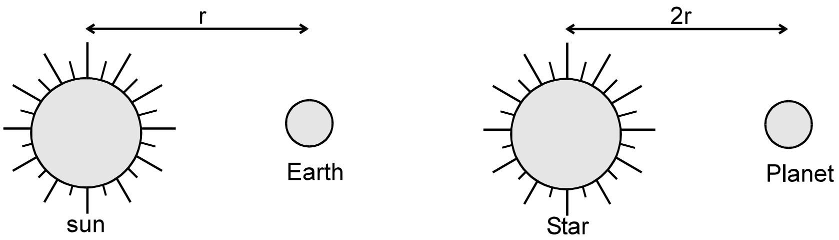
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. Let a star be much brighter than our sun but its mass is same as that of sun. If our earth has average life span of a man as 70 years. In the earth like planet of this star system having double the distance from our star find the average life span of a man on this planet in terms of year there. (Assuming same average life).



2. Consider a spacecraft in an elliptical orbit around the earth. At the lowest point or perigee, of its orbit it is 300 km above the earth's surface at the highest point or apogee, it is 3000 km above the earth's surface.
(a) What is the period of the spacecraft's orbit ?
(b) Find the ratio of the spacecraft's speed at perigee to its speed at apogee.
(c) Find the speed at perigee and the speed at apogee.
(d) It is derised to have the spacecraft escape from the earth completely. If the spacecraft's rockets are fired at perigee, by how much would the speed have to be increased to achieve this? What if the rockets were fired at apogee ? Which point in the orbit is the most efficient to use?

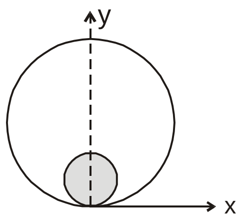
3. A planet A moves along an elliptical orbit around the Sun. At the moment when it was at the distance r_0 from the Sun its velocity was equal to v_0 and the angle between the radius vector r_0 and the velocity vector v_0 was equal to α. Find the maximum and minimum distance that will separate this planet from the Sun during its orbital motion. (Mass of Sun = M_S)

4. A satellite is put into a circular orbit with the intention that it hover over a certain spot on the earth's surface. However, the satellite's orbital radius is erroneously made 1.0 km too large for this to happen. At what rate and in what direction does the point directly below the satellite move across the earth's surface?
R = Radius of earth = 6400 km
r = radius of orbit of geostationary satellite = 42000 km
T = Time period of earth about its axis = 24 hr.

5. What are : (a) the speed and (b) the period of a 220 kg satellite in an approximately circular orbit 640 km above the surface of the earth ? Suppose the satellite loses mechanical energy at the average rate of 1.4×10^5 J per orbital revolution. Adopting the reasonable approximation that due to atmospheric resistance force, the trajectory is a "circle of slowly diminishing radius". Determine the satellites
(c) altitude (d) speed & (e) period at the end of its 1500^{th} revolution. (f) Is angular momentum around the earth's centre conserved for the satellite or the satellite-earth system?

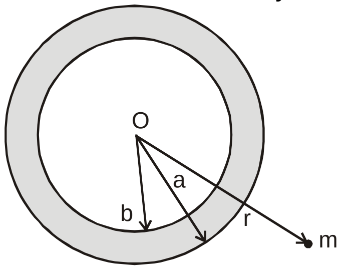
6. A planet of mass m moves along an ellipse around the Sun so that its maximum and minimum distance from the Sun are equal to r_1 and r_2 respectively. Find the angular momentum J of this planet relative to the centre of the Sun. (Mass of Sun = M_S)

7. A solid sphere of mass m and radius r is placed inside a hollow spherical shell of mass 4 m and radius 4r find gravitational field intensity at :



(a) $r < y < 2r$ (b) $2r < y < 8r$ (c) $y > 8r$
here y coordinate is measured from the point of contact of the sphere and the shell.

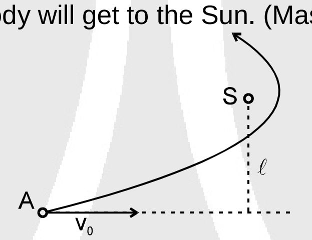
8. A sphere of density ρ and radius a has a concentric cavity of radius b, as shown in the figure.



