NEWTON'S LAWS OF MOTION

1. FORCE

A pull or push which changes or tends to change the state of rest or of uniform motion or direction of motion of any object is called force. Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Effect of resultant force :

(1) may change only speed
(2) may change only direction of motion.
(3) may change both the speed and direction of motion.
(4) may change size and shape of a body

Unit of force : Newton and $\frac{kg \cdot m}{s^2}$ (MKS System)

dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 Newton = 10^5 dyne

Kilogram force (kgf) : The force with which earth attracts a 1kg body towards its centre is called kilogram force, thus

kgf = $\frac{\text{Forece in newton}}{g}$

Dimensional Formula of force : $[MLT^{-2}]$

1.1 Fundamental Forces

All the forces observed in nature such as muscular force, tension, reaction, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

(A) Gravitational Force : The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.

$$\vec{F} = -G\frac{m_1m_2}{r^3}\vec{r} \qquad S \xrightarrow{\vec{r}} T$$

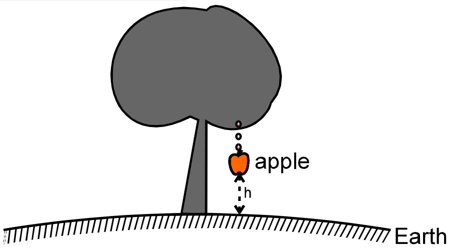
= position vector of test particle 'T' with respect to source particle 'S'. and G = universal gravitational constant = 6.67×10^{-11} Nm²/kg².

(i) It is the weakest force and is always attractive.

(ii) It is a long range force as it acts between any two particles situated at any distance in the universe.

(iii) It is independent of the nature of medium between the particles.

An apple is freely falling as shown in figure, When it is at a height h, force between earth and apple is given by



$$F = \frac{GM_em}{(R_e + h)^2}$$

where M_e – mass of earth, R_e – radius of earth. It acts towards earth's centre. Now rearranging above result,

$$F = m\frac{GM_e}{R_e^{\ 2}} \cdot \left(\frac{R_e}{R_e + h}\right)^2.$$

$$F = mg\left(\frac{R_e}{R_e + h}\right)^2 \left\{g = \frac{GM_e}{{R_e}^2}\right\}$$

Here h << R_e, so $\frac{R_e}{R_e + h} \approxeq 1$ $\therefore$ F = mg

This is the force exerted by earth on any particle of mass m near the earth surface. The value of g = 9.81 m/s^2 $\simeq$ 10 m/s^2 π^2 m/s^2 $\simeq$ 32 ft/s^2. It is also called acceleration due to gravity near the surface of earth.

(B) Electromagnetic Force : Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.

(b) These are long range forces

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

(C) Nuclear Force : It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

(D) Weak Force : It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 Fermi :

$F_N : F_{EM} : F_W : F_G : : 1 : 10^{-2} : 10^{-7} : 10^{-38}$

1.2 Classification of forces on the basis of contact :

(A) Field Force : Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force. Examples

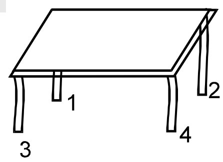
(a) Gravitation force (b) Electromagnetic force

(B) Contact Force : Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

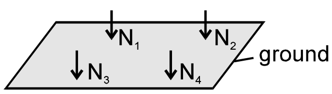
Examples :

(a) Normal force (N) :

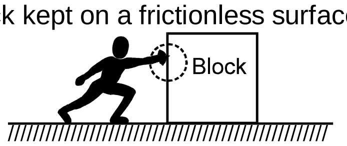
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on Earth as shown in figure



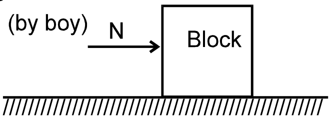
Here table presses the earth so normal force exerted by four legs of table on earth are as shown in figure.



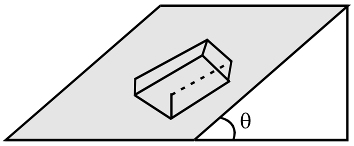
Now a boy pushes a block kept on a frictionless surface.



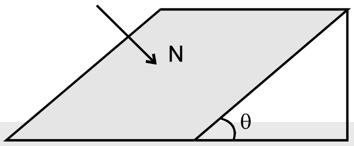
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



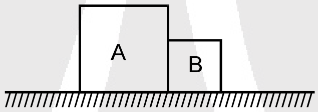
Normal force exerted by block on the surface of inclined plane is shown in figure.



Force acts perpendicular to the surface

Solved Examples

Example 1. Two blocks are kept in contact on a smooth surface as shown in figure. Draw normal force exerted by A on B.

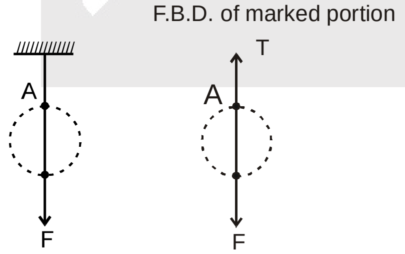


Solution : In above problem, block A does not push block B, so there is no molecular interaction between A and B. Hence normal force exerted by A on B is zero.

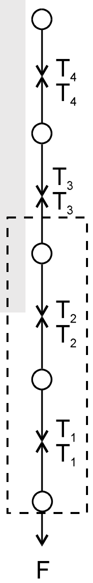
Note : Normal is a dependent force, it comes in role when one surface presses the other.

(b) Tension :

Tension in a string is a electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force 'F' as shown in figure, for calculating the tension at point 'A' we draw F.B.D. of marked portion of the string; Here string is massless.

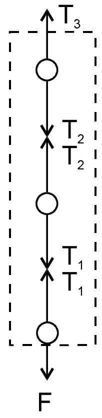


$\Rightarrow$ T = F



String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to Newton's third law.

For calculating tension at any segment, we consider two or more than two parts as a system.



$\boxed{T_3 = F}$

Here interaction between segments are considered as internal forces, so they are not shown in F.B.D.

(C) Frictional force : It is the component of contact force tangential to the surface. It opposes the relative motion (or attempted relative motion) of the two surfaces in contact.

2. THIRD LAW OF MOTION :

To every action, there is always an equal and opposite reaction. Newton's law from an 1803 translation from Latin as Newton wrote

"To every action there is always opposed an equal and opposite reaction : to the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

2.1 Important points about the Third Law

(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force'. A simple and clear way of stating the Third Law is as follows : Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on A by B and the force on B by A act at the same instant. Any one of them may be called action and the other reaction.

(c) Action and reaction forces act on different bodies, not on the same body. Thus if we are considering the motion of any one body (A or B), only one of the two forces is relevant. It is an error to add up the two forces and claim that the net force is zero.

However, if you are considering the system of two bodies as a whole, F_{AB} (force on A due to B) and F_{BA} (force on B due to A) are internal forces of the system (A + B). They add up to give a null force. Internal forces in a body or a system of particles thus cancel away in pairs. This is an important fact that enables the Second Law to be applicable to a body or a system of particles.

3. SYSTEM :

Two or more than two objects which interact with each other form a system.

3.1 Classification of forces on the basis of boundary of system :

(A) Internal Forces : Forces acting each with in a system among its constituents.

(B) External Forces : Forces exerted on the constituents of a system by the outside surroundings are called as external forces.

(C) Real Force : Force which acts on an object due to other object is called as real force. An isolated object (far away from all objects) does not experience any real force.

4. FREE BODY DIAGRAM

A free body diagram consists of a diagrammatic representations of a single body or a subsystem of bodies isolated from its surroundings showing all the forces acting on it.

4.1 Steps for F.B.D.

Step 1 : Identify the object or system and isolate it from other objects clearly specifying its boundary.

Step 2 : First draw non-contact external force in the diagram. Generally it is weight.

Step 3 : Draw contact forces which acts at the boundary of the object or system. Contact forces are normal, friction, tension and applied force.

In F.B.D, internal forces are not drawn, only external are drawn.

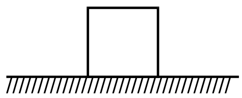
Solved Examples

Example 2. A block of mass 'm' is kept on the ground as shown in figure.

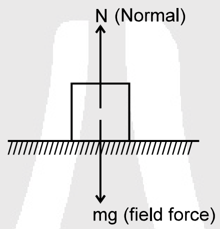
(i) Draw F.B.D. of block.

(ii) Are forces acting on block action–reaction pair.

(iii) If answer is no, draw action reaction pair.

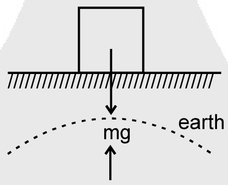


Solution : (i) F.B.D. of block

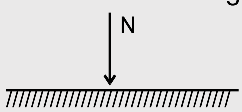


(ii) 'N' and mg are not action-reaction pair. Since pair act on different bodies, and they are of same nature.

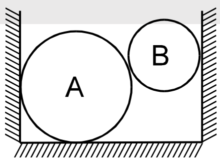
(iii) Pair of 'mg' of block acts on earth in opposite direction.



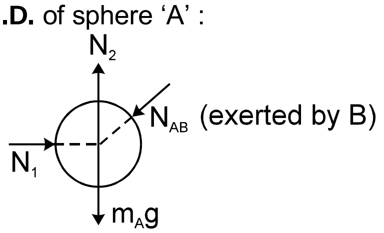
and pair of 'N' acts on surface as shown in figure.

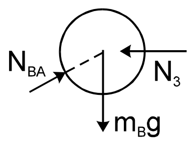


Example 3. Two sphere A and B are placed between two vertical walls as shown in figure. Draw the free body diagrams of both the spheres.



Solution : **F.B.D.** of sphere 'A' :





F.B.D. of sphere 'B' : (exerted by A)

Note : Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

5. NEWTON'S LAWS OF MOTION :

5.1 First Law of Motion

Each body continues to be in its state of rest or of uniform motion in a straight line unless compelled by some external force to act otherwise.

Newton's first law is really a statement about reference frames in that it defines the types of reference frames in which the laws of Newtonian mechanics hold. From this point of view the first law is expressed as:

If the net force acting on a body is zero, it is possible to find a set of reference frames in which that body has no acceleration.

