



High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A particles of mass m is attached at one end of a light, inextensible string of length ℓ whose other end is fixed at the point C. At the lowest point the particle is given minimum velocity to complete the circular path in the vertical plane. As it moves in the circular path the tension in the string changes with θ . θ is defined in the figure. As θ varies from '0' to ' 2π ' (i.e. the particle completes one revolution) plot the variation of tension 'T' against ' θ '.

2. A person stands on a spring balance at the equator. (a) By what percentage is the balance reading less than his true weight ? (b) If the speed of earth's rotation is increased by such an amount that the balance reading is half the true weight, what will be the length of the day in this case ?

3. A track consists of two circular parts ABC and CDE of equal radius 100 m and joined smoothly as shown in fig. Each part subtends a right angle at its centre. A cycle weighing 100 kg together with the rider travels at a constant speed of 18 km/h on the track. (a) Find the normal contact force by the road on the cycle when it is at B and D. (b) Find the force of friction exerted by the track on the tyres when the cycle is at B, C and D. (c) Find the normal force between the road and the cycle just, before and just after the cycle crosses C. (d) What should be the minimum friction coefficient between the road and the tyre, which will ensure that the cyclist can move with constant speed ? Take $g = 10\text{m/s}^2$.

4. A ring of radius R is placed such that it lies in a vertical plane. The ring is fixed. A bead of mass m is constrained to move along the ring without any friction. One end of the spring is connected with the mass m and other end is rigidly fixed with the topmost point of the ring. Initially the spring is in un-extended position and the bead is at a vertical distance R from the lowermost point of the ring. The bead is now released from rest. (a) What should be the value of spring constant K such that the bead is just able to reach bottom of the ring. (b) The tangential and centripetal accelerations of the bead at initial and bottommost position for the same value of spring constant K .

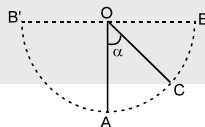
5. Wheel A of radius $r_A = 10\text{cm}$ is coupled by a belt C to another wheel of radius $r_B = 25\text{ cm}$ as in the figure. The belt does not slip. At time $t = 0$ wheel A increases it's angular speed from rest at a uniform rate of $\pi/2\text{ rad/sec}^2$. Find the time in which wheel B attains a speed of 100 rpm (wheel are fixed).

6. A rod AB is moving on a fixed circle of radius R with constant velocity ' v ' as shown in figure. P is the point of intersection of the rod and the circle. At an instant the rod is at a distance $x = \frac{3R}{5}$ from centre of the circle. The velocity of the rod is perpendicular to the rod and the rod is always parallel to the diameter CD. (a) Find the speed of point of intersection P. (b) Find the angular speed of point of intersection P with respect to centre of the circle.

7. A chain of mass m forming a circle of radius R is slipped on a smooth round cone with half-angle θ . Find the tension of the chain if it rotates with a constant angular velocity ω about a vertical axis coinciding with the symmetry axis of the cone.

