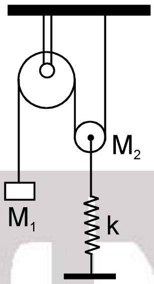
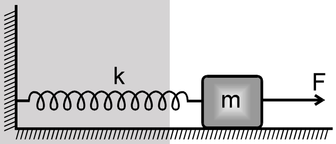
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

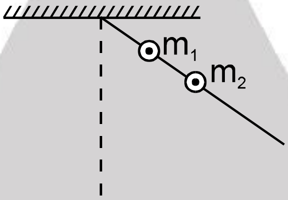
1. What would be the period of the free oscillations of the system shown here if mass M_1 is pulled down a little force constant of the spring is k, mass of fixed pulley is negligible and movable pulley is smooth



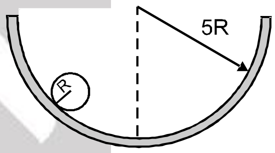
2. A constant force produces maximum velocity V on the block connected to the spring of force constant K as shown in the fig. When the force constant of spring becomes 4K, then find maximum velocity of the block. Assume that initially the spring is in relaxed state.



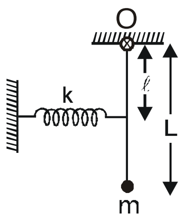
3 Two point masses m_1 and m_2 are fixed to a light rod hinged at one end. The masses are at distances ℓ_1 and ℓ_2 respectively from the hinge. Find the time period of oscillation (small amplitude) of this system in seconds if $m_1 = m_2$, $\ell_1 = 1$m, $\ell_2 = 3$m.



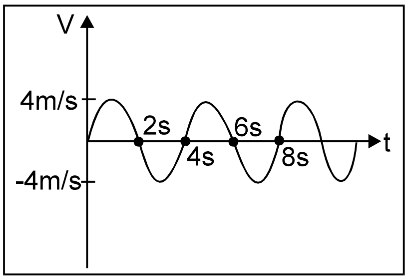
4. A solid sphere (radius = R) rolls without slipping in a cylindrical vessel (radius = 5R). Find the angular frequency of small oscillations of the sphere in s^{-1}. Take R = $\frac{1}{14}$ m and g = 10 m/s^2. (Axis of cylinder is fixed and horizontal).



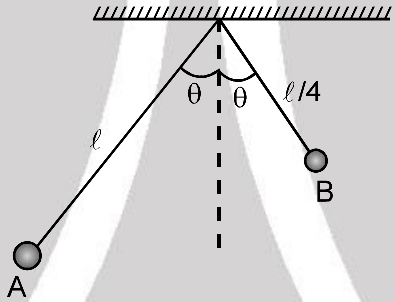
5. A particle of mass m is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point O. The spring is undeformed when the rod is vertical as shown in fig. If the period of oscillation of the system is $\pi\sqrt{\frac{L}{n}}$, when it is slightly displaced from its mean position then find n. Take k = $\frac{9mgL}{\ell^2}$ and g = 10m/s^2.



6. If velocity of a particle moving along a straight line changes sinusoidally with time as shown in the given graph. Find the average speed over time interval t = 0 to t = 2 (2n – 1) seconds, n being any positive integer.



7. Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle θ is very small.



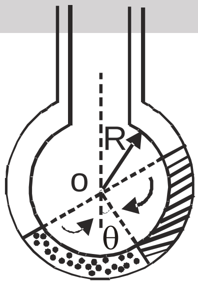
8. A particle of mass 'm' is moving in the x-y plane such that its x and y coordinate vary according to the law x = a sin ω t and y = a cos ω t where 'a' and 'ω' are positive constants and 't' is time. Find

(a) equation of the path. Name the trajectory (path)

(b) whether the particle moves in clockwise or anticlockwise direction

(c) magnitude of the force on the particle at any time t.

