High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A string that is stretched between fixed supports separated by 75.0 cm has resonant frequencies of 420 and 315 Hz with no intermediate resonant frequencies. What are (a) the lowest resonant frequencies and (b) the wave speed?

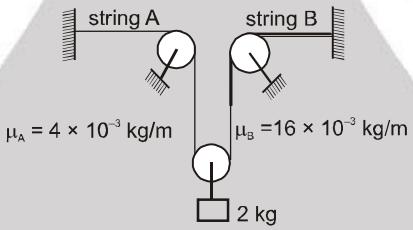
2. A steel wire of length 50 $\sqrt{3}$ cm is connected to an aluminium wire of length 60 cm and stretched between two fixed supports. The tension produced is 104 N, if the cross section area of each wire is 1mm^2. If a transverse wave is set up in the wire, find the lowest frequency for which standing waves with node at the joint are produced. (density of aluminium = 2.6 gm/cm^3 and density of steel = 7.8 gm/cm^3)

3. An aluminium wire of cross-sectional area 1×10^{-6} m^2 is joined to a steel wire of same cross-sectional area. This compound wire is stretched on a sonometer, pulled by a weight of 10 kg. The total length of the compound wire between the bridges is 1.5 m, of which the aluminium is 0.6 m and the rest is steel wire. Transverse vibrations are set up in the wire by using an external force of variable frequency. Find the lowest frequency of excitation for which standing waves are formed such that the joint in the wire is a node. What is the total number of nodes observed at this frequency, excluding the two at the ends of the wire ? The density of aluminium is 2.6×10^3 kg/m^3 and that of steel is 1.04×10^4 kg/m^3. **[REE - 1983]**

4. The fundamental frequency of a sonometer wire increases by 6 Hz if its tension is increased by 44 % keeping the length constant. Find the change in the fundamental frequency of the sonometer when the length of the wire is increased by 20 % keeping the original tension in the wire. **[JEE - 1997; 5]**

5. A metal wire with volume density ρ and young's modulus Y is streatched between rigid supports. At temperature T, the speed of a transverse wave is found to be v_1. When temperature decreases T – ΔT, the speed increases to v_2. Determine coefficient of linear expansion of wire.

6. Find velocity of wave in string A & B.



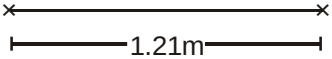
7. A string of length 50 cm is vibrating with a fundamental frequency of 400 Hz in horizontal position. In the fundamental mode the maximum displacement at the middle is 2 cm from equilibrium position and the tension in the string is 10 N. Determine the maximum value of the vertical component of force on the end supporting.

8. A string fixed at both ends is vibrating in the lowest mode of vibration for which a point at quarter of its length from one end is a point of maximum displacement. The frequency of vibration in this mode is 100 Hz. What will be the frequency emitted when it vibrates in the next higher mode such that this point is again a point of maximum displacement.

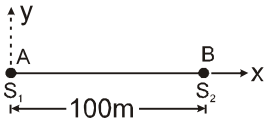
9. A guitar string is 180 cm long and has a fundamental frequency of 90 Hz. Where should it be pressed to produce a fundamental frequency of 135 Hz?

10. A piano wire weighing 4 g and having a length of 90.0 cm emits a fundamental frequency corresponding to the "Middle C" (v = 125 Hz). Find the tension in the wire.

11. Length of a sonometer wire is 1.21 m. Find the length of the three segments for fundamental frequencies to be in the ratio 1 : 2 : 3.



12. In the figure shown A and B are two ends of a string of length 100m. S_1 and S_2 are two sources due to which points 'A' and 'B' oscillate in 'y' and 'z' directions respectively according to the equation $y = 2 \sin (100\ \pi\ t + 30°)$ and $z = 3 \sin (100\ \pi\ t + 60°)$ where t is in sec and y is in mm. The speed of propagation of disturbance along the string is 50 m/s. Find the instantaneous position vector (in mm) and velocity vector in (m/s) of a particle 'P' of string which is at 25m from A. You have to find these parameters after both the disturbances from S_1 and S_2 have reached 'P'. Also find the phase difference between the waves at the point 'P' when they meet at 'P' first time.



13. A string of mass m is fixed at both ends. The fundamental tone oscillations are excited with angular frequency ω and maximum displacement amplitude a_{max}. Find :
(a) the maximum kinetic energy of the string;
(b) the mean kinetic energy of the string averaged over one oscillation period.

HLP Answers

1. (a) 105 Hz; (b) 157.5 m/s **2.** 1000/3Hz **3.** 162 vibrations/sec, 3

4. Decrease by 5Hz **5.** $\alpha = \frac{\rho(v_2^2 - v_1^2)}{Y\Delta T}$ **6.** 50 m/sec; 25 m/sec

7. $\frac{2\pi}{5}$ N **8.** 300 Hz **9.** 120 cm from an end.

10. 225 N **11.** 0.66 m, 0.33 m, 0.22 m

12. $\vec{r}_{(\text{in mm})} = 25000\hat{i} + 2\sin(100\pi t + 30°)\hat{j} + 3\sin(100\pi t + 60°)\hat{k}$

$\vec{v}(\text{in m/s}) = 0.2\pi\cos(100\pi t + 30°)\hat{j} + 0.3\pi\cos(100\pi t + 60°)\hat{k}$

Phase difference at time 't' is 30° constant always after they meet at 'P'.

13. (a) $T_{max} = 1/4\ m\ \omega^2 a^2_{max}$; (b) $T = 1/8\ m\omega^2\ a^2_{max}$.