EXERCISE–1
PART - I

Section (A)

A-1. $Q_1 = 30\mu C$, $C_1 = 5\mu F$

(i) $V_1 = \frac{Q_1}{C_1} = \frac{30}{5} = 6V$ **Ans.**

(ii) $U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(30\times10^{-6})^2}{(5\times10^{-6})} = 90\ \mu J$ **Ans.**

(iii) $Q_2 = 50\mu C$, $C_2 = 10\ \mu F$, $V_2 = \frac{Q_2}{C_2} = \frac{50}{10} = 5V$.

(a) Common potential $V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{30+50}{5+10} = \frac{16}{3}V$ **Ans.**

(b) $\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2 = \frac{1}{2}\frac{5\times10}{5+10}(6-5)^2 = \frac{5}{3}\mu J$ **Ans.**

(c) $\frac{Q_1'}{Q_2'} = \frac{C_1}{C_2} = \frac{5}{10} = \frac{1}{2}$ **Ans.**

(d) $Q_1' = C_1V = 5\times\frac{16}{3} = \frac{80}{3}\mu C$ $\quad Q_2' = C_2V = 10\times\frac{16}{3} = \frac{160}{3}\mu C$.

A-2. $F_{attraction} = \frac{q^2}{2\epsilon_0 A}$; $F_{Repulsion} = kx$

$|F_{attraction}| = |F_{Repulsion}| \Rightarrow \frac{q^2}{2\epsilon_0 A} = kx$

$x = \frac{q^2}{2k\epsilon_0 A}$

A-3. $W = q\Delta V$

$W_{by\ field} = q(V_i - V_f) = Q(V_1 - V_2)$

A-4. The electric force between the plates will be balanced by the additional weight

hence $mg = \frac{Q^2}{2A\epsilon_0} = \frac{C^2V^2}{2A\epsilon_0}$

$mg = \frac{\epsilon_0 AV^2}{2d^2}$

$m = \frac{\epsilon_0 AV^2}{2d^2 g} = \frac{\epsilon_0\times 100\times 10^{-4}\ (5000)^2}{2\ (5\times 10^{-3})^2\times 10}$

$m = 4.425$ g **Ans.**

A-5. Work done = change in potential energy

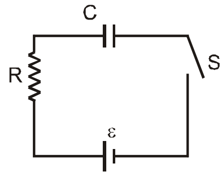
(i) $W = U_f - U_i = \frac{q^2}{2C_f} - \frac{q^2}{2C_i} = \frac{q^2}{2}\left\{\frac{x_2}{S\epsilon_o} - \frac{x_1}{S\epsilon_o}\right\} = \frac{q^2\ (x_2 - x_1)}{2\ S\epsilon_o}$ s

(ii) $W_{ex} + W_B = U_f - U_i = \left\{\frac{S\epsilon_o}{x_2} - \frac{S\epsilon_o}{x_1}\right\} = \frac{\epsilon_o\ SV^2}{2}\left\{\frac{1}{x_2} - \frac{1}{x_1}\right\}$

$W_{ex} = (U_f - U_i) - W_B = \frac{\Delta CV^2}{2} - \Delta CV^2 = -\frac{\Delta CV^2}{2} = (-)\frac{\epsilon_0 SV^2\left(\frac{1}{x_2} - \frac{1}{x_1}\right)}{2}$

Section (B)

B-1.



at t = 0, C is replace by wire.

(a) $V_{rmax} = \varepsilon$

(b) $i = \varepsilon/R$

at $t \rightarrow \infty$, C is replace by broken wire and now current in circuit = 0, so

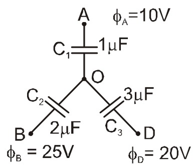
(c) $V_C = \varepsilon$

(d) $U_C = \frac{1}{2}C\varepsilon^2$

(e) $P_{battery} = i\,V = \frac{\varepsilon}{R}\varepsilon = \frac{\varepsilon^2}{R}$

(f) $\Delta H = \frac{\varepsilon^2}{R}$.

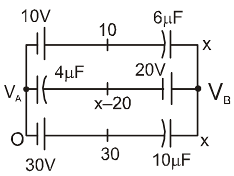
B-2.



$(V_A - V_o)C_1 + (V_B - V_o)C_2 + (V_D - V_o)C_3 = 0$

$V_o = \frac{V_AC_1 + V_BC_2 + V_DC_3}{C_1 + C_2 + C_3} = \frac{10 \times 1 + 25 \times 2 + 20 \times 3}{1 + 2 + 3} = 20$ V**Ans.**

B-3.



Assume माना ($V_A = 0$), ($V_B = x$)

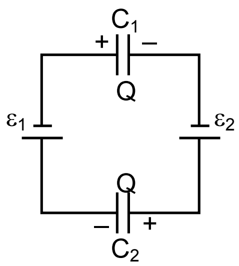
From conservation of charge on plates of capacitor

$4(x - 20) + (x - 10)6 + (x - 30)10 = 0$

$x = 22$

$V_A - V_B = 0 - 22 = -22$ V.

B-4.



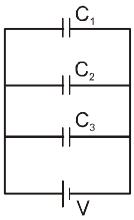
$Q = (\varepsilon_2 - \varepsilon_1)\frac{C_1C_2}{C_1 + C_2}$;

$$V_1 = \frac{Q}{C_1} = \frac{\varepsilon_2 - \varepsilon_1}{1 + \frac{C_1}{C_2}}$$

$$V_2 = -\frac{Q}{C_2} = \frac{\varepsilon_1 - \varepsilon_2}{1 + \frac{C_2}{C_1}}$$

Section (C)

C-1.



$Q_1 = C_1V = 1 \times 20 = 20\ \mu C$

$Q_2 = C_2V = 2 \times 20 = 40\ \mu C$

$Q_3 = C_3V = 3 \times 20 = 60\ \mu C$

$Q_t = 120\ \mu C$

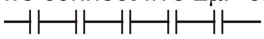
$W_{battery} = Q_tV = 120 \times 20 = 2400\ \mu J$

$U_C = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \frac{1}{2}C_3V^2 = \frac{1}{2}\ (C_1 + C_2 + C_3)V^2$

$= \frac{1}{2}\ (1 + 2 + 3)\ (20)^2 = 1200\ \mu J.$

C-2. (a) $C_{eq} = 0.4\ \mu F$,

we connect five $2\mu F$ capacitors in series



$\Rightarrow \quad C_{eq} = \frac{C}{5} = \frac{2}{5} = 0.4\ \mu F$

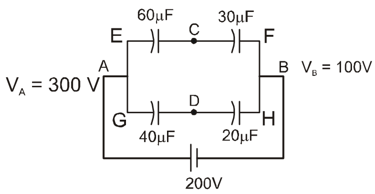
(b) $C_{eq} = 1.2\ \mu F$, capable of with standing 1000 volts

3 parallel rows, each consisting of five $2.0\ \mu F$ capacitors in series.

C-3. There three circuit are equivalent and all these are balance wheat stone bridge. For all given circuits.

$C_{eq} = \frac{5 \times 10}{5 + 10} + \frac{10 \times 20}{10 + 20} = 10\mu F$ **Ans.**

C-4.



Charge on $30\mu F = 200 \times 20 = 4000\mu C$

so $V_C - V_B = \frac{4000\mu C}{30\mu F} \quad \Rightarrow \quad V_C = 100 + \frac{400}{3} = \frac{700}{3}$ V

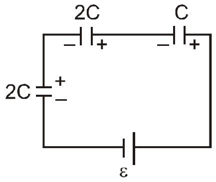
also charge on $20\mu F$

$= 200 \times \frac{40}{3} = \frac{8000}{3}\ \mu C$

so अत: $V_D - V_B = \frac{8000/3}{20}$

$V_D = 100 + \frac{400}{3} = \frac{700}{3}$ V $\quad \Rightarrow \quad V_C - V_D = 0$

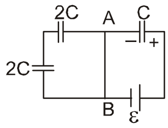
C-5. Situation when S is open



$C_{eq.} = C/2$

Charge supplied $Q = C_{eq.}\varepsilon = \frac{C\varepsilon}{2}$

Situation when S is closed



$C_{eq.} = C$

Charge on capacitor $C = C\varepsilon$

After S is closed, voltage across 2C capacitor become zero, so charge on it also become zero.

(a) So charge flown through the battery when the switch S is closed = $C\varepsilon - \frac{C\varepsilon}{2} = \frac{C\varepsilon}{2}$

(b) the charge flown through the switch S is $\frac{C\varepsilon}{2} + \frac{C\varepsilon}{2} = C\varepsilon$ (A to B) = $-C\varepsilon$ (B to A)

(c) Work done by the battery = $Q_{supplied}\, V = \frac{C\varepsilon}{2}\,\varepsilon = \frac{C\varepsilon^2}{2}$

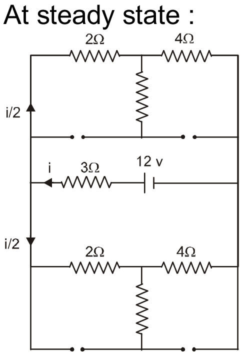
(d) change in energy stored in the capacitors

energy stored in 2nd case – energy stored in 1st case = $\frac{1}{2}C_{eq2}\,V^2 - \frac{1}{2}C_{eq1}V^2 = \frac{1}{2}.\left(C - \frac{C}{2}\right)\varepsilon^2 = \frac{C\varepsilon^2}{4}$

(e) Heat developed in the system = Work done by battery – change in energy in the capacitors

$= \frac{C\varepsilon^2}{2} - \frac{C\varepsilon^2}{4} = \frac{C\varepsilon^2}{4}$

C-6. At steady state :



Current in wire AB at steady state = I/2 = 1 amp.
Charge on 2μF capacitor = $2 \times 2 \times 10^{-6} = 4\mu C$
Charge on 4μF capacitor = $4 \times 4 \times 10^{-6} = 16\mu C$
Charge on 6μF capacitor = $2 \times 6 \times 10^{-6} = 12\mu C$
Charge on 8μF capacitor = $4 \times 8 \times 10^{-6} = 32\mu C$.

Section (D)

D-1. $V = V_0(1 - e^{-t/RC})$

$4 = 12(1 - e^{-1\times10^{-6}/10C})$

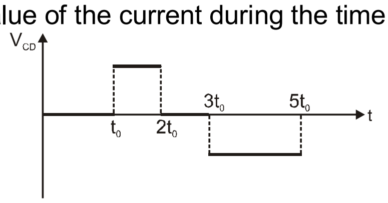
$\ell n\frac{3}{2} = \frac{10^{-7}}{C} \quad \Rightarrow \quad C = \frac{10^{-7}}{\ell n3/2}\,F = 0.25\ \mu F.$

D-2. (i) (a) Time constant = t = RC = $10^7 \times 1 \times 10^{-6}$
= 10 sec.
(b) $Q_0 = CV = 1 \times 10^{-6} \times 2 = 2\ \mu C$
(c) $Q = \dfrac{Q_0}{2} = Q_0(1 - e^{-t/Rc})$
$t/RC = \ell n2 \quad \Rightarrow \quad t = 10\ \ell n2 = 6.93$ sec.
(ii) $q = q_0 e^{-t/RC}$
$= 2 \times 10^{-6}\ e^{-50/10} = 2 \times 10^{-6}\ e^{-5} = 1.348 \times 10^{-8}$ C.

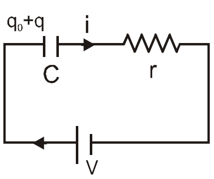
D-3. For charging
$q_1 = CV(1 - e^{-t/RC}) = 20 \times 10^{-4}\ (1 - e^{-16/200 \times 10^{-6} \times 40 \times 10^3}) = 20 \times 10^{-4}\ (1 - e^{-2})$
For discharging
$q_2 = q_1 e^{-t/RC}$
$= 20 \times 10^{-4}\ (1 - e^{-2})$
$= 20 \times 10^{-4}\ (1 - e^{-2})\ e^{-2} = 20 \times 10^{-4}\ \dfrac{(e^2 - 1)}{e^4}$
$= 233.55\ \mu C$ **Ans.**

D-4. $i_0 = \dfrac{20}{5} \times \dfrac{1}{5} = \dfrac{4}{5}$ amp.
$i = i_0 e^{-t/RC}$
$$H = \int i^2 R\,dt = \int_{25\mu S}^{50\mu s} \frac{16}{5} e^{-2t/25\times10^{-6}} dt$$
$$= \frac{16}{5}\left(-\frac{25}{2} \times 10^{-6}\right)\left[e^{-2t/25\times10^{-6}}\right]_{25\mu s}^{50\mu s}$$
$$= 40 \times 10^{-6}\left[\frac{e^2 - 1}{e^4}\right]$$
$= 4.7\ \mu J$ **Ans.**

D-5. It can be seen that during the time interval from 0 to t_0, the voltage across the capacitor is zero, the charge on it is also zero, there is no current through it and hence V_{CD} is zero during this time interval (fig.). During the time interval from t_0 to $2t_0$, the voltage across the capacitor and hence the charge on its plates, grows linearly and hence a direct current passes through the circuit. This means that the voltage V_{CD} is constant. During the time interval from $2t_0$ to $3t_0$, the voltage across the capacitor does not change. Hence current does not flow, and V_{CD} is zero. Finally, during the time interval from $3t_0$ to $5t_0$, the capacitor is discharged, the current through the resistor is negative and constant and its magnitude is half the value of the current during the time interval from t_0 to $2t_0$.



D-6.



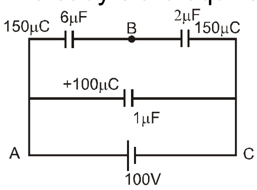
at time t using KVL
$$-\frac{q_0 + q}{C} - ir + V = 0 \quad \Rightarrow \quad -\frac{q_0 + q}{C} - \frac{dq}{dt} r + V = 0$$

$\Rightarrow \quad \int_0^q \frac{dq}{(CV - q_0) - q} = \int_0^t \frac{dt}{rC}$

by using integration $q = CV\,(1 - e^{-t/Cr}) + q_0\, e^{-t/Cr} - q_0$

$\Rightarrow$ So charge on capacitor = $q_0 + q = CV(1 - e^{-t/Cr}) + q_0e^{-t/Cr}$

D-7. In steady state equivalent circuit is



$\mathbf{C_{eq}} = \frac{5}{2}\mu F \qquad Q = 250\mu F$

$V_A = \frac{150}{6} + V_B \quad \Rightarrow \quad V_A - V_B = 25\ V$

$V_B = \frac{150}{2} + V_C \quad \Rightarrow \quad V_B - V_C = 75\ V$

Section (E)

E-1. $V = \frac{Q}{C} = \frac{Q}{\frac{K \epsilon_0 A}{d}}$; $\qquad R = \frac{\rho d}{A}$

(we can treat dielectric as a resistance between the capacitor plates)

$i = \frac{V}{R} = \frac{Q}{\frac{K \epsilon_0 A}{d}} \cdot \frac{A}{\rho d} = \frac{Q}{K \epsilon_0 \rho}$ **Ans**

E-2. (i) $C_0 = \frac{\epsilon_0 A}{d} = \frac{0.2 \epsilon_0}{10^{-2}} = 20\ \epsilon_0$

$= 20 \times 9 \times 10^{-12} = 180$ pF

(ii) $Q = C_0V = 180 \times 10^{-12} \times 3000 = 5.4 \times 10^{-7}$ C

(iii) $C_1 = \frac{Q}{V_1} = \frac{5.4 \times 10^{-7}}{1000} = 540$ pF

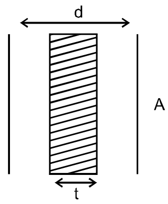
(iv) $K = \frac{C_1}{C_0} = \frac{540}{180} = 3$

(v) $\epsilon = \epsilon_r\epsilon_0 = K\epsilon_0 = 3 \times 9 \times 10^{-12} = 27 \times 10^{-12}\ C^2N^{-1}m^{-2}$

(vi) $E_0 = \frac{V}{d} = \frac{3000}{10^{-2}} = 3 \times 10^5$ V/m

(vii) $E = \frac{V_1}{d} = \frac{1000}{10^{-2}} = 1 \times 10^5$ V/m.