Newton's first law is sometimes called the law of inertia and the reference frames that it defines are called inertial reference frames.

Newton's law from an 1803 translation from Latin as Newton wrote

"Every body preserves in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon."

Examples of this law :

(a) A bullet fired on a glass window makes a clean hole through it while a stone breaks the whole of it. The speed of bullet is very high. Due to its large inertia of motion, it cuts a clean hole through the glass. When a stone is thrown, it inertia is much lower so it cannot cut through the glass.

(b) A passenger sitting in a bus gets a jerk when the bus starts or stops suddenly.

5.2 Second Law of Motion :

The rate of change of momentum of a body is proportional to the applied force and takes place in the direction in which the force acts.

Newton's law from an 1803 translation from Latin as Newton wrote

"The alteration of motion is ever proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."

Mathematically

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Or $\vec{F} = m\vec{a}$

where $\vec{p} = m\vec{v}$, $\vec{p}$ = Linear momentum.

In case of two particles having linear momentum $\vec{P}_1$ and $\vec{P}_2$ and moving towards each other under mutual forces, from Newton's second law ;

$$\frac{d}{dt}\left(\vec{p}_1 + \vec{p}_2\right) = \vec{F} = 0 \quad \Rightarrow \quad \frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0$$

$$\vec{F}_1 + \vec{F}_2 = 0 \quad \Rightarrow \quad \vec{F}_2 = -\vec{F}_1$$ which is Newton's third law.

5.3 Important points about second law

(a) The Second Law is obviously consistent with the First Law as F = 0 Implies a = 0.

(b) The Second Law of motion is a vector law. It is actually a combination of three equations, one for each component of the vectors :

$$F_x = \frac{dp_x}{dt} = ma_x \qquad F_y = \frac{dp_y}{dt} = ma_y \qquad F_z = \frac{dp_z}{dt} = ma_z$$

This means that if a force is not parallel to the velocity of the body, but makes some angle with it, it changes only the component of velocity along the direction of force. The component of velocity normal to the force remains unchanged.

(c) The Second Law of motion given above is strictly applicable to a single point mass. The force **F** in the law stand for the net external force on the particle and a stands for the acceleration of the particle. Any internal forces in the system are not to be included in F.

(d) The Second Law of motion is a local relation. What this means is that the force F at a point in space (location of the particle) at a certain instant of time is related to a at the same point at the same instant. That is acceleration here and now is determined by the force here and now not by any history of the motion of the particle.

5.4 Applications of Newton’s Laws

(a) When objects are in equilibrium

To solve problems involving objects in equilibrium:

Step 1 : Make a sketch of the problem.

Step 2 : Isolate a single object and then draw the **free-body diagram** for the object. Label all external forces acting on it.

Step 3 : Choose a convenient coordinate system and resolve all forces into rectangular components along x and y direction.

Step 4 : Apply the equations $\sum F_x = 0$ and $\sum F_y = 0$.

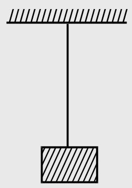
Step 5 : Step 4 will give you two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

Step 6 : If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps.

Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Solved Examples

Example 4. A ‘block’ of mass 10 kg is suspended with string as shown in figure. Find tension in the string. ($g = 10\ m/s^2$)

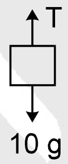


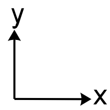
Solution : F.B.D. of block

$\Sigma F_y = 0$

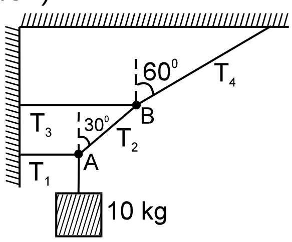
$T - 10\ g = 0$

$\therefore \quad T = 100\ N$





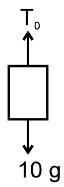
Example 5. The system shown in figure is in equilibrium. Find the magnitude of tension in each string ; T_1, T_2, T_3 and T_4. ($g = 10\ m/s^{-2}$)



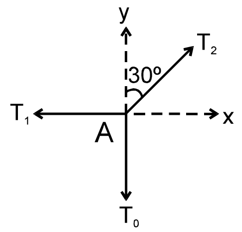
Solution : **F.B.D. of block 10 kg**

$T_0 = 10\ g$

$T_0 = 100\ N$



F.B.D. of point 'A'



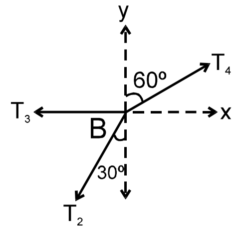
$\Sigma F_y = O$

$T_2 \cos 30^o = T_0 = 100$ N $\qquad T_2 = \dfrac{200}{\sqrt{3}}$ N

$\Sigma F_x = O$

$T_1 = T_2 \sin 30^o = \dfrac{200}{\sqrt{3}} \cdot \dfrac{1}{2} = \dfrac{100}{\sqrt{3}}$ N.

F.B.D. of point 'B'

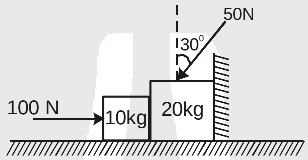


$\Sigma F_y = O \Rightarrow T_4 \cos 60^0 = T_2 \cos 30^0$

and $\Sigma F_x = O \Rightarrow T_3 + T_2 \sin 30^0 = T_4 \sin 60^0$

$T_3 = \dfrac{200}{\sqrt{3}}$ N, $\qquad T_4 = 200$ N

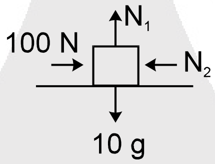
Example 6. Two blocks are kept in contact as shown in figure. Find



(a) forces exerted by surfaces (floor and wall) on blocks.

(b) contact force between two blocks.

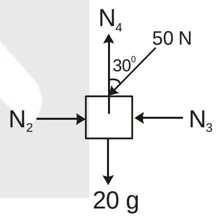
Solution : **F.B.D. of 10 kg block**



$N_1 = 10\,g = 100$ N .....(1)

$N_2 = 100$ N .....(2)

F.B.D. of 20 kg block



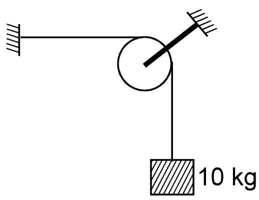
$N_2 = 50 \sin 30^o + N_3$

$\therefore \quad N_3 = 100 - 25 = 75$ N .....(3)

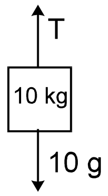
and $N_4 = 50 \cos 30° + 20\,g$

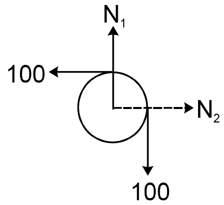
$N_4 = 243.30$ N

Example 7. Find magnitude of force exerted by string on pulley.



Solution : **F.B.D. of 10 kg block :** T = 10 g = 100 N





F.B.D. of pulley : Since string is massless, so tension in both sides of string is same. Force exerted by string

= $\sqrt{(100)^2+(100)^2}$ = $\sqrt{2}$ 100 N

Note : Since pulley is in equilibrium position, so net forces on it is zero.

Hence force exerted by hinge on it is 100 $\sqrt{2}$ N.

(b) Accelerating Objects

To solve problems involving objects that are in accelerated motion :

Step 1 : Make a sketch of the problem.

Step 2 : Isolate a single object and then draw the **free-body diagram** for that object. Label all external forces acting on it. Be sure to include all the forces acting on the chosen body, but be equally carefully not to include any force exerted by the body on some other body. Some of the forces may be unknown; label them with algebraic symbols.

Step 3 : Choose a convenient coordinate system, show location of coordinate axes explicitly in the free-body diagram, and then determine components of forces with reference to these axes and resolve all forces into x and y components.

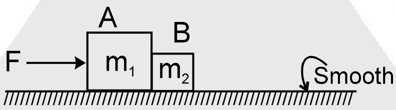
Step 4 : Apply the equations $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$

Step 5 : Step 4 will give two equations with several unknown quantities. If you have only two unknown quantities at this point, you can solve the two equations for those unknown quantities.

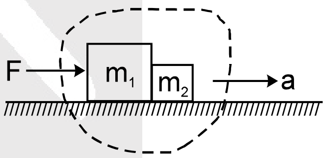
Step 6 : If step 5 produces two equations with more than two unknowns, go back to step 2 and select another object and repeat these steps. Eventually at step 5 you will have enough equations to solve for all unknown quantities.

Solved Examples

Example 8. A force F is applied horizontally on mass m_1 as shown in figure. Find the contact force between m_1 and m_2.



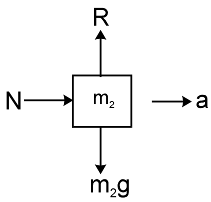
Solution : Considering both blocks as a system to find the common acceleration. Common acceleration



$$a = \frac{F}{(m_1+m_2)} \qquad .....(1)$$

To find the contact force between 'A' and 'B' we draw F.B.D. of mass m_2.

F.B.D. of mass m_2



$\Sigma F_x = ma_x$

$N = m_2 .a$

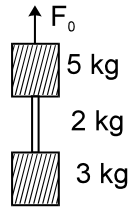
$$N = \frac{m_2F}{(m_1+m_2)}$$

Example 9. The velocity of a particle of mass 2 kg is given by $\vec{v} = at\hat{i} + bt^2\hat{j}$. Find the force acting on the particle.

Solution : From second law of motion :

$$\vec{F} = \frac{d\vec{P}}{dt} = \frac{d}{dt}(m\vec{v}) = 2.\frac{d}{dt}\ (at\hat{i}+bt^2\hat{j}) \Rightarrow \ \vec{F} = 2a\hat{i} + 4bt\hat{j}$$

Example 10. A 5 kg block has a rope of mass 2 kg attached to its underside and a 3 kg block is suspended from the other end of the rope. The whole system is accelerated upward at 2 m/s^2 by an external force F_0.

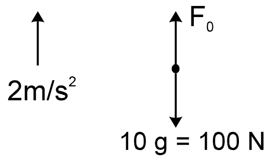


(a) What is F_0 ?
(b) What is the net force on rope ?
(c) What is the tension at middle point of the rope ? (g = 10 m/s^2)

Solution : For calculating the value of F_0, consider two blocks with the rope as a system.

