High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A ball of density d is dropped onto a horizontal solid surface. It bounces elastically from the surface and returns to its original position in a time t_1. Next, the ball is released and it falls through the same height before striking the surface of a liquid of density d_L. **[JEE-1992, 8 Marks]**

(a) If $d < d_L$, obtain an expression (in terms of d, t_1 and d_L) for the time t_2 the ball takes to come back to the position from which it was released.

(b) Is the motion of the ball simple harmonic?

(c) If $d = d_L$, how does the speed of the ball depend on its depth inside the liquid ?
Neglect all frictional and other dissipative forces. Assume the depth of the liquid to be large.

2. Two identical cylindrical vessels with their bases at the same level each contain a liquid of density ρ as shown in figure. The height of the liquid in one vessel is h_2 and other vessels h_1, the area of either base is A. Find the work done by gravity in equalizing the levels when the two vessels are connected.

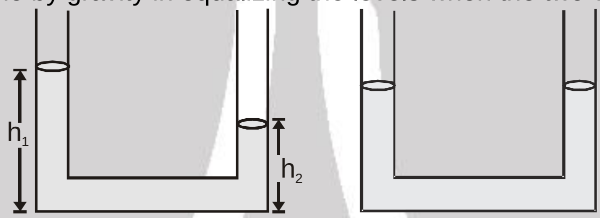
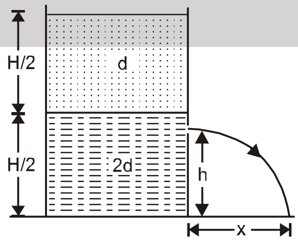


Figure (1) Figure (2)

3. A cyllindrical wooden stick of length L, and radius R and density ρ has a small metal piece of mass m (of negligible volume) attached to its one end. Find the minimum value for the mass m (in terms of given parameters) that would make the stick float vertically in equilibrium in a liquid of density σ (>ρ). **[JEE - 1999, 10/100]**

4. A container of large uniform cross–sectional area A resting on a horizontal surface, holds two immiscible, non–viscous and incompressible liquids of densities d and 2d, each of height $\frac{H}{2}$ as shown in figure. The lower density liquid is open to the atmosphere having pressure P_0. **[JEE - 1995, 5 + 5M]**

(a) A homogeneous solid cylinder of length $L\left(L < \frac{H}{2}\right)$ cross–sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical at the liquid–liquid interface with the length $\frac{L}{4}$ in the denser liquid. Determine:



(i) The density D of the solid and (ii) The total pressure at the bottom of the container.

(b) The cylinder is removed and the original arrangement is restored. A tiny hole of area s (s << A) is punched on the vertical side of the container at a height $h\left(h < \left(\frac{H}{2}\right)\right)$. Determine :

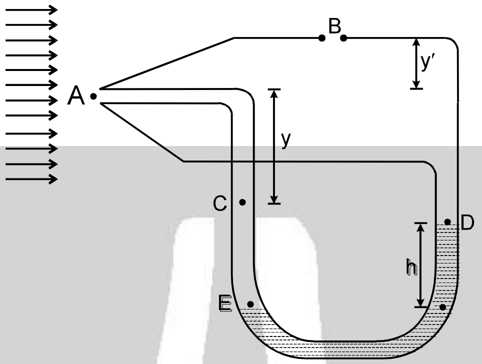
(i) The initial speed of efflux of the liquid at the hole

(ii) The horizontal distance x travelled by the liquid initially and

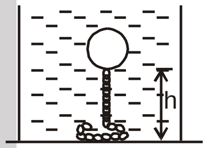
(iii) The height h_m at which the hole should be punched so that the liquid travels the maximum distance x_m initially. Also calculate x_m. [Neglect air resistance in these calculations]

5. A container of cross-section area ‘S’ and height ‘h’ is filled with mercury up to the brim. Then the container is sealed airtight and a hole of small cross section area ' S/n ' (where ‘n’ is a positive constant) is punched in its bottom. Find out the time interval upto which the mercury will come out from the bottom hole. [Take the atmospheric pressure to be equal to h_0 height of mercury column: $h > h_0$]

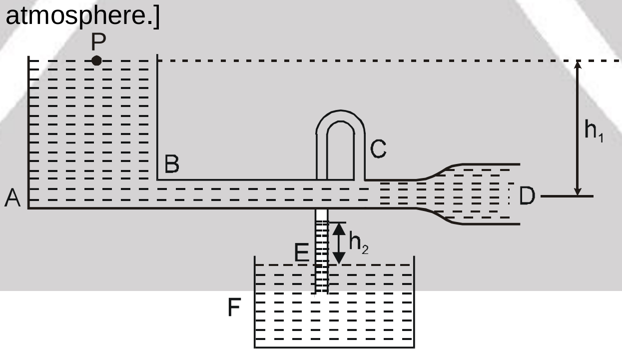
6. A Pitot tube is shown in figure. Wind blows in the direction shown. Air at inlet A is brought to rest, whereas its speed just outside of opening B is unchanged. The U tube contains mercury of density ρ_m. Find the speed of wind with respect to Pitot tube. Neglect the height difference between A and B and take the density of air as ρ_a.



