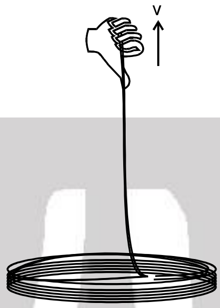
High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

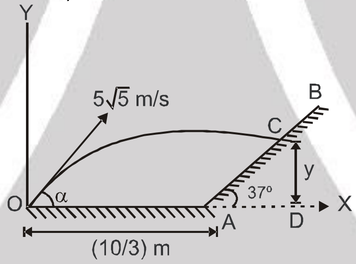
SUBJECTIVE QUESTIONS

1. A uniform rope of linear mass density λ and length ℓ is coiled on a smooth horizontal surface. One end is pulled up with constant velocity v. Then find average power applied by the external agent in pulling the entire rope just off the ground?



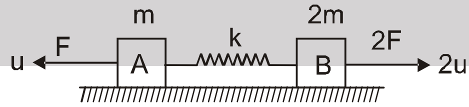
2. In the above question the maximum power delivered by the agent in pulling up the rope is

3. A particle is projected from point O on the ground with velocity $u = 5\sqrt{5}$ m/s at angle $\alpha = \tan^{-1}(0.5)$. It strikes at a point C on a fixed smooth plane AB having inclination of 37º with horizontal as shown in figure. If the particle does not rebound, calculate.



(a) coordinates of point C in reference to coordinate system as shown in the figure.
(b) maximum height from the ground to which the particle rises. ($g = 10$ m/s^2)

4. Two blocks A & B of mass 'm' & 2m respectively are joined to the ends of an undeformed massless spring of spring constant ' k '. They can move on a horizontal smooth surface. Initially A & B have velocities ' u ' towards left and '2 u' towards right respectively. Constant forces of magnitudes F and 2 F are always acting on A and B respectively in the directions shown. Find the maximum extension in the spring during the motion.



5. Two identical buggies 1 and 2 with one man in each move without friction due to inertia along the parallel rails toward each other. When the buggies got opposite to each other, the men exchange their places by jumping in the direction perpendicular to the direction of motion. As a consequence, buggy 1 stops and buggy 2 keeps moving in the same direction, with its velocity becoming equal to v. Find the initial velocities of the buggies v_1 and v_2 if the mass of each buggy (without a man) equals M and the mass of each man is m.

6. Two identical buggies move one after the other due to inertia (without friction) with the same velocity v_0. A man of mass m rides the rear buggy. At a certain moment the man jumps into the front buggy with a velocity u relative to his buggy. Knowing that the mass of each buggy is equal to M, find the velocities with which the buggies will move after that.

7. Two men, each of mass m, stand on the edge of a stationary buggy of mass M. Assuming the friction to be negligible, find the velocity of the buggy after both men jump off with the same horizontal velocity u relative to the buggy: (i) simultaneously; (ii) one after the other. In what case will the velocity of the buggy be greater and how many times?

8. A stationary pulley carries a rope whose one end supports a ladder with a man and the other end the counterweight of mass M. The man of mass m climbs up a distance ℓ' with respect to the ladder and then stops. Neglecting the mass of the rope and the friction in the pulley axle, find the displacement ℓ of the centre of inertia of this system.

9. A particle of mass m_1 experienced a perfectly elastic collision with a stationary particle of mass m_2. What fraction of the kinetic energy does the striking particle lose, if
(a) it recoils at right angles to its original motion direction;
(b) the collision is a head-on one?

10. Body 1 experiences a perfectly elastic collision with a stationary Body 2. Determine their mass ratio, if
(a) after a head-on collision the particles fly apart in the opposite directions with equal, velocities;
(b) the particles fly apart symmetrically relative to the initial motion direction of particle 1 with the angle of divergence $\theta = 60°$.

11. A ball moving transitionally collides with another stationary ball of the same mass. At the moment of impact the angle between the straight line passing through the centres of the balls and the direction of the initial motion of the striking ball is equal to $\alpha = 45°$. Assuming the balls to be smooth, find the fraction η of the kinetic energy of the striking ball that turned into potential energy at the moment of the maximum deformation.

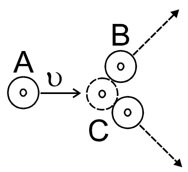
12. Particle 1 moving with velocity v = 10 m/s experienced a head-on collision with a stationary particle 2 of the same mass. As a result of the collision, the. kinetic energy of the system decreased by $\eta = 1.0\%$. Find the magnitude and direction of the velocity of particle 1 after the collision.