F.B.D. of whole system

(a)



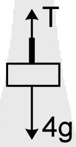
$F_0 - 100 = 10 \times 2$

F = 120 N .....(1)

(b) According to Newton's second law, net force on rope.

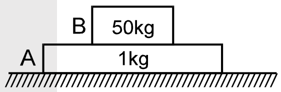
F = ma = (2) (2) = 4 N .....(2)

(c) For calculating tension at the middle point we draw F.B.D. of 3 kg block with half of the rope (mass 1 kg) as shown.



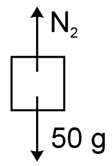
T – 4g = 4.(2) ; T = 48 N

Example 11. A block of mass 50 kg is kept on another block of mass 1 kg as shown in figure. A horizontal force of 10 N is applied on the 1kg block. (All surface are smooth). Find (g = 10 m/s^2)



(a) Acceleration of block A and B.
(b) Force exerted by B on A.

Solution : **(a) F.B.D. of 50 kg**



$N_2 = 50$ g = 500 N

along horizontal direction, there is no force $a_B = 0$

(b) F.B.D. of 1 kg block : along horizontal direction

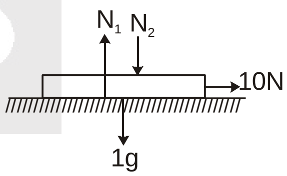
$10 = 1\, a_A$.

$a_A = 10$ m/s^2

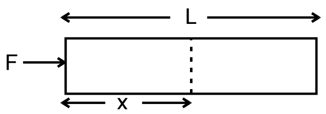
along vertical direction

$\therefore\ N_1 = N_2 + 1g$

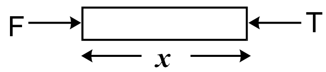
$= 500 + 10 = 510$ N



Example 12. A horizontal force is applied on a uniform rod of length L kept on a frictionless surface. Find the tension in rod at a distance 'x' from the end where force is applied.



Solution : Considering rod as a system, we find acceleration of rod a = $\frac{F}{M}$

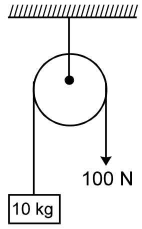


now draw F.B.D. of rod having length 'x' as shown in figure.

Using Newton's second law

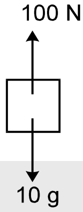
$$F - T = \left(\frac{M}{L}\right) x.a \quad \Rightarrow \quad T = F - \frac{M}{L} x . \frac{F}{M} \quad \Rightarrow \quad T = F\left(1 - \frac{x}{L}\right) .$$

Example 13. One end of string which passes through pulley and connected to 10 kg mass at other end is pulled by 100 N force. Find out the acceleration of 10 kg mass. ($g = 9.8\ m/s^2$)



Solution : Since string is pulled by 100 N force. So tension in the string is 100 N.

F.B.D. of 10 kg block

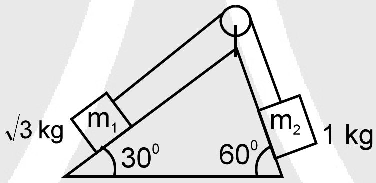


$100 - 10\ g = 10\ a$

$100 - 10 \times 9.8 = 10\ a$

$a = 0.2\ m/s^2$.

Example 14. Two blocks m_1 and m_2 are placed on a smooth inclined plane as shown in figure. If they are released from rest. Find :

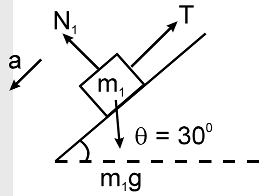


(i) Acceleration of mass m_1 and m_2

(ii) Tension in the string

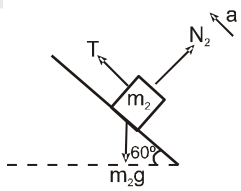
(iii) Net force on pulley exerted by string

Solution : **F.B.D. of m_1 :** $m_1 g\sin\theta - T = m_1 a$



$$\frac{\sqrt{3}}{2}\, g - T = \sqrt{3}\, a \qquad .....(1)$$

F.B.D. of m_2 : $T - m_2 g\sin\theta = m_2 a$



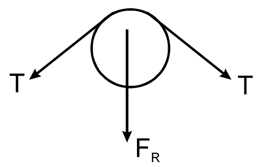
$$T - 1.\ \frac{\sqrt{3}}{2}\, g = 1.a \qquad .....(2)$$

Adding eq.(1) and (2) we get a = 0

Putting this value in eq.(i) we get

$$T = \frac{\sqrt{3}g}{2},$$

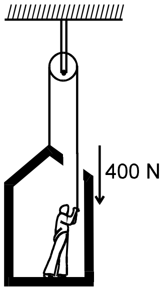
F.B.D. of pulley



$F_R = \sqrt{2}\, T$

$F_R = \sqrt{\frac{3}{2}}\, g$

Example 15. A 60 kg painter stands on a 15 kg platform. A rope attached to the platform and passing over an overhead pulley allows the painter to raise himself along with the platform.

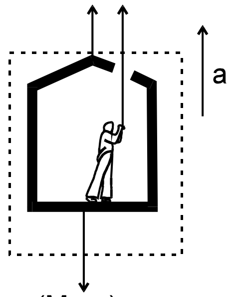


(i) To get started, he pulls the rope down with a force of 400 N. Find the acceleration of the platform as well as that of the painter.

(ii) What force must he exert on the rope so as to attain an upward speed of 1 m/s in 1s?

(iii) What force should he apply now to maintain the constant speed of 1 m/s?

Solution : The free body diagram of the painter and the platform as a system can be drawn as shown in the figure. Note that the tension in the string is equal to the force by which he pulls the rope.



(i) Applying Newton's Second Law

$2T - (M + m)g = (M + m)a$

or $a = \dfrac{2T - (M+m)g}{M+m}$

Here M = 60 kg; m = 15 kg; T = 400 N

g = 10 m/s^2

$a = \dfrac{2(400) - (60+15)(10)}{60+15} = 0.67$ m/s^2

(ii) To attain a speed of 1 m/s in one second, the acceleration a must be 1 m/s^2.

Thus, the applied force is

$F = \dfrac{1}{2}(M + m)(g + a) = \dfrac{1}{2}(60 + 15)(10 + 1) = 412.5$ N

(iii) When the painter and the platform move (upward) together with a constant speed, it is in a state of dynamic equilibrium.

Thus, $2F - (M + m)g = 0$

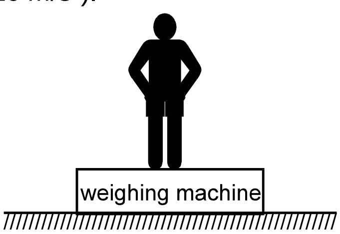
or $F = \dfrac{(M+m)g}{2} = \dfrac{(60+15)(10)}{2} = 375$ N

6. WEIGHING MACHINE :

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

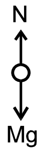
Solved Examples

Example 16. A man of mass 60 Kg is standing on a weighing machine placed on ground. Calculate the reading of machine (g = 10 m/s^2).

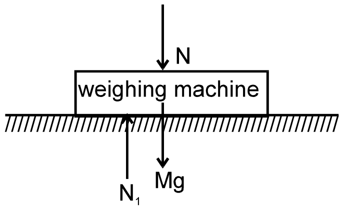


Solution : For calculating the reading of weighing machine, we draw F.B.D. of man and machine separately.

F.B.D. of man F.B.D. of weighing machine



$N = Mg$



Here force exerted by object on upper surface is N

Reading of weighing machine

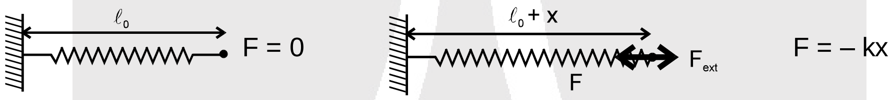
$N = Mg = 60 \times 10$

$N = 600$ N.

7. SPRING FORCE :

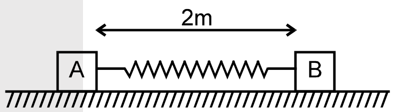
Every spring resists any attempt to change its length; when it is compressed or extended, it exerts force at its ends. The force exerted by a spring is given by $F = -kx$, where x is the change in length and k is the stiffness constant or spring constant (unit Nm^{-1}).

When spring is in its natural length, spring force is zero.



Solved Examples

Example 17. Two blocks are connected by a spring of natural length 2 m. The force constant of spring is 200 N/m. Find spring force in following situations :



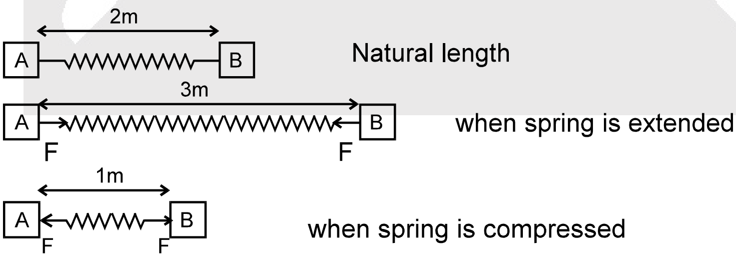
(a) If block 'A' and 'B' both are displaced by 0.5 m in same direction.

(b) If block 'A' and 'B' both are displaced by 0.5 m in opposite direction.

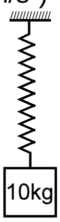
Solution : (a) Since both blocks are displaced by 0.5 m in same direction, so change in length of spring is zero. Hence, spring force is zero.

(b) In this case, change in length of spring is 1 m. In case of extension or compression of spring, spring force is $F = Kx = (200).(1)$

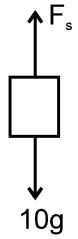
$F = 200$ N



Example 18. Force constant of a spring is 100 N/m. If a 10 kg block attached with the spring is at rest, then find extension in the spring. ($g = 10$ m/s^2)



Solution : In this situation, spring is in extended state so spring force acts in upward direction. Let x be the extension in the spring.



