ELECTROMAGNETIC INDUCTION

Contents

JEE (ADVANCED) SYLLABUS

Electro Magnetic induction : Faraday's law, Lenz's law; Self and mutual inductance; RC, LR and LC circuits with d.c. and a.c. sources.

JEE (MAIN) SYLLABUS

Electromagnetic induction : Faraday’s law, induced emf and current ; Lenz’s Law, Eddy currents. Self and mutual inductance.

ELECTROMAGNETIC INDUCTION

1. FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

(i) When magnetic flux passing through a loop changes with time or magnetic lines of force are cut by a conducting wire then an emf is produced in the loop or in that wire. This emf is called induced emf.
If the circuit is closed then the current will be called induced current.

magnetic flux = $\int \vec{B}.\vec{ds}$

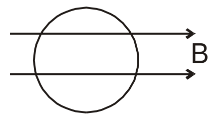
(ii) The magnitude of induced emf is equal to the rate of change of flux w.r.t. time in case of loop. In case of a wire it is equal to the rate at which magnetic lines of force are cut by a wire

$$E = -\frac{d\phi}{dt}$$

(–) sign indicates that the emf will be induced in such a way that it will oppose the change of flux.
SI unit of magnetic flux = Weber.

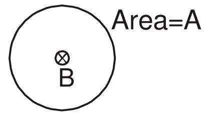
Solved Example

Example 1. A coil is placed in a constant magnetic field .The magnetic field is parallel to the plane of the coil as shown in figure. Find the emf induced in the coil.



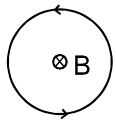
Solution : ϕ = 0 (always) since area is perpendicular to magnetic field. $\therefore$ emf = 0

Example 2. Find the emf induced in the coil shown in figure. The magnetic field is perpendicular to the plane of the coil and is constant.



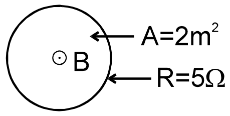
Solution : ϕ = BA (always)
= const. $\therefore$ emf = 0

Example 3. Find the direction of induced current in the coil shown in figure. Magnetic field is perpendicular to the plane of coil and it is increasing with time.



Solution : Inward flux is increasing with time. To opposite it outward magnetic field should be induced. Hence current will flow anticlockwise.

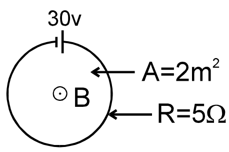
Example 4. Figure shows a coil placed in decreasing magnetic field applied perpendicular to the plane of coil .The magnetic field is decreasing at a rate of 10T/s. Find out current in magnitude and direction



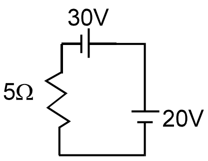
Solution : ϕ = B.A $\Rightarrow$ emf = A . $\frac{dB}{dt}$ = 2 × 10 = 20 v

$\therefore$ i = 20/ 5 = 4 amp. From Lenz's law direction of current will be anticlockwise.

Example 5. Figure shows a coil placed in a magnetic field decreasing at a rate of 10T/s. There is also a source of emf 30 V in the coil. Find the magnitude and direction of the current in the coil.

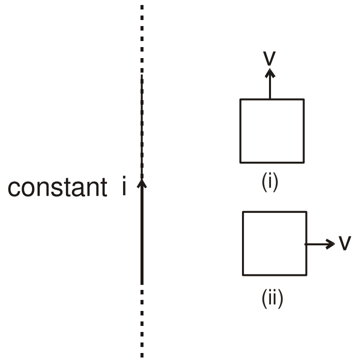


Solution :



Induce emf = 20V
equivalent i = 2A clockwise

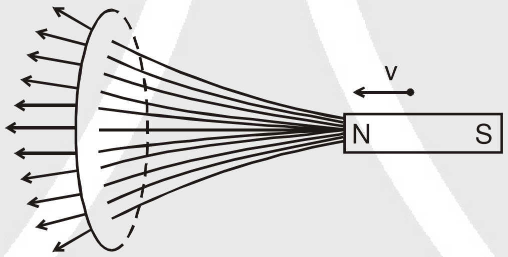
Example 6. Figure shows a long current carrying wire and two rectangular loops moving with velocity v. Find the direction of current in each loop.



Solution : In loop (i) no emf will be induced because there is no flux change.
In loop (ii) emf will be induced because the coil is moving in a region of decreasing magnetic field inward in direction. Therefore to oppose the flux decrease in inward direction, current will be induced such that its magnetic field will be inwards. For this direction of current should be clockwise.

2. LENZ'S LAW (CONSERVATION OF ENERGY PRINCIPLE)

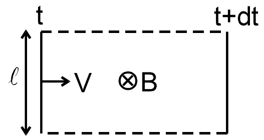
According to this law, emf will be induced in such a way that it will oppose the cause which has produced it.
Figure shows a magnet approaching a ring with its north pole towards the ring.



We know that magnetic field lines come out of the north pole and magnetic field intensity decreases as we move away from magnet. So the magnetic flux (here towards left) will increase with the approach of magnet. This is the cause of flux change. To oppose it, induced magnetic field will be towards right. For this the current must be anticlockwise as seen by the magnet.
If we consider the approach of North pole to be the cause of flux change, the Lenz's law suggests that the side of the coil towards the magnet will behave as North pole and will repel the magnet. We know that a current carrying coil will behave like North pole if it flows anticlockwise. Thus as seen by the magnet, the current will be anticlockwise.
If we consider the approach of magnet as the cause of the flux change, Lenz's law suggest that a force opposite to the motion of magnet will act on the magnet, whatever be the mechanism.
Lenz's law tells that if the coil is set free, it will move away from magnet, because in doing so it will oppose the 'approach' of magnet.
If the magnet is given some initial velocity towards the coil and is released, it will slow down .It can be explained as the following.
The current induced in the coil will produce heat. From the energy conservation, if heat is produced there must be an equal decrease of energy in some other form, here it is the kinetic energy of the moving magnet. Thus the magnet must slow down. So we can justify that the **Lenz's law is conservation of energy principle**.

3. MOTIONAL EMF

We can find emf induced in a moving rod by considering the number of lines cut by it per sec assuming there are 'B' lines per unit area. Thus when a rod of length ℓ moves with velocity v in a magnetic field B, as shown, it will sweep area per unit time equal to ℓv and hence it will cut B ℓ v lines per unit time.

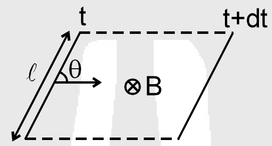


Hence emf induced between the ends of the rod = $Bv\ell$

Also emf = $d\phi/dt$. Here ϕ denotes flux passing through the area, swept by the rod.The rod sweeps an area equal to ℓ vdt in time interval dt.Flux through this area = $B\ell$ vdt. Thus $\frac{d\phi}{dt} = \frac{B\ell vdt}{dt} = Bv\ell$

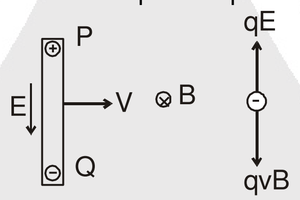
If the rod is moving as shown in the following figure, it will sweep area per unit time = $v\ell\sin\theta$ and hence it will cut B v ℓ $\sin\theta$ lines per unit time.

Thus emf = $Bv\ell\sin\theta$.



3.1 EXPLANATION OF EMF INDUCED IN ROD ON THE BASIS OF MAGNETIC FORCE:

If a rod is moving with velocity v in a magnetic field B ,as shown , the free electrons in a rod will experience a magnetic force in downward direction and hence free electrons will accumulate at the lower end and there will be a deficiency of free electrons and hence a surplus of positive charge at the upper end. These charges at the ends will produce an electric field in downward direction which will exert an upward force on electron. If the rod has been moving for quite some time enough charges will accumulate at the ends so that the two forces qE and qvB will balance each other. Thus E = v B.



$V_P - V_Q = VB\ell$

The moving rod is equivalent to the following diagram, electrically.

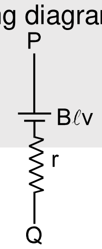
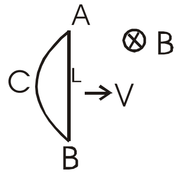
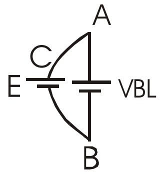


Figure shows a closed coil ABCA moving in a uniform magnetic field **B** with a velocity **v. The** flux passing through the coil is a constant and therefore the induced emf is zero.



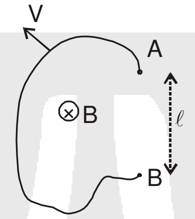
Now consider rod AB, which is a part of the coil. Emf induced in the rod =B L v
Suppose the emf induced in part ACB is E, as shown.



Since the emf in the coil is zero, Emf (in ACB) + Emf (in BA) = 0
or $-E + vBL = 0$ or $E = vBL$
Thus emf induced in any path joining A and B is same, provided the magnetic field is uniform. Also the equivalent emf between A and B is BLv (here the two emf's are in parallel)

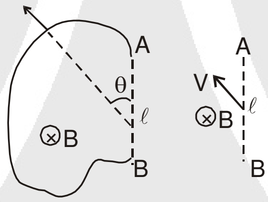
Solved Examples

Example 7. Figure shows an irregular shaped wire AB moving with velocity v, as shown.



Find the emf induced in the wire.

Solution : The same emf will be induced in the straight imaginary wire joining A and B , which is $Bv\ell \sin\theta$

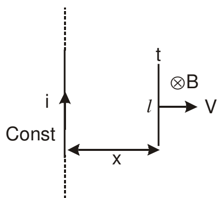


Example 8. A rod of length l is kept parallel to a long wire carrying constant current i. It is moving away from the wire with a velocity v. Find the emf induced in the wire when its distance from the long wire is x.

Solution : $E = B\,l\,V = \dfrac{\mu_0 i l v}{2\pi x}$

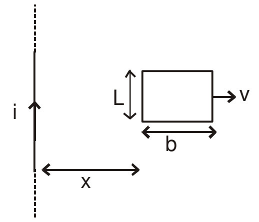
OR

Emf is equal to the rate with which magnetic field lines are cut. In dttime the area swept by the rod is lv dt. The magnetic field lines cut in dt time $= B\,l\,vdt = \dfrac{\mu_0 i l vdt}{2\pi x}$.

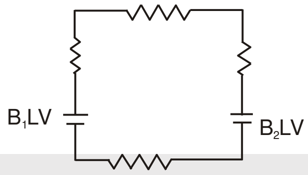


$\therefore$ The rate with which magnetic field lines are cut $= \dfrac{\mu_0 i l v}{2\pi x}$

Example 9. A rectangular loop, as shown in the figure, moves away from an infinitely long wire carrying a current i. Find the emf induced in the rectangular loop.



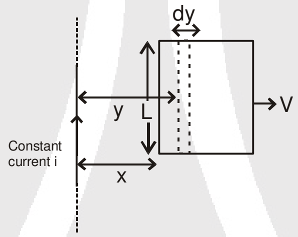
Solution : $E = B_1 LV - B_2Lv = \frac{\mu_0 i}{2\pi x}Lv - \frac{\mu_0 i}{2\pi(x+b)}Lv = \frac{\mu_0 iLbv}{2\pi x(x+b)}$



Aliter :

Consider a small segment of width dy at a distance y from the wire.

Let flux through the segment be $d\phi$.

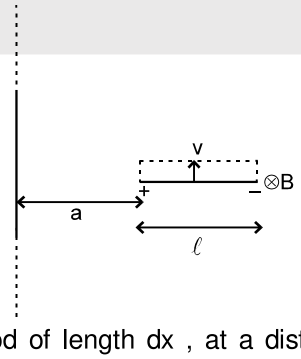


$\therefore \quad d\phi = \frac{\mu_0 i}{2\pi y} L\, dy \qquad \therefore \qquad \phi = \frac{\mu_0 iL}{2\pi}\int_x^{x+b}\frac{dy}{y} = \frac{\mu_0 iL}{2\pi}\left(\ell n(x+b) - \ell nx\right)$

Now $\frac{d\phi}{dt} = \frac{\mu_0 iL}{2\pi}\left[\frac{1}{x+b}\frac{dx}{dt} - \frac{1}{x}\frac{dx}{dt}\right]v = \frac{-\mu_0 ibLv}{2\pi x(x+b)}$

$\therefore \quad$ induced emf $= \frac{\mu_0 ibLv}{2\pi x(x+b)}$

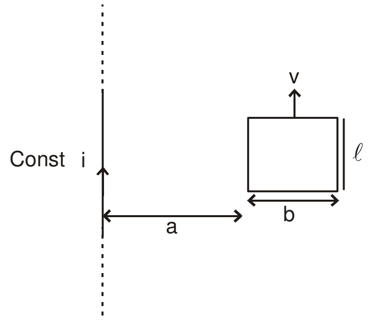
Example 10. A rod of length l is placed perpendicular to a long wire carrying current i. The rod is moved parallel to the wire with a velocity v. Find the emf induced in the rod, if its nearest end is at a distance 'a' from the wire.



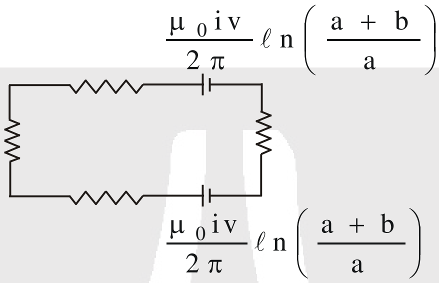
Solution : Consider a segment of rod of length dx , at a distance x from the wire. Emf induced in the segment

$dE = \frac{\mu_0 i}{2\pi x}dx.v \qquad \therefore \qquad E = \int_a^{a+\ell}\frac{\mu_0 ivdx}{2\pi x} = \frac{\mu_0 iv}{2\pi}\ell n\left(\frac{\ell + a}{a}\right)$

Example 11. A rectangular loop is moving parallel to a long wire carrying current i with a velocity v. Find the emf induced in the loop, if its nearest end is at a distance 'a' from the wire. Draw equivalent electrical diagram.



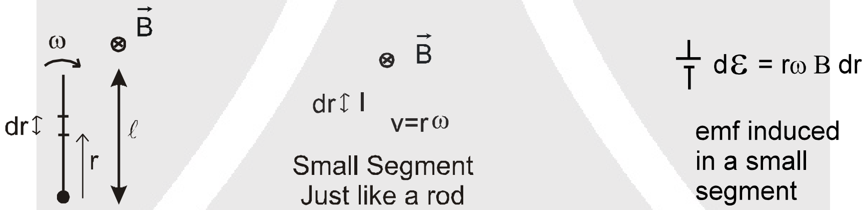
Solution : emf = 0 ;



4. INDUCED EMF DUE TO ROTATION

4.1 ROTATION OF THE ROD

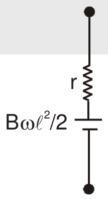
Consider a conducting rod of length l rotating in a uniform magnetic field.



Emf induced in a small segment of length dr, of the rod = v B dr = rω B dr

$\therefore$ emf induced in the rod = $\omega B\int_0^{\ell} r dr = \frac{1}{2}B\omega\ell^2$

equivalent of this rod is as following



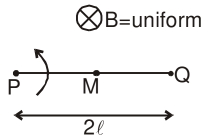
or $\varepsilon = \frac{d\Phi}{dt}$

$$\varepsilon = \frac{d\Phi}{dt} = \frac{\text{flux through the area swept by the rod in time dt}}{dt}$$

$$= \frac{B\frac{1}{2}\ell^2\omega dt}{dt} = \frac{1}{2}B\omega\ell^2$$

Solved Example

Example 12. A rod PQ of length 2ℓ is rotating about one end P in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Point M is the mid point of the rod. Find the induced emf between M & Q if that between P & Q = 100V .

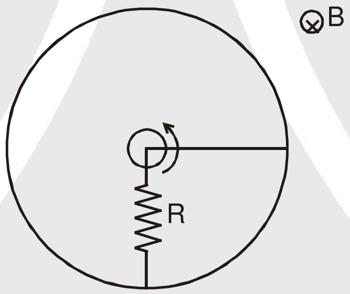


Solution : $E_{MQ} + E_{PM} = E_{PQ}$

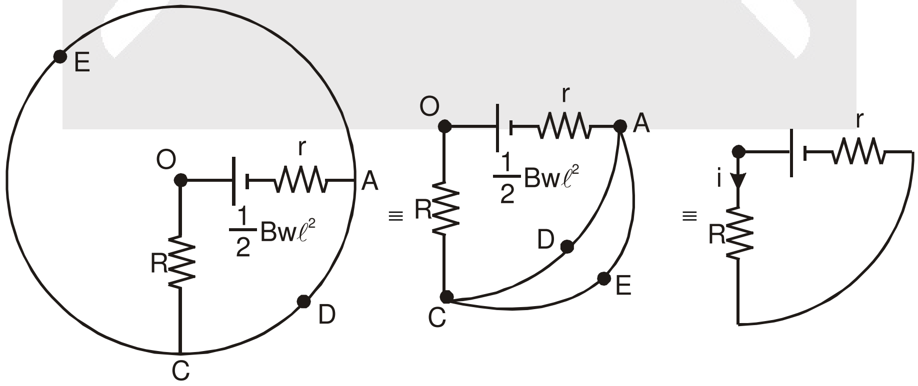
$$E_{PM} = \frac{Bw\ell^2}{2} = 100$$

$$E_{MQ} + \frac{B\omega\left(\frac{\ell}{2}\right)^2}{2} = \frac{B\omega\ell^2}{2} \quad \Rightarrow \quad E_{MQ} = \frac{3}{8}\,B\omega\ell^2 = \frac{3}{4} \times 100\text{ V} = 75\text{ V}$$

Example 13. A rod of length ℓ and resistance r rotates about one end as shown in figure. Its other end touches a conducting ring a of negligible resistance. A resistance R is connected between centre and periphery. Draw the electrical equivalence and find the current in the resistance R. There is a uniform magnetic field B directed as shown.

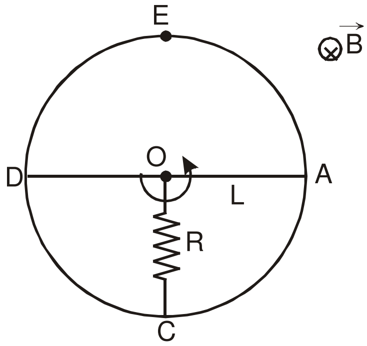


Solution :

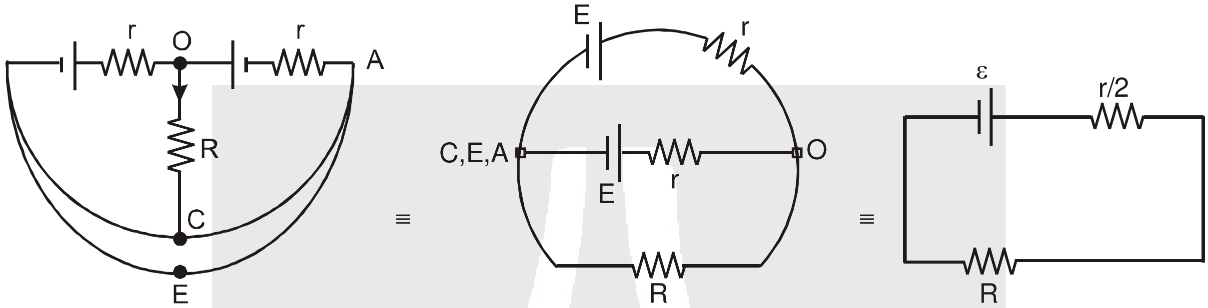


$$\text{current } i = \frac{\frac{1}{2}B\omega\ell^2}{R+r}$$

Example 14. Solve the above question if the length of rod is 2L and resistance 2r and it is rotating about its centre. Both ends of the rod now touch the conducting ring

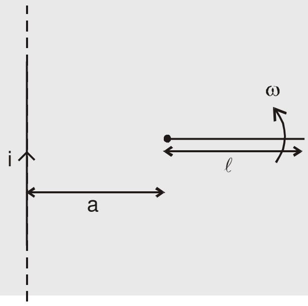


Solution :

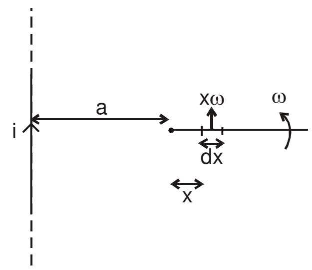


$$i = \frac{\varepsilon}{R + \frac{r}{2}} = \frac{\frac{1}{2}B\omega L^2}{R + \frac{r}{2}}$$

Example 15. A rod of length l is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.

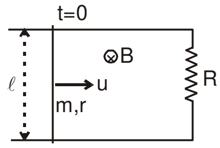


Solution : Consider a small segment of rod of length dx, at a distance x from one end of the rod. Emf induced in the segment

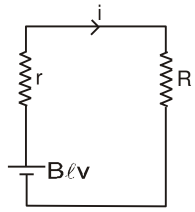


$$dE = \frac{\mu_0 i}{2\pi(x+a)}\ (x\omega)dx \qquad \therefore \qquad E = \int_0^{\ell} \frac{\mu_0 i}{2\pi(x+a)}\ (x\omega)dx = \frac{\mu_0 i\omega}{2\pi}\left[\ell - a.\ell n\left(\frac{\ell + a}{a}\right)\right].$$

Example 16. A rod of mass m and resistance r is placed on fixed, resistanceless, smooth conducting rails (closed by a resistance R) and it is projected with an initial velocity u .Find its velocity as a function of time.



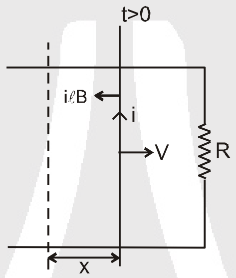
Solution : Let at an instant the velocity of the rod be v .The emf induced in the rod will be vBl. The electrically equivalent circuit is shown in the following diagram.



∴ Current in the circuit $i = \dfrac{B\ell v}{R+r}$

At time t

Magnetic force acting on the rod is F = iℓB, opposite to the motion of the rod.



$$i\ell B = -m\frac{dV}{dt} \qquad ...(1)$$

$$i = \frac{B\ell v}{R+r} \qquad ...(2)$$

Now solving these two equation

$$\frac{B^2\ell^2 V}{R+r} = -m \cdot \frac{dV}{dt}$$

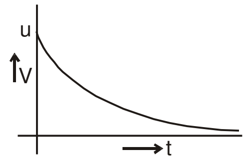
$$-\frac{B^2\ell^2}{(R+r)m} \cdot dt = \frac{dV}{V}$$

let $\dfrac{B^2\ell^2}{(R+r)m} = k$

$$-K \cdot dt = \frac{dV}{V}$$

$$\int_u^v \frac{dV}{V} = \int_0^t -K.dt$$

$V = ue^{-Kt}$



$$\ln\left(\frac{v}{u}\right) = -Kt$$

$V = ue^{-Kt}$

Example 17 In the above question find the force required to move the rod with constant velocity v, and also find the power delivered by the external agent .

Solution : The force needed to keep the velocity constant

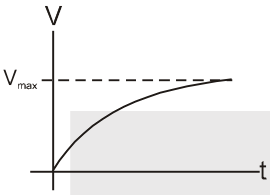
$$F_{ext} = i\ell B = \frac{B^2\ell^2 v}{R+r}$$

$$\text{Power due to external force} = \frac{B^2\ell^2 v^2}{R+r} = \frac{\varepsilon^2}{R+r} = i^2(R+r)$$

Note that the power delivered by the external agent is converted into joule heating in the circuit. That means magnetic field helps in converting the mechanical energy into joule heating.

Example 18 In the above question if a constant force F is applied on the rod. Find the velocity of the rod as a function of time assuming it started with zero initial velocity.

