

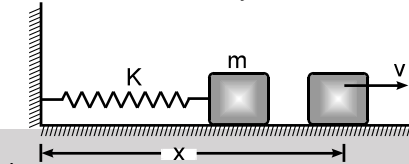


# High Level Problems (HLP)

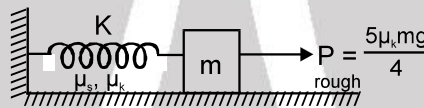
Marked Questions can be used as Revision Questions.

## SUBJECTIVE QUESTIONS

1. A block of mass 'm' is pushed against a spring of spring constant 'k' fixed at one end to a wall. The block can slide on a frictionless table as shown in the figure. The natural length of the spring is  $L_0$  and it is compressed to one-fourth of natural length and the block is released. Find its velocity as a function of its distance (x) from the wall and maximum velocity of the block. The block is not attached to the spring.



2. A block of mass m rests on a rough horizontal plane having coefficient of kinetic friction  $\mu_k$  and coefficient of static friction  $\mu_s$ . The spring is in its natural length, when a constant force of magnitude  $P = \frac{5\mu_k mg}{4}$  starts acting on the block. The spring force F is a function of extension x as  $F = kx^3$ . (Where k is spring constant)

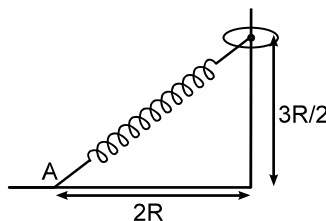


- (a) Comment on the relation between  $\mu_s$  and  $\mu_k$  for the motion to start.  
 (b) Find the maximum extension in the spring (Assume the force P is sufficient to make the block move).
3. A particle of mass  $m = 1$  kg lying on x-axis experiences a force given by law

$$\vec{F} = x(3x - 2) \hat{i} \text{ Newton,}$$

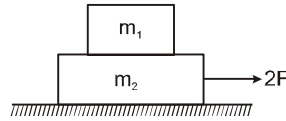
where x is the x-coordinate of the particle in meters.

- (a) Locate the points on x-axis where the particle is in equilibrium.  
 (b) Draw the graph of variation of force F (y-axis) with x-coordinate of the particle (x-axis). Hence or otherwise indicate at which positions the particle is in stable or unstable equilibrium.  
 (c) What is the minimum speed to be imparted to the particle placed at  $x = 4$  meters such that it reaches the origin.
4. A ring of mass 'm' can slide along a fixed rough vertical rod as shown in fig. The ring is connected by a spring of spring constant  $k = \frac{4mg}{R}$  where  $2R$  is the natural length of spring. The other end of spring is fixed to the ground at point A at a horizontal distance of  $2R$  from the base of the rod. If the ring is released from a height of  $3R/2$  & it reaches the ground with a speed  $\sqrt{3gR}$ , find co-efficient of friction between the rod & ring.

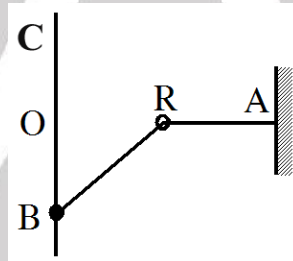




5. A block of mass  $m_1$  is kept over another block of mass  $m_2$  and the system rests on a horizontal surface (as shown in figure). A constant horizontal force  $2F$  acting on the lower block produces an acceleration  $\frac{F}{(m_1 + m_2)}$  in the system, the two blocks always move together. (a) Find the coefficient of kinetic friction between the bigger block and the horizontal surface. (b) Find the frictional force acting on the smaller block. (c) Find the work done by the force of friction on the smaller block by the bigger block during a displacement  $x$  of the system.



