GEOMETRICAL OPTICS

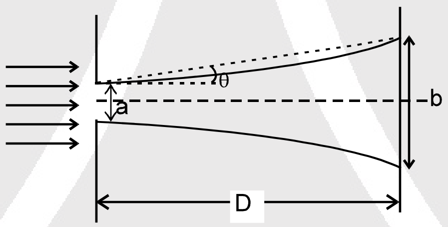
INTRODUCTION :

Blue lakes, ochre deserts, green forest, and multicolored rainbows can be enjoyed by anyone who has eyes with which to see them. But by studying the branch of physics called **optics**, which deals with the behaviour of light and other electromagnetic waves, we can reach a deeper appreciation of the visible world. A knowledge of the properties of light allows us to understand the blue color of the sky and the design of optical devices such as telescopes, microscopes, cameras, eyeglasses, and the human eyes. The same basic principles of optics also lie at the heart of modern developments such as the laser, optical fibers, holograms, optical computers, and new techniques in medical imaging.

1. CONDITION FOR RECTILINEAR PROPAGATION OF LIGHT

(Only for information not in IIT-JEE syllabus)

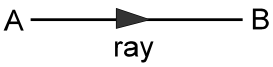
Some part of the optics can be understood if we assume that light travels in a straight line and it bends abruptly when it suffers reflection or refraction.
The assumption that the light travels in a straight line is correct if
(i) the medium is isotropic, i.e. its behaviour is same in all directions and (ii) the obstacle past which the light moves or the opening through which the light moves is not very small.
Consider a slit of width 'a' through which monochromatic light rays pass and strike a screen, placed at a distance D as shown.



It is found that the light strikes in a band of width 'b' more than 'a'. This bending is called **diffraction**.
Light bends by (b-a)/2 on each side of the central line .It can be shown by wave theory of light that $\sin\theta = \frac{\lambda}{a}$ ....(A), where θ is shown in figure.

This formula indicates that the **bending is considerable only when a $\simeq$ λ** . Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.

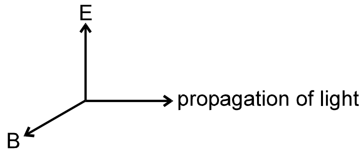
It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$. If this condition is fulfilled, light is said to move rectilinearly. In most of the situations including geometrical optics the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted.
Thus geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A **ray** of light is the straight line path of transfer of light energy. Arrow represents the direction of propagation of light.
Figure shows a ray which indicates light is moving from A to B.



2. PROPERTIES OF LIGHT

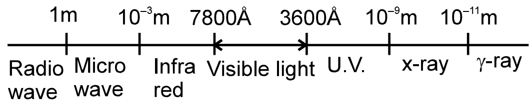
(i) Speed of light in vacuum, denoted by c, is equal to 3×10^8 m/s approximately.

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.



(iii) Light carries energy and momentum.

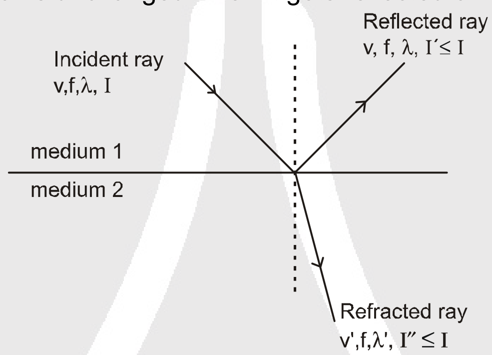
(iv) The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

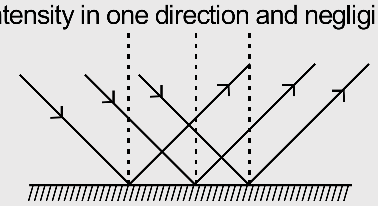


3. REFLECTION OF LIGHT

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

(a) Regular Reflection:

When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.



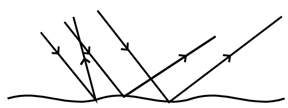
Regular Reflection

(b) Diffused Reflection

When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position.

Such a reflection is called as **diffused reflection**.

For example reflection from a wall, from a news paper etc. This is why you can not see your face in news paper and in the wall.

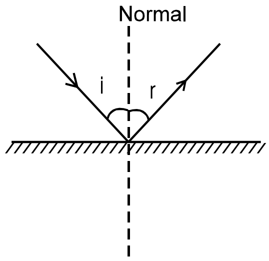


Diffused Reflection

3.1 Laws of Reflection

(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**. This condition can be expressed mathematically as $\vec{R}\cdot(\vec{I}\times\vec{N}) = \vec{N}\cdot(\vec{I}\times\vec{R}) = \vec{I}\cdot(\vec{N}\times\vec{R}) = 0$ where $\vec{I}$, $\vec{N}$ and $\vec{R}$ are vectors of any magnitude along incident ray, the normal and the reflected ray respectively.

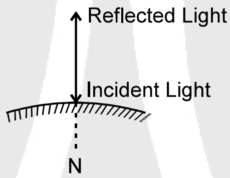
(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e.,



$\angle i = \angle r$

Special Cases :

Normal Incidence : In case light is incident normally,

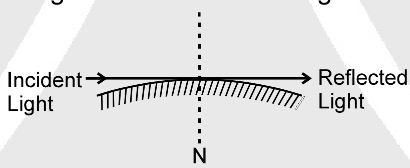


$i = r = 0$

$\delta = 180°$

Note : We say that the ray has retraced its path.

Grazing Incidence : In case light strikes the reflecting surface tangentially,



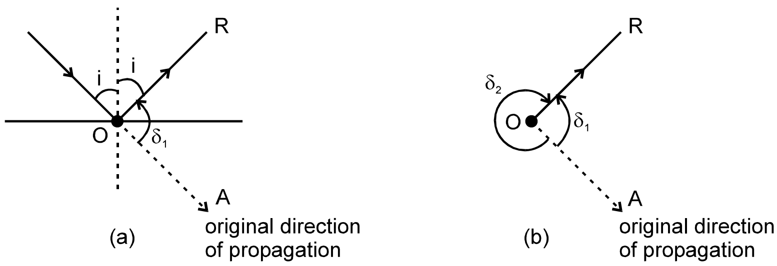
$i = r = 90$; deviation, $\delta = 0°$ or $360°$

Note : In case of reflection speed (magnitude of velocity) of light remains unchanged but in grazing incidence velocity remains unchanged.

Solved Example

Example 1. Show that for a light ray incident at an angle 'i' on getting reflected the angle of deviation is $\delta = \pi - 2i$ or $\pi + 2i$.

Solution :



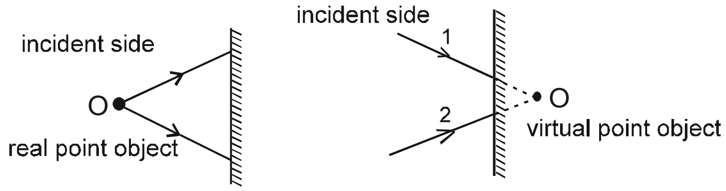
From figure (b) it is clear that light ray bends either by δ_1 anticlockwise or by δ_2 (= $2\pi - \delta_1$) clockwise.

From figure (a) $\delta_1 = \pi - 2i$.

$\therefore\ \delta_2 = \pi + 2i$.

3.2 Object and Image

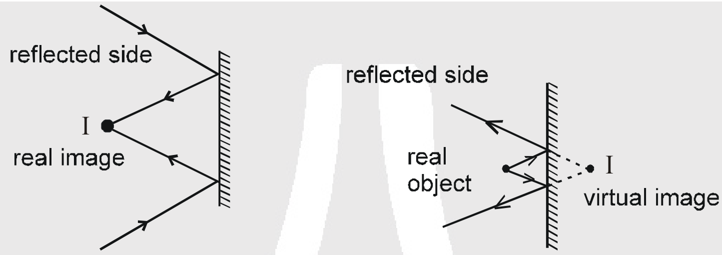
(a) Object (O) : Object is defined as point of intersection of **incident** rays.



Let us call the side in which incident rays are present as incident side and the side in which reflected (refracted) rays are present, as reflected (refracted) side.

Note : An object is called **real** if it lies on incident side otherwise it is called **virtual**. (In case of plane mirror only)

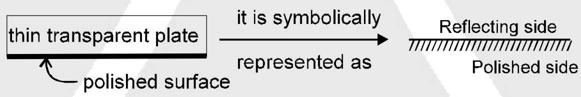
(b) Image (I) : Image is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction).



Note : An image is called **real** if it lies on reflected or refracted side otherwise it is called **virtual.**

4. PLANE MIRROR

Plane mirror is formed by polishing one surface of a plane thin glass plate .It is also said to be silvered on one side.

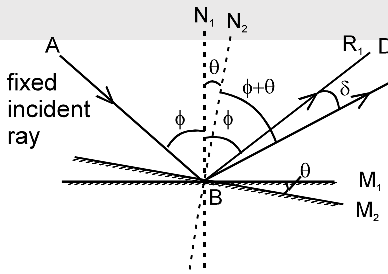


PLANE MIRROR

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Solved Example

Example 2. For a fixed incident light ray, if the mirror be rotated through an angle θ (about an axis which lies in the plane of mirror and perpendicular to the plane of incidence), show that the reflected ray turns through an angle 2θ in same sense.



Solution : See figure M_1, N_1 and R_1 indicate the initial position of mirror, initial normal and initial direction of reflected light ray respectively. M_2, N_2 and R_2 indicate the final position of mirror, final normal and final direction of reflected light ray respectively. From figure it is clear that ABC = $2\phi + \delta = 2(\phi + \theta)$ or $\delta = 2\theta$.

Note : Keeping the mirror fixed if the incident ray is rotated by angle θ about the normal then reflected ray rotates by same angle in the same direction of rotation

4.1 Point object

Characteristics of image due to reflection by a plane mirror :

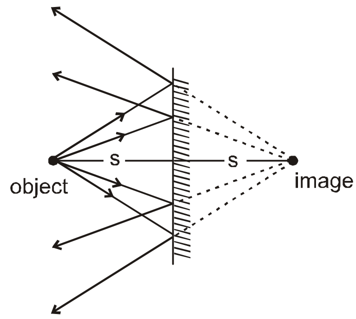
(i) Distance of object from mirror = Distance of image from the mirror.

(ii) All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

(iii) The line joining a point object and its image is normal to the reflecting surface.

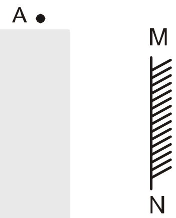
(iv) For a real object the image is virtual and for a virtual object the image is real

(v) The region in which observer's eye must be present in order to view the image is called **field of view**.

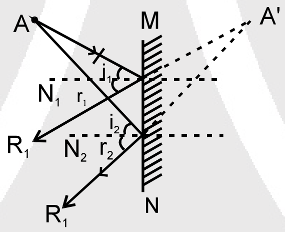


Solved Example

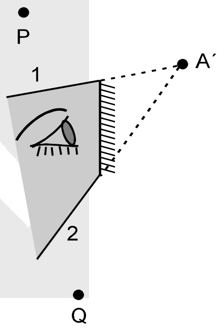
Example 3. Figure shows a point object A and a plane mirror MN. Find the position of image of object A, in mirror MN, by drawing ray diagram. Indicate the region in which observer's eye must be present in order to view the image. (This region is called **field of view**).