E-3.



$C' = \dfrac{A\in_o}{\frac{t}{k}+d-t}$

(i) Without dielectric, $C = \dfrac{A\in_o}{d}$

With dielectric, $C' = \dfrac{A\in_o}{\frac{t}{2}+d-t} = \dfrac{3C}{2} = \dfrac{3}{2}\dfrac{A\in_o}{d} \Rightarrow \dfrac{d}{t} = \dfrac{2}{3}$

(ii) Energy $= \dfrac{Q^2}{2C}$

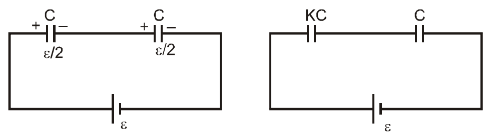
Energy in 1st case $= E_1 = \dfrac{q^2}{2C}$

Energy is 2nd case $= E_2 = \dfrac{q^2}{2C'}$

$$\frac{E_1}{E_2} = \frac{C'}{C} = \frac{3}{2}$$

(iii) $\Delta E = E_2 - E_1 = \dfrac{q^2}{2C'} - \dfrac{q^2}{2C} = \dfrac{q^2}{2}\left[\dfrac{2}{3C} - \dfrac{1}{C}\right] = -\dfrac{q^2}{6C} = -\dfrac{q^2 d}{6A\in_0}$

E-4.



$V_1 = \dfrac{Q_1}{kC} = \dfrac{\varepsilon}{k+1} \quad \Rightarrow \quad E_1 = \dfrac{V_1}{d} = \dfrac{1}{k+1}.\dfrac{\varepsilon}{d} \quad \Rightarrow \quad E = \dfrac{Q}{dC} = \dfrac{\varepsilon}{2d}$

$E_1 = \dfrac{2}{(k+1)} E \quad \Rightarrow \quad \dfrac{1}{2}$ (1 + k) time decrease

$\Delta q = Q_1 - Q = \dfrac{kC\varepsilon}{k+1} - \dfrac{C\varepsilon}{2} = \dfrac{C\varepsilon\ (2k-k-1)}{2(k+1)} \quad \Rightarrow \quad \Delta q = \dfrac{1}{2} C\varepsilon\ \dfrac{(k-1)}{(k+1)}.$

E-5. Work done on the system = change in potential energy

charge on plate $= Q = CV = \dfrac{A\in_o V}{d}$

Initial potential energy $= \dfrac{Q^2}{2\ C}$

where $C = \dfrac{A\in_o}{d}$

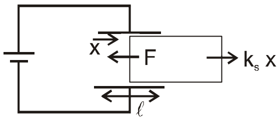
final potential energy $= \dfrac{Q^2}{2\ C'}$

where $C' = \dfrac{2KA\in_o}{d}$

Charge in potential energy = final potential energy – initial potential energy,

$= \dfrac{Q^2}{2}\left\{\dfrac{1}{C'} - \dfrac{1}{C}\right\} = \dfrac{\in_0 AV^2}{2d}\left\{\dfrac{1}{2K} - 1\right\}$

E-6.



Potential energy = $1/2(C_1 + C_2)\,V^2$

$$= \frac{1}{2}\left\{\frac{\epsilon_o\, xb}{d} + \frac{\epsilon_o\, K(\ell - x)b}{d}\right\} V^2$$

$$F = -\frac{\partial U}{\partial x} = -\frac{V^2}{2}\frac{dC}{dx}$$

$$F = \frac{-1}{2}\frac{dC}{dx} V^2$$

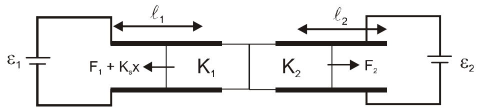
where , $\frac{dC}{dx} = \frac{\epsilon_o\, b}{d}\{1 - K\}$

$$F = \frac{1}{2}\frac{V^2 \epsilon_o\, b(K-1)}{d}$$

at equilibrium , $F = K_s\, x$

$$x = \frac{F}{k_s} = \frac{\epsilon_0\, bv^2(K-1)}{2dK_S}$$

E-7.



Force on dielectric $F = \frac{\partial U}{\partial x} = -\frac{V^2}{2}\frac{dC}{dx}$

$$F = \frac{V^2 \epsilon_0\, b(K-1)}{2d}$$

At equilibrium

$F_2 = F_1 + K_s x$

$$x = \frac{\epsilon_0\, b}{2dK_S}\left[(K_2 - 1)\varepsilon_2^{\,2} - (K_1 - 1)\ \ \varepsilon_1^{\,2}\right]$$

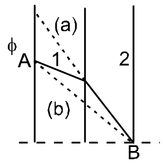
E-8. When dielectric slab is released from rest constant force act on slab towards the mean position after mean position same opposite force is act on slab which retard it come in rest position. Therefore motion of slab is periodic.

$F = \frac{1}{2}\frac{\epsilon_0\, 2\ell}{b}\ (K–1)\ V^2$ $\qquad$ $a = \frac{\epsilon_0\, \ell}{mb}(K–1)\ V^2$

by $\quad S = \frac{1}{2}\ (\text{acc.})\ t^2$ $\qquad$ $(\ell) = \frac{1}{2}\frac{\epsilon_0\, \ell}{mb}(K–1)\ V^2\ .\ t^2$

$$t = \sqrt{\frac{2\ell\ \ mb}{\epsilon_0\ \ell\ \ (K-1)\ \ V^2}}$$

$$\Rightarrow \qquad t = 2\sqrt{\frac{mb}{\epsilon_0\ \ (K-1)\ \ V^2}} \qquad \Rightarrow \qquad T = 4t = 4\sqrt{\frac{2bm}{\epsilon_0\ V^2(K-1)}}$$

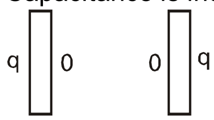
E-9. 1st part has di-electric; 

(a) by $E = \dfrac{Q}{A \in_0}$; Q remain constant so electric field in region 1 varies same as region 2, so now electric field graph of obtain by extend E of region 2

(b) now V is constant so by E = V/D ; electric field graph directly obtain by joining A and B.

E-10. (i) $C = \dfrac{\varepsilon_0 A}{d}$

Capacitance is independent of charge stored

(ii) 

on outer surfaces charge = $\dfrac{Q_1 + Q_2}{2} = \dfrac{q+q}{2} = q$

on inner surfaces charge = $\dfrac{Q_1 - Q_2}{2} = \dfrac{q-q}{2} = 0$

(iii) E = 0

electric field due to charges on outer surfaces cancel each other

(iv) $\Delta V = 0$

becouse electric field between the plates is zero

(v) U = 0

becouse electric field between the plates is zero

PART - II

Section (A)

A-1. $Q_t = Q_1 + Q_2 = 150\mu C$

$\dfrac{Q_1'}{Q_2'} = \dfrac{C_1}{C_2} = \dfrac{1}{2} \Rightarrow Q_1' = 50\mu C$

$Q_2' = 100\mu C$

$25\mu C$ charge will flow from smaller to bigger sphere.

A-2. Charge is flow until potential are equal and in charge flow energy is decrease

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q_1R_2 = Q_2R_1 .$$

A-3. (i) (C) $E = \dfrac{V}{d} = \dfrac{300}{2.5 \times 10^{-2}} = 12 \times 10^3$ V/m

(ii) (D) $\Delta U = U_f - U_i = \dfrac{1}{2} C_f V^2 - \dfrac{1}{2} C_i V^2$

$$= \frac{1}{2}\left(\frac{\in_0 A}{d_f} - \frac{\in_0 A}{d_i}\right) V^2$$

$$= \frac{1}{2}\left(\frac{1}{2.5} - \frac{1}{2}\right) \frac{9 \times 10^{-12} \times 100 \times 10^{-4}}{10^{-2}} (300)^2 = -405 \times 10^{-10} \text{J}.$$

(iii) **(D)** $E = \frac{Q}{A \in_0}$ = Constant

$= \frac{V}{d_i} = \frac{300}{2 \times 10^{-2}} = 15 \times 10^3$ V/m.

(iv) **(C)** $Q = \frac{A \in_0}{d_i} V$ = constant

$$\Delta U = \frac{1}{2}\frac{Q^2}{C_f} - \frac{Q^2}{C_i} = \frac{1}{2} A \in_0 V^2 \left(\frac{d_f}{d_i^2} - \frac{d_i}{d_i^2}\right)$$

$$= \frac{1}{2}\frac{A \in_0}{d_i^2} V^2(d_f - d_i) = \frac{1}{2}\frac{100 \times 10^{-4} \times 9 \times 10^{-12} \times (300)^2 (2.5 - 2) \times 10^{-2}}{(2 \times 10^{-2})^2} = 5.0625 \times 10^{-8} \text{J } \textbf{Ans.}$$

A-4. **(B)** Isolated capacitor $\Rightarrow$ Q = constant
separation d increase $\Rightarrow$ C = decrease
Q = CV $\Rightarrow$ V = increase

A-5. The charge on the capacitot remains constants

Capacitance	$C = \frac{\varepsilon_0 A}{d}$	d↑	C↓
Energy	$U = \frac{1}{2}\frac{Q^2}{C}$	C↓	U↑
Potential	$V = \frac{Q}{C}$	C↓	V↑

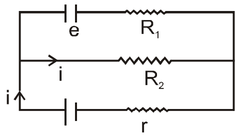
Section (B)

B-1. $W = U_f - U_i = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_i^2 = \frac{1}{2}C(40^2 - 20^2)$ W = 600 C

$W_1 = \frac{1}{2} C (50^2 - 40^2) = \frac{900}{2} C$

$W_1 = \frac{900}{2}.\frac{W}{600} = \frac{3}{4} W$ **Ans**

B-2. Charge on capacitor = CV = capacitance × (voltage across it)
In steady state, there will be no current through capacitor.



voltage across capacitor $V = iR_2 = \frac{E \ R_2}{R_2 + r}$

Charge on capacitor $= CiR_2 = \frac{C \ E \ R_2}{R_2 + r}$

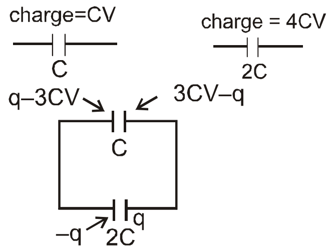
B-3. As battery is disconnected, charge remains constant in the work process.
Work done = final potential energy – initial potential energy

$= \frac{Q^2}{2 \ C'} - \frac{Q^2}{2 \ C} = \frac{Q^2}{2}\left\{\frac{1}{C'} - \frac{1}{C}\right\}$

Where , $Q = CV = \frac{A \in_0 V}{d}$, $C = \frac{A \in_0}{d}$ & $C' = \frac{A \in_0}{2 \ d}$

Now, work done $= \frac{\in_0 AV^2}{2d}$ Ans. is (D)

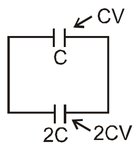
B-4.



Total charge = 4 CV – CV = 3 CV

Now, let it is distributed as shown, potential across the capacitors is same

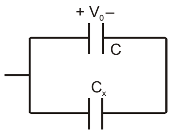
So, $\frac{q}{2C} = \frac{3CV - q}{C} \Rightarrow q = 2\,CV$



Total potential energy $= \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} = \frac{C^2V^2}{2C} + \frac{4C^2V^2}{2 \times 2C} = \frac{3CV^2}{2}$

B-5. Common potential

$V = \frac{V_0C + 0}{C + C_x}$



$V(C + C_x) = V_0C$

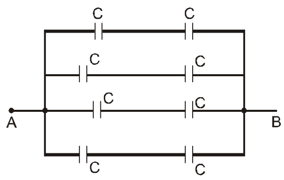
$C + C_x = \frac{V_0}{V}C$

$C_x = C$

$C_x = \frac{C(V_0 - V)}{V}$

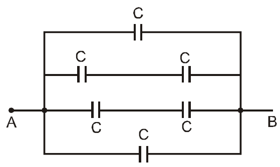
Section (C)

C-1.



$C_{eq} = \frac{4C}{2} = 2C.$

C-2.



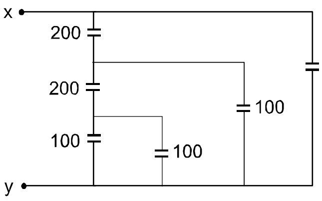
$C_{eq} = C + 2C/2 + C = 3C.$

C-3. $\frac{1}{C_1} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$

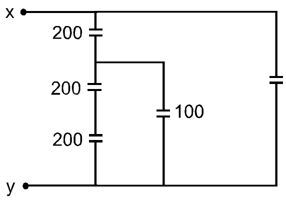
$\Rightarrow$ $C_1 = 1\ \mu F$, $C_2 = 2 + 1 = 3\ \mu F$

$C_{eq} = 1\mu F$.

C-4.

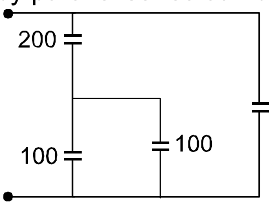


$\Rightarrow$

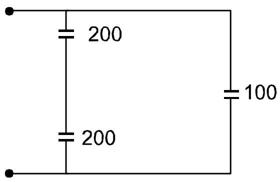


solving by parallel series combinations,

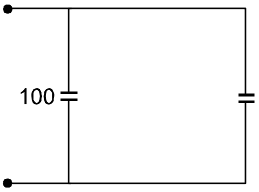
$\Rightarrow$



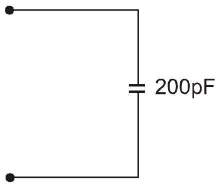
$\Rightarrow$



$\Rightarrow$

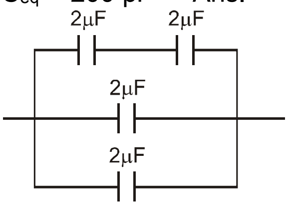


$\Rightarrow$



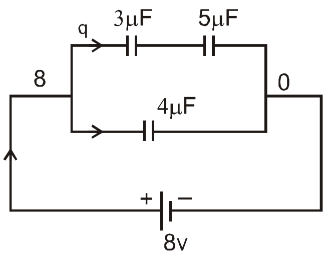
$C_{eq} = 200$ pF Ans.

C-5.



Minimum 4 capacitors required as shown in figure

C-6.



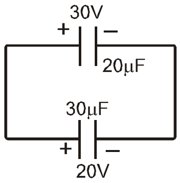
$C_{eq} = \frac{15}{8} + 4 = \frac{47}{8}\ \mu F$

$\frac{q}{3} + \frac{q}{5} = 8 \quad \Rightarrow \quad q = 15\mu C$

Charge on $2\mu F$

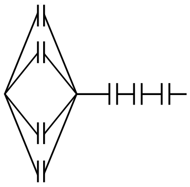
$\frac{q_1}{2} = \frac{15 - q_1}{3} \quad \Rightarrow \quad q_1 = \frac{30}{5} = 6.0\mu C$ **Ans.**

C-7.



Common potential $V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2} = \dfrac{600 + 600}{20 + 30} = 24$ V

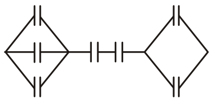
C-8.



