



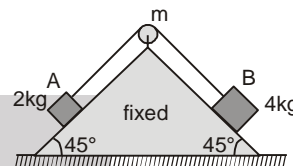
# High Level Problems (HLP)

## SUBJECTIVE QUESTIONS

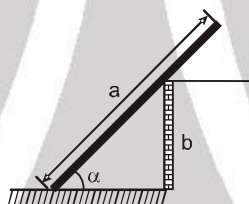
1. Find the M.I. of a rod about (i) an axis perpendicular to the rod and passing through left end. (ii) An axis through its centre of mass and perpendicular to the length whose linear density varies as  $\lambda = ax$  where  $a$  is a positive constant and 'x' is the position of an element of the rod relative to its left end. The length of the rod is  $\ell$ .

2. The pulley (uniform disc) shown in figure has, radius 10 cm and moment of inertia about its axis  $I = 0.5 \text{ kgm}^2$  (B and A both move)

- (a) Assuming all the plane surfaces are smooth and there is no slipping between pulley and string, calculate the acceleration of the mass 4kg.  
 (b) The friction coefficient between the block A and the plane below is  $\mu = 0.5$  and the plane below the B block is frictionless. Assuming no slipping between pulley and string find acceleration of 4kg block

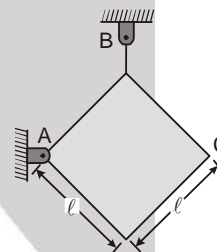


3. A uniform rod of length 'a' rests against a frictionless wall as shown in figure. Find the friction coefficient between the horizontal surface and the lower end if the minimum angle that the rod can make with the horizontal is  $\alpha$ , without slipping.

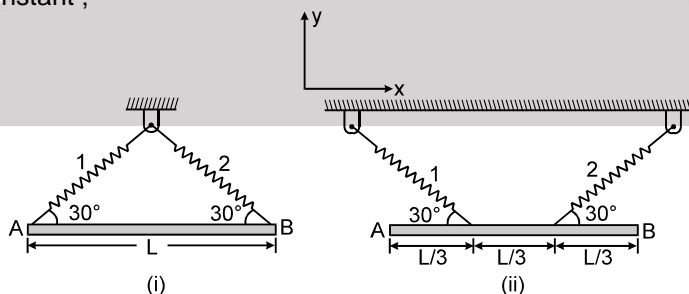


4. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that moment;

- (a) The angular acceleration of the plate.  
 (b) The acceleration of corner C.  
 (c) The reaction at A.

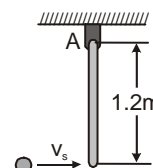


5. A uniform slender rod AB of mass m is suspended from two springs as shown. If spring 2 breaks, determine at that instant ;



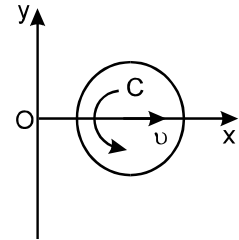
- (a) The angular acceleration of the bar. (b) The acceleration of point A.  
 (c) The acceleration of point B.

6. A 2 kg sphere moving horizontally to the right with an initial velocity of 5 m/s strikes the lower end of an 8 kg rigid rod AB. The rod is suspended from a hinge at A and is initially at rest. Knowing that the coefficient of restitution between the rod and sphere is 0.80, determine the angular velocity of the rod and the velocity of the sphere immediately after the impact.

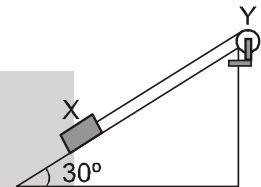




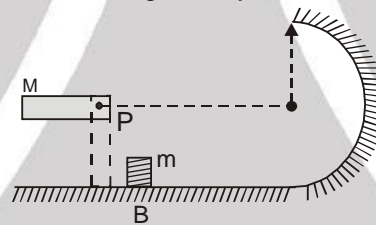
7. A rotating disc (figure) moves in the positive direction of the x-axis. Find the equation  $y(x)$  describing the position of the instantaneous axis of rotation, if at the initial moment the axis C of the disc was located at the point O after which it moved
- (a) With a constant velocity  $v$ , while the disc started rotating counter clockwise with a constant angular acceleration  $\beta$  (the initial angular velocity is equal to zero);
- (b) With a constant acceleration  $a$  (and the zero initial velocity), while the disc rotates counterclockwise with a constant angular velocity  $\omega$ .



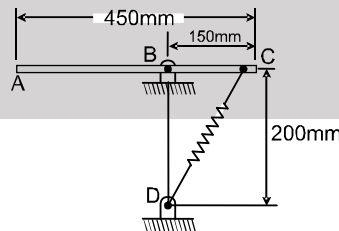
8. A block X of mass 0.5 kg is held by a long massless string on a fixed frictionless inclined plane inclined at  $30^\circ$  to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and radius 0.2 m as shown in figure. The drum is given an initial angular velocity such that block X starts moving up the plane. [JEE - 1994]
- (a) Find the tension in the string during motion.
- (b) At a certain instant of time the magnitude of the angular velocity of Y is  $10 \text{ rad s}^{-1}$ . Calculate the distance travelled by X from that instant of time until it comes to rest.



