ A particle moves with deceleration along the circle of radius R so that at any moment of time its tangential and normal accelerations are equal in magnitude. At the initial moment t = 0 the speed of the particle equals v_0, then :

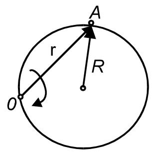
(i) the speed of the particle as a function of the distance covered s will be

(A) $v = v_0\, e^{-s/R}$ (B) $v = v_0\, e^{s/R}$ (C) $v = v_0\, e^{-R/s}$ (D) $v = v_0\, e^{R/s}$

(ii) the total acceleration of the particle as function of velocity.

(A) $a = \sqrt{2}\dfrac{v^2}{R}$ (B) $a = \dfrac{v^2}{R}$ (C) $a = \dfrac{2v^2}{R}$ (D) $a = \dfrac{2\sqrt{2}v^2}{R}$

6.✍ A particle A moves along a circle of radius R = 50 cm so that its radius vector r relative to the fixed point O (Figure) rotates with the constant angular velocity $\omega = 0.40$ rad/s. Then modulus v of the velocity of the particle, and the modulus a of its total acceleration will be

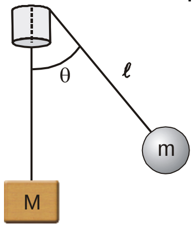


(A) v = 0.4 m/s, a = 0.4 m/s^2 (B) v = 0.32 m/s, a = 0.32 m/s^2
(C) v = 0.32 m/s, a = 0.4 m/s^2 (D) v = 0.4 m/s, a = 0.32 m/s^2

7. A boy whirls a stone in a horizontal circle 1.8 m above the ground by means of a string with radius 1.2 m. It breaks and stone flies off horizontally, striking the ground 9.1 m (horizontal range) away. The centripetal acceleration during the circular motion was nearly: (use g = 9.8 m/s^2)

(A) 94 m/s^2 (B) 141 m/s^2 (C) 188 m/s^2 (D) 282 m/s^2

8. A large mass M hangs stationary at the end of a light string that passes through a smooth fixed ring to a small mass m that moves around in a horizontal circular path. If ℓ is the length of the string from m to the top end of the tube and θ is angle between this part and vertical part of the string as shown in the figure, then time taken by m to complete one circle is equal to



(A) $2\pi\sqrt{\frac{\ell}{g\sin\theta}}$ (B) $2\pi\sqrt{\frac{\ell}{g\cos\theta}}$ (C) $2\pi\sqrt{\frac{m\ell}{gM\sin\theta}}$ (D) $2\pi\sqrt{\frac{\ell m}{gM}}$

9. Three identical particles are joined together by a thread as shown in figure. All the three particles are moving on a smooth horizontal plane about point O. If the speed of the outermost particle is v_0, then the ratio of tensions in the three sections of the string is : (Assume that the string remains straight)

(A) 3 : 5 : 7 (B) 3 : 4 : 5 (C) 7 : 11 : 6 (D) 6 : 5 : 3

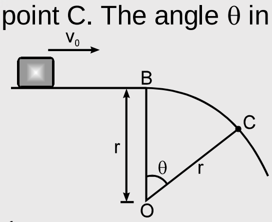
10. A Toy cart attached to the end of an unstretched string of length a, when revolved moves on a smooth horizontal table in a circle of radius 2a with a time period T. Now the toy cart is speeded up until it moves in a circle of radius 3a with a period T'. If Hook's law holds then (Assume no friction) :

(A) $T' = \sqrt{\frac{3}{2}}\,T$ (B) $T' = \left(\frac{\sqrt{3}}{2}\right)T$ (C) $T' = \left(\frac{3}{2}\right)T$ (D) T' = T

11. A stone of mass 1 kg tied to a light inextensible string of length $L = \frac{10}{3}$ m, whirling in a circular path in a vertical plane. The ratio of maximum tension in the string to the minimum tension in the string is 4, If g is taken to be 10 m/s^2, the speed of the stone at the highest point of the circle is :

