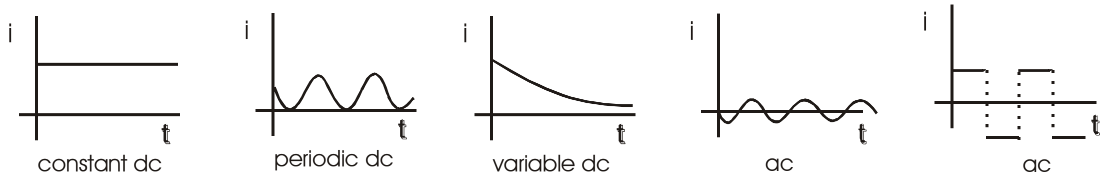
# ALTERNATING CURRENT

## 1. AC AND DC CURRENT :

A current that changes its direction periodically is called alternating current (AC). If a current maintains its direction constant it is called direct current (DC).



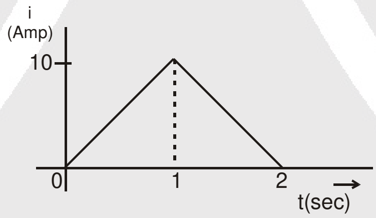

If a function suppose current, varies with time as $i = I_m \sin(\omega t+\phi)$, it is called sinusoidally varying function. Here $I_m$ is the peak current or maximum current and i is the instantaneous current. The factor $(\omega t+\phi)$ is called phase. $\omega$ is called the angular frequency, its unit rad/s. Also $\omega = 2\pi$ f where f is called the frequency, its unit $s^{-1}$ or Hz. Also frequency f = 1/T where T is called the time period.

## 2. AVERAGE VALUE :

Average value of a function, from $t_1$ to $t_2$, is defined as $\langle f \rangle = \dfrac{\int_{t_1}^{t_2} f\,dt}{t_2 - t_1}$. We can find the value of $\int_{t_1}^{t_2} f\,dt$ graphically if the graph is simple. It is the area of f-t graph from $t_1$ to $t_2$.

## Solved Example

**Example 1.** Find the average value of current shown graphically, from t = 0 to t = 2 sec.

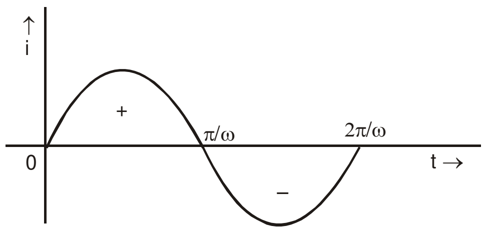



**Solution :** From the i – t graph, area from t = 0 to t = 2 sec = $\frac{1}{2} \times 2 \times 10$ = 10 Amp. sec.

$\therefore$ Average Current = $\frac{10}{2}$ = 5 Amp.

**Example 2.** Find the average value of current from t = 0 to t = $\frac{2\pi}{\omega}$ if the current varies as $i = I_m \sin \omega t$.

**Solution :** $\langle i \rangle = \dfrac{\int_0^{2\pi/\omega} I_m \sin\omega t\,dt}{\frac{2\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1-\cos\omega\frac{2\pi}{\omega}\right)}{\frac{2\pi}{\omega}} = 0.$

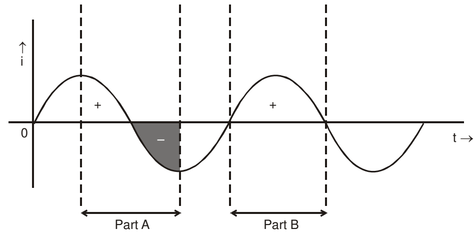



It can be seen graphically that the area of i – t graph of one cycle is zero.

$\therefore$ $\langle i \rangle$ in one cycle = 0.

**Example 3.** Show graphically that the average of sinusoidally varying current in half cycle may or may not be zero.

**Solution :** Figure shows two parts A and B, each half cycle.
In part A we can see that the net area is zero
$\therefore$ < i > in part A is zero.
In part B,
area is positive hence in this part < i > $\neq$ 0.




**Example 4.** Find the average value of current i = $I_m \sin \omega t$ from (i) t = $\frac{\pi}{\omega}$ 0 to t = $\frac{\pi}{2\omega}$ (ii) t = to t = $\frac{3\pi}{2\omega}$.

**Solution :** (i) $\langle i \rangle = \dfrac{\int_0^{\pi/\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = \dfrac{\frac{I_m}{\omega}\left(1 - \cos \omega \frac{\pi}{\omega}\right)}{\frac{\pi}{\omega}} = \dfrac{2I_m}{\pi}$ (ii) $\langle i \rangle = \dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin \omega t dt}{\frac{\pi}{\omega}} = 0.$

**Example 5.** Current in an A.C. circuit is given by i = $2\sqrt{2}\sin(\pi t + \pi/4)$, then the average value of current during time t = 0 to t = 1 sec is:

**Solution :** $\langle i \rangle = \dfrac{\int_0^1 i \; dt}{1} = 2\sqrt{2}\int_0^1 \sin\left(\pi t + \frac{\pi}{4}\right) = \dfrac{4}{\pi}$ **Ans.**

## 3. ROOT MEAN SQUARE VALUE :

Root Mean Square Value of a function, from $t_1$ to $t_2$, is defined as $f_{rms} = \sqrt{\dfrac{\int_{t_1}^{t_2} f^2 dt}{t_2 - t_1}}$.

## *Solved Example*

**Example 6.** Find the rms value of current from t = 0 to t = $\frac{2\pi}{\omega}$ if the current varies as i = $I_m \sin \omega t$.

**Solution :** $i_{rms} = \sqrt{\dfrac{\int_0^{2\pi/\omega} {I_m}^2 \sin^2 \omega t dt}{\frac{2\pi}{\omega}}} = \sqrt{\dfrac{{I_m}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

**Example 7.** Find the rms value of current i = $I_m \sin \omega t$ from (i) t = 0 to t = $\frac{\pi}{\omega}$ (ii) t = $\frac{\pi}{2\omega}$ to t = $\frac{3\pi}{2\omega}$.

**Solution :** (i) $i_{rms} = \sqrt{\dfrac{\int_0^{\pi/\omega} {I_m}^2 \sin^2 \omega t dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{{I_m}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

(ii) $\langle i \rangle = \sqrt{\dfrac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m^2 \sin^2 \omega t \; dt}{\frac{\pi}{\omega}}} = \sqrt{\dfrac{{I_m}^2}{2}} = \dfrac{I_m}{\sqrt{2}}$

**Note:**
- The r m s values for one cycle and half cycle (either positive half cycle or negative half cycle) is same.
- From the above two examples note that for sinusoidal functions **rms value** (Also called **effective value**) = $\dfrac{\text{peak value}}{\sqrt{2}}$ or $I_{rms} = \dfrac{I_m}{\sqrt{2}}$

**Example 8.** Find the effective value of current i = 2 sin 100 $\pi$t + 2 cos (100 $\pi$t + 30º).

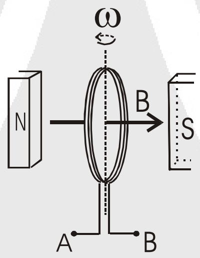
**Solution :** The equation can be written as i = 2 sin 100 $\pi$t + 2 sin (100 $\pi$t + 120º)

so phase difference $\phi$ = 120º $\qquad I_m)_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$= \sqrt{4+4+2\times2\times2\left(-\frac{1}{2}\right)}$ = 2, so effective value or rms value = 2 / $\sqrt{2}$ = $\sqrt{2}$ A

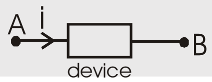
## 4. AC SINUSOIDAL SOURCE :

Figure shows a coil rotating in a magnetic field. The flux in the coil changes as $\varphi$ = NBA cos ($\omega$t + $\phi$). Emf induced in the coil, from Faraday's law is $\dfrac{-d\varphi}{dt}$ = N B A $\omega$ sin ($\omega$t + $\phi$). Thus the emf between the points A and B will vary as E = $E_0$ sin ($\omega$t +$\phi$) .The potential difference between the points A and B will also vary as V = $V_0$ sin ($\omega$t + $\phi$). The symbolic notation of the above arrangement is A—~—B. We do not put any + or – sign on the AC source.




## 5. POWER CONSUMED OR SUPPLIED IN AN AC CIRCUIT:

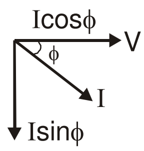
Consider an electrical device which may be a source, a capacitor, a resistor, an inductor or any combination of these. Let the potential difference be V = $V_A$–$V_B$ = $V_m$ sin $\omega$t. Let the current through it be i = $I_m$ sin($\omega$t + $\phi$). Instantaneous power P consumed by the device = V i = ($V_m$ sin $\omega$t ) ($I_m$ sin($\omega$t + $\phi$))




Average power consumed in a cycle = $\dfrac{\int_0^{2\pi/\omega} P\,dt}{\frac{2\pi}{\omega}}$ = $V_m I_m$ cos $\phi$ = $\dfrac{V_m}{\sqrt{2}} \cdot \dfrac{I_m}{\sqrt{2}}$ . cos $\phi$ = $V_{rms} I_{rms}$ cos $\phi$.

Here cos $\phi$ is called **power factor**.

**Note :** Isin$\phi$ is called "wattless current".



## Solved Examples

**Example 9.** When a voltage $v_s = 200\sqrt{2}\sin(\omega t + 15^\circ)$ is applied to an AC circuit the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$ then average power consumed in the circuit is

(A) 200 watt (B) $400\sqrt{2}$ watt (C) $100\sqrt{6}$ watt (D) $200\sqrt{2}$ watt

**Solution :** $P_{av} = v_{rms}\, I_{rms} \cos\phi = \frac{200\sqrt{2}}{\sqrt{2}} \cdot \frac{2}{\sqrt{2}} \cos(30^\circ) = 100\sqrt{6}$ watt

## 6. SOME DEFINITIONS :

The factor $\cos\phi$ is called Power factor.

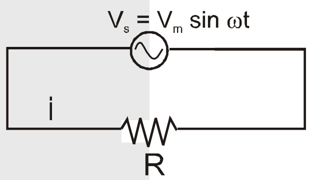
$I_m \sin\phi$ is called wattless current.

Impedance Z is defined as $Z = \frac{V_m}{I_m} = \frac{V_{rms}}{I_{rms}}$

$\omega L$ is called inductive reactance and is denoted by $X_L$. $\frac{1}{\omega C}$ is called capacitive reactance and is denoted by $X_C$.

## 7. PURELY RESISTIVE CIRCUIT:

Writing KVL along the circuit,




$V_s - iR = 0$

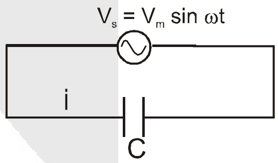
or $i = \frac{V_s}{R} = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$

⇒ We see that the phase difference between potential difference across resistance, $V_R$ and $i_R$ is 0.

$I_m = \frac{V_m}{R}$ ⇒ $I_{rms} = \frac{V_{rms}}{R}$ ; $\langle P \rangle = V_{rms} I_{rms} \cos\phi = \frac{V_{rms}^2}{R}$

## 8. PURELY CAPACITIVE CIRCUIT:

Writing KVL along the circuit, $V_s - \frac{q}{C} = 0$

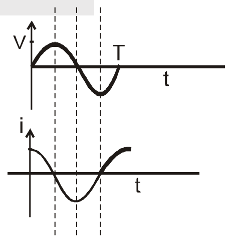



or $i = \frac{dq}{dt} = \frac{d(CV)}{dt} = \frac{d(CV_m \sin\omega t)}{dt}$

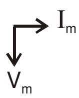
$= CV_m\omega \cos\omega t = \frac{V_m}{1/\omega C} \cos\omega t = \frac{V_m}{X_C} \cos\omega t = I_m \cos\omega t.$

$X_C = \frac{1}{\omega C}$ and is called capacitive reactance. Its unit is ohm Ω.