9. A rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the small block B of mass m and stops. Find the ratio of masses such that the block B completes the circular track of radius R. Neglect any friction.



10. A 3 kg uniform rod rotates in a vertical plane about a smooth pivot at B. A spring of constant  $k = 300 \text{ N/m}$  and of unstretched length 100 mm is attached to the rod as shown. Knowing that in the position shown the rod has an angular velocity of  $4 \text{ rad/s}$  clockwise, determine the angular velocity of the rod after it has rotated through. [ $g = 10 \text{ m/s}^2$ ]

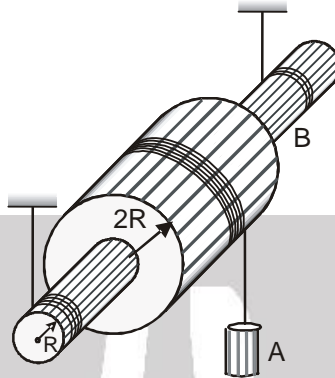


(D is vertically below B)

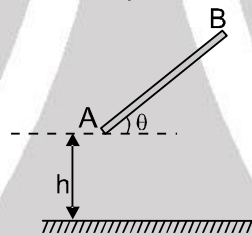
- (a)  $90^\circ$                       (b)  $180^\circ$
11. The angular momentum of a particle relative to a certain ' point O varies with time as  $\vec{M} = \vec{a} + \vec{b}t^2$ , where  $\vec{a}$  and  $\vec{b}$  are constant vectors, with  $\vec{a} \perp \vec{b}$ . Find the force moment N relative to the point O acting on the particle when the angle between the vectors N and M equals  $45^\circ$ .
12. A plank of mass  $m_1$  with a uniform sphere of mass  $m_2$  placed on it rests on a smooth horizontal plane. A constant horizontal force F is applied to the plank. With what accelerations will the plank and the centre of the sphere move provided there is no sliding between the plank and the sphere ?



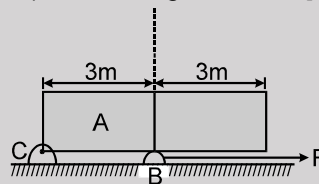
13. In the arrangement shown in the figure weight A possesses mass  $m$ , a pulley B possesses mass  $M$ . Also known are the moment of inertia  $I$  of the pulley relative to its axis and the radii of the pulley are  $R$  and  $2R$  respectively. Consider the mass of the threads is negligible. Find the acceleration of weight A after the system is set free. (Assume no slipping takes place anywhere and axis of cylinder remains horizontal)



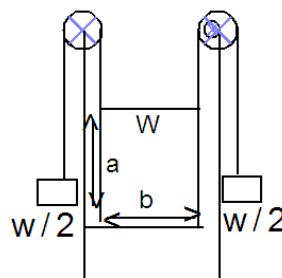
14. A uniform rod AB of length  $\ell$  is released from rest with AB inclined at angle  $\theta$  with horizontal. It collides elastically with smooth horizontal surface after falling through a height  $h$ . What is the height upto which the centre of mass of the rod rebounds after impact?



15. A uniform block A of mass 25 kg and length 6m is hinged at C and is supported by a small block B as shown in the Figure. A constant force  $F$  of magnitude 400N is applied to block B horizontally. What is the speed of B after it moves 1.5 m ? The mass of block B is 2.5 kg & the coefficient of friction for all contact surfaces is 0.3. [Use  $\ln(3/2) = 0.41$  and  $g = 10 \text{ ms}^{-2}$ ]

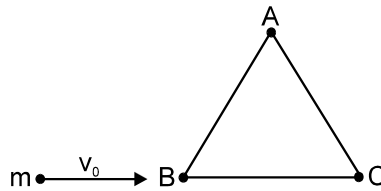


16. A window (of weight  $w$ ) is supported by two strings passing over two smooth pulleys in the frame of the window in which window just fits in, the other ends of the string being attached to weights each equal to half the weight of the window. One thread breaks and the window moves down. Find acceleration of the window if  $\mu$  is the coefficient of friction, and 'a' is the height and 'b' the breadth of the window.

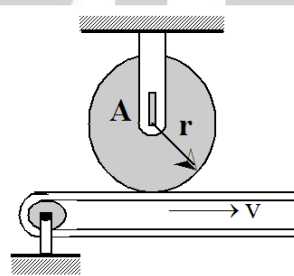




17. Three particles A, B, C of mass  $m$  each are joined to each other by massless rigid rods to form an equilateral triangle of side  $a$ . Another particle of mass  $m$  hits B with a velocity  $v_0$  directed along BC as shown. The colliding particle stops immediately after impact.