Solution :



$$m\frac{dv}{dt} = F - i\ell B \quad ....(1) \qquad i = \frac{B\ell v}{R+r} \quad ....(2)$$

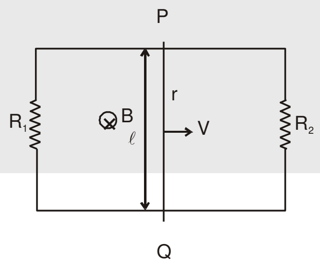
$$m\frac{dv}{dt} = F - \frac{B^2\ell^2 v}{R+r}$$

$$\text{let } K = \frac{B^2\ell^2}{R+r} \Rightarrow \int_0^v \frac{dV}{F-Kv} = \int_0^t \frac{dt}{m}$$

$$-\left[\ell n(F-KV)\right]_0^V = \frac{t}{m} \Rightarrow \ell n\left(\frac{F-kV}{F}\right) = -\frac{Kt}{m}$$

$$F - KV = F e^{-kt/m} \Rightarrow V = \frac{F}{K}(1 - e^{-kt/m})$$

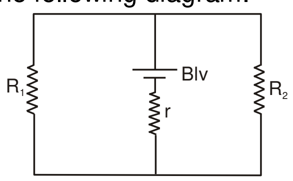
Example 19. A rod PQ of mass m and resistance r is moving on two fixed, resistanceless, smooth conducting rails (closed on both sides by resistances R_1 and R_2) .Find the current in the rod at the instant its velocity is v.



Solution :

$$i = \frac{B\ell V}{r + \frac{R_1R_2}{R_1+R_2}}$$

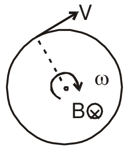
this circuit is equivalent to the following diagram.



4.2. EMF INDUCED DUE TO ROTATION OF A COIL

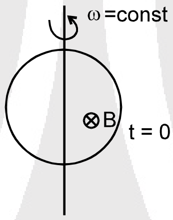
Solved Example

Example 20. A ring rotates with angular velocity ω about an axis perpendicular to the plane of the ring passing through the center of the ring. A constant magnetic field B exists parallel to the axis. Find the emf induced in the ring



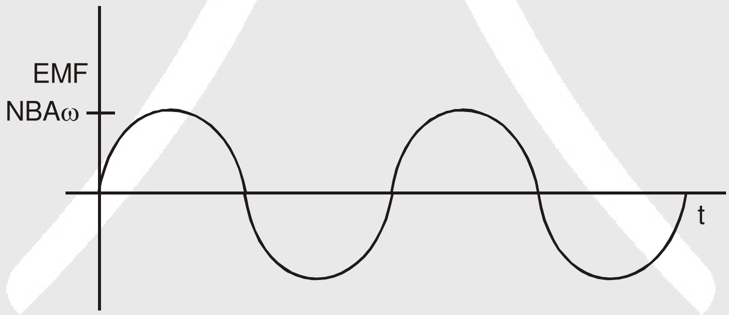
Solution : Flux passing through the ring ϕ = B.A is a constant here, therefore emf induced in the coil is zero. Every point of this ring is at the same potential, by symmetry.

Example 21. A ring rotates with angular velocity ω about an axis in the plane of the ring and which passes through the center of the ring. A constant magnetic field B exists perpendicular to the plane of the ring. Find the emf induced in the ring as a function of time.



Solution : At any time t , $\phi = BA \cos\theta = BA \cos \omega t$

Now induced emf in the loop



$e = \frac{-d\phi}{dt} = BA\,\omega \sin \omega t$

If there are N turns

emf = $BA\omega N \sin \omega t$

$BA\,\omega N$ is the amplitude of the emf

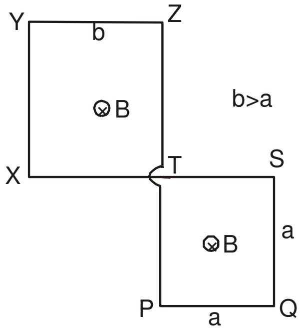
$e = e_m \sin \omega t$

$i = \frac{e}{R} = \frac{e_m}{R} \sin wt = i_m \sin wt$

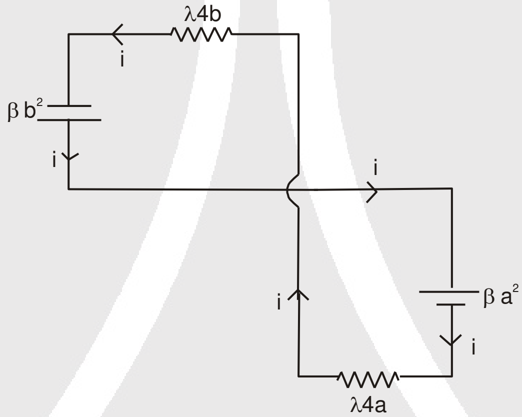
$i_m = \frac{e_m}{R}$

The rotating coil thus produces a sinusoidally varying current or alternating current. This is also the principle used in generator.

Example 22. Figure shows a wire frame PQSTXYZ placed in a time varying magnetic field given as $B=\beta t$, where β is a positive constant. Resistance per unit length of the wire is λ. Find the current induced in the wire and draw its electrical equivalent diagram.



Solution : Induced emf in part PQST = $\beta\ a^2$ (in anticlockwise direction, from Lenz's Law)
Similarly Induced emf in part TXYZ = $\beta\ b^2$ (in anticlockwise direction, from Lenz's Law)
Total resistance of the part PQST =$\lambda 4a$.
Total resistance of the part TXYZ = $\lambda 4b$. The equivalent circuit is as shown in the following diagram.



writing KVL along the current flow

$\beta b^2 - \beta a^2 - \lambda\ 4ai - \lambda\ 4bi = 0$

$i = \dfrac{\beta}{4\lambda}\ (b - a)$

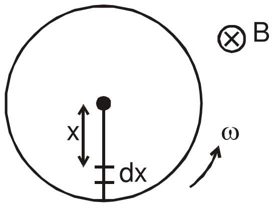
4.3 EMF INDUCED IN A ROTATING DISC :

Consider a disc of radius r rotating in a magnetic field B.
Consider an element dx at a distance x form the centre. This element is moving with speed $v = \omega x$.

$\therefore$ Induced emf across dx

$= B(dx)\ v = Bdx\omega x = B\omega xdx$

$\therefore$ emf between the centre and the edge of disc.



$= \int_0^r B\omega xdx = \dfrac{B\omega r^2}{2}$

5. FIXED LOOP IN A VARYING MAGNETIC FIELD

Now consider a circular loop, at rest in a varying magnetic field. Suppose the magnetic field is directed inside the page and it is increasing in magnitude. The emf induced in the loop will be

$\varepsilon = -\frac{d\phi}{dt}$. Flux through the coil will be $\phi = -\pi r^2 B$; $\frac{d\phi}{dt} = -\pi r^2 \frac{dB}{dt}$; $\varepsilon = -\frac{d\phi}{dt}$ $\therefore$ $\varepsilon = \pi r^2 \frac{dB}{dt}$

$\therefore \quad E\, 2\pi r = \pi r^2 \frac{dB}{dt}$ or $E = \frac{r}{2}\frac{dB}{dt}$

Thus changing magnetic field produces electric field which is non conservative in nature. The lines of force associated with this electric field are closed curves.

6. SELF INDUCTION

Self induction is induction of emf in a coil due to its own current change.Total flux $N\phi$ passing through a coil due to its own current is proportional to the current and is given as $N\phi = Li$ where L is called coefficient of self induction or inductance**.**The inductance L is purely a geometrical property i.e., we can tell the inductance value even if a coil is not connected in a circuit. Inductance depends on the shape and size of the loop and the number of turns it has.

If current in the coil changes by ΔI in a time interval Δt, the average emf induced in the coil is given as

$$\varepsilon = -\frac{\Delta(N\phi)}{\Delta t} = -\frac{\Delta(LI)}{\Delta t} = -\frac{L\Delta I}{\Delta t}.$$

The instantaneous emf is given as $\varepsilon = -\frac{d(N\phi)}{dt} = -\frac{d(LI)}{dt} = -\frac{LdI}{dt}$

S.I Unit of inductance is wb/amp or Henry(H)

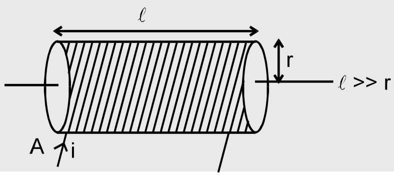
L - self inductance is +ve quantity .

L depends on : (1) Geometry of loop

(2) Medium in which it is kept. L does not depend upon current.

L is a scalar quantity.

6.1 SELF INDUCTANCE OF SOLENOID



Let the volume of the solenoid be V, the number of turns per unit length be n.

Let a current I be flowing in the solenoid.Magnetic field in the solenoid is given as $B=\mu_0 ni$.The magnetic flux through one turn of solenoid $\phi = \mu_0 niA$.

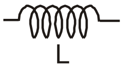
The total magnetic flux through the solenoid = $N\phi = N\mu_0 ni\, A = \mu_0 n^2 i\, A\, \ell$

$\therefore \quad L = \mu_0 n^2 \ell A = \mu_0 n^2 V \qquad \Rightarrow \qquad \phi = \mu_0\, n\, i\, \pi r^2\, (n\ell) \qquad \Rightarrow \qquad L = \frac{\phi}{i} = \mu_0\, n^2\, \pi r^2 \ell.$

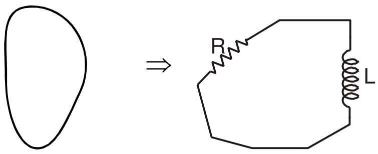
Inductance per unit volume = $\mu_0 n^2$.

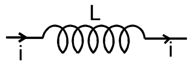
Self inductance is the physical property of the loop due to which it opposes the change in current that means it tries to keep the current constant.Current can not change suddenly in the inductor.

7. INDUCTOR :

It is represent by 

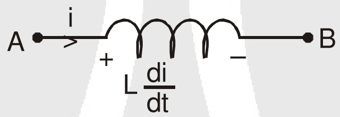
electrical equivalence of loop





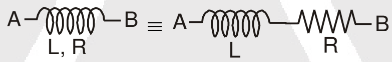
If current i through the inductor is increasing the induced emf will oppose the **increase** in current and hence will be opposite to the current.If current i through the inductor is decreasing the induced emf will oppose the **decrease** in current and hence will be in the direction of the current.

Over all result



$$V_A - L\frac{di}{dt} = V_B$$

Note : If there is a resistance in the inductor (resistance of the coil of inductor) then :



Solved Example

Example 23. A B is a part of circuit. Find the potential difference $v_A - v_B$ if

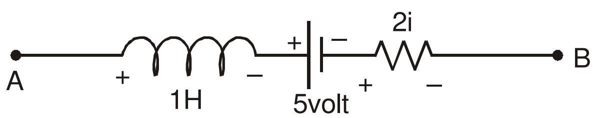
(i) current i = 2A and is constant

(ii) current i = 2A and is increasing at the rate of 1 amp/sec.

(iii) current i = 2A and is decreasing at the rate 1 amp/sec.

Solution : $L\frac{di}{dt} = 1\frac{di}{dt}$

writing KVL from A to B



$$V_A - 1\frac{di}{dt} - 5 - 2i = V_B.$$

(i) Put i = 2, $\frac{di}{dt} = 0$

$V_A - 5 - 4 = V_B$ $\quad\therefore\quad$ $V_A - V_B = 9$ volt

(ii) Put i = 2, $\frac{di}{dt}$ = 1 ; $V_A - 1 - 5 - 4 = V_B$ or $V_A - V_B = 10\ V_0$

(iii) Put i = 2 , $\frac{di}{dt}$ = – 1 ; $V_A + 1 - 5 - 2 \times 2 = V_B$ or V_A = 8 volt.

7.1 ENERGY STORED IN AN INDUCTOR:

If current in an inductor at an instant is i and is increasing at the rate di/dt, the induced emf will oppose the current. Its behaviour is shown in the figure.

Power consumed by the inductor = $i\,L\frac{di}{dt}$

Energy consumed in dt time = $i\,L\frac{di}{dt}dt$

∴ total energy consumed as the current increases from 0 to I = $\int_0^I iLdi = \frac{1}{2}LI^2 = \frac{1}{2}Li^2 \Rightarrow U = \frac{1}{2}LI^2$

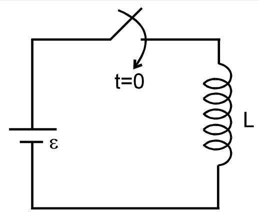
Note : This energy is stored in the magnetic field with energy density

$$\frac{dU}{dV} = \frac{B^2}{2\mu} = \frac{B^2}{2\mu_0\mu_r}$$

Total energy U = $\int \frac{B^2}{2\mu_0\mu_r}dV$

Solved Example

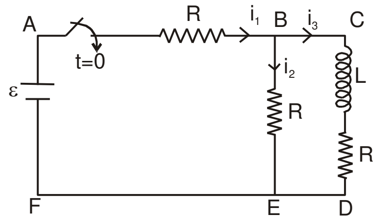
Example 24. A circuit contains an ideal cell and an inductor with a switch. Initially the switch is open .It is closed at t = 0.Find the current as a function of time.



Solution : $\varepsilon = L\frac{di}{dt} \Rightarrow \int_0^i \varepsilon dt = \int_0^i Ldi$

$\varepsilon t = Li \Rightarrow i = \frac{\varepsilon t}{L}$

Example 25. In the following circuit, the switch is closed at t = 0. Find the currents i_1, i_2, i_3 and $\frac{di_3}{dt}$ at t = 0 and at t = ∞. Initially all currents are zero.



Solution : At t = 0

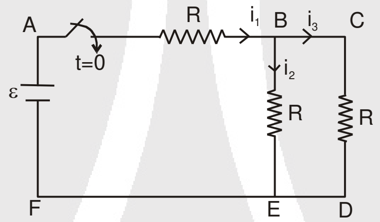
i_3 is zero ,since current cannot suddenly change due to the inductor.

∴ $i_1 = i_2$ (from KCL)

applying KVL in the part ABEF we get $i_1 = i_2 = \frac{\varepsilon}{2R}$.

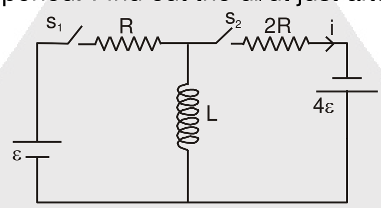
At t = ∞

i_3 will become constant and hence potential difference across the inductor will be zero.It is just like a simple wire and the circuit can be solved assuming it to be like shown in the following diagram.

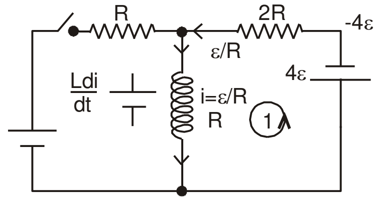


$i_2 = i_3 = \frac{\varepsilon}{3R}$, $i_1 = \frac{2\varepsilon}{3R}$.

Example 26. In the circuit shown in the figure, S_1 remains closed for a long time and S_2 remains open. Now S_2 is closed and S_1 is opened. Find out the di/dt just after that moment.



Solution : Before S_2 is closed and S_1 is opened current in the left part of the circuit = $\frac{\varepsilon}{R}$. Now when S_2 closed S_1 opened, current through the inductor can not change suddenly, current $\frac{\varepsilon}{R}$ will continue to move in the inductor.



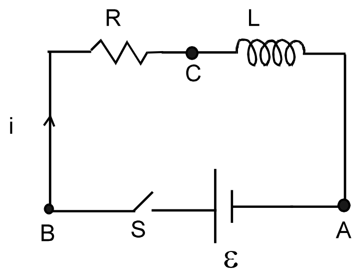
Applying KVL in loop 1.

$$L\frac{di}{dt} + \frac{\varepsilon}{R}(2R) + 4\varepsilon = 0$$

$$\frac{di}{dt} = -\frac{6\varepsilon}{L}$$

7.2 GROWTH OF CURRENT IN SERIES R–L CIRCUIT :

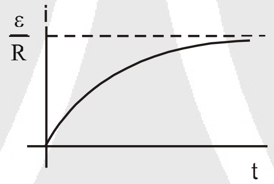
Figure shows a circuit consisting of a cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t=0.Suppose at an instant current in the circuit be i which is increasing at the rate di/dt.



Writing KVL along the circuit, we have $\varepsilon - L\frac{di}{dt} - iR = 0$

On solving we get, $i = \frac{\varepsilon}{R}(1 - e^{\frac{-Rt}{L}})$

The quantity L/R is called time constant of the circuit and is denoted by τ.The variation of current with time is as shown.

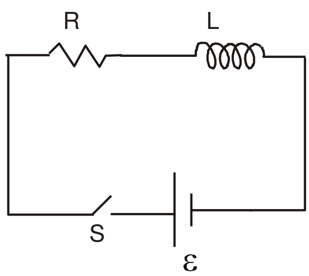


Note : 1. Final current in the circuit = $\frac{\varepsilon}{R}$, which is independent of L.

2. After one time constant, current in the circuit = 63% of the final current (verify yourself)
3. More time constant in the circuit implies slower rate of change of current.
4. If there is any change in the circuit containing inductor then there is no instantaneous effect on the flux of inductor.
   $L_1 i_1 = L_2 i_2$

Solved Examples

Example 27. At t = 0 switch is closed (shown in figure) after a long time suddenly the inductance of the inductor is made η times lesser ($\frac{L}{\eta}$) then its initial value, find out instant current just after the operation.

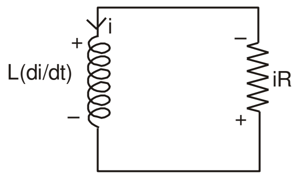


Solution : Using above result (note 4)

$L_1 i_1 = L_2 i_2 \Rightarrow i_2 = \dfrac{\eta\varepsilon}{R}$

DECAY OF CURRENT IN THE CIRCUIT CONTAINING RESISTOR AND INDUCTOR:

Let the initial current in the circuit be I_0. At any time t, let the current be i and let its rate of change at this instant be $\dfrac{di}{dt}$.



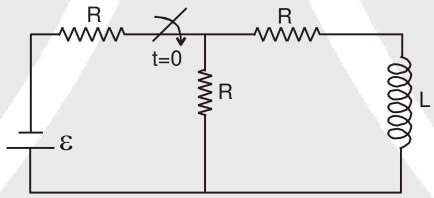
$L.\dfrac{di}{dt} + iR = 0, \qquad \dfrac{di}{dt} = -\dfrac{iR}{L}$

$\int_{I_0}^{i} \dfrac{di}{i} = -\int_0^t \dfrac{R}{L}.dt \Rightarrow \ln\left(\dfrac{i}{I_0}\right) = -\dfrac{Rt}{L} \qquad \text{or} \qquad i = I_0 e^{\frac{-Rt}{L}}$

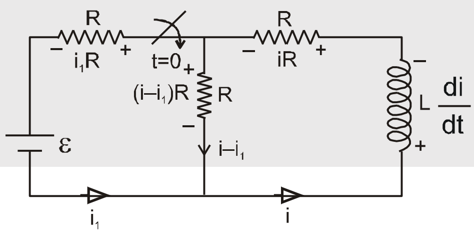
Current after one time constant : i = I_0 = 0.37% of initial current.

Solved Examples

Example 28 In the following circuit the switch is closed at t = 0. Intially there is no current in inductor. Find out current the inductor coil as a function of time.



Solution :



At any time t

$-\varepsilon + i_1R - (i - i_1)R = 0$

$-\varepsilon + 2i_1R - iR = 0$

$i_1 = \dfrac{iR + \varepsilon}{2R}$ Now, $-\varepsilon + i_1R + iR + L.\dfrac{di}{dt} = 0$

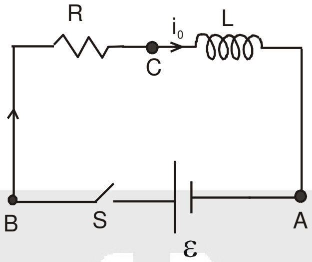
$-\varepsilon + \left(\dfrac{iR+\varepsilon}{2}\right) + iR + L.\dfrac{di}{dt} = 0 \Rightarrow -\dfrac{\varepsilon}{2} + \dfrac{3iR}{2} = -L.\dfrac{di}{dt}$

$\left(\dfrac{-\varepsilon + 3iR}{2}\right)dt = -L.di \Rightarrow -\dfrac{-dt}{2L} = \dfrac{di}{-\varepsilon + 3iR}$

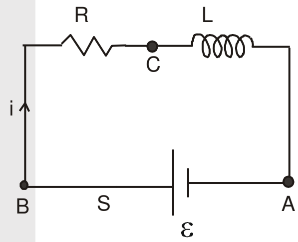
$$-\int_0^t \frac{dt}{2L} = \int_0^i \frac{di}{-\varepsilon + 3iR} \Rightarrow -\frac{t}{2L} = \frac{1}{3R}\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right)$$

$$-\ln\left(\frac{-\varepsilon + 3iR}{-\varepsilon}\right) = \frac{3Rt}{2L} \Rightarrow i = +\frac{\varepsilon}{3R}\left(1 - e^{-\frac{3Rt}{2L}}\right)$$

Example 29. Figure shows a circuit consisting of a ideal cell, an inductor L and a resistor R, connected in series. Let the switch S be closed at t = 0. Suppose at t = 0 current in the inductor is i_0 then find out equation of current as a function of time



Solution : Let an instant t current in the circuit is i which is increasing at the rate di/dt.

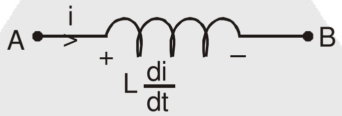


Writing KVL along the circuit , we have $\varepsilon - L\frac{di}{dt} - iR = 0$

$$\Rightarrow L\frac{di}{dt} = \varepsilon - iR \Rightarrow \int_{i_0}^{i} \frac{di}{\varepsilon - iR} = \int_0^t \frac{dt}{L}$$

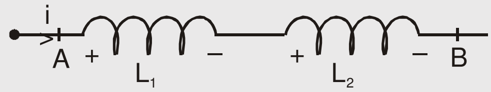
$$\Rightarrow \ln\left(\frac{\varepsilon - iR}{\varepsilon - i_0R}\right) = -\frac{Rt}{L} \Rightarrow \varepsilon - iR = (\varepsilon - i_0R)\, e^{-Rt/L} \Rightarrow i = \frac{\varepsilon - (\varepsilon - i_0R)e^{-Rt/L}}{R}$$

Equivalent self inductance :



$$L = \frac{V_A - V_B}{di/dt} \qquad ..(1)$$

Series combination

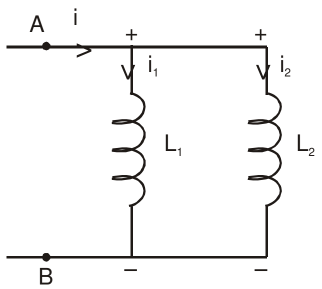


$$V_A - L_1\frac{di}{dt} - L_2\frac{di}{dt} = V_B \qquad ....(2)$$

from (1) and (2)

$L = L_1 + L_2$ (neglecting mutual inductance)

Parallel Combination :



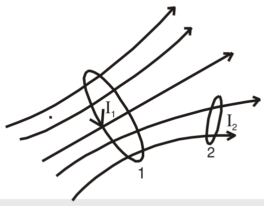
From figure $V_A - V_B = L_1 \frac{di_1}{dt} = L_2 \frac{di_2}{dt}$ ..... (3)

also $i = i_1 + i_2$

or $\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$ or $\frac{V_A - V_B}{L} = \frac{V_A - V_B}{L_1} + \frac{V_A - V_B}{L_2}$

$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ (Neglecting mutual inductance)

8. MUTUAL INDUCTANCE



Consider two arbitrary conducting loops 1 and 2. Suppose that I_1 is the instantaneous current flowing around loop 1. This current generates a magnetic field $\mathbf{B_1}$ which links the second circuit, giving rise to a magnetic flux ϕ_2 through that circuit. If the current I_1doubles, then the magnetic field $\mathbf{B_1}$ doubles in strength at all points in space, so the magnetic flux ϕ_2 through the second circuit also doubles. Furthermore, it is obvious that the flux through the second circuit is zero whenever the current flowing around the first circuit is zero. It follows that the flux ϕ_2 through the second circuit is directly proportional to the current I_1 flowing around the first circuit. Hence, we can write $\phi_2=M_{21}I_1$ where the constant of proportionality M_{21} is called the mutual inductance of circuit 2 with respect to circuit 1. Similarly, the flux ϕ_2 through the first circuit due to the instantaneous current I_2 flowing around the second circuit is directly proportional to that current, so we can write $\phi_1=M_{12}I_2$ where $M_{12}$is the mutual inductance of circuit 1 with respect to circuit 2. It can be shown that **$M_{21}= M_{12}$ (Reciprocity Theorem)**. Note that M is a purely geometric quantity, depending only on the size, number of turns, relative position, and relative orientation of the two circuits. The S.I. unit of mutual inductance is called Henry (H). One Henry is equivalent to a volt-second per ampere.