13. A particle of mass m having collided with a stationary particle of mass M deviated by an angle $\pi/2$ whereas the particle M start moving at an angle $\theta = 30°$ to the direction of the initial motion of the particle m. How much (in percent) and in what way has the kinetic energy of this system changed after the collision, if M / m = 5.0?

14. A closed system consists of two particles of masses m_1 and m_2 which move at right angles to each other with velocities v_1 and v_2 Find:
(a) the momentum of each particle and
(b) the total kinetic energy of the two particles in the reference frame fixed to their centre of inertia.

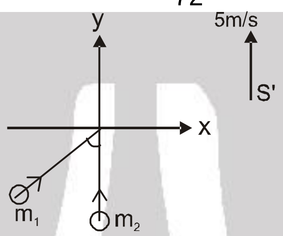
15. A particle of mass m_1 collides elastically with a stationary particle of mass m_2 ($m_1 > m_2$). Find the maximum angle through which the striking particle may deviate as a result of the collision.

16. Three identical discs A, B, and C as shown in figure rest on a smooth horizontal plane. The disc A is set in motion with velocity v after which it experiences an elastic collision simultaneously with discs B and C. The distance between the centres of the latter discs prior to the collision is η times the diameter of each disc Find the velocity of the disc A after the collision. At what value of η will the disc A recoil after the collision; stop; move on?



17. A spaceship of mass m_0 moves in the absence of external ' forces with a constant velocity v_0. To change the motion direction a jet engine is switched on. It starts ejecting a gas jet with velocity u, which is constant relative to the spaceship and directed at right angles to the spaceship motion. The engine is shut down when the mass of the spaceship decreases to m. Through what angle α did the motion direction of the spaceship deviate due to the jet engine operation?

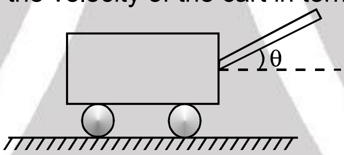
18. Two smooth spheres of the same radius, but which have different masses m_1 & m_2 collide inelastically. Their velocities before collision are 13 m/s & 5 m/s respectively along the directions shown in the figure in which $\cot\theta = \frac{5}{12}$. An observer S′ moving parallel to the positive y - axis with a constant speed of 5m/s observes this collision. He finds the final velocity of m_1 to be 5m/s along the y - direction and the total loss in the kinetic energy of the system to be $\frac{1}{72}$ of its initial value. Determine;



(a) The ratio of the masses

(b) The velocity of m_2 after collision with (respect to observer)

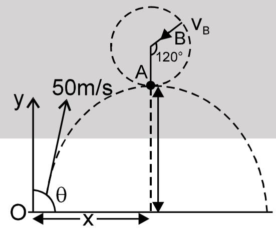
19. A cart of total mass M_0 is at rest on a rough horizontal road. It ejects bullets at rate of λ kg/s at an angle θ with the horizontal and at velocity 'u' (constant) relative to the cart. The coefficient of friction between the cart and the ground is μ. Find the velocity of the cart in terms of time 't'. The cart moves with sliding.



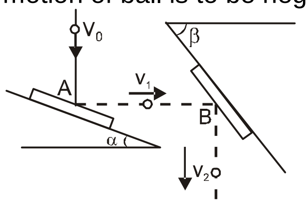
20. A bullet of mass M is fired with a velocity 50 m/s at an angle with the horizontal. At the highest point of its trajectory, it collides head on with a bob of mass 3M suspended by a massless rod of length $\frac{10}{3}$m and gets embedded in the bob. After the collision the rod moves through an angle 120°. Find

(a) The angle of projection.

(b) The vertical and horizontal coordinates of the initial position of the bob with respect to the point of firing of the bullet. ($g = 10 m/s^2$)



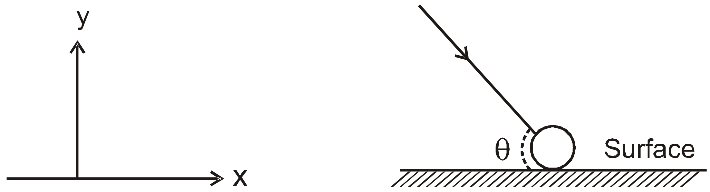
21. A steel ball falling vertically strikes a fixed rigid plate A with velocity v_0 and rebounds horizontally. The ball then strikes a second fixed rigid plate B and rebounds vertically as shown. Assuming smooth surface and the effect of gravity on motion of ball is to be neglected. Determine



(a) The required angles α and β.

