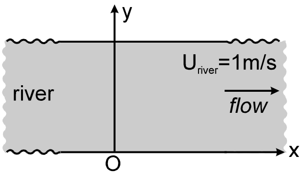
High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

1. A man can swim in still water with a speed of 3 m/s. x and y axis are drawn along and normal to the bank of river flowing to right with a speed of 1 m/s. The man starts swimming from origin O at t = 0 second. Assume size of man to be negligible. Find the equation of locus of all the possible points where man can reach at t = 1 sec.



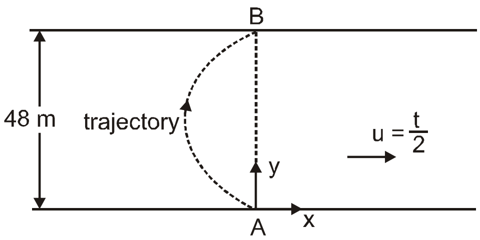
2. Two swimmers 'A' & 'B', one located on one side and other on the another side of a river are situated at a distance 'D' from each other. Line joining them is making an angle θ with the direction perpendicular to flow. The speed of each swimmer with respect to still water is 'u' & speed of river flow is v_r. Both A & B start swimming at the same time in the direction parallel to line AB towards each other and they keep on swimming in same direction. Then,
(a) Find the time after which they will meet.
(b) Find the speed of river (v_r) so that the path of the two swimmers with respect to the ground becomes perpendicular to each other.

3. Two particles start simultaneously from the same point and move along two straight lines making an angle α with each other. One move with uniform velocity u and the other with constant acceleration a and initial velocity zero.
(a) Find the least relative velocity of one with respect to other.
(b) At the same time, find the distance between the two particles.

4. A train of length ℓ = 350 m starts moving rectilinearly with constant acceleration $\omega = 3.0 \times 10^{-2}$ m/s^2; t = 30 s after the start the locomotive headlight is switched on (event 1), and τ = 60 sec after that event the train signal light is switched on (event 2). Find the distance between these events in the reference frames fixed to the train and to the Earth. How and at what constant velocity V relative to the Earth must a certain reference frame K move for the two events to occur in it at the same point?

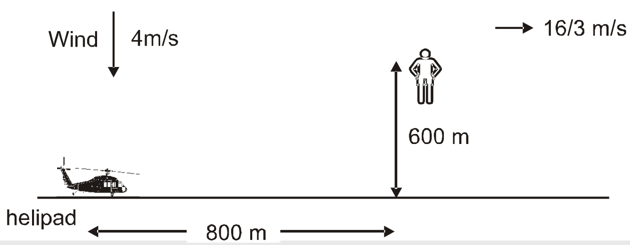
5. Find the time an aeroplane having velocity v (relative to air), takes to fly around a square with side a and the wind blowing at a velocity u, in the two cases,
(a) if the direction of wind is along one side of the square,
(b) If the direction of wind is along one of the diagonals of the square

6. A man starts swimming at time t = 0 from point A on the ground and he wants to reach the point B directly opposite the point A. His velocity in still water is 5 $\frac{m}{sec}$ and width of river is 48 m. River flow velocity 'u' varies with time t (in seconds) as u = $\frac{t}{2}\frac{metre}{sec}$. He always tries to swim in particular fixed direction with river flow. Find the (Given $\sin^{-1}\left(\frac{24}{25}\right)$ = 74º)



(a) direction (with line AB) in which he should make stroke and the time taken by man to cross the river.
(b) trajectory of path.

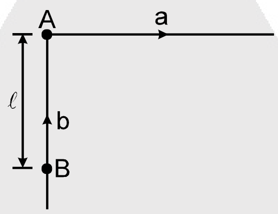
7. A child in danger of drowing in a river is being carried downstream by a current that flows uniformly at a speed of $\frac{16}{3}$ m/sec. The child is 600m from shore and 800m downstream of a helipad when the rescue helicopter sets out and the helicopter follows the same direction throughout the motion. If the helicopter proceeds at its maximum speed of $\frac{80}{3}$ m/sec. with respect to air and air is blowing with velocity of 4m/sec perpendicular to river flow velocity as shown, then



(i) Heading at what angle with the shore should the helicopter take off?
(ii) Calculate the time taken by helicopter to reach the child.

