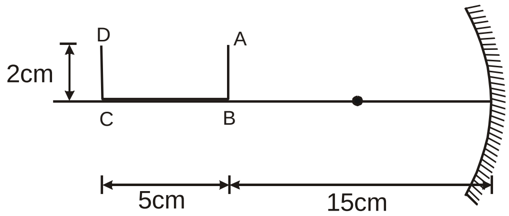
High Level Problems (HLP)

Marked Questions can be used as Revision Questions.

SUBJECTIVE QUESTIONS

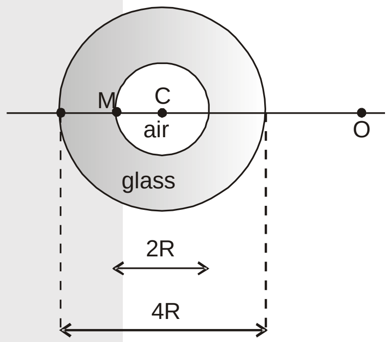
1. A U-shaped wire is placed before a concave mirror having radius of curvature 20 cm as shown in figure. Find the total length of the image.



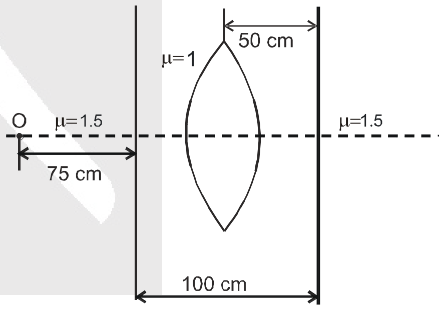
2. (i) A paper weight of refractive index n = 3/2 in the form of a hemisphere of radius 3.0 cm is used to hold down a printed page. An observer looks at the page vertically through the paperweight. At what height above the page will the printed letters near the centre appear to the observer?

(ii) Solve the previous problem if the paperweight is inverted at its place so that the spherical surface touches the paper.

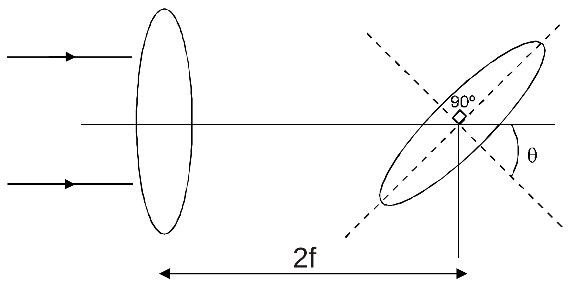
3. In the given figure, a hollow sphere of glass of refractive index n has a small mark M on its interior surface which is observed by an observer O from a point outside the sphere. C is centre of the sphere. The inner cavity (air) is concentric with the external surface and thickness of the glass is everywhere equal to the radius of the inner surface. Find the distance by which the mark will appear nearer than it really is, in terms of n and R assuming paraxial rays.



4. Two media each of refractive index 1.5 with plane parallel boundaries are separated by 100 cm. A convex lens of focal length 60 cm is placed midway between them with its principal axis normal to the boundaries. A luminous point object O is placed in one medium on the axis of the lens at a distance 125 cm from it. Find the position of its image formed as a result of refraction through the system.



5. Two converging lenses of the same focal length f are separated by distance 2 f as shown in figure. The axis of the second lens is inclined at small angle θ with respect to the axis of the first lens. A parallel paraxial beam of light is incident from left side on the lens. Find the coordinates of the final image with respect to the origin of the first lens.

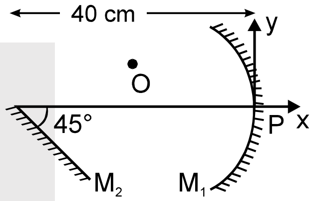


6. Two plane mirrors form an angle of 120°. The distance between the two images of a point source formed in them is 20 cm. Determine the distance from the light source to the point where the mirrors touch if it lies on the bisector of angle formed by the mirrors.