$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$$

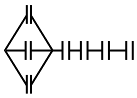
$$= \frac{1}{8} + \frac{3 \times 4}{2 \times 4}$$

$$\frac{1}{C_{eq}} = \frac{13}{8} \Rightarrow C_{eq.} = \frac{8}{13}\,\mu F$$



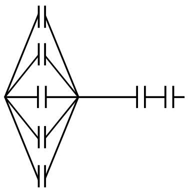
$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4}$$

$$\frac{1}{C_{eq}} = \frac{17}{12} \Rightarrow C_{eq.} = \frac{12}{17}$$



$$\frac{1}{C_{eq}} = \frac{1}{6} + \frac{1}{2} \times 4$$

$$\frac{1}{C_{eq}} = \frac{13}{6} \Rightarrow C_{eq} = \frac{6}{13}\,\mu F$$



$$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{2} + \frac{1}{2}$$

$$\frac{1}{C_{eq}} = \frac{10}{11}\,\mu F$$

Section (D)

D-1. (i) **(C)** $q_0 = 4\mu C$

$$i = \frac{dq}{dt} = \frac{q_0}{RC}\, e^{-t/RC} = \frac{4 \times 10^{-6}}{1 \times 10^{-6} \times 3 \times 10^{6}} e^{-1/3} = \frac{4}{3} e^{-1/3}\ \mu C/sec$$

(ii) **(A)** $U = \dfrac{q_0^2}{2C}\,(1 - e^{-t/RC})^2$

$$\frac{dU}{dt} = \frac{q_0^2}{RC^2}\,(1 - e^{-t/RC})\, e^{-t/RC}$$

$$= \frac{(4 \times 10^{-6})^2}{3 \times 10^{6} \times (1 \times 10^{-6})^2}\,(1 - e^{-1/3})\, e^{-1/3} = 16/3\,(1 - e^{-1/3})\, e^{-1/3}\ \mu J/sec.$$

(iii) **(C)** $H = \int i^2 R dt \Rightarrow \frac{dH}{dt} = i^2 R$

$$\frac{dH}{dt} = i_0^2 Re^{-2t/RC} = \left(\frac{4}{3\times10^6}\right)^2 3 \times 10^6\ e^{-2/3} = 16/3\ e^{-2/3}\ \mu J/s$$

(iv) **(C)** $U = qV \Rightarrow \frac{dU}{dt} = V\frac{dq_0}{dt}(1 - e^{-t/RC})$

$$\frac{dU}{dt} = \frac{q_0 V}{RC} e^{-t/RC}$$

$$= \frac{4\times10^{-6}\times4}{3\times10^6\times1\times10^6} e^{-1/3} = \frac{16}{3} e^{-1/3}\ \mu J/sec.$$

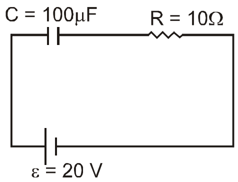
D-2. (i) (A) $i_0 = \frac{V}{R} = \frac{6}{24} = 0.25$ A

(ii) (B) $i = i_0\ e^{-t/RC}$

$= 0.25\ e^{-1}$

$= \frac{0.25}{e} = 0.09$ A.

D-3.



Energy stored in capacitor $= \frac{Q^2}{2\ C}$

Rate at which energy is stored $= \frac{d}{dt}\left(\frac{Q^2}{2\ C}\right) = \frac{Q}{C} \cdot \frac{dQ}{dt} = \frac{Qi}{C}$

$Q = \varepsilon C\ \{1 - e^{-t/RC}\}$

$i = \frac{\varepsilon\ e^{-t/RC}}{R}$

Rate of energy storage $= \frac{\varepsilon^2}{R}\{1 - e^{-t/RC}\}\{e^{-t/RC}\} = \frac{\varepsilon^2}{R}\{e^{-t/RC} - e^{-2t/RC}\}$ .... (1)

It will be maximum when, $e^{-t/RC} - e^{-2tRC}$ will be maximum let $y(t) = e^{-t/RC} - e^{-2t/RC}$

for maximum , $y'(t) = 0$

$$y'(t) = \frac{-e^{-t/RC}}{R\ C} + \frac{2\ e^{-2t/RC}}{R\ C}$$

$$e^{-t/RC} = \frac{1}{2}$$

Putting it back in eq. (1)

(i) Maximum rate of energy storage

$$= \frac{\varepsilon^2}{R}\left\{\frac{1}{2} - \left(\frac{1}{2}\right)^2\right\} = \frac{\varepsilon^2}{4\ R} = \frac{(20)^2}{4\times10} = 10\ J/s$$ **Ans. is (A)**

(ii) This will occur when, $e^{-t/RC} = 1/2$

$\frac{-t}{RC} = \ell n\ \frac{1}{2}$

$t = RC\ \ell n\ 2 = 10 \times 100 \times 10^{-6} \times \ell n 2 = (\ell n\ 2)$ ms **Ans. is (C)**

D-4. $q = \dfrac{q_1}{2} = \dfrac{8\times10^{-6}\times10}{2}\left(1-e^{-\frac{0.16\times10^{-3}}{8\times10^{-6}\times20}}\right)$

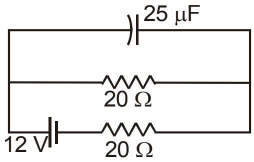
$q = 40(1 - e^{-1})\mu C = 40\,(1 - 0.37) = 25.2\mu C$ **Ans.**

D-5. (i) **(B)** at t_0 ; $q = q_0 = 60\ \mu C$

(ii) **(C)** $q = q_0e^{-t/RC} = 60 \times 10^{-6}\ e^{-100\times10^{-6}/10\times10^{-6}\times10} = \dfrac{60}{e}\ \mu C = 22\ \mu C.$

(iii) **(A)** $q = q_0e^{-t/RC} = 60 \times 10^{-6}\ e^{-1\times10^{-3}/10\times10^{-6}\times10} = \dfrac{60}{e^{10}}\ \mu C = 0.003\ \mu C.$

D-6. (C)



Switch is kept closed for a long time,

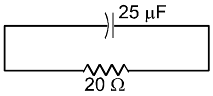
Current through 20Ω resistor $i = \dfrac{12}{40}$

Charge on the capacitor at steady state,

$q_0 = 25 \times \dfrac{12}{40} \times 20 = 150\ \mu C$

at t = 0 , switch is opened, $i = i_0\ e^{-t/\tau}$

$\tau = RC = 20 \times 25 = 500\ \mu S$

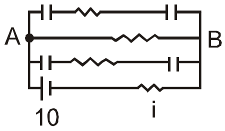


Current $i = \dfrac{q_0}{\tau}\ e^{-\frac{0.25\ \times\ 10^{-3}}{500\ \times\ 10^{-6}}}$

$i = \dfrac{150}{500}\ e^{-1/2} = 0.189\ A$

D-7. If S_1 is closed and S_2 is open then, condenser C is fully charged at potential V.

D-8. Charge on each capacitor will be same. In steady state current through capacitor will be zero



current in steady state = i = 10/5 = 2 amp
potential across AB = iR = 2 × 4 = 8 V.
Potential across each capacitor = 4V
on each plate Q = C V = 3 × 4 = 12 μC

D-9. $\dfrac{q_1}{C_1} = \dfrac{q_2}{C_2}$

$\dfrac{I_1}{C_1} = \dfrac{I_2}{C_2}$ $\quad C_1 = C_2$

$I = I_1 + I_2$

$I_1 = \dfrac{IC_1}{C_1 + C_2}$

Section (E)

E-1. $C' = \dfrac{\epsilon_0 A}{d/2} = \dfrac{2\epsilon_0 A}{d} = 2C.$

E-2. Q = constant

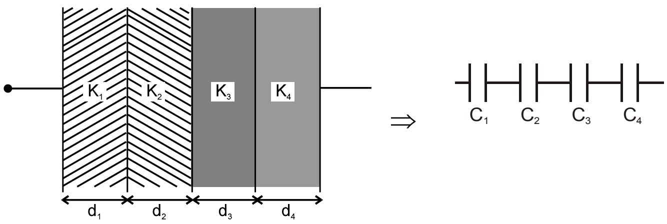
New capacitance = KC (increases)

$V' = \dfrac{V}{K}$ (decreases)

$U' = \dfrac{Q^2}{2CK}$ (decreases)

$E = \dfrac{Q}{A\epsilon_0} \Rightarrow E' = \dfrac{Q}{KA\epsilon_0}$ (decreases)

E-3.



$C_1 = \dfrac{K_1\varepsilon_0 A}{d_1}$, $C_2 = \dfrac{K_2\varepsilon_0 A}{d_2}$, $C_3 = \dfrac{K_3\varepsilon_0 A}{d_3}$, $C_4 = \dfrac{K_4\varepsilon_0 A}{d_4}$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \frac{1}{C_4}$$

$$\frac{1}{C_{eq}} = \frac{d_1}{K_1\varepsilon_0 A} + \frac{d_2}{K_2\varepsilon_0 A} + \frac{d_3}{K_3\varepsilon_0 A} + \frac{d_4}{K_4\varepsilon_0 A}$$

$$C_{eq} = \frac{\varepsilon_0 A}{\left[\frac{d_1}{K_1} + \frac{d_2}{K_2} + \frac{d_3}{K_3} + \frac{d_4}{K_4}\right]}$$

E-4. $V_{C_2} = V_{C_2} = V$

$C_1 = C$

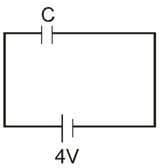
$C_2 = KC$

$q_1 = C_1 V_{C_1} = CV$

$q_2 = C_2 V_{C_2} = KCV$

$q_1 < q_2$.

E-5.



Here, Potential difference on the capacitor will depend on emf of battery i.e., 4V (C)

E-6. Charge on battery = Q = CV = 4 C

Now charge remains same, as battery is disconnected new capacitance = C´ = KC = 8C

$C'V' = Q \qquad V' = \dfrac{Q}{C'} = \dfrac{4C}{8C} = \dfrac{1}{2}$ V (A)

E-7. $U_0 = \frac{1}{2}CV^2$ (given) Now energy = $U' = \frac{1}{2}C'V^2$

$C' = CK$

$U' = \frac{1}{2}CV^2K = U_oK$ **Ans. is (A)**

E-8. Now, charge remains same on the plates.

$U_0 = \frac{Q^2}{2C}$ (given)

Now energy = $U' = \frac{Q^2}{2C'} = \frac{Q^2}{2CK} = \frac{U_o}{K}$ **(C) Ans**

PART - III

1. The initial charge on capacitor = $CV_i = 2 \times 1\ \mu C = 2\ \mu C$

The final charge on capacitor = $CV_f = 4 \times 1\ \mu C = 4\ \mu C$

$\therefore$ Net charge crossing the cell of emf 4V is

$q_f - q_i = 4 - 2 = 2\ \mu C$

The magnitude of work done by cell of emf 4V is

$W = (q_f - q_i)\,4 = 8\ \mu J$

The gain in potential energy of capacitor is

$$\Delta U = \frac{1}{2}C(V_f^2 - V_i^2) = \frac{1}{2}\ 1 \times [4^2 - 2^2]\ \mu J = 6\ \mu J$$

Net heat produced in circuit is

$$\Delta H = \frac{1}{2}C(V_f^2 - V_i^2)\ W - \Delta U = 8 - 6 = 2\ \mu J$$

SECTION (D)

2. (A) For potential difference across each cell to be same

$E_1 - ir = E_2 + ir$ or $i = \frac{E_1 - E_2}{2r}\left(< \frac{E_1 - E_2}{2\ r + R}\right)$

Hence potential difference across both cells cannot be same.

Cell of lower emf charges up.

For potential difference across cell of lower emf to be zero

$E_2 + ir = 0$ which is not possible.

Current in the circuit cannot be zero $\because E_1 \neq E_2$.

(B) For potential difference across each cell to be same

$E_1 - ir = E_2 - ir$ which is not possible

No cell charges up.

For potential difference across cell of lower emf to be zero

$E_2 - ir = 0$ and $E_1 - i\,(r + R) = 0$

or $\frac{E_1}{r+R} = \frac{E_2}{r}$ which is possible. $\because E_1 > E_2$.

Current in the circuit cannot be zero.

(C) Situation is same as in (A) except current decreases from $\frac{E_1 - E_2}{2r + R}$ to zero.

Hence the only option that shall changes is 'current shall finally be zero.'

(D) Situation is same as in (B) except current decreases from $\frac{E_1 + E_2}{2r + R}$ to zero.

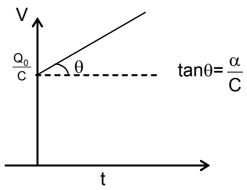
Hence the only option that shall changes is 'current shall finally be zero.

EXERCISE –2
PART - I

1. x = Vt $\Rightarrow$ $d \propto t$ $C = \frac{\in_0 A}{Vt}$ $\frac{dc}{dt} = -\frac{\in_0 A}{V}\frac{1}{t^2}$ $\frac{dc}{dt} \propto \frac{1}{d^2}$ **Ans**

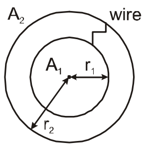
2. Given Q = (αt + Q_0)

$V = \frac{Q}{C} = \frac{\alpha t + Q_0}{C} = \frac{\alpha t}{C} + \frac{Q_0}{C}$



3. Theoritical capacitance = ∞, because d become zero

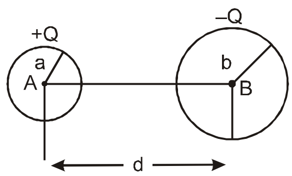
4.



All given charge of A_1 goes to A_2

Therefore C = $4\pi\in_0 r_2$

5.



$V_B = \frac{KQ}{d} - \frac{KQ}{b}$

$V_A = \frac{KQ}{a} - \frac{KQ}{d}$

$V_A - V_B = KQ\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$

$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right]$

or $\frac{Q}{V_A - V_B} = C = \frac{4\pi\varepsilon_0}{\left(\frac{1}{a} + \frac{1}{b} - \frac{2}{d}\right)}$

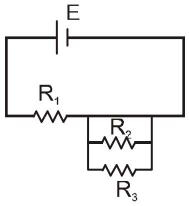
6. Charge on C_0, $Q_1 = C_0V_0$,
Initial charge on C_1, $Q_2 = 0$

Common potential $V_1 = \dfrac{Q_1+Q_2}{C_1+C_2} = \dfrac{C_0V_0}{C+C_0} \Rightarrow Q_1 = C_0V_1 = \dfrac{C_0^2}{C+C_0}\ V_0$

Similarly $V_2 = \dfrac{C_0V_1}{C+C_0} = \left(\dfrac{C_0}{C+C_0}\right)^2 V_0 \Rightarrow Q_2 = C_0V_2 = \dfrac{C_0^3}{(C+C_0)^2}\ V_0$

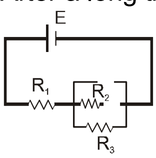
for n times $V_n = \left(\dfrac{C_0}{C+C_0}\right)^n V_0 = V \quad \Rightarrow \quad C = \left[\left(\dfrac{V_0}{V}\right)^{1/n} - 1\right] C_0$ **Ans**

7. Immediately after the key is closed, capacitor behave like a conducting wire, therefore.



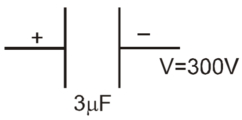
$i = \dfrac{E}{R_1}$ **Ans**

After a long time interval, capacitor behave like a open circuit. Therefore.

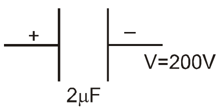


$i = \dfrac{E}{R_1+R_3}$ **Ans**

8.

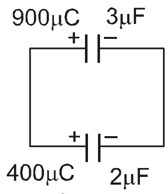


$Q_1 = C_1V_1 = 900\mu C$



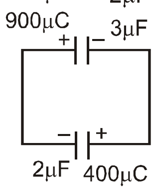
$Q_2 = C_2V_2 = 400\mu C$

(i) (D)



$V = \dfrac{Q_1+Q_2}{C_1+C_2} = \dfrac{900+400}{3+2} = 260V$

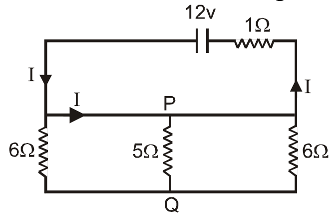
(i) (A)



$V = \dfrac{Q_1+Q_2}{C_1+C_2} = \dfrac{900-400}{3+2} = 100V$

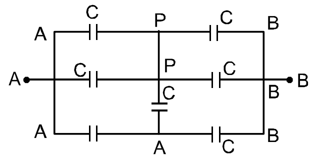
Charge on $3\mu F = C_1V = 300\mu C$
amount of charge flow is $= 900\mu C - 300\mu C = 600\mu C = 6 \times 10^{-4}$ C

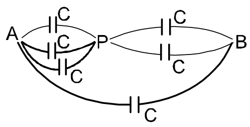
9. Just after switch closing



current through resistor PQ is zero just after closing the switch.

10.