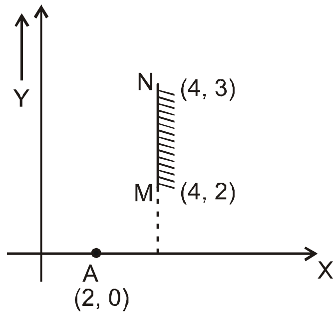
Solution See figure, consider any two rays emanating from the object. N_1 and N_2 are normals ;
$i_1 = r_1$ and $i_2 = r_2$



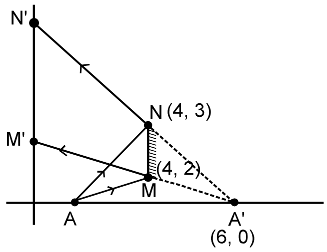
The meeting point of reflected rays R_1 and R_2 is image A'. Though only two rays are considered it must be understood that all rays from A reflect from mirror such that their meeting point is A´. To obtain the region in which reflected rays are present, join A´ with the ends of mirror and extend. The following figure shows this region as shaded. In figure, there are no reflected rays beyond the rays 1 and 2, therefore the observers P and Q cannot see the image because they do not receive any reflected ray.



Example 4. Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.

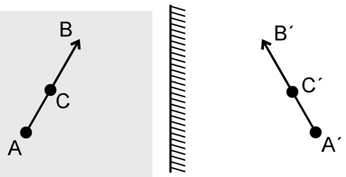


Solution : The image of point A, in the mirror is at A’ (6, 0).
Join A’ M and extend to cut Y axis at M’ (Ray originating from A which strikes the mirror at M gets reflected as the ray MM’ which appears to come from A’). Join A’N and extend to cut Y axis at N’ (Ray originating from A which strikes the mirror at N gets reflected as the ray NN’ which appears to come from A’).
From geometry.
M’ ≡ (0, 6)
N’ ≡ (0, 9). M’N’ is the region on Y axis in which reflected rays are present.



4.2 Extended object :

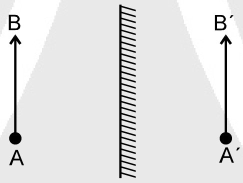
An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A´ will be image of A, C´ will be image of C, B´ will be image of B etc. All point images together form extended image. Thus extended image is formed of an extended object.



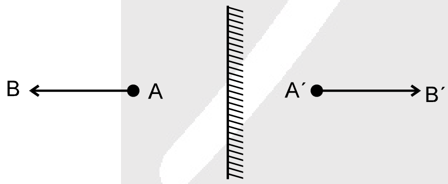
Properties of image of an extended object, formed by a plane mirror :

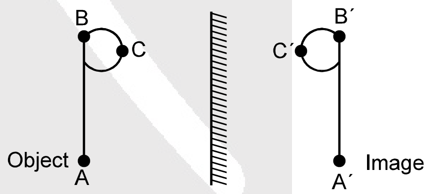
(1) Size of extended object = Size of extended image.

(2) The image is erect, if the extended object is placed parallel to the mirror.

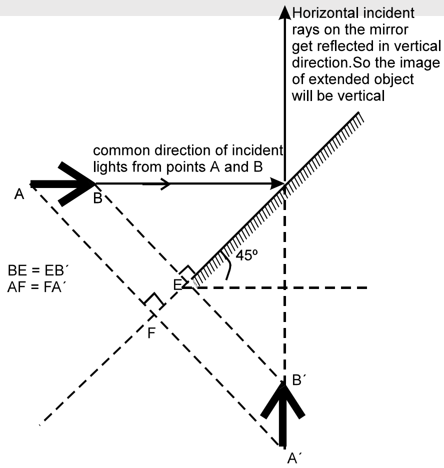


(3) The image is inverted if the extended object lies perpendicular to the plane mirror.





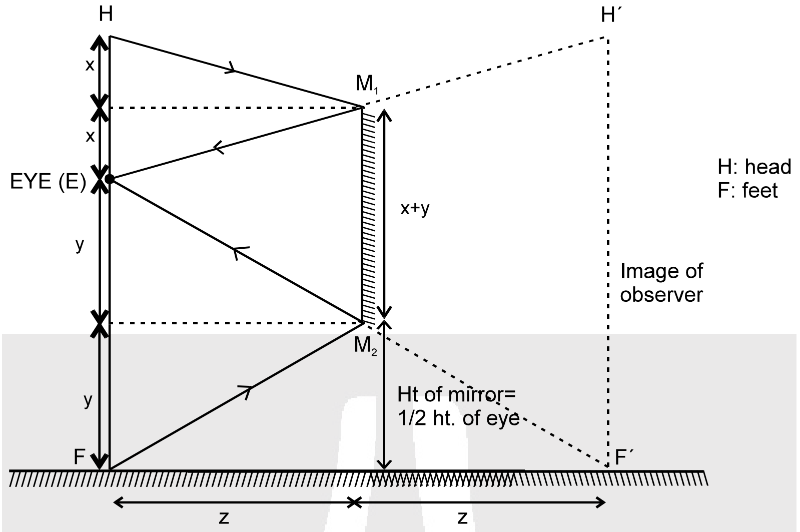
(4) If an extended horizontal object is placed infront of a mirror inclined 45° with the horizontal, the image formed will be vertical. See figure.



Solved Example

Example 5. Show that the minimum size of a plane mirror, required to see the full image of an observer is half the size of that observer.

Solution : See the following figure. It is self explanatory if you consider lengths 'x' and 'y' as shown in figure.



Alter :

ΔEM_1M_2 and $\Delta E\,H'F'$ are similar

$\therefore \quad \dfrac{M_1M_2}{H'F'} = \dfrac{z}{2z}$

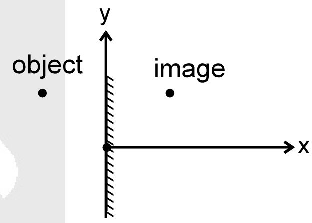
or $\quad M_1M_2 = H'F'/2 = HF/2$

4.3 Relation between velocity of object and image :

From mirror property :

$x_{im} = -x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror. Similarly others have corresponding meaning.



Differentiating w.r.t time, we get

$v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z}$,

$\Rightarrow$ For x axis $v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$

but For y axis and z axis $v_{iG} - v_{mG} = (v_{oG} - v_{mG})$ or $v_{iG} = v_{oG}$.

here: v_{iG} = velocity of image with respect to ground.

Solved Example

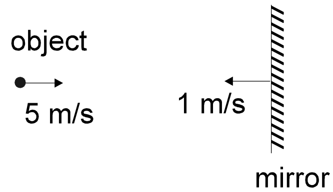
Example 6. An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.

Solution : Take $\longrightarrow$ as + direction. $v_i - v_m = v_m - v_0$

$v_i - (-1) = (-1) - 5$

$\therefore \quad v_i = -7$m/s.

$\Rightarrow$ 7 m/s and direction towards left.



Example 7. There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

Solution : We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$

or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$.

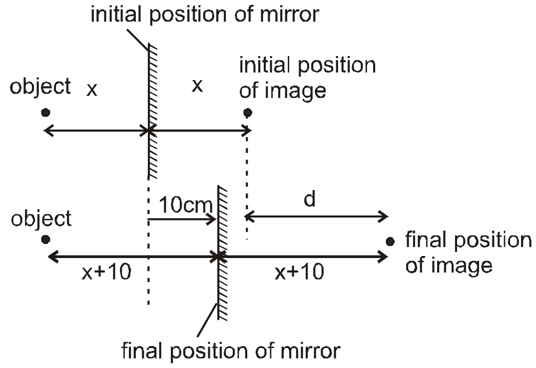
In this question $\Delta x_o = 0$; $\Delta x_m = 10$ cm.

Therefore $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

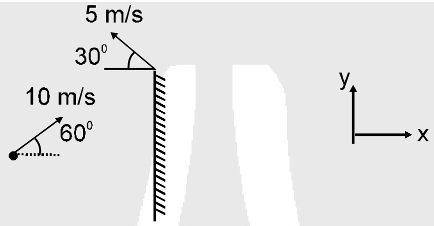
Alter :

$2(x + 10) = 2x + d$

$\therefore \quad d = 20$ cm



Example 8. In the situation shown in figure, find the velocity of image.



Solution : Along x direction, applying $v_i - v_m = -(v_0 - v_m)$

$v_i - (-5\cos 30^o) = -(10\cos 60^o - (-5\cos 30^o))$

$\therefore \; v_i = -5(1+\sqrt{3})$ m/s

Along y direction $v_0 = v_i$

$\therefore \quad v_i = 10 \sin 60° = 5\sqrt{3}$ m/s

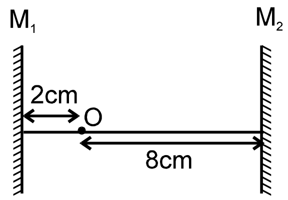
$\therefore \quad$ Velocity of the image $= -5(1+\sqrt{3})\,\hat{i} + 5\sqrt{3}\,\hat{j}$ m/s.

4.4 Images formed by two plane mirrors :

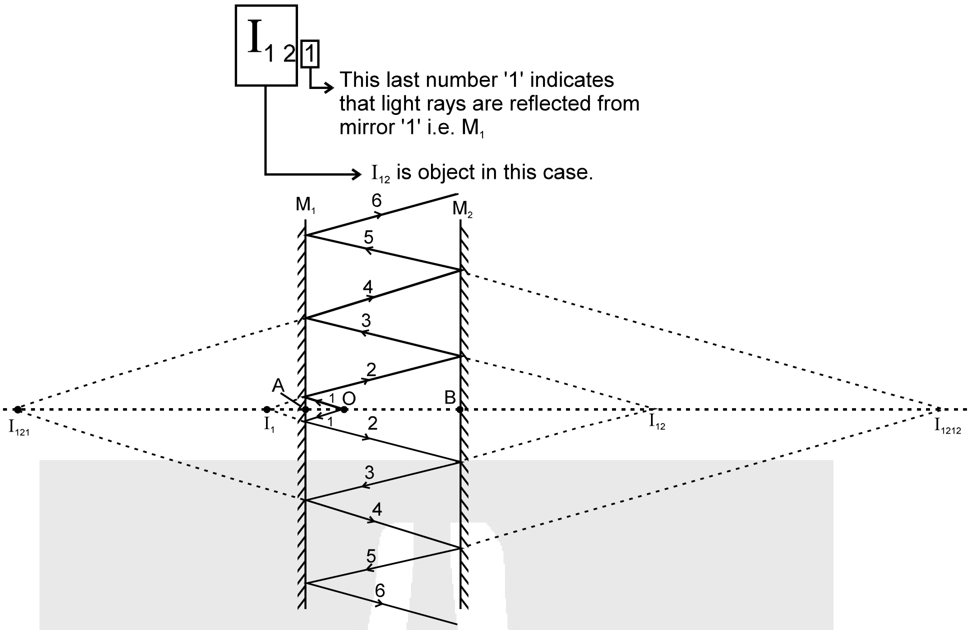
If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Solved Example

Example 9. Figure shows a point object placed between two parallel mirrors. Its distance from M_1 is 2 cm and that from M_2 is 8 cm. Find the distance of images from the two mirrors considering reflection on mirror M_1 first.



Solution : To understand how images are formed see the following figure and table. You will require to know what symbols like I_{121} stands for. See the following diagram.