From the graph of current versus time and voltage versus time $\frac{T}{4}$, it is clear that current attains its peak value at a time before the time at which voltage attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference




$= \omega\Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$. $i_C$ leads $v_C$ by $\pi/2$ Diagrammatically (phasor diagram) it is represented as




Since $\phi = 90^\circ$, $\langle P \rangle = V_{rms} I_{rms} \cos\phi = 0$

## Solved Examples

**Example 10.** An alternating voltage E = 200 $\sqrt{2}$ sin (100 t) V is connected to a 1μF capacitor through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

**Solution :** Comparing E = 200 $\sqrt{2}$ sin (100 t) with E = $E_0$ sin ωt we find that,

$E_0 = 200\sqrt{2}$ V and ω = 100 (rad/s)

So, $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4\,\Omega$

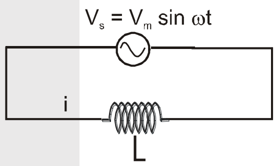
And as ac instruments reads rms value, the reading of ammeter will be,

$$I_{rms} = \frac{E_{rms}}{X_C} = \frac{E_0}{\sqrt{2}X_C} \quad \left[\text{as} \quad E_{rms} = \frac{E_0}{\sqrt{2}}\right]$$

i.e. $I_{rms} = \frac{200\sqrt{2}}{\sqrt{2} \times 10^4} = 20\text{mA}$ **Ans.**

## 9. PURELY INDUCTIVE CIRCUIT:

Writing KVL along the circuit,

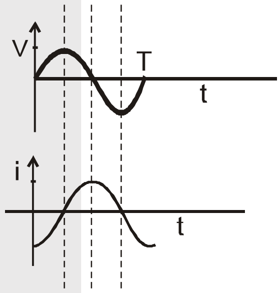



$V_s - L\frac{di}{dt} = 0 \quad \Rightarrow \quad L\frac{di}{dt} = V_m \sin \omega t$

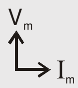
$\int L di = \int V_m \sin \omega t \, dt \Rightarrow i = -\frac{V_m}{\omega L}\cos \omega t + C$

$\langle i \rangle = 0 \qquad C = 0$

$\therefore \; i = -\frac{V_m}{\omega L}\cos \omega t \quad \Rightarrow \quad I_m = \frac{V_m}{X_L}$

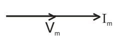
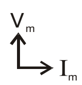
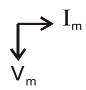



From the graph of current versus time and voltage versus time $\frac{T}{4}$, it is clear that voltage attains its peak value at a time before the time at which current attains its peak value. Corresponding to $\frac{T}{4}$ the phase difference = $\omega \Delta t = \frac{2\pi}{T}\frac{T}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$.

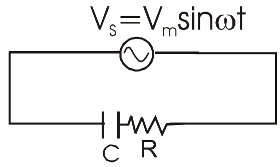
Diagrammatically (phasor diagram) it is represented as  . $i_L$ lags behind $v_L$ by π/2. Since $\phi = 90°$,

$\langle P \rangle = V_{rms} I_{rms} \cos \phi = 0$

**Summary :**

| AC source connected with | φ | | Z | Phasor Diagram |
|---|---|---|---|---|
| Pure Resistor | 0 | $V_R$ is in same phase with $i_R$ | R |  |
| Pure Inductor | π/2 | $V_L$ leads $i_L$ | $X_L$ |  |
| Pure Capacitor | π/2 | $V_C$ lags $i_C$ | $X_C$ |  |

## 10. RC SERIES CIRCUIT WITH AN AC SOURCE :




Let $i = I_m \sin(\omega t + \phi)$ $\Rightarrow$ $V_R = iR = I_mR\sin(\omega t + \phi)$

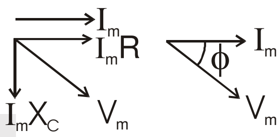
$V_C = (I_m X_C)\sin(\omega t + \phi - \frac{\pi}{2})$ $\Rightarrow$ $V_S = V_R + V_C$

or $V_m \sin(\omega t + \phi) = I_mR\sin(\omega t + \phi) + I_m X_C \sin(\omega t + \phi - \frac{\pi}{2})$

$$V_m = \sqrt{(I_mR)^2 + (I_mX_C)^2 + 2(I_mR)(I_mX_C)\cos\frac{\pi}{2}}$$

OR $I_m = \dfrac{V_m}{\sqrt{R^2 + X_C^2}}$ $\Rightarrow$ $Z = \sqrt{R^2 + X_C^2}$

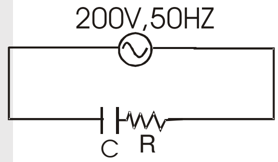
Using phasor diagram also we can find the above result.




$$\tan\phi = \frac{I_mX_C}{I_mR} = \frac{X_C}{R}.$$

## Solved Examples

**Example 11.** In an RC series circuit, the rms voltage of source is 200V and its frequency is 50 Hz. If R = 100 Ω and C= $\frac{100}{\pi}$ μF, find




| | |
|---|---|
| (i) Impedance of the circuit | (ii) Power factor angle |
| (iii) Power factor | (iv) Current |
| (v) Maximum current | (vi) voltage across R |
| (vii) voltage across C | (viii) max voltage across R |
| (ix) max voltage across C | (x) $<P>$ |
| (xi) $<P_R>$ | (xii) $<P_C>$ |

**Solution :** $X_C = \dfrac{10^6}{\frac{100}{\pi}(2\pi 50)} = 100\ \Omega$

(i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{100^2 + (100)^2} = 100\sqrt{2}\ \Omega$

(ii) $\tan\phi = \dfrac{X_C}{R} = 1$ $\therefore \phi = 45^{\circ}$

(iii) Power factor $= \cos\phi = \dfrac{1}{\sqrt{2}}$

(iv) Current $I_{rms} = \dfrac{V_{rms}}{Z} = \dfrac{200}{100\sqrt{2}} = \sqrt{2}$ A

(v) Maximum current $= I_{rms}\sqrt{2} = 2$A

(vi) voltage across R = $V_{R,rms} = I_{rms}R = \sqrt{2} \times 100$ Volt

(vii) voltage across C = $V_{C,rms} = I_{rms}X_C = \sqrt{2} \times 100$ Volt

(viii) max voltage across R = $\sqrt{2}\, V_{R,rms} = 200$ Volt

(ix) max voltage across C = $\sqrt{2}\, V_{C,rms} = 200$ Volt

(x) $<P> = V_{rms}I_{rms}\cos\phi = 200 \times \sqrt{2} \times \dfrac{1}{\sqrt{2}} = 200$ Watt

(xi) $<P_R> = I_{rms}^2R = 200$ W

(xii) $<P_C> = 0$

**Example 12.** In the above question if $V_s(t) = 220\sqrt{2}\ \sin(2\pi\ 50\ t)$, find (a) i (t), (b) $v_R$ and (c) $v_C$ (t)

**Solution :** (a) $i(t) = I_m \sin(\omega t + \phi) = 2\sin(2\pi\ 50\ t + 45^º)$

(b) $V_R = i_R\ .\ R = i(t)\ R = 2\times\ 100 \sin(100\ \pi t + 45^º)$

(c) $V_C\ (t) = i_C X_C$ (with a phase lag of 90º) $= 2\times 100 \sin(100\ \pi t + 45 - 90)$

**Example 13.** An ac source of angular frequency $\omega$ is fed across a resistor R and a capacitor C in series. The current registered is I. If now the frequency of source is changed to $\omega/3$ (but maintaining the same voltage), the current in the circuit is found to be halved. Calculate the ratio of reactance to resistance at the original frequency $\omega$.

**Solution :** According to given problem,

$$I = \frac{V}{Z} = \frac{V}{[R^2 + (1/C\omega)^2]^{1/2}} \quad .....(1)$$

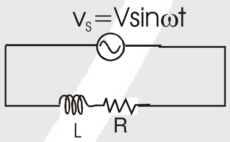
and,
$$\frac{I}{2} = \frac{V}{[R^2 + (3/C\omega)^2]^{1/2}} \quad .....(2)$$

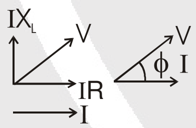
Substituting the value of I from Equation (1) in (2),

$$4\left(R^2 + \frac{1}{C^2\omega^2}\right) = R^2 + \frac{9}{C^2\omega^2}\ .\ \text{i.e.,}\ \frac{1}{C^2\omega^2} = \frac{3}{5}R^2$$

So that,
$$\frac{X}{R} = \frac{(1/C\omega)}{R} = \frac{\left(\frac{3}{5}R^2\right)^{1/2}}{R} = \sqrt{\frac{3}{5}}$$ **Ans.**

## 11. LR SERIES CIRCUIT WITH AN AC SOURCE :




From the phasor diagram $V = \sqrt{(IR)^2 + (IX_L)^2} = I\sqrt{(R)^2 + (X_L)^2} = IZ \Rightarrow \tan\phi = \frac{IX_L}{IR} = \frac{X_L}{R}$

## Solved Examples

**Example 14.** A $\frac{9}{100\pi}$ H inductor and a 12 ohm resistance are connected in series to a 225 V, 50 Hz ac source. Calculate the current in the circuit and the phase angle between the current and the source voltage.

**Solution :** Here $X_L = \omega L = 2\pi\ f\ L = 2\pi \times 50 \times \frac{9}{100\pi} = 9\ \Omega$

So, $Z = \sqrt{R^2 + {X_L}^2} = \sqrt{12^2 + 9^2} = 15\ \Omega$

So (a) $I = \frac{V}{Z} = \frac{225}{15} = 15$ A **Ans.**

and (b) $\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{9}{12}\right) = \tan^{-1} 3/4 = 37^º$

i.e., the current will lag the applied voltage by 37º in phase. **Ans.**

**Example 15.** When an inductor coil is connected to an ideal battery of emf 10 V, a constant current 2.5 A flows. When the same inductor coil is connected to an AC source of 10 V and 50 Hz then the current is 2A. Find out inductance of the coil.

**Solution :** When the coil is connected to dc source, the final current is decided by the resistance of the coil .

$\therefore \quad r = \frac{10}{2.5} = 4\ \Omega$

When the coil is connected to ac source, the final current is decided by the impedance of the coil.

$\therefore \quad Z = \frac{10}{2} = 5\,\Omega$

But $Z = \sqrt{(r)^2 + (X_L)^2}$ $\qquad X_L^2 = 5^2 - 4^2 = 9$

$X_L = 3\ \Omega$

$\therefore \quad \omega L = 2\pi fL = 3$

$\therefore \quad 2\pi\, 50\, L = 3 \qquad \therefore \quad L = 3/100\pi$ Henry

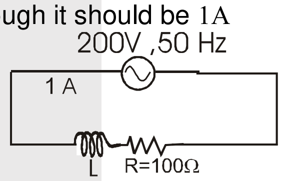
**Example 16.** A bulb is rated at 100 V, 100 W, it can be treated as a resistor .Find out the inductance of an inductor (called choke coil) that should be connected in series with the bulb to operate the bulb at its rated power with the help of an ac source of 200 V and 50 Hz.

**Solution :** From the rating of the bulb , the resistance of the bulb is $R = \frac{V_{rms}^{\ 2}}{P} = 100\ \Omega$

For the bulb to be operated at its rated value the rms current through it should be 1A

Also, $I_{rms} = \frac{V_{rms}}{Z}$

$\therefore \quad 1 = \frac{200}{\sqrt{100^2 + (2\pi 50 L)^2}} \quad ; L = \frac{\sqrt{3}}{\pi}$ H




**Example 17.** A choke coil is needed to operate an arc lamp at 160 V (rms) and 50 Hz. The arc lamp has an effective resistance of 5 Ω when running of 10 A (rms). Calculate the inductance of the choke coil. If the same arc lamp is to be operated on 160 V (dc), what additional resistance is required? Compare the power losses in both cases.

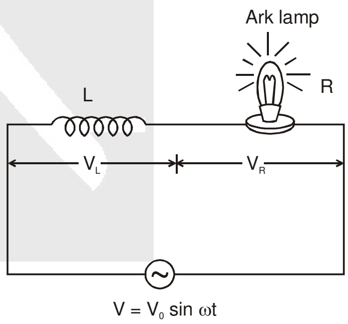
**Solution :** As for lamp $V_R = IR = 10 \times 5 = 50$ V, so when it is connected to 160 V ac source through a choke in series,

$V^2 = V_R^2 + V_L^2$,

$V_L = \sqrt{160^2 - 50^2} = 152$ V

and as, $V_L = IX_L = I\omega L = 2\pi fLI$

So, $L = \frac{V_L}{2\pi f I} = \frac{152}{2 \times \pi \times 50 \times 10} = 4.84 \times 10^{-2}$ H **Ans.**




Now the lamp is to be operated at 160 V dc; instead of choke if additional resistance r is put in series with it,

$V = I(R + r)$, i.e., $160 = 10(5 + r)$ i.e., $r = 11\ \Omega$ **Ans.**

In case of ac, as choke has no resistance, power loss in the choke will be zero while the bulb will consume,

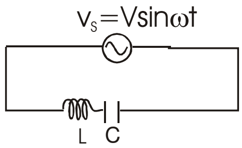
$P = I^2 R = 10^2 \times 5 = 500$ W

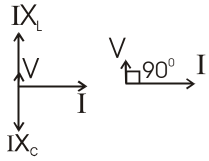
However, in case of dc as resistance r is to be used instead of choke, the power loss in the resistance r will be.

$PL = 10^2 \times 11 = 1100$ W

while the bulb will still consume 500 W, i.e., when the lamp is run on resistance r instead of choke more than double the power consumed by the lamp is wasted by the resistance r.