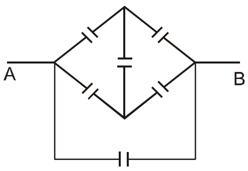


Get

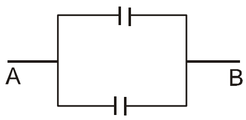
$C_{eq} = \dfrac{11c}{5}$

Charge flow = $C_{eq}\,\varepsilon = \dfrac{11\,c\varepsilon}{5}$.

11.



$\equiv$

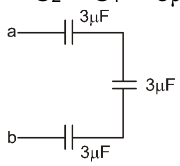


$C_{eq} = 2C$

12. (i) $\dfrac{1}{C_1'} = \dfrac{1}{C_1} + \dfrac{1}{C_1} + \dfrac{1}{C_2}$ $\Rightarrow$ $C_1' = 1\mu F$

$C_2' = C_2 + C_1' = 3\mu F$ $\Rightarrow$ $C_{eq} = 1\mu F$ **Ans**

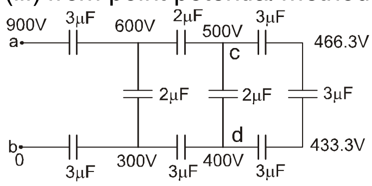
(ii)



$C_{eq} = 1\mu F$ $\quad Q = C_{eq}\,V = 900\mu F$

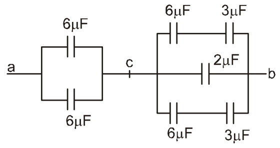
charge on nearest capacitor = $900\mu F$ **Ans**

(iii) from point potential method



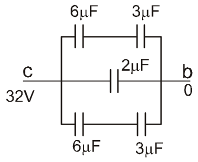
$V_c - V_d = 100V$ **Ans**

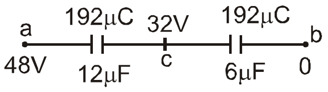
13. **(i)**



$C_{ac} = 6 + 6 = 12\mu F$ $\quad C_{cb} = 2\left(\frac{6.3}{6+3}\right) + 2 = 6\mu F$

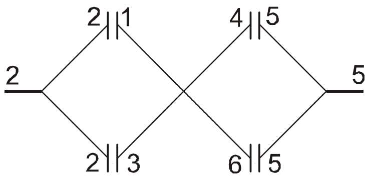
$C_{eq} = \frac{12 \times 6}{12+6} = 4\mu F$ **Ans**





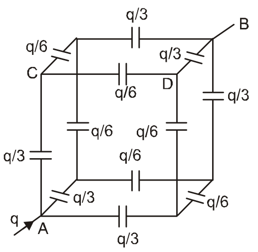
(ii)
Charge on $2\mu F$ capacitor $\Rightarrow$ $Q = CV$ $\quad Q = 2 \times 32 = 64\mu C$ **Ans**

14.



$C_{eq.} = \frac{C}{2} + \frac{C}{2} = C = \frac{\epsilon_0 A}{d}$ **Ans.**

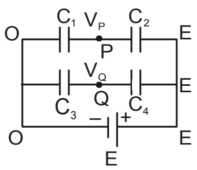
15.



Due to symmetric charge distribution as shown
for loop ACDB

$V_A - \frac{q}{3C} - \frac{q}{6C} - \frac{q}{3C} = V_B$ $\Rightarrow$ $V_A - V_B = \frac{5q}{6C}$ $\Rightarrow V_A - V_B = \frac{q}{C_{eq}}$ $\Rightarrow$ $C_{eq} = \frac{6C}{5}$ **Ans**

16.



C_1 and C_2 are in series, charge on each will remain same.

$(V_P - 0) \cdot C_1 = (E - V_P)\, C_2$

$V_P = \frac{C_2 E}{C_1 + C_2}$

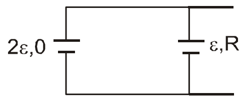
C_3 & C_4 are in series, charge on each will remain same,

$(V_Q - 0) \cdot C_3 = (E - V_Q) \cdot C_4$

$V_Q = \frac{C_4 E}{(C_3 + C_4)}$

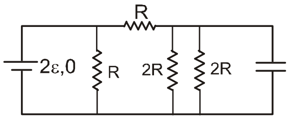
Hence $V_P - V_Q = \frac{(C_2C_3 - C_1C_4)\ E}{(C_1 + C_2)\ (C_3 + C_4)}$

17.

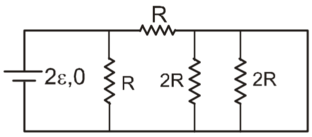


$\Rightarrow \quad E = \dfrac{\dfrac{E_1}{r_1}+\dfrac{E_2}{r_2}}{\dfrac{1}{r_1}+\dfrac{1}{r_2}} = \dfrac{E_1\ r_2 + E_2\, r_1}{r_1+r_2} = \dfrac{2\varepsilon R + \varepsilon \times 0}{0+R}$

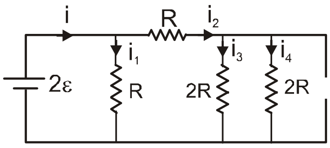
$\Rightarrow \quad E = 2\varepsilon\ ,\ r_{eq} = \dfrac{r_1\ r_2}{r_1+r_2} = 0$



Equivalent battery



$i_{max} = \dfrac{2\varepsilon}{R}$



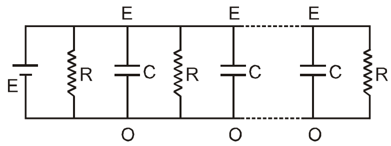
for Q_{max} $\Rightarrow$

$i = \dfrac{2\varepsilon}{2R/3} = \dfrac{3\varepsilon}{R}$ $\qquad i_2 = \dfrac{\varepsilon}{R}$, $\qquad i_1 = \dfrac{2\varepsilon}{R}$, $\qquad i_3 = i_4 = \dfrac{\varepsilon}{2R}$

potential on C = potential on 2R resistance $= i_3 \times 2R = \varepsilon$

charge on capacitor, $Q_{max} = CV = C\varepsilon$ $\qquad \tau = \dfrac{Q_{max}}{i_{max}} = \dfrac{C\varepsilon}{2\varepsilon/R} = \dfrac{RC}{2}$ **Ans**

18.



$Q_{first} = Q_{last} = CE$

Ratio $= \dfrac{Q_{first}}{Q_{last}} = 1.$

19. From the given conditions, resistance of analog voltmeter = 10 kΩ

Initial current = $\dfrac{1.5}{10}$mA

$= \dfrac{1500}{10}\mu A \ = 150\mu A$

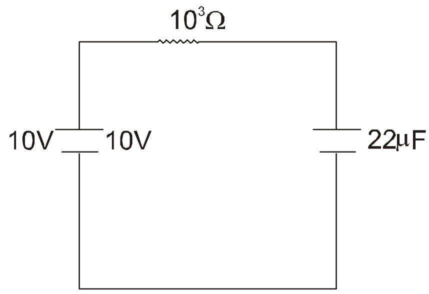
Using, $0.5 = 1.5\, e^{-t/RC} = 1.5\, e^{-t/10}$

$\dfrac{1}{3} = e^{-\frac{t}{10}}$

$\ell n\ 3 = t$

$t = \ell n\ 3$

20. **(A)**



$RC = 10^3 \times 22 \times 10^{-6}$

$RC = 10^3 \times 22 \times 10^{-6} = 22 \times 10^{-3}$ sec

At t = 1.5 sec

steady state V_C = 10V

While discharging $V = 10e^{\frac{-t\times10^3}{15}} = 1.226$

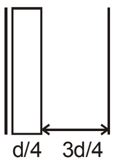
$e^{-\frac{10^3 t}{15}} = 0.1226$

$t = \frac{-15}{100}\ln(0.1226)$

= 330 ms.

21

(B)

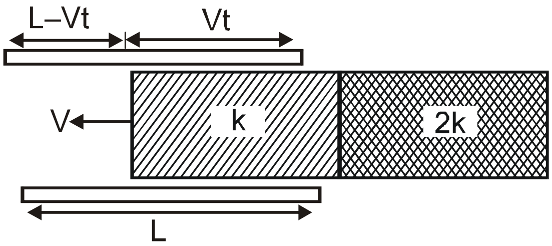


Capacitance of capacitor without dielectric, $C_0 = \frac{\varepsilon_0 A}{d}$

Capacitance of capacitor with dielectric, C

$$C = \frac{\varepsilon_0 A}{\frac{3d}{4} + \frac{d}{4\varepsilon_r}} = \frac{4\varepsilon_0 A.\ \varepsilon_r}{((3\varepsilon_r + 1)d} \Rightarrow \frac{C}{C_0} = \frac{4\varepsilon_r}{(3\varepsilon_r + 1)}$$

22. **Case – I** When dielectric slab of dielectric constant K enters in to the capacitor.



At any time t, there will be two capacitors are in parallel combination - one with air and other with dielectric slab.

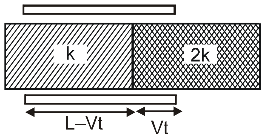
$C(t) = C_{air} + C_{slab}$

$= \frac{\epsilon_0 A\ (L - Vt)}{Ld} + \frac{K \epsilon_0 A\ (Vt)}{Ld}$

$= \frac{\epsilon_0 A}{Ld}$ [L – (K – 1) Vt] (linear function of t)

Its slope = M C(t) = $\frac{\epsilon_0 A}{Ld}$ (K – 1) V

Case – II When dielectric slab of dielectric constant 2K also enters into the capacitor.



C' (t) = $C_{slab\ 1} + C_{slab\ 2}$

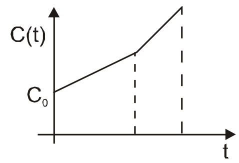
$$= \frac{\in_0 AK\ (L-Vt)}{Ld} + \frac{\in_0 A2K\ \ Vt}{Ld}$$

$$= \frac{K \in_0 A}{Ld}\ [L + Vt] \qquad \text{(linear function of t)}$$

Its slope = MC' (t) = $\frac{\in_0 AKV}{Ld}$

As = M C' (t) > MC (t)

and both C(t) and C'(t) are linear function of 't' hence variation of capacitance with time be best represented by (B)



23. U = $\frac{Q^2}{2C}$ since C will become k times

So U will become $\frac{1}{k}$ times

24. $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

$$\frac{1}{\frac{A\varepsilon_0}{\frac{d}{2}}} + \frac{1}{\frac{KA\varepsilon_0}{\frac{d}{2}}}$$

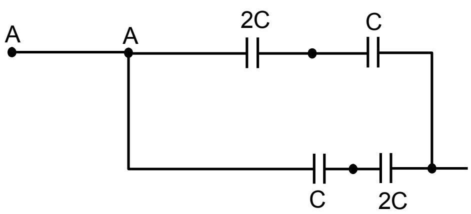
$$= \frac{d}{2A\varepsilon_0} + \frac{d}{2KA\varepsilon_0}$$

$$\frac{1}{C_{eq}} = \frac{d}{2A\varepsilon_0}\left(\frac{K+1}{K}\right)$$

$$C_{eq} = \frac{2KA\varepsilon_0}{d(K+1)}$$

$$\frac{C_{eq}}{C_{eq}'} = \frac{2KA\varepsilon_0}{d(K+1)} \cdot \frac{2d}{A\varepsilon_0(K+1)} = \frac{4K}{(K+1)^2}$$

25. Rearrange the circuit



C_{eq} = $\frac{2C}{3} \times 2$

PART - II

1. $C_{eq.}$ can be written as $C_{eq.} = \frac{C_1C_2}{C_1+C_2} = \frac{a^2 \in_o}{3d}$

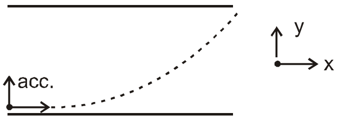
charge on plate $Q = C_{eq}V = \frac{a^2 \in_o V}{3d}$

surface charge density $= \sigma = \frac{Q}{a^2} = \frac{\in_o V}{3d}$

electric field $= \frac{\sigma}{2\in_o} + \frac{\sigma}{2\in_o} = \frac{\sigma}{\in_o} = \frac{V}{3d}$

electric force $= \frac{Ve}{3d}$

acceleration of electrons $= \frac{Ve}{(3d)m}$



in X axis $a = ut$ $t = \frac{a}{u}$

in Y axis $\Rightarrow$ $\frac{1}{2} \times \text{acceleration} \times t^2 = d$

$\frac{Vet^2}{2(3d)m} = d$ $\frac{Vea^2}{6dmu^2} = d$

$u = \left\{\frac{Vea^2}{6md^2}\right\}^{\frac{1}{2}}$

2. $C_{eq.} = \frac{2C}{3}$

$Q = \frac{2CV}{3}$

surface charge density $= \sigma = \frac{Q}{A} = \frac{2CV}{3A}$

Electric field between the plates of capacitor

$= \frac{\sigma}{2\in_o} + \frac{\sigma}{2\in_o} = \frac{\sigma}{\in_o} = \frac{2CV}{3A\in_o}$

Force बल $= qE = \frac{2CVq}{3A\in_o}$

for equilibrium this electrostatic force must be equal to mg $\Rightarrow$ $\frac{2CVq}{3A\in_o} = mg$

$V = \frac{3mgA\in_o}{2Cq}$

3. $Q = CV = 2 \times 12 = 24\mu C$

$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{1}{2}$, $Q_1 + Q_2 = 24\mu C$, $V = \frac{Q_1 + Q_2}{C_1 + C_2}$ = 4 Volt

(a) $Q_1 = 8\mu C$, $Q_2 = 16\mu C$

initial charge on $4\mu F = 0$

the charge flow from connecting wire = 16 μC

Ans 16

(b) $U_1 = \frac{1}{2} C_1V^2 = \frac{1}{2} \times 2 \times (4)^2 = 16\mu J$

$U_2 = \frac{1}{2} C_2V^2 = \frac{1}{2} \times 4 \times (4)^2 = 32\mu J$

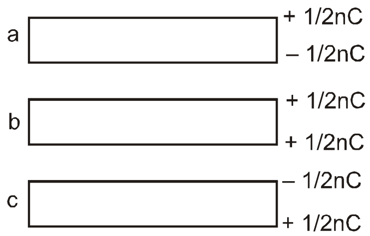
Total energy stored = 48 μJ

Ans 48

(c) $\Delta H = (U_i)$system– (U_f) system = $\frac{1}{2} \times 2 \times 12^2 - (16 + 32) = 96\mu J$

Ans 96

4. Charge after distribution



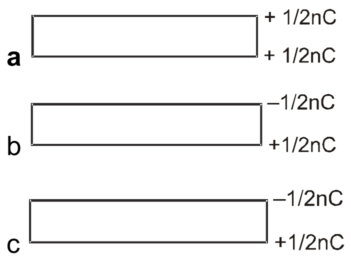
Charge on outer plate = $\frac{\Sigma q}{2} = \frac{1}{2}$ nC ;

$C = \frac{A \in_0}{d} = \frac{500 \times 10^{-4} \times 8.85 \times 10^{-12}}{8.85 \times 10^{-3}}$ = 50 pF

charge on outer surface of upper plate is = $\frac{1}{2}$nC = 0.5nC **Ans**

$V = \frac{q}{C} = \frac{0.5 \times 10^{-9}}{50 \times 10^{-12}}$ = 10V **Ans**

5.



charge on outer most plate = $\frac{\Sigma q}{2} = \frac{1}{2}$ nC

$V_{ab} = \frac{q}{C} = \frac{1/2nC}{50pF}$ = 10 V **Ans**

$V_{cd} = \frac{q}{C} = \frac{1/2nC}{50pF}$ = 10V **Ans**

6. When switch is open = $C_{eq.} = \frac{15}{2} \mu F$

$q_i = C_{eq.} V = \frac{15}{2} \times 2 = 15 \mu C$

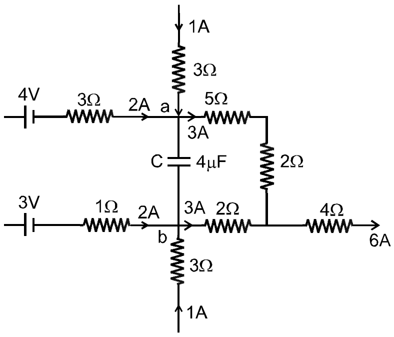
When switch is closed

$C_{eq.} = 30 \mu F$

$q_f = 30 \times 2 = 60 \mu C$

Charge flow through AB = $q_f - q_i = 45\mu C$

7. Using Kirchhoff's first law at junctions a and b, we have found the current in other wires of the circuit on which currents were not shown.