Suppose that the current flowing around circuit 1 changes by an amount ΔI_1 in a small time interval Δt. The flux linking circuit 2 changes by an amount $\Delta\phi_2=M\Delta I_1$in the same time interval. According to Faraday's law, an emf $\varepsilon_2 = -\frac{\Delta\phi_2}{\Delta t}$ is generated around the second circuit due to the changing magnetic flux linking that circuit. Since, $\Delta\phi_2 = M\Delta I_1$, this emf can also be written $\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t}$.

Thus, the emf generated around the second circuit due to the current flowing around the first circuit is directly proportional to the rate at which that current changes. Likewise, if the current I_2 flowing around the second circuit changes by an amount ΔI_1in a time interval Δt then the emf generated around the first circuit is $\varepsilon_1 = -M\frac{\Delta I_2}{\Delta t}$ Note that there is no direct physical connection(coupling) between the two circuits: the coupling is due entirely to the magnetic field generated by the currents flowing around the circuits.

Note : (1) $M \le \sqrt{L_1 L_2}$

(2) For two coils in series if mutual inductance is considered then
$L_{eq} = L_1 + L_2 \pm 2M$

Solved Example

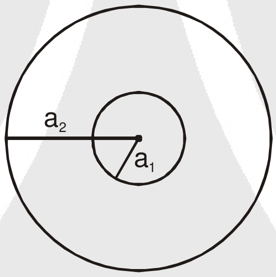
Example 30. Two insulated wires are wound on the same hollow cylinder, so as to form two solenoids sharing a common air-filled core. Let I be the length of the core, A the cross-sectional area of the core, N_1 the number of times the first wire is wound around the core, and N_2 the number of turns the second wire is wound around the core. Find the mutual inductance of the two solenoids, neglecting the end effects.

Solution : If a current I_1 flows around the first wire then a uniform axial magnetic field of strength $B_1 = \frac{\mu_0 N_1 I_1}{\ell}$ is generated in the core. The magnetic field in the region outside the core is of negligible magnitude. The flux linking a single turn of the second wire is B_1A. Thus, the flux linking all N_2 turns of the second wire is

$$\phi_2 = N_2 B_1 A = \frac{\mu_0 N_1 N_2 A I_1}{\ell} = MI_1 \qquad \therefore \qquad M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

As described previously, M is a geometric quantity depending on the dimensions of the core and the manner in which the two wires are wound around the core, but not on the actual currents flowing through the wires.

Example 31. Find the mutual inductance of two concentric coils of radii a_1 and a_2 ($a_1 << a_2$) if the planes of coils are same.



Solution : Let a current i flow in coil of radius a_2.

Magnetic field at the centre of coil $= \frac{\mu_0 i}{2a_2} \pi a_1^2$

or $\quad Mi = \frac{\mu_0 i}{2a_2} \pi a_1^2 \qquad$ or $\qquad M = \frac{\mu_0 \pi a_1^2}{2a_2}$

Example 32. Solve the above question, if the planes of coil are perpendicular.

Solution : Let a current i flow in the coil of radius a_1. The magnetic field at the centre of this coil will now be parallel to the plane of smaller coil and hence no flux will pass through it, hence M = 0.

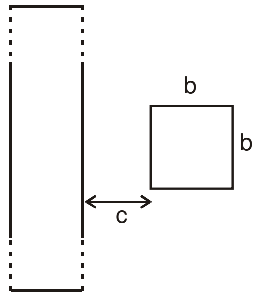
Example 33. Solve the above problem if the planes of coils make θ angle with each other.

Solution : If i current flows in the larger coil, magnetic field produced at the centre will be perpendicular to the plane of larger coil.

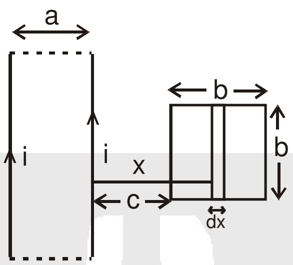
Now the area vector of smaller coil which is perpendicular to the plane of smaller coil will make an angle θ with the magnetic field.

Thus flux $= \vec{B}.\vec{A} = \frac{\mu_0 i}{2a_2}.\pi a_1^2.\cos\theta \qquad$ or $\qquad M = \frac{\mu_0 \pi a_1^2 \cos\theta_1}{2a_2}$

Example 34. Find the mutual inductance between two rectangular loops, shown in the figure



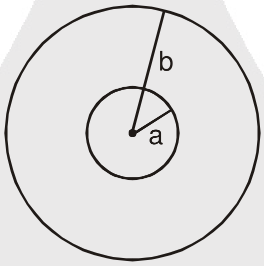
Solution :



Let current i flow in the loop having ∞-by long sides. Consider a segment of width dx at a distance x as shown flux through the regent

$$d\phi = \left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx \Rightarrow \quad \phi = \int_c^{c+b}\left[\frac{\mu_0 i}{2\pi x} - \frac{\mu_0 i}{2\pi(x+a)}\right] b\,dx = \frac{\mu_0 i b}{2\pi}\left[\ell n\frac{c+b}{c} - \ell n\frac{a+b+c}{a+c}\right].$$

Example 35. Figure shows two concentric coplanar coils with radii a and b (a << b). A current i = 2t flows in the smaller loop. Neglecting self inductance of larger loop



(a) Find the mutual inductance of the two coils

(b) Find the emf induced in the larger coil

(c) If the resistance of the larger loop is R find the current in it as a function of time

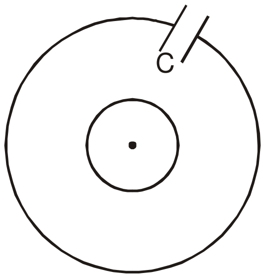
Solution : (a) To find mutual inductance, it does not matter in which coil we consider current and in which flux is calculated (Reciprocity theorem) Let current i be flowing in the larger coil. Magnetic field at the centre = $\frac{\mu_0 i}{2b}$.

flux through the smaller coil = $\frac{\mu_0 i}{2b}\pi a^2$ $\quad\therefore\quad$ $M = \frac{\mu_0}{2b}\pi a^2$

(b) $\left|\text{emf induced in larger coil}\right| = M\left[\left(\frac{di}{dt}\right)\text{ in smaller coil}\right] = \frac{\mu_0}{2b}\pi a^2\ (2) = \frac{\mu_0 \pi a^2}{b}$

(c) current in the larger coil $= \frac{\mu_0 \pi a^2}{b\ \ R}$.

Example 36. If the current in the inner loop changes according to i = $2t^2$ then, find the current in the capacitor as a function of time.



Solution : $M = \frac{\mu_0}{2b}\pi a^2$

$|\text{emf induced in larger coil}| = M\left[\left(\frac{di}{dt}\right) \text{in smaller coil}\right] \Rightarrow e = \frac{\mu_0}{2b}\pi a^2 \ (4t) = \frac{2\mu_0 \pi a^2 t}{b}$

Applying KVL :-

$+e - \frac{q}{c} - iR = 0$

$\frac{2\mu_0 \pi a^2 t}{b} - \frac{q}{c} - iR = 0$

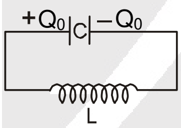
differentiate wrt time :-

$\frac{2\mu_0 \pi a^2}{b} - \frac{i}{c} - \frac{di}{dt}R = 0$

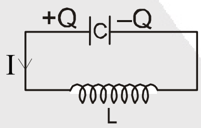
on solving it

$i = \frac{2\mu_0 \pi a^2 C}{b}\left[1 - e^{-t/RC}\right]$

9. LC OSCILLATIONS



At t = 0



At t = t

When capacitor C is completely charged upto Q_0 and connected to an inductor L at t = 0 then at t = t

$L\frac{dI}{dt} - \frac{Q}{C} = 0, \qquad -L\frac{d^2Q}{dt^2} - \frac{Q}{C} = 0, \qquad Q = -LC\frac{d^2Q}{dt^2}$

$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0$ therefore charge Q oscillates with $Q = Q_0 \cos \omega t$

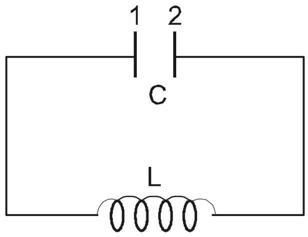
Hence initial phase of oscillation is $\frac{\pi}{2}$ and angular frequency $\omega = \frac{1}{\sqrt{LC}}$

One can prove that the energy in the system remains conserved.

Therefore $\frac{Q_0^2}{2C} + 0 = \frac{Q^2}{2C} + \frac{1}{2}LI^2 = \frac{1}{2}LI_0^2 + 0$

Solved Examples

Example 37. Consider a L – C oscillation circuit. Circuit elements has zero resistance. Initially at t = 0 all the energy is stored in the form of electric field and plate-1 is having positive charge :



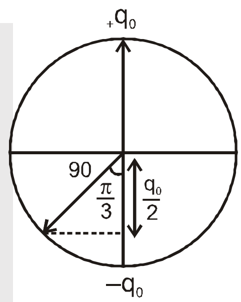
at time $t = t_1$ plate-2 attains half of the maximum +ve change for the first time. Value of t_1 is :

(A) $\frac{2\pi}{3}\sqrt{LC}$ (B) $\frac{\pi}{3}\sqrt{LC}$ (C) $\frac{4\pi}{3}\sqrt{LC}$ (D) $\pi\sqrt{LC}$

Solution : $q_1 = q_0 \sin(\omega t + \pi/2)$

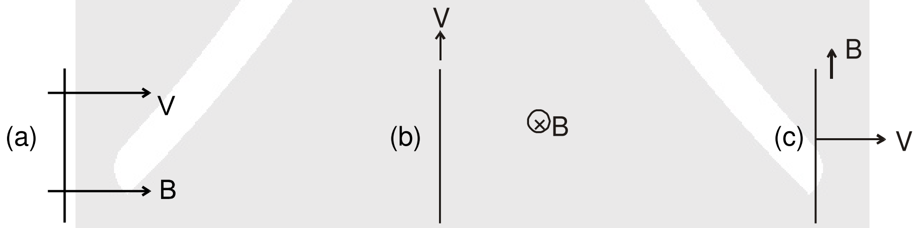
at $t = t_1$ $q_1 = -\frac{q_0}{2}$

$$t_1 = \frac{\pi - \frac{\pi}{3}}{\omega} = \frac{2\pi}{3\omega} = \frac{2\pi}{3}\sqrt{LC}$$



Solved Miscellaneous Problems

Problem 1. Find the emf induced in the rod in the following cases. The figures are self explanatory.

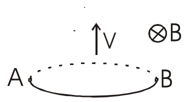


Solution : (a) here $\vec{v} \parallel \vec{B}$ so $\vec{v} \times \vec{B} = 0$ emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(b) here $\vec{v} \parallel \vec{\ell}$ so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

(c) here $\vec{B} \parallel \vec{\ell}$ so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

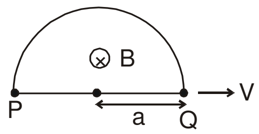
Problem 2. A circular coil of radius R is moving in a magnetic field **B** with a velocity **v** as shown in the figure.



Find the emf across the diametrically opposite points A and B.

Solution : emf = $BV l_{effective}$
= 2RvB

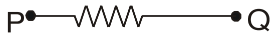
Problem 3. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown.Also draw the electrical equivalent circuit of each branch.

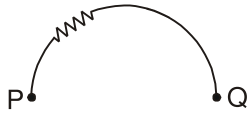


Solution : Here $\vec{v} \parallel \vec{\ell}$

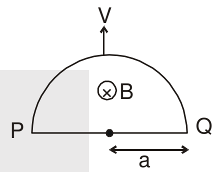
so emf = $\vec{\ell} \cdot (\vec{v} \times \vec{B}) = 0$

Induced emf = 0

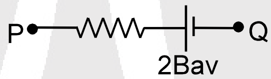


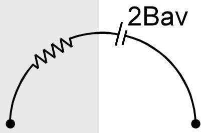


Problem 4. Find the emf across the points P and Q which are diametrically opposite points of a semicircular closed loop moving in a magnetic field as shown. Also draw the electrical equivalence of each branch.

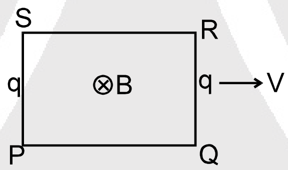


Solution : Induced emf = 2Bav

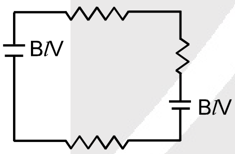




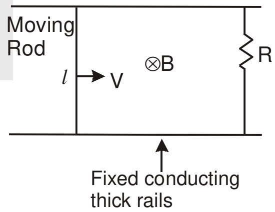
Problem 5. Figure shows a rectangular loop moving in a uniform magnetic field .Show the electrical equivalence of each branch.



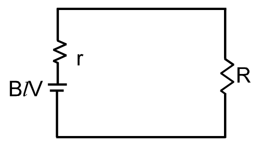
Solution :



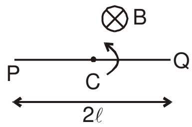
Problem 6. Figure shows a rod of length l and resistance r moving on two rails shorted by a resistance R.A uniform magnetic field B is present normal to the plane of rod and rails .Show the electrical equivalence of each branch.



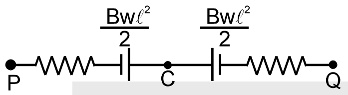
Solution :



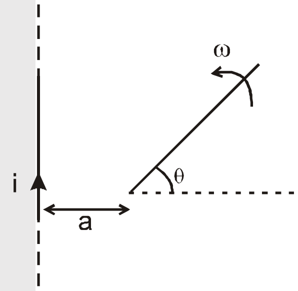
Problem 7. A rod PQ of length 2ℓ is rotating about its mid point C, in a uniform magnetic field B which is perpendicular to the plane of rotation of the rod. Find the induced emf between P Q and PC. Draw the circuit diagram of parts PC and CQ.



Solution : $emf_{PQ} = 0$; $emf_{PC} = \dfrac{B\omega\ell^2}{2}$



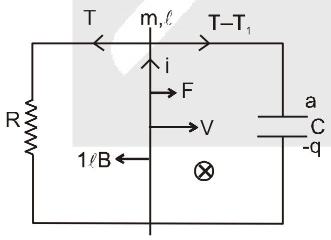
Problem 8. A rod of length ℓ is rotating with an angular speed ω about its one end which is at a distance 'a' from an infinitely long wire carrying current i. Find the emf induced in the rod at the instant shown in the figure.



Solution : $E = \displaystyle\int \frac{\mu_0 i}{2\pi\ (a + r\cos\theta)} \times (r\omega)\ .\ (dr)$

$$E = \frac{\mu_0 \omega i}{2\pi}\int_0^{\ell}\frac{r}{a + r\cos\theta}\,dr \Rightarrow E = \frac{\mu_0 i\omega}{2\pi\cos\theta}\left[\ell - \frac{a}{\cos\theta}\,\ell n\left(\frac{a + \ell\cos\theta}{a}\right)\right]$$

Problem 9.



,

Find the velocity of the moving rod at time t if the initial velocity of the rod is zero and a constant force F is applied on the rod. Neglect the resistance of the rod.

Solution : At any time t, let the velocity of the rod be v .

Applying Newtons law: $F - ilB = ma$ ... (1)

Also $B\,l\,v = i_1R = \frac{q}{c}$

Applying Kcl, $i = i_1 + \frac{dq}{dt} = \frac{B\ell V}{R} + \frac{d}{dt}(BlvC)$ or $i = \frac{B\ell V}{R} + B\ell C\,a$

Putting the value of i in eq (1), $F - \frac{B^2\ell^2V}{R} = (m + B^2\ell^2C)a = (m + B^2\ell^2C)\frac{dv}{dt}$

$$(m + B^2\ell^2C)\frac{dv}{F - \frac{B^2l^2v}{R}} = dt$$

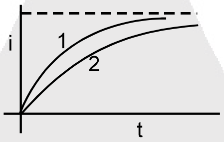
Integrating both sides, and solving we get $v = \frac{FR}{B^2\ell^2}\left(1 - e^{-\frac{tB^2\ell^2}{R(m+CB^2\ell^2)}}\right)$

Problem 10. A rod PQ of length ℓ is rotating about end P, with an angular velocity ω. Due to centrifugal forces the free electrons in the rod move towards the end Q and an emf is created. Find the induced emf.

Solution : The accumulation of free electrons will create an electric field which will finally balance the centrifugal forces and a steady state will be reached. In the steady state $m_e\omega^2x = e\,E$.

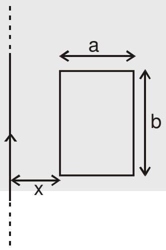
$$V_P - V_Q = \int_{x=0}^{x=\ell} \bar{E}.d\bar{x} = \int_0^{\ell} \frac{m_e\omega^2x}{e}dx = \frac{m_e\omega^2\ell^2}{2e}$$

Problem 11. Which of the two curves shown has less time constant.



Solution : Curve1

Problem 12. Find the mutual inductance of a straight long wire and a rectangular loop, as shown in the figure



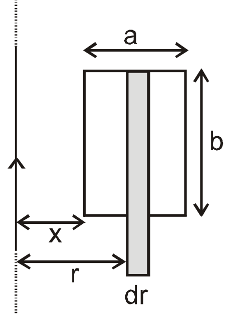
Solution :

$$d\phi = \frac{\mu_0 i}{2\pi r} \times bdr$$

$$\phi = \int_x^{x+a} \frac{\mu_0 i}{2\pi r} \times bdr$$

$$M = \phi/i$$

$$M = \frac{\mu_0 b}{2\pi}\ln\left(1 + \frac{a}{x}\right)$$



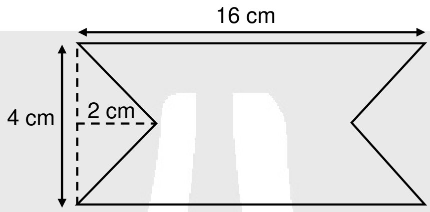
3. Consider a circular coil of wire carrying constant current **I**, forming a magnetic dipole. The magnetic flux through an infinite plane that contains the circular coil and excluding the circular coil area is given by ϕ_i. The magnetic flux through the area of the circular coil area is given by ϕ_0. Which of the following option is correct ?

(1) $\phi_i > \phi_0$ (2) $\phi_i < \phi_0$ (3*) $\phi_i = -\phi_0$ (4) $\phi_i = \phi_0$

Sol. As magnetic field lines always form a closed loop, hence every magnetic field line creating magnetic flux in the inner region must be passing through the outer region. Since flux in two regions are in opposite direction,

$\therefore \quad \phi_i = -\phi_0$

8. At time t = 0 magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is:



(1) 28 μV (2) 36 μV (3) 48 μV (4*) 56 μV

Sol. $\varepsilon = \left|-\frac{d\varphi}{dt}\right| = \left|-\frac{AdB}{dt}\right|$

$= .(16 \times 4 - 4 \times 2)\frac{(1000-500)}{5} \times 10^{-4} \times 10^{-4}$

$= 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6}$ V

25. In a fluorescent lamp choke (a small transformer) 100 V of reverse voltage is produced when the choke current changes uniformly from 0.25 A to 0 in a duration of 0.025 ms. The self–inductance of the choke (in mH) is estimated to be ....

Ans. **10**

Sol. $100 = \frac{L(0.25)}{0.025} \times 10^3$

$\therefore \quad L = 100 \times 10^{-4}$ H

$= 10$ mH

Exercise-1

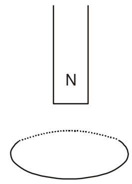
✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Flux and Faraday's laws of electromagnetic induction

A-1. If flux in a coil changes by $\Delta\phi$, and the resistance of the coil is R, prove that the charge flown in the coil during the flux change is $\Delta\phi/R$. (**Note :** It is independent of the time taken for the change in flux)

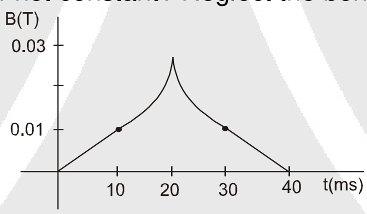
A-2. The north pole of a magnet is brought down along the axis of a horizontal circular coil (figure). As a result the flux through the coil changes from 0.4 Weber to 0.9 Weber in an interval of half of a second. Find the average emf induced during this period. Is the induced current clockwise or anticlockwise as you look into the coil from the side of the magnet?



A-3.✍ The flux of magnetic field through a closed conducting loop of resistance 0.4 Ω changes with time according to the equation $\Phi = 0.20t^2 + 0.40t + 0.60$ where t is time in seconds. Find (i) the induced emf at t = 2s. (ii) the average induced emf in t = 0 to t = 5 s. (iii) charge passed through the loop in t = 0 to t = 5s (iv) average current in time interval t = 0 to t = 5 s (v) heat produced in t = 0 to t = 5s.

(i) 1.2 Volt (ii) 1.4 volt (iii) 17.5 C (iv) 3.5 A
(v) 86/3 joule.

A-4. (a) The magnetic field in a region varies as shown in figure. Calculate the average induced emf in a conducting loop of area 10^{-3} m^2 placed perpendicular to the field in each of the 10 ms intervals shown.
(b) In which interval(s) is the emf not constant? Neglect the behavior near the ends of 10 ms intervals.



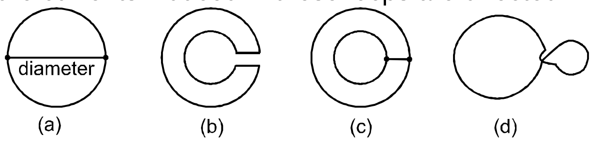
A-5. A conducting loop confined in a plane is rotated in its own plane with some angular velocity. A uniform and constant magnetic field exist in the region. Find the current induced in the loop.

A-6. A metallic ring of area 25 cm^2 is placed perpendicular to a magnetic field of 0.2T. It is removed from the field in 0.2 s. Find the average emf produced in the ring during this time.

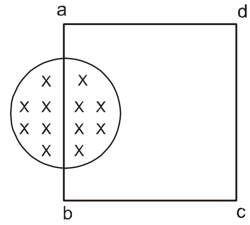
A-7.✍ A solenoid has a cross sectional area of 6.0×10^{-4} m^2, consists of 400 turns per meter, and carries a current of 0.40 A. A 10 turn coil is wrapped tightly around the circumference of the solenoid. The ends of the coil are connected to a 1.5 Ω resistor. Suddenly, a switch is opened, and the current in the solenoid dies to zero in a time 0.050 s. Find the average current in the coil during this time.

A-8. A heart pacing device consists of a coil of 50 turns & radius 1 mm just inside the body with a coil of 1000 turns & radius 2 cm placed concentrically and coaxially just outside the body. Calculate the average induced EMF in the internal coil, if a current of 1A in the external coil collapses in 10 milliseconds.