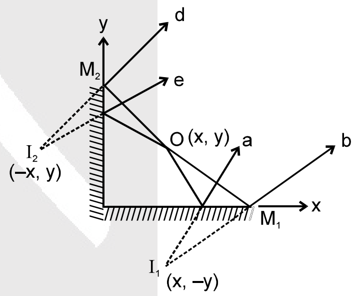


Incident rays	Reflected by	Reflected rays	Object	Image	Object distance	Image distance
Rays 1	M_1	Rays 2	O	I_1	AO = 2cm	AI_1 = 2 cm
Rays 2	M_2	Rays 3	I_1	I_{12}	BI_1 = 12 cm	BI_{12} = 12 cm
Rays 3	M_1	Rays 4	I_{12}	I_{121}	AI_{12} = 22cm	AI_{121} = 22cm
Rays 4	M_2	Rays 5	I_{121}	I_{1212}	BI_{121} =32cm	BI_{1212}=32cm

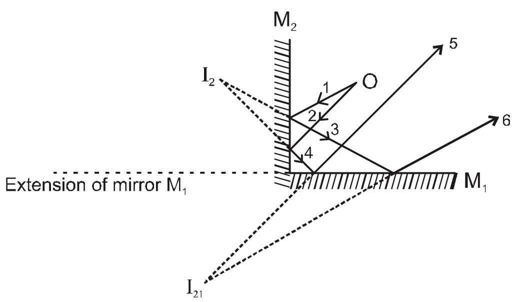
Similarly images will be formed by the rays striking mirror M_2 first. Total number of images = ∞.

Example 10. Consider two perpendicular mirrors. M_1 and M_2 and a point object O. Taking origin at the point of intersection of the mirrors and the coordinate of object as (x, y), find the position and number of images.

Solution Rays 'a' and 'b' strike mirror M_1 only and these rays will form image I_1 at (x, –y), such that O and I_1 are equidistant from mirror M_1. These rays do not form further image because they do not strike any mirror again. Similarly rays 'd' and 'e' strike mirror M_2 only and these rays will form image I_2 at (–x, y), such that O and I_2 are equidistant from mirror M_2.



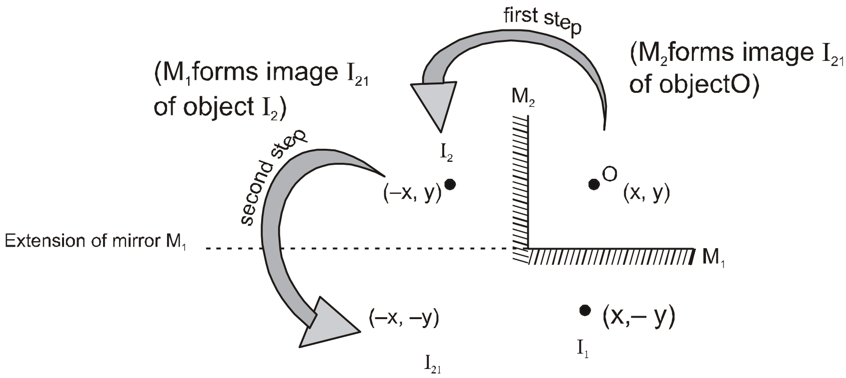
Now consider those rays which strike mirror M_2 first and then the mirror M_1.



For incident rays 1, 2 object is O, and reflected rays 3, 4 form image I_2.

Now rays 3, 4 incident on M_1 (object is I_2) which reflect as rays 5, 6 and form image I_{21}. Rays 5, 6 do not strike any mirror, so image formation stops.

I_2 and I_{21}, are equidistant from M_1. To summarize see the following figure



For rays reflecting first from M_1 and then from M_2, first image I_1 (at (x, –y)) will be formed and this will function as object for mirror M_2 and then its image I_{12} (at (–x, –y)) will be formed.

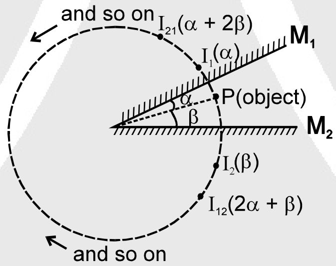
I_{12} and I_{21} coincide.

$\therefore$ Three images are formed

4.5 Locating all the images formed by two plane mirrors:

Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.



Now I_2 will act as an object for M_1 which is at an angle $(\alpha + 2\beta)$ from M_1. Its image will be formed at an angle $(\alpha + 2\beta)$ on the opposite side of M_1. This image will be denoted as I_{21}, and so on. Think when this will process stop.

Hint : The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360^\circ}{\theta}$ = even number ; number of image = $\frac{360^\circ}{\theta} - 1$

(ii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360^\circ}{\theta}$ = odd number ; number of image = $\frac{360^\circ}{\theta}$, if the object is not placed on the angle bisector.

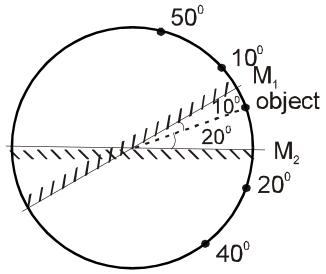
(iv) If $\frac{360^\circ}{\theta} \neq$ integer, then count the number of images as explained above.

Solved Example

Example 11. Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

Solution : Figure is self explanatory.

Number of images



(i) Using direct formula : $\frac{360^\circ}{30^\circ}$ = 12 (even number)

$\therefore$ number of images = 12 – 1 = 11

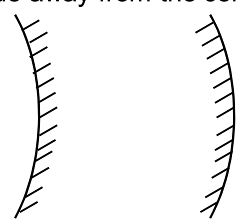
(ii) By counting. See the following table

Image formed by Mirror M_1 (angles are measured from the mirror M_1.)		Image formed by Mirror M_2 (angles are measured from the mirror M_2.)
10°	+30°	20°
50°	+30°	40°
70°	+30°	80°
110°	+30°	100°
130°	+30°	140°
170°		160°
Stop because next angle will be more than 180°		Stop because next angle will be more than 180°

To check whether the final images made by the two mirrors coincide or not : add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide. Here last angles made by the mirrors + the angle between the mirrors = 160° + 170° + 30° = 360°. Therefore in this case the last images coincide. Therefore the number of images = number of images formed by mirror M_1 + number of images formed by mirror M_2 -1 (as the last images coincide)= 6 + 6 –1 = 11.

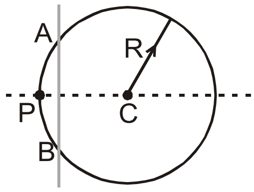
5. SPHERICAL MIRRORS

Spherical Mirror is formed by polishing one surface of a part of sphere. Depending upon which part is shining the spherical mirror is classified as (a) Concave mirror, if the side towards center of curvature is shining and (b) Convex mirror if the side away from the center of curvature is shining.



Concave mirror   Convex mirror

5.1 Important terms related with spherical mirrors :



A spherical shell with the center of curvature, pole aperture and radius of curvature identified

(a) Center of Curvature (C) :

The center of the sphere from which the spherical mirror is formed is called the center of curvature of the mirror. It is represented by C and is indicated in figure.

(b) Pole (P) :

The center of the mirror is called as the Pole. It is represented by the point P on the mirror APB in figure.

(c) Principal Axis :

The Principal Axis is a line which is perpendicular to the plane of the mirror and passes through the pole. The Principal Axis can also be defined as the line which joins the Pole to the Center of Curvature of the mirror.

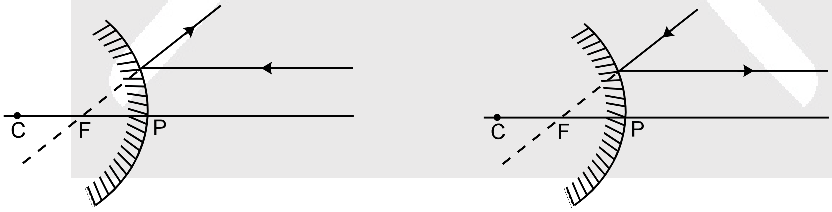
(d) Aperture (A) :

The aperture is the segment or area of the mirror which is available for reflecting light. In figure. APB is the aperture of the mirror.

(e) Principle focus (F) :

It is the point of intersection of all the reflected rays for which the incident rays strike the mirror (with small aperture) parallel to the principal axis. In concave mirror it is real and in the convex mirror it is virtual. The distance from pole to focus is called **focal length.**

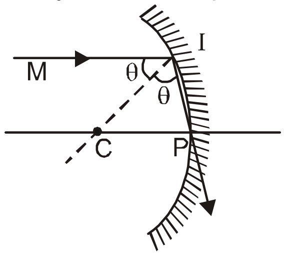
Concave mirror



Convex mirror

Solved Example

Example 12. Find the angle of incidence of ray for which it passes through the pole, given that MI || CP.



Solution : $\angle$ MIC = $\angle$CIP = θ

MI || CP $\angle$MIθ = $\angle$ICP = θ

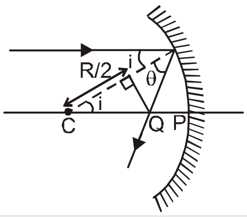
CI = CP

$\angle$CIP = $\angle$CPI = θ

$\therefore$ In ΔCIP all angle are equal

$3\theta = 180^\circ \Rightarrow \theta = 60^\circ$

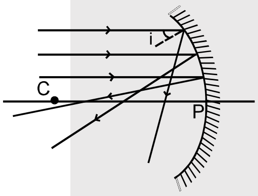
Example 13. Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also find the distance CQ if $i \to 0$.



Solution : $\cos i = \dfrac{R}{2CQ} \Rightarrow CQ = \dfrac{R}{2\cos i}$

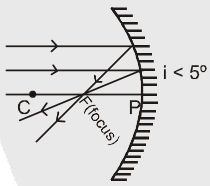
As i increases cos i decreases.

Hence CQ increases



If i is a small angle $\cos i \approx 1$

$\therefore$ CQ = R/2

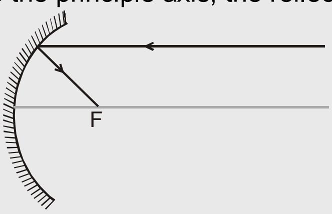


So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.

5.1 Ray tracing :

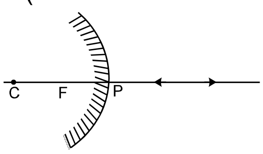
Following facts are useful in ray tracing.

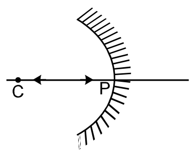
(i) If the incident ray is parallel to the principle axis, the reflected ray passes through the focus.



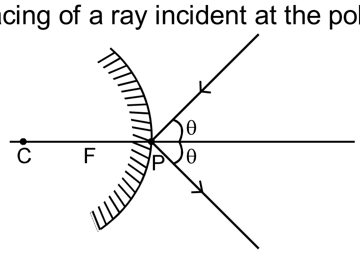
(ii) If the incident ray passes through the focus, then the reflected ray is parallel to the principle axis.

(iii) Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).





(iv) It is easy to make the ray tracing of a ray incident at the pole as shown in below.



5.2 Sign Convention

We are using co–ordinate sign convention.

(i) Take origin at pole (in case of mirror) or at optical centre (in case of lens). Take X axis along the Principal Axis, taking **positive direction along the incident light.**
u, v, R and **f** indicate the x coordinate of object, image, centre of curvature and focus respectively.

(ii) y-coordinates are taken positive above Principle Axis and negative below Principle Axis' h_1 and h_2 denote the y coordinates of object and image respectively.

Note :
- This sign convention is used for reflection from mirror, reflection through flat or curved surfaces or lens.

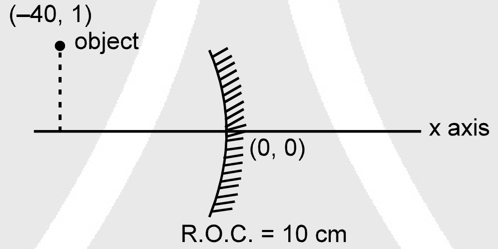
5.3 Formula for Reflection from spherical mirrors :

(a) Mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{2}{R}=\frac{1}{f}$

X-coordinate of centre of curvature and focus of concave mirror are negative and those for convex mirror are positive. In case of mirrors since light rays reflect back in X-direction, therefore **-ve sign of v indicates real image and +ve sign of v indicates virtual image.**

Solved Example

Example 14. Figure shows a spherical concave mirror with its pole at (0, 0) and principal axis along x axis. There is a point object at (–40 cm, 1cm), find the position of image.