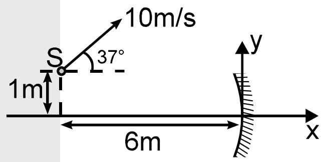
7. A kid of height 1.1 ft is sleeping straight between focus and centre of curvature along the principal axis of a concave mirror of small aperture. His head is towards the mirror and is 0.5 ft from the focus of the mirror. How a plane mirror should be placed so that the image formed by it due to reflected light from concave mirror looks like a person of height 5.5 ft standing vertically. Draw the ray diagram. Find the focal length of the concave mirror.

8. The average size of an Indian face is 24 × 16 cm^2. Find the minimum size of a plane mirror required to see the face completely by:
(i) one eyed man **(ii)** two eyed man. (Distance between eyes is = 4 cm)

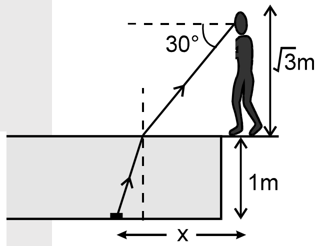
9. As shown in the figure, an object O is at the position (– 10, 2) with respect to the origin P. The concave mirror M_1 has radius of curvature 30 cm. A plane mirror M_2 is kept at a distance 40 cm infront of the concave mirror. Considering first reflection on the concave mirror M_1 and second on the plane mirror M_2. Find the coordinates of the second image w.r.t. the origin P.



10. A point source S is moving with a speed of 10 m/s in x-y plane as shown in the figure. The radius of curvature of the concave mirror is 4m. Determine the velocity vector of the image formed by paraxial rays.



11. A man is standing at the edge of a 1m deep swimming pool, completely filled with a liquid of refractive index $\sqrt{3/2}$. The eyes of the man are $\sqrt{3}$ m above the ground. A coin located at the bottom of the pool appears to be at an angle of depression of 30^0 with reference to the eye of man. Then find horizontal distance (represented by x in the figure) of the coin from the eye of the man.

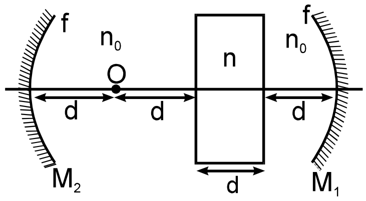


12. An object lies in front of a thick parallel glass slab, the bottom of which is polished. If the distance between first two images formed by bottom surface is 4cm then find the thickness of the slab.
[Assume n_{glass} = 3/2 and paraxial rays]

13. A beam of parallel rays of diameter ' b ' propagates in glass at an angle θ to its plane. Find the diameter of the beam when it goes to air through this face. (n_{glass} = n)

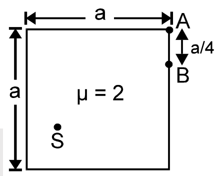
14. A small ball is thrown from the edge of one bank of a river of width 100 m to just reach the other bank. The ball was thrown in the vertical plane (which is also perpendicular to the banks) at an angle 37^o to the horizontal. Taking the starting point as the origin O, vertically upward direction as positive y-axis and the horizontal line passing through the point O and perpendicular to the bank as x-axis find:
(a) equation of trajectory of the image formed by refraction by the water surface
(water surface is at the level y = 0)
(b) instantaneous velocity of the image formed due to refraction. [Use g = 10 m/s^2, R. I. of water =4/3]

15. Two concave mirrors each of focal length 'f' are placed infront of each other co-axially at a distance of 4d in a medium of refractive index n_0. A plane glass slab of refractive index 'n' & thickness 'd' is placed at a distance of 'd' from M_1. A point object O is placed at a distance of 'd' from M_2 as shown in the figure. Consider first reflection by M_2, then refraction on slab and then reflection by M_1. Determine the distance of this image after reflection from M_1.

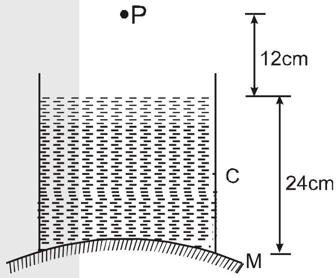


16. An observer observes a fish moving upwards in a cylindrical container of cross section area 1 m^2 filled with water up to a height of 5 m. A hole is present at the bottom of the container having cross section area 1/1000 m^2. Find out the speed of the image of fish observed by observer when the bottom hole is just opened. (Given: The fish is moving with the speed of 6 m/s towards the observer, μ of water = 4/3)

17. The figure shows the square front face (of side 'a') of a transparent cuboidal block. The thickness or the third dimension of the block is negligible in comparison to 'a'. The block has uniform refractive index μ equal to 2. A point source S which can emit light in all directions can move inside the block. It is desired that no light of 'S' should pass through AB. Sketch the region in which S should be present to satisfy this condition.