8. A boy sitting at the rear end of a railway compartment of a train, running at a constant acceleration on horizontal rails, throws a ball towards the fore end of the compartment with a muzzle velocity of 20 m/sec at an angle 37º above the horizontal, when the train is running at a speed of 10 m/sec. If the same boy catches the ball without moving from his seat and at the same height of projection, find the speed of the train at the instant of his catching the ball. [$g = 10$ m/sec^2; sin 37º = 3/5]

9. In the figure shown A and B are two particles which start from rest. A has constant acceleration ' a ' in the direction shown. B also increases its speed at a constant rate ' b ', but the direction of velocity is always towards A. Find the time after which B meets A. Also find the total distance travelled by B. (b > a)



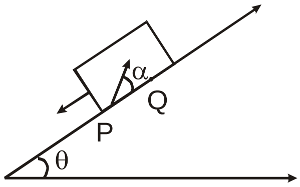
10. Two swimmers start from point A on one bank of a river to reach point B on the other bank, lying directly opposite to point A. One of them crosses the river along the straight line AB, while the other swims at right angles to the stream and then walks the distance which he has been carried away by the stream to get to point B. What was the velocity (assumed uniform) of his walking if both the swimmers reached point B simultaneously? Velocity of each swimmer in still water is 2.5 km h^{-1} and the stream velocity is 2 km h^{-1}.

11. A ship moves along the equator to the east with velocity v_0 = 30 km/hour. The southeastern wind blows at an angle ϕ = 60° to the equator with velocity v = 15 km/hour. Find the wind velocity v' relative to the ship and the angle ϕ' between the equator and the wind direction in the reference frame fixed to the ship.

12. Two cars A and B are racing along straight line. Car A is leading, such that their relative velocity is directly proportional to the distance between the two cars. When the lead of car A is ℓ_1 = 10 m, it's running 10 m/s faster than car B. If the time car A will take to increase its lead to ℓ_2 = 20 m from car B is t = ($\log_e$n) sec, then find n.

13. A large heavy box is sliding without friction down a smooth plane of inclination θ from a point P on the bottom of the box, a particle is thrown inside box. The initial speed of the particle with respect to the box is u and the direction of projection makes an angle α with the bottom as shown in the figure :

[JEE-1998, 5 + 3/120]



(a) Find the distance along the bottom of the box between the point of projection P and the point Q where the particle lands (Assume that the particle does not hit any other surface of the box. Neglect air resistance.)

(b) If the horizontal displacement of the particle as seen by an observer on the ground is zero, find the speed of the box with respect to the ground at the instant when the particle was projected.

14. A motorboat going downstream overcame a raft (A wooden block) at a point A; τ = 60 min later it turned back and after some time passed the boat meets raft at a distance ℓ = 6.0 km from the point A. Find the flow velocity assuming the duty of the engine to be constant.

15. A boat moves relative to water with a velocity half of the river flow velocity. If the angle from the direction of flow at which the boat must move relative to stream direction to minimize drift is $\frac{2\pi}{n}$, then find n.

HLP Answers

1. $(x-1)^2 + y^2 = 9$

2. (a) $\frac{D}{2u}$ (b) $v_r = u$

3. (a) $(v_{rel})_{least} = u \sin\alpha$ (b) separation = $\frac{u^2 \cos\alpha}{2a}\sqrt{1+3\sin^2\alpha}$

4. $x_1 - x_2 = \ell - w\tau(t + \tau/2) = \frac{6}{25}$ km, Towards the train with velocity v = 4 m/s

5. (a) $\frac{2a}{v^2-u^2}\left(v+\sqrt{v^2-u^2}\right)$ (b) $2\sqrt{2}\,a\left(\frac{\sqrt{2v^2-u^2}}{v^2-u^2}\right)$

6. (a) 37° and 53°, 12 sec. and 16 sec. (b) $x = y^2/64 - \frac{3y}{4}$

7. (i) 37° (ii) 50 sec

8. 42 m/s

9. $t = \sqrt{\frac{2\ell b}{b^2-a^2}}, \frac{b^2\ell}{b^2-a^2}$

10. 3 km/h towards B

11. $v' = \sqrt{v_0^2+v^2+2v_0 v\cos\phi} \approx 40$ km per hour, $\phi' = 19°$

12. 2

13. (a) PQ = $(u^2 \sin 2\alpha)/g \cos\theta$ (b) $v = \frac{u\cos(\alpha+\theta)}{\cos\theta}$

14. $v_B = \ell/2\tau$ = 3.0 km per hour

15. 3