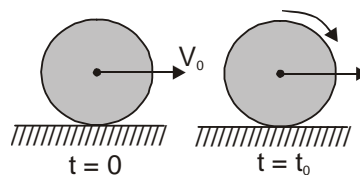
- (a) Calculate the time required by the triangle ABC to complete half revolution in its subsequent motion.  
 (b) What is the net displacement of point B during this interval ?
18. Disk A has a mass of 4 kg and a radius  $r = 75$  mm, it is at rest when it is placed in contact with the belt, which moves at a constant speed  $v = 18$  m/s. Knowing that  $\mu_k = 0.25$  between the disk and the belt, determine the number of revolutions executed by the disk before it reaches a constant angular velocity. (Assume that the normal reaction by the belt on the disc is equal to weight of the disc) .



19. A 160 mm diameter pipe of mass 6 kg rests on a 1.5 kg plate. The pipe and plate are initially at rest when a force  $P$  of magnitude 25 N is applied for 0.75 s. Knowing that  $\mu_{s1} = 0.25$  &  $\mu_{k1} = 0.20$  between the plate and both the pipe and the floor, determine ;



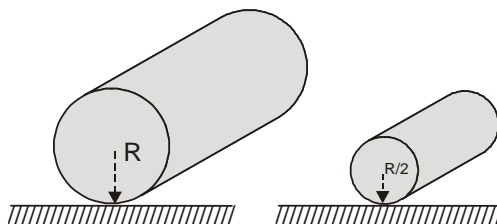
- (a) whether the pipe slides with respect to the plate.  
 (b) the resulting velocities of the pipe and of the plate.
20. A uniform disc of mass  $m$  and radius  $R$  is rolling up a rough inclined plane, which makes an angle of  $30^\circ$  with the horizontal. If the coefficients of static and kinetic friction are each equal to  $\mu$  and only the forces acting are gravitational, normal reaction and friction, then the magnitude of the frictional force acting on the disc is \_\_\_\_\_ and its direction is \_\_\_\_\_ (write 'up' or 'down') the inclined plane. [JEE - 1997]
21. A uniform disc of mass  $m$  and radius  $R$  is projected horizontally with velocity  $v_0$  on a rough horizontal floor so that it starts off with a purely sliding motion at  $t = 0$ . After  $t_0$  seconds, it acquires a purely rolling motion as shown in figure. [JEE - 1997]



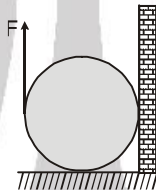
- (a) Calculate the velocity of the centre of mass of the disc at  $t_0$  .  
 (b) Assuming the coefficient of friction to be  $\mu$ , calculate  $t_0$ . Also calculate the work done by the frictional force as a function of time & the total work done by it over a time  $t$  much longer than  $t_0$ .



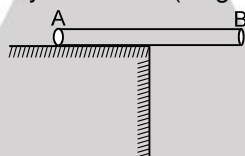
22. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to  $R/2$ . [JEE - 1990]



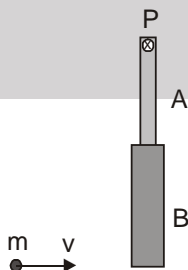
23. Figure shows a vertical force  $F$  that is applied tangentially to a uniform cylinder of weight  $W$ . The coefficient of static friction between the cylinder and all surfaces is 0.5. Find in terms of  $W$ , the maximum force that can be applied without causing the cylinder to rotate.



24. A drinking straw of mass  $2m$  is placed on a smooth table orthogonally to the edge such that half of it extends beyond the table. A fly of mass  $m$  lands on the A end of the straw and walks along the straw until it reaches the B end. It does not tip even when another fly gently lands on the top of the first one. Find the largest mass that the second fly can have. (Neglect the friction between straw and table).



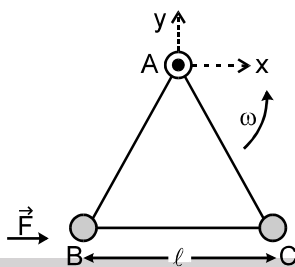
25. Two uniform thin rods A & B of length 0.6 m each and of masses 0.01 kg & 0.02 kg respectively are rigidly joined, end to end. The combination is pivoted at the lighter end P as shown in figure such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg moving horizontally hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position? [JEE - 1994]



26. A uniform cube of side 'a' and mass  $m$  rests on a rough horizontal table. A horizontal force  $F$  is applied normal to one of the faces at a point directly above the centre of the face, at a height  $\frac{3a}{4}$  above the base. (i) What is the minimum value of  $F$  for which the cube begins to tip about an edge? (ii) What is the minimum value of  $\mu_s$  so that toppling occurs. (iii) If  $\mu = \mu_{\min}$ , find minimum force for toppling. (iv) Minimum  $\mu_s$  so that  $F_{\min}$  (as in part-(i)) can cause toppling.