(b) The magnitude of the velocity v_1 & v_2. Consider coefficient of restitution for both plates as e.

22. A small spherical ball undergoes an elastic collision with a rough horizontal surface. Before the collision, it is moving at an angle θ to the horizontal (see Fig). You may assume that the frictional force obeys the law $f = \mu N$ during the contact period, where N is the normal reaction on the ball and μ is the coefficient of friction.



(a) Obtain $\theta_m(\mu)$ so that the subsequent horizontal range of the ball after leaving the horizontal surface is maximized.

(b) Find the allowed range for θ_m.

23. Two skaters (A and B), each of mass 70 kg, are approaching each another on a frictionless surface, each with a speed of 1 ms^{-1}. Skater A carries a ball of mass 10 kg. Both skaters can toss the ball at 5 ms^{-1} relative to themselves such that when A tosses the ball at t = 0 s to B then the ball leaves at 6 ms^{-1} with respect to the ground. Further, they start (t = 0s) tossing the ball back and forth when they are 10m apart (see Fig. (1). Assume that the motion is one dimensional, all collisions are completely inelastic and that the time delay between receiving the ball and tossing it back is 1s.

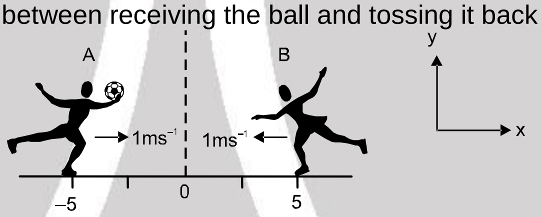


Figure-1

(a) State initial momenta of skaters (just before t = 0 s).

$\vec{P}_A$ = ; $\vec{P}_B$ =

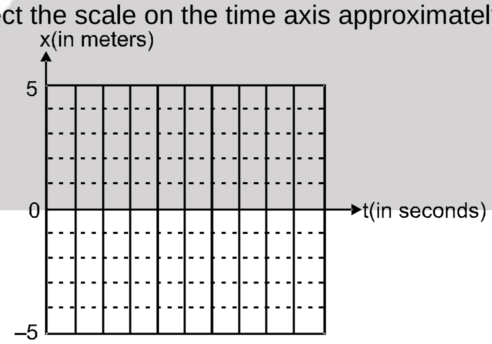
(b) At t = 0 s skater A tosses the ball to skater B. State momenta of both the skaters immediately after B catches the ball.

$\vec{P}_A$ = ; $\vec{P}_B$ =

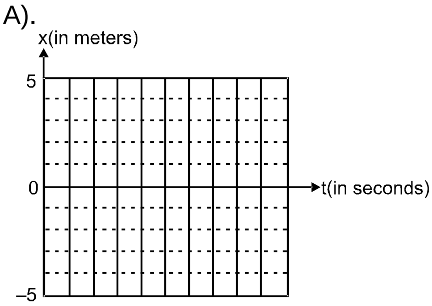
(c) Indicates the minimum number of tosses by each skater required to avoid collision.

Number of tosses by A = ; Number of tosses by B =

(d) Indicate motion of each skater on the following x–t plot if no tosses are made. [Note : For this and next part you must select the scale on the time axis approximately. You may use a pencil for sketching].



(e) Indicate motion of each skater on the following x–t plot from t = 0 s till just after one round trip by the ball (from A to B and back to A).

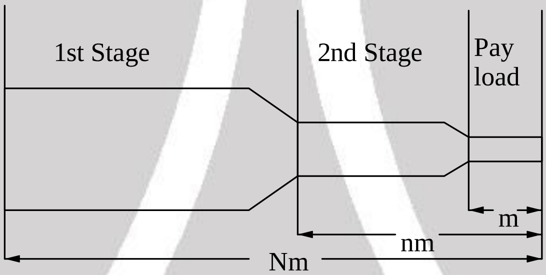


24. A neutron is scattered through (≡ deviation from its original direction) θ degree in an elastic collision with an initially stationary deutron. If the neutron loses $\frac{2}{3}$ of its initial K.E. to the deutron then find the value of θ. (In atomic mass unit, the mass of a neutron is 1u and mass of a deutron is 2u).

25. A shell flying with velocity v = 500 m/s bursts into three identical fragments so that the kinetic energy of the system increases η = 1.5 times. What maximum velocity can one of the fragments obtain?