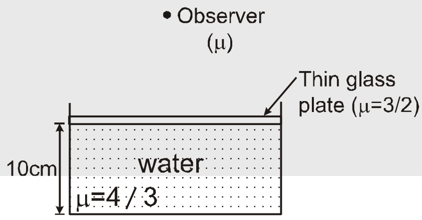


18. An insect at point 'P' sees its two images in the water-mirror system as shown in the figure. One image is formed due to direct reflection from water surface and the other image is formed due to refraction, reflection & again refraction by water mirror system in order. Find the separation between the two images. M has focal length 60 cm. (n_w = 4/3)

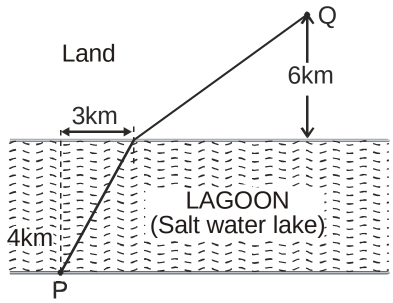


19. A ray of light is incident on a surface in a direction given by vector $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$. The normal to that surface passing through the point of incidence is along the vector $\vec{N} = \hat{j} - 2\hat{k}$. The unit vector in the direction of reflected ray is given by $\vec{R} = a\hat{i} + b\hat{j} + c\hat{k}$. Find three equations in terms of a, b, c using which we can find the values of a, b & c.

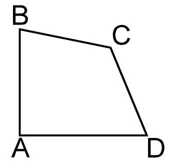
20. In the given figure if observer sees the bottom of vessel at 8 cm, find the refractive index of the medium in which observer is present.



21. A man starting from point P crosses a 4 km wide lagoon and reaches point Q in the shortest possible time by the path shown in the figure. If the person swims at a speed of 3 km/hr and walks at a speed of 4 km/hr, then find his time of journey (in minutes).



22. In the given figure, the faces of prism ABCD made of glass with a refractive index n form dihedral angles $\angle A = 90°$, $\angle B = 75°$, $\angle C = 135°$ & $\angle D = 60°$ (The Abbe's prism). A beam of light falls on face AB & after total internal reflection from face BC escapes through face AD. Find the range of n and angle of incidence α of the beam onto face AB, if a beam that has passed through the prism in this manner is perpendicular to the incident beam.



23. A point source of light is placed at a distance h below the surface of a large deep lake.

(a) Show that the fraction f of the light energy that escapes directly from the water surface is independent of h and is given by $f = \frac{1}{2} - \frac{1}{2n}\sqrt{n^2 - 1}$ where n is the index of refraction of water.

(**Note:** Absorption within the water and reflection at the surface; except where it is total, have been neglected)

(b) Evaluate this ratio for n = 4/3.

24. A glass prism with a refracting angle of 60° has a refractive index 1.52 for red and 1.6 for violet light. A parallel beam of white light is incident on one face at an angle of incidence, which gives minimum deviation for red light. Find :

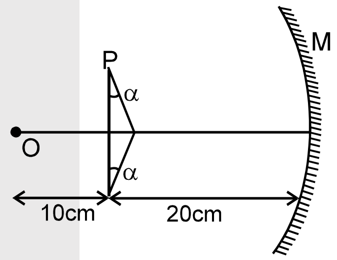
(a) the angle of incidence

(b) angular width of the spectrum

(c) the length of the spectrum if it is focussed on a screen by lens of focal length 100 cm.

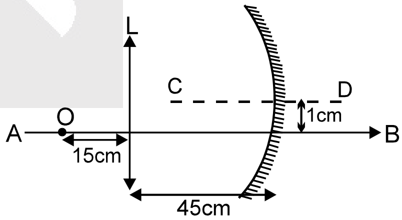
[**Use:** sin (49.7º) = 0.760; sin (31.6º) = 0.520 ; sin (28.4º) = 0.475; sin (56º) = 0.832 ; = 22/7]

25. In the given figure, O is a point object kept on the principal axis of a concave mirror M of radius of curvature 20 cm. P is a prism of angle α = 1.8°. Light falling on the prism (at small angle of incidence) get refracted through the prism and then fall on the mirror. Refractive index of prism is 3/2. Find the distance between the images formed by the concave mirror due to this light.