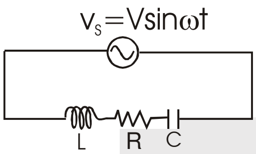
## 12. LC SERIES CIRCUIT WITH AN AC SOURCE :

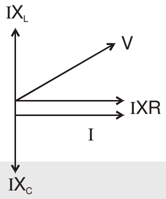


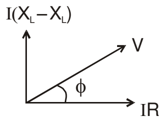




From the phasor diagram $V = I\left|(X_L - X_C)\right| = IZ$ $\qquad \phi = 90^\circ$

## 13. RLC SERIES CIRCUIT WITH AN AC SOURCE :




From the phasor diagram

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2} = I\sqrt{(R)^2 + (X_L - X_C)^2} = IZ \quad Z = \sqrt{(R)^2 + (X_L - X_C)^2}$$

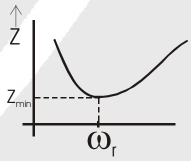
$$\tan\phi = \frac{I(X_L - X_C)}{IR} = \frac{(X_L - X_C)}{R}$$

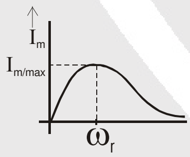
### 13.1 Resonance :

Amplitude of current (and therefore $I_{rms}$ also) in an RLC series circuit is maximum for a given value of $V_m$ and R , if the impedance of the circuit is minimum, which will be when $X_L$-$X_C$ =0. This condition is called **resonance**.

So at resonance: $X_L$-$X_C$ =0.

or $\omega L = \frac{1}{\omega C}$ or $\omega = \frac{1}{\sqrt{LC}}$. Let us denote this ω as $\omega_r$.




## Solved Examples

**Example 18.** In the circuit shown in the figure, find :

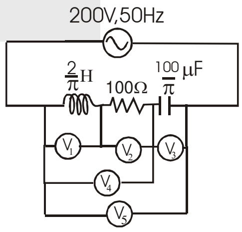
(a) the reactance of the circuit .

(b) impedance of the circuit

(c) the current

(d) readings of the ideal AC voltmeters

(these are hot wire instruments and read rms values).




**Solution :** (a) $X_L = 2\pi f L = 2\pi \times 50 \times \frac{2}{\pi} = 200\Omega$

$$X_C = \frac{1}{2\pi 50 \frac{100}{\pi} \times 10^{-6}} = 100\Omega$$

∴ The reactance of the circuit $X = X_L – X_C$ = 200-100 = 100 Ω

Since $X_L > X_C$, the circuit is called inductive.

(b) impedance of the circuit $Z = \sqrt{R^2 + X^2} = \sqrt{100^2 + 100^2} = 100\Omega$

(c) the current $I_{rms} = \frac{V_{rms}}{Z} = \frac{200}{100\sqrt{2}} = \sqrt{2}$ A

(d) readings of the ideal voltmeter

$V_1$ : $I_{rms}X_L = 200\sqrt{2}$ Volt

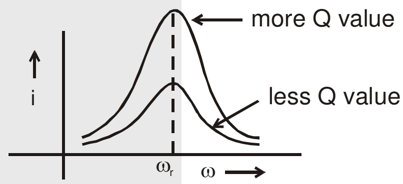
$V_2$ : $I_{rms}R = 100\sqrt{2}$ Volt

$V_3$: $I_{rms}X_C = 100\sqrt{2}$ Volt

$V_4$: $I_{rms}\sqrt{R^2 + {X_L}^2} = 100\sqrt{10}$ Volt

$V_5$: $I_{rms}Z = 200$ Volt, which also happens to be the voltage of source.

## 13.2 Q VALUE (QUALITY FACTOR) OF LCR SERIES CIRCUIT (NOT IN IIT SYLLABUS) :




Q value is defined as $\frac{X_L}{R}$ where $X_L$ is the inductive reactance of the circuit, at resonance. More Q value implies more sharpness of I v/s ω curve.

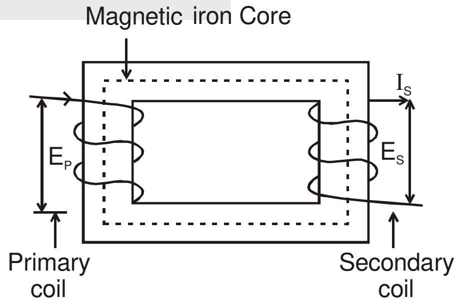
Quality factor : $Q = \frac{X_L}{R} = \frac{X_C}{R}$

$$Q = \frac{\text{Resonance freq.}}{\text{Band width}} = \frac{\omega_R}{\Delta\omega} = \frac{f_R}{f_2 - f_1}$$

where $f_1$ & $f_2$ are half power frequencies.

## 14. TRANSFORMER

**A transformer** changes an alternating potential difference from one value to another of greater or smaller value using the principle of mutual induction. Two coils called the primary and secondary windings, which are not connected to one another in any way, are wound on a complete soft iron core. When an alternating voltage $E_P$ is applied to the primary, the resulting current produces a large alternating magnetic flux which links the secondary and induces an emf $E_S$ in it. It can be shown that for an ideal transformer




$$\frac{E_s}{E_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s} \ ;$$

$\frac{N_s}{N_p}$ = turns ratio of the transformer.

$E_S$, N and I are the emf, number of turns and current in the coils.

$N_S > N_P \Rightarrow E_S > E_P$ → step up transformer.

$N_S < N_P \Rightarrow E_S < E_P$ → step down transformer.

**Note :** Phase difference between the primary and secondary voltage is $\pi$.

## 15. ENERGY LOSSES IN TRANSFORMER

Although transformers are very efficient devices, small energy losses do occur in them due to four main causes.

### 15.1. RESISTANCE OF THE WINDINGS :

The copper wire used for the windings has resistance and so $I^2R$ heat losses occur.

### 15.2. EDDY CURRENT :

Eddy current is induced in a conductor when it is placed in a changing magnetic field or when a conductor is moved in a magnetic field and/or both. Any imagined circuit within the conductor will change its magnetic flux linkage and the subsequent induced emf. will drive current around the circuit. Thus the alternating magnetic flux induces eddy currents in the iron core and causes heating. The effect is reduced by **laminating** the core, i.e., the core is made of this sheets of iron with insulating sheets between them so that the circuits for the eddy currents are broken.

### 15.3. HYSTERESIS :

The magnetization of the core is repeatedly reversed by the alternating magnetic field. The resulting expenditure of energy in the core appears as heat and is kept to a minimum by using a magnetic material which has a low hysteresis loss.

### 15.4. FLUX LEAKAGE :

The flux due to the primary may not all link the secondary if the core is badly designed or has air gaps in it .Very large transformers have to be oil cooled to prevent overheating.

## *Solved Example*

**Example 19.** In a step-up transformer the turns ratio is 10. If the frequency of the current in the primary coil is 50 Hz then the frequency of the current in the secondary coil will b

(A) 500 Hz (B) 5 Hz (C) 60 Hz (D*) 50 Hz

**Solution :** Frequency of the current remains same, only magnitudes of current changes in a tranformer.

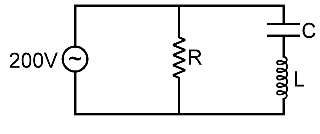
**Example 20.** A power transformer is used to step up an alternating emf of 220 volt to11 kv to transmit 4.4 kw of power. If the primary coil has 1000 turns, what is the current in the secondary?

(A) 4 A (B) 0.4 A (C) 0.04 A (D) 0.2 A

**Answer :** **(C)**

**Solution :** $I_s = P_S/V_S \Rightarrow I_S = \dfrac{4.4\times10^3}{11\times10^3} = 0.4A$ **Ans.**

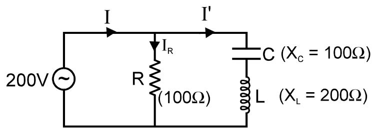
**Example 21.** In the circuit diagram shown, $X_C = 100\Omega$, $X_L = 200\Omega$ & $R = 100\Omega$. The effective current through the source is:

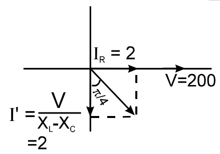



(A) 2 A (B) $2\sqrt{2}$ A

(C) 0.5 A (D) $\sqrt{0.4}$ A

**Solution :** $I_R = \frac{V}{R} = \frac{200}{100} = 2A$

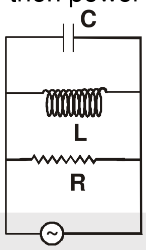



$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2A$

$I = \sqrt{{I_R}^2 + I'^2} = 2\sqrt{2}$ Amp.

**Example 22.** If for above circuit the capacitive reactance is two times of Inductive Reactance, and resistance R is equal to Inductive Reactance then power factor of circuit is.




(A) 1 (B) Zero (C) $\frac{1}{2}$ (D) $\frac{2}{\sqrt{5}}$

**Answer :** **(D)**

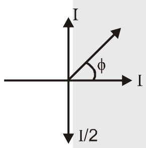
**Solution :** $X_C = 2X_L = 2R$

$$\frac{1}{z^2} = \frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2 = \frac{1}{R^2} + \left(\frac{1}{2R} - \frac{1}{R}\right)^2 = \frac{5}{4R^2}$$

$$z = \frac{2R}{\sqrt{5}}$$

Power factor in parallel combination = $\cos\theta = \frac{z}{R} = \frac{2}{\sqrt{5}}$.

**Alternate solution:**




$$\cos\phi = \frac{I}{\sqrt{I^2 + \frac{I^2}{4}}} = \frac{2}{\sqrt{5}}$$

## Solved Miscellaneous Problems

**Problem 1.** The peak voltage in a 220 V AC source is

(A) 220 V (B) about 160 V (C) about 310 V (D) 440 V

**Solution :** $V_0 = \sqrt{2}\, V_{rms} = \sqrt{2} \times 220 \simeq 330$ V

**Ans.** is (C)

**Problem 2.** An AC source is rated 220V, 50 Hz. The average voltage is calculated in a time interval of 0.01 s. It

(A) must be zero (B) may be zero (C) is never zero (D) is (220/√2)V

**Solution :** May be zero

**Ans.** is (B)

**Problem 3.** Find the effective value of current i = 2 + 4 cos 100 πt.

**Solution :** $I_{rms} = \left[\int_0^T \frac{(2 + 4\cos 100\pi t)^2\, dt}{T}\right]^{1/2} = 2\sqrt{3}$

**Problem 4.** The peak value of an alternating current is 5 A and its frequency is 60 Hz. Find its rms value. How long will the current take to reach the peak value starting from zero?

**Solution :** $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{5}{\sqrt{2}}$ A, $t = \frac{T}{4} = \frac{1}{240}$ s

**Problem 5.** An alternating current having peak value 14 A is used to heat a metal wire. To produce the same heating effect, a constant current i can be used where i is

(A) 14 A (B) about 20 A (C) 7 A (D) about 10 A

**Solution :** $I_{RMS} = \frac{I_0}{\sqrt{2}} = \frac{14}{\sqrt{2}} \simeq 10$

**Ans.** is (D)

**Problem 6.** Find the average power concumed in the circuit if a voltage $v_s = 200\sqrt{2}\sin\omega t$ is applied to an AC circuit and the current in the circuit is found to be $i = 2\sin(\omega t + \pi/4)$ .

**Solution :** $P = V_{RMS} I_{RMS} \cos\phi = \frac{200\sqrt{2}}{\sqrt{2}} \times \frac{2}{\sqrt{2}} \times \cos\frac{\pi}{4} = 200$ W

**Problem 7.** A capacitor acts as an infinite resistance for

(A) DC (B) AC (C) DC as well as AC (D) neither AC nor DC

**Solution :** $x_C = \frac{1}{\omega c}$ for DC $\omega = 0$. So, $x_C = \infty$

**Ans.** is (A)

**Problem 8.** A 10 μF capacitor is connected with an ac source $E = 200\sqrt{2}\sin(100\ t)$ V through an ac ammeter (it reads rms value). What will be the reading of the ammeter?

**Solution :** $I_0 = \frac{V_0}{x_C} = \frac{200\sqrt{2}}{1/\omega C}$ ; $I_{RMS} = \frac{I_0}{\sqrt{2}} = 200$ mA

**Problem 9.** Find the reactance of a capacitor (C = 200 μF) when it is connected to (a) 10 Hz AC source, (b) a 50 Hz AC source and (c) a 500 Hz AC source.

**Solution :** (a) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 80\Omega$ for f = 10 Hz AC source,

(b) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 16\Omega$ for f = 50 Hz and

(c) $x_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} \simeq 1.6\Omega$ for f= 500 Hz.

**Problem 10.** An inductor (L = 200mH) is connected to an AC source of peak current. What is the instantaneous voltage of the source when the current is at its peak value?