(A) 10 m/s (B) $5\sqrt{2}$ m/s (C) $10\sqrt{3}$ m/s (D) 20 m/s

12. A small frictionless block slides with velocity $0.5\sqrt{gr}$ on the horizontal surface as shown in the Figure. The block leaves the surface at point C. The angle θ in the Figure is :

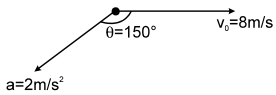


(A) $\cos^{-1}(4/9)$ (B) $\cos^{-1}(3/4)$ (C) $\cos^{-1}(1/2)$ (D) none of the above

13. A sphere of mass m is suspended by a thread of length 'ℓ' is oscillating in a vertical plane, the angular amplitude being θ_0. What is the tension in the thread when it makes an angle θ with the vertical during oscillations ? If the thread can support a maximum tension of 2 mg, then what can be the maximum angular amplitude of oscillation of the sphere without breaking the rope?

(A) $3mg\cos\theta - 2mg\cos\theta_0$, $\theta_0 = 60^0$ (B) $3mg\cos\theta + 2mg\cos\theta_0$, $\theta_0 = 60^0$
(C) $2mg\cos\theta - 3mg\cos\theta_0$, $\theta_0 = 30^0$ (D) $2mg\cos\theta + 3mg\cos\theta_0$, $\theta_0 = 30^0$

14. The figure shows the velocity and acceleration of a point like body at the initial moment of its motion. The acceleration vector of the body remains constant. The minimum radius of curvature of trajectory of the body is



(A) 2 meter (B) 4 meter (C) 8 meter (D) 16 meter.

15. A particle is projected horizontally from the top of a tower with a velocity v_0. If v be its velocity at any instant, then the radius of curvature of the path of the particle at that instant is directly proportional to:
(A) v^3 (B) v^2 (C) v (D) 1/v

16. A racing car moves along circular track of radius b. The car starts from rest and its speed increases at a constant rate α. Let the angle between the velocity and the acceleration be θ at time t. Then ($\cos\theta$) is :
(A) 0 (B) $\alpha t^2/b$ (C) $\frac{b}{\left(b+\alpha t^2\right)}$ (D) $\frac{b}{\left(b^2+\alpha^2 t^4\right)^{\frac{1}{2}}}$

PART - II : NUMERICAL VALUE TYPE

1. A solid body rotates with deceleration about a stationary axis with an angular deceleration $\beta \propto \sqrt{\omega}$ where ω is its angular velocity. If at the initial moment of time its angular velocity was equal to ω_0 then the mean angular velocity of the body averaged over the whole time of rotation till it comes to rest is $\frac{\omega_0}{n}$ where n is.

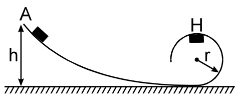
2. A particle moves clockwise in a circle of radius 1 m with centre at (x, y) = (1m, 0). It starts at rest at the origin at time t = 0. Its speed increases at the constant rate of $\left(\frac{\pi}{2}\right)$ m/s^2. If the net acceleration at t = 2 sec is $\frac{\pi}{2}\sqrt{\left(1+N\pi^2\right)}$ then what is the value of N ?

3. Two particles A and B move anticlockwise with the same speed v in a circle of radius R and are diametrically opposite to each other. At t = 0, A is imparted a tangential acceleration of constant magnitude $a_t = \frac{72v^2}{25\pi R}$. If the time in which A collides with B is $\frac{5\pi R}{N_1 v}$, the angle traced by A during this time is $\frac{11\pi}{N_2}$, its angular velocity is $\frac{17v}{N_3 R}$ and radial acceleration at the time of collision is $\frac{289\ v^2}{5RN_4}$. Then calculate the value of $N_1 + N_2 + N_3 + N_4$.

4. A block of mass m = 1kg moves on a horizontal circle against the wall of a cylindrical room of radius R = $2\sqrt{2}$ m. The floor of the room on which the block moves is smooth but the friction coefficient between the wall and the block is μ= 1. The block is given an initial speed v_0. If speed at a instant is v = 2m/s then calculate resultant acceleration of block in m/s^2 at that instant

5. A car goes on a horizontal circular road of radius R = $\sqrt{27}$ meter, the speed increasing at a constant rate dv/dt = a = 1 m/s^2, starting from rest. The friction coefficient between the road and the tyre is μ = 0.2. Find the time at which the car will skid.