(a) Sketch the gravitational force F exerted by the sphere on the particle of mass m, located at a distance r from the centre of the sphere as a function of r in the range $0 \le r \le \infty$.
(b) Sketch the corresponding curve for the potential energy u (r) of the system.

9. (a) What is the escape speed for an object in the same orbit as that of Earth around sun (Take orbital radius R) but far from the earth ? (Mass of the sun = M_s)
(b) If an object already has a speed equal to the earth's orbital speed, what minimum additional speed must it be given to escape as in (a) ?

10. A cosmic body A moves towards the Sun with velocity v_0 (when far from the Sun) and aiming parameter ℓ, the direction of the vector v_0 relative to the centre of the Sun as shown in the figure. Find the minimum distance by which this body will get to the Sun. (Mass of Sun = M_S)

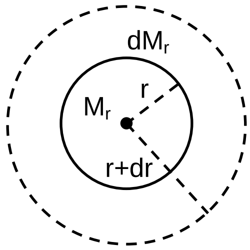


11. Two stars of mass M_1 & M_2 are in circular orbits around their centre of mass. The star of mass M_1 has an orbit of radius R_1, the star of mass M_2 has an orbit of radius R_2. (Assume that their centre of mass is not accelerating and distance between stars is fixed)
(a) Show that the ratio of the orbital radii of the two stars equals the reciprocal of the ratio of their masses, that is $R_1/R_2 = M_2/M_1$.
(b) Explain why the two stars have the same orbital period and show that the period,

$$T = 2\pi \frac{(R_1 + R_2)^{3/2}}{\sqrt{G(M_1 + M_2)}}.$$

12. **Linked questions (12-16)**
A star can be considered as a spherical ball of hot gas of radius R. Inside the star, the density of the gas is ρ_r at a radius r and mass of the gas within this region is M_r. The correct differential equation for variation of mass with respect to radius $\left(\frac{dM_r}{dr}\right)$ is (refer to the adjacent figure)

[OLYMPIAD 2016_STAGE-1_(ASTRONOMY)]



13. A star in its prime age is said to be under equilibrium due to gravitational pull and outward radiation pressure (p). Consider the shell of thickness dr as in the figure of question (12). If the pressure on this shell is dp then the correct equation is (G is universal gravitational constant)

[OLYMPIAD 2016_STAGE-1_(ASTRONOMY)]

14. In astronomy order of magnitude estimation plays an important role. The derivative $\frac{dp}{dr}$ can be taken difference ratio $\frac{\Delta P}{\Delta r}$. Consider the star has a radius R, pressure at its centre is P_C and pressure at outer layer is zero if the average mass is $\frac{M_0}{2}$ and average radius $\frac{R_0}{2}$ then the expression for P_C is

[OLYMPIAD 2016_STAGE-1_(ASTRONOMY)]

15. The value of mass and radius of sun are given by $M_0 = 2 \times 10^{30}$ kg and $R_0 = 7 \times 10^5$ km respectively. The pressure at the centre is about ($G = 6.67 \times 10^{-11}$ m^3. kg^{-1}. s^{-2})

[OLYMPIAD-2016_STAGE-1_(ASTRONOMY)]

16. Assuming that the gas inside the sun behaves very much like the perfect gas, the temperature at the centre of the sun is nearly (the number density of gas particles $= \frac{2\rho}{M_H}$), Boltzmann constant $k_B = 1.4 \times 10^{-23}$ J.K^{-1} and mass of proton $M_H = 1.67 \times 10^{-27}$ kg)

[OLYMPIAD-2016_STAGE-1_(ASTRONOMY)]

HLP Answers

1. $\frac{70}{(2)^{3/2}} \approx 25$ years.

2. (a) $T = \frac{4\pi}{R\sqrt{g}} a^{3/2} = 7.16 \times 10^3$ sec. (b) $\frac{v_1}{v_2} = \frac{94}{67} = 1.4$

(c) $V_p = 896 \times 10^2 \sqrt{\frac{94}{67 \times 161}}$ m/sec. $= 8.35 \times 10^3$ m/s,

$V_a = 896 \times 10^2 \sqrt{\frac{67}{94 \times 161}}$ m/sec $= 5.95 \times 10^3$ m/s

(d) $\Delta V = 14 \times 10^2 \sqrt{67} - V_P = 3.09 \times 10^3$ m/s, perigee

3. $r_m = \frac{r_0}{2-\eta} [1 \pm \sqrt{1-(2-\eta)\eta \sin^2\alpha}]$, where $\eta = r_0 v_0^2/GM_S$.