**Solution :** Because phase difference between voltage and current is $\pi/2$ for pure inductor.

So, Ans. is zero

**Problem 11.** An AC source producing emf $E = E_0[\cos(100\pi\ s^{-1})t + \cos(500\ \pi s^{-1})t]$ is connected in series with a capacitor and a resistor. The current in the circuit is found to be

$i = i_1\cos[(100\ \pi s^{-1})t + \varphi_1] + i_2\cos[(500\ \pi s^{-1})t + \varphi_1]$

(A) $i_1 > i_2$ (B) $i_1 = i_2$ (C) $i_1 < i_2$

(D) the information is insufficient to find the relation between $i_1$ and $i_2$

**Solution :** Impedence z is given by $z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

For higher $\omega$, z will be lower so current will be higher. Ans is (C)

**Problem 12.** An alternating voltage of 220 volt r.m.s. at a frequency of 40 cycles/sec is supplied to a circuit containing a pure inductance of 0.01 H and a pure resistance of 6 ohms in series. Calculate (i) the current, (ii) potential difference across the resistance, (iii) potential difference across the inductance, (iv) the time lag, (v) power factor.

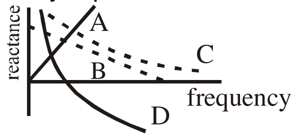
**Solution :** (i) $z = \sqrt{(\omega L)^2 + R^2} = \sqrt{(2\pi \times 40 \times 0.01^2)^2 + 6^2} = \sqrt{(42.4)}$

$I_{rms} = \dfrac{220}{z} = 33.83$ amp.

(ii) $V_{rms} = I_{rms} \times R = 202.98$ volts (iii) $\omega L \times I_{rms} = 96.83$ volts

(iv) $t = T\dfrac{\phi}{2\pi} = 0.01579$ sec (v) $\cos\phi = \dfrac{R}{Z} = 0.92$

**Problem 13.** Which of the following plots may represents the reactance of a series LC combination ?




**Answer :** (D)

**Problem 14.** A series AC circuit has resistance of 4Ω and a reactance of 3Ω. The impedance of the circuit is

(A) 5Ω (B) 7Ω (C) 12/7 Ω (D) 7/12 Ω

**Solution :** $Z = \sqrt{4^2 + 3^2} = 5\Omega$

**Ans.** is (A)

# Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

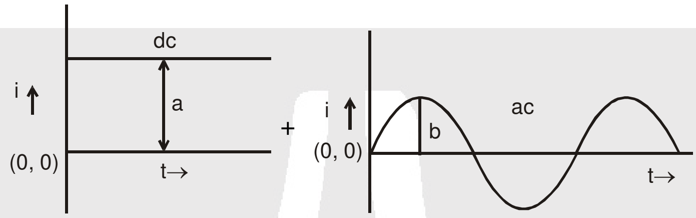
## PART - I : SUBJECTIVE QUESTIONS

### Section (A) : Average, peak and RMS value

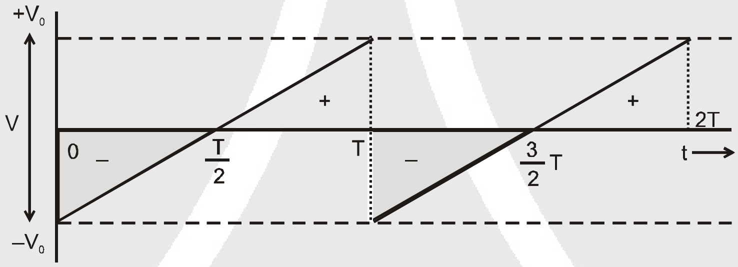
**A-1.** The household supply of electricity is at 220V rms value and 50 Hz .Calculate the peak voltage and the minimum possible time in which the voltage can change from the rms value to zero.

**A-2.** In a LR circuit discharging current is given by $I = I_0e^{-t/\tau}$ where $\tau$ is the time constant of the circuit find the rms current for the period t = 0 to t = $\tau$.

**A-3.** If a direct current of value 'a' ampere is superimposed on an alternating current $I = b \sin \omega t$ flowing through a wire, what is the effective(rms) value of the resulting current in the circuit?




**A-4.** Find the average for the saw-tooth voltage of peak value $V_0$ from t = 0 to t = 2T as shown in figure.




### Section (B) : Power consumed in an ac circuit

**B-1.** A bulb is designed to operate at 12 volts constant direct current. If this bulb is connected to an alternating current source and gives same brightness. What would be the peak voltage of the source ?

**B-2.** A resistor of resistance 100Ω is connected to an AC source $\varepsilon$ = (12V) sin (250$\pi$s$^{-1}$)t. Find the power consumed by the bulb.

**B 3.**✍ In an ac circuit the instantaneous values of current and applied voltage are respectively i = 2(Amp) sin (250 $\pi$s$^{-1}$)t and $\varepsilon$ = (10V) sin [(250 $\pi$s$^{-1}$)t + $\frac{\pi}{3}$]. Find the instantaneous power drawn from the source at t = $\frac{2}{3}$ ms and its average value.

### Section (C) : AC source with R, L, C connected in series

**C-1.** The dielectric strength of air is 3.0 × 10$^6$ V/m. A parallel plate air capacitor has area 20 cm$^2$ and plate separation $\sqrt{2}$ mm. Find the maximum rms voltage of an AC source which can be safely connected to this capacitor.

**C-2.** An electric bulb is designed to consume 55 W when operated at 110 volts. It is connected to a 220 V, 50 Hz line through a choke coil in series. What should be the inductance of the coil for which the bulb gets correct voltage ?

**C 3.** A resistor, a capacitor and an inductor (R = 300 Ω, C = 20 μF, L = 1.0 henry) are connected in series with an AC source of, $E_{rms}$ = 50 V and $\nu = \frac{50}{\pi}$ Hz. Find (a) the rms current in the circuit and (b) the rms potential differences across the capacitor, the resistor and the inductor.

**C-4.** Consider the situation of the previous problem calculate the average electric field energy stored in the capacitor and the average magnetic field energy stored in the inductor coil.

**C-5.** A 20 volts 5 watt lamp (lamp to be treated as a resistor) is used on AC mains of 200 volts and $\frac{50}{\pi}\sqrt{11}$ c.p.s. Calculate the (i) capacitance of the capacitor, or inductance of the inductor, to be put in series to run the lamp. (ii) How much pure resistance should be included in place of the above device so that the lamp can run on its rated voltage. (iii) which is more economical (the capacitor, the inductor or the resistor).

**C-6.** A circuit has a resistance of 50 ohms and an inductance of $\frac{3}{\pi}$ henry. It is connected in series with a condenser of $\frac{40}{\pi}$ μF and AC supply voltage of 200 V and 50 cycles/sec. Calculate
(i) the impedance of the circuit,
(ii) the p.d. across inductor coil and condenser.
(iii) Power factor

**C-7.** A coil draws a current of 1.0 ampere and a power of 100 watt from an A.C. source of 110 volt and $\frac{5\sqrt{22}}{\pi}$ hertz. Find the inductance and resistance of the coil.

**C-8.** An inductor 2/π Henry, a capacitor 100/π μF and a resistor 75 Ω are connected in series across a source of emf V = 10 sin 100 π t. Here t is in second. (a) find the impedance of the circuit. (b) find the energy dissipated in the circuit in 20 minutes.

## Section (D) : Resonance

**D-1.** A series circuit consists of a resistance, inductance and capacitance. The applied voltage and the current at any instant are given by E = 141.4 cos (5000 t – 10º) and I = 5 cos (5000 t – 370º). The inductance is 0.01 henry. Calculate the value of capacitance and resistance.

**D-2.** An inductance of 2.0 H, a capacitance of 18 μF and a resistance of 10 kΩ are connected to an AC source of 20V with adjustable frequency (a) What frequency should be chosen to maximise the current(RMS) in the circuit? (b) What is the value of this maximum current (RMS) ?

**D-3.** An inductor-coil, a capacitor are connected in series with an AC source of rms voltage 24 V. When the frequency of the source is varied a maximum rms current of 6.0 A is observed. If this inductor coil is connected to a DC source of 12 V and having internal resistance 4.0 Ω, what will be the current in steady state?

**D-4.** An electro magnetic wave of wavelength 300 metre can be transmitted by a transmission centre. A condenser of capacity 2.5 μF is available. Calculate the inductance of the required coil for a resonant circuit.Use $\pi^2$=10.

## Section (E) : Transformer

**E-1** A transformer has 50 turns in the primary and 100 turns in the secondary. If the primary is connected to a 220 V DC supply, what will be the voltage across the secondary ?

**E-2.** In a transformer ratio of secondary turns ($N_2$) and primary turns ($N_1$) i.e. $\frac{N_2}{N_1} = 4$ . If the voltage applied in primary is 200 V, 50 Hz, find (a) voltage induced in secondary (b) If current in primary is 1A, find the current in secondary if the transformer is (i) ideal and (ii) 80% efficient and there is no flux leakage.

# PART - II : ONLY ONE OPTION CORRECT TYPE

## Section (A) : Average, peak and RMS values and RMS values

**A-1.** r.m.s. value of current $i = 3 + 4 \sin(\omega t + \pi/3)$ is:

(A) 5 A (B) $\sqrt{17}$ A (C) $\frac{5}{\sqrt{2}}$ A (D) $\frac{7}{\sqrt{2}}$ A

**A-2.** The peak value of an alternating e.m.f given by $E = E_0 \cos \omega t$, is 10 volt and frequency is 50 Hz. At time t = (1/600) sec, the instantaneous value of e.m.f is :

(A) 10 volt (B) $5\sqrt{3}$ volt (C) 5 volt (D) 1 volt

**A-3.** The voltage of an AC source varies with time according to the equation, $V = 100 \sin 100\pi t \cos 100\pi t$. Where t is in second and V is in volt. Then :

(A) the peak voltage of the source is 100 volt (B) the peak voltage of the source is $(100/\sqrt{2})$ volt
(C) the peak voltage of the source is 50 volt (D) the frequency of the source is 50 Hz

**A-4.** An alternating voltage is given by : $e = e_1 \sin \omega t + e_2 \cos \omega t$. Then the root mean square value of voltage is given by :

(A) $\sqrt{e_1^2 + e_2^2}$ (B) $\sqrt{e_1 e_2}$ (C) $\sqrt{\frac{e_1 \ e_2}{2}}$ (D) $\sqrt{\frac{e_1^2 + e_2^2}{2}}$

**A-5.** An AC voltage is given by $E = E_0 \sin \frac{2\pi t}{T}$. Then the mean value of voltage calculated over time interval of T/2 seconds :

(A) is always zero (B) is never zero (C) is $(2E_0/\pi)$ always (D) may be zero

**A-6.** An AC voltage of $V = 220\sqrt{2} \sin\left(100\pi t + \frac{\pi}{2}\right)$ is applied across a DC voltmeter, its reading will be:

(A) $220\sqrt{2}$ V (B) $\sqrt{2}$ V (C) 220 V (D) zero

## Section (B) : Power consumed in an AC circuit

**B-1.** The average power delivered to a series AC circuit is given by (symbols have their usual meaning) :

(A) $E_{rms} I_{rms}$ (B) $E_{rms} I_{rms} \cos\phi$ (C) $E_{rms} I_{rms} \sin\phi$ (D) zero

**B-2.** Energy dissipates in LCR circuit in :

(A) L only (B) C only (C) R only (D) all of these

**B-3.** The potential difference V across and the current I flowing through an instrument in an AC circuit are given by :

$V = 5 \cos \omega t$ volt
$I = 2 \sin \omega t$ Amp.

The power dissipated in the instrument is :

(A) zero (B) 5 watt (C) 10 watt (D) 2.5 watt

**B-4.** A direct current of 2 A and an alternating current having a maximum value of 2 A flow through two identical resistances. The ratio of heat produced in the two resistances in the same time interval will be:

(A) 1 : 1 (B) 1 : 2 (C) 2 : 1 (D) 4 : 1

**B-5.** A sinusoidal AC current flows through a resistor of resistance R. If the peak current is $I_p$, then average power dissipated is :

(A) $I_p^2 R \cos\theta$ (B) $\frac{1}{2} I_p^2 R$ (C) $\frac{4}{\pi} I_p^2 R$ (D) $\frac{1}{\pi^2} I_p^2 R$

**B-6.** What is the rms value of an alternating current which when passed through a resistor produces heat, which is thrice that produced by a D.C. current of 2 ampere in the same resistor in the same time interval?

(A) 6 ampere (B) 2 ampere (C) $2\sqrt{3}$ ampere (D) 0.65 ampere

**B-7.** A resistor and a capacitor are connected to an AC supply of 200 volt, 50 Hz in series. The current in the circuit is 2 ampere. If the power consumed in the circuit is 100 watt, then the resistance in the circuit is:

(A) 100 $\Omega$ (B) 25 $\Omega$ (C) $\sqrt{125 \times 75}$ $\Omega$ (D) 400 $\Omega$

**B-8.** The impedance of a series circuit consists of 3 ohm resistance and 4 ohm reactance. The power factor of the circuit is :

(A) 0.4 (B) 0.6 (C) 0.8 (D) 1.0

**B-9.** A coil of inductance 5.0 mH and negligible resistance is connected to an alternating voltage V = 10 sin (100 t). The peak current in the circuit will be :

(A) 2 amp (B) 1 amp (C) 10 amp (D) 20 amp

**B-10.** An electric bulb and a capacitor are connected in series with an AC source. On increasing the frequency of the source, the brightness of the bulb :

(A) increase (B) decreases
(C) remains unchanged (D) sometimes increases and sometimes decreases

**B-11.** By what percentage the impedance in an AC series circuit should be increased so that the power factor changes from (1/2) to (1/4) (when R is constant) ?