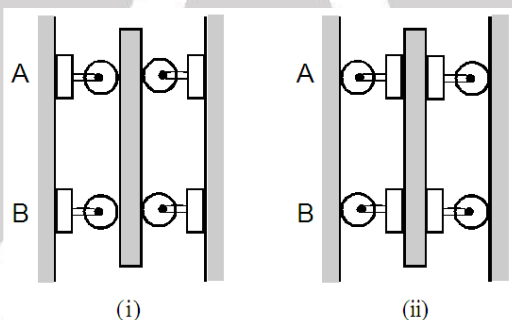


27. Three particles A, B and C each of mass  $m$  are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side  $\ell$ . This body is placed on a horizontal frictionless table ( $x - y$  plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A as shown in figure. The body is set into rotational motion on the table about A with a constant angular velocity  $\omega$

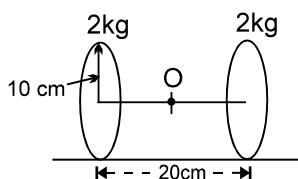


- (a) Find the magnitude of the horizontal force exerted by the hinge on the body .  
 (b) At time  $T$ , when the side  $BC$  is parallel to the  $x -$  axis, a force  $F$  is applied on  $B$  along  $BC$  as shown. Obtain the  $x -$  component and the  $y -$  component of the force exerted by the hinge on the body, immediately after time  $T$ . [JEE Mains 02, (1+4)/60]

28. A bar of mass  $m$  is held as shown between 4 disks , each of mass  $M$  & radius  $r = 75$  mm Determine the acceleration of the bar immediately after it has been released from rest, knowing that the normal forces exerted on the disks are sufficient to prevent any slipping and assuming that ;  
 In (i) case the discs are attached to the fixed support on wall. In (ii) case the discs are attached to the bar.



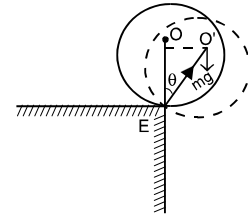
- (a)  $m = 5$  kg and  $M = 2$  kg .  
 (b) the mass of  $M$  of the disks is negligible.  
 (c) the mass of  $m$  of the bar is negligible .
29. Two thin circular discs of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. The friction with the floor of the truck is large enough, so that object can roll on the truck without slipping. Take  $x$ -axis as the direction of motion of the truck and  $z$ -axis as the vertically upward direction. If the truck has an acceleration of  $9 \text{ m/s}^2$ , calculate [JEE - 1997' 5/100]



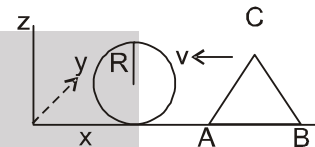
- (a) the force of friction on each disc.  
 (b) the magnitude & direction of frictional torque acting on each disk about the centre of mass  $O$  of the object. Express the torque in the vector form in terms of unit vectors  $\hat{i}$ ,  $\hat{j}$  &  $\hat{k}$  along  $x$ ,  $y$  &  $z$  direction.



30. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius  $r$  is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine :
- The angle  $\theta_c$  through which the cylinder rotates before it leaves contact with the edge.
  - The speed of the centre of mass of the cylinder before leaving contact with the edge.
  - The ratio of translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge.
- [JEE - 1995]

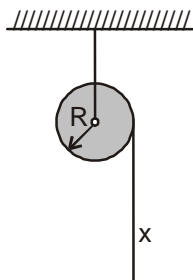


31. A wedge of mass 'm' and triangular cross section ( $AB = BC = CA = 2R$ ) is moving with a constant velocity  $-v \hat{i}$  towards a sphere of radius  $R$  fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time  $\Delta t$ , during which the sphere exerts a constant force  $\vec{F}$  on the wedge. The sphere is always fixed.



- Find the force  $\vec{F}$  and also the normal force  $\vec{N}$  exerted by the table on the wedge during the time  $\Delta t$ .
  - Let 'h' denote the perpendicular distance between the centre of mass of the wedge and the line of action of force  $\vec{F}$ . Find the magnitude of the torque due to the normal force  $\vec{N}$  about the centre of the wedge, during the time  $\Delta t$ .
- [JEE - 1998, 8]