4. $V_{rel} = \frac{3\Delta r R \pi}{rT} = \frac{\pi}{189}$ m/sec ≈ 1.66 cm/sec., to the east along equator

5. (a) $\frac{448}{\sqrt{3520}}$ km/s = 7.527 km/s (b) $\frac{220\pi}{7}\sqrt{3520}$ sec. ≈ 1.63 hour

(c) $\left[\frac{22 \times 14 \times 64^2 \times 7040}{22 \times 14 \times 64^2 + 7040 \times 6} - 6400\right]$ km ≈ 411.92 km (d) $\frac{448}{\sqrt{3406}}$ km/sec. ≈ 7.67 km/s

(e) $\frac{1703\pi}{56}\sqrt{3406}$ sec. ≈ 1.55 hour (f) No

6. $J = m\sqrt{2GM_S r_1 r_2/(r_1+r_2)}$

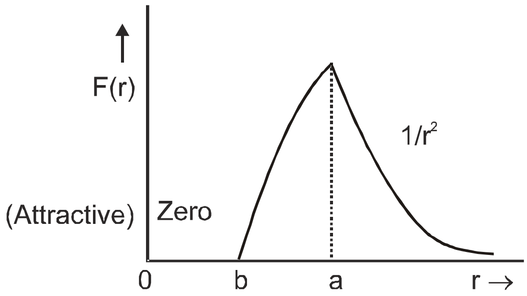
7. (a) $\left(\frac{Gm\ \ (y-r)}{r^3}(-\hat{j}\ )\right)$ (b) $\left(\frac{Gm}{(y-r)^2}(-\hat{j}\ )\right)$ (c) $\left(\frac{4Gm}{(y-4r)^2} + \frac{Gm}{(y-r)^2}\right)(-\hat{j})$

8. (a) Force will be due to the mass of the sphere upto the radius r

In case (i) $0 < r < b$; Mass M = 0, therefore F(r) = 0

In case (ii) $b < r < a$; Mass $M = \frac{4}{3}\pi\rho\ (r^3 - b^3)$, therefore $F(r) = \frac{4}{3}\pi G\rho m\left(r - \frac{b^3}{r^2}\right)$

(iii) $a < r < \infty$; Mass $\frac{4}{3}\pi\rho\ M = (a^3 - b^3)$, therefore $F(r) = \frac{4}{3}\pi G\rho m\left(\frac{a^3 - b^3}{r^2}\right)$

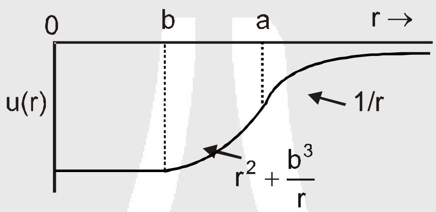


(b) $U_f - U_i = -\int_{r_1}^{r_2} \vec{F}_c . \vec{dr}$

(i) $0 < r < b$; $u(r) = -2\pi G\rho m(a^2 - b^2)$

(ii) $b < r < a$; $u(r) = \dfrac{-2\pi G\rho m}{3r}(3ra^2 - 2b^3 - r^3)$

(iii) $a < r < \infty$; $u(r) = \dfrac{-4\pi G\rho m}{3r}(a^3 - b^3)$



9. (a) $\sqrt{\dfrac{2GM_s}{R}}$ (b) $\left(\sqrt{2}-1\right)\sqrt{\dfrac{GM_s}{R}}$

10. $r_{min} = (GM_S / v_0^2)\ [\sqrt{1+(\ell v_0^2/GM_S)^2} - 1]$

11. $M_\alpha = \dfrac{4\pi^2[1.5\times10^{12}]^3}{3G[44.5\times365\times86400]^2} = 3.376 \times 10^{29}$ kg, $M_\beta = 2M_\alpha = 6.75 \times 10^{29}$ kg

12. $\dfrac{dM_r}{dr} = \rho_r 4\pi r^2$ **13.** $\dfrac{dp}{dr} = -\dfrac{GM_r}{r^2}\rho_r$ **14.** $P_C = \dfrac{GM_0^2}{R_0^4} \times \left(\dfrac{3}{2\pi}\right)$

15. 5×10^{14} N/m^2 **16.** 2.10×10^7