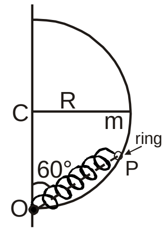
6. A small body of mass m = 0.5 kg is allowed to slide on an inclined frictionless track from rest position as shown in the figure. (g = 10 m/s^2)



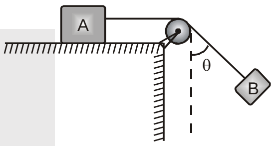
If h is double of that minimum height required to complete the loop successfully, calculate resultant force on the block at position H in newton

7. A nail is located at a certain distance vertically below the point of suspension of a simple pendulum. The pendulum bob is released from the position where the string makes an angle of 60° from the vertical. Calculate the value of x if distance of the nail from the point of suspension is x/10 such that the bob will just perform revolution with the nail as centre. Assume the length of pendulum to be 1m.

8. A smooth semicircular wire-track of radius R is fixed in a vertical plane shown in fig. One end of a massless spring of natural length (3R/4) is attached to the lower point O of the wire track. A small ring of mass m, which can slide on the track, is attached to the other end of the spring. The ring is held stationary at point P such that the spring makes an angle of 60° with the vertical. The spring constant K = mg/R. Consider the instant when the ring is released. If the tangential acceleration of the ring is $\frac{x\sqrt{3}g}{8}$ and the normal reaction is $\frac{y\ \ mg}{8}$ then calculate value of x + y.



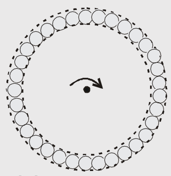
9. Two particles A and B each of mass m are connected by a massless string. A is placed on the rough table. The string passes over a small, smooth peg. B is left from a position making an $\angle\theta$ with the vertical. If the minimum coefficient of friction between A and the table is $\mu_{min} = 3 - N\cos\theta$ so that A does not slip during the motion of mass B. Then calculate the value of N



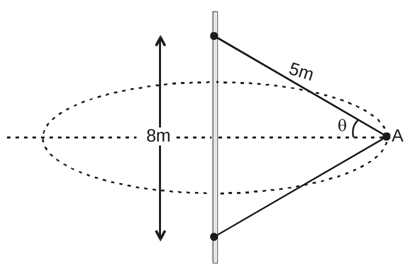
10. A particle moves along the plane trajectory y (x) with velocity v whose modulus is constant. Find the curvature radius of the trajectory at that point x = 0, if the trajectory has the form of a parabola $y = \frac{1}{10}x^2$.

11. A particle of mass m is suspended by string of length ℓ from a fixed rigid support. A sufficient horizontal velocity $v_0 = \sqrt{3g\ell}$ is imparted to it suddenly. Calculate the angle (in degree) made by the string with the vertical when the acceleration of the particle is inclined to the string by 45º.

12. A uniform metallic chain in a form of circular loop of mass m = 3 kg with a length ℓ = 1 m rotates at the rate of n = 5 revolutions per second. Find the tension T (in Newton) in the chain.



13. A 4 kg block is attached to a vertical rod by means of two strings of equal length. When the system rotates uniformly about the axis of the rod, the strings are extended as shown in figure. If tension in upper and lower chords are 200 newton and 10x newton respectively and angular velocity of particle is $\sqrt{\frac{y}{2}}$ than calculate value of x + y.



14. A simple pendulum is suspended from the ceiling of a car taking a turn of radius 10 m at a speed of 36 km/h. Find the angle (in degree) made by the string of the pendulum with the vertical if this angle does not change during the turn. Take g = 10 m/s^2.

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A stone is projected from level ground at t = 0 sec such that its horizontal and vertical components of initial velocity are 10 m/s and 20 m/s respectively. Then the instant of time at which magnitude of tangential and magnitude of normal components of acceleration of stone are same is: (neglect air resistance) g = 10 m/s^2.