Now, to calculate the energy stored in the capacitor we will have to first find the potential difference V_{ab} across it.

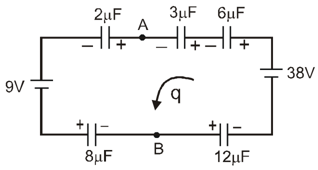
$V_a - 3 \times 5 - 3 \times 2 + 3 \times 2 = V_b$

$\therefore$ $V_a - V_b = V_{ab} = 15$ volt

$\therefore$ $U = \frac{1}{2} CV^2_{ab}$

$= 1/2\ (0.4 \times 10^{-6})\ (15)^2\ J = 45\ \mu J$ **Ans.**

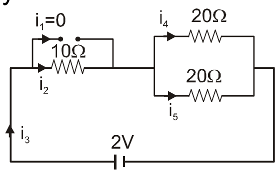
8.



$38 - \frac{q}{6} - \frac{q}{3} - \frac{q}{2} - 9 - \frac{q}{8} - \frac{q}{12} = 0 \quad \Rightarrow \quad q = 24\mu C$

$V_A - \frac{24}{2} - 9 - \frac{24}{8} = V_B \qquad V_A - V_B = 24\ V$ **Ans**

9. at steady state



$R_{eq} = 20\Omega \qquad i_1 = 0$ **Ans**

$i_3 = i_2 = \frac{2}{20} = \frac{1}{10}$ amp **Ans**

$i_4 = i_5 = \frac{i_3}{2} = \frac{1}{20}$ amp **Ans**

charge on capacitor $6\mu F$ is $= 6 \times 10^{-6} \times \frac{1}{10} \times 10 = 6\mu C$ **Ans**

10. In steady state situation no current will flow through the capacitor, 2Ω and 3Ω are in parallel. Therefore, their combined resistance will be

$R = \frac{2 \times 3}{2+3} = 1.2\ \Omega$

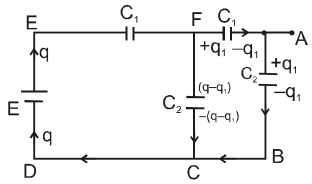
Net current through the battery :

$i = \frac{60}{1.2 + 2.8} = 15\ A$

This current will distribute in inverse ratio of their resistance 2Ω and 3Ω.

$\therefore \quad \frac{i_2}{i_3} = \frac{3}{2} \qquad$ or $\qquad i_2 = \left(\frac{3}{3+2}\right)(15) = 9\ A$

11.



The distribution of charges is shown in figure In closed loop (CDEFC)

$$+E-\frac{q}{C_1}-\frac{(q-q_1)}{C_2}=0 \qquad .....(i)$$

In closed loop (ABCFA) $-\frac{q_1}{C_1}-\frac{q_1}{C_2}+\frac{q-q_1}{C_2}=0$

or $\quad -\frac{q_1}{C_1}-\frac{q_1}{C_2}-\frac{q_1}{C_2}+\frac{q}{C_2}=0$

or $\quad \frac{q}{C_2}=q_1\left(\frac{2}{C_2}+\frac{1}{C_1}\right)$

or $\quad q=\left(\frac{2C_1+C_2}{C_1}\right)\ q_1 \qquad .....(ii)$

From Eq.(i), we get $E-\frac{q}{C_1}-\frac{q}{C_2}+\frac{q_1}{C_2}=0$

or $\quad E+\frac{q_1}{C_2}=q\left(\frac{C_1+C_2}{C_1C_2}\right)$

or $\quad E+\frac{q_1}{C_2}=\left(\frac{2C_1+C_2}{C_1}\right)\ q_1\left(\frac{C_1+C_2}{C_1C_2}\right)$

or $\quad EC_2+q_1=\frac{(2C_1+C_2)\ (C_1+C_2)\ q_1}{C_1^2}$

or $\quad EC_2=\frac{(2C_1^2+2C_1C_2+C_1C_2+C_2^2)\ q_1}{C_1^2}-q_1$

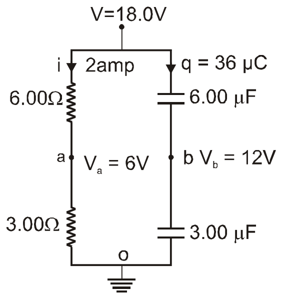
$\quad EC_2=\left\{\frac{2C_1^2+3C_1C_2+C_2^2-C_1^2}{C_1^2}\right\}\ q_1$

$\therefore \quad q_1=\frac{EC_2C_1^2}{C_1^2+3C_1C_2+C_2^2}$

$\quad V_A-V_B=\left|\frac{-q_1}{C_2}\right|=\frac{q_1}{C_2}$

$$=\frac{EC_1^2}{C_1^2+3C_1C_2+C_2^2}=\frac{E}{1+3\frac{C_2}{C_1}+\frac{C_2^2}{C_1^2}}=\frac{E}{1+3\eta+\eta^2}=10V\left[\because\ \frac{C_2}{C_1}=\eta=2\right]$$

12.



(i) When switch s is closed potential at b point is same as potential at a point

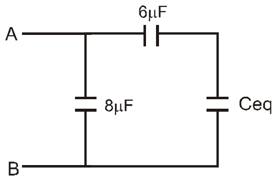
$V_b = V_a = 6.0$ Volt

(ii) Final charge on 6.0μF is $q = 6 \times 12 = 72\mu C$ Therefore charge flow through wire b to a is

$= 72 - 36 = 54\ \mu C$

13. Let equivalent capacitance = C_{eq}.

Infinite ladder can be shown as :



Now C_{eq} of this ladder,

$$C_{eq} = \frac{6\ \ C_{eq}}{6 + C_{eq}} + 8$$

by solving it,

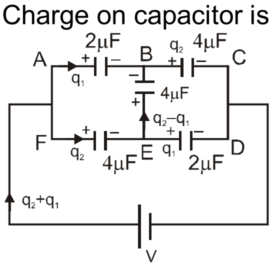
$C_{eq}^2 - 8\,C_{eq} - 48 = 0$

$C_{eq} = 12\ \mu F$ or $-4\ \mu F$

neglecting –ve answer,

$C_{eq} = 12\ \mu F$.

14. Charge on capacitor is assume as according to reverse symmetry.



$q_1 + q_2 = C_{eq}\,V$ .....(i)

from loop AB EFA

$$-\frac{q_1}{2} + \frac{q_2 - q_1}{4} + \frac{q_2}{4} = 0 \Rightarrow 2q_2 - 3\,q_1 = 0 \quad .....(ii)$$

from loop ABCA

$$-\frac{q_1}{2} - \frac{q_2}{4} + V = 0 \Rightarrow 4V = 2q_1 + q_2 \quad .....(iii)$$

from (i), (ii) and (iii) we get

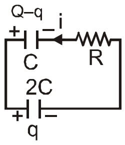
$$q_1 + \frac{3q_1}{2} = C_{eq}\left[\frac{2q_1 + \frac{3q_1}{2}}{4}\right] \Rightarrow C_{eq} = \frac{20}{7}\ \mu F$$ **Ans**

15. $E = E_0\, e^{-t/RC} \Rightarrow \frac{E_0}{3} = E_0 e^{-t/RC}$

$\frac{1}{3} = e^{-4.4\times10^{-6}/R\times2\times10^{-6}}$

$\ell n\, 3 = \frac{11}{R5} \quad \Rightarrow \quad R = \frac{11}{5\ell n3} = 2.0\ \Omega.$

16.



At time t using KVL $\frac{Q-q}{C} - \frac{q}{2C}$ $-iR = 0$

$\frac{2Q-3q}{2C} - \frac{dq}{dt}\, R = 0 \quad \Rightarrow \quad \int_0^q \frac{dq}{2Q-3q} = \int_0^t \frac{dt}{2RC}$

$\frac{1}{-3}\ell n \left(\frac{2Q-3q}{2Q}\right) = \frac{t}{2RC} \quad \Rightarrow \quad \frac{2Q-3q}{2Q} = e^{-\frac{3t}{2RC}} \quad \Rightarrow \quad q = \frac{2Q}{3}\left(1-e^{-\frac{3t}{2RC}}\right)$

17. Let distance between the plates = d

$18 \times 10^6 \times d = 4000$

$d = \frac{4000}{18\times10^6}$

Now, $C = \frac{\epsilon_r A \epsilon_o}{d}$

$7.0 \times 10^{-2} \times 10^{-6} = \frac{\frac{10^{-9}}{36\pi}\times A \times 2.8}{\frac{4000}{18\times10^6}}$

Solving, We get $A = \frac{\pi}{5}$

18. Let a be the side of the square plate.

As shown in figure, C_1 and C_2 are in parallel. Therefore, total capacity of capacitors in the position shown is

$C = C_1 + C_2$

$C = \frac{\epsilon_0\, a(a-x)}{d} + \frac{K\epsilon_0\, ax}{d}$

$\therefore \quad q = CV = \frac{\epsilon_0\, aV}{d}\ (a - x + Kx)$

As plates are lowered in the oil, C increases hence charge stored will increase.

Therefore , $i = \frac{dq}{dt} = \frac{\epsilon_0\, aV}{d}\ (K-1) \cdot \frac{dx}{dt}$

Substituting the values

$\epsilon_0 = 8 \times 10^{-12}\ C^2/N\text{–}m^2$

a = 1m, V = 500 volt, d = 0.01m, K = 11 and $\frac{dx}{dt}$ = speed of plate = 0.001 m/s

We get current

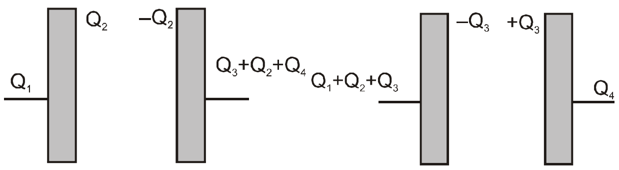
$i = \frac{(8\times10^{-12})1\ (500)\ \ (11-1)\ \ (0.001)}{(0.01)}$ Amp.

i = 4

PART - III

1. Electric field in the capacitor is same at every where which is equal to V/d. so that force at C and B point is same.
Electric field out side the capacitor is zero so that force at A point is zero.

2.



Charge on outer surfaces are equal so $Q_1 = Q_3 + Q_2 + Q_4$ ....(i)
and $Q_1 + Q_2 + Q_3 = Q_4$ ....(ii)

$V = \left|\frac{Q_2}{C}\right|$ or $V = \left|\frac{Q_1 - Q_3 - Q_4}{C}\right|$

$V = \left|\frac{Q_3}{C}\right|$ or $V = \left|\frac{Q_1 - Q_2 - Q_4}{C}\right|$

Adding (i) and (ii)
$Q_1 = Q_4$ and $Q_2 = -Q_3$

3. (i) $E_0 = \frac{2Q}{2\varepsilon_0 A} - \frac{Q}{2\varepsilon_0 A} = \frac{Q}{2\varepsilon_0 A}$

$E_{in} = \frac{2Q}{2A\varepsilon_0} + \frac{Q}{2A\varepsilon_0} \Rightarrow E_{in} = \frac{3Q}{2A\varepsilon_0}$

$E_{in} = \frac{3}{2}\frac{Q}{Cd} \Rightarrow E_{in}d = \frac{3Q}{2C} = V$

(ii) $F = EQ$

$F = \left(\frac{2Q}{2A\varepsilon_0}\right) \times (-Q) = -\frac{Q^2}{A\varepsilon_0}$

$F = \frac{Q^2}{A\varepsilon_0}$

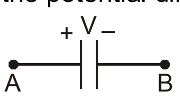
(iii) Energy $= \frac{1}{2}\varepsilon_0 E^2 Ad = \frac{1}{2}\varepsilon_0\left(\frac{3Q}{2Cd}\right)^2 A\ d = \frac{9}{8}\frac{Q^2}{C}$.

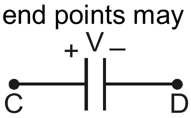
4. equivalent capacitance before switch closed is $C_{eq} = \frac{2C}{3}$,

Total charge flow through the cell is $q = \frac{2CE}{3}$

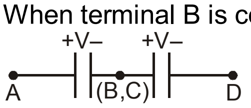
equivalent capacitance after switch S closed is $C_{eq} = 2C$
Total charge flow through the cell is $q = 2CE$
Therefore some positive charge flow through the cell after closing the switch is $= q_f - q_i = 2CE - \frac{2CE}{3} = \frac{4CE}{3}$

5. the potential difference between end points may becomes zero.
the potential difference between end points may becomes 2V.



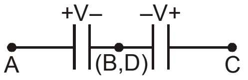


When terminal B is connected with terminal C



$V_A - V_D = 2V$

When terminal B is connected with terminal D



$V_A - V_C = 0$

The energy stored in the system remains same.

6. $\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} = \frac{3+2+4}{60}$

$C_{eq} = \frac{60}{9} = \frac{20}{3}\mu F$

Total charge in this series combination is = $\frac{20}{3}$ × 90

q = 600μC

Potential difference between the plate of C_1 is = $\frac{q}{C_1}$ $\frac{600}{20}$ = 30V

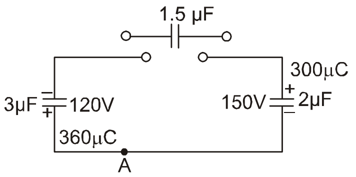
Potential difference between the plate of C_2 is = $\frac{q}{C_2}$ = $\frac{600}{30}$ = 20V

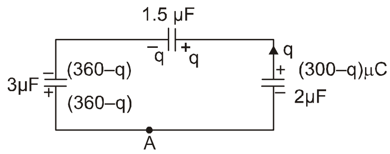
Potential difference between the plate of C_3 is = $\frac{q}{C_3}$ = $\frac{600}{15}$ = 40V

$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{15} = \frac{3+2+4}{60}$

$C_{eq} = \frac{60}{9} = \frac{20}{3}\mu F$

7.





$V_A + \frac{(300-q)}{2} - \frac{q}{1.5} + \frac{360-q}{3} = V_A$ ....(i)

by solve this equation we get

⇒ q = 180μC

Charge on 1.5μF capacitor is = 150μC

Charge on 2μF capacitor is = 300 – 180 = 120μC

Therefore charge flows through A from left to right .

8. Magnitude of charge on the charged capacitor decreases and total charge is conserved.

At $V_1 = V_2$ ⇒ no further flow of charge occurs i.e. condition of steady state.

In charge flow energy is consumed in heat.

9. C = 2μF

$C_{eq} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + ...$

$C_{eq} = C\left(\frac{1}{1-1/2}\right) = 2\left(\frac{1}{1/2}\right) = 4\mu F$ **Ans**

Charge on first row capacitor is q_1 = 2 × 10μC = 20μC

Charge on second row capacitor is q_2 = 1 × 10μC = 10μC

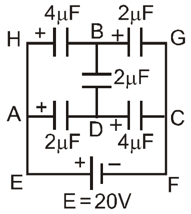
Charge on third row capacitor is q_3 = 1/2 × 10μC = 5μC

Therefore charge on the capacitor in the first row is more than on any other capacitor.