Solution : According to sign convention,
u = –40 cm
h_1 = +1 cm
f = – 5 cm.

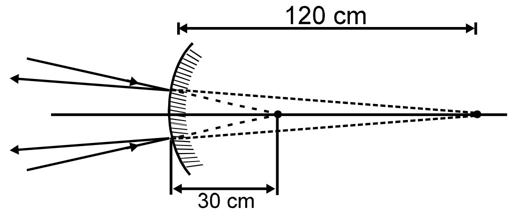
$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{v}+\frac{1}{-40}=\frac{1}{-5}$; $v=\frac{-40}{7}$ cm. ; $\frac{h_2}{h_1}=\frac{-v}{u}$

$\Rightarrow h_2 = -\frac{-v}{u} \times h_1 = \frac{-\left(-\frac{40}{7}\right)\times 1}{-40} = -\frac{1}{7}$ cm.

$\therefore$ The position of image is $\left(\frac{-40}{7}\text{cm}, -\frac{1}{7}\text{cm}\right)$

Example 15. Converging rays are incident on a convex spherical mirror so that their extensions intersect 30 cm behind the mirror on the optical axis. The reflected rays form a diverging beam so that their extensions intersect the optical axis 1.2 m from the mirror. Determine the focal length of the mirror.

Solution : In this case u = + 30

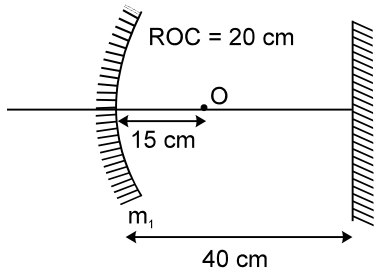


$\Rightarrow$ v = + 120

$\therefore \frac{1}{f}=\frac{1}{v}+\frac{1}{u}=\frac{1}{120}+\frac{1}{30}$

f = 24 cm

Example 16. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution : **I reflection :** Focus of mirror = – 10 cm ⇒ u = – 15 cm

Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ ⇒ v = – 30 cm.

For II reflection on plane mirror : u = – 10 cm

∴ v = 10 cm

For III reflection on curved mirror again : u = – 50 cm ; f = – 10 cm

Applying mirror formula : $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; v = – 12.5 cm.

(b) Lateral magnification (or transverse magnification) denoted by m is defined as $m = \frac{h_2}{h_1}$ and is related as $m = -\frac{v}{u}$. From the definition of m, positive sign of m indicates erect image and negative sign indicates inverted image.

(c) In case of successive reflection from mirrors, the overall lateral magnification is given by $\mathbf{m_1 \times m_2 \times m_3}$ ......, where m_1, m_2 etc. are lateral magnifications produced by individual mirrors.
$\mathbf{h_1}$ and $\mathbf{h_2}$ denote the y coordinate of object and image respectively.

Note :

- Using **5.3(a)** and **5.3(b)** the following conclusions can be made **(check yourself)**.

Nature of Object	**Nature of Image**	**Inverted or erect**
Real	Real	Inverted
Real	Virtual	Erect
Virtual	Real	Erect
Virtual	Virtual	Inverted

From **5.3(a)** and **5.3(b)**; we get $m=\frac{f}{f-u}=\frac{f-v}{f}$ .... (just a time saving formula)

Solved Example

Example 17. An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 cm at a distance of 15 cm from pole. Find the lateral magnification produced.

Solution u = – 15 cm f = – 10 cm

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we get, v = – 30 cm

∴ $m = -\frac{v}{u} = -2.$

Aliter : $m = \frac{f}{f-u} = \frac{-10}{-10-(-15)} = -2$

Example 18. A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 cm then find the nature and radius of curvature of the mirror.

Solution : Person will see his face only when the image is virtual. Virtual image of real object is erect.
Hence m = 2

$\therefore \quad \frac{-v}{u} = 2 \quad \Rightarrow \quad v = 40$ cm

Applying $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$; f = – 40 cm or R = –80 cm (concave)

$\therefore$ R.O.C. = 80 cm

Alter : $m = \frac{f}{f-u} \quad \Rightarrow \quad 2 = \frac{f}{f-(-20)}$

$\Rightarrow$ f = – 40 cm or R = –80cm (concave)

$\therefore$ R.O.C. = 80 cm

Example 19. An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 cm then the image become triple its size. Find the nature and ROC of the mirror.

Solution : Since the images formed on screen it is real. Real object and real image implies concave mirror.

Applying $m = \frac{f}{f-u}$ or $-2 = \frac{f}{f-(u)}$ .....(1)

After shifting $-3 = \frac{f}{f-(u+5)}$ .....(2)

[Why u + 5 ?, why not u – 5 : In a concave mirror, the size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its x coordinate will increase]
From (1) & (2) we get,
f = – 30 cm or R = –60 cm (concave) and R.O.C. = 60cm

(d) Velocity of image

(i) Object moving perpendicular to principal axis : From the relation in 5.3.(b) we have

$\frac{h_2}{h_1} = -\frac{v}{u}$ or $h_2 = -\frac{v}{u} \cdot h_1$

If a point object moves perpendicular to the principal axis, x coordinate of both the object & the image become constant. On differentiating the above relation w.r.t. time, we get,

$$\frac{dh_2}{dt} = -\frac{v}{u}\frac{dh_1}{dt}$$

Here, $\frac{dh_1}{dt}$ denotes velocity of object perpendicular to the principal axis and $\frac{dh_2}{dt}$ denotes velocity of image perpendicular to the principal axis.

(ii) Object moving along principal axis : On differentiating the mirror formula with respect to time we get $\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$, where $\frac{dv}{dt}$ is the velocity of image along principal axis and $\frac{du}{dt}$ is the velocity of object along principal axis. Negative sign implies that the image, in case of mirror, always moves in the direction opposite to that of object. This discussion is for velocity with respect to mirror and along the x axis.

(iii) Object moving at an angle with the principal axis : Resolve the velocity of object along and perpendicular to the principal axis and find the velocities of image in these directions separately and then find the resultant.

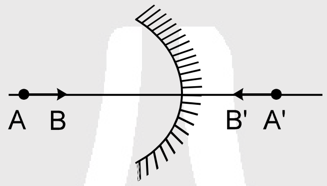
(e) Optical power of a mirror (in Dioptre) = $-\frac{1}{f}$

f = focal length with sign and in meters.

(f) If object lying along the principal axis is not of very small size, the **longitudinal magnification** = $\frac{v_2 - v_1}{u_2 - u_1}$ (it will always be inverted)

(g) If the size object is very small as compared to its distance from Pole then

On differentiating the mirror formula we get $\frac{dv}{du} = -\frac{v^2}{u^2}$ **:** Mathematically 'du' implies small change in position of object and 'dv' implies corresponding small change in position of image. If a small object lies along principal axis, du may indicate the size of object and dv the size of its image along principal axis (Note that the focus should not lie in between the initial and final points of object). In this case $\frac{dv}{dt}$ is called longitudinal magnification. **Negative sign indicates inversion of image irrespective of nature of image and nature of mirror.**



Solved Example

Example 20. A point object is placed 60 cm from pole of a concave mirror of focal length 10 cm on the principle axis. Find

(a) the position of image

(b) If object is shifted 1 mm towards the mirror along principle axis find the shift in image. Explain the result.

Solution : (a) u = – 60 cm

f = – 10cm

$$v = \frac{fu}{u-f} = \frac{-10\ \ (-60)}{-60-(-10)} = \frac{600}{-50} = -12 \text{ cm.}$$

(b) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

Differentiating, we get $dv = -\frac{v^2}{u^2}du = -\left(\frac{-12}{-60}\right)^2$ [1 mm] $= -\frac{1}{25}$ mm

[$\because$ du = 1mm; sign of du is + because it is shifted in +ve direction defined by sign convention.]

(a) –ve sign of dv indicates that the image will shift towards negative direction.

(b) The sign of v is negative. Which implies the image is formed on negative side of pole.

(a) and (b) together imply that the image will shift away from pole.

Note that differentials dv and du denote small changes only.

(h) Newton's Formula: **XY = f** ^{2}

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

6. REFRACTION OF LIGHT

When the light changes its medium, some changes occurs in its properties, the phenomenon is known as refraction.

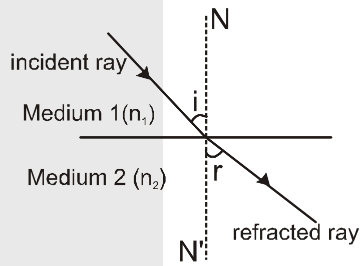
- If the light is incident at an angle (0º < i < 90º) then it deviates from its actual path. It is due to change in speed of light as light passes from one medium to another medium.
- If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.
- Refractive index of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum $\mu = \frac{c}{v} = \frac{\text{speed of light in vacuum}}{\text{speed of light in medium}}$.

More (less) refractive index implies less (more) speed of light in that medium, which therefore is called optical denser (rarer) medium.

6.1 Laws of Refraction

(a) The incident ray, the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\text{Sin } i}{\text{Sin } r}$ = Constant for any pair of media and for light of a given wave length. This is known as *Snell's* Law.



Also, $\frac{\text{Sin } i}{\text{Sin } r} = \frac{n_2}{n_1} = \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$

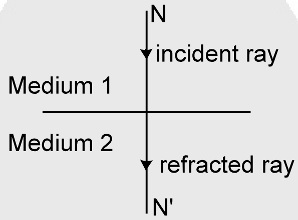
For applying in problems remember **$n_1 \sin i = n_2 \sin r$**

$\frac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

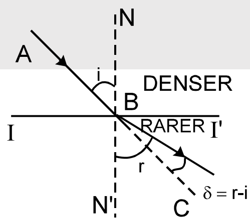
c = speed of light in air (or vacuum) = 3×10^8 m/s.

Special cases :

- Normal incidence : i = 0 ; from Snell's law : r = 0

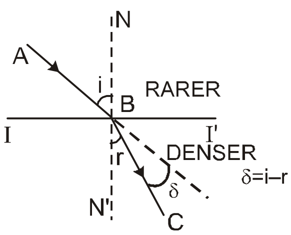


- When light moves from denser to rarer medium it bends away from normal.



r > i

- When light moves from rarer to denser medium it bends towards the normal.

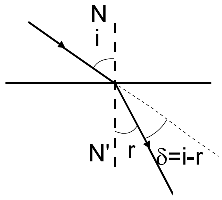


Note :
- Higher the value of R.I., denser (optically) is the medium.
- Frequency of light does not change during refraction.
- Refractive index of the medium relative to vacuum = $\sqrt{\mu_r \in_r}$

n_{vacuum} = 1 ; n_{air} = $\gtrsim$ 1; n_{water} (average value) = 4/3 ; n_{glass} (average value)= 3/2

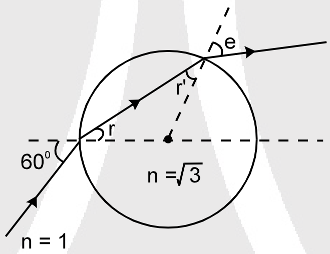
6.2 Deviation of a Ray Due to Refraction

Deviation (δ) of ray incident at $\angle$ i and refracted at $\angle$ r is given by δ = **|i – r|.**



Solved Example

Example 21. A light ray is incident on a glass sphere at an angle of incidence 60^0 as shown. Find the angles r, r', e and the total deviation after two refractions.



Solution : Applying Snell's law $1\sin 60° = \sqrt{3}\ \sin r$

$\Rightarrow$ r = 30^0

From symmetry r' = r = 30°.