(A) $\frac{1}{2}$ sec (B) 1 sec (C) 3 sec (D) 4 sec.

2. A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60^o with the vertical. Then (g = 9.8 m/s^2)

(A) its period of revolution is $\frac{4\pi}{7}$ sec.

(B) the tension in the string is double the weight of the particle

(C) the speed of the particle = $2.8\sqrt{3}$ m/s

(D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s^2.

3. A car of mass M is travelling on a horizontal circular path of radius r. At an instant its speed is v and tangential acceleration is a :

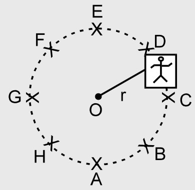
(A) The acceleration of the car is towards the centre of the path

(B) The magnitude of the frictional force on the car is greater than $\frac{mv^2}{r}$

(C) The friction coefficient between the ground and the car is not less than a/g.

(D) The friction coefficient between the ground and the car is $\mu = \tan^{-1}\frac{v^2}{rg}$

4. A machine, in an amusement park, consists of a cage at the end of one arm, hinged at O. The cage revolves along a vertical circle of radius r (ABCDEFGH) about its hinge O, at constant linear speed $v = \sqrt{gr}$. The cage is so attached that the man of weight 'w' standing on a weighing machine, inside the cage, is always vertical. Then which of the following is/are correct



(A) the reading of his weight on the machine is the same at all positions

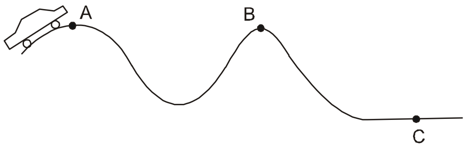
(B) the weight reading at A is greater than the weight reading at E by 2 w.

(C) the weight reading at G = w

(D) the ratio of the weight reading at E to that at A = 0

(E) the ratio of the weight reading at A to that at C = 2.

5. A car is moving with constant speed on a road as shown in figure. The normal reaction by the road on the car is N_A, N_B and N_C when it is at the points A, B and C respectively.



(A) $N_A = N_B$ (B) $N_A > N_B$ (C) $N_A < N_B$ (D) $N_C > N_A$

6. Assuming the motion of Earth around the Sun as a circular orbit with a constant speed of 30 km/s.
(A) The average velocity of the earth during a period of 1 year is zero
(B) The average speed of the earth during a period of 1 year is zero.
(C) The average acceleration during first 6 monts of the year is zero
(D) The instantaneous acceleration of the earth points towards the Sun.

7. A car of mass m attempts to go on the circular road of radius r, which is banked for a speed of 36 km/hr. The friction coefficient between the tyre and the road is negligible.
(A) The car cannot make a turn without skidding.
(B) If the car turns at a speed less than 36 km/hr, it will slip down
(C) If the car turns at the constant speed of 36 km/hr, the force by the road on the car is equal to $\frac{mv^2}{r}$
(D) If the car turns at the correct speed of 36 km/hr, the force by the road on the car is greater than mg as well as greater than $\frac{mv^2}{r}$.

8. A particle is attached to an end of a rigid rod. The other end of the rod is hinged and the rod rotates always remaining horizontal. It's angular speed is increasing at constant rate. The mass of the particle is 'm'. The force exerted by the rod on the particle is $\vec{F}$, then :
(A) $F > mg$
(B) F is constant
(C) The angle between $\vec{F}$ and horizontal plane decreases.
(D) The angle between $\vec{F}$ and the rod decreases.