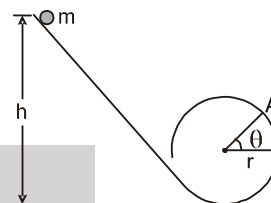
32. The surface mass density (mass/area) of a circular disc of radius 'R' depends on the distance from the centre  $x$  given as,  $\sigma(x) = \alpha + \beta x$ . Where  $\alpha$  and  $\beta$  are positive constant find its moment of inertia about the line perpendicular to the plane of the disc through its centre.
33. Calculate the moment of inertia of a uniform solid cone relative to its symmetry axis, if the mass of the cone is equal to  $m$  and the radius of its base to  $R$ .
34. A force  $\vec{F} = A \hat{i} + B \hat{j}$  is applied to a point whose radius vector relative to the origin of coordinates  $O$  is equal to  $\vec{r} = a \hat{i} + b \hat{j}$ , where  $a, b$  &  $A, B$  are constants, and  $\hat{i}, \hat{j}$  are the unit vectors of the  $x$  and  $y$  axes. Find the moment  $N$  (torque of  $\vec{F}$ ) and the arm  $\ell$  of the force relative to the point  $O$ .
35. A uniform cylinder of radius  $R$  and mass  $M$  can rotate freely about a stationary horizontal axis  $O$  Fig. A thin cord of length  $\ell$  and mass  $m$  is wound on the cylinder in a single layer. Find the angular acceleration of the cylinder as a function of the length  $x$  of the hanging part of the cord. The wound part of the cord is supposed to have its centre of gravity on the cylinder axis.



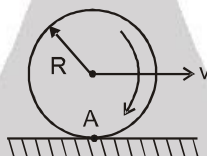


36. A vertically oriented uniform rod of mass  $M$  and length  $\ell$  can rotate about a fixed horizontal smooth axis passing through its upper end. A horizontally flying bullet of mass  $m$  strikes the lower end of the rod and gets stuck in it; as a result, the rod swings through an angle  $\alpha$  ( $\alpha < 90^\circ$ ). Assuming that  $m \ll M$ , find :
- the velocity of the flying bullet ;
  - the momentum increment in the system "bullet + rod" during the impact; what causes the change of that momentum ;
  - at what distance  $x$  : from the upper end of the rod the bullet must strike for the momentum of the system "bullet-rod" to remain constant during the impact.

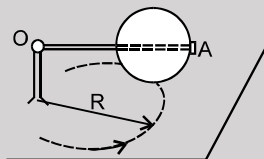
37. A small spherical ball of mass  $m$  is rolling without slipping down the loop track as shown in the figure. The ball is released from rest on the linear portion at a vertical height  $h$  from the lowest point. The circular part as shown in figure has a radius  $r$ . [ $g = 10 \text{ ms}^{-2}$ ]
- Find the kinetic energy of the ball when it is at a point A where the radius make an angle  $\theta$  with the horizontal
  - Find the radial and the tangential accelerations of the centre when the ball is at A.
  - Find the normal force and the frictional force acting on the ball if  $h = 50 \text{ cm}$ ,  $r = 10 \text{ cm}$ ,  $\theta = 0$  and  $m = 70 \text{ g}$ .



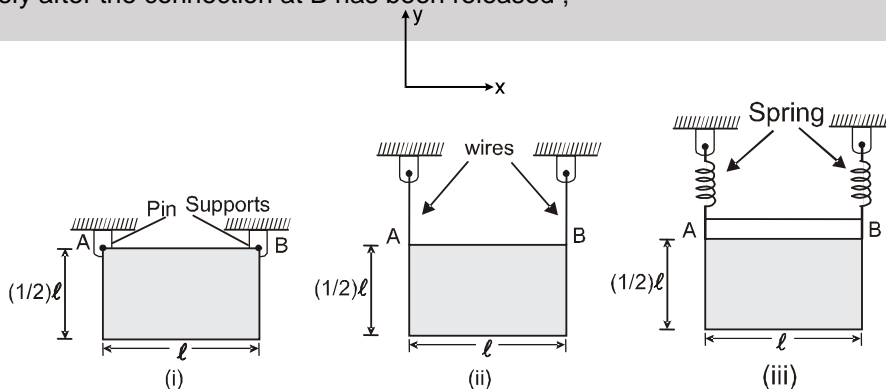
38. A point A is located on the rim of a wheel of radius  $R = 0.50 \text{ m}$  which rolls without slipping along a horizontal surface with velocity  $v = 1.00 \text{ m/s}$  as shown in figure. Find:
- the modulus and the direction of the acceleration vector of the point A ;
  - the total distance  $s$  traversed by the point A between the two successive moments at which it touches the surface.



39. A uniform sphere of mass  $m$  and radius  $r$  rolls without sliding over a horizontal plane, rotating about a horizontal axle OA. In the process, the centre of the sphere moves with velocity along a circle of radius  $R$  Find the kinetic energy of the sphere.