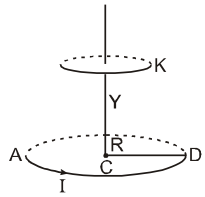
A-9. Figure illustrates plane figures made of thin conductors which are located in a uniform magnetic field directed away from a reader beyond the plane of the drawing. The magnetic induction starts diminishing. Find how the currents induced in these loops are directed.



A-10. A uniform magnetic field B exists in a cylindrical region of radius 1 cm as shown in figure. A uniform wire of length 16 cm and resistance 4.0 Ω is bent into a square frame and is placed with one side along a diameters of the cylindrical region. If the magnetic field increases at a constant rate of 1 T/s find the current induced in the frame.

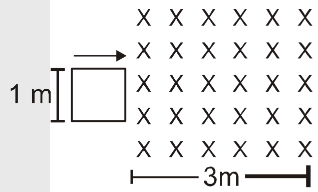


A-11. A coil ACD of N turns & radius R carries a current of I Amp & is placed on a horizontal table. K is a very small horizontal conducting ring of radius r placed at a distance Y_0 from the centre of the coil vertically above the coil ACD. Find an expression for the EMF established when the ring K is allowed to fall freely. Express the EMF in terms of instantaneous speed v & height Y.



A-12. A closed circular loop of 200 turns of mean diameter 50 cm & having a total resistance of 10 Ω is placed with its plane at right angle to a magnetic field of strength 10^{-2} Tesla. Calculate the quantity of electric charge passed through it when the coil is turned through 180º about an axis in its plane.

A-13. Figure shows a square loop of resistance 1 Ω of side 1 m being moved towards right at a constant speed of 1 m/s. The front edge enters the 3 m wide magnetic field (B = 1 T) at t = 0. Draw the graph of current induced in the loop as time passes. (Take anticlockwise direction of current as positive)



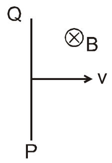
A-14. Find the total heat produced in the loop of the previous problem during the interval 0 to 5 s

Section (B) : Lenz's Law

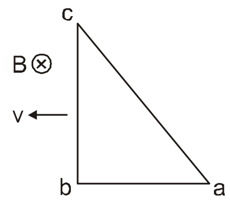
B-1. Two straight long parallel conductors are moved towards each other. A constant current i is flowing through one of them. What is the direction of the current induced in other conductor? What is the direction of induced current when the conductors are drawn apart.

Section (C) : induced EMF in a moving rod in uniform magnetic field

C-1. A metallic wire PQ of length 1 cm moves with a velocity of 2 m/s in a direction perpendicular to its length and perpendicular to a uniform magnetic field of magnitude 0.2 T. Find the emf induced between the ends of the wire. Which end will be positively charged.



C-2. A right angled triangle abc, made of a metallic wire, moves at a uniform speed v in its plane as shown in the figure. A uniform magnetic field B exists in the perpendicular direction of plane of triangle. Find the emf induced (a) in the loop abc, (b) in the segment bc, (c) in the segment ac and (d) in the segment ab.

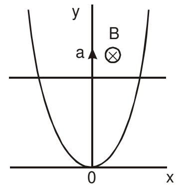


C-3. A metallic metre stick translates in a direction making an angle of 60º with its length. The plane of motion is perpendicular to a uniform magnetic field of 0.1 T that exists in the space. Find the emf induced between the ends of the rod if the speed of translation is 0.2 m/s.

C-4 The two rails, separated by 1m, of a railway track are connected to a voltmeter. What will be the reading of the voltmeter when a train travels on the rails with speed 5 m/s. The earth's magnetic field at the place is 4×10^{-4} T, and the angle of dip is 30º.

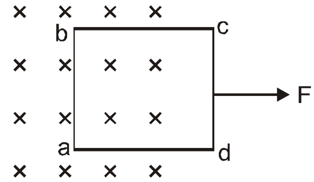
C-5. A circular conducting-ring of radius r translates in its plane with a constant velocity v. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the ring. Consider different pairs of diametrically opposite points on the ring. (a) Between which pair of points is the emf maximum? (b) Between which pair of points is the emf minimum? What is the value of this minimum emf ?

C-6. A wire bent as a parabola $y = kx^2$ is located in a uniform magnetic field of induction B, the vector B being perpendicular to the plane xy. At the moment t = 0 a connector starts sliding translation wise from the parabola apex with a constant acceleration a (figure). Find the emf of electromagnetic induction in the loop thus formed as a function of y.



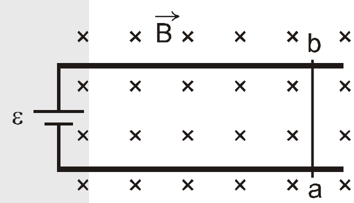
Section (D) : Circuit Problems and Mechanics

D-1. A square frame of wire abcd of side 1 m has a total resistance of 4 Ω. It is pulled out of a magnetic field B = 1 T by applying a force of 1 N (figure). It is found that the frame moves with constant speed. Find (a) this constant speed, (b) the emf induced in the loop, (c) the potential difference between the points a and b and (d) the potential difference between the points c and d.

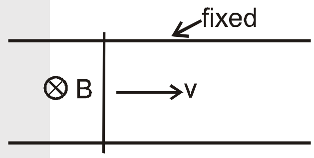


D-2. Consider the situation shown in figure. The wire CD has a negligible resistance and is made to slide on the three rails with a constant speed of 50 cm/s. Find the current in the 10 Ω resistor when the switch S is thrown to (a) the middle rail (b) bottom rail. (Neglect resistance of rails)

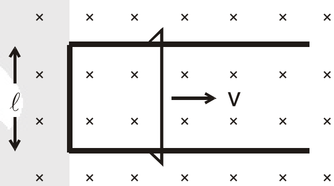
D-3. Figure shows a smooth pair of thick metallic rails connected across a battery of emf ε having a negligible internal resistance. A wire ab of length ℓ and resistance r can slide smoothly on the rails. The entire system lies in a horizontal plane and is immersed in a uniform vertical magnetic field B. At an instant t, the wire is given a small velocity v towards right. (a) Find the current in the wire at this instant. (b) What is the force acting on the wire at this instant. (c) Show that after some time the wire ab will slide with a constant velocity. Find this velocity.



D-4. Figure shows a wire of resistance R sliding on two parallel, conducting fixed thick rails placed at a separation ℓ. A magnetic field B exists in a direction perpendicular to the plane of the rails. The wire is moving with a constant velocity v. Find current through the wire

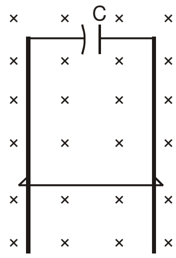


D-5. A long U-shaped wire of width ℓ placed in a perpendicular uniform and constant magnetic field B (figure). A wire of length ℓ is slid on the U-shaped wire with a constant velocity v towards right. The resistance of all the wires is r per unit length. At t = 0, the sliding wire is close to the left edge of the fixed U-shaped wire. Draw an equivalent circuit diagram at time t, showing the induced emf as a battery. Calculate the current in the circuit.



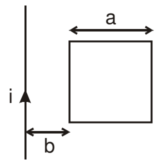
D-6. Consider the situation of the previous problem. (a) Calculate the force needed to keep the sliding wire moving with a constant velocity v. (b) If the force needed just after t = 0 is F_0, find the time at which the force needed will be $F_0/2$.

D-7. A wire of mass m and length ℓ can slide freely on a pair of fixed, smooth, vertical rails (figure). A magnetic field B exists in the region in the direction perpendicular to the plane of the rails. The rails are connected at the top end by an initially uncharged capacitor of capacitance C. Find the velocity of the wire at any time (t) after released. Neglecting any electric resistance. (initial velocity of wire is zero)



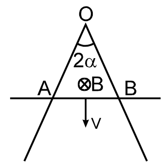
Section (E) : EMF Induced in a rod or loop in non uniform magnetic field

E-1. Figure shows a fixed square frame of wire having a total resistance r placed coplanarly with a long, straight wire. The wire carries a current i given by $i = i_0 \cos (2\pi t/T)$. Find (a) the flux of the magnetic field through the square frame, (b) the emf induced in the frame and (c) the heat developed in the frame in the time interval 0 to 10 T.



E-2. The magnetic field in a region is given by $\vec{B} = \frac{B_0}{L} x\hat{k}$, where L is a fixed length. A conducting rod of length L lies along the X-axis between the origin and the point (L, 0, 0). If the rod moves with a velocity $\vec{v} = v_0 \hat{j}$, find the emf induced between the ends of the rod.

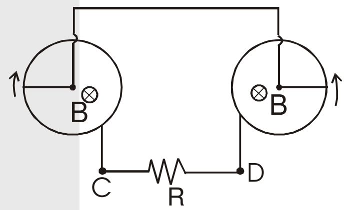
E-3. A straight wire with a resistance of r per unit length is bent to form an angle 2α. A rod of the same wire perpendicular to the angle bisector (of 2α) forms a closed triangular loop. This loop is placed in a uniform magnetic field of induction B. Calculate the current in the wires when the rod moves at a constant speed V.



Section (F) : Induced emf in a rod, Ring, Disc rotating in a uniform magnetic field

F-1. A metal rod of length 15×10^{-2} m rotates about an axis passing through one end with a uniform angular velocity of 60 rad s^{-1}. A uniform magnetic field of 0.1 Tesla exists in the direction of the axis of rotation. Calculate the EMF induced between the ends of the rod.

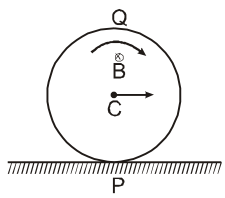
F-2. In the figure there are two identical conducting rods each of length 'a' rotating with angular speed ω in the directions shown. One end of each rod touches a conducting ring. Magnetic field B exists perpendicular to the plane of the rings. The rods, the conducting rings and the lead wires are resistanceless. Find the magnitude and direction of current in the resistance R.



F-3. A bicycle is resting on its stand in the east-west direction and the rear wheel is rotated at an angular speed of 50 revolutions per minute. If the length of each spoke is 30.0 cm and the horizontal component of the earth's magnetic field is 4×10^{-5} T, find the emf induced between the axis and the outer end of a spoke. Neglect centripetal force acting on the free electrons of the spoke.

F-4. A thin wire of negligible mass & a small spherical bob constitute a simple pendulum of effective length ℓ. If this pendulum is made to swing through a semi-vertical angle θ, under gravity in a plane normal to a uniform magnetic field of induction B, find the maximum potential difference between the ends of the wire.

F-5. A conducting disc of radius R is rolling without sliding on a horizontal surface with a constant velocity 'v'. A uniform magnetic field of strength B is applied normal to the plane of the disc. Find the EMF induced between (at this moment)
(a) P & Q (b) P & C (c) Q & C
(C is centre, P&Q are opposite points on vertical diameter of the disc)



F-6. A closed coil having 50 turns is rotated in a uniform magnetic field $B = 2 \times 10^{-4}$ T about a diameter which is perpendicular to the field. The angular velocity of rotation is 300 revolutions per minute. The area of the coil is 100 cm^2 and its resistance is 4 Ω. Find (a) the average emf developed in half a turn from a position where the coil is perpendicular to the magnetic field, (b) the average emf in a full turn, (c) the net charge flown in part (a) and (d) the emf induced as a function of time if it is zero at t=0 and is increasing in positive direction. (e) the maximum emf induced. (f) the average of the squares of emf induced over a long period

Section (G) : Fixed loop in a time varying magnetic field & Induced electric field

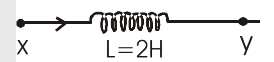
G-1. A circular loop of radius 1m is placed in a varying magnetic field given as B = 6t Tesla, where t is time in sec.
(a) Find the emf induced in the coil if the plane of the coil is perpendicular to the magnetic field.
(b) Find the electric field in the tangential direction, induced due to the changing magnetic field.
(c) Find the current in the loop if its resistance is 1Ω/m.

G-2 The current in an ideal, long solenoid is varied at a uniform rate of 0.01 A/s. The solenoid has 2000 turns/m and its radius is 6.0 cm. (a) Consider a circle of radius 1.0 cm inside the solenoid with its axis coinciding with the axis of the solenoid. Write the change in the magnetic flux through this circle in 2.0 seconds. (b) Find the electric field induced at a point on the circumference of the circle. (c) Find the electric field induced at a point outside the solenoid at a distance 8.0 cm from its axis.

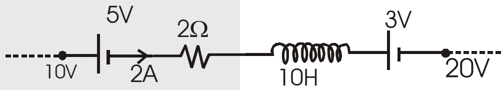
G-3. A uniform field of induction B is changing in magnitude at a constant rate dB/dt. You are given a mass m of copper which is to be drawn into a wire of radius r & formed into a circular loop of radius R. Show that the induced current in the loop does not depend on the size of the wire or of the loop. Assuming B perpendicular to the loop prove that the induced current $i = \frac{m}{4\pi\rho\delta}\frac{dB}{dt}$, where ρ is the resistivity and δ the density of copper.

Section (H) : Self induction, Self inductance self induced emf & Magnetic energy density

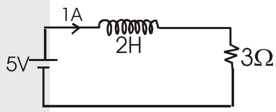
H-1. The figure shows an inductor of 2 H through which a current increasing at the rate of 5A/sec, is flowing. Find the potential difference $V_X - V_Y$.



H-2. Figure shows a part of a circuit. Find the rate of change of the current, as shown.



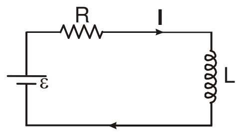
H-3. In the circuit shown find (a) the power drawn from the cell, (b) the power consumed by the resistor which is converted into heat and (c) the power given to the inductor.



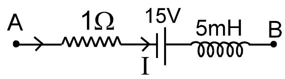
H-4. Find the energy stored in the magnetic field inside a volume of 1.00 mm^3 at a distance of 10.0 cm from a long wire carrying a current of 4 A.

H-5. What is the magnetic energy density (in terms of standard constant & r) at the centre of a circulating electron in the hydrogen atom in first orbit. (Radius of the orbit is r)

H-6. Suppose the EMF of the battery, the circuit shown varies with time t so the current is given by i (t) = 3 + 5 t, where i is in amperes & t is in seconds. Take R = 4 Ω, L = 6 H & find an expression for the battery EMF as a function of time.



H-7. The network shown in Fig. is a part of a complete circuit. What is the potential difference $V_B - V_A$, when the current I is 5A and is decreasing at a rate of 10^3 (A/s)?



Section (I) : Circuit containing inductance, Resistance & battery, Growth and decay Of Current in a circuit containing inductor

I -1. A coil having resistance 20 Ω and inductance 2 H is connected to a battery of emf 4.0 V. Find (a) the current at 0.20 s after the connection is made and (b) the magnetic field energy in the coil at this instant.

I–2. A solenoid of resistance 50 Ω and inductance 80 Henry is connected to a 200 V battery. How long will the current take to reach 50 % of its final equilibrium value? Calculate the maximum energy stored.

I-3. A solenoid has an inductance of 10 Henry and a resistance of 2Ω. It is connected to a 10 volt battery. How long will it take for the magnetic energy to reach $1/4^{th}$ of its maximum value?

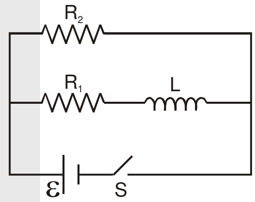
I-4. A coil of resistance 4 Ω is connected across a 0.4 V battery. The current in the coil is 63 mA. 1sec after the battery is connected. Find the inductance of the coil. [$e^{-1} \approx 0.37$]

I-5. A coil of negligible resistance and inductance 5H, is connected in series with a 100 Ω resistor and a battery of emf 2.0 V. Find the potential difference across the resistor 20 ms after the circuit is switched on. ($e^{-0.4} = 0.67$)

I-6. An LR circuit has L = 1.0 H and R = 20 Ω. It is connected across an emf of 2.0 V at t = 0. Find di/dt and Ldi/dt at t = 50 ms.

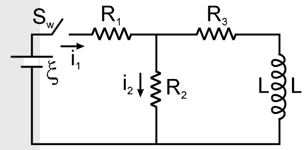
I-7. An inductor-coil of inductance 20 mH having resistance 10 Ω is joined to an ideal battery of emf 5.0 V. Find the rate of change of the magnitude of induced emf at (a) t = 0, (b) t = 10 ms.

I–8. Consider the circuit shown in figure. (a) Find the current through the battery a long time after the switch S is closed. (b) Suppose the switch is opened at t = 0. What is the time constant of the decay circuit? (c) Find the current through the inductor after one time constant.

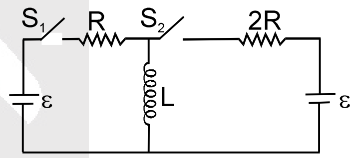


I–9. A superconducting loop of radius R has self inductance L. A uniform & constant magnetic field B is applied perpendicular to the plane of the loop. Initially current in this loop is zero. The loop is rotated about its diameter by 180º. Find the current in the loop after rotation.

I-10. In figure, ξ = 100 V, R_1 = 10 Ω, R_2 = 20 Ω, R_3 = 30 Ω and L = 2 H. Find i_1 & i_2.
(a) immediately after switch S_w is closed
(b) a long time after
(c) immediately after S_w is opened again
(d) a long time later.



I -11. In the circuit shown S_1 & S_2 are switches. S_2 remains closed for a long time and S_1 open. Now S_1 is also closed. Just after S_1 is closed. The potential difference (V) across R and $\frac{di}{dt}$ (with sign) in L.



I -12. Show that if two inductors with equal inductance L are connected in parallel then the equivalent inductance of the combination is L/2. The inductors are separated by a large distance.

I-13. Two inductances L_1 & L_2 are connected in series & are separated by a large distance
(a) Show that their equivalent inductance is $L_1 + L_2$.
(b) Why must their separation be large?

Section (J) : Mutual Induction & Mutual inductance

J-1. The average emf induced in the secondary coil is 0.1 V when the current in the primary coil changes from 1 to 2 A in 0.1 s. What is the mutual inductance of the coils?

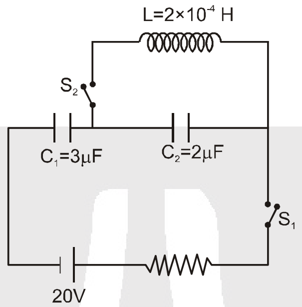
J-2. The mutual inductance between two coils is 0.5 H. It the current in one coil is changed at the rate of 5 A/s, what will be the emf induced in the other coil?

J-3. A small square loop of wire of side ℓ is placed inside a large square loop of wire of side L(L >> ℓ). The loops are co – planar and their centres coincide. Find the mutual inductance of the system.

Section (K) : LC Oscillations

K-1. An LC circuit contains a 20 mH inductor and a 50μF capacitor with an initial charge of 10 mC. The resistance of the circuit is negligible. Let the instant the circuit is closed be t = 0.
(a) What is the total energy stored initially? Is it conserved during LC oscillations?
(b) What is the natural frequency of the circuit?
(c) At what time is the energy stored
(i) Completely electric (i.e., stored in the capacitor)? (ii) Completely magnetic (i.e., stored in the inductor)?
(d) At what times is the total energy shared equally between the inductor and the capacitor?

K-2. The circuit shown in figure is in the steady state with switch S_1 closed. At t = 0, S_1 is opened and switch S_2 is closed.



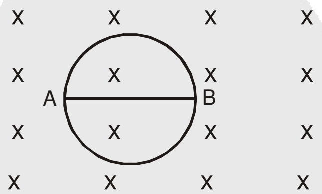
Find the first instant t, when energy in inductor becomes one third of that in capacitor

K-3. A radio can tune over the frequency range of a portion of MW broadcast band: (800 kHz to 1200 kHz). If its LC circuit has an effective inductance of 200 μH, what must be the range of its variable capacitor?

PART - II : ONLY ONE OPTION CORRECT TYPE

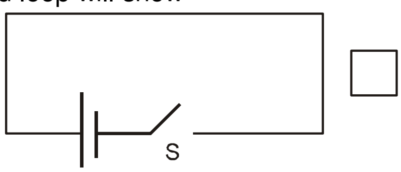
Section (A) : Flux and Faraday's laws of electromagnetic induction

A-1. The radius of the circular conducting loop shown in figure is R. Magnetic field is decreasing at a constant rate α. Resistance per unit length of the loop is ρ. Then current in wire AB is (AB is one of the diameters)



(A) $\frac{R\alpha}{2\rho}$ from A to B (B) $\frac{R\alpha}{2\rho}$ from B to A (C) $\frac{2R\alpha}{\rho}$ from A to B (D) Zero

A-2. Consider the conducting square loop shown in fig. If the switch is closed and after some time it is opened again, the closed loop will show

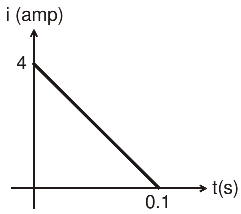


(A) a clockwise current-pulse
(B) an anticlockwise current-pulse
(C) an anticlockwise current-pulse and then a clockwise current-pulse
(D) a clockwise current-pulse and then an anticlockwise current-pulse

A-3. Solve the previous question if the square loop is completely enclosed in the circuit containing the switch.
(A) a clockwise current-pulse
(B) an anticlockwise current-pulse
(C) an anticlockwise current-pulse and then a clockwise current-pulse
(D) a clockwise current-pulse and then an anticlockwise current-pulse

A-4. A small, circular loop of wire is placed inside a long solenoid carrying a current. The plane of the loop contains the axis of the solenoid. If the current in the solenoid is varied, the current induced in the loop is -
(A) anticlockwise (B) clockwise (C) zero
(D) clockwise or anticlockwise depending on whether the resistance in increased or decreased.

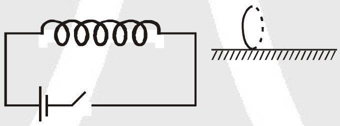
A-5. Some magnetic flux is changed in a coil of resistance 10 ohm. As a result an induced current is developed in it, which varies with time as shown in figure. The magnitude of change in flux through the coil in Webers is (Neglect self inductance of the coil)



(A) 2 (B) 4
(C) 6 (D) 8

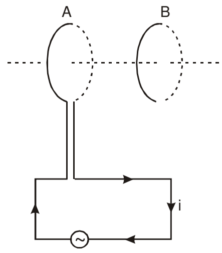
Section (B) : Lenz's Law

B-1. A horizontal solenoid is connected to a battery and a switch (figure). A conducting ring is placed on a frictionless surface, the axis of the ring being along the axis of the solenoid. As the switch is closed, the ring will



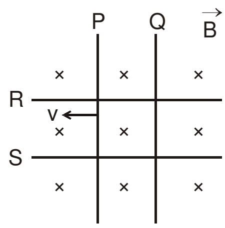
(A) move towards the solenoid
(B) remain stationary
(C) move away from the solenoid
(D) move towards the solenoid or away from it depending on which terminal (positive or negative) of the battery is connected to the left end of the solenoid.

B-2. Two circular coils A and B are facing each other as shown in figure. The current i through A can be altered



(A) there will be repulsion between A and B if i is increased
(B) there will be attraction between A and B if i is increased
(C) there will be neither attraction nor repulsion when i is changed
(D) attraction or repulsion between A and B depends on the direction of current. It does not depend whether the current is increased or decreased.