9. A particle starting form rest at the highest point slides down the outside of a smooth vertical circular track of radius 0.3 m. When it leaves the track its vertical fall is h and the linear velocity is v. The angle made by the radius at that position of the particle with the vertical is θ. Now consider the following observation : (g = 10 m/s^2)
(I) h = 0.1 m and cos θ = 2/3. (II) h = 0.2 m and cosθ = 1/3. (III) v = $\sqrt{2}$ m/s^{-1}. (IV) After leaving the circular track the particle will describe a parabolic path. Therefore,
(A) (I) and (III) both are correct
(B) only (II) is incorrect
(C) only (III) is correct
(D) (IV) is correct

10. A particle moves along a horizontal circle such that the radial force acting on it is directly proportional to square of time. Then choose the correct option :
(A) tangential force acting on it is directly proportional to time
(B) power developed by total force is directly proportional to time
(C) average power developed by the total force over first t second from rest is directly proportional to time
(D) angle between total force and radial force decreases with time

PART - IV : COMPREHENSION

Comprehension-1

A particle undergoes uniform circular motion. The velocity and angular velocity of the particle at an instant of time is $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s and $\vec{\omega} = x\hat{i} + 6\hat{j}$ rad/sec.

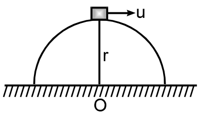
1. The value of x in rad/s is
(A) 8 (B) – 8 (C) 6 (D) can't be calculated

2. The radius of circle in metres is
(A) 1/2 m (B) 1 m (C) 2 m (D) can't be calculated

3. The acceleration of particle at the given instant is
(A) $-50\hat{k}$ (B) $-42\hat{k}$ (C) $2\hat{i} + 3\hat{j}$ (D) $50\hat{k}$

Comprehension-2

A small block of mass m is projected horizontally from the top of the smooth and fixed hemisphere of radius r with speed u as shown. For values of $u \geq u_0$, ($u_0 = \sqrt{gr}$) it does not slide on the hemisphere. [i.e., leaves the surface at the top itself]



4. For $u = 2\,u_0$, it lands at point P on ground. Find OP.

(A) $\sqrt{2}\ r$ (B) 2 r (C) 4r (D) $2\sqrt{2}\ r$

5. For $u = u_0/3$, find the height from the ground at which it leaves the hemisphere.

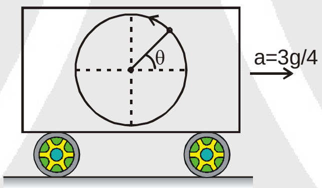
(A) $\frac{19\,r}{9}$ (B) $\frac{19\,r}{27}$ (C) $\frac{10r}{9}$ (D) $\frac{10r}{27}$

6. Find its net acceleration at the instant it leaves the hemisphere.

(A) g/4 (B) g/2 (C) g (D) g/3

Comprehension - 3

A bus is moving with a constant acceleration a = 3g/4 towards right. In the bus, a ball is tied with a rope of length ℓ and is rotated in vertical circle as shown.



7. At what value of angle θ, tension in the rope will be minimum

(A) θ = 37º (B) θ = 53º (C) θ = 30º (D) θ = 90º

8. At above mentioned position, find the minimum possible speed V_{min} during whole path to complete the circular motion :

(A) $\sqrt{5g\ell}$ (B) $\frac{5}{2}\sqrt{g\ell}$ (C) $\frac{\sqrt{5g\ell}}{2}$ (D) $\sqrt{g\ell}$

9. For above value of V_{min} find maximum tension in the string during circular motion.

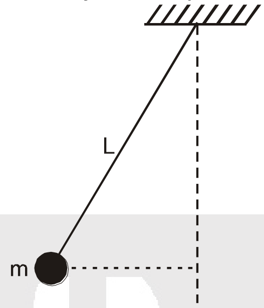
(A) 6 mg (B) $\frac{117}{20}$mg (C) $\frac{15}{2}$mg (D) $\frac{17}{2}$mg

Exercise-3

✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

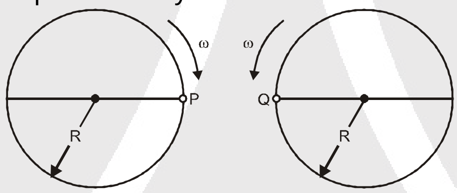
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