(A) 200% (B) 100% (C) 50% (D) 400%

**B-12.** If the frequency of the source e.m.f. in an AC circuit is n, the power varies with a frequency :

(A) n (B) 2 n (C) n/2 (D) zero

## Section (C) : AC source with R, L, C connected in series

**C-1.** A 0.21-H inductor and a 88-$\Omega$ resistor are connected in series to a 220-V, 50-Hz AC source. The current in the circuit and the phase angle between the current and the source voltage are respectively. (Use $\pi$ = 22/7)

(A) 2 A, $\tan^{-1}$ 3/4 (B) 14.4 A, $\tan^{-1}$ 7/8 (C) 14.4 A, $\tan^{-1}$ 8/7 (D) 3.28 A, $\tan^{-1}$ 2/11

**C-2.** A 100 volt AC source of angular frequency 500 rad/s is connected to a LCR circuit with L = 0.8 H, C = 5 $\mu$F and R = 10 $\Omega$, all connected in series. The potential difference across the resistance is

(A) $\frac{100}{\sqrt{2}}$ volt (B) 100 volt (C) 50 volt (D) $50\sqrt{3}$

**C-3.** A pure resistive circuit element X when connected to an AC supply of peak voltage 200 V gives a peak current of 5 A which is in phase with the voltage. A second circuit element Y, when connected to the same AC supply also gives the same value of peak current but the current lags behind by 90°. If the series combination of X and Y is connected to the same supply, what will be the rms value of current ?

(A) $\frac{10}{\sqrt{2}}$ amp (B) $\frac{5}{\sqrt{2}}$ amp (C) $\frac{5}{2}$ amp (D) 5 amp

**C-4.** In an AC circuit, a resistance of R ohm is connected in series with an inductance L. If phase angle between voltage and current be 45°, the value of inductive reactance will be.

(A) R/4 (B) R/2
(C) R (D) cannot be found with the given data

**C-5.** In an AC circuit the potential differences across an inductor and resistor joined in series are respectively 16 V and 20 V. The total potential difference across the circuit is

(A) 20 V (B) 25.6 V (C) 31.9 V (D) 53.5 V

**C-6.** An AC voltage source $V = 200\sqrt{2} \sin 100t$ is connected across a circuit containing an AC ammeter(it reads rms value) and capacitor of capacity 1 $\mu$F. The reading of ammeter is :
(A) 10 mA (B) 20 mA (C) 40 mA (D) 80 mA

**C-7.** When 100 V DC is applied across a solenoid, a steady current of 1 A flows in it. When 100 V AC is applied across the same solenoid, the current drops to 0.5 A. If the frequency of the AC source is $150/\sqrt{3}\,\pi$ Hz, the impedance and inductance of the solenoid are :
(A) 200 $\Omega$ and 1/3 H (B) 100 $\Omega$ and 1/16 H (C) 200 $\Omega$ and 1.0 H (D) 1100 $\Omega$ and 3/117 H

**C-8.** If in a series LCR AC circuit, the rms voltage across L, C and R are $V_1$, $V_2$ and $V_3$ respectively, then the voltage of the source is always :
(A) equal to $V_1 + V_2 + V_3$ (B) equal to $V_1 - V_2 + V_3$
(C) more than $V_1 + V_2 + V_3$ (D) none of these is true

## Section (D) : Resonance

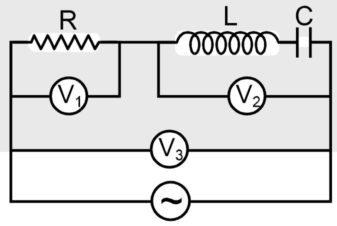
**D-1.** The value of power factor $\cos\phi$ in series LCR circuit at resonance is :
(A) zero (B) 1 (C) 1/2 (D) 1/2 ohm

**D-2.** A series LCR circuit containing a resistance of 120 ohm has angular resonance frequency $4 \times 10^3$ rad $s^{-1}$. At resonance, the voltage across resistance and inductance are 60V and 40 V respectively. The values of L and C are respectively :
(A) 20 mH, 25/8 $\mu$F (B) 2mH, 1/35 $\mu$F (C) 20 mH, 1/40 $\mu$F (D) 2mH, 25/8 nF

**D-3.** In an LCR circuit, the capacitance is made one-fourth, when in resonance. Then what should be the change in inductance, so that the circuit remains in resonance ?
(A) 4 times (B) 1/4 times (C) 8 times (D) 2 times

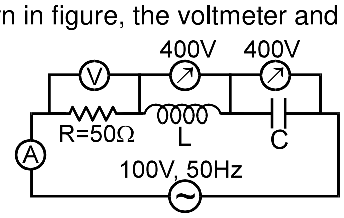
**D-4.** A resistor R, an inductor L and a capacitor C are connected in series to an oscillator of frequency $\upsilon$. If the resonant frequency is $\upsilon_r$, then the current lags behind voltage, when :
(A) $\upsilon = 0$ (B) $\upsilon < \upsilon_r$ (C) $\upsilon = \upsilon_r$ (D) $\upsilon > \upsilon_r$

**D-5.** A resistor R, an inductor L, a capacitor C and voltmeters $V_1$, $V_2$ and $V_3$ are connected to an oscillator in the circuit as shown in the adjoining diagram. When the frequency of the oscillator is increased, upto resonance frequency, the voltmeter reading (at resonance frequency) is zero in the case of :




(A) voltmeter $V_1$ (B) voltmeter $V_2$ (C) voltmeter $V_3$ (D) all the three voltmeters

**D-6.** In the series LCR circuit as shown in figure, the voltmeter and ammeter readings are :




(A) V = 100 volt, I = 2 amp (B) V = 100 volt, I = 5 amp
(C) V = 1000 volt, I = 2 amp (D) V = 300 volt, I = 1 amp

## Section (E) : Transformer

**E-1.** A power transformer (step up) with an 1 : 8 turn ratio has 60 Hz, 120 V across the primary; the load in the secondary is $10^4\ \Omega$. The current in the secondary is
(A) 96 A (B) 0.96 A (C) 9.6 A (D) 96 mA

**E-2.** A transformer is used to light a 140 watt, 24 volt lamp from 240 V AC mains. The current in the main cable is 0.7 amp. The efficiency of the transformer is :
(A) 48% (B) 63.8% (C) 83.3% (D) 90%

**E-3.** In a step-up transformer the voltage in the primary is 220 V and the current is 5A. The secondary voltage is found to be 22000 V. The current in the secondary (neglect losses) is
(A) 5 A (B) 50 A (C) 500 A (D) 0.05 A

**E-4.** The core of a transformer is laminated to reduce
(A) eddy current loss (B) hysteresis loss (C) copper loss (D) magnetic loss

## Section (F) : Miscellaneous

**F-1.** A capacitor is a perfect insulator for :
(A) constant direct current (B) alternating current
(C) direct as well as alternating current (D) variable direct current

**F-2.** A choke coil sould have :
(A) high inductance and high resistance (B) low inductance and low resistance
(C) high inductance and low resistance (D) low inductance and high resistance

**F-3.** A choke coil is preferred to a rheostat in AC circuit as :
(A) it consumes almost zero power (B) it increases current
(C) it increases power (D) it increases voltage

**F-4.** With increase in frequency of an AC supply, the inductive reactance :
(A) decreases (B) increases directly proportional to frequency
(C) increases as square of frequency (D) decreases inversely with frequency

**F-5.** With increase in frequency of an AC supply, the capacitive reactance :
(A) varies inversely with frequency (B) varies directly with frequency
(C) varies directly as square of frequency (D) remains constant

**F-6.** An AC ammeter is used to measure current in a circuit. When a given direct constant current passes through the circuit, the AC ammeter reads 3 ampere. When an alternating current passes through the circuit, the AC ammeter reads 4 ampere. Then the reading of this ammeter if DC and AC flow through the circuit simultaneously, is :
(A) 3 A (B) 4 A (C) 7 A (D) 5 A

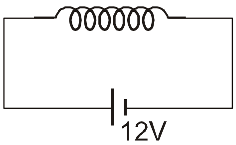
**F-7.** In an a.c. circuit consisting of resistance R and inductance L, the voltage across R is 60 volt and that across L is 80 volt. The total voltage across the combination is
(A) 140 V (B) 20 V (C) 100 V (D) 70 V

# PART - III : MATCH THE COLUMN

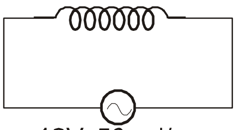
**1.** Match the Physical quantities given in column-I with the parameters they depend on as given in column-II.

| Column I | Column II |
|---|---|
| (A) Inductance of a coil | (p) Depends on resistivity |
| (B) Capacitance | (q) Depends on shape |
| (C) Impedance of a coil | (r) Depends on medium inserted |
| (D) Reactance of a capacitor | (s) Depends on external AC voltage source |

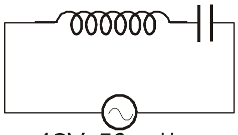
**2.** ✍ A steady current 4 A flows in an inductor coil when connected to a 12 V dc source as shown in figure 1. If the same coil is connected to an ac source of 12 V, 50 rad/s, a current of 2.4 A flows in the circuit as shown in figure 2. Now after these observations, a capacitor of capacitance $\frac{1}{50}$ F is connected in series with the coil as shown in figure 3 with the same AC source :



(Figure 1)



(Figure 2)



(Figure 3)

| Column-I | Column-II (in S.I units) |
|---|---|
| (A) The inductance of the coil (nearly) | (p) 24 |
| (B) The resistance of the coil | (q) 3 |
| (C) Average power (nearly) | (r) 0.08 |
| (D) Total reactance | |

## Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

### PART - I : ONLY ONE OPTION CORRECT TYPE

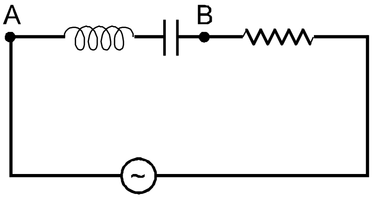
**1.** A coil has an inductance of $\frac{2.2}{\pi}$ H and is joined in series with a resistance of 220Ω. When an alternating e.m.f. of 220 V at 50 c.p.s. is applied to it, then the wattless component of the rms current in the circuit is

(A) 5 ampere (B) 0.5 ampere (C) 0.7 ampere (D) 7 ampere

**2.** The current in a circuit containing a capacitance C and a resistance R in series leads over the applied voltage of frequency $\frac{\omega}{2\pi}$ by. **[REE - 1991]**

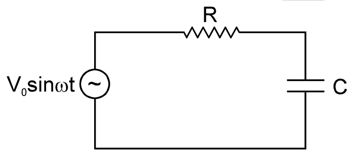
(A) $\tan^{-1}\left(\frac{1}{\omega CR}\right)$ (B) $\tan^{-1}(\omega CR)$ (C) $\tan^{-1}\left(\omega\frac{1}{R}\right)$ (D) $\cos^{-1}(\omega CR)$

**3.** ✍ An inductor $\left(L = \frac{1}{100\pi}H\right)$, a capacitor $\left(C = \frac{1}{500\pi}F\right)$ and a resistance (3Ω) is connected in series with an AC voltage source as shown in the figure. The voltage of the AC source is given as V = 10 cos (100 πt) volt. What will be the potential difference between A and B ?




(A) 8 cos(100 πt – 127º) volt (B) 8 cos(100 πt – 53º) volt
(C) 8 cos(100 πt – 37º) volt (D) 8 cos(100 πt + 37º) volt

**4.** An ac voltage source $V = V_0 \sin \omega t$ is connected across resistance R and capacitance C as shown in figure. It is given that $R = \frac{1}{\omega C}$. The peak current is $I_0$. If the angular frequency of the voltage source is changed to $\frac{\omega}{\sqrt{3}}$ keeping R and C fixed, then the new peak current in the circuit is :




(A) $\frac{I_0}{2}$ (B) $\frac{I_0}{\sqrt{2}}$ (C) $\frac{I_0}{\sqrt{3}}$ (D) $\frac{I_0}{3}$

**5.** For a LCR series circuit with an A.C. source of angular frequency $\omega$.

(A) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$ (B) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$

(C) power factor of circuit will by unity if capacitive reactance equals inductive reactance

(D) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

**6.** An LCR series circuit with 100 $\Omega$ resistance is connected to an AC source of 200 V and angular frequency 300 radians per second. When only the capacitance is removed, the current lags the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60º. Then the current and power dissipated in LCR circuit are respectively

(A) 1A, 200 watt. (B) 1A, 400 watt. (C) 2A, 200 watt. (D) 2A, 400 watt.