6. A box having mass 400 kg is to be slowly slide through 10 m on a horizontal straight track having friction coefficient 0.2 with the box. (a) Find the work done by the person pulling the box with a rope at an angle  $\theta$  with the horizontal. (b) Find the work when the person has chosen a value of  $\theta$  which ensures him the minimum magnitude of the force. [ $g = 10 \text{ m/s}^2$ ]
7. A small bead 'B' of mass  $m$  is free to slide on a fixed smooth vertical wire, as indicated in the diagram. One end of a light elastic string, of unstretched length  $a$  and force constant  $2mg/a$  is attached to B. The string passes through a smooth fixed ring R and the other end of the string is attached to the fixed point A, AR being horizontal. The point O on the wire is at same horizontal level as R and  $AR = RO = a$ .  
 (i) In the equilibrium position, find OB.  
 (ii) The bead B is raised to a point C of the wire above O, where  $OC = a$  and is released from rest. Find the speed of the bead as it passes O and find the greatest depth below O of the bead in the subsequent motion.



8. A particle of mass  $m$  approaches a region of force starting from  $r = +\infty$ . The potential energy function in terms of distance  $r$  from the origin is given by,

$$U(r) = \frac{K}{2a^3} (3a^2 - r^2) \text{ for } 0 \leq r \leq a$$

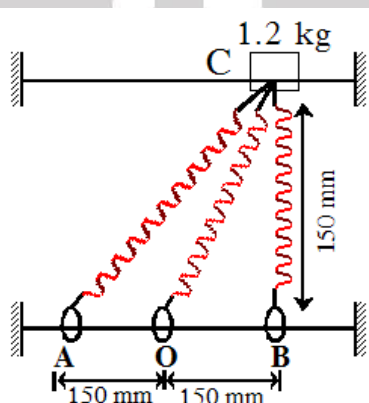
$$= K/r \text{ for } r \geq a$$

where  $K > 0$  (positive constant)

- (a) Derive the force  $F(r)$  and determine whether it is repulsive or attractive.  
 (b) With what velocity should the particle start at  $r = \infty$  to cross over to other side of the origin.  
 (c) If the velocity of the particle at  $r = \infty$  is  $\sqrt{\frac{2K}{am}}$  towards the origin describe the motion.
9. A uniform string of mass 'M' and length  $2a$ , is placed symmetrically over a smooth and small pulley and has particles of masses 'm' and 'm'' attached to its ends; show that when the string runs off the peg its velocity is  $\sqrt{\left\{ \frac{M + 2(m - m')}{M + m + m'} ag \right\}}$  Assume that  $m > m'$ .



10. A single conservative force  $F(x)$  acts on a 1.0 kg particle that moves along the x-axis. The potential energy
- $$U(x) \text{ is given by : } U(x) = 20 + (x - 2)^2$$
- where  $x$  is in meters. At  $x = 5.0$  m the particle has a kinetic energy of 20 J.
- What is the mechanical energy of the system ?
  - Make a plot of  $U(x)$  as a function of  $x$  for  $-10 \text{ m} < x < 10 \text{ m}$ , and on the same graph draw the line that represents the mechanical energy of the system.  
Use part (ii) to determine
    - The least value of  $x$  and
    - The greatest value of  $x$  between which the particle can move.
    - The maximum kinetic energy of the particle and
    - The value of  $x$  at which it occurs.
  - Determine the equation for  $F(x)$  as a function of  $x$ .
  - For what value of  $x$  does  $F(x) = 0$  ?
11. A 1.2 kg collar C may slide without friction along a fixed smooth horizontal rod. It is attached to three springs each of constant  $K = 400 \text{ N/m}$  and 150 mm undeformed length. Knowing that the collar is released from rest in the position shown. Determine the maximum velocity it will reach in its motion. [Here A, O, B are fixed points.]