26. Light travelling in air falls at an incidence angle of 2° on one refracting surface of a prism of refractive index 1.5 and angle of refraction 4º. The medium on the other side is water (n = 4/3). Find the deviation produced by the prism.

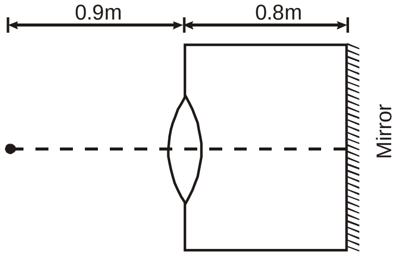
27. In the figure shown L is a converging lens of focal length 10cm and M is a concave mirror of radius of curvature 20cm. A point object O is placed in front of the lens at a distance 15cm. AB and CD are optical axes of the lens and mirror respectively. Find the distance of the final image formed by this system from the optical centre of the lens. The distance between CD & AB is 1 cm.



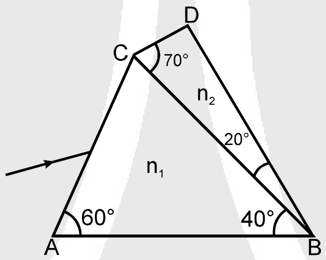
28. An object is kept at rest on the principal axis of a lens. Initially the object is at a distance three times the focal length ' f ' of the lens. The lens runs towards the object at a constant speed u, until the distance between the object and its real image becomes 4 f. If the image always forms on a moving screen then express the velocity of the screen as a function of time.

29. A convex lens produces an image of a candle flame upon a screen whose distance from candle is D. When the lens is displaced through a distance x, (the distance between the candle and the screen is kept constant), it is found that a sharp image is again produced upon the screen. Find the focal length of the lens in terms of D and x.

30. A thin equiconvex lens of glass of refractive index $\mu = 3/2$ & of focal length 0.3 m in air is sealed into an opening at one end of a tank filled with water ($\mu = 4/3$). On the opposite side of the lens, a mirror is placed inside the tank on the tank wall perpendicular to the lens axis, as shown in figure. The separation between the lens and the mirror is 0.8 m. A small object is placed outside the tank infront of the lens at a distance of 0.9 m from the lens along its axis. Find the position (relative to the lens) of the image of the object formed by the system.



31. A prism of refractive index n_1 and another prism of refractive index n_2 are stuck together without a gap as shown in the figure. The angles of the prisms are as shown. n_1 and n_2 depend on λ, the wavelength of light according to $n_1 = 1.20 + \frac{10.8 \times 10^4}{\lambda^2}$ and $n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda^2}$ where λ is in nm.

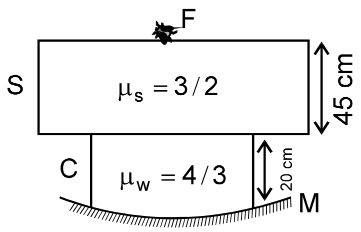


(i) Calculate the wavelength λ_0 for which rays incident at any angle on the interface BC pass through without bending at that interface.

(ii) For light of wavelength λ_0, find the angle of incidence i on the face AC such that the deviation produced by the combination of prisms is minimum.

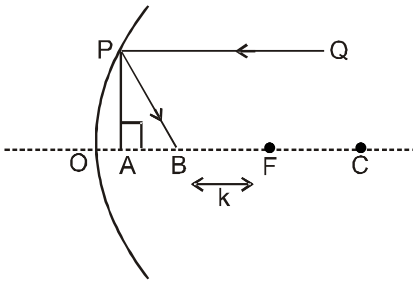
32. A pole of length 2.00 m stands half dipped in a swimming pool with water level 1 m higher than the bed (bottom). The refractive index of water is 4/3 and sunlight is coming at an angle of 37° with the vertical. Find the length of the shadow of the pole on the bed.
Use $\sin^{-1}(0.45) = 26.8°$, $\tan(26.8°) = 0.5$

33. A fly F is sitting on a glass slab S 45cm thick & of refractive index 3/2. The slab covers the top of a container C containing water (R.I. 4/3) upto a height of 20 cm. Bottom of container is closed by a concave mirror M of radius of curvature 40 cm. Locate the final image formed by all refractions & reflection assuming paraxial rays.