Again applying Snell's law at second surface $1\sin e = \sqrt{3}\ \sin r$

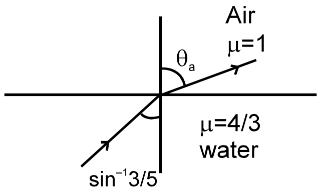
$\Rightarrow$ e = 60^0

Deviation at first surface = i – r = $60^0 - 30^0 = 30^0$

Deviation at second surface = e – r' = $60^0 - 30^0 = 30^0$

Therefore total deviation = 60^0.

Example 22 : Find the angle θ_a made by the light ray when it gets refracted from water to air, as shown in figure.



Solution : Snell's Law

$\mu_W \sin \theta_W = \mu_a \sin \theta_a \Rightarrow \frac{4}{3} \times \frac{3}{5} = 1\ \sin \theta_a$

$\sin \theta_a = \frac{4}{5}$ $\qquad \theta_a = \sin^{-1}\frac{4}{5}$

Example 23. Find the speed of light in medium 'a' if speed of light in medium 'b' is $\frac{c}{3}$ where c = speed of light in vacuum and light refracts from medium 'a' to medium 'b' making 45º and 60º respectively with the normal.

Solution : Snell's Law $\mu_a \sin\theta_a = \mu_b \sin\theta_b$

$$\frac{c}{v_a}\sin\theta_a = \frac{c}{v_b}\sin\theta_b.$$

$$\frac{c}{v_a}\sin 45^\circ = \frac{c}{c/3}\sin 60^\circ.$$

$$v_a = \frac{\sqrt{2}c}{3\sqrt{3}}$$

6.3 Principle of Reversibility of Light Rays

(a) A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.
(b) A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus the incident and refracted rays are mutually reversible.

7. REFRACTION THROUGH A PARALLEL SLAB

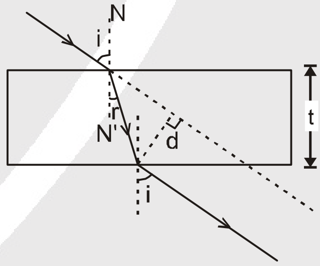
When light passes through a parallel slab, having same medium on both sides, then

(a) Emergent ray is parallel to the incident ray.

Note :

- Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

(b) Light is shifted laterally, given by (students should be able to derive it)



$$\mathbf{d} = \frac{t\sin(i-r)}{\cos r}\;;\quad t = \text{thickness of slab}$$

Solved Example

Example 24. Find the lateral shift of light ray while it passes through a parallel glass slab of thickness 10 cm placed in air. The angle of incidence in air is 60º and the angle of refraction in glass is 45º.

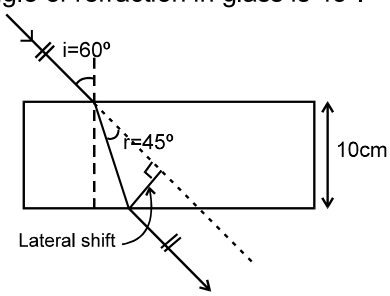
Solution :

$$d = \frac{t\sin(i-r)}{\cos r}$$

$$= \frac{10\sin(60^\circ - 45^\circ)}{\cos 45^\circ}$$

$$= \frac{10\sin 15^\circ}{\cos 45^\circ}$$

$$= 10\sqrt{2}\sin 15^\circ \text{cm}$$



7.1 Apparent Depth and shift of Submerged Object

At near normal incidence (small angle of incidence i) apparent depth (d′) is given by:

$$\mathbf{d' =} \frac{d}{n_{relative}} \quad \text{and} \quad v' = \frac{v}{n_{relative}}$$

where $n_{relative} = \dfrac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$

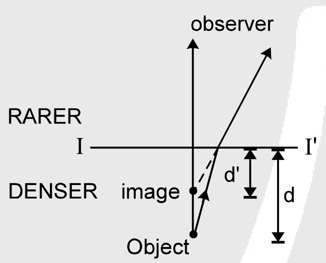
d = distance of object from the interface = real depth

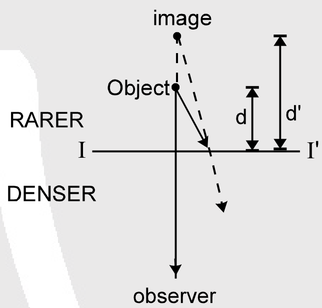
d’ = distance of image from the interface = apparent depth

v = velocity of object perpendicular to interface relative to surface.

v´ = velocity of image perpendicular to interface relative to surface.

This formula can be easily derived using Snell’s law and applying the condition of nearly normal incidence.... (try it or see in text book).





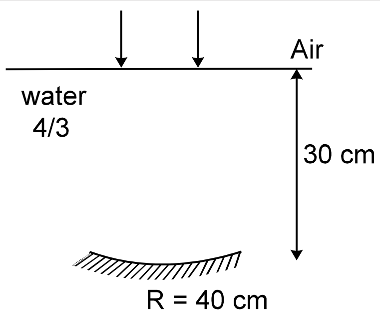
Apparent shift = d $\left(1 - \dfrac{1}{n_{rel}}\right)$

Solved Example

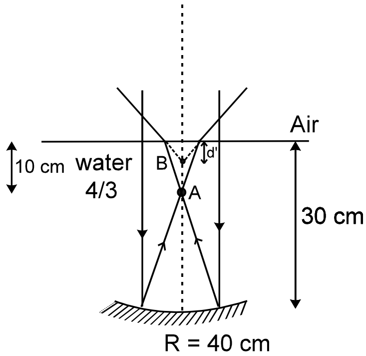
Example 25. An object lies 100 cm inside water. It is viewed from air nearly normally. Find the apparent depth of the object.

Solution : $d' = \dfrac{d}{n_{relative}} = \dfrac{100}{\dfrac{4/3}{1}} = 75 \text{ cm}$

Example 26. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Solution: The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 cm from the mirror.

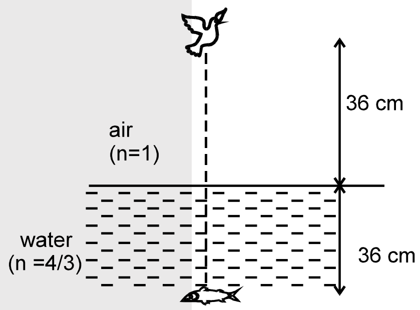


Now for the interface between water and air, d = 10 cm.

$\therefore \quad d' = \dfrac{d}{\left(\dfrac{n_w}{n_a}\right)} = \dfrac{10}{\left(\dfrac{4/3}{1}\right)} = 7.5$ cm.

Example 27. See the figure

(i) Find apparent height of the bird.
(ii) Find apparent depth of fish.
(iii) At what distance will the bird appear to the fish?
(iv) At what distance will the fish appear to the bird?
(v) If the velocity of bird is 12 cm/sec downward and the fish is 12 cm/sec in upward direction, then find out their relative velocities with respect to each other.



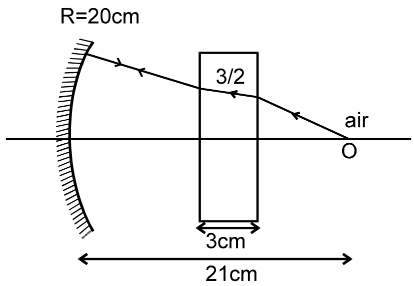
Solution (i) $d'_B = \dfrac{36}{\dfrac{1}{\left(\dfrac{4}{3}\right)}} = \dfrac{36}{3/4} = 48$ cm (ii) $d'_F = \dfrac{36}{4/3} = 27$ cm

(iii) For fish : $d_B = 36 + 48 = 84$ cm ; $d_B = 36 + 48 = 84$ cm
(iv) For bird : $d_F = 27 + 36 = 63$ cm. ; $d_F = 27 + 36 = 63$ cm.

(v) Velocity of fish with respect to bird = $12 + \left(\dfrac{12}{4/3 \big/ 1/1}\right) = 21$ cm/sec.

Velocity of bird with respect to fish = $12 + \left(\dfrac{12}{3/4 \big/ 1/1}\right) = 28$ cm/sec.

Example - 28 See the figure. Find the distance of final image formed by mirror



Solution : Shift = $3\left(1 - \dfrac{1}{3/2}\right)$

For mirror object is at a distance

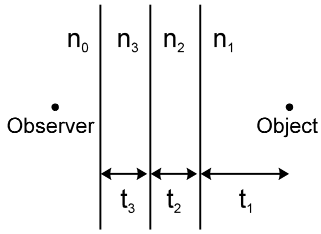
$= 21 - 3\left(1 - \dfrac{1}{3/2}\right) = 20$ cm

$\therefore$ Object is at the centre of curvature of mirror. Hence the light rays will retrace and image will be formed on the object itself.

7.2 Refraction through a composite slab (or refraction through a number of parallel media, as seen from a medium of R. I. n_0)

Apparent depth (distance of final image from final surface)

$$= \frac{t_1}{n_{1\,rel}} + \frac{t_2}{n_{2\,rel}} + \frac{t_3}{n_{3\,rel}} + \ldots\ldots + \frac{t_n}{n_{n\ \ rel}}$$



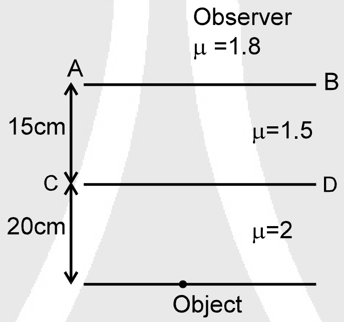
Apparent shift

$$= t_1\left[1-\frac{1}{n_{1\,rel}}\right] + t_2\left[1-\frac{1}{n_{2\,rel}}\right] + \ldots\ldots + \left[1-\frac{n}{n_{n\ rel}}\right]t_n$$

Where ' t ' represents thickness and ' n ' represents the R.I. of the respective media, relative to the medium of observer. (i.e. $n_{1rel} = n_1/n_0$, $n_{2\,rel} = n_2/n_0$ etc.)

Solved Example

Example 29. See the figure. Find the apparent depth of object seen below surface AB.



Solution : $D_{app} = \sum \frac{d}{\mu} = \frac{20}{\left(\frac{2}{1.8}\right)} + \frac{15}{\left(\frac{1.5}{1.8}\right)} = 18 + 18 = 36$ cm.

8. CRITICAL ANGLE AND TOTAL INTERNAL REFLECTION (T. I. R.)

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90º. When angle in denser medium is more than critical angle, then the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the figure

O = Object

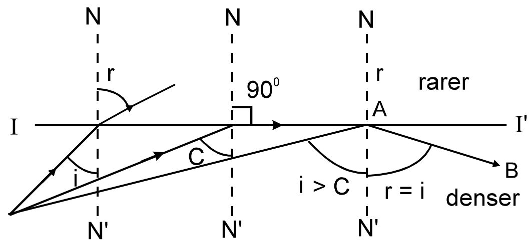
NN′ = Normal to the interface

II′ = Interface

C = Critical angle;

AB = reflected ray due to T. I. R.

When i = C then r = 90º

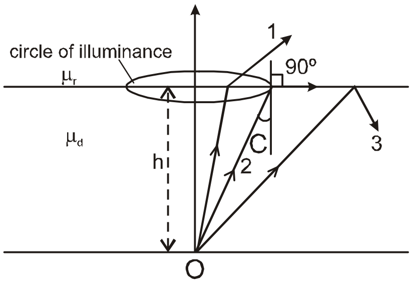


$\therefore$ **C = sin**$^{-1}\frac{n_r}{n_d}$

8.1 Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle (i > C). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r & d stand for rarer and denser medium respectively.