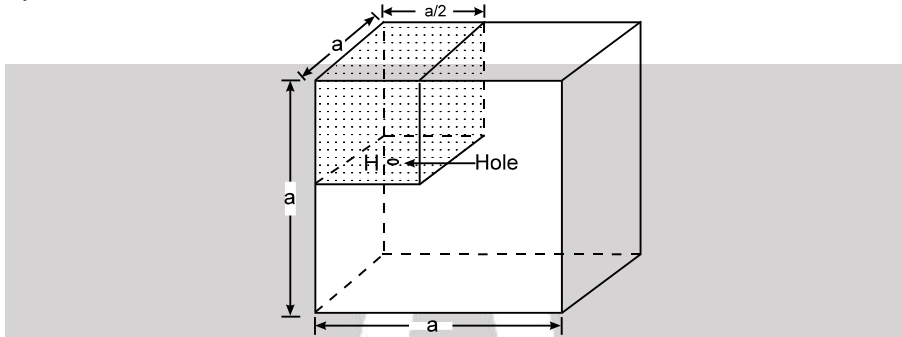


12. A block of mass 4 kg is moved from rest on a smooth inclined plane of inclination  $53^\circ$  by applying a constant force of 40 N parallel to the incline. The force acts for two seconds. (a) Show that the work done by the applied force is not less than 160 J. (b) Find the work done by the force of gravity in that two seconds if the work done by the applied force is 160 J. (c) Find the kinetic energy of the block at the instant the force ceases to act in case (b). [Take  $g = 10 \text{ m/s}^2$ ]
13. There is a vertically suspended spring, mass system. When block of mass 10 kg is suspended from lower end of the spring, it is stretched by 20 cm under the load of block at equilibrium position. When an upward speed of 4 m/s is imparted to the block by giving a sharp impulse from below, how much high
14. A certain spring is found not to obey Hooke's law, it exerts a restoring force  $F(x) = -\alpha x - \beta x^2$  if it is stretched or compressed, where  $\alpha = 48 \text{ N/m}$  and  $\beta = 24 \text{ N/m}^2$ . The mass of the spring is negligible. An object with mass 1 kg on a frictionless, horizontal surface is attached to the spring, pulled a distance 1m to the right to stretch the spring and released. The speed of the object when it is 0.5m to the right of the  $x = 0$  equilibrium position is
15. Wind entering in a wind mill with a velocity of 20 m/sec facing area of the windmill is  $10 \text{ m}^2$  and density of air is  $1.2 \text{ kg/m}^3$ . If wind energy is converted into electrical energy with 33.3% efficiency, then find electrical power produced by the wind mill in kw.
16. An engine can pull 4 coaches at a maximum speed of 20 m/s. Mass of the engine is twice the mass of every coach. Assuming resistive forces to be proportional to the weight, approximate maximum speeds of the engine when it pulls 12 and 6 coaches are (power of engine remains constant) :
17. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain "n" times water from the same pipe in the same time, the factor by which the power of the motor should be increased is:

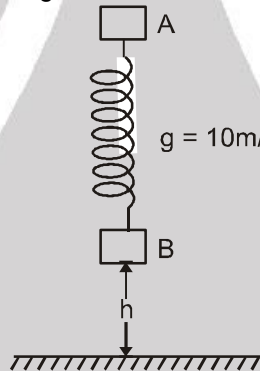


18. A chain of mass  $M$  and length  $\ell$  is held vertically such that its bottom end just touches the surface of a horizontal table. The chain is released from rest. Assume that the portion of chain on the table does not form a heap. The momentum of the portion of the chain above the table after the top end of the chain falls down by a distance  $\frac{\ell}{8}$ .

19. The figure shows a hollow cube of side 'a' of volume  $V$ . There is a small chamber of volume  $\frac{V}{4}$  in the cube as shown. This chamber is completely filled by  $m$  kg of water. Water leaks through a hole  $H$  and spreads in the whole cube. Then the work done by gravity in this process assuming that the complete water finally lies at the bottom of the cube is :

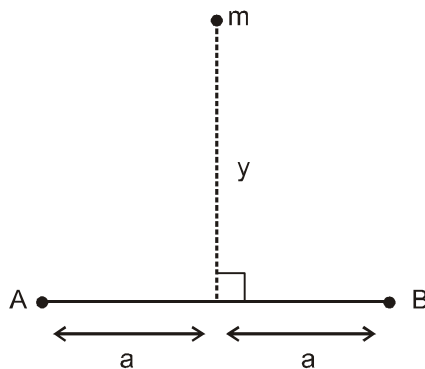


20. From what minimum height 'h' in metre must the system be released when spring is in its natural length as shown in the figure. So that after perfectly inelastic collision. ( $e = 0$ ), of block B, with ground, B may be lifted off ground. (Take  $k = 40 \text{ N/m}$ ,  $g = 10 \text{ m/s}^2$ ,  $m_A = 2\text{kg}$ ,  $m_B = 4\text{kg}$ )