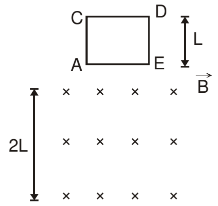
B-3. Two identical conductors P and Q are placed on two frictionless fixed conducting rails R and S in a uniform magnetic field directed into the plane. If P is moved in the direction shown in figure with a constant speed, then rod Q



(A) will be attracted towards P
(B) will be repelled away from P
(C) will remain stationary
(D) may be repelled or attracted towards P

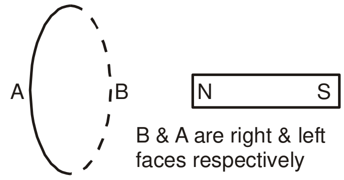
B-4. Two identical coaxial circular loops carry a current i each circulating in the same direction. If the loops approach each other
(A) the current in each loop will decrease
(B) the current in each loop will increase
(C) the current in each loop will remain the same
(D) the current in one loop will increase and in the other loop will decrease

B-5. A square coil ACDE with its plane vertical is released from rest in a horizontal uniform magnetic field $\vec{B}$ of length 2L. The acceleration of the coil is



(A) less than g for all the time till the loop crosses the magnetic field completely
(B) less than g when it enters the field and greater than g when it comes out of the field
(C) g all the time
(D) less than g when it enters and comes out of the field but equal to g when it is within the field

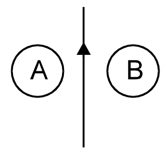
B-6. In the figure shown, the magnet is pushed towards the fixed ring along the axis of the ring and it passes through the ring.



(A) when magnet goes towards the ring the face B becomes south pole and the face A becomes north pole
(B) when magnet goes away from the ring the face B becomes north pole and the face A becomes south pole
(C) when magnet goes away from the ring the face A becomes north pole and the face B becomes south pole
(D) the face A will always be a north pole.

B-7. A metallic ring with a small cut is held horizontally and a magnet is allowed to fall vertically through the ring then the acceleration of the magnet is :
(A) always equal to g
(B) initially less than g but greater than g once it passes through the ring
(C) initially greater than g but less than g once it passes through the ring
(D) always less than g

B-8. A and B are two metallic rings placed at opposite sides of an infinitely long straight conducting wire as shown. If current in the wire is slowly decreased, the direction of induced current will be :



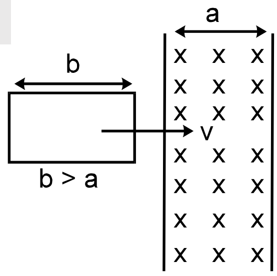
(A) clockwise in A and anticlockwise in B
(B) anticlockwise in A and clockwise in B
(C) clockwise in both A and B
(D) anticlockwise in both A & B

Section (C) : induced EMF in a moving rod in uniform magnetic field

C-1. A wire of length ℓ is moved with a constant velocity $\vec{\upsilon}$ in a magnetic field. A potential difference appears across the two ends

(A) if $\vec{\upsilon} \,|\, \vec{\ell}\,|$ (B) if $\vec{\upsilon} \,||\, \vec{B}$ (C) if $\vec{\ell} \,||\, \vec{B}$ (D) none of these

C-2. In the given arrangement, the loop is moved with constant velocity v in a uniform magnetic field B in a restricted region of width a. The time for which the emf is induced in the circuit is:



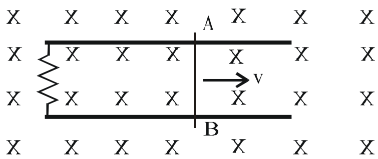
(A) $\frac{2b}{v}$ (B) $\frac{2a}{v}$

(C) $\frac{(a+b)}{v}$ (D) $\frac{2(a-b)}{v}$

C-3. A uniform magnetic field exists in region given by $\vec{B} = 3\hat{i} + 4\hat{j} + 5\hat{k}$. A rod of length 5 m is placed along y- axis is moved along x-axis with constant speed 1 m/sec. Then induced e.m.f. in the rod will be:

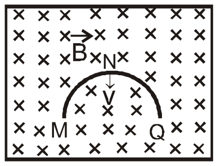
(A) zero (B) 25 v (C) 20 v (D) 15 v

C-4. The resistanceless wire AB (in figure) is slid on the fixed rails with a constant velocity. If the wire AB is replaced by a resistanceless semicircular wire, the magnitude of the induced current will



(A) decrease
(B) remain the same
(C) increase
(D) increase or decrease depending on whether the semicircle bulges towards the resistance or away from it.

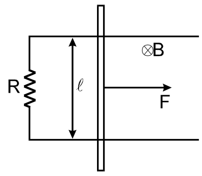
C-5. A thin semicircular conducting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$. At the position MNQ the speed of the ring is v then the potential difference developed across the ring is:



(A) zero
(B) $\frac{Bv\pi R^2}{2}$ and M is at higher potential
(C) π RBV and Q is at higher potential
(D) 2 RBV and Q is at higher potential.

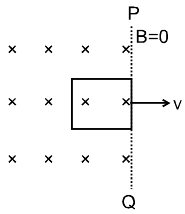
Section (D) : Circuit Problems with dynamics

D-1. A constant force F is being applied on a rod of length 'ℓ' kept at rest on two parallel conducting rails connected at ends by resistance R in uniform magnetic field B as shown.



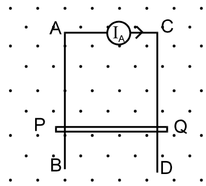
(A) the power delivered by force will be constant with time
(B) the power delivered by force will be increasing first and then it will decrease
(C) the rate of power delivered by the external force will be increasing continuously
(D) the rate of power delivered by external force will be decreasing continuously before becoming zero.

D-2. Figure shows a square loop of side 1 m and resistance 1 Ω. The magnetic field on left side of line PQ has a magnitude B = 1.0T. The work done in pulling the loop out of the field uniformly in 1 s is



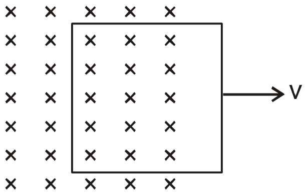
(A) 1 J
(B) 10 J
(C) 0.1 J
(D) 100 J

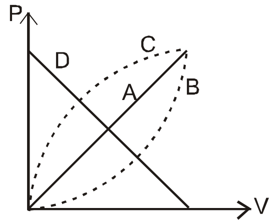
D-3. AB and CD are fixed conducting smooth rails placed in a vertical plane and joined by a constant current source at its upper end. PQ is a conducting rod which is free to slide on the rails. A horizontal uniform magnetic field exists in space as shown. If the rod PQ is released from rest then :



(A) The rod PQ may move downward with constant acceleration
(B) The rod PQ may move upward with constant acceleration
(C) The rod will move downward with decreasing acceleration and finally acquire a constant velocity
(D) either A or B.

D-4. Fig. shows a conducting loop being pulled out of a magnetic field with a constant speed v. Which of the four plots shown in fig. may represent the power delivered by the pulling agent as a function of the constant speed v.

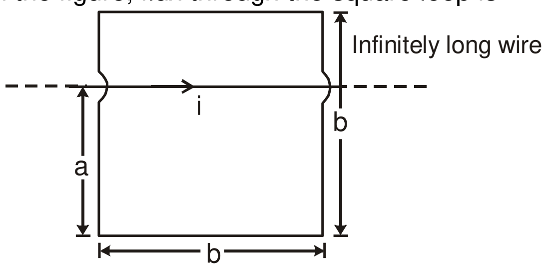




(A) A
(B) B
(C) C
(D) D

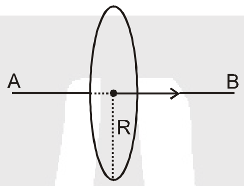
Section (E) : EMF Induced in a rod or loop in nonuniform magnetic field

E-1. For the situation shown in the figure, flux through the square loop is



(A) $\left(\frac{\mu_0 ia}{2\pi}\right)\ell n\left(\frac{a}{2a-b}\right)$ (B) $\left(\frac{\mu_0 ib}{2\pi}\right)\ell n\left(\frac{a}{2b-a}\right)$ (C) $\left(\frac{\mu_0 ib}{2\pi}\right)\ell n\left(\frac{a}{b-a}\right)$ (D) $\left(\frac{\mu_0 ia}{2\pi}\right)\ell n\left(\frac{2a}{b-a}\right)$

E-2. A long conductor AB lies along the axis of a circular loop of radius R. If the current in the conductor AB varies at the rate of I ampere/second, then the induced emf in the loop is



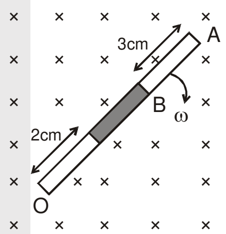
(A) $\frac{\mu_0 IR}{2}$ (B) $\frac{\mu_0 IR}{4}$ (C) $\frac{\mu_0 \pi IR}{2}$ (D) zero

Section (F) : Induced emf in a rod, Ring, Disc rotating in a uniform magnetic field

F-1. A conducting rod of length ℓ rotates with a uniform angular velocity ω about its perpendicular bisector. A uniform magnetic field B exists parallel to the axis of rotation. The potential difference between the two ends of the rod is

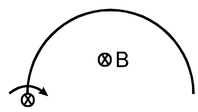
(A) $2B\omega\ell^2$ (B) $1/2\ \omega B\ell^2$ (C) $B\omega\ell^2$ (D) zero

F-2. A rod of length 10 cm made up of conducting and non-conducting material (shaded part is non-conducting). The rod is rotated with constant angular velocity 10 rad/sec about point O, in constant and uniform magnetic field of 2 Tesla as shown in the figure. The induced emf between the point A and B of rod will be



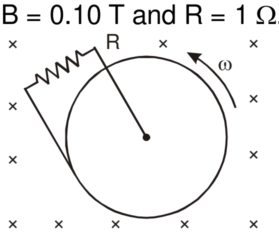
(A) 0.029 v (B) 0.1 v
(C) 0.051 v (D) 0.064 v

F-3. A semicircular wire of radius R is rotated with constant angular velocity ω about an axis passing through one end and perpendicular to the plane of the wire. There is a uniform magnetic field of strength B. The induced e.m.f. between the ends is:



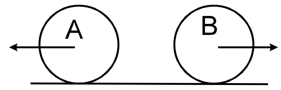
(A) $B\ \omega R^2/2$ (B) $2\ B\ \omega R^2$ (C) is variable (D) none of these

F-4. Figure shows a conducting disc rotating about its axis in a perpendicular uniform and constnat magnetic field B. A resistor of resistance R is connected between the centre and the rim. The radius of the disc is 5.0 cm, angular speed ω = 40 rad/s, B = 0.10 T and R = 1 Ω. The current through the resistor is



(A) 5 mA (B) 50 A (C) 5 A (D) 10 mA

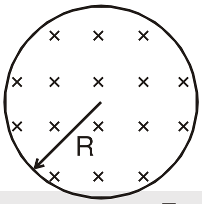
F-5. Two identical conducting rings A & B of radius r are in pure rolling over a horizontal conducting plane with same speed (of center of mass) υ but in opposite direction. A constant magnetic field B is present pointing inside the plane of paper. Then the potential difference between the highest points of the two rings, is :

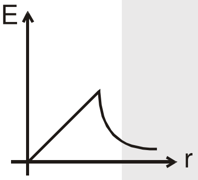


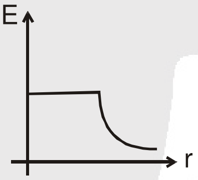
(A) zero (B) 2 Bvr (C) 4Bvr (D) none of these

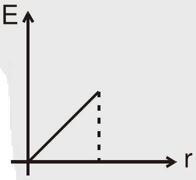
Section (G) : Fixed loop in a time varying magnetic field & Induced electric field

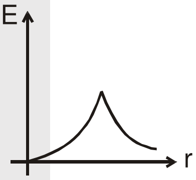
G-1. A cylindrical space of radius R is filled with a uniform magnetic induction B parallel to the axis of the cylinder. If B changes at a constant rate, the graph showing the variation of induced electric field with distance r from the axis of cylinder is



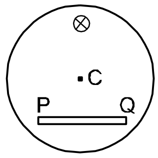
(A) 

(B) 

(C) 

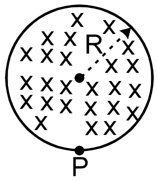
(D) 

G-2. In a cylindrical region uniform magnetic field which is perpendicular to the plane of the figure is increasing with time and a conducting rod PQ is placed in the region. If C is the centre of the circle then



(A) P will be at higher potential than Q.
(B) Q will be at higher potential than P.
(C) Both P and Q will be equipotential.
(D) no emf will be developed across rod as it is not crossing / cutting any line of force.

G-3. A uniform magnetic field of induction B is confined to a cylindrical region of radius R. The magnetic field is increasing at a constant rate of $\frac{dB}{dt}$ (Tesla/second). An electron of charge q, placed at the point P on the periphery of the field experiences an acceleration :



(A) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward left (B) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ toward right

(C) $\frac{eR}{m}\frac{dB}{dt}$ toward left (D) $\frac{1}{2}\frac{eR}{m}\frac{dB}{dt}$ zero

G-4. A neutral metallic ring is placed in a circular symmetrical uniform magnetic field with its plane perpendicular to the field. If the magnitude of field starts increasing with time, then:
(A) the ring starts translating (B) the ring starts rotating about its axis
(C) the ring slightly contracts (D) the ring starts rotating about a diameter

G-5. A bar magnet is released at one end from rest coaxially along the axis of a very long fixed, vertical copper tube. After some time the magnet
(A) will move with an acceleration g (B) will move with almost constant speed
(C) will stop in the tube (D) will oscillate

Section (H) : Self induction, self inductance self induced emF & Magnetic energy density

H-1. A wire of fixed length is wound on a solenoid of length 'ℓ' and radius 'r'. Its self inductance is found to be L. Now if another wire is wound on a solenoid of length $\ell/2$ and radius r/2, then the self inductance will be:
(A) 2 L (B) L (C) 4 L (D) 8 L

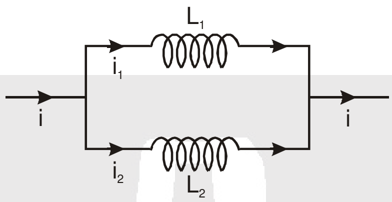
H-2. The number of turns, cross-sectional area and length for four solenoids are given in the following table.

Solenoid	Total Turns	Area	Length
1	2N	2A	ℓ
2	2N	A	ℓ
3	3N	3A	2ℓ
4	2N	2A	$\ell/2$

The solenoid with maximum self inductance is :
(A) 1 (B) 2 (C) 3 (D) 4

Section (I) : Circuit containing inductance, Resistance & battery, Growth and decay Of Current in a circuit containing inductor

I -1. Two inductors L_1 and L_2 are connected in parallel and a time varying current i flows as shown. The ratio of currents i_1/i_2 at any time t is

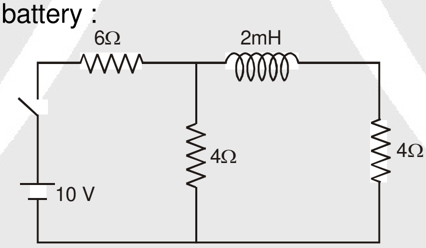


(A) L_1/L_2 (B) L_2/L_1 (C) $\frac{L_1^2}{(L_1+L_2)^2}$ (D) $\frac{L_2^2}{(L_1+L_2)^2}$

I-2. In an LR circuit current at t = 0 is 20 A. After 2s it reduces to 18 A. The time constant of the circuit is (in second):

(A) $\ln\left(\frac{10}{9}\right)$ (B) 2 (C) $\frac{2}{\ln\left(\frac{10}{9}\right)}$ (D) $2\ln\left(\frac{10}{9}\right)$

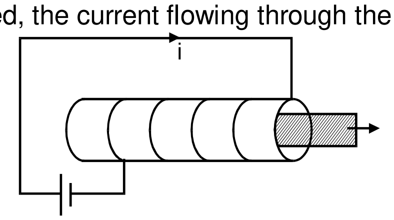
I-3. In the given circuit find the ratio of i_1 to i_2. Where i_1 is the initial (at t = 0) current, and i_2 is steady state (at t = ∞) current through the battery :



(A) 1.0 (B) 0.8 (C) 1.2 (D) 1.5

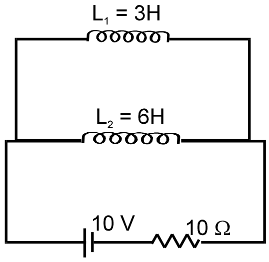
I-4. In a series L–R growth circuit, if maximum current and maximum induced emf in an inductor of inductance 3mH are 2A and 6V respectively, then the time constant of the circuit is :
(A) 1 ms. (B) 1/3 ms. (C) 1/6 ms (D) 1/2 ms

I-5. A solenoid having an iron core has its terminals connected across an ideal DC source and it is in steady state. If the iron core is removed, the current flowing through the solenoid just after removal of rod



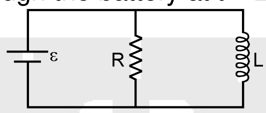
(A) increases (B) decreases (C) remains unchanged (D) nothing can be said

I-6. Two inductor coils of self inductance 3H and 6H respectively are connected with a resistance 10Ω and a battery 10 V as shown in figure. The ratio of total energy stored at steady state in the inductors to that of heat developed in resistance in 10 seconds at the steady state is (neglect mutual inductance between L_1 and L_2) :



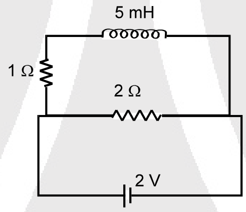
(A) 1/10 (B) 1/100 (C) 1/1000 (D) 1

I -7. The battery shown in the figure is ideal. The values are ε = 10 V, R = 5Ω, L = 2H. Initially the current in the inductor is zero. The current through the battery at t = 2s is



(A) 12 A (B) 7 A (C) 3 A (D) none of these

I-8. When induced emf in inductor coil is 50% of its maximum value then stored energy in inductor coil in the given circuit at that instant will be



(A) 2.5 mJ (B) 5mJ (C) 15 mJ (D) 20 mJ

I-9. An inductor coil stores energy U when a current i is passed through it and dissipates heat energy at the rate of P. The time constant of the circuit when this coil is connected across a battery of zero internal resistance is :

(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

Section (J) : Mutual Induction & Mutual Inductance

J-1. Two coils are at fixed locations. When coil 1 has no current and the current in coil 2 increases at the rate 15.0 A/s the e.m.f. in coil 1 in 25.0 mV, when coil 2 has no current and coil 1 has a current of 3.6 A, flux linkage in coil 2 is

(A) 16 mWb (B) 10 mWb (C) 4.00 mWb (D) 6.00 mWb

J-2. A rectangular loop of sides 'a' and ' b ' is placed in xy plane. A very long wire is also placed in xy plane such that side of length 'a' of the loop is parallel to the wire. The distance between the wire and the nearest edge of the loop is ' d '. The mutual inductance of this system is proportional to:

(A) a (B) b (C) 1/d (D) current in wire

J-3. Two coils of self inductance 100 mH and 400 mH are placed very close to each other. Find the maximum mutual inductance between the two when 4 A current passes through them

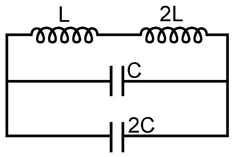
(A) 200 mH (B) 300 mH (C) $100\sqrt{2}$ mH (D) none of these

J-4. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is

(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0

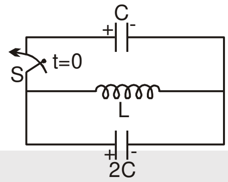
Section (K) : LC Oscillations

K-1. The frequency of oscillation of current in the inductor is:



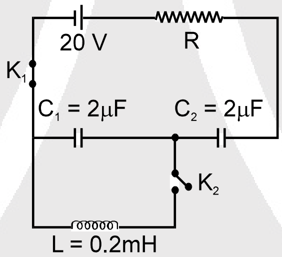
(A) $\frac{1}{3\sqrt{LC}}$ (B) $\frac{1}{6\pi\sqrt{LC}}$ (C) $\frac{1}{\sqrt{LC}}$ (D) $\frac{1}{2\pi\sqrt{LC}}$

K-2. In the given LC circuit if initially capacitor C has charge Q on it and 2C has charge 2Q .The polarities are as shown in the figure. Then after closing switch S at t = 0



(A) energy will get equally distributed in both the capacitor just after closing the switch.
(B) initial rate of growth of current in inductor will be $2Q/3CL$
(C) maximum energy in the inductor will be $3Q^2/2C$
(D) none of these

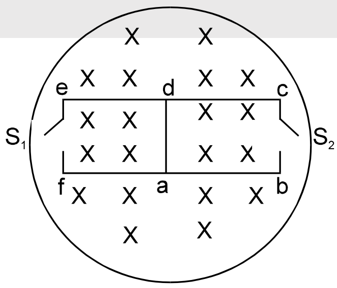
K-3. A circuit containing capacitors C_1 and C_2 as shown in the figure are in steady state with key K_1 closed. At the instant t = 0, if K_1 is opened and K_2 is closed then the maximum current in the circuit will be :



(A) 1 A (B) A (C) 2 A (D) None of these

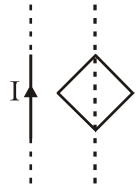
PART - III : MATCH THE COLUMN

1. The magnetic field in the cylindrical region shown in figure increases at a constant rate of 10.0 mT/s Each side of the square loop abcd and defa has a length of 2.00 cm and resistance of 2.00 Ω . Correctly match the current in the wire 'ad' in four different situations as listed in column-I with the values given in column-II.



Column-I	Column-II
(A) The switch S_1 is closed but S_2 is open	(p) 5×10^{-7} A, d to a
(B) S_1 is open but S_2 is closed	(q) 5×10^{-7} A, a to d
(C) Both S_1 and S_2 are open	(r) 2.5×10^{-8} A, d to a
(D) Both S_1 and S_2 are closed.	(s) 2.5×10^{-8} A, a to d
	(t) No current flows

2. A square loop of conducting wire is placed symmetrically near a long straight current carrying wire as shown. Match the statements in column-I with the corresponding results in column-II.



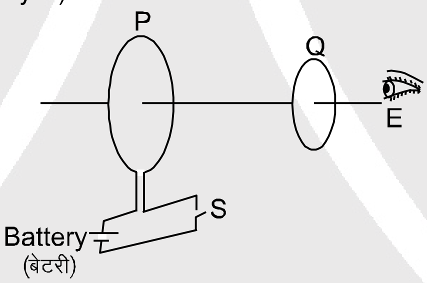
Column-I	Column-II
(A) If the magnitude of current I is increased in the loop	(p) Current will induce in clockwise direction
(B) If the magnitude of current I is decreased in the loop	(q) Current will induce in anticlockwise direction
(C) If the loop is moved away from the wire	(r) wire will attract the loop
(D) If the loop is moved towards the wire	(s) wire will repel the loop
	(t) loop will rotate when current changes.

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

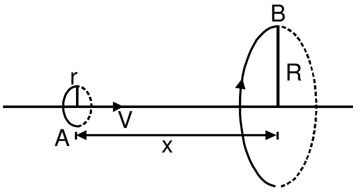
1. As shown in the fig. P and Q are two coaxial conducting loops separated by some distance. When the switch S is closed, a clockwise current I_P flows in P (as seen by E) and an induced current I_{Q1} flows in Q. The switch remains closed for a long time. When S is opened, a current I_{Q2} flows in Q. Then the directions of I_{Q1} and I_{Q2} (as seen by E) are **[JEE 2002 (Screening) 3/90, –1]**



(A) respectively clockwise and anti-clockwise
(B) both clockwise
(C) both anti-clockwise
(D) respectively anti-clockwise and clockwise.