In the figure, ray 1 strikes the surface at an angle less than critical angle C and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance**. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object can not be seen from the rarer medium. Radius of C.O.I can be easily found.

Solved Example

Example 30. Find the maximum angle that can be made in glass medium (μ = 1.5) if a light ray is refracted from glass to vacuum.

Solution : 1.5 sin C = 1 sin 90º, where C = critical angle.

sin C = 2/3

C = $\sin^{-1}$ 2/3

Example 31. Find the angle of refraction in a medium (μ = 2) if light is incident in vacuum, making angle equal to twice the critical angle.

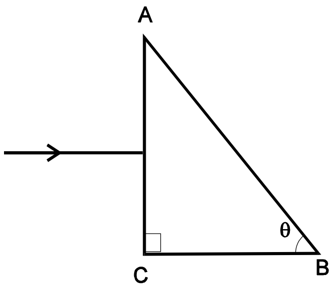
Solution : Since the incident light is in rarer medium. Total Internal Reflection can not take place.

$C = \sin^{-1}\frac{1}{\mu} = 30º \qquad \therefore \qquad i = 2C = 60º$

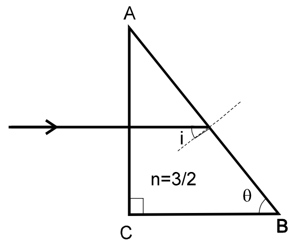
Applying Snell's Law. $\qquad$ 1 sin 60º = 2 sin r

$\sin r = \frac{\sqrt{3}}{4} \qquad \Rightarrow \qquad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right).$

Example 32. What should be the value of angle θ so that light entering normally through the surface AC of a prism (n = 3/2) does not cross the second refracting surface AB?



Solution : Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.



$i = 90 - \theta$

For the required condition : $90° - \theta > c$

or $\sin(90° - \theta) > \sin c$ or $\cos\theta > \sin c = \frac{1}{3/2} = \frac{2}{3}$

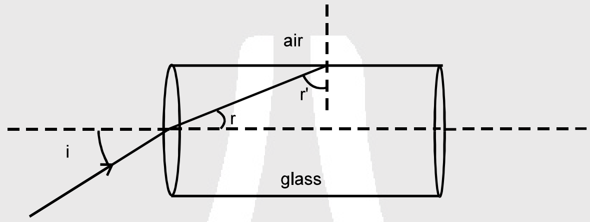
or $\theta < \cos^{-1}.\ \frac{2}{3}$

Example 33. What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

Solution : It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle .....(A)

Angle r' is minimum when r is maximum i.e. C (why ?). Therefore the minimum value of r' is 90 – C.

From condition (A) :



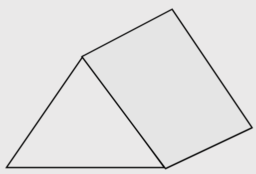
$90° - C > C$ or $C < 45°$

$\sin C < \sin 45°$; $\frac{1}{n} < \frac{1}{\sqrt{2}}$ or $n > \sqrt{2}$.

9. CHARACTERISTICS OF A PRISM

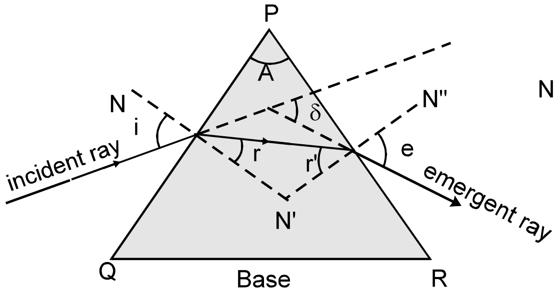
(a) A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism :

3-D view



Refraction through a prism:

View from one side



NN' & N'N'' are normals

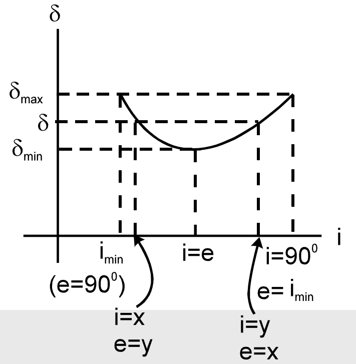
(b) PQ and PR are refracting surfaces.

(c) $\angle QPR = A$ is called refracting angle or the angle of prism (also called Apex angle).

(d) δ = angle of deviation

(e) For refraction of a monochromatic (single wave length) ray of light through a prism;
$\delta = (i + e) - (r_1 + r_2)$ and $r_1 + r_2 = A$
$\therefore$ **$\delta = i + e - A$.**

(f) Variation of δ versus i (shown in diagram).
For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light



Note :
(i) For application of above result medium on both sides of prism must be same.
(ii) Based on above graph we can also derive following result, which says that i and e can be interchanged for a particular deviation in other words there are two angle of incidence for a given deviation (except minimum deviation).

i	r_1	r_2	e	δ
θ_1	θ_2	θ_3	θ_4	θ_5
θ_4	θ_3	θ_2	θ_1	θ_5

(g) There is one and only one angle of incidence for which the angle of deviation is minimum.

(h) When $\delta = \delta_{min}$, the angle of minimum deviation, then i = e and $r_1 = r_2$, the ray passes symmetrically w.r.t. the refracting surfaces. We can show by simple calculation that
$\delta_{min} = 2i_{min} - A$
where i_{min} = angle of incidence for minimum deviation, and r = A/2.

$$\therefore \quad n_{rel} = \frac{\sin\left[\frac{A + \delta_m}{2}\right]}{\sin\left[\frac{A}{2}\right]}, \text{ where } n_{rel} = \frac{n_{prism}}{n_{surroundings}}$$

Also $\delta_{min} = (n - 1) A$ (for small values of $\angle A$)

(i) For a thin prism ($A \le 10^o$) and for small value of i, all values of

$$\delta = (n_{rel} - 1) A \quad \text{where } n_{rel} = \frac{n_{prism}}{n_{surrounding}}$$

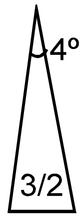
Solved Example

Example 34. Refracting angle of a prism A = 60º and its refractive index is, n = 3/2, what is the angle of incidence i to get minimum deviation? Also find the minimum deviation. Assume the surrounding medium to be air (n = 1).

Solution : For minimum deviation, $r_1 = r_2 = \frac{A}{2} = 30^o$.
applying Snell's law at I surface

$$1 \times \sin i = \frac{3}{2} \sin 30^o \quad \Rightarrow \quad i = \sin^{-1}\left(\frac{3}{4}\right) \quad \Rightarrow \quad \delta_{min} = 2\sin^{-1}\left(\frac{3}{4}\right) - 60^o$$

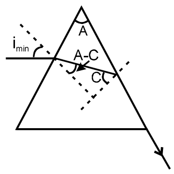
Example 35. See the figure. Find the deviation caused by a prism having refracting angle 4º and refractive index $\frac{3}{2}$.



Solution : $\delta = (\frac{3}{2} - 1) \times 4° = 2°$

Example 36. For a prism, A = 60º, n = $\sqrt{\frac{7}{3}}$. Find the minimum possible angle of incidence, so that the light ray is refracted from the second surface. Also find δ_{max}.

Solution : In minimum incidence case the angles will be as shown in figure Applying snell's law :



$1 \times \sin i_{min} = \sqrt{\frac{7}{3}} \sin(A - C)$

$= \sqrt{\frac{7}{3}} (\sin A \cos C - \cos A \sin C) = \sqrt{\frac{7}{3}} \left(\sin 60 \sqrt{1 - \frac{3}{7}} - \cos 60 \sqrt{\frac{3}{7}} \right) = \frac{1}{2}$

$\therefore \; i_{min} = 30º \qquad \therefore \; \delta_{max} = i_{min} + 90° - A = 30° + 90° - 60° = 60°$.

Example 37. Show that if A > A_{max} (= 2 C), then total internal reflection occurs at second refracting surface PR of the prism for any value of 'i'.

Solution : For T.I.R. at second surface

$r' > C \Rightarrow (A - r) > C$ or $A > (C + r)$

The above relation will be fulfilled if

or $A > C + r_{max}$ or $A > C + C$ or $A > 2C$

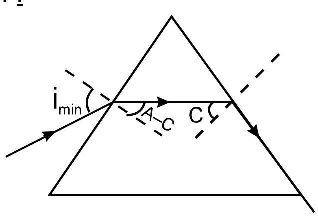
(j) On the basis of above example and similar reasoning, it can be shown that (you should try the following cases (ii) and (iii) yourself.)

(i) If A > 2C, all rays are reflected back from the second surface.

(ii) If A ≤ C, no rays are reflected back from the second surface i.e. all rays are refracted from second surface.

(iii) If 2C ≥ A > C, some rays are reflected back from the second surface and some rays are refracted from second surface, depending on the angle of incidence.

(k) δ is maximum for two values of i



⇒ i_{min} (corresponding to e = 90º) and i = 90º (corresponding to e_{min}).

For i_{min} : $n_s \sin i_{min} = n_p \sin(A - C)$

If $i < i_{min}$ then T.I.R. takes place at second refracting surface PR.

10. DISPERSION OF LIGHT

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

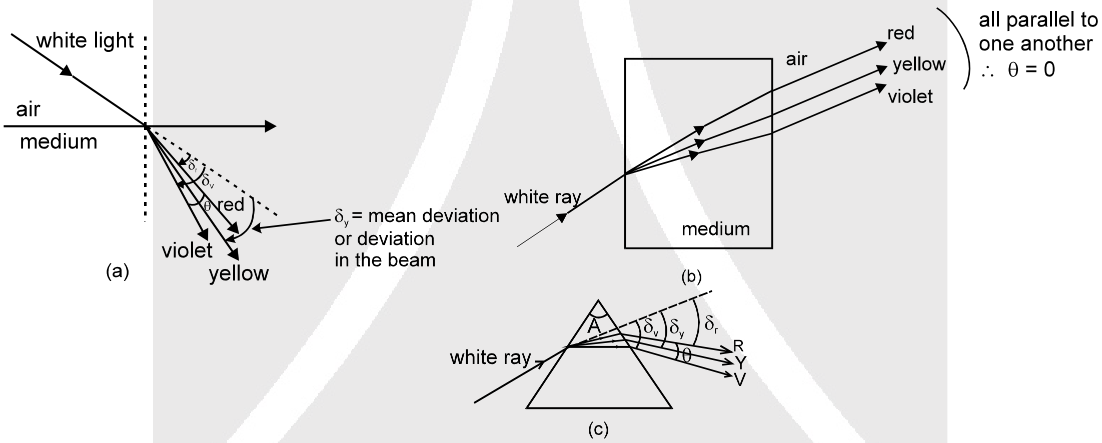
Cauchy's formula $n(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

Note :

- Such phenomenon is not exhibited by sound waves.

  Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.** $\theta = \delta_v - \delta_r$ (Fig. (a))

  Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



For prism of small 'A' and with small 'i' :

$\theta = \delta_v - \delta_r = (n_v - n_r) A$

Deviation of beam (also called mean deviation)

$\delta = \delta_y = (n_y - 1) A$

n_v, n_r and n_y are R. I. of material for violet, red and yellow colours respectively.

Solved Example

Example 38. The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Solution : Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion = $\delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 5^0 = 0.095^0$.

Note :

- Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

  Dispersive power (ω) of the medium of the material of prism is given by : $\omega = \dfrac{n_v - n_r}{n_y - 1}$

- ω is the property of a medium.

  For small angled prism ($A \le 10^o$) with light incident at small angle i :

  $$\frac{n_v - n_r}{n_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$

  [$n_y = \dfrac{n_v + n_r}{2}$ if n_y is not given in the problem]

- $n - 1$ = refractivity of the medium for the corresponding colour.

Example 39. Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively. Find
(a) the refractive index for yellow colour, approximately
(b) Dispersive power of the medium.