F.B.D. of 10 kg block :

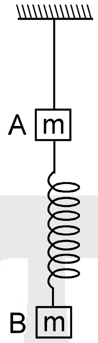
$F_s = 10g$

$\Rightarrow$ $Kx = 100$

$\Rightarrow$ $(100)x = (100)$

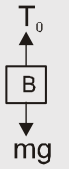
$\Rightarrow$ $x = 1m$

Example 19. Two blocks 'A' and 'B' of same mass 'm' attached with a light spring are suspended by a string as shown in figure. Find the acceleration of block 'A' and 'B' just after the string is cut.



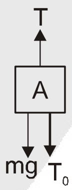
Solution : When block A and B are in equilibrium position

F.B.D of 'B'



$T_0 = mg$ .....(1)

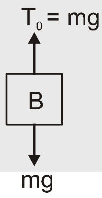
F.B.D of 'A'



$T = mg + T_0$
$T = 2\,mg$ .....(2)

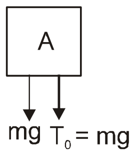
When string is cut, tension T becomes zero. But spring does not change its shape just after cutting. So spring force acts on mass B, again draw F.B.D. of blocks A and B as shown in figure

F.B.D. of 'B'



$T_0 - mg = ma_B$
$a_B = 0$

F.B.D. of 'A'

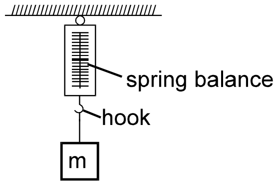


$mg + T_0 = ma_A$
$2mg = ma_A$
$a_A = 2g$ (downwards)

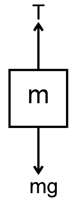
7.1 Spring Balance :

It does not measure the weight. It measures the force exerted by the object at the hook.

Symbolically, it is represented as shown in figure. A block of mass 'm' is suspended at hook.



When spring balance is in equilibrium, we draw the F.B.D. of mass m for calculating the reading of balance.



F.B.D. of 'm'.

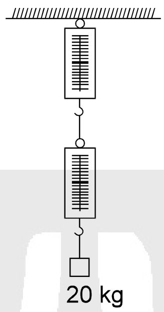
mg – T = 0

T = mg

Magnitude of T gives the reading of spring balance.

Solved Examples

Example 20. A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the

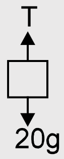


(1) reading of spring balance (1).

(2) reading of spring balance (2).

Solution : For calculating the reading, first we draw F.B.D. of 20 kg block.

F.B.D of 20 kg.



mg – T = 0

T = 20 g = 200 N

Since both balances are light so, both the scales will read 20 kg

8. CONSTRAINED MOTION:

8.1 String Constraint :

When two objects are connected through a string and if the string have the following properties:

(a) The length of the string remains constant i.e. inextensible string.

(b) Always remains tight, does not slacks.

Then the parameters of the motion of the objects along the length of the string and in the direction of extension have a definite relation between them.

Steps for String Constraint

Step 1. Identify all the objects and number of strings in the problem.

Step 2. Assume variable to represent the parameters of motion such as displacement, velocity acceleration etc.

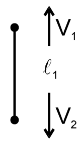
(i) Object which moves along a line can be specified by one variable.

(ii) Object moving in a plane are specified by two variables.

(iii) Objects moving in 3-D requires three variables to represent the motion.

Step 3. Identify a single string and divide it into different linear sections and write in the equation format. $\ell_1 + \ell_2 + \ell_3 + \ell_4 + \ell_5 + \ell_6 = \ell$

Step 4. Differentiate with respect to time



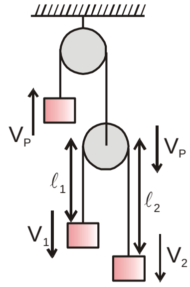
$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} + \frac{d\ell_3}{dt} + ... = 0$$

$\frac{d\ell_1}{dt}$ = represents the rate of increment of the portion 1, end points are always in contact with some object so take the velocity of the object along the length of the string $\frac{d\ell_1}{dt} = V_1 + V_2$

Take positive sign if it tends to increase the length and negative sign if it tends to decrease the length. Here $+V_1$ represents that upper end is tending to increase the length at rate V_1 and lower end is tending to increase the length at rate V_2.

Step 5. Repeat all above steps for different-different strings.

Let us consider a problem given below



Here $\ell_1 + \ell_2$ = constant

$$\frac{d\ell_1}{dt} + \frac{d\ell_2}{dt} = 0$$

$(V_1 - V_P) + (V_P - V_2) = 0$

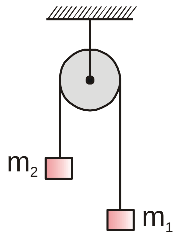
$V_p = \frac{V_1 + V_2}{2}$ Similarly, $a_P = \frac{a_1 + a_2}{2}$ Remember this result

Solved Examples

Example 21. Two blocks of masses m_1 and m_2 are attached at the ends of an inextensible string which passes over a smooth massless pulley. If $m_1 > m_2$, find :

(i) the acceleration of each block

(ii) the tension in the string.

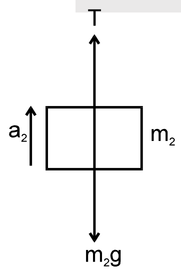


Solution : The block m_1 is assumed to be moving downward and the block m_2 is assumed to be moving upward. It is merely an assumption and it does not imply the real direction.

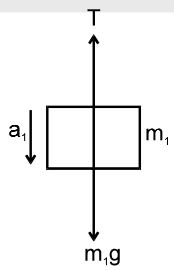
If the values of a_1 and a_2 come out to be positive then only the assumed directions are correct; otherwise the body moves in the opposite direction. Since the pulley is smooth and massless, therefore, the tension on each side of the pulley is same.

The free body diagram of each block is shown in the figure.

F.B.D. of m_2 **F.B.D. of m_1**



F.B.D. of m_2



F.B.D. of m_1

Applying Newton's second Law on blocks m_1 and m_2

Block m_1 $m_1g - T = m_1a_1$ ....(1)

Block m_2 $-m_2g + T = m_2a_2$ ....(2)

Number of unknowns : T, a_1 and a_2 (three)

Number of equations: only two

Obviously, we require one more equation to solve the problem. Note that whenever one finds the number of equations less than the number of unknowns, one must think about the constraint relation. Now we are going to explain the mathematical procedure for this.

How to determine Constraint Relation ?

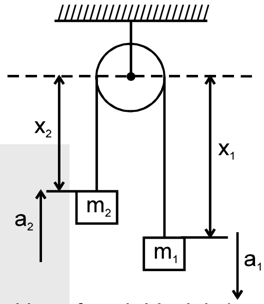
(1) Assume the direction of acceleration of each block, e.g. a_1 (downward) and a_2 (upward) in this case.

(2) Locate the position of each block from a fixed point (depending on convenience), e.g. centre of the pulley in this case.

(3) Identify the constraint and write down the equation of constraint in terms of the distance assumed. For example, in the chosen problem, the length of string remains constant is the constraint or restriction.

Thus, $x_1 + x_2$ = constant

Differentiating both the sides w.r.t. time we get $\frac{dx_1}{dt} + \frac{dx_2}{dt} = 0$



Position of each block is located w.r.t. centre of the pulley

Each term on the left side represents the velocity of the blocks.

Since we have to find a relation between accelerations, therefore we differentiate it once again w.r.t. time.

Thus $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} = 0$

Since, the block m_1 is assumed to be moving downward (x_1 is increasing with time)

$\therefore \quad \frac{d^2x_1}{dt^2} = +a_1$

and block m_2 is assumed to be moving upward (x_2 is decreasing with time)

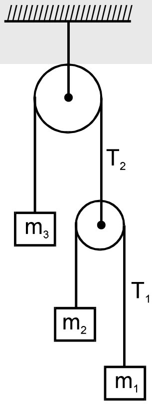
$\therefore \quad \frac{d^2x_2}{dt^2} = -a_2$

Thus $a_1 - a_2 = 0$ or $a_1 = a_2 = a$ (say) is the required constraint relation.

Substituting $a_1 = a_2 = a$ in equations (1) and (2) and solving them, we get

(i) $a = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] g$ (ii) $T = \left[\frac{2m_1m_2}{m_1 + m_2}\right] g$

Example 22. A system of three masses m_1, m_2 and m_3 are shown in the figure. The pulleys are smooth and massless; the strings are massless and inextensible.

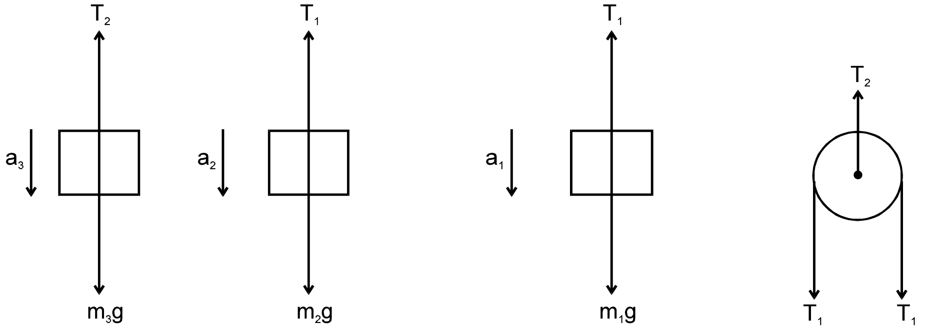


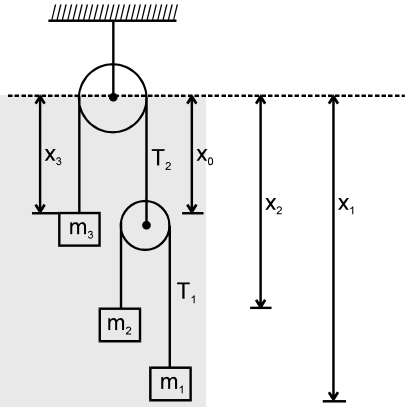
(i) Find the tensions in the strings.

(ii) Find the acceleration of each mass.

Solution : All the blocks are assumed to be moving downward and the free body diagram of each block is shown in figure.