**7.** In an L-R series circuit ($L = \frac{175}{11}$ mH and $R = 12\Omega$), a variable emf source ($V = V_0 \sin \omega t$) of $V_{rms} = 130\sqrt{2}$ V and frequency 50 Hz is applied. The current amplitude in the circuit and phase of current with respect to voltage are respectively (Use $\pi = 22/7$)

(A) 14.14A, 30° (B) $10\sqrt{2}$ A, $\tan^{-1}\frac{5}{12}$ (C) 10 A, $\tan^{-1}\frac{5}{12}$ (D) 20 A, $\tan^{-1}\frac{5}{12}$

**8.** In LCR circuit at resonance current in the circuit is $10\sqrt{2}$ A. If now frequency of the source is changed such that now current lags by 45° than applied voltage in the circuit. Which of the following is correct :

(A) Frequency must be increased and current after the change is 10 A

(B) Frequency must be decreased and current after the change is 10 A

(C) Frequency must be decreased and current is same as that of initial value

(D) The given information is insufficient to conclude anything

**9.** The overall efficiency of a transformer is 90%. The transformer is rated for an output of 9000 watt. The primary voltage is 1000 volt. The ratio of turns in the primary to the secondary coil is 5 : 1. The iron losses at full load are 700 watt. The primary coil has a resistance of 1 ohm.

**(i)** The voltage in secondary coil is :

(A) 1000 volt (B) 5000 volt (C) 200 volt (D) zero volt

**(ii)** In the above, the current in the primary coil is :

(A) 9 amp (B) 10 amp (C) 1 amp (D) 4.5 amp

**(iii)** In the above, the copper loss in the primary coil is :

(A) 100 watt (B) 700 watt (C) 200 watt (D) 1000 watt

**(iv)** In the above, the copper loss in the secondary coil is :

(A) 100 watt (B) 700 watt (C) 200 watt (D) 1000 watt

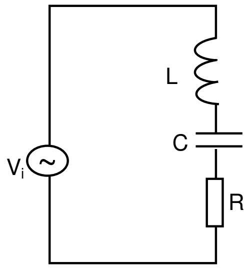
**(v)** In the above, the current in the secondary coil is :

(A) 45 amp (B) 46 amp (C) 10 amp (D) 50 amp

**(vi)** In the above, the resistance of the secondary coil is approximately :

(A) 0.01 $\Omega$ (B) 0.1 $\Omega$ (C) 0.2 $\Omega$ (D) 0.4 $\Omega$

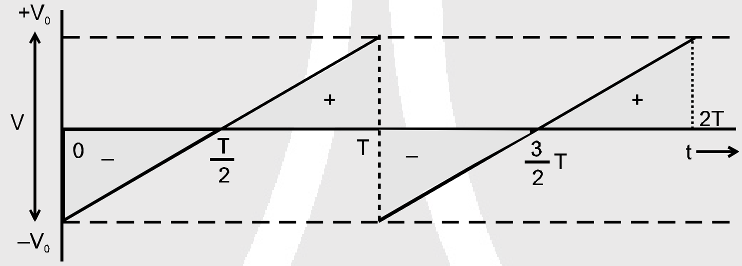
**10.**_ In the following circuit the current is in phase with the applied voltage. Therefore, the current in the circuit and the frequency of the source voltage respectively, are **[Olympiad (Stage-1) 2017]**



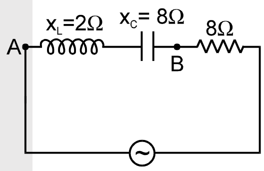

(A) $\frac{v_i}{R}$ and $\frac{1}{2\pi\sqrt{LC}}$ (B) zero and $\frac{1}{\sqrt{LC}}$ (C) $\sqrt{\frac{C}{L}}v_i$ and $\frac{2}{\pi\sqrt{LC}}$ (D) $4\sqrt{\frac{C}{LR^2}}$ and $\frac{2}{\sqrt{LC}}$

## PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

**1.** The rms value for the saw-tooth voltage of peak value $V_0$ from t = 0 to t = 2T as shown in figure is $\frac{V_0}{\sqrt{x}}$.
Find the value of x.




**2.** An inductor ($x_L$ = 2Ω) a capacitor ($x_C$ = 8Ω) and a resistance (8Ω) is connected in series with an ac source. The voltage output of A.C source is given by v = 10 cos 100πt. The instantaneous p.d. between A and B is equal to x × $10^{-1}$ volt, when it is half of the voltage output from source at that instant Find out value of x .

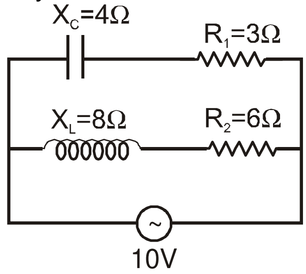



**3.** A 2000 Hz, 20 volt source is connected to a resistance of 20 ohm, an inductance of 0.125/π H and a capacitance of 500/π nF all in series. Calculate the time (in seconds) in which the resistance (thermal capacity = 100 joule/ºC) will get heated by 10º C. (Assume no loss of heat)

**4.** A series LCR circuit containing a resistance of 120 ohm has angular resonance frequency 4 × $10^5$ rad $s^{-1}$. At resonance, the voltage across resistance and inductance are 60V and 40 V respectively. At frequency the current in the circuit lags the voltage by 45º is equal to x × $10^5$ rad/sec. Find value of x **[REE - 1995]**

**5.** An LCR circuit has L = 10 mH, R = 150 Ω and C = 1 μF connected in series to a source of 150 $\sqrt{2}$ cos ωt volt. At a frequency that is 50% of the resonant frequency, calculate the average power (in watt) dissipated per cycle.

**6.** In the figure shown an ideal alternative current (A.C.) source of 10 Volt is connected. Find half of the total average power (in watts) given by the cell to the circuit.



## PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Average power consumed in an A.C. series circuit is given by (symbols have their usual meaning) :
(A) $E_{rms} I_{rms} \cos\phi$ (B) $(I_{rms})^2 R$ (C) $\frac{E_{max}{}^2 R}{2(|z|)^2}$ (D) $\frac{I_{max}{}^2 |z| \cos\phi}{2}$

2. An AC source supplies a current of 10 A (rms) to a circuit, rms voltage of source is 100 V. The average power delivered by the source :
(A) must be 1000 W (B) may be less than 1000 W
(C) may be greater than 1000 W (D) may be 1000 W

3. Which of the following quantities have zero average value over a cycle. If an inductor coil having some resistance is connected to a sinusoidal AC source.
(A) induced emf in the inductor (B) current
(C) joule heat (D) magnetic energy stored in the inductor

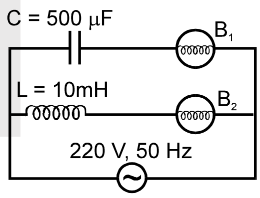
4. In a series LCR circuit with an AC source ($E_{rms}$ = 50 V and $\nu = 50/\pi$ Hz), R = 300 Ω, C = 0.02 mF, L = 1.0 H, which of the following is correct
(A) the rms current in the circuit is 0.1 A
(B) the rms potential difference across the capacitor is 50 V
(C) the rms potential difference across the capacitor is 14.1 V
(D) the rms current in the circuit is 0.14 A

5. In an AC series circuit when the instantaneous source voltage is maximum, the instantaneous current is zero. Connected to the source may be a
(A) pure capacitor
(B) pure inductor
(C) combination of pure an inductor and pure capacitor
(D) pure resistor

6. A coil of inductance 5.0 mH and negligible resistance is connected to an oscillator giving an output voltage E = (10V) sin ωt. Which of the following is correct
(A) for ω = 100 $s^{-1}$ peak current is 20 A (B) for ω = 500 $s^{-1}$ peak current is 4 A
(C) for ω = 1000 $s^{-1}$ peak current is 2 A (D) for ω = 1000 $s^{-1}$ peak current is 4 A

7. A pure inductance of 1 henry is connected across a 110 V, 70Hz source. Then correct option are (Use π = 22/7):
(A) reactance of the circuit is 440 Ω (B) current of the circuit is 0.25 A
(C) reactance of the circuit is 880 Ω (D) current of the circuit is 0.5 A

8. In the circuit shown in figure, if both the bulbs $B_1$ and $B_2$ are identical :

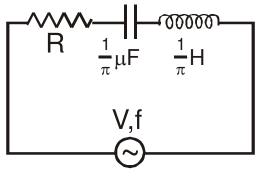



(A) their brightness will be the same
(B) $B_2$ will be brighter than $B_1$
(C) as frequency of supply voltage is increased the brightness of bulb $B_1$ will increase and that of $B_2$ will decrease.
(D) only $B_2$ will glow because the capacitor has infinite impedance

9. A circuit is set up by connecting L = 100 mH, C = 5 μF and R =100 Ω in series. An alternating emf of $(150\sqrt{2})$ volt, $\frac{500}{\pi}$ Hz is applied across this series combination. Which of the following is correct
(A) the impedance of the circuit is 141.4 Ω
(B) the average power dissipated across resistance 225 W
(C) the average power dissipated across inductor is zero.
(D) the average power dissipated across capacitor is zero.

10. In a series RC circuit with an AC source (peak voltage $E_0$ = 50 V and f = 50/π Hz), R = 300 Ω, C = 25 μF. Then
(A) the peak current is 0.1 A (B) the peak current is 0.7 A
(C) the average power dissipated is 1.5 W (D) the average power dissipated is 3 W

**11.** In the AC circuit shown below, the supply voltage has constant rms value V but variable frequency f. At resonance, the circuit :




(A) has a current I given by I = V/R
(B) has a resonance frequency 500 Hz
(C) has a voltage across the capacitor which is $180^0$ out of phase with that across the inductor
(D) has a current given by $I = \dfrac{V}{\sqrt{R^2 + \left(\frac{1}{\pi} + \frac{1}{\pi}\right)^2}}$

**12.** A town situated 20 km away from a power house at 440 V, requires 600 KW of electric power at 220 V. The resistance of transmission line carrying power is 0.4 Ω per km. The town gets power from the line through a 3000 V–220 V step-down transformer at a substation in the town. Which of the following is/are correct
(A) The loss in the form of heat is 640 kW (B) The loss in the form of heat is 1240 kW
(C) Plant should supply 1240 kW (D) Plant should supply 640 kW

**13.** 11 kW of electric power can be transmitted to a distant station at (i) 220 V or (ii) 22000 V. Which of the following is correct
(A) first mode of transmission consumes less power (B) second mode of transmission consumes less power
(C) first mode of transmission draws less current (D) second mode of transmission draws less current

**14.** Power factor may be equal to 1 for :
(A) pure inductor (B) pure capacitor (C) pure resistor (D) An LCR circuit

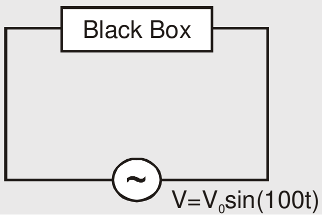
**15._** In a series R-C circuit the supply voltage (Vs) is kept constant at 2V and the frequency f of the sinusoidal voltage is varied from 500 Hz to 2000 Hz. The voltage across the resistance R = 1000 ohm is measured each time as $V_R$. For the determination of the C a student wants to draw a linear graph and try to get C from the slope. Then she may draw a graph of **[Olympiad (Stage-1) 2017]**

(A) $f^2$ against $V_R^2$ (B) $\frac{1}{f^2}$ against $\frac{V_S^2}{V_R^2}$ (C) $\frac{1}{f^2}$ against $\frac{1}{V_R^2}$ (D) f against $\frac{V_R}{\sqrt{V_s^2 - V_R^2}}$

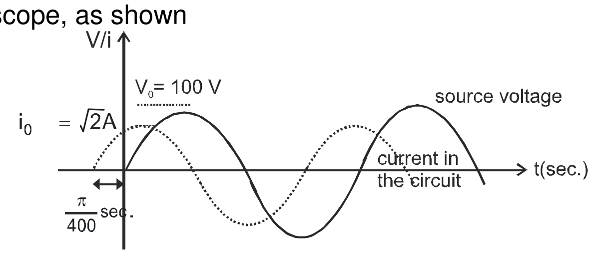
# PART - IV : COMPREHENSION

**Comprehension-1**

A voltage source V = $V_0$ sin (100 t) is connected to a black box in which there can be either one element out of L, C, R or any two of them connected in series.




At steady state the variation of current in the circuit and the source voltage are plotted together with time, using an oscilloscope, as shown



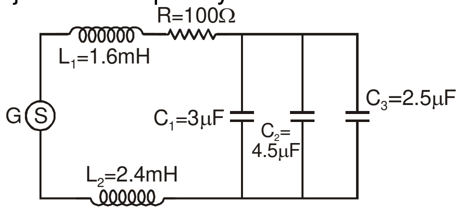

**1.** The element(s) present in black box is/are :
(A) only C (B) L and C (C) L and R (D) R and C

**2.** Values of the parameters of the elements, present in the black box are -
(A) R = 50Ω, C = 200 μf (B) R = 50Ω, L = 2mμ (C) R = 400Ω, C = 50 μf (D) None of these

**3.** If AC source is removed, the circuit is shorted for some time so that capacitor is fully discharged and then a battery of constant EMF is connected across the black box, at t = 0. The current in the circuit will
(A) increase exponentially with time constant = 0.02 sec.
(B) decrease exponentially with time constant = 0.01 sec.
(C) oscillate with angular frequency 20 rad/sec
(D) first increase and then decrease

**Comprehension-2**

An ac generator G with an adjustable frequency of oscillation is used in the circuit, as shown.