21. Potential energy of a particle of mass  $m$ , depends on distance  $y$  from line AB according to given relation  $U = \frac{K}{\sqrt{y^2 + a^2}}$ , where  $K$  is a positive constant. A particle of mass  $m$  is projected from  $y = \sqrt{3}a$  towards

line AB (perpendicular to it) then minimum velocity so that it cannot return to its initial point is  $\sqrt{\frac{K}{aNm}}$ , calculate  $N$ .



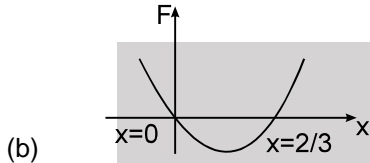


# HLP Answers

1.  $v = \sqrt{\frac{k}{m} \left[ \left( \frac{3L_0}{4} \right)^2 - (L_0 - x)^2 \right]}$  when  $x < L_0$  ;  $v_{\max} = \frac{3L_0}{4} \sqrt{\frac{k}{m}}$  when  $x \geq L_0$

2. (a)  $5\mu_k > 4\mu_s$  ; (b)  $x = \left( \frac{\mu_k mg}{K} \right)^{1/3}$

3. (a)  $x = 0$  and  $x = \frac{2}{3} \text{ m}$



(b) The particle is in stable equilibrium at  $x = 0$  metre and unstable equilibrium at  $x = \frac{2}{3}$  metre

(c)  $v = \sqrt{\frac{2600}{27}} \text{ m/s}$

4.  $\frac{1}{8(3 - 4 \ln 2)}$

5. (a)  $\frac{F}{(m_1 + m_2)g}$

(b)  $\frac{m_1 F}{(m_1 + m_2)}$  (c)  $\frac{m_1 F x}{(m_1 + m_2)}$

6. (a)  $\frac{40000}{5 + \tan \theta} \text{ J}$

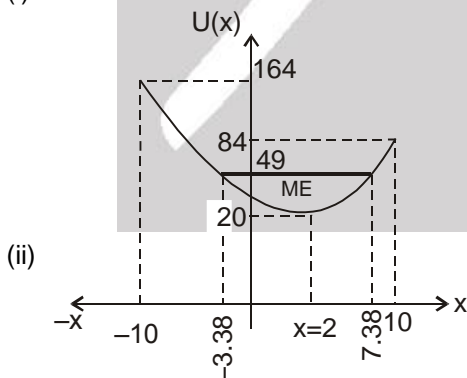
(b) 7692.31 J 7690 J

7. (i)  $OB = a/2$  (ii)  $v = \sqrt{4ag}$  ,  $d = 2a$

8. (a) repulsive (b)  $v > \sqrt{\frac{3k}{am}}$  (c) stops at  $r = a$  & then reaches to  $r = \infty$ .

9.  $v = \sqrt{ag \frac{M + 2(m - m')}{M + m + m'}}$

10. (i) 49 J



(iii)  $-\sqrt{29} + 2 \approx -3.38 \text{ m}$  (iv)  $-\sqrt{29} + 2 \approx 7.38 \text{ m}$  (v) 29 J (vi)  $x = 2 \text{ m}$   
 (vii)  $F = 2(2 - x)$  (viii)  $x = 2$

11.  $\frac{15}{2} \left\{ (\sqrt{5} - 1)^2 - (\sqrt{2} - 1)^2 \right\}^{1/2} \text{ m/s} = 3.189 \text{ m/s}$  12. (b) -128 J (c) 32 J 13. 0.56 m = 56 cm

14.  $5\sqrt{2} \text{ m/s}$  15. 16 16. 8.5 m/s and 15 m/s respectively 17.  $n^3$

18.  $\frac{7}{16} M \sqrt{gl}$  19.  $5/8 \text{ mga}$  20. 2 21. 1