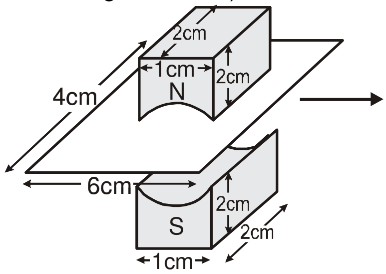
2. A close loop is placed in a time-varying magnetic field. Electrical power is dissipated due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved keeping the radius of the loop unchanged, the electrical power dissipated would be:
(A) halved (B) the same (C) doubled (D) quadrupled

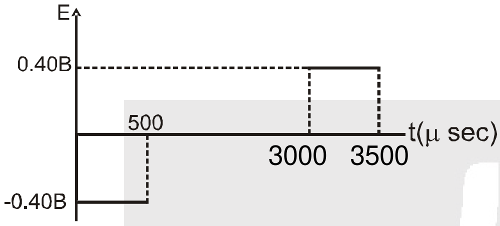
3. Loop A of radius r(r << R) moves towards loop B with a constant velocity V in such a way that their planes are always parallel. What is the distance between the two loops (x) when the induced emf in loop A is maximum

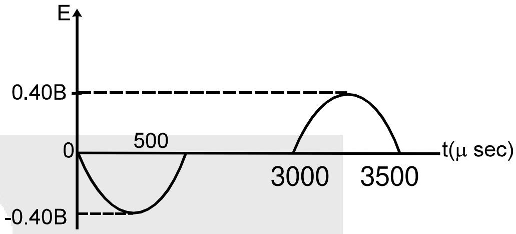


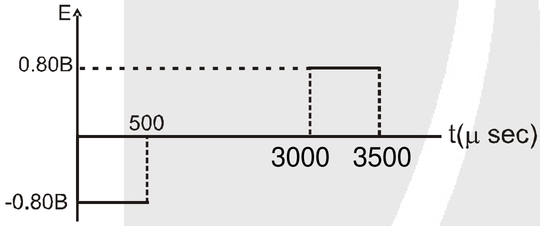
(A) R (B) $\frac{R}{\sqrt{2}}$ (C) $\frac{R}{2}$ (D) $R\left(1-\frac{1}{\sqrt{2}}\right)$

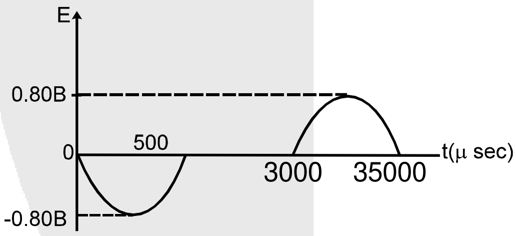
4. A magnetic field (B), uniform between two magnets can be determined measuring the induced voltage in the loop as it is pulled through the gap at uniform speed 20 m/sec. Size of magnet and coil is 2cm × 1cm × 2cm and 4cm × 6cm as shown in figure. The correct variation of induced emf with time is : Assume at t = 0, the coil enters in the magnetic field) :



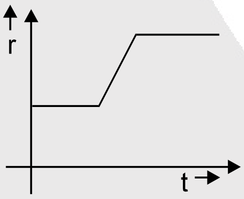
(A)


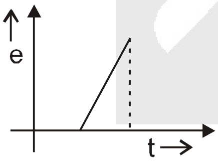
(B)


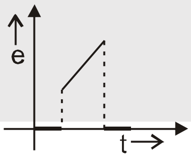
(C)


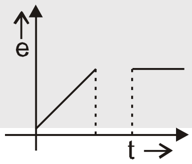
(D)


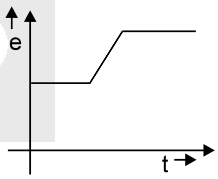
5. Radius of a circular ring is changing with time and the coil is placed in uniform magnetic field perpendicular to its plane. The variation of ' r ' with time ' t ' is shown in the figure. Then induced e.m.f. ε with time will be best represented by :



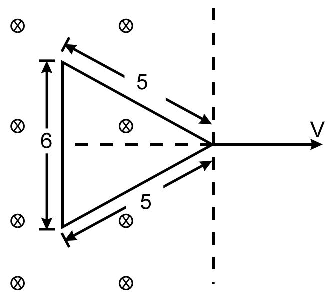
(A)


(B)


(C)


(D)


6. A triangular loop as shown in the figure is started to being pulled out at t = 0 from a uniform magnetic field with a constant velocity v. Total resistance of the loop is constant and equals to R. Then the variation of power produced in the loop with time will be:



(A) linearly increasing with time till whole loop comes out
(B) increases parabolically till whole loop comes out
(C) P α t^3 till whole loop come out
(D) will be constant with time

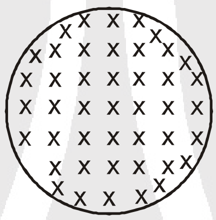
7. A metal rod of resistance 20 Ω is fixed along a diameter of conducting ring of radius 0.1 m and lies in x-y plane. There is a magnetic field $\vec{B} = (50T)\ \hat{k}$. The ring rotates with an angular velocity ω = 20 rad/s about its axis. An external resistance of 10 Ω is connected across the centre of the ring and rim. The current through external resistance is

(A) $\frac{1}{4}$A (B) $\frac{1}{2}$A (C) $\frac{1}{3}$A (D) zero

8. Earth is a spherical conductor with a uniform surface charge density σ. It rotates about its axis with angular velocity ω. Suppose the magnetic field due to Sun at Earth at some instant is a uniform field B pointing along earth's axis. Then the emf developed between the pole and equator of earth due to this field is. (R_e = radius of earth)

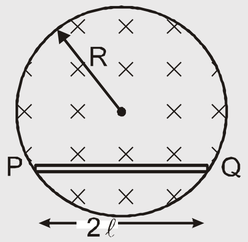
(A) $\frac{1}{2}B\omega R_e^2$ (B) $B\omega R_e^2$ (C) $\frac{3}{2}B\omega R_e^2$ (D) zero

9. A non conducting ring of radius R and mass m having charge q uniformly distributed over its circumference is placed on a rough horizontal surface. A vertical time varying uniform magnetic field $B = 4t^2$ is switched on at time t = 0. The coefficient of friction between the ring and the table, if the ring starts rotating at t =2 sec, is :



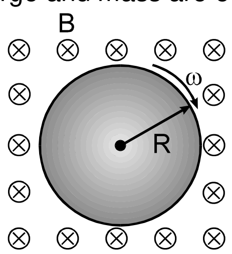
(A) $\frac{4qmR}{g}$ (B) $\frac{2qmR}{g}$ (C) $\frac{8qR}{mg}$ (D) $\frac{qR}{2mg}$

10. A uniform magnetic field, $B = B_0 t$ (where B_0 is a positive constant), fills a cylindrical volume of radius R, then the potential difference in the conducting rod PQ due to electrostatic field is :



(A) $B_0\ell\ \sqrt{R^2+\ell^2}$ (B) $B_0\ell\ \sqrt{R^2-\frac{\ell^2}{4}}$ (C) $B_0\ell\ \sqrt{R^2-\ell^2}$ (D) $B_0R\ \sqrt{R^2-\ell^2}$

11. A conducting disc of radius R is placed in a uniform and constant magnetic field B parallel to the axis of the disc. With what angular speed should the disc be rotated about its axis such that no electric field develops in the disc. (The electronic charge and mass are e and m)

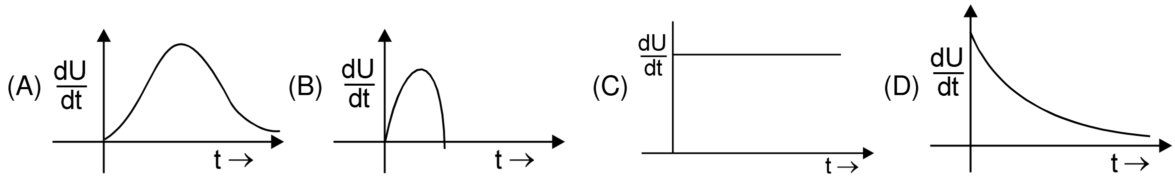


(A) $\frac{eB}{2m}$ (B) $\frac{eB}{m}$ (C) $\frac{2\pi m}{eB}$ (D) $\frac{\pi m}{eB}$

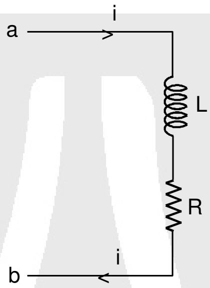
12. When the current in a certain inductor coil is 5.0 A and is increasing at the rate of 10.0 A/s, the of potential difference across the coil is 140 V. When the current is 5.0 A and decreasing at the rate of 10.0 A/s, the potential difference is 60 V. The self inductance of the coil is :

(A) 2H (B) 4H (C) 10H (D) 12H

13. Rate of increment of energy in an inductor with time in series LR circuit getting charge with battery of e.m.f. E is best represented by: [Inductor has initially zero current]

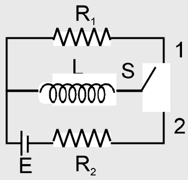


14. When the current in the portion of the circuit shown in the figure is 2A and increasing at the rate of 1A/s, the measured potential difference $V_a - V_b = 8V$. However when the current is 2A and decreasing at the rate of 1A/s, the measured potential difference $V_a - V_b = 4V$. The values of R and L are :



(A) 3 ohm and 2 Henry respectively (B) 2 ohm and 3 Henry respectively
(C) 10 ohm and 6 Henry respectively (D) 6 ohm and 1 Henry respectively

15. In the circuit shown switch S is connected to position 2 for a long time and then joined to position 1. The total heat produced in resistance R_1 is :

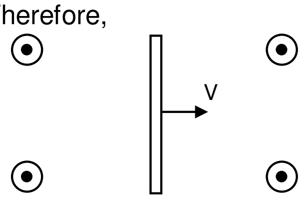


(A) $\frac{L\ \ E^2}{2R_2^2}$ (B) $\frac{L\ \ E^2}{2R_1^2}$ (C) $\frac{L\ \ E^2}{2R_1R_2}$ (D) $\frac{L\ \ E^2(R_1+R_2)^2}{2R_1^2R_2^2}$

16. Two identical coils each of self-inductance L, are connected in series and are placed so close to each other that all the flux from one coil links with the other. The total self-inductance of the system is :

(A) L (B) 2L (C) 3L (D) 4L

17. A neutral metal bar moves at a constant velocity v to the right through a region of uniform magnetic field directed out the page, as shown. Therefore,

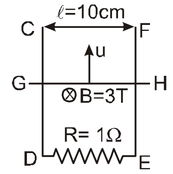


(A) positive charges accumulate to the left side and negative charges to the right side of the rod
(B) negative charges accumulate to the left side and positive charges to the right side of the rod.
(C) positive charges accumulate to the top end and negative charges to the bottom end of the rod
(D) negative charges accumulate to the top end and positive charges to the bottom end of the rod

PART - II : NUMERICAL VALUE

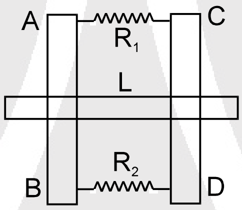
1. A plane spiral with a great number N of turns wound tightly to one another is located in a uniform magnetic field perpendicular to the spiral's plane. The outside radius of the spiral's turns is equal to 'a' and inner radius is zero. The magnetic induction varies with time as $B = B_0 \sin \omega t$, where B_0 and ω are constants. The amplitude of emf induced in the spiral is $\varepsilon_{im} = \frac{1}{x} \pi a^2 N \omega B_0$. Find out value of x.

2. In the figure, CDEF is a fixed conducting smooth frame in vertical plane. A conducting uniform rod GH of mass 'm' = 1 g can move vertically and smoothly without losing contact with the frame. GH always remains horizontal. It is given velocity 'u = 1 m/s' upwards and released. Taking the acceleration due to gravity as 'g' and assuming that no resistance is present other than 'R'. Time taken by rod to reach the highest point is equal to $\frac{\ell n 10}{x}$ second. Find out value of x.

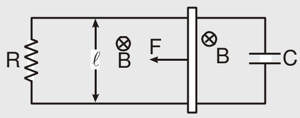


3. Two parallel vertical metallic rails AB and CD are separated by 1 m. They are connected at the two ends by resistance R_1 and R_2 as shown in the figure. A horizontal metallic bar L of mass 0.2 kg slides without friction, vertically down the rails under the action of gravity. There is a uniform horizontal magnetic field of 0.6T perpendicular to the plane of the rails. It is observed that when the terminal velocity is attained, the power dissipated in R_1 and R_2 are 0.76 W and 1.2 W respectively. If the terminal velocity of bar L is x m/s and R_1 is y Ω and R_2 is ZΩ then find the value of x + 76y + 10z. ($g = 9.8\ m/s^2$)

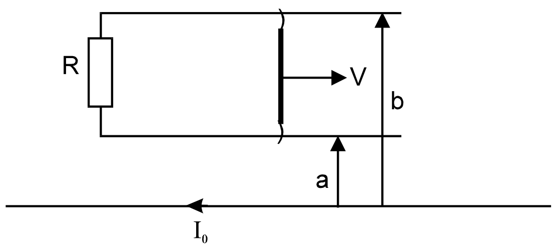
[JEE - 1994]



4. Two parallel long smooth conducting rails separated by a distance ℓ = 10 cm are connected by a movable conducting connector of mass m = 4mg. Terminals of the rails are connected by the resistor R = 2Ω & the capacitor C = 1μF as shown. A uniform magnetic field B = 20T perpendicular to the plane of the rails is switched on. The connector is dragged by a constant force F=10N. The speed of the connector as function of time if the force F is applied at t = 0 is equal to $v = 5(1 - e^{-x \times 10^4 \times t})$ m/s .Find the value of x.

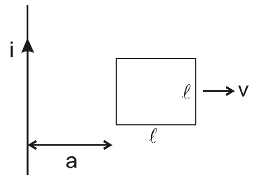


5. A long straight wire carries a current I_0. At distance a and b = 3a from it there are two other wires, parallel to the former one, which are interconnected by a resistance R (figure). A connector slides without friction along the wires with a constant velocity v. Assuming the resistances of the wires, the connector, the sliding contacts, and the self-inductance of the frame to be negligible ;

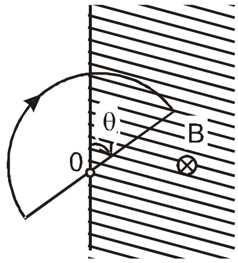


The point of application (distance from the long wire) of magnetic force on sliding wire due to the long wire is $\frac{2a}{\ell n x}$ from long wire. Then findout value of x.

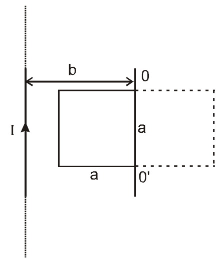
6. A square metallic loop of side ℓ is placed near a fixed long wire carrying a current i (figure). The loop is moved towards right perpendicular to the wire with a speed v in the plane containing the wire and the loop. The emf induced in the loop when the rear end of the loop is at a distance a = 2ℓ from the wire is $\frac{\mu_0 iv}{x\pi}$. Find out value of x.



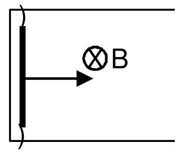
7. A wire loop enclosing a semi-circle of radius a = 2cm is located on the boundary of a uniform magnetic field of induction B=1T (Figure). At the moment t = 0 the loop is set into rotation with a constant angular acceleration β = 2 rad/sec^2 about an axis O coinciding with a line of vector B on the boundary. The emf induced in the loop as a function of time t is [$x \times 10^{-4} (-1)^n \times t$] volts, where n = 1, 2, ... is the number of the half-revolution that the loop performs at the given moment t. Find the value of x. (The arrow in the figure shows the emf direction taken to be positive, at t = 0 loop was completely outside)



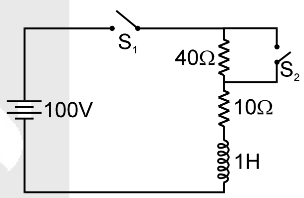
8. A square wire frame (initially current is zero) with side a and a straight conductor carrying a constant current I are located in the same plane (figure). The inductance and the resistance of the frame are equal to L and R respectively. The frame was turned through 180° about the axis OO' separated from the current-carrying conductor by a distance b = 2a. The total electric charge having flown through the frame if i = 0 at t = 0 in the loop is equal to $\frac{\mu_0 aI}{2\pi R}\ell n x$. Find the value of x.



9. A Π-shaped conductor is located in a uniform magnetic field perpendicular to the plane of the conductor and varying with time at the rate $\frac{dB}{dt}$ = 0.10 T/s. A conducting connector starts moving with a constant acceleration w = 10 cm/s^2 along the parallel bars of the conductor.
The length of the connector is equal to ℓ = 20 cm. Find the emf induced (in mV) in the loop t = 2.0 s after the beginning of the motion, if at the moment t = 0 the loop area and the magnetic induction (B) are equal to zero. The self inductance of the loop is to be neglected.



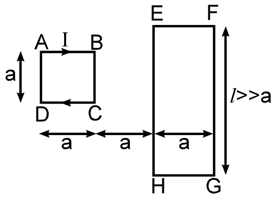
10. In the circuit diagram shown in the figure the switches S_1 and S_2 are closed at time t = 0. After time t = (0.1) ℓn 2 sec, switch S_2 is opened. The current in the circuit at time, t = (0.2) ln 2 sec is equal to $\frac{x}{32}$ amp. Findout value of x.



11. A closed circuit consists of a source of constant emf E and a choke coil of inductance L connected in series. The active resistance of the whole circuit is equal to R. It is in steady state. At the moment t = 0 the choke coil inductance was decreased abruptly 4 times. The current in the circuit as a function of time t is E/R [$1 + xe^{-4tR/L}$]. Findout value of x.

12. A very small circular loop of radius 'a' is initially coplanar & concentric with a much larger circular loop of radius b (>> a). A constant current I is passed in the large loop which is kept fixed in space & the small loop is rotated with constant angular velocity ω about a diameter. The resistance of the small loop is R & its inductance is negligible. The induced emf in the large loop due to current induced in smaller loop as a function of time is equal to $\frac{1}{x}\left(\frac{\pi a^2 \mu_0 \omega}{b}\right)^2 \frac{I\cos 2\omega t}{R}$. Findout value of x.

13. In the figure shown two loops ABCD & EFGH are in the same plane. The smaller loop carries time varying current I = bt, where b is a positive constant and t is time. The resistance of the smaller loop is r and that of the larger loop is R : (Neglect the self inductance of large loop)

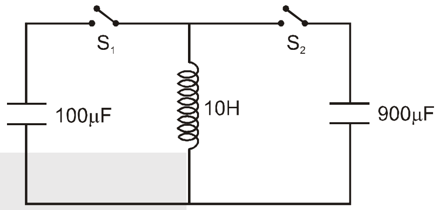


The magnetic force on the loop EFGH due to loop ABCD is $\frac{\mu_0^2 Iab}{x\,\pi^2 R}\ \ell n\ \frac{4}{3}$.

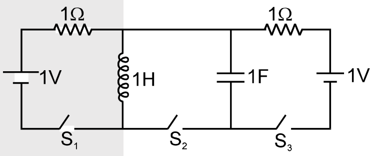
Findout value of x.

14. A solenoid of length 1 m, area of cross-section 4.0 cm^2 and having 4000 turns is placed inside another solenoid of 2000 turns having a cross-sectional area 6 cm ^{2} and length 2 m. The mutual inductance between the solenoids is x $\pi \times 10^{-5}$ H. Findout value of x.

15. Initially the 900 μF capacitor is charged to 100 V and the 100 μF capacitor is uncharged in the figure shown. Then the switch S_2 is closed for a time t_1, after which it is opened and at the same instant switch S_1 is closed for a time t_2 and then opened. It is now found that the 100 μF capacitor is charged to 300 V. If t_1 and t_2 minimum possible values of the time intervals, then findout t_1/t_2.

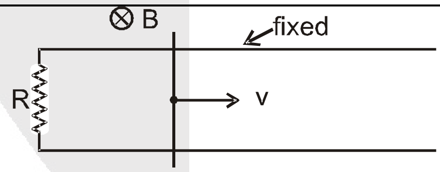


16. In the circuit shown switches S_1 and S_3 have been closed for 1 sec and S_2 remained open. Just after 1 second is over switch S_2 is closed and S_1, S_3 are opened. The charge on the upper plate of the capacitor as function of time taking the instant of switching on of S_2 and switching off all the switches to be t = 0 is q = x × 10^{-2} $\cos\left(t+\frac{\pi}{4}\right)$. Findout value of x. (Given $\left(1-\frac{1}{e}\right)\sqrt{2} \cong 0.89$)



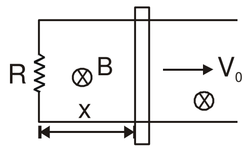
PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A resistance R is connected between the two ends of the parallel smooth conducting rails. A conducting rod lies on these fixed horizontal rails and a uniform constant magnetic field B exists perpendicular to the plane of the rails as shown in the figure. If the rod is given a velocity v and released as shown in figure, it will stop after some time, which option are correct:



(A) The total work done by magnetic field is negative.
(B) The total work done by magnetic field is positive.
(C) The total work done by magnetic field is zero.
(D) loss in kinetic energy of conducting rod is equal to heat generate between R.

2. A conducting rod of length ℓ is moved at constant velocity 'v_0' on two parallel, conducting, smooth, fixed rails, that are placed in a uniform constant magnetic field B perpendicular to the plane of the rails as shown in figure. A resistance R is connected between the two ends of the rail. Then which of the following is/are correct :

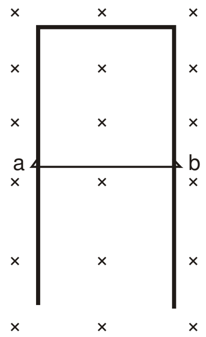


(A) The thermal power dissipated in the resistor is equal to rate of work done by external person pulling the rod.
(B) If applied external force is doubled than a part of external power increases the velocity of rod.
(C) Lenz's Law is not satisfied if the rod is accelerated by external force
(D) If resistance R is doubled then power required to maintain the constant velocity v_0 becomes half.

3. A conducting ring is placed in a uniform magnetic field with its plane perpendicular to the field. An emf is induced in the ring if
(A) it is rotated about its axis
(B) it is translated
(C) it is rotated about a diameter
(D) it is deformed

4. A conducting loop rotates with constant angular velocity about its fixed diameter in a uniform magnetic field in a direction perpendicular to that fixed diameter.
(A) The emf will be maximum at the moment when flux is zero.
(B) The emf will be '0' at the moment when flux is maximum.
(C) The emf will be maximum at the moment when plane of the loop is parallel to the magnetic field
(D) The phase difference between the flux and the emf is $\pi/2$

5. A copper wire ab of length ℓ, resistance r and mass m starts sliding at t = 0 down a smooth, vertical, thick pair of connected condcuting rails as shown in figure. A uniform magnetic field B exists in the space in a direction perpendicular to the plane of the rails which opctions are correct.



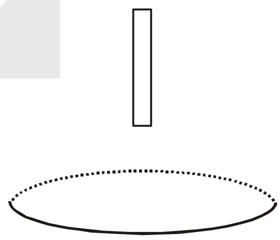
(A) The magnitude and direction of the induced current in the wire when speed of the wire v is $\frac{vB\ell}{r}$, a to b
(B) The downward acceleration of the wire at this instant g $-\frac{B^2\ell^2}{mr}$ v.
(C) The velocity of the wire as a function of time $v_m(1 - e^{-gt/v_m})$, (where $v_m = \frac{mgr}{B^2\ell^2}$)
(D) The displacement of the wire as a function of time $v_m t - \frac{{v_m}^2}{g}(1 - e^{-gt/v_m})$, (where $v_m = \frac{mgr}{B^2\ell^2}$)

6. A super conducting loop having an inductance 'L' is kept in a magnetic field which is varying with respect to time. If ϕ is the total flux, ε = total induced emf, then:
(A) ϕ = constant (B) I = 0 (C) $\varepsilon = 0$ (D) $\varepsilon \neq 0$

7. An LR series circuit with a battery is connected at t =0. Which of the following quantities are zero just after the connection ?
(A) current in the circuit (B) magnetic field energy in the inductor
(C) power delivered by the battery (D) emf induced in the inductor

8. An LR series circuit has L = 1 H and R = 1 Ω. It is connected across an emf of 2 V. The maximum rate at which energy is stored in the magnetic field is :
(A) The maximum rate at which energy is stored in the magnetic field is 1W
(B) The maximum rate at which energy is stored in the magnetic field is 2W
(C) The current at that instant is 1 A
(D) The current at that instant is 2 A

9. In figure a bar magnet is moved along the axis of a copper ring, an anticlockwise (as seen from the side of magnet) current is found to be induced in the ring. Which of the following may be true ?