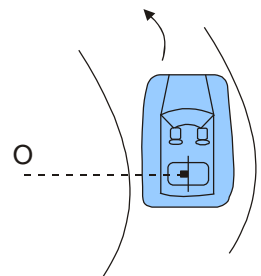


8. A small sphere of mass m suspended by a thread is first taken aside so that the thread forms the right angle with the vertical and then released, then :
- Find the total acceleration of the sphere and the thread tension as a function of θ , (the angle of deflection of the thread from the vertical)
 - Find the angle θ between the thread and the vertical at the moment when the total acceleration vector of the sphere is directed horizontally
 - Find the thread tension at the moment when the vertical component of the sphere's velocity is maximum
9. Find the magnitude and direction of the force acting on the particle of mass m during its motion in the plane xy according to the law $x = a \sin \omega t$, $y = b \cos \omega t$, where a , b and ω are constants.
10. A chain of length ℓ is placed on a smooth spherical surface of radius R with one of its ends fixed at the top of the sphere. What will be the acceleration a of each element of the chain when its upper end is released? It is assumed that the length of the chain $\ell < \frac{1}{2} \pi R$
11. A point moves in the plane so that its tangential acceleration $\omega_t = a$, and its normal acceleration $\omega_n = bt^4$, where a and b are positive constants, and t is time. At the moment $t = 0$, the point was at rest. Find how the curvature radius R of the point's trajectory and the total acceleration ω depend on the distance covered s .
12. A block of mass m is kept on a horizontal ruler. The friction coefficient between the ruler and the block is $\mu = 0.5$. The ruler is fixed at one end and the block is at a distance $L = 1$ m from the fixed end. The ruler is rotated about the fixed end in the horizontal plane through the fixed end. If the angular speed of the ruler is uniformly increased from zero at a constant angular acceleration $\alpha = 3 \text{ rad/sec}^2$. Find the angular speed at which block will slip. ($g = 10 \text{ m/s}^2$)
13. A particle moves along the plane trajectory $y(x)$ with velocity v whose modulus is constant. Find the acceleration of the particle at the point $x = 0$ and the curvature radius of the trajectory at that point if the trajectory has the form
- of a parabola $y = ax^2$.
 - of an ellipse $(x/a)^2 + (y/b)^2 = 1$; a and b are constants here.
14. A particle moves in the plane xy with velocity $v = a \hat{i} + bx \hat{j}$, where \hat{i} and \hat{j} are the unit vectors of the x and y axes and a and b are constants. At the initial moment of time the particle was located at the point $x = y = 0$. Find:
- the equation of the particle's trajectory $y(x)$;
 - the curvature radius of trajectory as a function of x .
15. A simple pendulum is vibrating with an angular amplitude of 90° as shown in the given figure. For what value of α , is the acceleration directed?



- (i) vertically upwards (ii) horizontally (iii) vertically downwards

16. A car moving at a speed of 36 km/hr is taking a turn on a circular road of radius 50 m . A small wooden plate is kept on the seat with its plane perpendicular to the radius of the circular road (figure). A small block of mass 100 g is kept on the seat which rests against the plate. The friction coefficient between the block and the plate is $\mu = \frac{1}{\sqrt{3}} = 0.58$.



- Find the normal contact force exerted by the plate on the block.
- The plate is slowly turned so that the angle between the normal to the plate and the radius of the road slowly increases. Find the angle at which the block will just start sliding on the plate



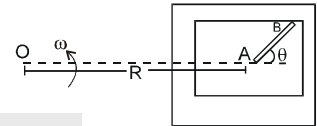
17. A hemispherical bowl of radius $r = 0.1\text{m}$ is rotating about its axis (which is vertical) with an angular velocity ω . A particle of mass 10^{-2}kg on the frictionless inner surface of the bowl is also rotating with the same ω . The particle is at a height h from the bottom of the bowl. (a) Obtain the relation between h and ω . What is the minimum value of ω needed in order to have a nonzero value of h . (b) It is desired to measure 'g' using this setup by measuring h accurately. Assuming that r and ω are known precisely and that the least count in the measurement of h is 10^{-4}m . What is minimum error Δg in the measured value of g . [$g = 9.8\text{m/s}^2$]

[JEE 1993]

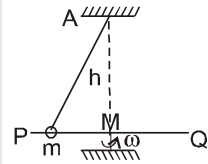
18. A block is placed inside a horizontal hollow cylinder. The cylinder is rotating with constant angular speed one revolution per second about its axis. The angular position of the block at which it begins to slide is 30° below the horizontal level passing through the center. Find the radius of the cylinder if the coefficient of friction is 0.6. What should be the minimum constant angular speed of the cylinder so that the block reaches the highest point of the cylinder?

[REE 2001]

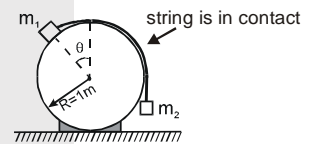
19. A table with smooth horizontal surface is fixed in a cabin that rotates with a uniform angular velocity ω in a circular path of radius R (figure). A smooth groove AB of length $L (< R)$ is made on the surface of the table. The groove makes an angle θ with the radius OA of the circle in which the cabin rotates. A small particle is kept at the point A in the groove and is released to move along AB. Find the time taken by the particle to reach the point B.