Energy stored in all capacitor is = $1/2\ C_{eq} V^2 = 1/2 \times 4 \times 10^{-6} \times (10)^2 = 0.2$ mJ **Ans**

C = 2μF

10.



given $V_C = 0$ in AEFC

$V_A - 20 = V_C \Rightarrow V_A = 20$ V **Ans**

by KCL, at point D

$2(V_A - V_D) + 2(V_B - V_D) + 4(V_C - V_D) = 0$

$2(V_A - V_D) + 2(V_B - V_D) = 4V_D$ ..... (i) **Ans**

by KCL, at point B

$4(V_A - V_B) + 2(V_D - V_B) + 2(V_C - V_B) = 0$

$4(V_A - V_B) + 2(V_B - V_D) = 2V_B$ ......(ii) **Ans**

Adding eq (i) and (ii)

$2(V_A - V_D) + 2(V_B - V_D) + 4(V_A - V_B) + 2(V_B - V_D) = 4V_D + 2V_B$

$\Rightarrow \quad 6V_A = 6V_D + 6V_B$

$\Rightarrow \quad V_A = V_D + V_B$

11. In shown fig. C_2 and C_3 are parallel capacitor therefore $V_2 = V_3$.

Charge Q_1 flow through battery and gone to C_1 and divided into C_2 and C_3

$Q_1 = Q_2 + Q_3$,

Total potential $V = V_1 + V_2 = V_1 + V_3 = V_1 + \frac{V_2 + V_3}{2}$

12. $q_{max} = q_{01} = q_{02}$ = Both capacitors are charged up to the same magnitude of charge

$t_2 > t_1$

$R_2C_2 > R_1C_1$

$q_{01} = C_1V_1 = q_{02} = C_2V_2$

$C_1 \neq C_2$

So $V_1 \neq V_2$.

13. $t_1 > t_2$

$R_1C_1 > R_2C_2$ for same q_{max}

$q_{01} = q_{02} \Rightarrow E_1C_1 = E_2C_2$

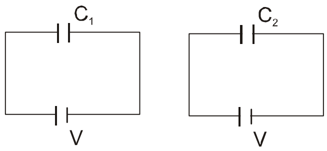
If यदि $E_1 = E_2 \Rightarrow C_1 = C_2 \Rightarrow R_1 = R_2$.

14. During decay of charge in RC circuit

$I = I_0 e^{-t/RC}$

where $I_0 = \frac{q_0}{RC}$

when $t = 0$, $I = I_0 = \frac{q_0}{RC}$



Since potential difference between the plates is same initially therefore I same in both the cases at t = 0 and is equal to

$$I = \frac{q_0}{RC} = \frac{V}{R}$$

Also $q = q_0 e^{-t/RC}$. When $q = \frac{q_0}{2}$ then $\frac{q_0}{2} = q_0\, e^{-t/RC}$

$\Rightarrow$ $e^{+t/RC} = 2$.

$\frac{t}{RC} = \ell n 2$

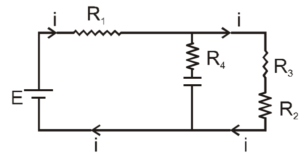
$\Rightarrow$ $t = RC \log_e 2$

$\Rightarrow$ $t \propto C$. Therefore time taken for the first capacitor ($1\mu F$) for discharging 50% of Initial charge will be less.

(B), (D) are the correct options.

15. A long time after closing the switch, system comes in steady state and no current flow through capacitor.

Circuit :



$$i = \frac{E}{R_1 + R_2 + R_3}$$

energy stored in battery $= \frac{1}{2} CV^2 = \frac{1}{2} C \left(\frac{E\ \ (R_3 + R_2)}{R_1 + R_2 + R_3} \right)^2$

16. $E = \frac{V}{d}$ $\Rightarrow$ remains constant

$C' = KC$ $\Rightarrow$ Increase

$Q' = KQ$ $\Rightarrow$ Increase

$U = \frac{1}{2} KCV^2 = KU$ $\Rightarrow$ Increase

17. $C = \frac{\epsilon_0 A}{d}$, $C' = \frac{K \epsilon_0 A}{d}$ $Q = CV = \frac{\epsilon_0 KAV}{d}$ **Ans**

$Q = CV = C_1V_1$ $\Rightarrow$ $V_1 = \frac{V}{K}$ $E = \frac{V_1}{d} = \frac{V}{Kd}$ **Ans**

$W = U_f - U_i = \frac{1}{2} CV^2 - \frac{1}{2} C_1V_1^2 = \frac{1}{2} \frac{\epsilon_0 AV^2}{d^2} - \frac{1}{2} \frac{K \epsilon_0 A}{d} \left(\frac{V}{K} \right)^2 = \frac{\epsilon_0 AV^2}{2d} \left(1 - \frac{1}{K} \right)$ **Ans**

18. Battery connected V = constant

$U' = \frac{1}{2} KCV^2 = KU$ $\Rightarrow$ Increase by K–times

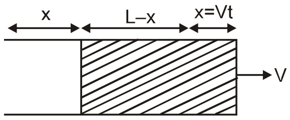
$E = \frac{V}{d}$ = constant

$F = \frac{Q^2}{2 \epsilon_0 A}$ $\Rightarrow$ $F = \frac{C^2V^2}{2 \epsilon_0 A}$ $\Rightarrow$ $F' = \frac{K^2C^2V^2}{2 \epsilon_0 A} = K^2F$

$\Rightarrow$ Increase by K^2–times

$Q = CV$ $\Rightarrow$ $Q' = KCV = KQ$ $\Rightarrow$ Increase by K–times.

19. Capacitance of capacitor is = $C_0 = \frac{k \in_0 \, a.L}{d}$



$C = \frac{\in_0 ax}{d} + \frac{k \in_0 a \ (L-x)}{d}$

$C = \frac{a \in_0}{d}\left[x + k(L-x)\right] = \frac{a \in_0}{d}\left[kL-(k-1)x\right] = \frac{a \in_0}{d}\left[kL-(k-1)vt\right]$

So, C decreases linearly with time

Charge on capacitor $Q = C_0V_0 = \frac{k \in_0 \, aL}{d} V_0$ = constant.

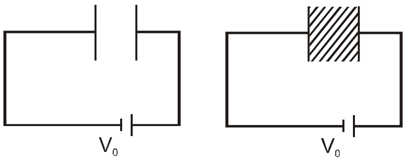
Potential difference across plate is

$V = \frac{Q}{C} = \frac{C_0V_0}{C} \quad \Rightarrow \quad V \propto \frac{1}{C}$

$V = \frac{V_0}{\frac{a \in_0}{d}[kL-(k-1)vt]}$

Potential energy $U = \frac{1}{2} QV = \frac{1}{2} C_0V_0 \cdot V \quad \Rightarrow \quad U \propto V$ **Ans**

20.



Potential difference = V_0
Capacitance = C

$Q_0 = CV_0$

Potential Energy $= \frac{1}{2} CV_0^2$

Potential difference = V_0
Capacitance = KC
[K is the dielectric constant of Slab K > 1]
New charge = KC V_0

New potential energy = $\frac{1}{2}$ KC V_0^2

Correct options are (A), (D).

21. In PQS process charge on capacitor is Q = CV

In PSQ process charge on capacitor is Q′ = KCV

Electric energy stored in PQS is = $\frac{1}{2} CV^2$

Electric energy stored in PSQ is = $\frac{1}{2} KCV^2$

$U_{PSQ} > U_{PQS}$

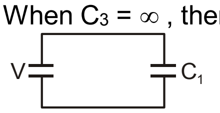
Electric field in PS is $E = \frac{V}{d}$

Electric field in SP is $E = \frac{V}{d}$

$E_{PS} = E_{SP}$

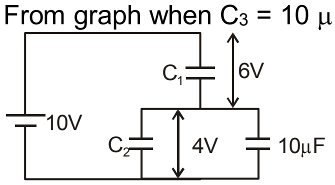
PART - IV

1 to 3. When $C_3 = \infty$, there will be no charge on C_2



As $V_1 = 10$ V therefore V = 10 V

From graph when $C_3 = 10\ \mu F$, $V_1 = 6$ V

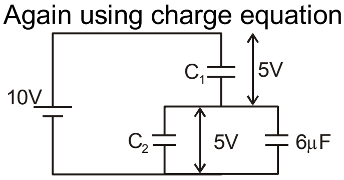


Charge on C_1 = Charge on C_2 + Charge on C_3

$6C_1 = 4C_2 + 40\ \mu C$ .... (1)

Also when $C_3 = 6\ \mu F$, $V_1 = 5V$

Again using charge equation



$5C_1 = 5C_2 + 30\ \mu C$ ....(2)

Solving (1) and (2)

$C_1 = 8\ \mu F$, $C_2 = 2\ \mu F$.

4 to 6. For t = 0 to t_o = RC seconds, the circuit is of charging type. The charging equation for this time is

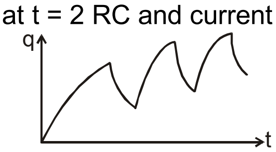
$$q = CE\ \ (1 - e^{-\frac{t}{RC}})$$

Therefore the charge on capacitor at time t_0 = RC is $q_o = CE(1 - \frac{1}{e})$

For t = RC to t = 2RC seconds, the circuit is of discharging type. The charge and current equation for this time are

$$q = q_o e^{-\frac{t-t_o}{RC}} \quad \text{and} \quad i = \frac{q_o}{RC} e^{-\frac{t-t_o}{RC}}$$

Hence charge at t = 2 RC and current at t = 1.5 RC are



$$q = q_o e^{-\frac{2RC-RC}{RC}} = \frac{q_o}{e} = \frac{1}{e} CE(1 - \frac{1}{e})$$

and $$i = \frac{q_o}{RC} e^{-\frac{1.5RC-RC}{RC}} = \frac{q_o}{\sqrt{e}RC} = \frac{E}{\sqrt{e}R}(1 - \frac{1}{e}) \text{ respectively}$$

Since the capacitor gets more charged up from t = 2RC to t = 3RC than in the interval t = 0 to t = RC, the graph representing the charge variation is as shown in figure

EXERCISE–3
PART - I

1. Equation of charging of capacitor,

$$V = V_0\left(1 - e^{-t/R_{eq}C_{eq}}\right)$$

$C_{eq} = 2 + 2 = 4\ \mu F$

$R_{eq} = 1\ M\Omega$

$$4 = 10\left(1 - e^{-\frac{t}{10^6 \times 4 \times 10^{-6}}}\right)$$

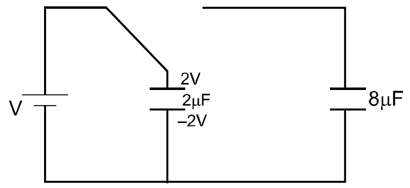
$e^{-t/4} = 0.6$

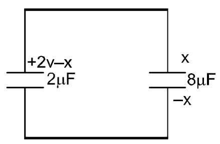
$\Rightarrow \quad e^{t/4} = \frac{5}{3} \quad \Rightarrow \quad \frac{t}{4} = \ell n\ 5 - \ell n\ 3$

$\Rightarrow \quad t = 0.5 \times 4$

$t = 2$ sec. **Ans.**

2.





$U_i = \frac{1}{2}(2)V^2$, $V_{common} = \frac{V}{5}$

$$U_f = \frac{1}{2}\ (2 + 8)\left(\frac{V}{5}\right)^2$$

$$\frac{U_i - U_f}{U_i} \times 100 = \frac{V^2 - \frac{V^2}{5}}{V^2} \times 100$$

$\frac{4}{5} \times 100 = 80\%$ Ans.

3. $q_3 = \frac{C_3}{C_2 + C_3}.Q$

$q_3 = \frac{3}{3+2} \times 80 = \frac{3}{5} \times 80 = 48\ \mu C$

4. When switch S_1 is released charge on C_1 is $2CV_0$ (on upper plate)

When switch S_2 is pressed charge on C_1 is CV_0 (on upper plate) and charge on C_2 is CV_0 (on upper plate)

When switch S_2 is released and switch S_3 pressed charge on C_1 is CV_0 (on upper plate) and charge on C_2 is $-CV_0$ (on upper plate)

5. $C = \frac{K\varepsilon_0 A}{3d} + \frac{2\varepsilon_0 A}{3d}$

$C_1 = \frac{K\varepsilon_0 A}{3d}$

$\frac{C}{C_1} = \frac{2+K}{K}$ Ans. (D)

$$E_1 = E_2 = \frac{V}{d}$$

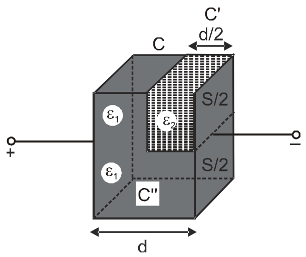
$\Rightarrow \quad \frac{E_1}{E_2} = 1 \qquad$ Ans. (A)

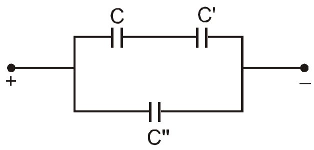
$$Q_1 = C_1V = \frac{K\varepsilon_0 A}{3d}V$$

$$Q_2 = C_2V = \frac{2\varepsilon_0 A}{3d}V$$

$\Rightarrow \quad \frac{Q_1}{Q_2} = \frac{K}{2}$

6.





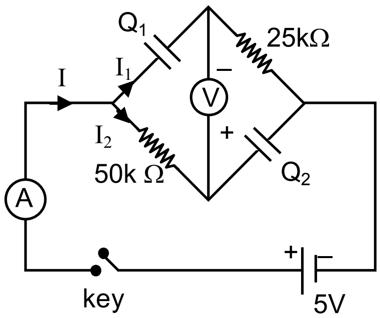
$C_1 = \frac{\epsilon_0 A}{d}$; $\qquad C = \frac{2\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{2\varepsilon_0 s}{d}$

$C' = \frac{4\varepsilon_0 \frac{s}{2}}{\frac{d}{2}} = \frac{4\varepsilon_0 s}{d}$ $\qquad ; \qquad C'' = \frac{2\varepsilon_0 \frac{s}{2}}{d} = \frac{\epsilon_0 s}{d}$

$C_2 = \frac{C \quad C'}{C + C'} + C'' = \frac{4}{3}\frac{\epsilon_0 s}{d} + \frac{\epsilon_0 s}{d} = \frac{7}{3}\frac{\epsilon_0 s}{d}$

$\frac{C_2}{C_1} = \frac{7}{3}$

7.



$q_1 = (200 \times 10^{-3}) \left[1 - e^{-\frac{t}{1}}\right]$

$q_2 = (100 \times 10^{-3}) \left[1 - e^{-\frac{t}{1}}\right]$

$$\frac{q_1}{C} = (50 \times 10^3)\frac{dq_2}{dt}$$

$$\frac{\left(200\times10^{-3}\right)\left(1-e^{-t}\right)}{40\times10^{-6}} = (50 \times 10^3)(100 \times 10^{-3})[e^{-t}]$$

$$(1 - e^{-t})\ \frac{10^6}{20} = 50 \times 10^3(e^{-t})$$

$$\frac{1}{2} = e^{-t}$$

$t = \ell n2$

$I = I_1 + I_2 = (200 \times 10^{-3})(e^{-t}) + (100 \times 10^{-3})e^{-t}$

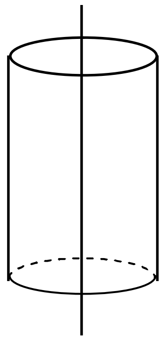
$= 100 \times 10^{-3}[2e^{-t} + e^{-t}]$

$= (300 \times 10^{-3})\ e^{-t}$

$$= \left(\frac{300\times10^{-3}}{e}\right)$$

At $t = \infty$, $I = 0$

8.