40. A uniform plate of mass 'm' is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;

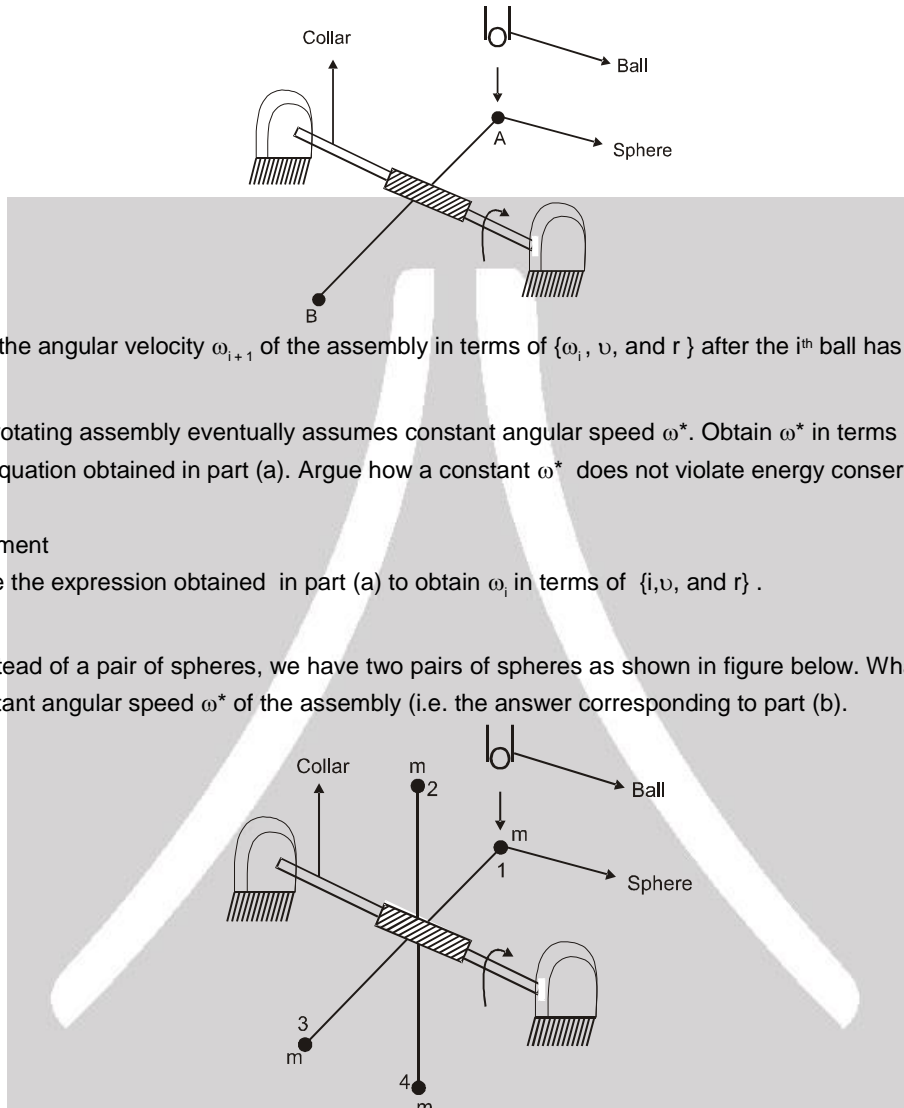


- The angular acceleration of the plate.
- The acceleration of its center of mass.

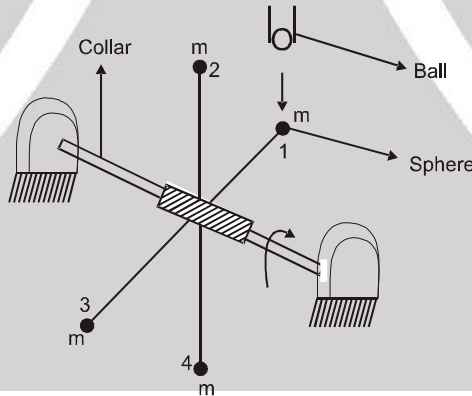




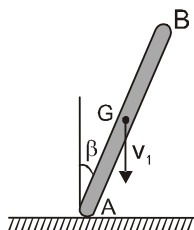
41. Figure (1) shows a mechanical system free of any dissipation. The two spheres (A and B) are each of equal mass  $m$ , and a uniform connecting rod AB of length  $2r$  has mass  $4m$ . The collar is massless. Right above the position of sphere A in Fig. (1) is a tunnel from which balls each of mass  $m$  fall vertically at suitable intervals. The falling balls cause the rods and attached spheres to rotate. Sphere B when reaches the position now occupied by sphere A, suffers a collision from another falling ball and so on. Just before striking, the falling ball has velocity  $v$ . All collision are elastic and the spheres as well as the falling balls can be considered to be point masses. [Olympiad\_2011]



- (a) Find the angular velocity  $\omega_{i+1}$  of the assembly in terms of  $\{\omega_i, v, \text{ and } r\}$  after the  $i^{\text{th}}$  ball has struck it.  
 $\omega_{i+1}$
- (b) The rotating assembly eventually assumes constant angular speed  $\omega^*$ . Obtain  $\omega^*$  in terms of  $v$  and  $r$  by solving the equation obtained in part (a). Argue how a constant  $\omega^*$  does not violate energy conservation.  
 $\omega^* =$   
 Argument
- (c) Solve the expression obtained in part (a) to obtain  $\omega_i$  in terms of  $\{i, v, \text{ and } r\}$ .  
 $\omega_i =$
- (d) If instead of a pair of spheres, we have two pairs of spheres as shown in figure below. What would be the new constant angular speed  $\omega^*$  of the assembly (i.e. the answer corresponding to part (b)).