34. A glass porthole is made at the bottom of a ship for observing sea life. The hole diameter D is much larger than the thickness of the glass. Determine the area S of the field of vision at the sea bottom for the porthole if the refractive index of water is μ_w and the sea depth is h.

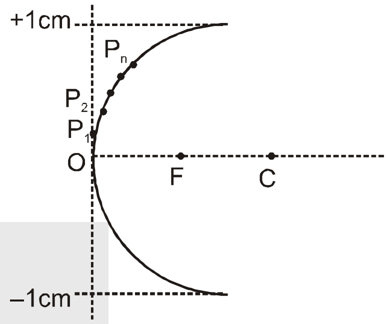
35. The figure below depicts a concave mirror with center mirror with center of curvature C focus F, and a horizontally drawn OFC as the optic axis. The radius of curvature is R (OC = R) and OF = R/2). A ray of ligth QP, parallel to the optical axis and at a perpendicular distance w($w \le R/2$) from it, is incident on the mirror at P. It is reflected to the point B on the optical axis, such that BF = k. Here k is a measure of lateral aberration.



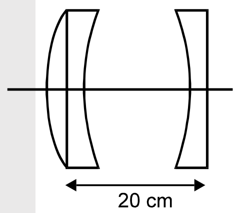
(a) Express k in terms of {w, R}. k =

(b) Sketch k vs w for $w \in [0, R/2]$

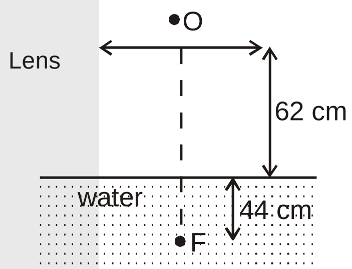
(c) Consider points P_1, P_2 ......P_n on the concave mirror which are increasingly further away from the optic centre O and approximately equidistant from each other(see figure below). Rays parallel to the optic axis are incident at P_1, P_2,.......P_n and reflected to points on the optic axis. Consider the points where these rays reflected from P_n, P_{n-1}, ...P_2 intersect the rays reflected from P_{n-1}, P_{n-2}, ... P_1 respectively. Qualitatively sketch the locus of these points in figure below for a mirror (shown with solid line) with radius of curvature 2 cm.



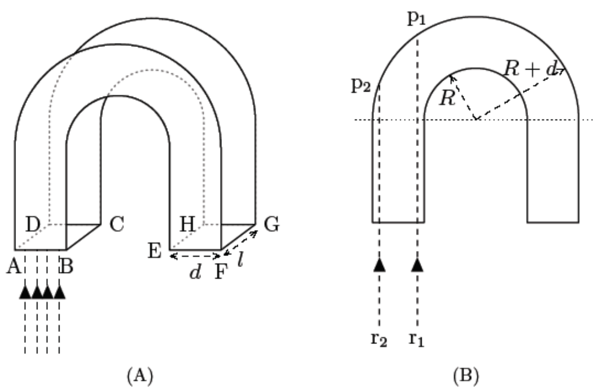
36. A symmetrical converging convex lens of focal length 10 cm & diverging concave symmetrical lens of focal length –20 cm are cut from the middle and perpendicularly and symmetrically to their principal axis. The parts thus obtained are arranged as shown in the figure. Find the focal length (in cm) of this arrangement



37. In the given figure, a stationary observer O looking at a fish F (in water of, $\mu = 4/3$) through a converging lens of focal length 90 cm. The lens is allowed to fall freely from a height 62.0 cm with its axis vertical. The fish and the observer are on the principal axis of the lens. The fish moves up with constant velocity 100 cm/s. Initially it was at a depth of 44.0 cm. The velocity with which the fish appears to move to the observer at t = 0.2 sec is$\left(x+\frac{3}{4}\right)$m/s. Find the value of x. ($g = 10\ m/s^2$)



38. A glass rod of refractive index 1.50 of rectangular cross section {$d \times \ell$} is bent into a "U" shape see Fig. (A). The cross sectional view of this rod is shown in Fig.(B).