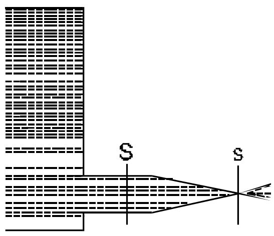
7. One end of a long iron chain of linear mass density λ is fixed to a sphere of mass m and specific density 1/3 while the other end is free. The sphere along with the chain is immersed in a deep lake. If specific density of iron is 7, the height h above the bed of the lake at which the sphere will float in equilibrium is (Assume that the part of the chain lying on the bottom of the lake exerts negligible force on the upper part of the chain.) :



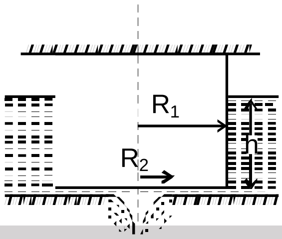
8. Two very large open tanks A & F both contain the same liquid. A horizontal pipe BCD, having a small constriction at C, leads out of the bottom of tank A, and a vertical pipe E containing air opens into the constriction at C and dips into the liquid in tank F. Assume streamline flow and no viscosity. If the cross section area at C is one-half that at D, and if D is at distance h_1 below the level of the liquid in A, to what height h_2 will liquid rise in pipe E? Express your answer in terms of h_1. [Neglect changes in atmospheric pressure with elevation. In the containers there is atmosphere above the water surface and D is also open to atmosphere.]



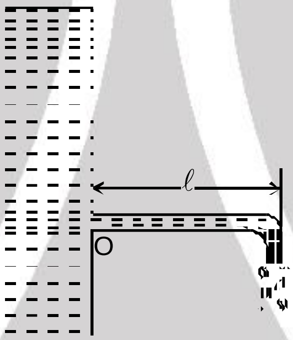
9. A side wall of a wide open tank is provided with a narrowing tube (as shown in figure) throught which water flows out. The cross-sectional area of the tube decrease from S = 3.0 cm^2 to s = 1.0 cm^2. The water level in the tank is h = 4.6 m higher than that in the tube. Neglecting the viscosity of the water, find the horizontal component of the force tending to pull the tube out of the tank.



10. The horizontal bottom of a wide vessel with an ideal fluid has a round orifice of radius R_1 over which a round closed cylinder is mounted, whose radius $R_2 > R_1$. The clearance between the cylinder and the bottom of the vessel is very small, the fluid density is ρ. Find the static pressure of the fluid in the clearance as a function of the distance r from the axis of the orifice (and the cylinder), if the height of the fluid is equal to h.



11. Water flows out of a big tank along a tube bent at right angles, the inside radius of the tube is equal to r = 0.50 cm. The length of the horizontal section of the tube is equal to ℓ = 22 cm.The water flow rate is Q = 0.50 litres per second. Find the moment of reaction forces of flowing water, acting on the tube's walls, relative to the point O.



HLP Answers

1. (a) $\frac{t_1 d_L}{d_L - d}$ (b) No (c) $v = g\frac{t_1}{2}$ = constant

2. $\frac{gA\rho}{4}(h_1 - h_2)^2$ **3.** $m \geq \pi r^2 L\,(\sqrt{\rho\sigma} - \rho)$

4. (a) (i) Density = $\frac{5}{4}d$

(ii) Pressure = $P_0 + \frac{1}{4}(6H + L)\,dg$

(b) (i) $v = \sqrt{\frac{g}{2}(3H - 4h)}$ (ii) $x = \sqrt{h(3H - 4h)}$

(iii) $x_{max} = \frac{3}{4}H$, $h_{max} = \frac{3H}{8}$

5. $t = n\sqrt{\frac{2}{g}(h - h_0)}$ **6.** $v = \sqrt{\frac{2(\rho_m - \rho_a)gh}{\rho_a}}$

7. $\frac{7m}{3\lambda}$ **8.** $h_2 = 3h_1$

9. $F = \rho gh\,(S - s)^2/S = 6N$

10. $p = p_0 + \rho gh\,(1 - R_1^2/r^2)$, where $R_1 < r < R_2$, p_0 is the atmospheric pressure.

11. $N = \rho\ell Q^2/\pi r^2 = 0.7$ N.m.