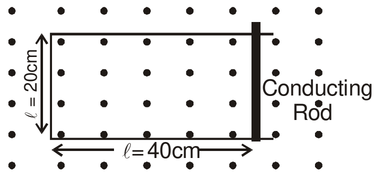
(A) The north pole faces the ring and the magnet moves away from it.
(B) The north pole faces the ring and the magnet moves towards it
(C) The south pole faces the ring and the magnet moves away from it.
(D) The south pole faces the ring and the magnet moves towards it

10. Two different coils have self-inductance L_1 = 8 mH, L_2 = 2 mH. The current in one coil is increased at a constant rate. The current in the second coil is also increased at the same rate. At a certain instant of time, the power given to the two coils is the same. At that time the current, the induced voltage and the energy stored in the first coil are i_1, V_1 and W_1 respectively. Corresponding values for the second coil at the same instant are i_2, V_2 and W_2 respectively. Then
(A) $\frac{i_1}{i_2} = \frac{1}{4}$ (B) $\frac{i_1}{i_2} = 4$ (C) $\frac{W_2}{W_1} = 4$ (D) $\frac{V_2}{V_1} = \frac{1}{4}$

PART - IV : COMPREHENSION

COMPREHENSION-1.

Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance 1Ω/m. Position of the conducting rod at t = 0 is shown. A time t dependent magnetic field B = 2t Tesla is switched on at t = 0.



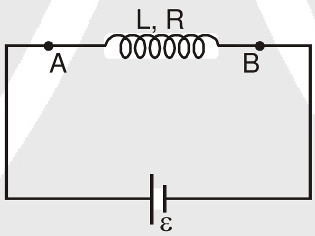
1. The current in the loop at t = 0 due to induced emf is
(A) 0.16 A, clockwise (B) 0.08 A, clockwise
(C) 0.08 A, anticlockwise (D) zero

2. At t = 0, when the magnetic field is switched on, the conducting rod is moved to the left at constant speed 5 cm/s by some external means. The rod moves remaining perpendicular to the rails. At t = 2s, induced emf has magnitude.
(A) 0.12 V (B) 0.08 V (C) 0.04 V (D) 0.02 V

3. Following situation of the previous question, the magnitude of the force required at the same instant t = 2s to move the conducting rod at constant speed 5 cm/s, is equal to
(A) 0.16 N (B) 0.12 N (C) 0.08 N (D) 0.06 N

COMPREHENSION 2

An inductor having self inductance L with its coil resistance R is connected across a battery of emf ε. When the circuit is in steady state at t = 0 an iron rod is inserted into the inductor due to which its inductance becomes n L (n > 1).



4. After insertion of rod which of the following quantities will change with time ?
(1) Potential difference across terminals A and B.
(2) Inductance.
(3) Rate of heat produced in coil
(A) only (1) (B) (1) & (3) (C) Only (3) (D) (1), (2) & (3)

5. After insertion of the rod, current in the circuit :
(A) Increases with time (B) Decreases with time
(C) Remains constant with time (D) First decreases with time then becomes constant

6. When again circuit is in steady state, the current in it is :
(A) $I < \varepsilon/R$ (B) $I > \varepsilon/R$ (C) $I = \varepsilon/R$ (D) None of these

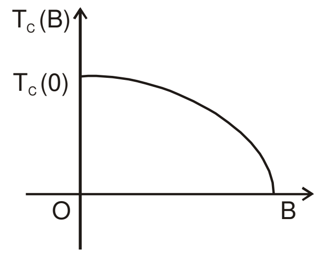
Exercise-3

✍ **Marked Questions may have for Revision Questions.**
*** Marked Questions may have more than one correct option.**

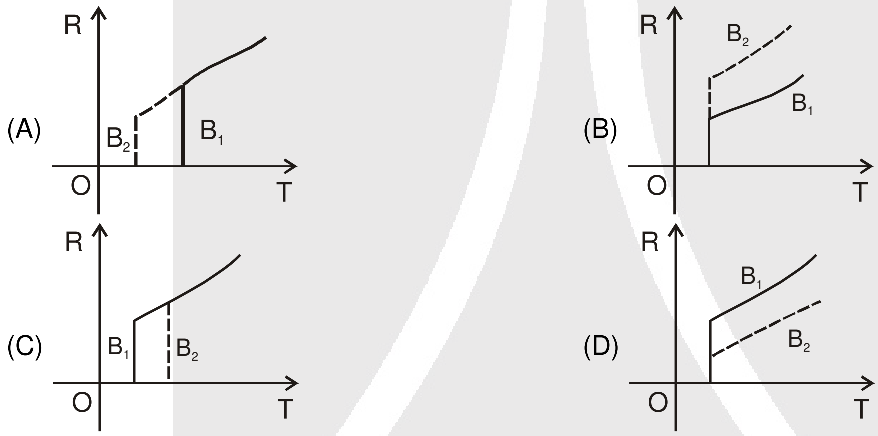
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

Paragraph for Question No. 1 to 2

Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature T_C (0). An interesting property of superconductors is that their critical temperature becomes smaller than T_C (0) if they are placed in a magnetic field, i.e., the critical temperature T_C (B) is a function of the magnetic field strength B. The dependence of T_C (B) on B is shown in the figure.



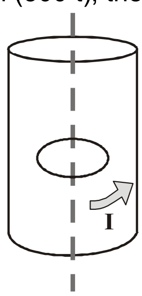
1. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1 (solid line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields? **[JEE 2010' 3/163, –1]**



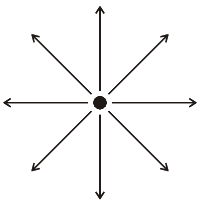
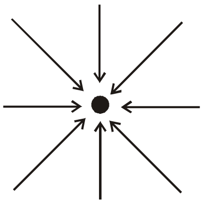
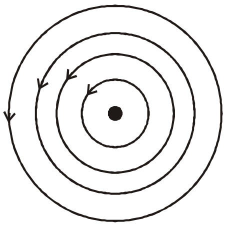
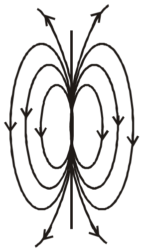
2. A superconductor has T_C (0) = 100 K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when : **[JEE 2010' 3/163, –1]**

(A) B = 5 Tesla, T_C (B) = 80 K
(B) B = 5 Tesla, 75 K < T_C (B) < 100 K
(C) B = 10 Tesla, 75 K < T_C (B) < 100 K
(D) B = 10 Tesla, T_C (B) = 70 K

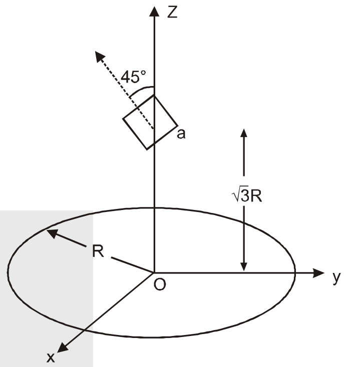
3.✍ A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0 \cos (300\ t)$ where I_0 is constant. If the magnetic moment of the loop is $N\ \mu_0\ I_0 \sin (300\ t)$, then 'N' is **[JEE 2011' 4/160]**



4. Which of the field patterns given below is valid for electric field as well as for magnetic field?
[JEE 2011' 3/160, –1]

(A)  (B)  (C)  (D) 

5. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side a (a << R) having two turns is placed with its center at $z = \sqrt{3}\,R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is
[IIT-JEE-2012, Paper-1; 4/70]



6. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement (s) is (are) : **[IIT-JEE-2012, Paper-2; 4/66]**
(A) the emf induced in the loop is zero if the current is constant.
(B) The emf induced in the loop is finite if the current is constant.
(C) The emf induced in the loop is zero if the current decreases at a steady rate.
(D) The emf induced in the loop is finite if the current decreases at a steady rate.

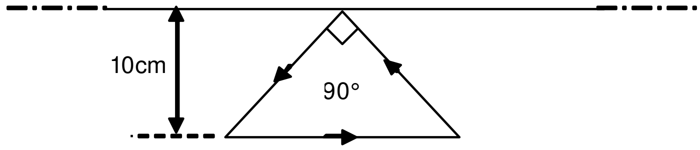
Paragraph for Questions 7 and 8

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that, for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionally constant γ.
[JEE (ADV.) 2013, 3×2/60]

7. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is :
(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

8. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change, is :
(A) $-\gamma BQR^2$ (B) $-\gamma\frac{BQR^2}{2}$ (C) $\gamma\frac{BQR^2}{2}$ (D) γBQR^2

9. A conducting loop in the shape of a right angled isosceles triangle of height 10cm is kept such that the 90° vertex is very closed to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 A s^{-1}. Which of the following statement (s) is (are) true ? **[JEE (Advanced) 2016 ; P-1, 4/62, –2]**



(A) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

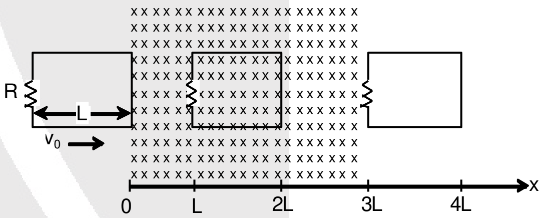
(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire.

(C) The induced current in the wire is in opposite direction to the current along the hypotenuse.

(D) There is a repulsive force between the wire and the loop.

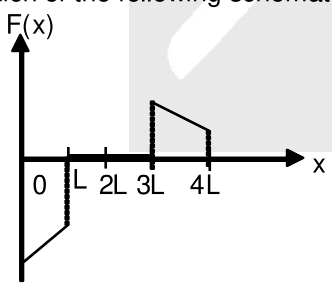
10. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2 mH, internal resistance 4 Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is : **[JEE (Advanced) 2016 ; P-1, 3/62]**

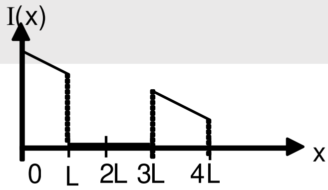
11. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper; as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region.

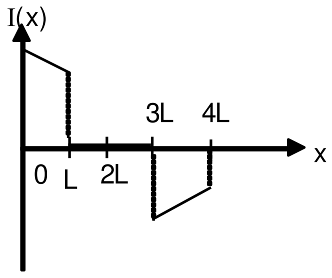


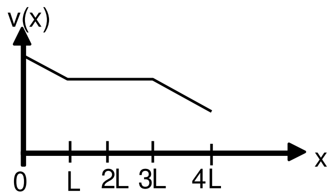
Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. **[JEE (Advanced) 2016 ; P-2, 4/62, –2]**

Which of the following schematic plot(s) is(are) correct? (Ignore gravity)

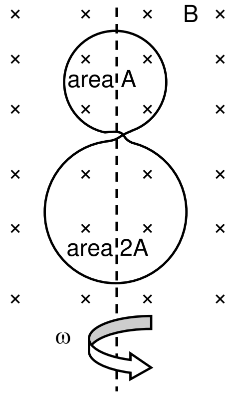
(A)


(B)


(C)


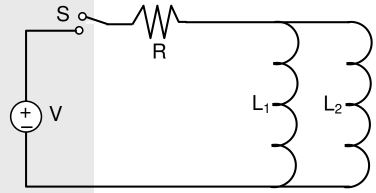
(D)


12*. A circular insulated copper wire loop is twisted to form two loops of are A and 2A as shown in the figure. At the point of crossing the wires remain electrically insulated from each other. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotation about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct ? **[JEE (Advanced) 2017 ; P-1, 4/61, –2]**



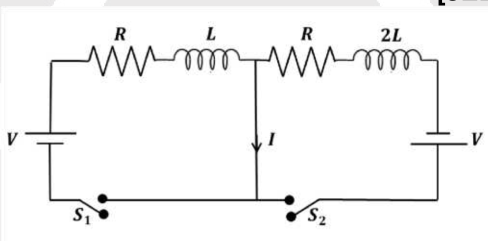
(A) The net emf induced due to both the loops is proportional to cos ωt
(B) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper
(C) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone
(D) The emf induced in the loop is proportional to the sum of the area of the two loops

13*. A source of constant voltage V is connected to a resistance R and two ideal inductor L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At t = 0, the switch is closed and current begins to flow. Which of the following options is/are correct ? **[JEE (Advanced) 2017 ; P-2, 4/61, –2]**



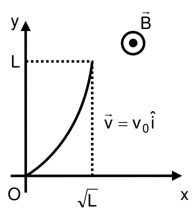
(A) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$
(B) At t = 0, the current through the resistance R is V/R
(C) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$
(D) The ratio of the currents through L_1 and L_2 is fixed at all times (t > 0)

14*. In the figure below, the switches S_1 and S_2 are closed simultaneously at t = 0 and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time t = τ. Which of the following statements is (are) true? **[JEE (Advanced) 2018 ; P-1, 4/60, –2]**



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

15. A conducting wire of parabolic shape, y = x^2, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statements(s) is/are :



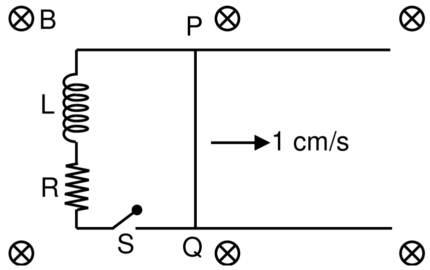
(1) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for β = 2 **[JEE (Advanced) 2019 ; P-1, 4/62, –1]**

(2) $|\Delta\phi|$ remains same if the parabolic wire is replaced by a straight wire, y = x, initially, of length $\sqrt{2}\ell$

(3) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for β = 0

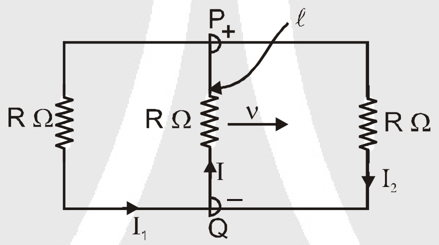
(4) $|\Delta\phi|$ is proportional to the length of wire projected on y-axis

16. A 10 cm long perfectly conducting wire PQ is moving with a velocity 1 cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is x × 10^{-3}A, where the value of x is _____________.
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed. Given : e^{-1} = 0.37, where e is base of the natural logarithm] **[JEE (Advanced) 2019 ; P-2, 3/62,]**



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. A rectangular loop has a sliding connector PQ of length ℓ and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are : **[AIEEE 2010, 8/144, –2]**



(1) $I_1 = -I_2 = \frac{B\ell v}{R}$, $I = \frac{2B\ell v}{R}$ (2) $I_1 = I_2 = \frac{B\ell v}{3R}$, $I = \frac{2B\ell v}{3R}$

(3) $I_1 = I_2 = I = \frac{B\ell v}{R}$ (4) $I_1 = I_2 = \frac{B\ell v}{6R}$, $I = \frac{B\ell v}{3R}$

2. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is : **[AIEEE 2011, 1 May, 4/120, –1]**

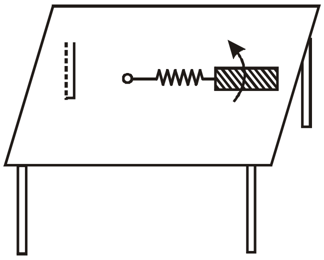
(1) $\pi\sqrt{LC}$ (2) $\frac{\pi}{4}\sqrt{LC}$ (3) $2\pi\sqrt{LC}$ (4) $\sqrt{LC}$

3. A boat is moving due east in a region where the earth's magnetic field is 5.0 × 10^{-5} $NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is : **[AIEEE - 2011, 1 May, 4/120, –1]**
(1) 1 mV (2) 0.75 mV (3) 0.50 mV (4) 0.15 mV

4. A horizontal straight wire 20 m long extending from to east to west falling with a speed of 5.0 m\s, at right angles to the horizontal component of the earth's magnetic field 0.30 × 10^{-4} Wb/m^2. The instantaneous Value of the e.m. f. induced in the wire will be : **[AIEEE 2011, 11 May; 4/120, –1]**
(1) 3 mV (2) 4.5 mV (3) 1.5 mV (4) 6.0mV

5. A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to : **[AIEEE 2012 ; 4/120, –1]**
(1) developement of air current when the plate is placed.
(2) induction of electrical charge on the plate
(3) shielding of magnetic lines of force as aluminium is a paramagnetic material.
(4) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping.

6. A metallic rod of length 'l' is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is: **[JEE (Main) 2013, 4/120, –1]**

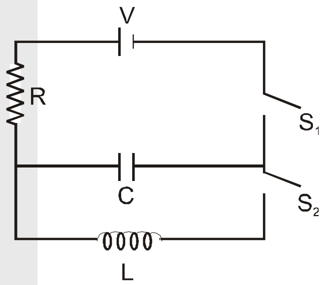


(1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$

7. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is : **[JEE (Main) 2013, 4/120]**

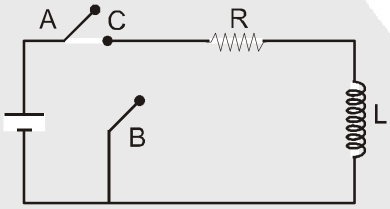
(1) 9.1×10^{-11} weber (2) 6×10^{-11} weber (3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber

8. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open. (q is charge on the capacitor and τ = RC is Capacitive time constant). Which of the following statement is correct ? **[JEE (Main) 2013, 4/120]**



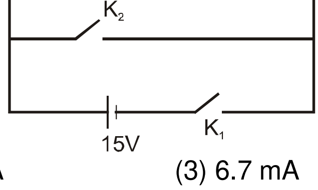
(1) Work done by the battery is half of the energy dissipated in the resistor
(2) At $t = \tau$, $q = CV/2$
(3) At $t = 2\tau$, $q = CV(1 - e^{-2})$
(4) At $t = \tau/2$, $q = CV(1 - e^{-1})$

9. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to : **[JEE (Main) 2014, 4/120, –1]**



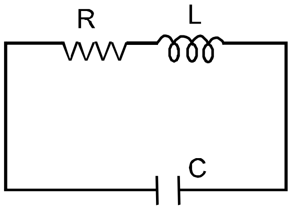
(1) $\frac{e}{1-e}$ (2) 1 (3) –1 (4) $\frac{1-e}{e}$

10. An inductor (L = 0.03H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1ms, the current in the circuit will be : ($e^5 \cong 150$) **[JEE(Main) 2015; 4/120, –1]**

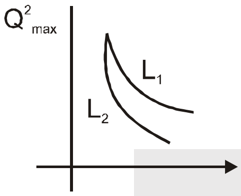


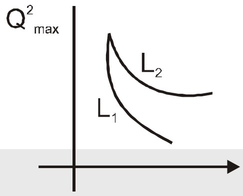
(1) 100 mA (2) 67 mA (3) 6.7 mA (4) 0.67 mA

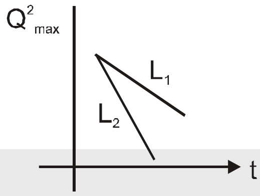
11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below : **[JEE(Main) 2015; 4/120, –1]**

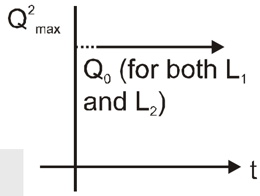


If a student plots graphs of the square of maximum charge (Q^2_{Max}) on the capacitor with time (t) for two different values L_1 and L_2($L_1 > L_2$) of L then which of the following represents this graph correctly ? (plots are schematic and not drawn to scale)

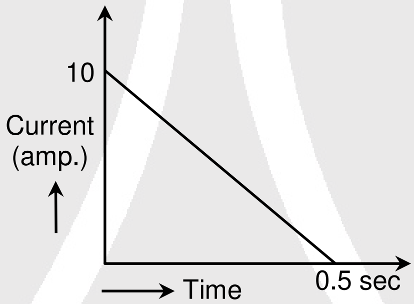
(1) 

(2) 

(3) 

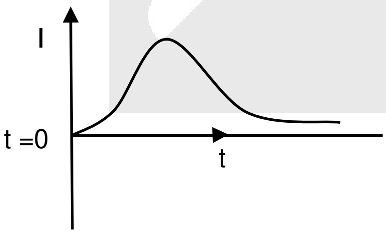
(4) 

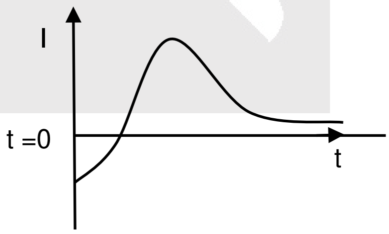
12. In a coil resistance 100Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is : **[JEE (Main) 2017, 4/120, –1]**

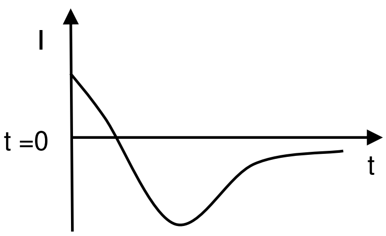


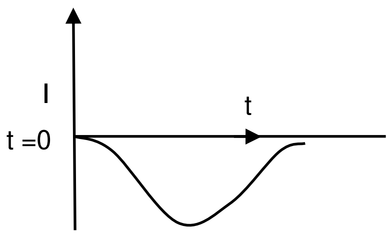
(1) 275 Wb (2) 200 Wb (3) 225 Wb (4) 250 Wb

13. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}$ ($k > 0$), as a function of time ($t \geq 0$). Counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly, as a function of time by : **[JEE (Main) 2019 April, 4/120, –1]**

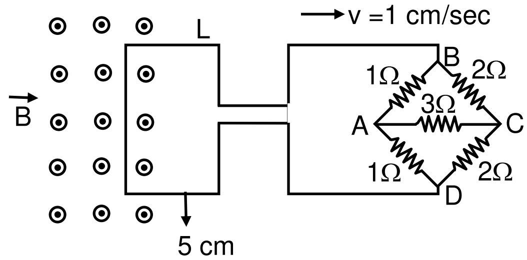
(1) 

(2) 

(3) 

(4) 

14. The figure shows a square loop L of side 5 cm which is connected to a network of resistance. The whole setup is moving towards right with a constant speed of 1 cm s^{-1}. At some instant, a part of L is in a uniform magnetic field of 1 T, perpendicular to the plane of the loop. If the resistance of L is 1.7 Ω, the current in the loop at that instant will be close to : **[JEE (Main) 2019 April, 4/120, –1]**



(1) 170 μA (2) 150μA (3) 115μA (4) 60μA

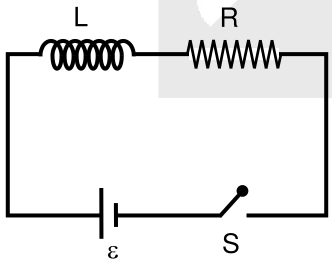
15. A long solenoid of radius R carries a time(t)- dependent current $I(t) = I_0t(1 - t)$. A ring of radius 2R is placed coaxially near its middle. During the time interval $0 \le t \le 1$, the induced current (I_R) and the induced EMF(V_R) in the ring change as : **[JEE (Main) 2020, 07 January; 4/100, –1]**
(1) Direction of I_R remains unchanged and V_R is zero at t = 0.25
(2) At t = 0.5 direction of I_R reverses and V_R is zero
(3) At t = 0.25 direction of I_R reverses and V_R is maximum
(4) Direction of I_R remains unchanged and V_R is maximum at t = 0.5

16. A loop ABCDEFA of straight edges has six corner points A(0, 0, 0), B(5, 0, 0), C(5, 5, 0), D(0, 5, 0), E(0, 5, 5) and F(0, 0, 5). The magnetic field in this region is $\vec{B} = \left(3\hat{i} + 4\hat{k}\right)T$. The quantity of flux through the loop ABCDEFA (in Wb) is ……. . **[JEE (Main) 2020, 07 January; 4/100]**

17. A planar loop of wire rotates in a uniform magnetic field. Initially at t = 0, the plane of the loop is perpendicular to the magnetic field. If it rotates with a period of 10 s about an axis in its plane then the magnitude of induced emf will be maximum and minimum, respectively at :
[JEE (Main) 2020, 07 January; 4/100, –1]
(1) 5.0 s and 7.5 s (2) 5.0 s and 10.0 s (3) 2.5 s and 7.5 s (4) 2.5 s and 5.0 s

18. An emf of 20 V is applied at time t = 0 to a circuit containing in series 10 mH inductor and 5 Ω resistor. The ratio of the currents at time t = ∞ and at t = 40 s is close to : (Take $e^2 = 7.389$)
[JEE (Main) 2020, 07 January; 4/100, –1]
(1) 1.46 (2) 1.06 (3) 0.84 (4) 1.15

19.