F.B.D. m_3 **F.B.D. m_2** **F.B.D. m_1** **F.B.D. of pulley**





Applying Newton's Second Law to

Block m_1 : $m_1g - T_1 = m_1a_1$ ....(1)

Block m_2 : $m_2g - T_1 = m_2a_2$ ....(2)

Block m_3 : $m_3g - T_2 = m_3a_3$ ....(3)

Pulley : $T_2 = 2T_1$ ....(4)

Number of unknowns a_1, a_2, a_3, T_1 and T_2 (Five)

Number of equations : Four

The constraint relation among accelerations can be obtained as follows

For upper string $x_3 + x_0 = c_1$

For lower string $x_2 - x_0) + (x_1 - x_0) = c_2$

$x_2 + x_1 - 2x_0 = c_2$

Eliminating x_0 from the above two relations,

we get $x_1 + x_1 + 2x_3 = 2c_1 + c_2$ = constant.

Differentiating twice with respect to time,

we get $\frac{d^2x_1}{dt^2} + \frac{d^2x_2}{dt^2} + 2\frac{d^2x_3}{dt^2} = 0$

or $a_1 + a_2 + 2a_3 = 0$ ......(5)

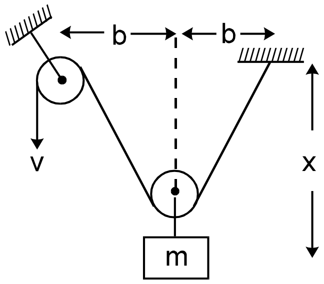
Solving equations (1) to (5), we get

(i) $T_1 = \left[\frac{4m_1m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $T_2 = 2T_1$

(ii) $a_1 = \left[\frac{4m_1m_2 + m_1m_3 - 3m_2m_3}{4m_1m_2 + m_3(m_1+m_2)}\right]g$; $a_2 = \left[\frac{3m_1m_3 - m_2m_3 - 4m_1m_2}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

$a_3 = \left[\frac{4m_1m_2 - m_3(m_1+m_2)}{4m_1m_2 + m_3(m_1+m_2)}\right]g$

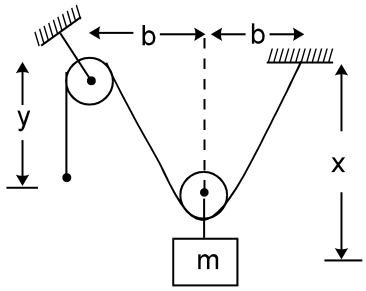
Example 23. The figure shows one end of a string being pulled down at constant velocity v. Find the velocity of mass 'm' as a function of 'x'.



Solution : Using constraint equation $2\sqrt{x^2+b^2}$ + y = length of string = constant

Differentiating w.r.t. time : $\frac{2}{2\sqrt{x^2+b^2}}$ $.2x\left(\frac{dx}{dt}\right)+\left(\frac{dy}{dt}\right)=0$

$\Rightarrow \left(\frac{dy}{dt}\right)=v \quad \Rightarrow \left(\frac{dx}{dt}\right)=-\frac{v}{2x}\sqrt{x^2+b^2}$



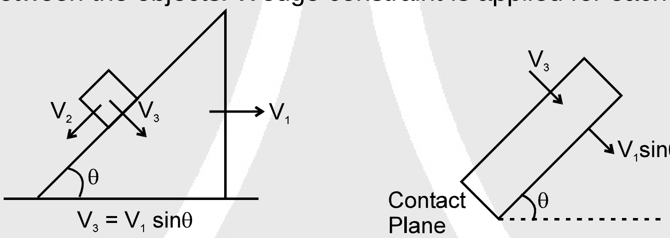
8.2 Wedge Constraint :

Conditions :

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.

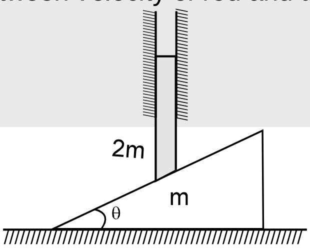


In other words,

Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Solved Examples

Example 24. A rod of mass 2m moves vertically downward on the surface of wedge of mass as shown in figure. Find the relation between velocity of rod and that of the wedge at any instant.



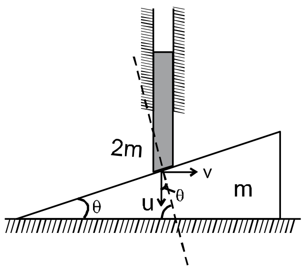
Solution : Using wedge constraint.

Component of velocity of rod along perpendicular to inclined surface is equal to velocity of wedge along that direction.

u cos θ = v sin θ

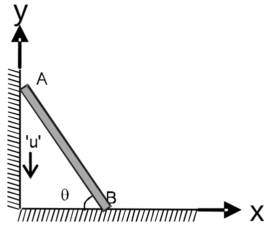
$\frac{u}{v}$ = tan θ

u = v tan θ



perpendicular to contact of two blocks

Example 25. The velocity of end 'A' of rigid rod placed between two smooth vertical walls is 'u' along vertical direction. Find out the velocity of end 'B' of that rod, rod always remains in contact with the vertical wall and also find the velocity of centre of rod. also find equation of path of centre of rod.



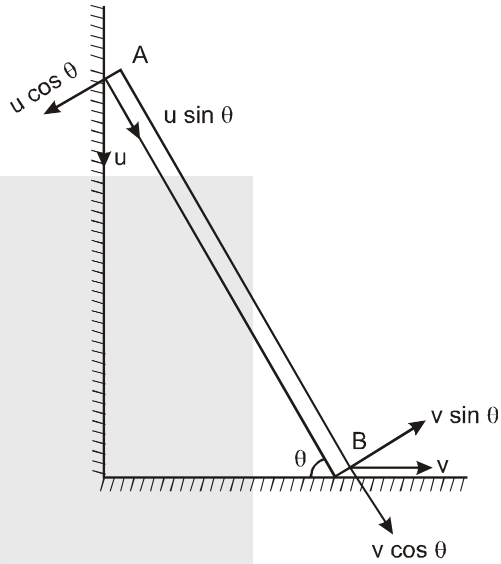
Solution : Since rod is rigid, its length can't increase.

$\therefore$ velocity of approach of A and B point of rod is zero.

$\Rightarrow \quad u \sin\theta - v\cos\theta = 0$

$\Rightarrow \quad v = u \tan\theta$

at any angle θ, x and y coordinates of center of mass are



Ans. $u \tan\theta.\ \left(\frac{u}{2}(-\hat{j}) + \frac{u\tan\theta}{2}\hat{i}\right)$, , $x^2 + y^2 = \left(\frac{\ell}{2}\right)^2$

9. NEWTON'S LAW FOR A SYSTEM

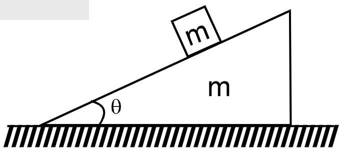
$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$

$\vec{F}_{ext} =$ Net external force on the system.

m_1, m_2, m_3 are the masses of the objects of the system and $\vec{a}_1, \vec{a}_2, \vec{a}_3$ are the acceleration of the objects respectively.

Solved Examples

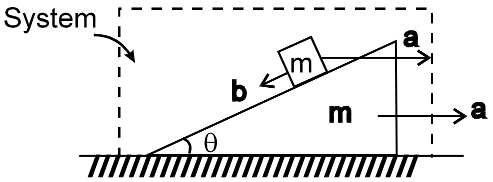
Example 26. The block of mass m slides on a wedge of mass 'm' which is free to move on the horizontal ground. Find the accelerations of wedge and block. (All surfaces are smooth).

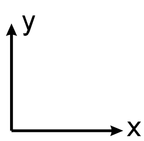


Solution : Let . a $\Rightarrow$ acceleration of wedge

b $\Rightarrow$ acceleration of block with respect to wedge

Taking block and wedge as a system and applying Newton's law in the horizontal direction



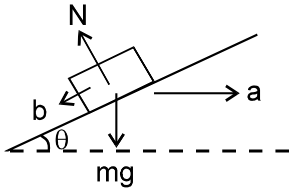


$F_x = m_1\ \vec{a}_{1x} + m_2\ \vec{a}_{2x} = 0$

$0 = ma + m(a - b\cos\theta)$ .....(1)

here 'a' and 'b' are two unknowns , so for making second equation, we draw F.B.D. of block.

F.B.D of block.



using Newton's second law along inclined plane

$mg\sin\theta = m\,(b - a\cos\theta)$ .....(2)

Now solving equations (1) and (2) we will get

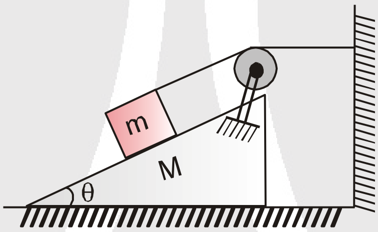
$$a = \frac{mg\sin\theta\cos\theta}{m(1+\sin^2\theta)} = \frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)} \text{ and } b = \frac{2g\sin\theta}{(1+\sin^2\theta)}$$

So in vector form :

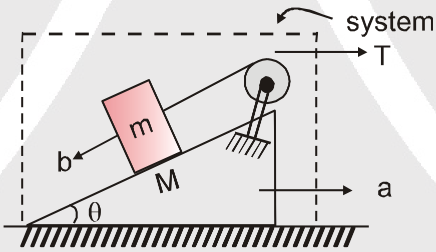
$$\vec{a}_{\text{wedge}} = a\hat{i} = \left(\frac{g\sin\theta\cos\theta}{1+\sin^2\theta}\right)\hat{i} \quad \Rightarrow \vec{a}_{\text{block}} = (a - b\cos\theta)\,\hat{i} - b\sin\theta\,\hat{j}$$

$$\vec{a}_{\text{block}} = -\frac{g\sin\theta\cos\theta}{(1+\sin^2\theta)}\,\hat{i} - \frac{2g\sin^2\theta}{(1+\sin^2\theta)}\,\hat{j}\ .$$

Example 27. For the arrangement shown in figure when the system is released, find the acceleration of wedge. Pulley and string are ideal and friction is absent.