26. A particle moves along a closed trajectory in a central field of force where the particle's potential energy $U = kr^2$ (k is positive constant, r is the distance of the particle from the centre O of the field). Find the mass of the particle if its minimum distant; from the point O equals r_1 and its velocity at the point farthest from O equals v_2-

27. This problem is designed to illustrate the advantage that can be obtained by the use of multiple-staged instead of single-staged rockets as launching vehicles. Suppose that the payload (e.g., a space capsule) has mass m and is mounted on a two-stage rocket (see figure). The total mass (both rockets fully fuelled, plus the payload) is Nm.



The mass of the second-stage rocket plus the payload, after first-stage burnout and separation, is nm. In each stage the ratio of container mass to initial mass (container plus fuel) is r, and the exhaust speed is V, constant relative to the engine. Note that at the end of each state when the fuel is completely exhausted, the container drops off immediately without affecting the velocity of rocket. Ignore gravity.

(a) Obtain the velocity v of the rocket gained from the first-stage burn, starting from rest in terms of {V,N,n,r}

(b) Obtain a corresponding expression for the additional velocity u gained from the second stage burn.

(c) Adding v and u, you have the payload velocity w in terms of N, n, and r. Taking N and r as constants, find the value of n for which w is a maximum. For this maximum condition obtain u/v.

(d) Find an expression for the payload velocity w_s of a single-stage rocket with the same values of N, r, and V

(e) Suppose that it is desired to obtain a payload velocity of 10 km/s, using rockets which V = 2.5km/s and r = 0.1. Using the maximum condition of part (c) obtain the value of N if the job is to be done with a two-stage rocket.

28. A (trolley + child) of total mass 200 kg is moving with a uniform speed of 36 km/h on a frictionless track. The child of mass 20 kg starts running on the trolley from one end to the other (10 m away) with a speed of 10 m s^{-1} relative to the trolley in the direction of the trolley's motion and jumps out of the trolley with the same relative velocity. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run and just before jump?

HLP Answers

1. $\frac{\lambda\ell \ vg}{2}+\lambda v^3$ 2. $\lambda\ell gv+v^3\lambda$ 3. (a) (5m, 1.25m) (b) 4.45 m.

4. $x_{max} = \frac{4F+\sqrt{16F^2+54mu^2k}}{3k}$ 5. $v_1 = -mv/(M-m)$, $v_2 = Mv/(M-m)$

6. $v_{rear} = v_0 - \frac{m}{M+m}u$; $v_{front} = v_0 + \frac{mMu}{(M+m)^2}$

7. (i) $v_1 = \frac{-2m\vec{u}}{(M+2m)}$ (ii) $v_2 = \frac{-m(2M+3m)\vec{u}}{(M+m)(M+2m)}$, $v_2 > v_1$ by a factor of $\frac{(2M+3m)}{(2M+2m)}$

8. $\ell = \ell' m / 2M$ 9. (a) $\eta = 2m_1/(m_1+m_2)$; (b) $\eta = \frac{4\,m_1\ m_2}{(m_1+m_2)^2}$

10. (a) $\frac{m_1}{m_2} = 1/3$; (b) $\frac{m_1}{m_2} = 1 + 2\cos\theta = 2.0$ 11. $\eta = \frac{1}{2}\cos^2\alpha = 0.25$

12. Will continue moving in the same direction, although this time with the velocity $v' = (1-\sqrt{1-2\eta})v/2$. For $\eta << 1$ the velocity $v' = \eta v/2 = 5$ cm/s.

13. $\Delta T/T = (1 + m/M)\tan^2\theta + m/M - 1 = -40\%$

14. (a) $p = \mu\sqrt{v_1^2+v_2^2}$; (b) $T = 1/2\mu(v_1^2+v_2^2)$. Here $\mu = m_1m_2/(m_1+m_2)$

15. $\sin\theta_{max} = m_2/m_1$

16. $v' = -v(2-\eta^2)/(6-\eta^2)$. Respectively at smaller η, equal, or greater than $\sqrt{2}$

17. $\alpha = (u/u_0)\ln(m_0/m)$ 18. (a) $\frac{m_1}{m_2} = \frac{9}{13}$ (b) 9 m/s

19. $v = (u\cos\theta - \mu u\sin\theta)\ell n\left(\frac{M_0}{(M_0-\lambda t)}\right) - \mu gt$ 20. (a) 37°, (b) x = 120 m and y = 45 m