As shown in the figure, a battery of emf ε is connected to an inductor L and resistance R in series. The switch is closed at t = 0. The total charge that flows from the battery, between t = 0 and t = t_c (t_c is the time constant of the circuit) is : **[JEE (Main) 2020, 08 January; 4/100, –1]**

(1) $\frac{\varepsilon L}{R^2}\left(1-\frac{1}{e}\right)$ (2) $\frac{\varepsilon R}{eL^2}$ (3) $\frac{\varepsilon L}{R^2}$ (4) $\frac{\varepsilon L}{eR^2}$

Answers

EXERCISE # 1
PART - I

Section (A)

A-2. 1.0 V, anticlockwise.

A-3. (i) 1.2 Volt (ii) 1.4 volt (iii) 17.5 C (iv) 3.5 A (v) 86/3 joule.

A-4. (a) –1 mV, –2 mV, 2 mV, 1mV
(b) 10 ms to 20 ms and 20 ms to 30 ms.

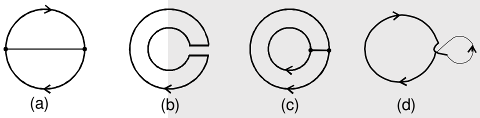
A-5. zero

A-6. 2.5 mV

A-7. 1.6×10^{-5}A

A-8. 493 μV

A-9.

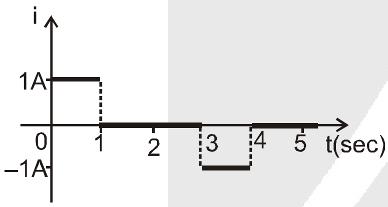


A-10. $\frac{\pi}{8}\times 10^{-4}$ A

A-11. $\frac{3}{2}\frac{\mu_0 \pi R^2 r^2 NI y v}{\left(R^2+y^2\right)^{5/2}}$

A-12. $25\pi\times10^{-3}$ C = 0.078 C

A-13.



A-14. 2 J

Section (B)

B-1. Opposite direction, Same direction.

Section (C)

C-1. 4 mV, Q

C-2. (a) zero (b) vB (bc), positive at b (c) vB(bc), positive at a (d) zero

C-3. $\sqrt{3}\times10^{-2}$ V

C-4 1 mV

C-5. (a) at the ends of the diameter perpendicular to the velocity, 2rvB (b) at the ends of the diameter parallel to the velocity, zero.

C-6. By $\sqrt{8a/k}$

Section (D)

D-1. (a) 4 m/s (b) 4 V (c) 3 V (d) 1 V.

D-2. (a) 0.1 mA (b) 0.2 mA

D-3. (a) 1/r ($\varepsilon - vB\ell$), from b to a
(b) $\frac{\ell B}{r}$ ($\varepsilon - vB\ell$) towards right (c) $\frac{\varepsilon}{B\ell}$.

D-4. zero

D-5. $i = \frac{Bv\ell}{2(\ell+vt)r}$

D-6. (a) $\frac{B^2\ell^2 v}{2\ \ r(\ell+vt)}$ (b) ℓ/v.

D-7. $\frac{mgt}{m+CB^2\ell^2}$

Section (E)

E-1. (a) $\phi = \frac{\mu_0 i a}{2\pi}\ \ell n\left(\frac{a+b}{b}\right)$;
(b) $\varepsilon = \frac{\mu_0 i_0 a}{T}\ \ell n\left(\frac{a+b}{b}\right)\sin\left(\frac{2\pi t}{T}\right)$
(c) heat = $\left(\frac{5\mu_0^2\ \ i_0^2\ \ a^2}{Tr}\right)\left[\ell n\left(\frac{a+b}{b}\right)\right]^2$

E-2. $\frac{B_0 v_0 L}{2}$

E-3. (BV sin α) / r(1 + sin α)

Section (F)

F-1. 67.5 mV

F-2. $\frac{B\omega a^2}{R}$ from C to D

F-3. $3\pi\times10^{-6}$ V

F-4. $B\ell\sqrt{g\ell}\sin\frac{\theta}{2}$

F-5. (a) 2BRv (b) $\frac{BRv}{2}$ (c) $\frac{3\ \ BRv}{2}$

F-6. (a) 2.0×10^{-3} V (b) zero
(c) 50 μC (d) $\pi\times10^{-3}\sin(10\pi t)$
(e) π mV (f) $\frac{\pi^2}{2}\times10^{-6}$ V

Section (G)

G-1. **(a)** 6π Volt **(b)**3 N/C **(c)** 3 A

G-2 (a) $16\pi^2\times10^{-10}$ = 1.6×10^{-8} Weber
(b) $4\pi\times10^{-8}$ V/m (c) $18\pi\times10^{-8}$
= 5.6×10^{-7} V/m

Section (H)

H-1. 10V

H-2. 2.2 A/s, decreasing

H-3. (a) 5 W (b) 3W (c) 2 W

H-4. 2.55×10^{-14} J

H-5. $\frac{\mu_0 \quad e^4}{128\pi^3\varepsilon_0 mR^5}$

H-6. 42 + 20 t volt

H-7. 15V

Section (I)

I-1. (a) $\frac{1}{5}(1-e^{-2}) \simeq 0.17$ A

(b) $\frac{1}{25}(1-e^{-2})^2 \quad 0.03$ J

I–2. (L/R) ln 2 = 1.109 s, 640 J

I-3. t = (L/R) ℓn 2 = 3.47 s **I-4.** 4.0 H

I-5. $2\,[1 - e^{-0.4}] = 0.66$ V **I-6.** $\frac{2}{e}$ A/s, 2/e

I-7. (a) -2.5×10^3 V/s (b) $-2.5\times 10^3 \times e^{-5}$ V/s

I–8. (a) $\frac{\varepsilon(R_1+R_2)}{R_1R_2}$ (b) $\frac{L}{R_1+R_2}$ (c) $\frac{\varepsilon}{R_1 e}$

I–9. $\frac{2\,B\,\pi\,R^2}{L}$

I-10. (a) $i_1 = i_2 = \frac{10}{3} = 3.33$ A

(b) $i_1 = \frac{50}{11} = 4.55$ A; $i_2 = \frac{30}{11} = 2.73$ A

(c) $i_1 = 0$, $i_2 = -\frac{20}{11} = -1.82$ A

(d) $i_1 = i_2 = 0$

I -11. $\frac{\varepsilon}{3}, \frac{2\,\varepsilon}{3\,L}$ **I -12.** $L_{eq} = \frac{L}{2}$

I-13. (b) Separation is large to neglect mutual inductance

Section (J)

J-1. 0.01 H **J-2.** 2.5 V

J-3. $\frac{2\sqrt{2}\mu_0\ell^2}{\pi L}$

Section (K)

K-1. (a) 1.0 J. Yes, sum of the energies stored in L and C is conserved if R = 0.

(b) $\omega = 10^3$ rads^{-1}, ν = 159 Hz

(c) $q = q_0 \cos \omega t$

(i) Energy stored is completely electrical at

$t = 0, \frac{T}{2}, T, \frac{3T}{2}, ...$

(ii) Energy stored is completely magnetic (i.e., electrical energy is zero) at

$t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4} ......,$ where $T = \frac{1}{\nu} = 6.3$ ms

(d) At $t = \frac{T}{8}, \frac{3T}{8}, \frac{5T}{8}, ...,$ because $q = q_0$

$\cos\omega t = \frac{q_0}{\sqrt{2}}$ (when energy shared equally between the inductor and the capacitor).

K-2. $\frac{\pi \times 10^{-5}}{3}$ sec. **K-3.** 88 pF to 198 pF

PART - II

Section (A)

A-1. (D) **A-2.** (D) **A-3.** (C)
A-4. (C) **A-5.** (A)

Section (B)

B-1. (C) **B-2.** (A) **B-3.** (A)
B-4. (A) **B-5.** (D) **B-6.** (C)
B-7. (A) **B-8.** (B)

Section (C)

C-1. (D) **C-2.** (B) **C-3.** (B)
C-4. (B) **C-5.** (D)

Section (D)

D-1. (D) **D-2.** (A) **D-3.** (D)
D-4. (B)

Section (E)

E-1. (C) **E-2.** (D)

Section (F)

F-1. (D) **F-2.** (C) **F-3.** (B)
F-4. (A) **F-5.** (C)

Section (G)

G-1. (A) **G-2.** (B) **G-3.** (A)
G-4. (C) **G-5.** (B)

Section (H)

H-1. (A) **H-2.** (D)

Section (I)

I -1.	(B)	**I-2.**	(C)	**I-3.**	(B)
I-4.	(A)	**I-5.**	(A)	**I-6.**	(B)
I -7.	(A)	**I-8.**	(A)	**I-9.**	(C)

Section (J)

J-1.	(D)	**J-2.**	(A)	**J-3.**	(A)
J-4.	(D)				

Section (K)

K-1.	(B)	**K-2.**	(C)	**K-3.**	(A)

PART - III

1. (A) q (B) p (C) t (D) t
2. (A) q,s (B) p,r (C) p,r (D) q,s

EXERCISE # 2

PART - I

1.	(D)	**2.**	(B)	**3.**	(C)
4.	(A)	**5.**	(B)	**6.**	(B)
7.	(C)	**8.**	(A)	**9.**	(C)
10.	(C)	**11.**	(B)	**12.**	(B)
13.	(A)	**14.**	(A)	**15.**	(A)
16.	(D)	**17.**	(D)		

PART - II

1.	03.00	**2.**	90.00	**3.**	40.00
4.	25.00	**5.**	03.00	**6.**	12.00
7.	04.00	**8.**	03.00	**9.**	12.00
10.	67.00	**11.**	03.00	**12.**	04.00
13.	12.00	**14.**	64.00	**15.**	03.00
16.	89.00				

PART - III

1.	(CD)	**2.**	(A)	**3.**	(CD)
4.	(ABCD)	**5.**	(ABCD)	**6.**	(AC)
7.	(ABC)	**8.**	(AC)	**9.**	(BC)
10.	(ACD)				

PART - IV

1.	(A)	**2.**	(B)	**3.**	(C)
4.	(C)	**5.**	(A)	**6.**	(C)

EXERCISE # 3

PART - I

1.	(A)	**2.**	(B)	**3.**	**6**
4.	(C)	**5.**	**7**	**6.**	(AC)
7.	(B)	**8.**	(B)	**9.**	(AD)
10.	(8)	**11.**	(CD)	**12.**	(BC)
13.	(ACD)	**14.**	(BD)	**15.**	(ABD)
16.	0.63				

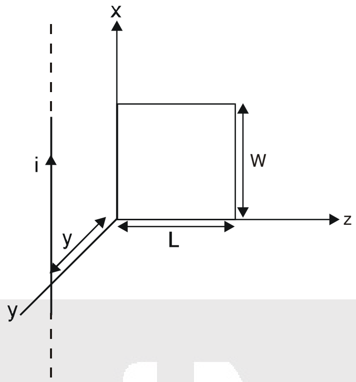
PART - II

1.	(2)	**2.**	(2)	**3.**	(4)
4.	(1)	**5.**	(4)	**6.**	(4)
7.	(1)	**8.**	(3)	**9.**	(3)
10.	(4)	**11.**	(1)	**12.**	(4)
13.	(2)	**14.**	(1)	**15.**	(2)
16.	175	**17.**	(4)	**18.**	(2)
19.	(4)				

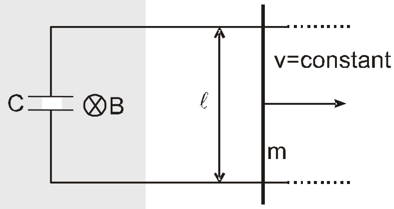
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

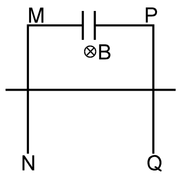
1. In the figure, a long thin wire carrying a varying current $i = i_0 \sin \omega t$ lies at a distance y above one edge of a rectangular wire loop of length L and width W lying in the x-z plane. What emf is induced in the loop.



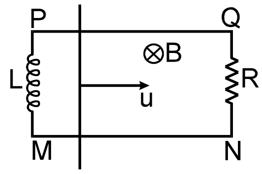
2. In the figure shown a conducting rod of length ℓ, resistance R and mass m is moved with a constant velocity v. The magnetic field B varies with time t as B = 5 t, where t is time in second. At t = 0 the area of the loop containing capacitor and the rod is zero and the capacitor is uncharged. The rod started moving at t = 0 on the fixed smooth conducting rails which have negligible resistance. Find the current in the circuit as a function of time t.



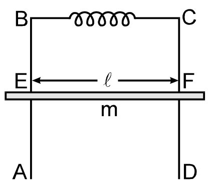
3. In the figure shown a conducting rod of length ℓ, resistance R & mass m can move vertically downward due to gravity. Other parts are kept fixed. B = constant = B_0. MN and PQ are vertical, smooth, conducting rails. The capacitance of the capacitor is C. The rod is released from rest. Find the maximum current in the circuit.



4. In the figure, a conducting rod of length ℓ = 1 meter and mass m = 1 kg moves with initial velocity u = 5 m/s. on a fixed horizontal frame containing inductor L = 2 H and resistance R = 1 Ω. PQ and MN are smooth, conducting wires. There is a uniform magnetic field of strength B = 1T. Initially there is no current in the inductor. Find the total charge in coulomb, flown through the inductor by the time velocity of rod becomes v_f = 1 m/s and the rod has travelled a distance x = 3 meter.

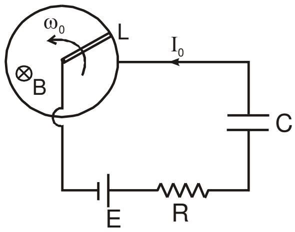


5. A conducting frame ABCD is kept fixed in a vertical plane. A conducting rod EF of mass m can slide smoothly on it remaining horizontal always. The resistance of the loop is negligible and inductance is constant having value L. The rod is left from rest and allowed to fall under gravity and inductor has no initial current. A uniform magnetic field of magnitude B is present throughout the loop pointing inwards. Determine.

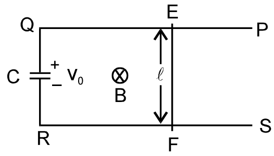


(a) position of the rod as a function of time assuming initial position of the rod to be x = 0 and vertically downward as the positive X-axis.
(b) maximum current in the circuit
(c) maximum velocity of the rod.

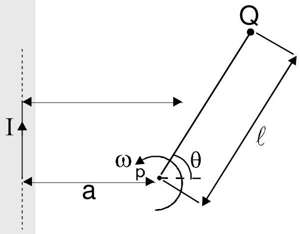
6. L is s smooth conducting loop of radius ℓ = 1.0 m & fixed in a horizontal plane. A conducting rod of mass m = 1.0 kg and length slightly greater than ℓ hinged at the centre of the loop can rotate in the horizontal plane such that the free end slides on the rim of the loop. There is a uniform magnetic field of strength B = 1.0 T directed vertically downward. The rod is rotated with angular velocity ω_0 = 1.0 rad/s and left. The fixed end of the rod and the rim of the loop are connected through a battery of e.m.f. E, a resistor of resistance R = 1.0 Ω, and initially uncharged capacitor of capacitance C = 1.0 F in series. Find :
(i) the time dependence of e.m.f. E such that the current I_0 = 1.0 A in the circuit is constant.
(ii) energy supplied by the battery by the time rod stops .



7. In the figure shown 'PQRS' is a fixed resistanceless conducting frame in a uniform and constant magnetic field of strength B. A rod 'EF' of mass 'm', length 'ℓ' and resistance R can smoothly move on this frame. A capacitor charged to a potential difference 'V_0' initially is connected as shown in the figure. Find the velocity of the rod as function of time 't' if it is released at t = 0 from rest.



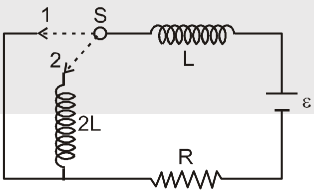
8. In the figure shown a long conductor carries constant current I. A rod PQ of length ℓ is in the plane of the rod. The rod is rotated about point P with constant angular velocity ω as shown in the figure. Find the e.m.f. induced in the rod in the position shown. Indicate which point is at high potential.



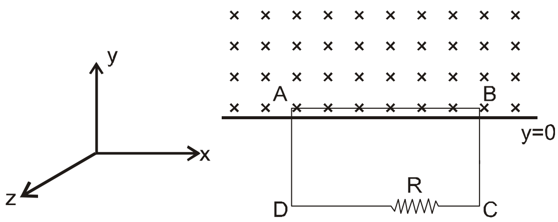
9. An infinitesimally small bar magnet of dipole moment M is moving with the speed v in the X-direction. A small closed circular conducting loop of radius 'a' and negligible self-inductance lies is the Y-Z plane with its centre at x = 0, and its axis coinciding with the X-axis. Find the force opposing the motion of the magnet, if the resistance of the loop is R. Assume that the distance x of the magnet from the centre of the loop is much greater than a.

10. A square loop of side a = 12 cm with its sides parallel to x, and y-axis is moved with velocity, V = 8 cm/s in the positive x direction in a magnetic field along the positive z-direction. The field is neither uniform in space nor constant in time. It has a gradient $\partial B/\partial x = -10^{-3}$ T/cm along the x-direction, and it is changing in time at the rate $\partial B/\partial t$ = 7T/sec in the loop if its resistance is R = 4.5 Ω. Find the current.

11. In the circuit shown, the switch S is shifted to position 2 from position 1 at t = 0, having been in position 1 for a long time. Find the current in the circuit as a function of time.



12. A square loop ABCD of side ℓ is moving in xy plane with velocity $\vec{v} = \beta t\ \hat{j}$. There exists a non-uniform magnetic field $\vec{B} = -B_0(1 + \alpha y^2)\ \hat{k}$ (y > 0), where B_0 and α are positive constants.Initially, the upper wire of the loop is at y = 0. Find the induced voltage across the resistance R as a function of time. Neglect the magnetic force due to induced current.

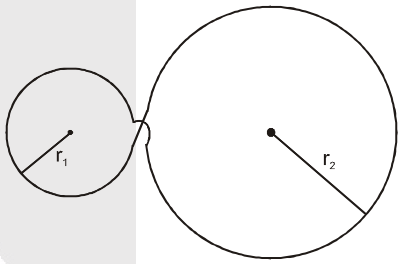


13. A thin wire ring of radius a and resistance r is located inside a long solenoid so that their axes coincide. The length of the solenoid is equal to ℓ, its cross-sectional radius, to b. At a certain moment the solenoid was connected to a source of a constant voltage V. The total resistance of the circuit is equal to R. Assuming the inductance of the ring to be negligible, find the maximum value of the radial force acting per unit length of the ring.

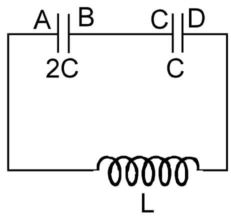
14. A long cylinder of radius a carrying a uniform surface charge rotates about its axis with an angular velocity ω. Find the magnetic field energy per unit length of the cylinder if the linear charge density equals λ and $\mu_r = 1$.

15. A long solenoid of length ℓ = 2.0m, radius r = 0.1m and total number of turns N = 1000 is carrying a current i_0 = 20.0A. The axis of the solenoid coincides with the z-axis.
(a) State the expression for the magnetic field of the solenoid and calculate its value?
Magnetic field
(b) Obtain the expression for the self-inductance (L) of the solenoid. Calculate its value.
Value of L
(c) Calculate the energy stored (E) when the solenoid carries this current?
(d) Let the resistance of the solenoid be R. It is connected to a battery of emf e. Obtain the expression for the current (i) in the solenoid.
(e) Let the solenoid with resistance R described in part (d) be stretched at a constant speed υ (ℓ is increased but N and γ are constant). State Kirchhoff's second law for this case. (Note: Do not solve for the current.)
(f) Consider a time varying current $i = i_0 \cos(\omega t)$ (where i_0 = 20.0A) flowing in the solenoid. Obtain an expression for the electric field due to the current in the solenoid. (Note: Part (e) is not operative, i.e. the solenoid is not being stretched.)
(g) Consider $t = \pi/2\omega$ and $\omega = 200/\pi$ rad–s^{-1} in the previous part. Plot the magnitude of the electric field as a function of the radial distance from the solenoid. Also, sketch the electric lines of force.

16. The wire loop shown in the figure lies in uniform magnetic induction $B = B_0\cos \omega t$ perpendicular to its plane. (Given r_1 = 10 cm and r_2 = 20 cm, B_0 = 20 mT and $\omega = 100\ \pi$). Find the amplitude of the current induced in the loop if its resistance is 0.1 Ω/m.



17. Two capacitors of capacitances 2C and C are connected in series with an inductor of inductance L. Initially capacitors have charge such that $V_B - V_A = 4V_0$ and $V_c – V_D = V_0$. Initial current in the circuit is zero. Find:
(a) Maximum current that will flow in the circuit.
(b) Potential difference across each capacitor at that instant.
(c) equation of current flowing towards left in the inductor.



HLP Answers

1. $\dfrac{\mu_0 i_0 W\omega\cos\omega t}{4\pi}\,\ell n\left(\dfrac{L^2}{Y^2}+1\right)$

2. $i = 10\,\ell\, v\, c\ (1 - e^{-t/Rc})$

3. $i_{max} = \dfrac{mg\,B\,\ell\, c}{m + B^2\,\ell^2\, c}$

4. $Q = \dfrac{-\dfrac{B^2\ \ell^2}{R}\ x - \ m\ \ (v_f\ -\ \ u)}{B\ \ell} = 1C.$

5. (a) $x = \dfrac{g}{\omega^2}[1-\cos\omega t]$, (b) $I_{max} = \dfrac{2mg}{B\ell}$, (c) $V_{max} = \dfrac{g}{\omega}$

6. **(i)** $\dfrac{1}{2}+\dfrac{7t}{4}$ (ii) $\dfrac{13}{18}$ J

7. $v = \dfrac{B\ell C\ \ V_0}{m+B^2\ell^2 C}\left(1\ -\ e^{-\left(\frac{B^2\ \ell^2}{mR}+\frac{1}{RC}\right)t}\right)$

8. $\dfrac{\mu_0 i\omega}{2\pi\cos\theta}\left[\ell - \dfrac{a}{\cos\theta}\ell n\left(\dfrac{a+\ell\cos\theta}{a}\right)\right]$

9. $\dfrac{9\mu_0{}^2M^2a^4v}{4\ \ Rx^8}$

10. 22.4 mA

11. $I = \dfrac{\varepsilon}{R}\left(1-\dfrac{2}{3}\times e^{\frac{-Rt}{3L}}\right)$

12. $\varepsilon = -B_0 I\beta\left(t+\dfrac{\alpha\times\beta^2 t^5}{4}\right)$

13. $\dfrac{\mu_0 a^2 V^2}{4rRIb^2}$

14. $\dfrac{\mu_0 a^2\omega^2\lambda^2}{8\pi}$

16. π ampere.

17. (a) $I_{max} = \left(\sqrt{\dfrac{6C}{L}}\right) v_0$; (b) $3v_0$, $3v_o$; (c) $i = I_{max}\sin\omega t$; $\omega = \left(\sqrt{\dfrac{3}{2LC}}\right)$