**4.** Current drawn from the ac source will be maximum if its angular frequency is -
(A) $10^5$ rad/s (B) $10^4$ rad/s (C) 5000 rad/s (D) 500 rad/s

**5.** To increase resonant frequency of the circuit, some of the changes in the circuit are carried out. Which change(s) would certainly result in the increase in resonant frequency ?
(A) R is increased.
(B) $L_1$ is increased and $C_1$ is decreased.
(C) $L_2$ is decreased and $C_2$ is increased.
(D) $C_3$ is removed from the circuit.

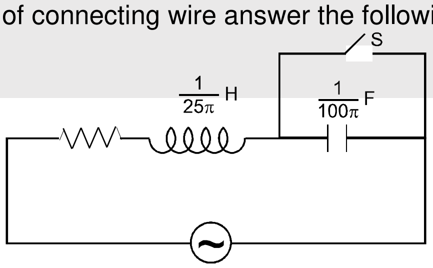
**6.** If the ac source G is of 100 V rating at resonant frequency of the circuit, then average power supplied by the source is -
(A) 50 W (B) 100 W (C) 500 W (D) 1000 W

**7.** Average energy stored by the inductor $L_2$ (Source is at resonance frequency) is equal to
(A) zero (B) 1.2 mJ (C) 2.4 mJ (D) 4 mJ

**8.** Thermal energy produced by the resistance R in time duration 1 μs, using the source at resonant condition, is
(A) 0 J
(B) 1 μJ
(C) 100 μJ
(D) not possible to calculate from the given information

**Comprehension-3**✍

In the LCR circuit shown in figure unknown resistance and alternating voltage source are connected. When switch 'S' is closed then there is a phase difference of $\frac{\pi}{4}$ between current and applied voltage and voltage accross resister is $\frac{100}{\sqrt{2}}$ V. When switch is open current and applied voltage are in same phase. Neglecting resistance of connecting wire answer the following questions :




**9.**✍ Peak voltage of applied voltage sources is :
(A) $200\sqrt{2}$ V (B) 100 V (C) $100\sqrt{2}$ V (D) $\frac{100}{\sqrt{2}}$ V

**10.**✍ Resonance frequency of circuit is :
(A) 50 Hz
(B) 25 Hz
(C) 75 Hz
(D) Data insufficient for caculation

**11.**✍ Average power consumption in the circuit when 'S' is open :
(A) 2500 W (B) 3000 W (C) 5000 W (D) 1250 W

# Exercise-3

✍ **Marked Questions can be used as Revision Questions.**

* **Marked Questions may have more than one correct option.**

## PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

**1.** An AC voltage source of variable angular frequency $\omega$ and fixed amplitude V connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When $\omega$ is increased :

**[JEE 2010; 3/163, –1]**

(A) the bulb glows dimmer
(B) the bulb glows brighter
(C) total impedence of the circuit is unchanged
(D) total impedence of the circuit increases

**2.** You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II.** When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage $V_1$ and $V_2$. (indicated in circuits) are related as shown in **Column I.** Match the two column. **[JEE 2010; 8/163]**

**Column I**

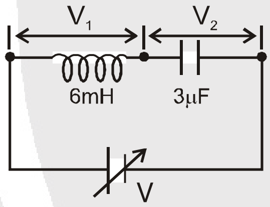
(A) $I \neq 0, V_1$ is proportional to $I$

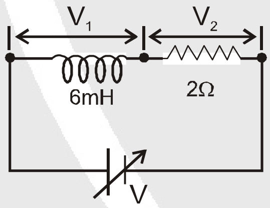
(B) $I \neq 0, V_2 > V_1$

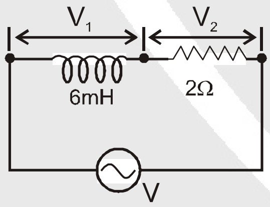
(C) $V_1 = 0,\ V_2 = V$

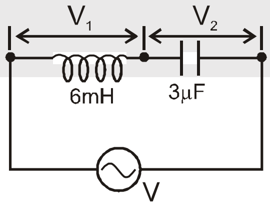
(D) $I \neq 0, V_2$ is proportional to $I$

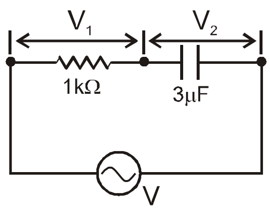
**Column II**

(p)



(q)



(r)



(s)


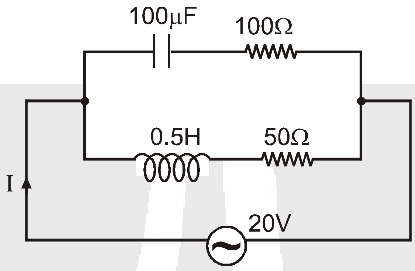

(t)


**3.*** A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current $I_R$ through the resistor and voltage $V_C$ across the capacitor are compared in the two cases. Which of the following is/are true? **[JEE 2011; 4/160]**

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$ (C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

**4.** A series R-C combination is connected to an AC voltage of angular frequency $\omega$ = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is **[JEE 2011; 4/160 ]**

**5.*** In the given circuit, the AC source has $\omega$ = 100 rad/s. considering the inductor and capacitor to be ideal, the correct choice (s) is(are) **[IIT-JEE-2012, Paper-2; 4/66]**




(A) The current through the circuit, I is approximately 0.3A

(B) The current through the circuit, I is $0.3\sqrt{2}$ A.

(C) The voltage across 100Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50Ω resistor = 10V

## Paragraph for Questions 6 and 7

A thermal power plant produces electric power of 600 kW at 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values. **[JEE(Advanced) 2013; 3/60]**

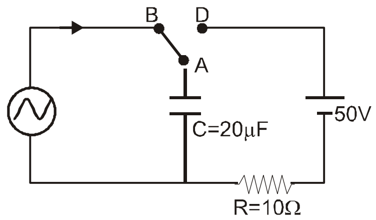
**6.** If the direct transmission method with a cable of resistance 0.4 Ω $km^{-1}$ is used, the power dissipation (in %) during transmission is :

(A) 20 (B) 30 (C) 40 (D) 60

**7.** In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is :

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

**8.*** At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and alternating current $I(t) = I_0\cos(\omega t,)$, with $I_0$ = 1A and $\omega$ = 500 rad $s^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If C = 20μ, R = 10Ω and the battery is ideal with emf of 50V, identify the correct statement (s) **[JEE (Advanced) 2014,P-1, 3/60]**



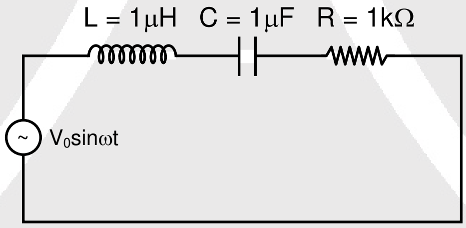

(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3}$ C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise

(C) Immediately after A is connected to D. the current in R is 10A.

(D) $Q = 2 \times 10^{-3}$ C.

**9*.** In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin\omega t$ as shown. Which of the following options is/are correct ?**[JEE (Advanced) 2017, P-1, 4/61, –2]**




(A) The current will be in phase with the voltage if $\omega = 10^4$ rad.$s^{-1}$

(B) At $\omega >> 10^6$ rad.$s^{-1}$, the circuit behaves like a capacitor

(C) The frequency at which the current will be in phase with the voltage is independent of R

(D) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero

**10*.** The instantaneous voltages at three terminals marked X, Y and Z are given by

$$V_X = V_0 \sin\omega t,\ V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading (s) of the voltmeter will be

**[JEE(Advanced) 2017 ; P-2, 4/61, –2]**

(A) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$ (B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) independent of the choice of the two terminals (D) $V_{XY}^{rms} = V_0$

# PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

**1.** A circuit has a resistance of 12 ohm and an impedance of 15 ohm. The power factor of the circuit will be : **[AIEEE 2005; 4/300]**

(1) 0.8 (2) 0.4 (3) 1.25 (4) 0.125

**2.** The phase difference between the alternating current and emf is $\pi/2$. Which of the following cannot be the constituent of the circuit? **[AIEEE 2005; 4/300]**

(1) C alone (2) R, L (3) L, C (4) L alone

**3.** In a series LCR circuit R = 200 $\Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30º. On taking out the inductor from the circuit the current leads the voltage by 30º. The power dissipated in the LCR circuit is **[AIEEE 2010; 4/144, –1]**

(1) 305 W (2) 210 W (3) 0 W (4) 242 W

**4.** An arc lamp requires a direct current of 10 A at 80 V to function. If it is connected to a 220V(rms), 50 Hz AC supply, the series inductor needed for it to work is close to : **[JEE (Main) 2016, 4/120, –1]**

(1) 0.08 H (2) 0.044 H (3) 0.065 H (4) 80 H

**5.** For an RLC circuit driven with voltage of amplitude $\upsilon_m$ and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by : **[JEE (Main) 2018; 4/120, –1]**

(1) $\frac{R}{(\omega_0 C)}$ (2) $\frac{CR}{\omega_0}$ (3) $\frac{\omega_0 L}{R}$ (4) $\frac{\omega_0 R}{L}$

**6.** In an a.c circuit, the instantaneous e.m.f and current are given by

e = 100 sin 30t

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c the average power consumed by the circuit and the wattless current are, respectively: **[JEE (Main) 2018; 4/120, –1]**

(1) $\frac{50}{\sqrt{2}}$,0 (2) 50,0 (3) 50,10 (4) $\frac{1000}{\sqrt{2}}$,10

# Answers

## EXERCISE-1

### PART – I

### Section (A) :

**A-1.** $220\sqrt{2}$ V, 2.5 ms

**A-2.** $\frac{I_o}{e}\sqrt{(e^2-1)/2}$

**A-3.** $I_{eff} = \left[a^2 + \frac{1}{2}b^2\right]^{1/2}$

**A-4.** 0

### Section (B) :

**B-1.** $12\sqrt{2}$ volts

**B-2.** 0.72 W

**B 3.** 10 W, 5 W

### Section (C) :

**C-1.** 3.0 kV

**C-2.** $\frac{2.2\sqrt{3}}{\pi}$ =1.2 H = $\frac{7\sqrt{3}}{10}$ H

**C 3.** (a) 0.1 A (b) 50 V, 30 V, 10 V

(Note that the sum of the rms potential differences across the three elements is greater than the rms voltage of the source.)

**C-4.** 25 mJ, 5mJ

**C-5.** (i) $\frac{125}{33}$ $\mu$F or 2.4 H (ii) 720 $\Omega$

(iii) It will be more economical to use inductance or capacitance in series with the lamp to run it as it.It consumes no power while there would be dissipation of power when resistance is inserted in series with the lamp.

**C-6.** $Z = 50\sqrt{2}$ ohm, $V_C = 500\sqrt{2}$ volt and

$V_L = 600\sqrt{2}$ volt, $\frac{1}{\sqrt{2}}$

**C-7.** $\sqrt{\frac{21}{22}}$ H, 100 Ω  **C-8.** 125Ω, 288J

## Section (D) :

**D-1.** 4μF, $R = \frac{141.4}{5}\Omega$

**D-2.** (a) $\frac{250}{3\pi}$ Hz (b) 2 mA  **D-3.** 1.5 A

**D-4.** $1\times10^{-8}$ henry

## Section (E) :

**E-1** zero

**E-2.** (a) 800 V

(b) (i) 0.25 A  (ii) 0.2 A.