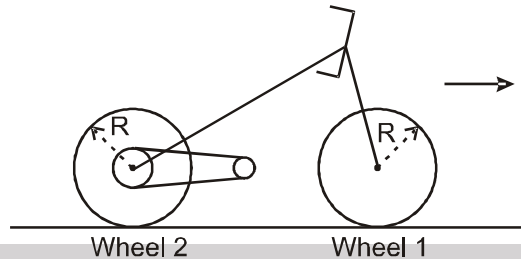
42. A rod of length  $L$  forming an angle  $\beta$  with the vertical strikes a frictionless floor at A with a vertical velocity  $v_1$  and no angular velocity. Assuming that the impact at A is perfectly elastic, derive an expression for the angular velocity of the rod immediately after the impact.





43. Consider a bicycle in vertical position accelerating forward without slipping on a straight horizontal road. The combined mass of the bicycle and the rider is  $M$  and the magnitude of the accelerating torque applied on the rear wheel by the pedal and gear system is  $\tau$ . The radius and the moment of inertia of each wheel is  $R$  and  $I$  (with respect to axis) respectively. The acceleration due to gravity is  $g$ . [INPhO-2013]

(a) Draw the free diagram of the system (bicycle and rider).



(b) Obtain the acceleration  $a$  in terms of the above mentioned quantities.

$a =$

(c) For simplicity assume that the centre of mass of the system is at height  $R$  from the ground and equidistant at  $2R$  from the center of each of the wheels. Let  $\mu$  be the coefficient of friction (both static and dynamic) between the wheels and the ground. Consider  $M \gg I/R^2$  and no slipping. Obtain the conditions for the maximum acceleration  $a_m$  of the bike.

$a_m =$

(d) For  $\mu = 1.0$  calculate  $a_m$ .

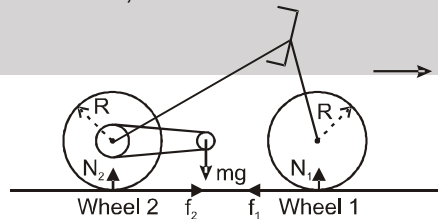
$a_m =$

## HLP Answers

1. (i)  $I = \frac{a\ell^4}{4}$  (ii)  $\frac{a\ell^4}{36}$       2. (a)  $0.25 \text{ m/s}^2$  (b)  $0.125 \text{ m/s}^2$       3.  $\frac{a \cos \alpha \sin^2 \alpha}{2b - a \cos^2 \alpha \sin \alpha}$
4. (a)  $\frac{3g}{2\sqrt{2}\ell}$  (cw) (b)  $\frac{3}{2}g \downarrow$  (c)  $\frac{Mg}{4} \uparrow$       5. (i) (a)  $3g/L$  (cw) (b)  $\left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)g = 1.323g \angle 49.1^\circ$
- (c)  $\left(\frac{\sqrt{3}}{2}\hat{i} - 2\hat{j}\right)g = 2.18g \angle -66.6^\circ$       (ii) (a)  $g/L$  (cw) (b)  $-\left(\frac{\sqrt{3}}{2}\right)g\hat{i}$  (c)  $\left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)g = 1.323g \angle -130.9^\circ$
6.  $\omega = \frac{45}{14} = 3.21 \text{ rad/s}$  (ccw),  $v_s = \frac{1}{7} = 0.143 \text{ m/s} \leftarrow$
7. (a)  $y = \frac{v^2}{\beta x}$  (Hyperbola); (b)  $y = \frac{\sqrt{2ax}}{\omega}$  (Parabola)      8. (a)  $1.633\text{N}$  (b)  $1.224 \text{ m}$
9.  $\frac{M}{m} = \sqrt{15}$       10. (a)  $\frac{2}{3}\sqrt{86} \text{ rad/s}$  (b)  $4 \text{ rad/s}$       11.  $N = 2b\sqrt{\frac{a}{b}}$
12.  $w_1 = F/(m_1 + 2/7m_2)$ ;  $w_2 = 2/7 w_1$       13.  $w = 3g(M + 3m) / (M + 9m + I/R^2)$
14.  $H = \left(\frac{1 - 3 \cos^2 \theta}{1 + 3 \cos^2 \theta}\right)^2 h$ ;  $h = \frac{49 \pi \ell}{144}$       15.  $\sqrt{323.4} \text{ m/s}$  or  $18.52 \text{ m/sec.}$       16.  $A = \left(\frac{a - \mu b}{3a + \mu b}\right)g$
17. (a)  $t = \frac{6a\pi}{\sqrt{3}v_0}$  (b)  $s = \frac{a}{\sqrt{3}}\sqrt{1 + (2\pi + \sqrt{3})^2}$       18.  $\frac{216}{\pi}$
19. (a) pipe rolls without sliding (b) pipe:  $\frac{5}{6} \text{ m/s} \rightarrow$ ,  $\frac{125}{24} \text{ rad/s}$  (ccw); plate:  $\frac{5}{3} \text{ m/s} \rightarrow$
20.  $\frac{mg}{6}$ , up      21. (a)  $v = \frac{2v_0}{3}$ ;  $t_0 = \frac{v_0}{3\mu g}$  (b)  $w = -\mu mg(v_0 t - \frac{3}{2}\mu g t^2)$ ;  $-\frac{1}{6}mv_0^2$