21. $\tan\alpha = \sqrt{e}$, $\nu_1 = \sqrt{e}\,\nu_0$, $\cot\beta = \sqrt{e}$, $\nu_2 = e\nu_0$

22. (a) $\theta_m = \frac{1}{2}\cot^{-1}(2\mu)$ (b) Possible range of θ_m : $\theta_m \in]0, \pi/4[$

23. (a) $\vec{P}_A = 80\,\hat{i}$ kg. m.s^{-1} or $\vec{P}_a = 70\,\hat{i}$ kg.m.s^{-1}; $\vec{P}_b = -70\,\hat{i}$ kg.m.s^{-1}

(b) $\vec{P}_A = 20\,\hat{i}$ kg.m.s^{-1}; $\vec{P}_b = -10\,\hat{i}$ kg.m.s^{-1} or $\vec{P}_b = -70/8\,\hat{i}$ kg.m.s^{-1}

(c) A = 1, B = 1

(d) See Fig.

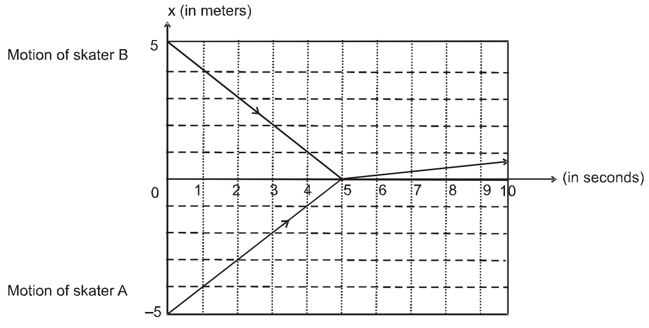


Figure 1:

(e) See Fig.

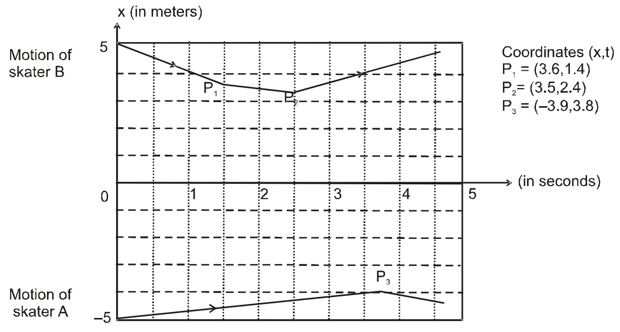


Figure 2:

24. 90 **25.** $v_{max} = v(1 + \sqrt{2(\eta - 1)}) = 1.0$ km per second किलोमीटर प्रति सेकण्ड **26.** $m = 2kr_1^2/v_2^2$.

27. **(a)** Variable mass equation gives

$$m\frac{d\vec{v}}{dt} = \vec{F}_{external} + V_{relative}\frac{dm}{dt}$$

No gravity hence $\vec{F}_{external} = 0$, $\vec{v}_{relative} = v$. Solving rocket equation

$$= v = V\ln\frac{m_i}{m_f} \qquad (1)$$

Here initially mass $m_i = Nm$ (2)

Final mass $m_f = [Nr + n(1 - r)]m$ (3)

(b) Now $m_i = nm$, $m_f = m(nr + 1 - r)$. Equation (1) yields

$$u = V\ln\frac{n}{nr + 1 - r} \qquad (4)$$

(c) From equation (3 to 4)

$$\omega = V\ln\frac{N_n}{[Nr + n(1-r)]\,[nr + 1 - r]} \quad V\ln f(n)$$

Maximizing ω is equivalent to maximizing f(n). Differentiating and setting equal to zero, we obtain

$$n = \sqrt{N} \Rightarrow \frac{u}{v} = \frac{\ln[\sqrt{N}/\{r\sqrt{N} + (1-r)\}]}{\ln[N/Nr + \sqrt{N}(1-r)\}]} = 1 \qquad (5)$$

where we have used equation (1 and 4)

(d) Here $m_i = Nm$ and $m_f = m + r(Nm - m)$. Using equation (1)

$$w_s = V\ln\frac{N}{Nr + 1 - r}$$

(e) Payload velocity $w = u + v = 2V\ln\frac{\sqrt{N}}{r\sqrt{N} + 1 - r}$

For the desired value of w, N = 649.4. Answer should be an integer number for the number of state. Hence N = 650.

28. 9m/s, 9m