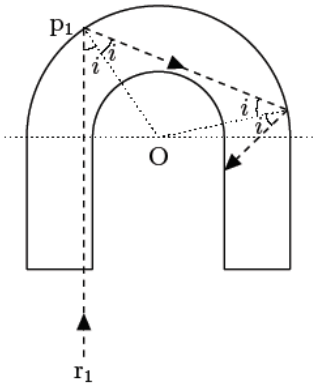


Bent portion of the rod is semi-circular with inner and outer radii R and R + d respectively. Parallel monochromatic beam of light is incident normally on face ABCD. **[Olympiad-2016, Stage-2]**

(a) Consider two monochromatic rays r_1 and r_2 in Fig. (B). State whether the following statements are True or False.

Statement	True/ False
If r_1 is total internally reflected from the semi circular section at the point p_1 then r_2 will necessarily be total internally reflected at the point p_2.	True
If r_2 is total internally reflected from the semi circular section at the point p_2 then r_1 will necessarily be total internally reflected at the point p_1	False

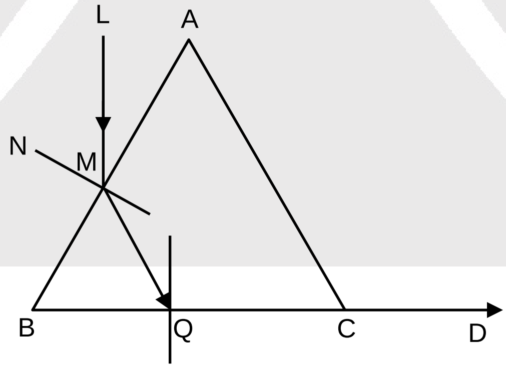
(b) Consider the ray r_1 whose point of incidence is very close to the edge BC. Assume it undergoes total internal reflection at p_1. In cross sectional view below, draw the trajectory of this reflected ray beyond the next glass-air boundary that it encounters.



(c) Obtain the minimum value of the ratio R/d for which any light ray entering the glass normally through the face ABCD undergoes at least one total internal reflection.

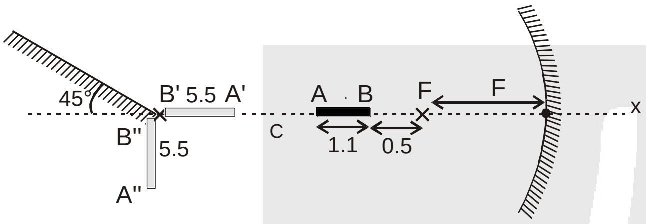
(d) A glass rod with the above computed minimum ratio of R/d, is fully immersed water of refractive index 1.33. What fraction of light flux entering the glass through the plane surface ABCD undergoes at least one total internal reflection?

39. The following figure shows the section ABC of an equilateral triangular prism. A ray of light enters the prism along LM and emerges along QD. If the refractive index of the material of the prism is 1.6, angle LMN is **[Olympiad 2017 (Stage–I)]**

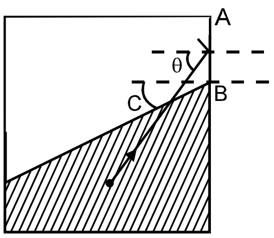


HLP Answers

1. 16 cm
2. (i) No shift is observed (ii) 1 cm
3. $(n-1)R/(3n-1)$
4. 200 cm, right of the lens
5. $\frac{f(1-2\cos\theta)}{1-\cos\theta}$, 0
6. $11.5 \text{ cm} = \frac{20}{\sqrt{3}}$
7. The plane mirror should be placed at an angle of 45º with negative x-axis; f = 2ft.