20. A smooth rod PQ is rotated in a horizontal plane about its mid point M which is $h = 0.1\text{m}$ vertically below a fixed point A at a constant angular velocity 14 rad/s . A light elastic string of natural length 0.1m requiring 1.47 N/cm has one end fixed at A and its other end attached to a ring of mass $m = 0.3\text{ kg}$ which is free to slide along the rod. When the ring is stationary relative to rod, then find inclination of string with vertical, tension in string, force exerted by ring on the rod. ($g = 9.8\text{ m/s}^2$)

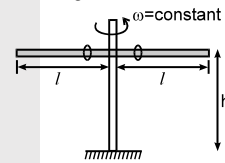


21. A mass m_1 lies on fixed, smooth cylinder. An ideal cord attached to m_1 passes over the cylinder and is connected to mass m_2 as shown in the figure. Find the value of θ in degree (shown in diagram) for which the system is in equilibrium if $m_1 = \sqrt{2}\text{ kg}$ and $m_2 = 1\text{ kg}$



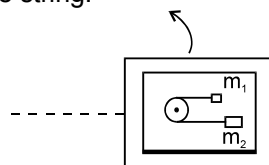
22. In above question if $m_1 = 5\text{ kg}$, $m_2 = 4\text{kg}$. The system is released from rest when $\theta = 30^\circ$. Find the value of N if the magnitude of acceleration of mass m_1 just after the system is released is $\frac{N}{9}\text{ m/s}^2$.

23. Two identical rings which can slide along the rod are kept near the mid point of a smooth rod of length 2ℓ ($\ell = 1\text{ m}$) The rod is rotated with constant angular velocity $\omega = 3\text{ radian/ sec}$ about vertical axis passing through its centre. The rod is at height $h = 5\text{ m}$ from the ground. Find the distance (in meter) between the points on the ground where the rings will fall after leaving the rods.



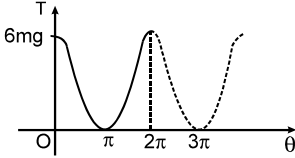
24. A table with smooth horizontal surface is placed in a cabin which moves in a circle of a large radius R (figure). A smooth pulley of small radius is fastened to the table. Two masses m and $2m$ placed on the table are connected through a string over the pulley. Initially the masses are held by a person with the string along the outward radius and then the system is released from rest (with respect to the cabin).

Find the value of $\frac{T}{ma}$ where a is the magnitude of the initial acceleration of the masses as seen from the cabin and 'T' is the tension in the string.





HLP Answers

1. 
2. (a) $\frac{\omega^2 R}{g} \times 100 = 0.34\%$, (b) $2\pi\sqrt{\frac{2R}{g}} = 2.0 \text{ hour}$
3. (a) 975N, 1025 N, (b) 0,707N, 0, (c) 682N , 732 N , (d) 1.037
4. (a) $K = \frac{mg}{R(3-2\sqrt{2})}$ (b) at initial instant $a_t = g$, $a_c = 0$ at bottommost position $a_t = 0$ $a_c = 0$
5. 50/3 sec.
6. (a) $V_P = \frac{5}{4} V$ (b) $\omega = \frac{V_P}{R} = \frac{5V}{4R}$
7. $T = (\cot\theta + \omega^2 R / g) mg / 2\pi$
8. (i) $g\sqrt{1+3\cos^2\theta}$, $T = 3mg \cos\theta$ (ii) $\cos\theta = \frac{1}{\sqrt{3}}$ (iii) $mg\sqrt{3}$
9. $\vec{F} = -m\omega^2 \vec{r}$, where \vec{r} is the radius vector of the particle relative to the origin of coordinates;
 $F = m\omega^2 \sqrt{x^2 + y^2}$
10. $a = [1 - \cos(\ell/R)] Rg/\ell$
11. $R = a^3 / 2bs$, $\omega = a\sqrt{1+(4bs^2/a^3)^2}$
12. 2 rad/sec.
13. (a) $\omega = 2av^2$, $R = \frac{1}{2a}$; (b) $\omega = bv^2 / a^2$, $R = a^2 / b$
14. (a) $y = (b/2a)y^2$, (b) $R = v^2 / \omega_n = v^2 / \sqrt{\omega^2 - \omega_\tau^2} = (a/b) [1 + (xb/a)^2]^{3/2}$
15. (i) 0° , (ii) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$, (iii) 90°
16. (a) 0.2N, (b) 30°
17. (a) $7\sqrt{2}$ rad/s (b) $-9.8 \times 10^{-3} \text{ m/s}^2$ 18. 0.24m, 8.9rad/sec
19. $\sqrt{\frac{2L}{\omega^2 R \cos\theta}}$ 20. $\cos\theta = 3/5$, $T = 9.8 \text{ N}$, $N = \frac{147}{50} = 2.94 \text{ N}$
21. 45 22. 15 23. 10
24. 4