22.  $v = \sqrt{\frac{14gR}{3}}$       23.  $\frac{3W}{8}$       24. 3m
25. 6.3 m/s      26. (i)  $\frac{2}{3} mg$ , (ii)  $\mu_{\min} = 0$ , (iii)  $F = 2 mg$ , (iv)  $\mu_s = \frac{2}{3}$
27. (a)  $\sqrt{3} m \omega^2 \ell$  (b)  $F_y = \sqrt{3} m \omega^2 \ell$   $F_x = -F/4$  ]
28. (i) (a)  $5g/9 \downarrow$  (b)  $g \downarrow$  (c) 0 (ii) (a)  $\frac{13g}{17} \downarrow$  (b)  $g \downarrow$  (c)  $\frac{2g}{3} \downarrow$
29. (a) 6 N (b)  $\vec{\tau}_1 = 0.6\hat{k} - 0.6\hat{j}$ ,  $\vec{\tau}_2 = -0.6\hat{k} - 0.6\hat{j}$
30. (a)  $\theta = \cos^{-1} \frac{4}{7}$  (b)  $v = \sqrt{\frac{4}{7}} gr$  (c)  $\frac{k_T}{k_R} = 6$
31. (a)  $\vec{F} = \frac{2mV}{\Delta t} \hat{i} - \frac{2mV}{\sqrt{3}\Delta t} \hat{k}$ ;  $\vec{N} = \left( \frac{2mV}{\sqrt{3}\Delta t} + mg \right) \hat{k}$ , (b)  $\vec{\tau} = -\left( \frac{4mVh}{\sqrt{3}\Delta t} \right) \hat{j}$       32.  $2\pi \left( \frac{\alpha R^4}{4} + \frac{\beta R^5}{5} \right)$
33.  $I = 3/10 mR^2$
34.  $N = (aB - bA) \hat{k}$ , where  $\hat{k}$  is the unit vector of the z axis  $\ell = |aB - bA|/\sqrt{A^2 + B^2}$
35.  $\beta = 2mgx/R\ell(M + 2m)$
36. (a)  $v = (M/m) \sqrt{2/3g\ell} \sin(\alpha/2)$ ; (b)  $\Delta p = M \sqrt{1/6g\ell} \sin(\alpha/2)$ ; (c)  $x \approx 2/3\ell$
37. (a)  $mg(h-r-r \sin\alpha)$ , (b)  $\frac{10}{7} g \left( \frac{h}{r} - 1 - \sin\alpha \right)$ ,  $-\frac{5}{7} g \cos\alpha$  (c) 4N, 0.2N upward
38. (a)  $\omega_A = \frac{v^2}{R} = 2.0 \text{ m/s}^2$ , the vector  $\omega_A$  is permanently directed to the centre of the wheel ;  
 (b)  $s = 8R = 4.0 \text{ m}$
39.  $T = 7/10 mv^2 (1 + 2/7r^2/R^2)$
40. (i) (a)  $\frac{1.2g}{\ell}$  (cw) (b)  $-0.3(\hat{i} + 2\hat{j}) g$  (ii) (a)  $24g/17 \ell$  (cw) (b)  $12g/17 \downarrow$  (iii)  $2.4g/\ell$  (cw) (b)  $0.5g \downarrow$
41. (a)  $\omega_{i+1} = \frac{7}{13} \omega_i + \frac{6}{13} \frac{v}{r}$       (b)  $\omega^* = \frac{v}{r}$  after this no further collision occurs  
 (c)  $\omega_{i+1} = \frac{v}{r} \left( 1 - \left( \frac{7}{13} \right)^i \right)$       (d)  $\omega^*$  will remain same as in case b.
42.  $\omega = \frac{v_1}{L} \frac{12 \sin\beta}{3 \sin^2\beta + 1}$  (cw)
43. (a) Here  $f_1, f_2$  are frictional forces and  $N_1, N_2$  are normal reactions



- (b)  $a = \frac{\tau}{MR^2 + 2I} R$       (c)  $a \leq \frac{\mu g/2}{(1 - \mu/4)}$       (d)  $a_m = 2g/3$