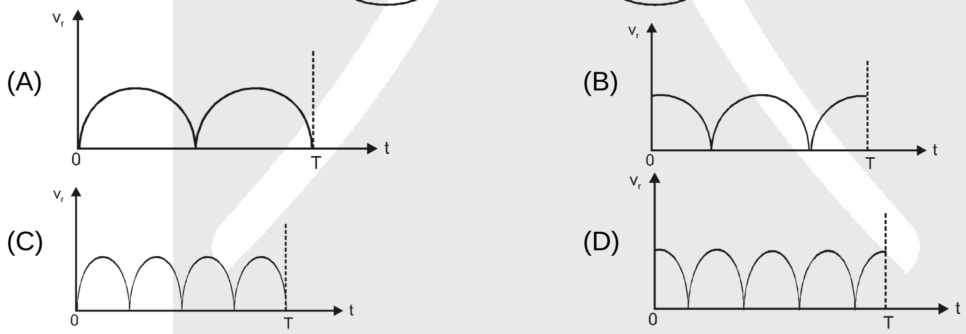
1.✍ A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is : **[JEE 2011, 3/160, –1]**



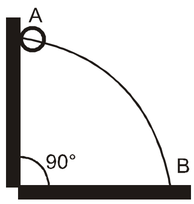
(A) 9 (B) 18 (C) 27 (D) 36

2. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The disc are in the same horizontal plane.At time t = 0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. as function of times best represented by **[IIT-JEE-2012, Paper-2; 3/66, –1]**





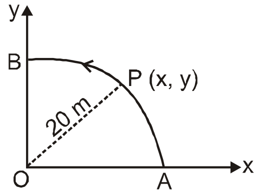
3.✍ A wire, which passes through the hole is a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is **[JEE (Advanced)-2014, 3/60, –1]**



(A) always radially outwards
(B) always radially inwards
(C) radially outwards initially and radially inwards later
(D) radially inwards initially and radially outwards later.

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly. **[AIEEE - 2010, 4/144]**



(1) 13 m/s^2 (2) 12 m/s^2 (3) 7.2 m/s^2 (4) 14 m/s^2

2. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P (R, θ) on the circle of radius R is (Here θ is measured from the x-axis) **[AIEEE - 2010, 4/144]**

(1) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (2) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

(3) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (4) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

3. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2, respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is : **[AIEEE 2012 ; 4/120, –1]**

(1) $m_1 r_1 : m_2 r_2$ (2) $m_1 : m_2$ (3) $r_1 : r_2$ (4) 1 : 1

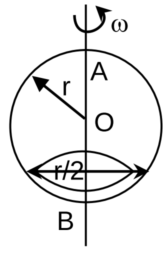
4. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then : **[JEE (Main) 2018; 4/120, –1]**

(1) $T \alpha R^{(n+1)/2}$ (2) $T \alpha R^{n/2}$ (3) $T \alpha R^{3/2}$ For any n. (4) $T \alpha R^{\frac{n}{2}+1}$

5. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is : **[JEE (Main) 2018; 4/120, –1]**

(1) zero (2) $-\frac{3}{2}\frac{k}{a^2}$ (3) $-\frac{k}{4a^2}$ (4) $\frac{k}{2a^2}$

6. A smooth wire of length 2πr is bent into a circle and kept in a vertical plane. A bead can slide smoothly on the wire. When the circle is rotating with angular speed ω about the vertical diameter AB, as shown in figure, the bead is at rest with respect to the circular ring at position P as shown. Then the value of ω^2 is equal to : **[JEE (Main) 2019; 4/120, –1]**



(1) $\frac{2g}{r}$ (2) $\frac{\sqrt{3}\,g}{2r}$

(3) $\frac{(g\sqrt{3})}{r}$ (4) $\frac{2g}{r\sqrt{3}}$

7. A spring mass system (mass m, spring constant k and natural length ℓ) rests in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω, (k>> mω^2) the relative change in the length of the spring is best given by the option : **[JEE (Main) 2020, 09 January; 4/100, –1]**

(1) $\sqrt{\frac{2}{3}}\left(\frac{m\omega^2}{k}\right)$ (2) $\frac{m\omega^2}{3k}$ (3) $\frac{2m\omega^2}{k}$ (4) $\frac{m\omega^2}{k}$

Answers

EXERCISE # 1

PART - I

Section (A) :

A-1. (a) first (b) third.