Suppose charge per unit length at any instant is λ and initially it is λ_0
electric field at a distance r any instant is

$$E = \frac{\lambda}{2\pi \in r}$$

$$J = \sigma\frac{\lambda}{2\pi \in r}$$

$$\frac{dq}{dt} = J(A) = -J \times 2\pi r\ell$$

$$\frac{d\lambda\ell}{dt} = -\frac{\lambda}{2\pi \in r}\times\sigma 2\pi r\ell$$

$$\lambda = \lambda_0 e^{-\frac{\sigma}{\in}t}$$

$$J = J_0 e^{-\frac{\sigma}{\in}t}$$

9. $E_C = \frac{1}{2}CV_0^2$; $E_D = V_0CV_0 - \frac{1}{2}CV_0^2$

$= \frac{1}{2}CV_0^2$

$\therefore\ E_C = E_D$

10. $E_{D_1} = \frac{V_0}{3}\left(\frac{CV_0}{3}\right) - \frac{1}{2}C.\left(\frac{V_0}{3}\right)^2 = \frac{CV_0^2}{9} - \frac{CV_0^2}{18}$

$= \frac{CV_0^2}{18}$

$E_{D_2} = \frac{2V_0}{3}\left[\frac{2CV_0}{3} - \frac{CV_0}{3}\right] - \left[\frac{1}{2}C\left(\frac{2V_0}{3}\right)^2 - \frac{1}{2}C.\left(\frac{V_0}{3}\right)^2\right]$

$= \frac{2V_0}{3}\left[\frac{CV_0}{3}\right] - \frac{1}{2}C\left[\frac{4V_0^2}{9} - \frac{V_0^2}{9}\right]$

$= \left(\frac{2}{9} - \frac{1}{2\times 9}\times 3\right)CV_0^2 = \left(\frac{2}{9} - \frac{1}{6}\right)CV_0^2 = \left(\frac{12-9}{9\times 6}\right)CV_0^2$

$E_{D_2} = \frac{1}{18}CV_0^2$

$E_{D_3} = V_0\left[CV_0 - \frac{2CV_0}{3}\right] - \left[\frac{1}{2}CV_0^2 - \frac{1}{2}C\left(\frac{2V_0}{3}\right)^2\right]$

$= \frac{1}{3}CV_0^2 - \frac{1}{2}CV_0^2\left[1 - \frac{4}{9}\right]$

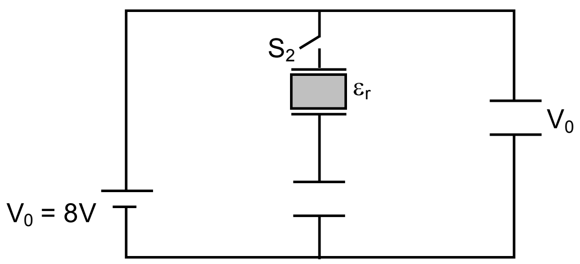
$= \left(\frac{1}{3} - \frac{5}{18}\right)CV_0^2 = \left(\frac{6-5}{18}\right)CV_0^2 = \left(\frac{1}{18}\right)CV_0^2$

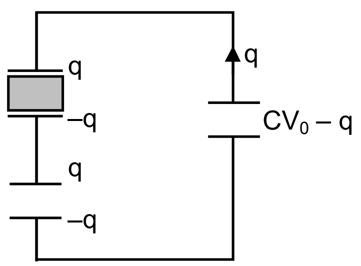
Total = $\left(\frac{1}{18} + \frac{1}{18} + \frac{1}{18}\right)CV_0^2$

$= \frac{3}{18}CV_0^2$

$E_D = \frac{3}{9}\left[\frac{1}{2}CV_0^2\right] = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$

11.





$\frac{CV_0 - q}{C} - \frac{q}{\varepsilon_r C} - \frac{q}{C} = 0$ $\qquad$ $\frac{CV_0 - q}{C} = 5$

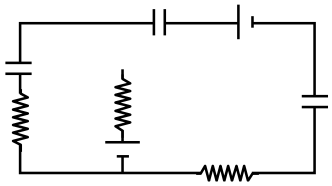
$5 = \frac{q}{C}\left(1 + \frac{1}{\varepsilon_r}\right)$ $\qquad$ $8C - q = 5C$

$= 3\left(1+\dfrac{1}{\varepsilon_r}\right)$ $\Rightarrow$ q = 3C

$\dfrac{5}{3} = 1 + \dfrac{1}{\varepsilon_r} \Rightarrow \dfrac{1}{\varepsilon_r} = \dfrac{2}{3}$

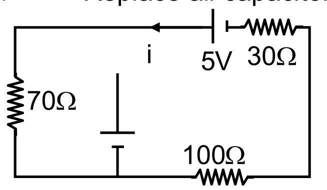
$\varepsilon_r = \dfrac{3}{2} = 1.5$

12.



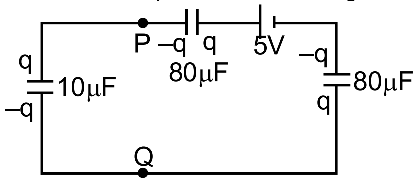
Just after closing of switch S_1 charge on capacitors is zero.

$\therefore$ Replace all capacitors with wire.



$i = \dfrac{5}{70+100+30} = \dfrac{5}{200} = 25mA$

Now S_1 is kept closed for long time circuit is in steady state



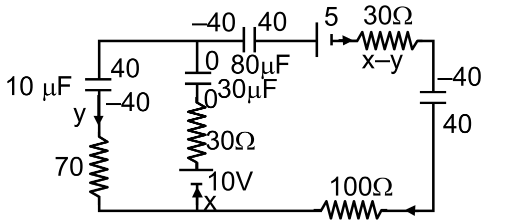
$\dfrac{q}{10} + \dfrac{q}{80} + \dfrac{q}{80} - 5 = 0$

$\dfrac{10q}{80} = 5$

$\therefore$ q = 40 μC

$\therefore$ V across C_1 = 40/10 = 4 volt

Now just after closing of S_2 charge on each capacitor remain same



KVL

–10 + x × 30 + 40/10 + y × 70 = 0

30x + 70y = 6 ....(1)

$-\dfrac{40}{80}$ + 5 + (x – y) 30 $-\dfrac{40}{80}$ + (x – y) × 100 – 10 + x × 30 = 0

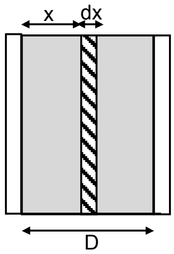
160x – 130y – 6 = 0 ....(2)

y = 96/1510

x = 0.05 amp.

Correct option – 2, 3

13.



$$\frac{x}{m} = \frac{D}{N}$$

$$d\left(\frac{1}{C}\right) = \frac{dx}{K_m\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(1+\frac{m}{N}\right)} = \frac{dx}{K\varepsilon_0 A\left(1+\frac{x}{D}\right)}$$

$$\frac{1}{C_{eq}} = \int d\left(\frac{1}{C}\right) = \int_0^D \frac{Ddx}{K\varepsilon_0 A(D+x)}$$

$$\frac{1}{C_{eq}} = \frac{D}{K\varepsilon_0 A}\ell n2$$

$C_{eq} = \dfrac{K\varepsilon_0 A}{D\ell n2}$. Therefore $\alpha = 1$

PART - II

1. $U_0 = \dfrac{q_0^2}{2C}$ $\quad U = \dfrac{q_0^2 e^{-2t_1/\tau}}{2C} = \dfrac{U_0}{2} = \dfrac{q_0^2}{4C}$ $\quad\Rightarrow\quad e^{-2t_1/\tau} = \dfrac{1}{2}$

$t_1 = \dfrac{\tau}{2}\ \ell n2$ ....(1)

and $q = q_0 e^{-t_2/\tau}$

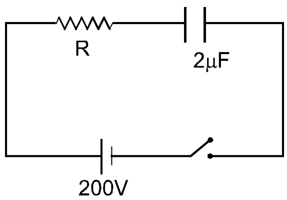
$\dfrac{q_0}{4} = q_0\ e^{-t_2/\tau}$,

$e^{-t_2/\tau} = \dfrac{1}{4}$

$t_2 = 2\tau \ln 2$ ....(2)

$\dfrac{t_1}{t_2} = \dfrac{1}{4}$

2.



$v = 200(1 - e^{-t/\tau})$

$120 = 200(1 - e^{-t/\tau})$

$e^{-t/\tau} = \dfrac{200-120}{200} = \dfrac{80}{200}$

$t/\tau = \log(2.5) = 0.4$

$5 = (0.4) \times R \times 2 \times 10^{-6}$

$\Rightarrow\quad R = \dfrac{5}{(0.4)\times 2\times 10^{-6}} = R = 2.7 \times 10^6$ **Ans.**

3. Time constant for parallel combination = 2RC

Time constant for series combination = $\dfrac{RC}{2}$

In first case :

$$V = V_0 e^{-\frac{t_1}{2RC}} = \frac{V_0}{2} \qquad \text{........(i)}$$

In second case :

$$V = V_0 e^{-\frac{t_2}{(RC/2)}} = \frac{V_0}{2} \qquad \text{........(ii)}$$

From (i) & (ii),

$$\frac{t_1}{2RC} = \frac{t_2}{(RC/2)} \quad \Rightarrow \quad t_2 = \frac{t_1}{4} = \frac{10}{4} = 2.5 \text{ sec.}$$

4. $Q = c\varepsilon_0\ e^{-t/cR}$

$4\varepsilon = 4\varepsilon_0 e^{-t/\tau}$

$\varepsilon = \varepsilon_0\ \varepsilon^{-t/\tau}$

When t = 0 $\Rightarrow$ $\varepsilon_0 = 25$

$\varepsilon = \varepsilon_0 = 25$

when t = 200 $\Rightarrow$ $\varepsilon = 5$

$5 = 25\ e^{-\frac{200}{\tau}}$

$\ln 5 = \frac{200}{\tau}$

$\tau = \dfrac{200}{\ell n5} = \dfrac{200}{\ell n10 - \ell n2}$

$= \dfrac{200}{\ell n10 - 0.693}$

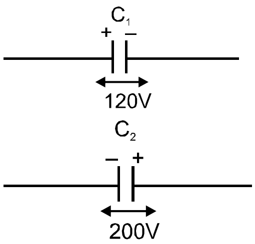
Alternative :

Time constant is the time in which 63% discharging is completed.

So remaining charge = 0.37 × 25 = 9.25 V

Which time in 100 < t < 150 sec.

5.



For potential to be made zero, after connection

$120C_1 = 200\ C_2$

$\Rightarrow \quad 3C_1 = 5C_2$

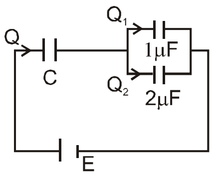
Ans. (2)

6. Electric field inside dielectric $\dfrac{\sigma}{K\varepsilon_0} = 3 \times 10^4$

$\Rightarrow \sigma = 2.2 \times 8.85 \times 10^{-12} \times 3 \times 10^4$

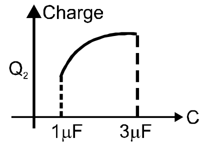
$= 6 \times 10^{-7}$ C/m^2

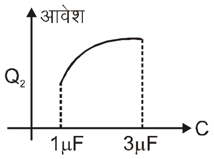
7.



$Q_2 = \frac{2}{2+1}$ $\qquad$ $Q = \frac{2Q}{3}$

$Q = E\left(\frac{C \times 3}{C+3}\right) \Rightarrow Q_2 = \frac{2}{3}\left(\frac{3CE}{C+3}\right) = \frac{2CE}{C+3}$





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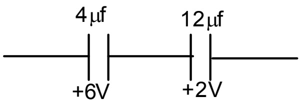


$Q_1 = 24\mu c$

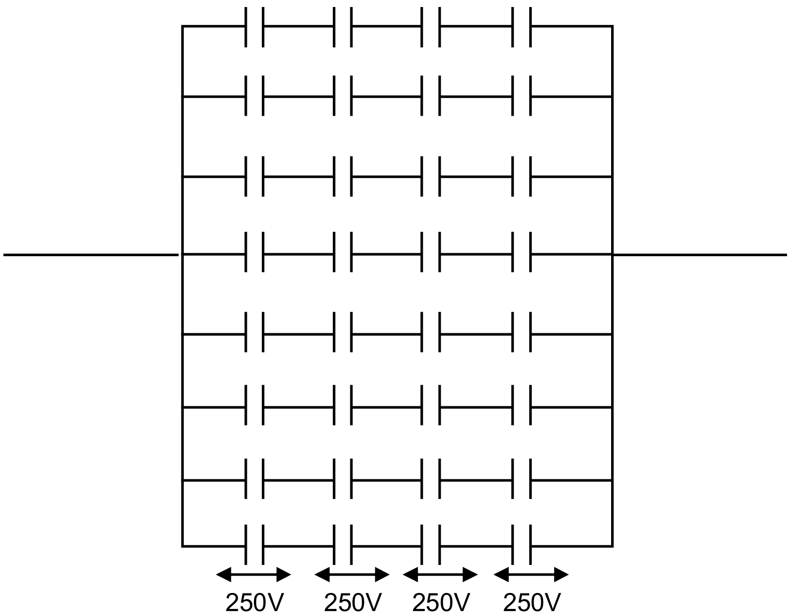
$Q_2 = 18\ \mu c$

$Q = 42\mu c$

$E = 10^7 \times 42 \times 10^{-6}$

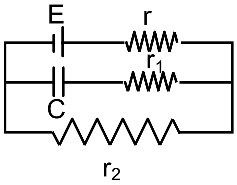
$E = 420$ N/C

9.

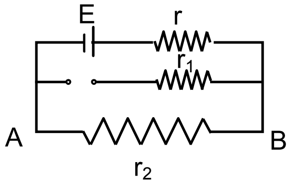


Minimum no. of capacitors required = 32

10.



at steady state



current in the circuit $I = \dfrac{E}{r + r_2}$

potential difference across AB= $Ir_2 = \dfrac{Er_2}{r + r_2}$

charge on capacitor = Q = $C(\Delta V)_{AB}$

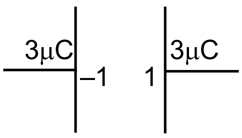
$Q = \dfrac{CEr_2}{r + r_2}$

11. $Q_{cap} = KC_0V$

$|Q_{polarised}| = \left|Q_{cap}\left(1 - \dfrac{1}{k}\right)\right|$

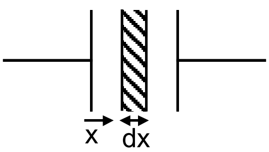
$= (90 \times 10^{-12})\ (20)\ \left(\dfrac{5}{3}\right)\left(1 - \dfrac{3}{5}\right)$ Coulomb = 1200×10^{-12} Coulomb = 1.2 nc

12.



$V = \dfrac{Q}{C} = \dfrac{1\mu C}{1\mu F} = 1\text{Volt}$

13. Capacitance of element $= \dfrac{k\varepsilon_0 A}{dx}$



Capacitance of element, C' = $\dfrac{K(1 + \alpha x)\varepsilon_0 A}{dx}$

$$\sum \frac{1}{C'} = \int_0^d \frac{dx}{K\varepsilon_0 A(1 + \alpha x)}$$

$$\frac{1}{C} = \frac{1}{K\varepsilon_0 A\alpha}\ell n(1 + \alpha d)$$

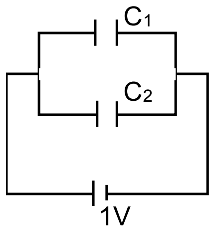
Given – $\alpha d << 1$

$$\frac{1}{C} = \frac{1}{K\varepsilon_0 A\alpha}\left(\alpha d - \frac{\alpha^2 d^2}{2}\right)$$

$$\frac{1}{C} = \frac{d}{K\varepsilon_0 A}\left(1 - \frac{\alpha d}{2}\right)$$

$$C = \frac{K\varepsilon_0 A}{d}\left(1 + \frac{\alpha d}{2}\right)$$

14.



Given $C_1 + C_2 = 10\mu F$ …(i)

$$4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$$

$\Rightarrow$ $4C_1 = C_2$ …(ii)

from equation (i) & (ii)

$C_1 = 2\mu F$

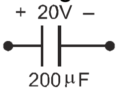
$C_2 = 8\mu F$

If they are in series

$C_{eq.} = \dfrac{C_1C_2}{C_1 + C_2} = 1.6\mu F$

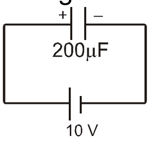
SOLUTIONS OF HIGH LEVEL PROBLEMS

1. (a) Charge on capacitor before connection



$Q_1 = CV_1 = 4000\ \mu C$

Charge on capacitor after connection



$Q_2 = CV_2 = 200 \times 10 = 2000\ \mu C$

(b) Charge flown through the 10V battery = 4000 – 2000 = 2000 µC

(c) Work is done on the battery.
Work done = $Q_2 \times V_2 = 2000 \times 10 = 20$mJ.

(d) The decrease in electrostatic field energy.