## PART - II

## Section (A) :

| | | | | | |
|---|---|---|---|---|---|
| **A-1.** | (B) | **A-2.** | (B) | **A-3.** | (C) |
| **A-4.** | (D) | **A-5.** | (D) | **A-6.** | (D) |

## Section (B) :

| | | | | | |
|---|---|---|---|---|---|
| **B-1.** | (B) | **B-2.** | (C) | **B-3.** | (A) |
| **B-4.** | (C) | **B-5.** | (B) | **B-6.** | (C) |
| **B-7.** | (B) | **B-8.** | (B) | **B-9.** | (D) |
| **B-10.** | (A) | **B-11.** | (B) | **B-12.** | (B) |

## Section (C) :

| | | | | | |
|---|---|---|---|---|---|
| **C-1.** | (A) | **C-2.** | (B) | **C-3.** | (C) |
| **C-4.** | (C) | **C-5.** | (B) | **C-6.** | (B) |
| **C-7.** | (C) | **C-8.** | (D) | | |

## Section (D) :

| | | | | | |
|---|---|---|---|---|---|
| **D-1.** | (B) | **D-2.** | (A) | **D-3.** | (A) |
| **D-4.** | (D) | **D-5.** | (B) | **D-6.** | (A) |

## Section (E) :

| | | | | | |
|---|---|---|---|---|---|
| **E-1.** | (D) | **E-2.** | (C) | **E-3.** | (D) |
| **E-4.** | (A) | | | | |

## Section (F) :

| | | | | | |
|---|---|---|---|---|---|
| **F-1.** | (A) | **F-2.** | (C) | **F-3.** | (A) |
| **F-4.** | (B) | **F-5.** | (A) | **F-6.** | (D) |
| **F-7.** | (C) | | | | |

## PART - III

**1.** (A) → q,r; (B) → q,r; (C) → p,q,r,s; (D)→q,r, s

**2.** (A) → r ; (B) → q ; (C) → p ; (D) → q

# EXERCISE-2

## PART - I

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (B) | **2.** | (A) | **3.** | (C) |
| **4.** | (B) | **5.** | (C) | **6.** | (D) |
| **7.** | (D) | **8.** | (A) | | |

**9.** (i) (C) (ii) (B) (iii) (A) (iv) (C) (v) (B) (vi) (B)

**10.** (A)

## PART - II

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 3 | **2.** | 48 | **3.** | 50 |
| **4.** | 8 | **5.** | 75 | **6.** | 9 |

## PART - III

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (ABCD) | **2.** | (BD) | **3.** | (AB) |
| **4.** | (AB) | **5.** | (ABC) | **6.** | (ABC) |
| **7.** | (AB) | **8.** | (BC) | **9.** | (ABCD) |
| **10.** | (AC) | **11.** | (ABC) | **12.** | (AC) |
| **13.** | (BD) | **14.** | (CD) | **15.** | (BCD) |

## PART - IV

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (D) | **2.** | (A) | **3.** | (B) |
| **4.** | (C) | **5.** | (D) | **6.** | (B) |
| **7.** | (B) | **8.** | (D) | **9.** | (C) |
| **10.** | (B) | **11.** | (C) | | |

# EXERCISE-3

## PART - I

**1.** (B)

**2.** (A)→r,s,t;(B)→q,r,s,t;(C)→p,q;(D)→q,r,s,t

| | | | | | |
|---|---|---|---|---|---|
| **3.** | (BC) | **4.** | 4 | **5.** | (AC) |
| **6.** | (B) | **7.** | (A) | **8.** | (CD) |
| **9.** | (CD) | **10.** | (AC) | | |

## PART - II

| | | | | | |
|---|---|---|---|---|---|
| **1.** | (1) | **2.** | (2) | **3.** | (4) |
| **4.** | (3) | **5.** | (3) | **6.** | (4) |

# High Level Problems (HLP)

## SUBJECTIVE QUESTIONS

**1.** A current of 4 A flows in a coil when connected to a 12 V d.c. source. If the same coil is connected to a 12 V, 50 rad/s, AC source, a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also, find the power developed in the circuit if a 2500 μF condenser is connected in series with coil. **[REE - 1993]**

**2.** A box P and a coil Q are connected in series with an AC source of variable frequency. The EMF of source is constant at 10 V. Box P contains a capacitance of 1 μF in series with a resistance of 32 Ω. Coil Q has a self inductance 4.9 mH and a resistance of 68 Ω. The frequency is adjusted so that the maximum current flows in P and Q. Find the impedance of P and Q at this frequency. Also find the voltage across P and Q respectively. **[REE - 1998]**

**3.** In a series LCR circuit with an ac source of 50 V, R = 300Ω, frequency $\nu = \frac{50}{\pi}$ Hz. The average electric field energy, stored in the capacitor and average magnetic energy stored in the coil are 25 mJ and 5 mJ respectively. The RMS current in the circuit is 0.10 A. Then find:
(a) Capacitance (c) of capacitor
(b) Inductance (L) of inductor.
(c) The sum of rms potential difference across the three elements.

**4.** An inductor $20 \times 10^{-3}$ Henry, a capacitor 100 μF and a resistor 50Ω are connected in series across a source of EMF V = 10 sin 314 t. Find the energy dissipated in the circuit in 20 minutes. If resistance is removed from the circuit and the value of inductance is doubled, then find the variation of current with time (t in second) in the new circuit. **[REE - 1999]**

**5.** The electric current in an AC circuit is given by $i = i_0 \sin \omega t$. What is the time taken by the current to change from its maximum value to the rms value? **[REE - 1999]**

**6.** A circuit containing a 0.1 H inductor and a 500 μF capacitor in series is connected to a 230 volt, 100/π Hz supply. The resistance of the circuit is negligible. (a) Obtain the current amplitude and rms values. (b) Obtain the rms values of potential drops across each element. (c) What is the average power transferred to the inductor? (d) What is the average power transferred to the capacitor? (e) What is the total average power absorbed by the circuit? ['Average' implies average over one cycle.]

**7.** A series LCR circuit with L = 0.125/π H, C = 500/π nF, R = 23 Ω is connected to a 230 V variable frequency supply.
(a) What is the source frequency for which current amplitude is maximum? Obtain this maximum value.
(b) What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of this maximum power.
(c) For what reactance of the circuit, the power transferred to the circuit is half the power at resonance? What is the current amplitude at this reactance ?
(d) If ω is the angular frequency at which the power consumed in the circuit is half the power at resonance, write an expression for ω
(e) What is the Q-factor (Quality factor) of the given circuit?

**8.** The maximum values of the alternating voltages and current are 400 V and 20 A respectively in a circuit connected to 50 Hz supply and these quantities are sinusoidal. The instantaneous values of the voltage and current are $200\sqrt{2}$ V and 10 A respectively. At that instant both are increasing positively. Determine the average power consumed in the circuit.

**9.** A 750 Hz, 20 V source is connected to a resistance of 100 Ω, a capacitance of 1.0 μF and an inductance of 0.18 H in series. Calculate the following quantities : **[Olympaid 2013-14]**

(a) Impedence of the circuit

(b) Draw an impedence diagram with suitable scale

(c) Power factor

(d) The time in which the resistance will get heated by 10ºC, provided that the thermal capacity of resistance = 2 J/ºC

**10.** In the given circuit

Calculate

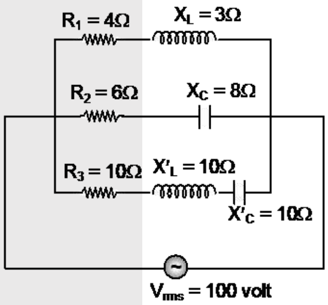
(a) Current in each branch.

(b) Power generated in each resistance.

(c) Total power generated in the circuit.

(d) Net current drawn from source.

(e) Net impedance of the circuit.




**11.** A metallic coil of N turns of radius a, resistance R, and inductance L is held fixed with its axis along a spatial uniform magnetic field $\vec{B}$ whose magnitude is given by $B_0 \sin(\omega t)$.

(a) Write the emf equation for the current i in the coil.

(b) Assuming that in the steady state i. oscillates with the same frequency ω as the magnetic field, obtain the expression for i.

(c) Obtain the force per unit length. Further obtain its oscillatory part and the time-averaged compressional part.

(d) Calculate the time-averaged compressional force per unit length given that $B_0$ = 1.00 tesla, N = 10, a = 10.0 cm, ω = 1000.0 rad-s$^{-1}$, R = 10.0 Ω , L = 100.0 mH.

## HLP Answers

**1.** .08 H; 17.28 watt **2.** P = 76.96 Ω, Q = 97.59Ω, P ≈ 7.7 V; Q = 9.8 V, net impedance =100 Ω

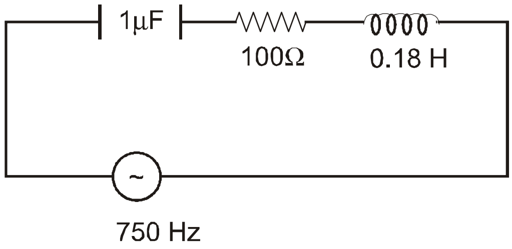
**3.** (a) C = 20 μF, (b) 1 H, (c) 90 V **4.** 951.52 J; 0.52 cos 314 t **5.** T/8 or $\frac{\pi}{4\omega}$

**6.** (a) $23\sqrt{2}$ A, 23 A, (b) 460 volt, 230 volt, (c) zero, (d) zero, (e) zero

**7.** (a) 2000 Hz, $10\sqrt{2}$ A, (b) 2000 Hz, 2300 watt, (c) 23 Ω, 10 A. (d) $\frac{0.125}{\pi}\omega - \frac{1\times10^9}{\omega\frac{500}{\pi}} = \pm$ 23 (e) 500/23

**8.** P = 3864 W

**9.** $z = \sqrt{(X_L - X_C)^2 + R^2}$




$\omega = 2n\pi = 2(750)\pi = 1500\pi$

$X_L = \omega_L = (1500\pi) \quad (0.18) = 848.5 \quad \Omega$

$X_C = \frac{1}{\omega C} = \frac{10^6}{1500\pi} = 212.12 \quad \Omega$

$z = \sqrt{(636.4)^2 + (100)^2} = 100\sqrt{(6.364)^2 + 12}$

$= 100 \times 6.44 \cong 644 \quad \Omega$

$\tan\phi = \frac{X_L - X_C}{R} = \frac{848.5 - 212.12}{100} = 6.36$

(c) $\cos\phi = \frac{R}{z} = \frac{100}{644} = 0.155$

(b) impedence is constant as n is constant

(d) $i_{rms} = \frac{\varepsilon_{rms}}{z} = \frac{20}{644}A$

$H = (i_{rms})^2 Rt = \left(\frac{20}{644}\right)^2 (100)t \quad \Rightarrow \quad (ms)\ (\Delta\theta) = \left(\frac{20}{644}\right)^2 (100)\ t$

$(2)\ (10) = \left(\frac{20}{644}\right)^2 (100)\ t \quad \Rightarrow \quad t = \left(\frac{644}{20}\right)^2 \times \frac{1}{100} \times 20 \Rightarrow \quad t = 207.36$ sec.

**10.** Impedence of each branch $Z_1 = \sqrt{R_1^2 + X_L^2} = \sqrt{4^2 + 3^2} = 5$

$Z_2 = \sqrt{6^2 + 8^2} = 10$

$Z_3 = \sqrt{(10)^2 + (10 - 10)^2} = 10$

(a) Current in each branch $I_1 = \frac{V}{Z_1} = \frac{100}{5} = 20$ Amp.

$I_2 = \frac{100}{10} = 10$ Amp.

$I_3 = \frac{100}{10} = 10$ Amp.

(b) Power in each branch.

$P_1 = (I_1)^2 R_1 = (20)^2 (4) = 1600$ watt

$P_2 = (I_2)^2 R_2 = (10)^2 (6) = 600$ watt

$P_3 = (I_3)^2 R_3 = (10)^2 (10) = 1000$ watt

(c) Net power of the circuit.

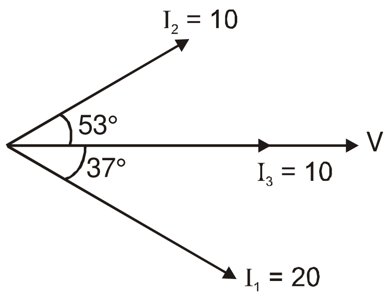
$P = P_1 + P_2 + P_3 = 3200$ watt

phase difference between voltage & current in each branch.

$\tan\phi_1 = \frac{X_L}{R_1} = \frac{3}{4} \Rightarrow \phi_1 = 37°$

$\tan\phi_2 = \frac{X_C}{R_2} = \frac{8}{6} \Rightarrow \phi_2 = 53°$

$\tan\phi_3 = 0 \Rightarrow \phi_3 = 0$




(d) Net current drawn from source.

$\sqrt{(I_1\cos 37° + I_2\cos 53° + I_3)^2 + (I_1\sin 37° - I_2\sin 53°)^2} = \sqrt{1040}$

(e) Net impedance of the circuit. $Z = \frac{V}{I} = \frac{100}{\sqrt{1040}}$

**11.** (a) $iR + L\frac{di}{dt} = -N\pi a^2 B_0 \omega \cos \omega t$

(b) $i = \frac{N\pi a^2 B_0\omega\,(R\cos\omega t + \omega L\sin\omega t)}{R^2 + \omega^2 L^2}$

(c) $\frac{dF}{d\ell} = -\frac{NB_0^2\ \pi a^2\omega}{R^2 + \omega^2 L^2}$ (R sin ωt cos ωt + ω L sin² ωt)

$= \left.\frac{dF}{d\ell}\right|_{av} = -\frac{NB_0^2\ \pi a^2\omega^2 L}{2(R^2 + \omega^2 L^2)}$

$= \left.\frac{dF}{d\ell}\right|_{osc} = -\frac{NB_0^2\ \pi a^2\omega}{2(R^2 + \omega^2 L^2)}$ (R sin 2 ωt – ωL cos 2 ωt)

(d) $\left.\frac{dF}{d\ell}\right|_{av} = 1.55\ N.m^{-1}$