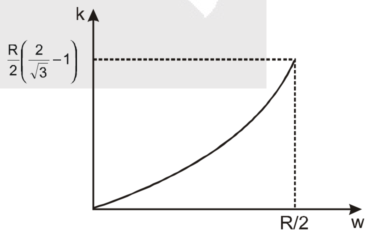


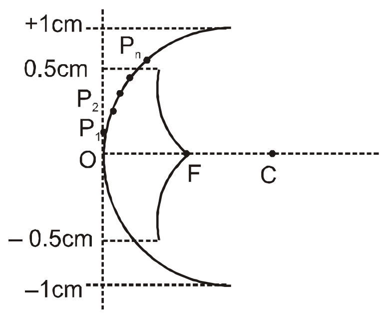
8. (i) $12 \times 8 \text{ cm}^2$ (ii) $12 \times 6 \text{ cm}^2$
9. Coordinates of I_2 w.r.t. P = (– 46, – 70)
10. $\vec{V}_i = V_{ix}\hat{i} + V_{iy}\hat{j} = -2\hat{i} - 4\hat{j}$
11. d = 4000 mm
12. t = 2 cm
13. $CD = \frac{b.\sqrt{1-n^2\cos^2\theta}}{\sin\theta}$
14. (a) $\left(x - \frac{x^2}{100}\right)$

    (b) $\vec{v}\,(t)_{image} = 20\sqrt{\frac{5}{3}}\,\hat{i} + 20\left[\sqrt{\frac{5}{3}} - \frac{2}{3}t\right]\hat{j}$
15. $\left|\frac{\left(\left(\frac{df}{f-d}-d\right)\left(1-\frac{n_0}{n}\right)+4d\right)f}{\left(\frac{df}{d-f}+d\right)\left(1-\frac{n_0}{n}\right)-4d+f}\right|$
16. 4.4975 m/s
17. 

18. 24 cm.
19. $a^2 + b^2 + c^2 = 1$; $3a + 4b + 2c = 0$; $b - 2c = 4/3$
20. $\frac{16}{15}$
21. 250
22. $\sqrt{2} < n \le 2$, and $45^\circ < \alpha \le 90^\circ$
23. **(b)** $(4-\sqrt{7})/8$
24. (a) 49.7º, (b) 56º – 49.7º = 6.3º (c) fθ = 11cm
25. $\frac{\pi}{20}$ cm.
26. 1°
27. $6\sqrt{26}$ cm
28. $v_i = u\left[1-\left\{\frac{f}{ut-2f}\right\}^2\right]$
29. $f = (D^2 - x^2)/4D$
30. 90 cm from the lens towards right
31. **(i)** $\lambda_0 = 600$ nm, n = 1.5

    (ii) $i = \sin^{-1}(0.75) = 48.59^\circ$
32. 1.25 m
33. $\left(\frac{135}{6}\right)$ cm = 22.5 cm below the upper surface of the glass slab
34. $\pi\left[\frac{h}{\sqrt{\mu_w^2-1}} + \frac{D}{2}\right]^2$
35. (a) $k = \frac{R}{2}\left[\frac{R}{(R^2-\omega^2)^{1/2}} - 1\right]$

    (b) 

    (c) 

36. 80 cm **37.** 22

38. For total internal reflection

$\sin i \geq \sin\theta_c$

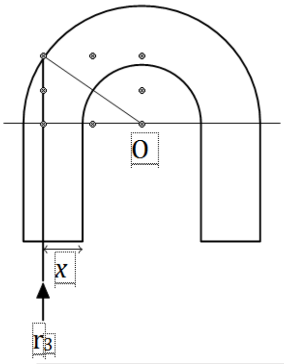
where is the incidence angle of the ray on bent portion of the rod (see figure in part (b)) and is the critical angle for glass-air boundary.

For a light ray close to edge BC

$$\frac{R}{R+d} \geq \frac{1}{n_{glass}}$$

For refractive index $n_{glass} = 1.5$ $R \geq 2d$

Minimum value of R/d = 2



Let the intensity of beam be I_0. Flux entering through glass slab will be $dI I_0$. Assume that any light ray up to distance x from the edge BC undergoes at least one total internal reflection. Then the flux going through at least one total internal reflection will be $(d - x)I I_0$. Also

$$\frac{R+x}{R+d} = \frac{n_{water}}{n_{glass}}$$

For R/ d = 2 and $n_{water} = 1.33$, x = 2d/ 3.

Fraction of light = 0.33

39. 35.6°