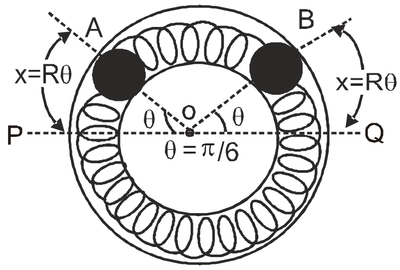
9. Two non–viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius R and small cross–section kept fixed in a vertical plane as shown in fig. Each liquid occupies one–fourth the circumference of the tube.



(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

10. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring–mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest.

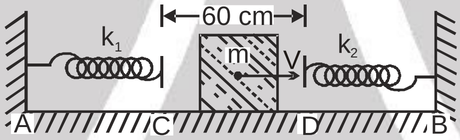


(i) Calculate the frequency of oscillation of ball B.

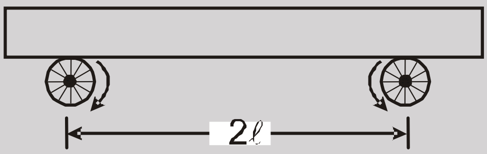
(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system ? **[1993 ; 6M]**

11. Two light springs of force constant k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed to rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block. (k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 g) **[1985 ; 6M]**



12. Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass M is placed on it symmetrically. The friction co-efficient between each wheel and the plank is μ. Find the frequency of oscillations, when plank is slightly displaced along its length and released.



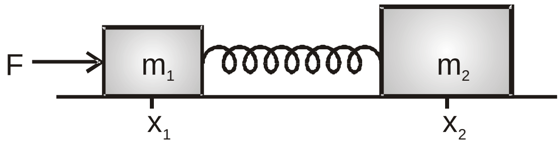
13. **The Cubic Potential :** Consider a particle of mass m moving in one dimension under the influence of potential energy $u(x) = \frac{m\omega^2 x^2}{2} - \delta x - \frac{\alpha x^3}{3}$

Here ω, δ and a are real and positive.

(a) Sketch typical plots of u(x) and identify extrema if any.

(b) Consider the case where (in appropriate units) we have m = 1, $\omega = \sqrt{2}$, α = 1 and δ = 1/2. Sketch the potential energy u(x). If the total energy of the particle moving in this one-dimensional potential is E = 0 (in same units), discuss the motion of the particle in terms of allowed regions, boundedness and periodicity.

14. Two blocks of masses m_1 = 1.0 kg and m_2 = 2.0 kg are connected by a massless elastic spring and are at rest on a smooth horizontal surface with the spring at its natural length.



A horizontal force of constant magnitude F = 6.0 N is applied to the block m_1 for a certain time t in which m_1 suffers a displacement Δx_1 = 0.1 m and Δx_2 = 0.05 m. Kinetic energy of the system with respect to center of mass is 0.1 J. The force F is then withdrawn.

(a) Calculate t.

(b) Calculate the speed and the kinetic energy of the center of mass after the force is withdrawn.

(c) Calculate the energy stored in the system

HLP Answers

1. $T = 2\pi\sqrt{\dfrac{M_2 + 4M_1}{k}}$ **2.** V/2 **3.** π **4.** 5

5. 25 **6.** $\dfrac{8}{\pi}$ m/s **7.** $\dfrac{\pi}{3}\sqrt{\dfrac{\ell}{g}}$

8. (a) $x^2 + y^2 = a^2$, circle (b) The particle moves in clock wise sense.

(c)The magnitude of force = m $\sqrt{a_x^2 + a_y^2}$ = $m\omega^2 a$

9. (a) $\tan^{-1}\left(\dfrac{1}{5}\right)$ (b) $2\pi\sqrt{\dfrac{R}{6.11}}$

10. (i) f = $\dfrac{1}{2\pi}\sqrt{\dfrac{4\times0.1}{0.1}}$ = $\dfrac{1}{\pi}$ Hz (ii) V = 0.0628 (iii) 3.9 × 10^{-4} J **11.** 2.82 s

12. $2\pi\sqrt{\dfrac{\ell}{\mu g}}$ **13.** (b) between x = 0 and x = $\dfrac{3-\sqrt{3}}{2}$. U is (–ve). So, K.E. is +ve.

14. (a) 0.26s (b) 0.52 ms^{-1}, 0.40 J (c) 0.20 J