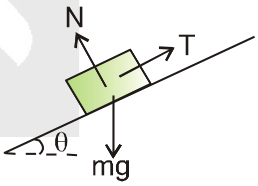


Solution : Considering block and wedge as a system and using. Newton's law for the system along x-direction



$T = Ma + m\,(a - b\cos\theta)$ .....(i)

F.B.D of m



along the inclined plane

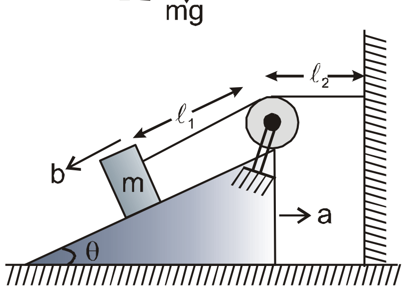
$mg\sin\theta - T = m\,(b - a\cos\theta)$ .....(ii)

using string constraint equation.

$\ell_1 + \ell_2 = \text{constant}$

$$\frac{d^2\ell_1}{dt^2} + \frac{d^2\ell_2}{dt^2} = 0$$

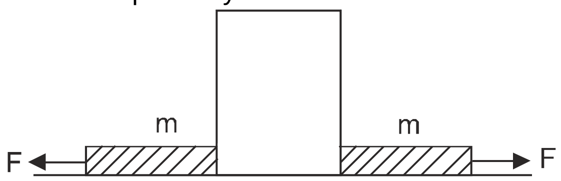
$b - a = 0$ .....(iii)



Solving above equations (i),(ii) & (iii) , we get

$$a = \frac{mg\sin\theta}{M + 2m(1-\cos\theta)}$$

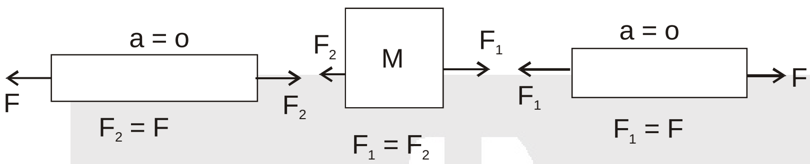
Example 28. A heavy block kept on a frictionless surface and being pulled by two ropes of equal mass m as shown in figure. At t = 0, the force on the left rope is withdrawn but the force on the right end continues to act. Let F_1 and F_2 be the magnitudes of the forces by the right rope and the left rope on the block respectively.



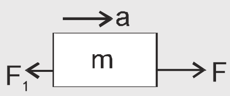
(A) $F_1 = F_2 = F$ for $t < 0$
(B) $F_1 = F_2 = F + mg$ for $t < 0$
(C) $F_1 = F$, $F_2 = F$ for $t > 0$
(D) $F_1 < F$, $F_2 = F$ for $t > 0$

Solution : For t < 0 As net force on system is zero, therefore acceleration of the system is zero

$\therefore \quad F_1 = F_2 = F \quad$ for $t < 0$

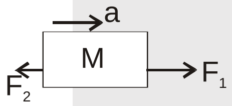


For t > 0 system is accelerated given by $a = \dfrac{F}{2m + M}$



$F - F_1 = ma$

$F_1 = F - ma$ .... (1)



$F_1 - F_2 = Ma$

$F_1 = F_2 + Ma$ .... (2)

$\boxed{F > F_1 > F_2}$

Ans. **(A)**

10. NEWTON'S LAW FOR NON INERTIAL FRAME :

$\vec{F}_{Real} + \vec{F}_{Pseudo} = m\vec{a}$

Net sum of real and pseudo force is taken in the resultant force.

$\vec{a}$ = Acceleration of the particle in the non inertial frame

$\vec{F}_{Pseudo} = -m\,\vec{a}_{Frame}$

Pseudo force is always directed opposite to the direction of the acceleration of the frame.

Pseudo force is an imaginary force and there is no action-reaction for it. So it has nothing to do with Newton's Third Law.

10.1 Reference Frame:

A frame of reference is basically a coordinate system in which motion of object is analyzed. There are two types of reference frames.

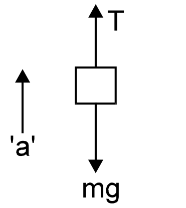
(a) **Inertial reference frame:** Frame of reference either stationary or moving with constant velocity.

(b) **Non-inertial reference frame:** A frame of reference moving with non-zero acceleration.

Solved Examples

Example 29. A lift having a simple pendulum attached with its ceiling is moving upward with constant acceleration 'a'. What will be the tension in the string of pendulum with respect to a boy inside the lift and a boy standing on earth, mass of bob of simple pendulum is m.

Solution : **F.B.D** . of bob (with respect to ground)



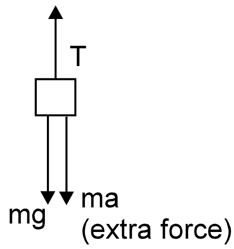
$T - mg = ma$

$T = mg + ma$ .....(i)

With respect to boy inside the lift, the acceleration of bob is zero. So he will write above equation in this manner.

$T - mg = m. (0)$ $\therefore$ $T = mg$

He will tell the value of tension in string is mg. But this is 'wrong'. To correct his result, he makes a free body diagram in this manner, and uses Newton's second law.



$T = mg + ma$ .....(ii)

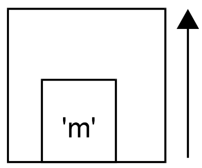
By using this **extra force**, equations (i) and (ii) give the same result. This **extra force** is called **pseudo force**. This **pseudo force** is used when a problem is solved with a accelerating frame (Non-inertial)

Note : Magnitude of Pseudo force = mass of system × acceleration of frame of reference.

Direction of force:

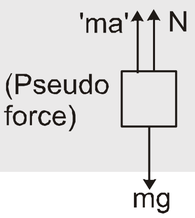
Opposite to the direction of acceleration of frame of reference, (not in the direction of motion of frame of reference)

Example 30. A box is moving upward with retardation 'a' <g, find the direction and magnitude of "pseudo force" acting on block of mass 'm' placed inside the box. Also calculate normal force exerted by surface on block



Solution : **Pseudo force** acts opposite to the direction of acceleration of reference frame.

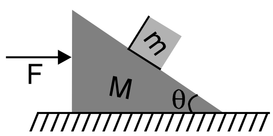
pseudo force = ma in upward direction F.B.D of 'm' w.r.t. box (non–inertial)



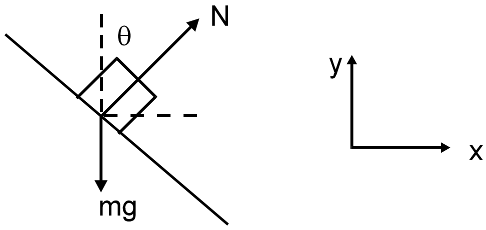
$N + ma = mg$

$N = mg - ma$

Example 31. All surfaces are smooth in the adjoining figure. Find F such that block remains stationary with respect to wedge.



Solution : Acceleration of (block + wedge) is $a = \dfrac{F}{(M+m)}$



Let us solve the problem by using both frames.

From inertial frame of reference (Ground)

F.B.D. of block w.r.t. ground (Apply real forces) : with respect to ground block is moving with an acceleration 'a' .

$\therefore\ \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....(i)

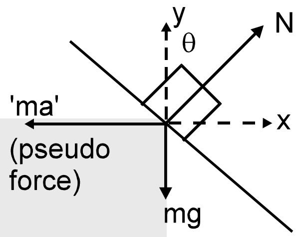
and $\Sigma F_x = ma \Rightarrow N\sin\theta = ma$ .....(ii)

From Eqs. (i) and (ii)

$a = g\tan\theta$

$\therefore\ F = (M + m)\,a \quad = (M + m)\,g\tan\theta$

From non–inertial frame of reference (Wedge) :



F.B.D. of block w.r.t. wedge (real forces + pseudo force)

w.r.t. wedge, block is stationary

$\therefore \Sigma F_y = 0 \Rightarrow N\cos\theta = mg$ ....(iii)

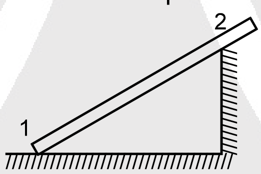
$\Sigma F_x = 0 \Rightarrow N\sin\theta = ma$ ....(iv)

From Eqs. (iii) and (iv), we will get the same result

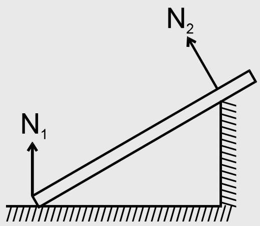
i.e. $F = (M + m)\,g\tan\theta$

Solved Miscellaneous Problems

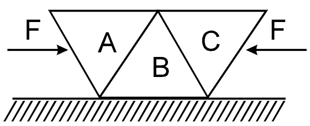
Problem 1. Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



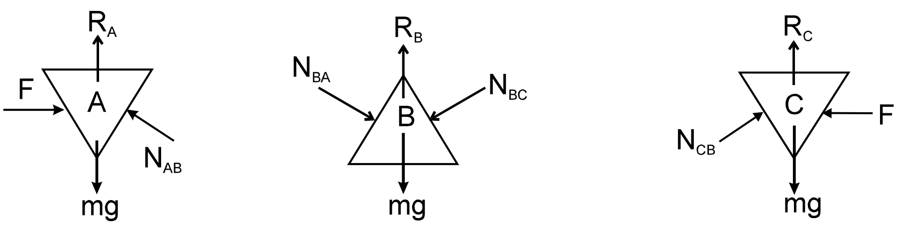
Solution : Normal force acts perpendicular to extended surface at point of contact..



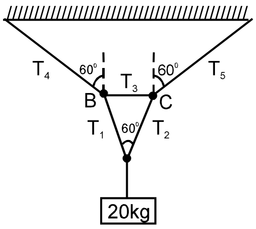
Problem 2. Three triangular blocks A, B and C of equal masses 'm' are arranged as shown in figure. Draw F.B.D. of blocks A,B and C. Indicate action–reaction pair between A,B and B,C.



Solution :

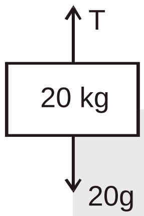


Problem 3. The system shown in figure is in equilibrium, find the tension in each string ; T_1, T_2, T_3, T_4 and T_5.