A-2. 12 : 1, $\frac{\pi}{30}$ rad/sec. **A-3.** 3 : 1

A-4. 24 rad/sec

A-5. $\frac{\pi\sqrt{2}}{30}$ cm/sec, $\frac{\pi\sqrt{2}}{30\times 15}$ cm/s^2

Section (B) :

B-1. $4\pi^2$

B-2. $\left(\frac{\sqrt{181}}{5} m/sec^2\right)$

B-3. (a) 4.0 cm/s^2 , (b) 2.0 cm/s^2, (c) $\sqrt{20}$ cm/s^2

Section (C)

C-1. $2\sqrt{\frac{6}{5}}$ sec., $\frac{13}{6}$ N (with $\pi^2 = 10$)

C-2. 2/5 **C-3.** 8 T_0

C-4. $\frac{15\pi^2}{10}$ = 14.8N, $\frac{15\pi^2}{10}$ = 14.8 N.

Section (D) :

D-1. $\frac{u^2 \sin^2\theta}{g}$ **D-2** $\frac{u^2\cos^2\theta}{g\cos^3(\theta/2)}$

Section (E) :

E-1. 5 rad/s **E-2.** 90°

E-3. $\frac{9}{2}$ mg **E-4.** $\sqrt{5g\ell}$

Section (F) :

F-1. $5\sqrt{2}$ ms^{-1} **F-2** $\sqrt{120}$ m/s

F-3. $\frac{2}{45}$ m **F-4.** $\tan^{-1}(1/4)$, 1/4

F-5. $100\sqrt{3}$ m/s **F-6.** 10 m/s^2

F-7. Between $\sqrt{\frac{50}{3}}\times\frac{18}{5}$ = 14.7 km/h and 54 km/hr

PART - II

Section (A) :

A-1. (C) **A-2.** (C) **A-3.** (C)
A-4. (D)

Section (B) :

B-1. (C) **B-2.** (C) **B-3.** (C)
B-4. (C)

Section (C) :

C-1. (D) **C-2.** (D) **C-3.** (B)
C-4. (C) **C-5.** (A)

Section (D) :

D-1. (C) **D-2.** (A)

Section (E) :

E-1. (B) **E-2.** (D) **E-3.** (C)
E-4. (C)

Section (F) :

F-1. (A) **F-2** (A)

PART - III

1. (A) q,s (B) p (C) p (D) q,r
2. (A) q (B) q, s (C) q, s (D) p, s

EXERCISE # 2

PART - I

1.	(D)	**2.**	(B)	**3.**	(A)
4.	(A)	**5.**	(A)	**6.**	(D)
7.	(C)	**8.**	(D)	**9.**	(D)
10.	(B)	**11.**	(A)	**12.**	(B)
13.	(A)	**14.**	(C)	**15.**	(A)
16.	(D)				

PART - II

1.	3.00	**2.**	4.00	**3.**	22.00
4.	2.00	**5.**	3.00	**6.**	30.00
7.	8.00	**8.**	8.00	**9.**	2.00
10.	5.00	**11.**	90.00	**12.**	75.00
13.	50.00	**14.**	45.00		

PART - III

1.	(BC)	**2.**	(ABCD)	**3.**	(BC)
4.	(BCDE)	**5.**	(BD)	**6.**	(AD)
7.	(BD)	**8.**	(ACD)	**9.**	(ABD)
10.	(BCD)				

PART - IV

1.	(B)	**2.**	(A)	**3.**	(A)
4.	(D)	**5.**	(B)	**6.**	(C)
7.	(B)	**8.**	(C)	**9.**	(C)

EXERCISE # 3

PART - I

1.	(D)	**2.**	(A)	**3.**	(D)

PART - II

1.	(4)	**2.**	(3)	**3.**	(3)
4.	(1)	**5.**	(1)	**6.**	(4)
7.	(4)				