$= U_i - U_f = \frac{1}{2}CV_1^2 - \frac{1}{2}CV_2^2 = \frac{1}{2} \times 200 \times (20)^2 - \frac{1}{2} \times 200\ (10)^2$

= 30 mJ

(e) $W = \Delta U + \Delta H$

-20 mJ $= -30$ mJ $+ \Delta H$

$[\Delta H = 10$ mJ$]$

2. Equivalent capacitance = 0.7 µF = $\frac{7}{10}$ µF

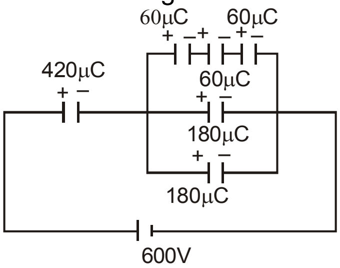
$$\frac{1}{C_{eq}} = \frac{1}{\frac{10}{7}} = \frac{1}{1+\frac{3}{7}} = \frac{1}{1+\frac{1}{7/3}} = \frac{1}{1+\frac{1}{2+\frac{1}{3}}}$$

$C_{eq} = 0.7\mu C$ can be obtain as shown in figure

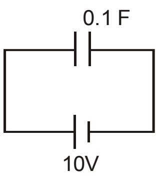
$\frac{1}{3} \rightarrow$ Three capacitors in series

$2 \rightarrow 2$ capacitor in parallel.

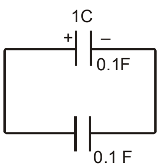
and now charge distaribute as shown in figure



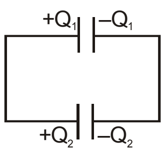
3. $Q = CV = 0.1 \times 10 = 1C$



$\Rightarrow$



$\Rightarrow$



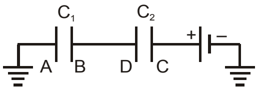
$Q_2 = Q_1 = Q/2$

$U_{initial} = \frac{1}{2} \times 0.1 \times (10)^2 = 5$ J

$U_{final} = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}\frac{Q^2}{C} = \frac{Q^2}{C}$

$$= \left(\frac{1}{2}\right)^2 \times 10 = \frac{10}{4} = \frac{5}{2} J$$

$$\frac{U_{initial}}{U_{final}} = \frac{5}{2.5} = 2$$

4. 

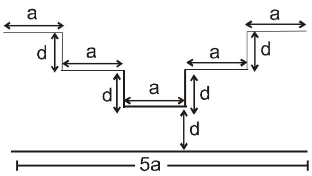
$$C_1 = \frac{\epsilon_0\ \pi \left(\frac{0.1}{2}\right)^2}{2\times10^{-3}} = \frac{5\pi\ \epsilon_0}{4}$$

$$C_2 = \frac{\epsilon_0\ \pi \left(\frac{0.12}{2}\right)^2}{3\times10^{-3}} = \frac{6\pi\ \epsilon_0}{5}$$

$$C_{eq} = \frac{C_1C_2}{C_1+C_2} = \frac{30\pi\ \epsilon_0}{49} = \frac{30}{49\times36\times10^9} \approx 17pF$$

The energy stored $U = \frac{1}{2} C_{eq} V^2$

$$= \frac{1}{2} \times 17 \times (120)^2 \times 10^{-12} = 122.4 \text{ n J}$$

5. 

This is combination of 5 capacitors connected parallel.

$$C = C_1 + C_2 + C_3 + C_4 + C_5 = \frac{(A/5)\epsilon_o}{d} + 2\frac{(A/5)\epsilon_o}{2\ d} + 2\frac{(A/5)\epsilon_o}{3\ d}$$

$$= \frac{A\epsilon_o}{5\ d}\frac{8}{3} = \frac{8\ \epsilon_o\ A}{15\ d}$$

6. $$\frac{1}{dC} = \int \frac{dx}{K(x)\ .\ A\epsilon_0} = \int_0^{d/2} \frac{dx}{\left[1+\frac{\beta x}{\epsilon_0}\right] A\epsilon_0} + \int_{\frac{d}{2}}^{d} \frac{dx}{\left[1+\frac{\beta x}{\epsilon_0}(d-x)\right] A\epsilon_0}$$

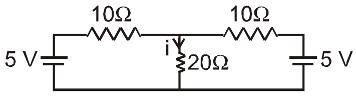
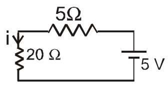
$$\int \frac{1}{dC} = \frac{\frac{1}{A\epsilon_0}\ \ell n\left[1+\frac{\beta x}{\epsilon_0}\right]_0^{d/2}}{\frac{\beta}{\epsilon_0}} + \frac{\frac{1}{A\epsilon_0}\ell n\left[1+\frac{\beta}{\epsilon_0}(d-x)\right]_{d/2}^{d}}{-\frac{\beta}{\epsilon_0}}$$

$$\int \frac{1}{dC} = \frac{1}{A\beta}\ell n\left[1+\frac{\beta d}{2\epsilon_0}\right] + \frac{1}{A\beta}\ \ell n\left[1+\frac{\beta d}{2\epsilon_0}\right]$$

$$C_{eq} = \frac{A\beta}{2\ell n\left[1+\frac{\beta d}{2\epsilon_0}\right]}$$

Now $C_{eq} = 2C_0$ (C_0 = capacitance when it is without any dielectric)

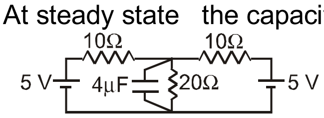
$$= \frac{A\beta}{2\ell n\left[1+\frac{\beta d}{2\epsilon_0}\right]} = \frac{2\epsilon_0 A}{d} \qquad \beta d = 4\epsilon_0 \ell n\left(1+\frac{\beta d}{2\epsilon_0}\right)$$

7. (a)  $\Rightarrow$ 

Both the batteries are in parallel

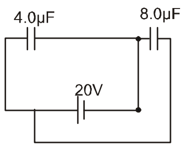
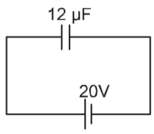
$i = \frac{5}{25} = \frac{1}{5}$ A

(b) At steady state the capacitor will be fully charged.



Potential difference across capacitor $v = \frac{1}{5} \times 20 = 4$ V

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 4 \times (4)^2 = 32$ μJ

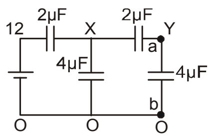
8.  $\Rightarrow$ 

(i) Charge flown through the battery $Q = C_{eq}.V = 240$ μC

(ii) Charge on 4 μF capacitor = $Q_{4\mu F} = 20 \times 4 = 80$ μC

Charge on 8 μF capacitor = $Q_{8\mu F} = 20 \times 8 = 160$ μC

9.

(a)  By KCL,

$$(x-12)2 + (x-0).\frac{4}{3} + (x-0).4 = 0$$

$$2x - 24 + \frac{4x}{3} + 4x = 0$$

$$6x + \frac{4x}{3} = 24$$

$$x = \frac{36}{11}\ V$$

$$(Y-0)4 = \left(\frac{36}{11} - Y\right) \times 2$$

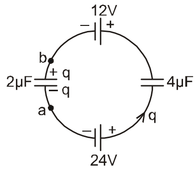
$$(Y-0)2 = \left(\frac{36}{11} - Y\right)$$

$$2Y = \frac{36}{11} - Y$$

$$\left[Y = \frac{12}{11}\ V\right]$$

$$V_a - V_b = \frac{12}{11} V$$ **Ans.**

(b) $-12 - \frac{q}{2} + 24 - \frac{q}{4} = 0$



$\frac{q}{2} + \frac{q}{4} = 12$

$\frac{3q}{4} = 12 \Rightarrow q = 16\ \mu C$

$V_a - V_b = -\frac{q}{c} = -\frac{16}{2} = -8\ V$ **Ans.**

10. Before opening the switch potential difference across both the capcitors is V, as they are in paralle. Hence, energy stored in them is,

$U_A = U_B = \frac{1}{2}CV^2 \qquad \therefore\ U_{Total} = CV^2 = U_i$ .... (1)

After opening the switch, potential difference across it is V and its capacity is 3C

$\therefore \qquad U_A = \frac{1}{2}\ (3C)V^2 = \frac{3}{2}CV^2$

In case of capacitor B, charge strored in it is q = CV and its capacity is also 3C. Therefore,

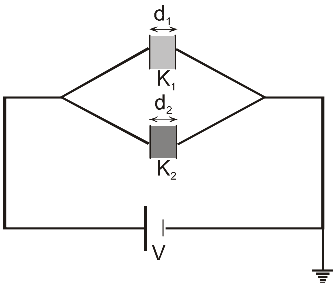
$U_B = \frac{q^2}{2(3C)} = \frac{CV^2}{6}$

$\therefore\ U_{Total} = \frac{3CV^2}{2} + \frac{CV^2}{6} = \frac{10}{6}CV^2 = \frac{5CV^2}{3} = U_f$ .... (2)

From Eqs.(1) and (2)

$\frac{U_i}{U_f} = \frac{3}{5}$

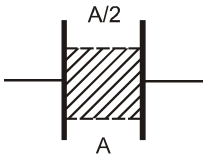
11.



(a) $C_{eq} = C_1 + C_2 = \frac{K_1 \in_0 A}{d_1} + \frac{K_2 \in_0 A}{d_2} = \in_0 A\left(\frac{K_1}{d_1} + \frac{K_2}{d_2}\right)$

(b) Surface charge denisity = $\frac{Q_1}{A} = \left(\frac{K_1 \in_0 A}{d_1}\right)\frac{V}{A} = \frac{K_1 \in_0}{d_1}V$ and $\frac{Q_2}{A} = \left(\frac{K_2 \in_0 A}{d_2}\right)\frac{V}{A} = \frac{K_2 \in_0}{d_2}V$

(c) Energy density in medium $K_1 = \frac{1}{2}K_1 \in_0 E^2 = \frac{\in_0 K_1 V^2}{2d_1^2}$

12. 

$$C_A = \frac{k \in_0 A}{2d} + \frac{\in_0 A}{2d}$$

(i) $C_A = \frac{\in_0 A}{2d}(k+1) = \frac{8.85 \times 10^{-12} \times 0.04}{2 \times 8.85 \times 10^{-4}}\ (9+1) = 2 \times 10^{-9}$ F = 2nF

$U_A = \frac{1}{2} C_A V^2 = \frac{1}{2} \times 2 \times 10^{-9} \times (110)^2 = 121 \times 10^{-7}$ J $= 12.1\mu$J **Ans**

(ii) $W = |U_f - U_i|$

$C_{air} = \frac{\in_0 A}{d} = \frac{C_A 2}{(k+1)} = \frac{2 \times 10^{-9} \times 2}{10} = 0.4$nF $\qquad Q = C_A V = 2 \times 10^{-9} \times 110 = 0.22\mu$C

$U_f = \frac{1}{2}\frac{Q^2}{C_{air}} = \frac{1}{2}\frac{(0.22 \times 10^{-6})^2}{(0.4 \times 10^{-9})} = 60.5\mu$J, $U_i = 12.1\ \mu$J $\Rightarrow W = 48.4\mu$J **Ans**

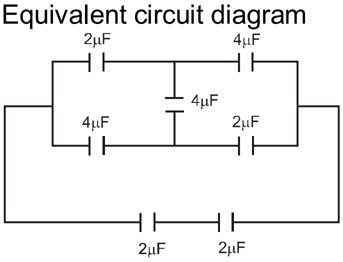
(iii) $C_A = 0.4$ nF, $Q_A = 0.22\mu$C, $\quad C_B = \frac{k \in_0 A}{d} = \frac{9 \times 8.85 \times 10^{-12} \times 0.02}{8.85 \times 10^{-4}} = 18 \times 10^{-10}$ F = 1.8 nF

common potential $V = \frac{\text{total charge}}{\text{total capacitance}} = \frac{0.22 \times 10^{-6}}{(0.4 + 1.8) \times 10^{-9}} = 100$V

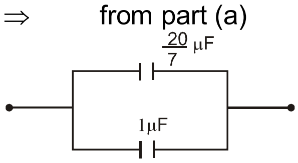
$U = \frac{1}{2} C_A V^2 + \frac{1}{2} C_B V^2 = \frac{1}{2}(C_A + C_B) V^2$

$= \frac{1}{2}(0.4 + 1.8) \times 10^{-9} (100)^2 = 11\mu$J **Ans**

13. Equivalent circuit diagram

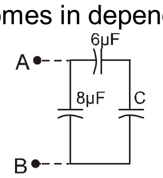


$\Rightarrow$ from part (a)



Resultant capacitance = $\frac{20}{7} + 1 = \frac{27}{7}\mu$F

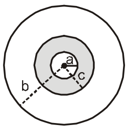
14. If capacitance of C is equal to 12μF. Then equivalent capacitance of the ladder between points A and B is becomes in dependent of the number of section in between points.



$\Rightarrow \quad \frac{6C}{6+C} + 8 = C$

$\Rightarrow \quad C^2 - 8C - 48 = 0 \qquad \Rightarrow \qquad C = 12\ \mu$F

15.



$\int d\,V = \int E.\,dr$

$$V = \int_a^c \frac{q\,dr}{4\pi\in_0 kr^2} + \int_c^b \frac{q}{4\pi\in_0}\frac{dr}{r^2} = \frac{q}{4\pi\in_0}\frac{1}{k}\left[-\frac{1}{r}\right]_a^c + \frac{q}{4\pi\in_0}\left[-\frac{1}{r}\right]_c^b$$

$$= \frac{q}{4\pi\in_0}\left[\frac{1}{k}\left(\frac{1}{a}-\frac{1}{c}\right)-\left(\frac{1}{c}-\frac{1}{b}\right)\right] = \frac{q}{4\pi\in_0}\left[\frac{bc-ab-kab+kac}{kabc}\right]$$

$$V = \frac{q}{4\pi\in_0}\left[\frac{ba(b-c)+b(c-a)}{kabc}\right];\ C_{eq} = \frac{q}{V}$$

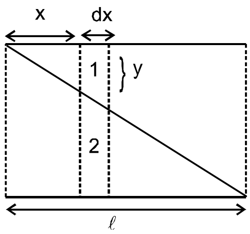
$$C_{eq} = \frac{4\pi\in_0\ kabc}{ka(b-c)+b(c-a)} = \frac{4\pi\in_0\ (3)\ (a)\ (3a)\ (2a)}{3a\ (3a-2a)+3a(2a-a)}$$

$C_{eq} = 3\ (4\pi\in_0 a)$

= 3 (capacitance of capacitor of isolated conducting shell of radius a)

$\Rightarrow\quad n = 3$

16.



take 'dx' element at x distance $\frac{x}{y} = \frac{\ell}{d}$; $y = \frac{x\ d}{\ell}$

$$dC_1 = \frac{K_2 bdx \in_o}{y} = \frac{K_2 \in_o b\ell dx}{xd}$$

$$dC_2 = \frac{K_1 b \in_o dx}{d-y} = \frac{K_1\ b\ \in_o\ dx\ \ell}{d(\ell - x)}$$

dC_1 and dC_2 are in series, so their equivalent

$$dC = \frac{dC_1\ dC_2}{dC_1 + dC_2} = \frac{K_1K_2 b\ell \in_o}{d}\ \frac{dx}{(K_1\ -K_2)x + K_2\ell}$$

Now, we can consider there parallel slabs to the parallel in circuit combination

$C_{eq.} = dC_1 + dC_2 + dC_3 + dC_4$

$$= \int dC$$

$$= \int_o^\ell \frac{K_1K_2\ b\ell \in_o}{d}\ \frac{d\ x}{(K_1-K_2)x + K_2\ell}$$

$$= \frac{K_1\ K_2\ b\ell\ \in_o}{d(K_1-K_2)}\ \{\ln \ell K_1 - \ln K_2 \ell\}$$

$$C = \frac{K_1\ K_2\ b\ell\ \in_0}{d(K_1-K_2)}\ \ln\frac{K_1}{K_2} = \frac{K_1\ K_2\ b\ell\ \in_0}{d(K_2-K_1)}\ \ell n\frac{K_2}{K_1}$$