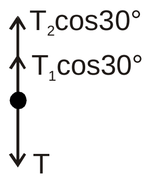


Answer : $T_1 = T_2 = \frac{200}{\sqrt{3}}$ N, $T_4 = T_5 = 200$ N, $T_3 = \frac{200}{\sqrt{3}}$ N.

Solution : **<u>FBD of 20 kg block</u>** $\rightarrow$



So, $T = 20 \times g = 200$ N .....(1)



<u>From figure</u> $\rightarrow$

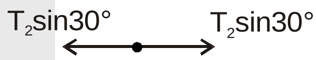
$T = T_1\cos30° + T_2\cos30°$ .....(2)

$T_1\sin30° = T_2\sin30°$

$T_1 = T_2$ .....(3)

so from equation (3) $T = 2T_1\cos30°$

$$\mathbf{T_1 = \frac{200}{\sqrt{3}} = T_2} \quad .....(4)$$



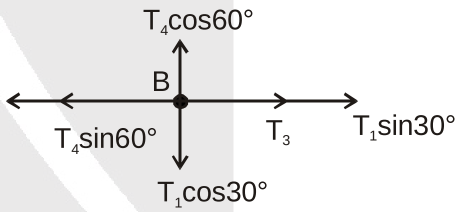
<u>From figure FBD of point B</u>

In vertical direction

So, $T_4\cos60° = T_1\cos30°$

$$T_4 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 200 \text{ N}$$

So, $\mathbf{T_4 = 200\ N}$



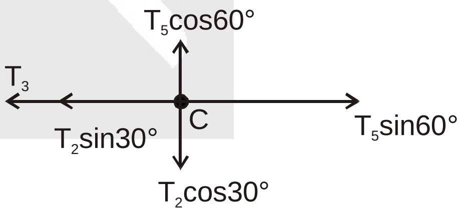
<u>FBD of point C</u>

Equating forces In vertical direction -

$T_5\cos60° = T_2\cos30°$

$$T_5 \times \frac{1}{2} = \frac{200}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$$

$\mathbf{T_5 = 200\ N}$



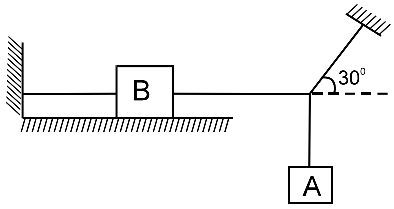
<u>For T_3</u> $\rightarrow$

Equating forces in horizontal direction -

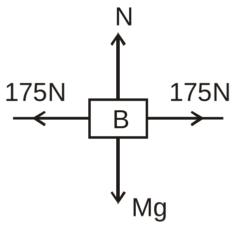
$T_3 + T_2\sin30° = T_5\sin60°$

$$T_3 = 200 \times \frac{\sqrt{3}}{2} - \frac{200}{\sqrt{3}} \times \frac{1}{2} \Rightarrow \mathbf{T_3 = \frac{200}{\sqrt{3}}\,N}$$

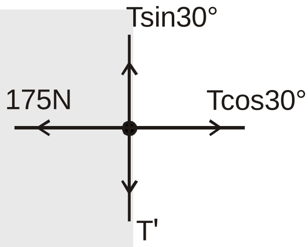
Problem 4. The breaking strength of the string connecting wall and block B is 175 N, find the magnitude of weight of block A for which the system will be stationary. The block B weighs 700 N. ($g = 10\ m/s^2$)



Solution : **<u>FBD of block B</u>** $\rightarrow$



<u>FBD of point in figure</u> $\rightarrow$



Equating forces in horizontal direction $\rightarrow$

$T\cos 30° = 175$

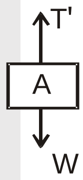
$T = \dfrac{175 \times 2}{\sqrt{3}}$ N

In vertical direction $\rightarrow$

$T\sin 30° = T'$

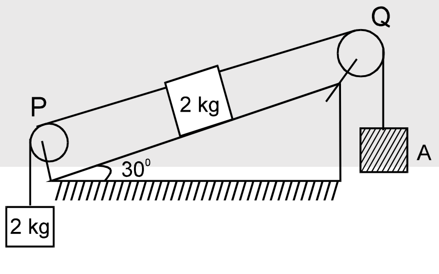
So, $T' = \dfrac{175 \times 2}{\sqrt{3}} \times \dfrac{1}{2} = \dfrac{175}{\sqrt{3}}$ N

<u>FBD of block A</u> $\rightarrow$



So, $\mathbf{T' = W = \dfrac{175}{\sqrt{3}}\ N}$

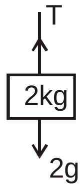
Problem 5. In the arrangement shown in figure, what should be the mass of block A so that the system remains at rest. Also find force exerted by string on the pulley Q. ($g = 10\ m/s^2$)



Answer : m = 3 kg, $30\sqrt{3}$ N.

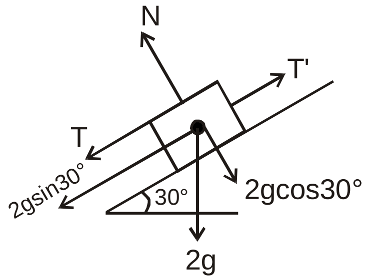
Solution : From figure

<u>FBD of 2 kg block hanging vertically</u> $\rightarrow$



T = 20 N .... (1)

FBD of 2kg block on incline plane

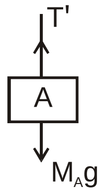


Along the plane →

T + 2gsin30° = T′

$T' = 20 + 20\times\frac{1}{2} = 30$ N

FBD of block A

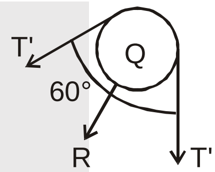


So $T' = M_A g$

$M_A = \frac{T'}{g} = \frac{30}{10} = 3$ kg

$M_A = 3$ kg

FBD of pulley Q



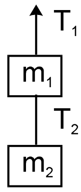
So, $R = 2T'\cos\frac{\theta}{2}$

$R = 2\times 30\cos 30°$

$R = 2\times 30\times\frac{\sqrt{3}}{2}$

$\mathbf{R = 30\sqrt{3}\ N}$

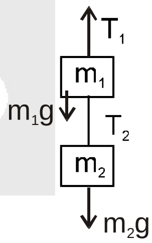
Problem 6. Two blocks with masses $m_1 = 0.2$ kg and $m_2 = 0.3$ kg hang one under other as shown in figure. Find the tensions in the strings (massless) in the following situations ($g = 10\ m/s^2$)



(a) the blocks are at rest
(b) they move upward at 5 m/s
(c) they accelerate upward at 2 m/s^2
(d) they accelerate downward at 2 m/s^2
(e) if maximum allowable tension is 10 N. What is maximum possible upward acceleration ?

Answer : (a) 5 N, 3 N (b) 5 N, 3 N (c) 6 N, 3.6 N (d) 4 N, 2.4 N (e) 10 m/s^2

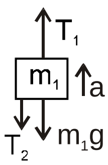
Solution : (a) At rest a = 0

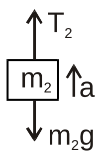


$T_2 = m_2g = 0.3 \times 10 = 3N$
$T_1 = m_1g + T_2$
$T_1 = 0.2 \times 10 + 3 = 5$ N

(b) same as above
$a = 0,\ T_2 = 3N,\ T_1 = 5N$

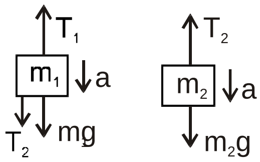
(c) $a = 2m/s^2$ ↑ (upward)





$T_2 - m_2g = m_2a$
$\Rightarrow\ T_2 - m_2g = m_2a$
$\Rightarrow\ T_2 - 0.3 \times 10 = 0.3 \times 2$
$\Rightarrow\ T_2 = 0.6 + 3 = 3.6$ N
$T_1 - m_1g - T_2 = m_1a$
$\Rightarrow\ T_1 - 0.2\times 10 - 3.6 = 0.2 \times 2$
$\Rightarrow\ T_1 = 0.4 + 5.6 = 6N$

(d) $a = 2\ m/s^2$ (downward)
$m_2g - T_2 = m_2a$
$\Rightarrow\ 0.3 \times 10 - T_2 = 0.3 \times 2$
$\Rightarrow\ T_2 = 3 - 0.6 = 2.4$ N



$T_2 + m_1g - T_1 = m_1a$

$\Rightarrow\ 2.4 + 2 - T_1 = 0.2 \times 2$

$\Rightarrow\ T_1 = 4.4 - 0.4 =$ **4 N Ans.**

(e) Chance of breaking is of upper string means

$T_1 < 10$ N

For m_1 –

$T_1 - m_1g - T_2 = m_1a$

$10 - 2 - T_2 = 0.2\ a$ .....(1)

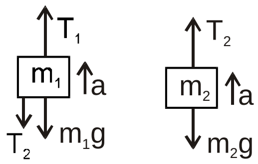
For m_2 –

$T_2 - m_2g = m_2a$

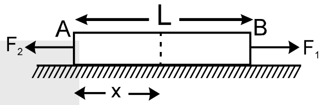
$\Rightarrow\ T_2 - 3 = 0.3\ a$ .....(2)

Adding equation (1) and (2)

$8 - 3 = 0.5\ a \quad \Rightarrow \quad a = \dfrac{5}{0.5} = \mathbf{10\ m/s^2}$

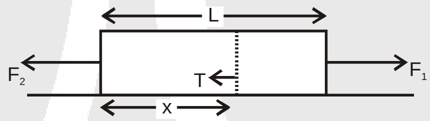


Problem 7. Two forces F_1 and F_2 ($F_2 > F_1$) are applied at the free ends of uniform rod kept on a horizontal frictionless surface. Find tension in rod at a distance x from end 'A',

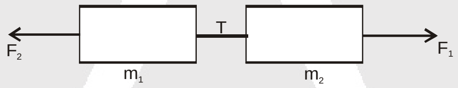


Answer : $T = F_2 - \dfrac{(F_2 - F_1)}{L}.x$

Solution : $a = \dfrac{F_2 - F_1}{m}$



$T - F_1 = m_2a$



$\Rightarrow\ T - F_1 = \dfrac{m}{L}(L - x)\dfrac{F_2 - F_1}{m} \qquad (m_2 = \dfrac{m}{L}(L - x)\ )$

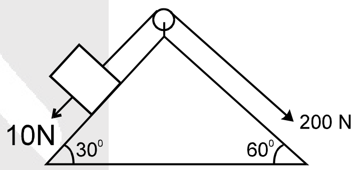
$\Rightarrow\ T = F_1 + \left(1 - \dfrac{x}{L}\right)(F_2 - F_1) = F_1 + F_2 - F_1 - \dfrac{x}{L}(F_2 - F_1) = F_2 - \dfrac{x}{L}(F_2 - F_1)$

Problem 8. A 10 kg block kept on an inclined plane is pulled by a string applying 200 N force. A 10 N force is also applied on 10 kg block as shown in figure.