Solution : (a) $\mu_y \simeq \dfrac{\mu_v + \mu_R}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$ (b) $\omega = \dfrac{\mu_v - \mu_R}{\mu_y - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

10.1 Dispersion without deviation (Direct Vision Combination)

The condition for direct vision combination is :

$$[n_y - 1]\ A = [n'_y - 1]A' \quad \Leftrightarrow \quad \left[\frac{n_v + n_r}{2} - 1\right] A = \left[\frac{n'_v + n'_r}{2} - 1\right] A'$$

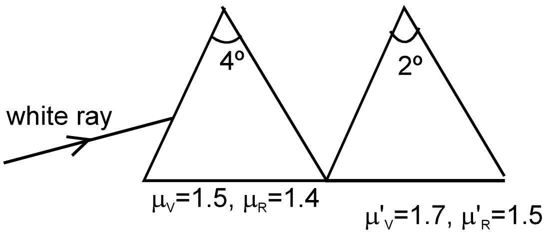
Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.

10.2 Deviation without dispersion (Achromatic Combination)

Condition for achromatic combination is: $(n_v - n_r)\ A = (n'_v - n'_r)\ A'$

Solved Example

Example 40. If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution : Both prisms will turn the light rays towards their bases and hence in same direction. Therefore turnings caused by both prisms are additive.

Total angular dispersion = $\theta + \theta' = (\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A'$

$= (1.5 - 1.4)\, 4^o + (1.7 - 1.5) 2^o = 0.8°$

Total deviation = $\delta + \delta'$

$= \left(\frac{\mu_V + \mu_R}{2} - 1\right) A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right) A' = \left(\frac{1.5 + 1.4}{2} - 1\right) 0.4^o + \left(\frac{1.7 + 1.5}{2} - 1\right) 0.2^o$

$= (1.45 - 1)\, 0.4^o + (1.6 - 1)\, 0.2^o = 0.45 \times 0.4^o + 0.6 \times 0.2^o = 1.80 + 1.2 =$ **3.0º** **Ans.**

Example 41. Two thin prisms are combined to form an achromatic combination. For I prism A = 4º, $\mu_R = 1.35$, $\mu_Y = 1.40$, $\mu_V = 1.42$ for II prism $\mu'_R = 1.7$, $\mu'_Y = 1.8$ and $\mu'_V = 1.9$ find the prism angle of II prism and the net mean deviation.

Solution : Condition for achromatic combination. $\theta = \theta'$

$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A' \quad \therefore \quad A' = \frac{(1.42 - 1.35)4^o}{1.9 - 1.7} = 1.4°$

$\delta_{Net} = \delta - \delta' = (\mu_Y - 1)A - (\mu'_Y - 1)A' = (1.40 - 1)\, 4^o - (1.8 - 1)\, 1.4° = 0.48^o$.

Example 42. A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes without deviation. Find (a) the apex angle of the flint glass prism needed and (b) the angular dispersion produced by the given combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.5, 1.6 and 1.7 respectively for crown glass and 1.8, 2.0 and 2.2 for flint glass.

Solution : The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is :

$\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions. Thus, the net deviation is :

$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'$. ......(1)

(a) If the net deviation for the mean ray is zero,

$(\mu - 1)A = (\mu' - 1)A' \quad$ or, $\quad A' = \frac{(\mu - 1)}{(\mu' - 1)} A = \frac{1.6 - 1}{2.0 - 1} \times 5° = 3°$

(b) The angular dispersion produced by the crown prism is : $\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is, $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is, $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A = (1.7 - 1.5) \times 5^0 - (2.2 - 1.8) \times 3° = -0.2°$.

The angular dispersion has magnitude 0.2°.

11. SPECTRUM

(Only for your knowledge and not of much use for JEE)

Ordered pattern produced by a beam emerging from a prism after refraction is called *Spectrum*. Types of spectrum:

11.1 Types of spectrum:

(a) Line spectrum : Due to source in atomic state.

(b) Band spectrum : Due to source in molecular state.

(c) Continuous spectrum : Due to white hot solid.

11.2 In Emission spectrum

Bright colours or lines, emitted from source are observed.

The spectrum emitted by a given source of light is called emission spectrum. It is a wavelength-wise distribution of light emitted by the source. The emission spectra are given by incandescent solids, liquids and gases which are either burnt directly as a flame (or a spark) or burnt under low pressure in a discharge tube.

11.3 In Absorption spectrum

Dark lines indicates frequencies absorbed.

When a beam of light from a hot source is passed through a substance (at a lower temperature), a part of the light is transmitted but rest of it is absorbed. With the help of a spectrometer, we can know the fraction of light absorbed corresponding to each wavelength. The distribution of the wavelength absorption of light by a substance is called an absorption spectrum. Every substance has its own characteristic absorption spectrum.

11.4 Spectrometer

Consists of a collimator (to collimate light beam), prism and telescope. It is used to observe the spectrum and also measure deviation.

12. REFRACTION AT SPHERICAL SURFACES

For paraxial rays incident on a spherical surface separating two media:

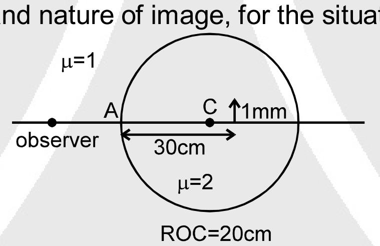
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R} \text{ .... (A)}$$

where light moves from the medium of refractive index n_1 to the medium of refractive index n_2.

Transverse magnification (m) (of dimension perpendicular to principal axis) due to refraction at spherical surface is given by $\mathbf{m} = \frac{v-R}{u-R} = \left(\frac{v/n_2}{u/n_1}\right)$

Solved Example

Example 43. Find the position, size and nature of image, for the situation shown in figure. Draw ray diagram.



Solution : For refraction near point A, u = – 30 ; R = – 20; $n_1 = 2$; $n_2 = 1$.

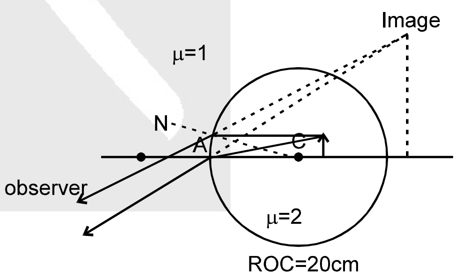
Applying refraction formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$

$$\frac{1}{v} - \frac{2}{-30} = \frac{1-2}{-20}$$

v = – 60 cm

$$m = \frac{h_2}{h_1} = \frac{n_1 v}{n_2 u} = \frac{2(-60)}{1(-30)} = 4$$

$\therefore$ h_2 = 4 mm.



Special case:

Refraction at plane Surfaces

Putting R = ∞ in the formula $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, we get ; $\mathbf{v} = \frac{n_2 u}{n_1}$

The same sign of v and u implies that the object and the image are always on the same side of the interface separating the two media. If we write the above formula as $v = \frac{u}{n_{rel}}$,

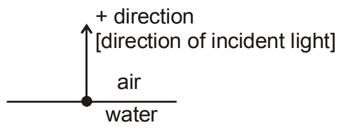
it gives the relation between the apparent depth and real depth, as we have seen before.

Solved Example

Example 44. Using formula of spherical surface or otherwise, find the apparent depth of an object placed 10 cm below the water surface, if seen near normally from air.

Solution : Put R = ∞ in the formula of the Refraction at Spherical Surfaces we get,

$v = \frac{un_2}{n_1}$



$u = -10$ cm

$n_1 = \frac{4}{3}$

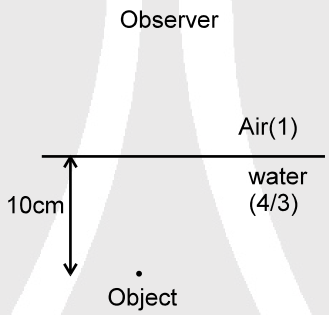
$n_2 = 1$

$v = -\frac{10 \times 1}{4/3} = -7.5$ cm

Negative sign implies that the image is formed in water.

Alter :

$d_{app} = \frac{d_{real}}{\mu_{rel}} = \frac{10}{4/3} = \frac{30}{4} = 7.5$ cm.

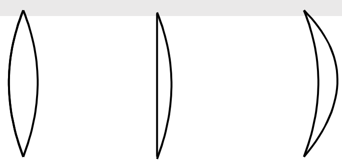


13. THIN LENS

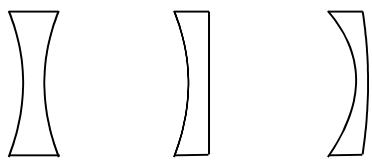
A thin lens is called convex if it is thicker at the middle and it is called concave if it is thicker at the ends. One surface of a convex lens is always convex. Depending on the other surface a convex lens is categorized as

(a) biconvex or convexo convex, if the other surface is also convex,
(b) Plano convex if the other surface is plane and
(c) Concavo convex if the other surface is concave.

Similarly concave lens is categorized as concavo-concave or biconcave, plano-concave and convexo-concave.



Bi convex Plano convex Concavo convex



Bi concave Plano concave Convexo concave

For a spherical, thin lens having the same medium on both sides:

$$\frac{1}{v} - \frac{1}{u} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad ....(a),$$

where $n_{rel} = \frac{n_{lens}}{n_{medium}}$ and R_1 and R_2 are x coordinates of the centre of curvature of the 1st surface and 2nd surface respectively.

$$\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \text{ Lens Maker's formula} \qquad ....(b)$$

From (a) and (b)

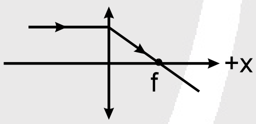
$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

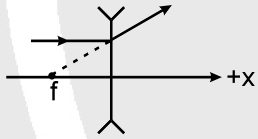
Lens has two Focii :

If $u = \infty$, then $\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$ $\Rightarrow$ $v = f$

$\Rightarrow$ If incident rays are parallel to principal axis then its refracted ray will cut the principal axis at 'f'. It is called 2nd focus.

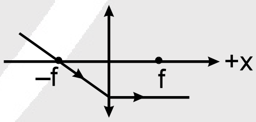
In case of converging lens it is positive and in case of diverging lens it is negative.

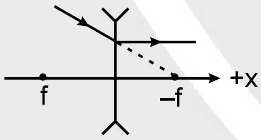




If $v = \infty$ that means $\frac{1}{\infty} - \frac{1}{u} = \frac{1}{f}$ $\Rightarrow$ $u = -f$

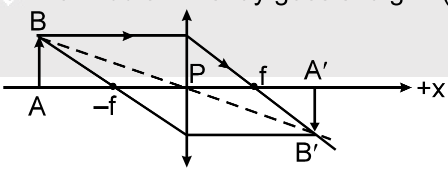
$\Rightarrow$ If incident rays cuts principal axis at – f then its refracted ray will become parallel to the principal axis. It is called 1st focus. In case of converging lens it is negative ($\because$ f is positive) and in the case of diverging lens it positive ($\because$ f is negative)

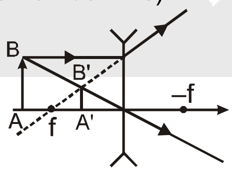




use of – f & + f is in drawing the ray diagrams.

Notice that the point B, its image B′ and the pole P of the lens are collinear. It is due to parallel slab nature of the lens at the middle. This ray goes straight. (Remember this)





From the relation $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ it can be seen that the second focal length depends on two factors.

(A) The factor $\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ is

- **(i)** **Positive** for all types of **convex** lenses and
- **(ii)** **Negative** for all types of **concave** lenses.