Find : (a) tension in the string.

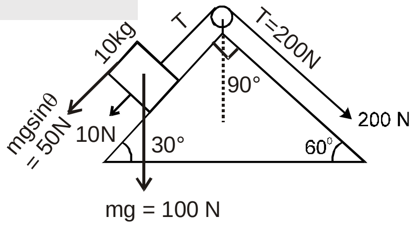
(b) acceleration of 10 kg block.

(c) net force on pulley exerted by string



Answer : (a) 200 N, (b) 14 m/s^2, (c) $200\sqrt{2}$ N

Solution : (a) T = **200 N**

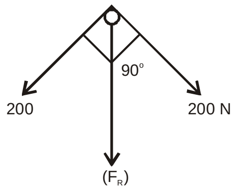


(b) $T - 10 - mg\sin\theta = ma$

$\Rightarrow\ T - 10 - 50 = 10a$

$\Rightarrow\ 200 - 60 = 10a$

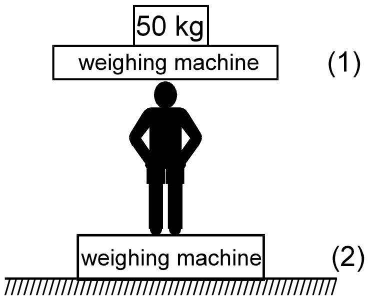
$\Rightarrow\ a = \dfrac{140}{10} = \mathbf{14\ m/s^2}$



(c) $(F_R) = \sqrt{(200)^2 + (200)^2}$

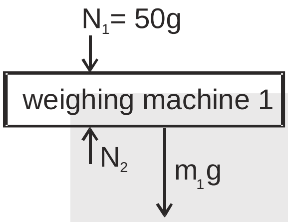
$= \mathbf{200\sqrt{2}\ N}$ **Ans.**

Problem 9. A man of mass 60 kg is standing on a weighing machine (2) of mass 5kg placed on ground. Another similar weighing machine is placed over man's head. A block of mass 50kg is put on the weighing machine (1). Calculate the readings of weighing machines (1) and (2) ($g = 10\ m/s^2$)



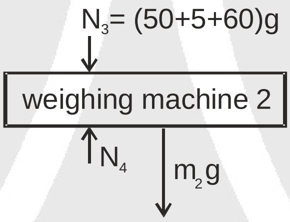
Answer : **500 N, 1150 N**

Solution :



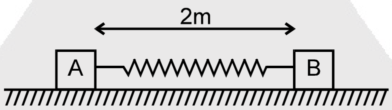
$R_1 = N_1 = 50 \times g = 500$ N

where R_1 = reading in weighing machine 1



$R_2 = N_3 = (50 + 5 + 60)\ g = 115 \times 10 = 1150$ N where R_2 = reading in weighing machine 2

Problem 10. Two blocks are connected by a spring of natural length 2 m.



The force constant of spring is 200 N/m. Find spring force in following situations :

(a) A is kept at rest and B is displaced by 1 m in right direction.

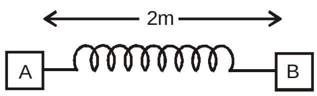
(b) B is kept at rest and A is displaced by 1m in left direction.

(c) A is displaced by 0.75 m in right direction, and B is 0.25 m in left direction.

Answer : (a) F = 200 N, (b) 200 N, (c) 200 N

Solution : (a) Extension in spring = 1 m.

$\therefore$ $F_{spring} = Kx = 200 \times 1 = 200$ N

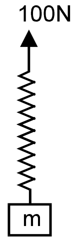


(b) Same extension same spring force in both directions $F_{spring} = 200$ N.

(c) Both displacements of A of B are compressing the spring total compressing = 0.75 + 0.25 = 1 m.

$\therefore$ $F_{spring} = kx = 200 \times 1 = 200$ N.

Problem 11. If force constant of spring is 50 N/m. Find mass of the block, if it is at rest in the given situation. ($g = 10\ m/s^2$)

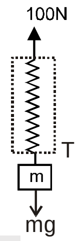


Answer : m = 10 kg

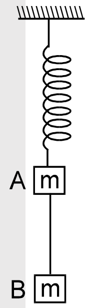
Solution : T = 100 N

$\Rightarrow$ mg = 100 N

$m = \frac{100}{g} = 10$ kg.

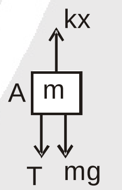


Problem 12. Two blocks 'A' and 'B' of same mass 'm' attached with a light string are suspended by a spring as shown in figure Find the acceleration of block 'A' and 'B' just after the string is cut..



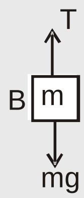
Answer : g (upwards), g (downwards)

Solution : **<u>When string is not cut</u> :**



<u>FBD of 'A' block</u>

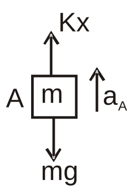
$kx = mg + T$ ......(i)



<u>F. B.D of 'B' block</u>

$T = mg$ ......(ii)

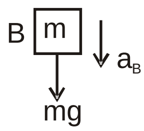
<u>When string is cut :</u>



<u>FBD of 'A' block</u>

$kx - mg = ma_A$

$2mg - mg = ma_A = a_A = g$ (upwards)

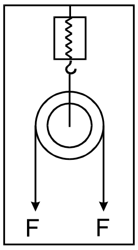


<u>FBD of 'B' block</u>

$ma_B = mg$

$a_B = g$ (downwards)

Problem 13. Find the reading of spring balance in the adjoining figure, pulley and strings are ideal.

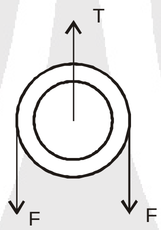


Answer : 2F

Solution : **FBD of spring balance**

R = T .....(i)

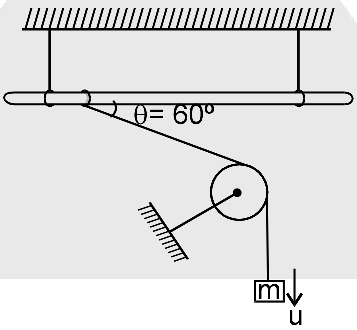
FBD of pulley



T = 2 F .....(ii)

R = 2F

Problem 14. The figure shows mass m moves with velocity u. Find the velocity of ring at that moment. Ring is restricted to move on smooth rod.



Answer : $V_R = \frac{u}{\cos\theta}$, $V_R = 2u$

Solution : Velocity along string remains same .

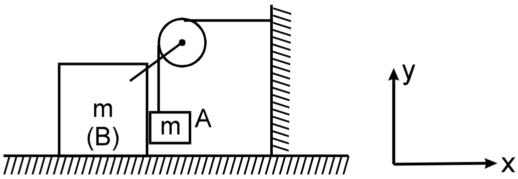
$V_R \cos\theta = u$

$V_R = \frac{u}{\cos\theta}$

$\theta = 60^o$

$V_R = 2u$

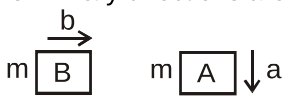
Problem 15. In the system shown in figure, the block A is released from rest. Find :



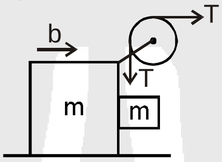
(i) The acceleration of both blocks 'A' and 'B'.
(ii) Tension in the string.
(iii) Contact force between 'A' and 'B'.

Answer : (i) $\frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$, $\frac{g}{3}\hat{i}$ (ii) $\frac{2mg}{3}$ (iii) $\frac{mg}{3}$

Solution : (i) Let acceleration of blocks in x & y directions are

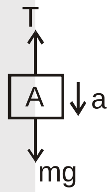


<u>Taking both blocks as a system</u>



$T = 2mb$ .....(i)

<u>Taking A block :</u>



$mg - T = ma$ .....(ii)

From equations (i) & (ii) ; $ma + 2\,mb = mg$

$a + 2\,b = g$ .....(iii)

From string constraint ;

$a = b$ .....(iv)

From equations (iii) & (iv);

$a = b = \frac{g}{3}$

Hence, acceleration of block A

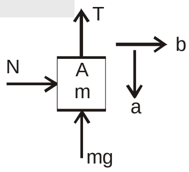
$a_A = b\hat{i} - a\hat{j}$ $\qquad a_A = \frac{g}{3}\hat{i} - \frac{g}{3}\hat{j}$

Acceleration of block B $a_B = b\hat{i} = \frac{g}{3}\hat{i}$

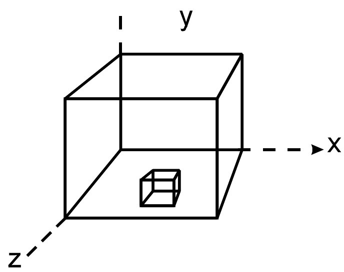
(ii) $T = 2mb = \frac{2mg}{3}$

(iii) For contact force between 'A' and 'B'

<u>FBD of block 'A'</u> : $N = mb$ $\qquad N = \frac{mg}{3}$



Problem 16. A block of mass 2 kg is kept at rest on a big box moving with velocity $2\hat{i}$ and having acceleration $-3\hat{i} + 4\hat{j}$ m/s^2. Find the value of 'Pseudo force' acting on block with respect to box



Answer : $\vec{F}_{pseudo} = -m\vec{a}_{frame} = -2(-3\hat{i} + 4\hat{j})$

$F = 6\hat{i} - 8\hat{j}$.