(B) The factor $(n_{rel} - 1)$ is

- **(i)** **Positive** when **surrounding medium is rarer than the medium of lens.**
- **(ii)** **Negative** when **surrounding medium is denser than the medium of lens.**

(C) So a lens is **converging** if **f is positive** which happens when **both the factors (A) and (B) are of same sign**.

(D) And a lens is **diverging** if **f is negative** which happens when the **factors (A) and (B) are of opposite signs**.

(E) Focal length of the lens depends on medium of lens as well as surrounding.

(F) It also depends on wavelength of incident light. Incapability of lens to focus light rays of various wavelengths at single point is known as **chromatic aberration**.

Solved Example

Example 45. Find the behaviour of a concave lens placed in a rarer medium.

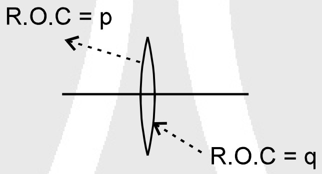
Solution : Factor (A) is **negative**, because the lens is **concave**.

Factor (B) is **positive**, because the **lens is placed in a rarer medium**.

Therefore the focal length of the lens, which depends on the product of these factors, is negative and hence the lens will behave as diverging lens.

Example 46. Show that the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (and therefore focal length) does not depend on which surface of the lens light strike first.

Solution : Consider a convex lens of radii of curvature p and q as shown.

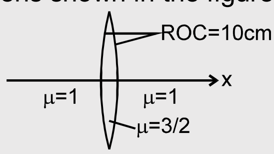


CASE 1 : Suppose light is incident from left side and strikes the surface with radius of curvature p, first. Then $R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{p}-\frac{1}{-q}\right)$

CASE 2 : Suppose light is incident from right side and strikes the surface with radius of curvature q, first. Then $R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{q}-\frac{1}{-p}\right)$

Though we have shown the result for biconvex lens, it is true for every lens.

Example 47. Find the focal length of the lens shown in the figure.

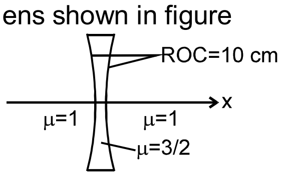


converging lens

Solution : $\because \frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

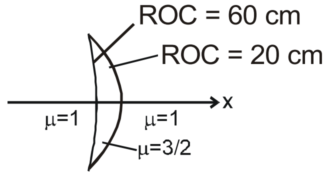
$\Rightarrow \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10}-\frac{1}{(-10)}\right) \Rightarrow \frac{1}{f} = \frac{1}{2} \times \frac{2}{10} \Rightarrow f = +10$ cm.

Example 48. Find the focal length of the lens shown in figure



Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{-10}-\frac{1}{10}\right)$; f = – 10 cm

Example 49. Find the focal length of the lens shown in figure



(a) If the light is incident from left side. (b) If the light is incident from right side.

Solution : (a) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$; f = 60 cm

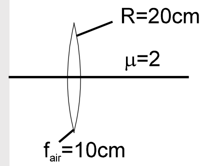
(b) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{60}\right)$; f = 60 cm

Example 50. Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss about the position of the image formed.

Solution : $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0$ $[\because R_1 = R_2]$

$\frac{1}{v} - \frac{1}{u} = 0$ or v = u i.e. rays pass without appreciable bending.

Example 51. Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index $\mu = 2$. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



Solution : Let radius of I surface be R_1 and refractive index of lens be μ . Let parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \qquad .....(1)$$

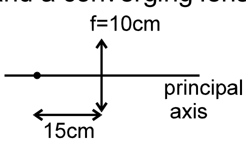
At II surface $\frac{2}{v} - \frac{\mu}{v_1} = \frac{2 - \mu}{-20}$ .....(2)

Adding (1) and (2)

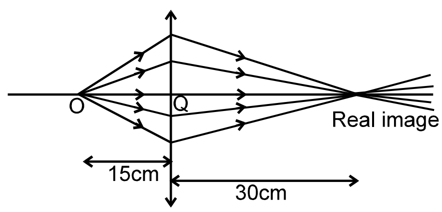
$$\frac{\mu}{v_1} - \frac{1}{\infty} + \frac{2}{v} - \frac{\mu}{v_1} = \frac{\mu - 1}{R_1} + \frac{2 - \mu}{-20}$$

$$= (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{\mu - 1}{20} - \frac{2 - \mu}{20} = \frac{1}{f} \text{ (in air)} + \frac{1}{20} - \frac{2}{20} \Rightarrow v = 40 \text{ cm} \Rightarrow f = 40 \text{ cm}$$

Example - 52 Figure shows a point object and a converging lens. Find the final image formed.



Solution :



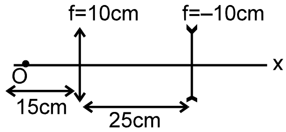
$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$

$\frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{1}{30}$

v = + 30 cm

Example 53. See the figure Find the position of final image formed.



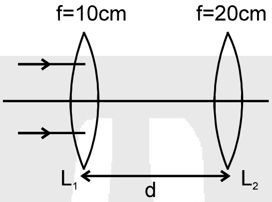
Solution : For converging lens

$u = -15$ cm, $f = 10$ cm $\qquad v = \frac{fu}{f+u} = 30$ cm

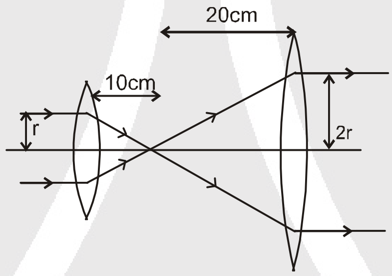
For diverging lens, $\qquad u = 5$ cm

$f = -10$ cm $\qquad v = \frac{fu}{f+u} = 10$ cm

Example 54. Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.



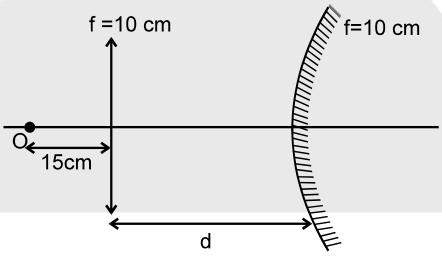
Solution : Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2.



$d = 10 + 20 = 30$ cm

Here the diameter of ray beam becomes wider.

Example 55. See the figure Find the position of final image formed.



Solution : For lens, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \qquad v = +30$ cm

Hence it is object for mirror u = – 15 cm

$\frac{1}{v} + \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \qquad v = -30$ cm

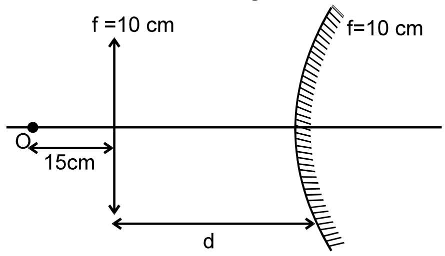
Now for second time it again passes through lens u = – 15 cm

v = ? ; f = 10 cm

$\frac{1}{v} - \frac{1}{-15} = \frac{1}{10} \qquad \Rightarrow \qquad v = +30$

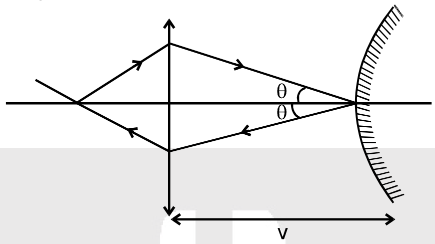
Hence final image will form at a distance 30 cm from the lens towards left.

Example 56. What should be the value of d so that image is formed on the object itself.

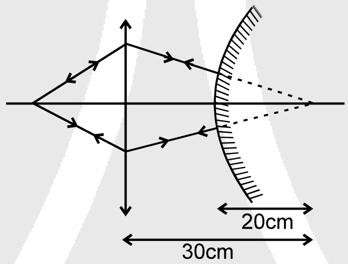


Solution : For lens : $\frac{1}{v} - \frac{1}{-15} = \frac{1}{10}$ v = + 30 cm

Case I : If d = 30, the object for mirror will be at pole and its image will be formed there itself.



Case II : If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself



∴ d = 30 – 20 = 10 cm

13.2 Transverse magnification (m)

Transverse magnification (m) (perpendicular to principal axis) is given by $m = \frac{v}{u}$. If the lens is thick or/and the medium on both sides is different, then we have to apply the formula given for refraction at spherical surfaces step by step.

Solved Example

Example 57. An extended real object of size 2 cm is placed perpendicular to the principal axis of a converging lens of focal length 20 cm. The distance between the object and the lens is 30cm.

(i) Find the lateral magnification produced by the lens.

(ii) Find the height of the image.

(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Solution : Using $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ and $m = \frac{v}{u}$ we get $m = \frac{f}{f+u}$ ......(A)

∴ $m = \frac{+20}{+20+(-30)} = \frac{+20}{-10} = -2$

–ve sign implies that the image is inverted.

(ii) $\frac{h_2}{h_1} = m$ ∴ $h_2 = mh_1 = (-2)(2) = -4$ cm

(iii) Differentiating (A) we get

$$dm = \frac{-f}{(f+u)^2}\,du \;=\; \frac{-(20)}{(-10)^2}\,(0.1) = \frac{-2}{100} = -.02$$

Note that the method of differential is valid only when changes are small.

Alternate method : u (after displacing the object) = –(30 + 0.1) = – 29.9 cm

Applying the formula $m = \frac{f}{f+u}$

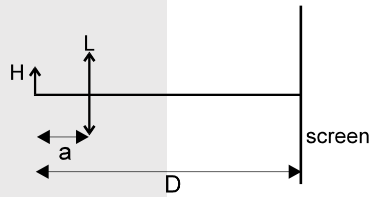
$$m = \frac{20}{20+(-29.9)} = -2.02$$

∴ change in 'm' = – 0.02.

Since in this method differential is not used, this method can be used for any changes, small or large.

13.3 Displacement Method to find Focal length of Converging Lens :

Fix an object of small height H and a screen at a distance D from object (as shown in figure). Move a converging lens from the object towards the screen. Let a sharp image forms on the screen when the distance between the object and the lens is 'a'. From lens formula we have



$$\frac{1}{D-a} - = \frac{1}{f} \quad \text{or} \quad a^2 - D\,a + f\,D = 0 \quad ...(A)$$

This is quadratic equation and hence two values of 'a' are possible. Call them a_1 and a_2. Thus a, and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

∴ $a_1 + a_2 = D \Rightarrow a_1a_2 = f\,D$

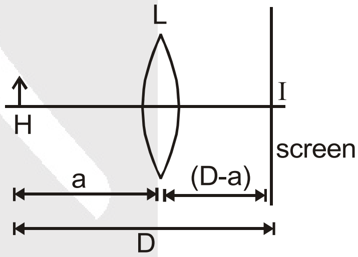
Also $(a_1 - a_2) = \sqrt{(a_1+a_2)^2 - 4a_1a_2} = \sqrt{D^2 - 4fD}$ = d (suppose).

'd' physically means the separation between the two position of lens.

The focal length of lens in terms of D and d.

so, $a_1 - a_2 = \sqrt{(a_1+a_2)^2 - 4a_1a_2}$

$$\sqrt{D^2 - 4fd} = d \Rightarrow \boxed{f = \frac{D^2 - d^2}{4D}}$$



condition, d = 0, i.e. the two position coincide

$$f = \frac{D^2}{4D}$$

∴ $\boxed{D = 4f}$

Roots of the equation $a^2 - Da + f\,D = 0$, become imaginary if

$b^2 - 4ac < 0.$

$= D^2 - 4fD < 0$

$= D\,(D - 4f) < 0.$

$= \boxed{D - 4f < 0}$

for real value of a in equation $a^2 - Da + f\,D = 0$

$b^2 - 4ac \geq 0. \quad = D^2 - 4f\,D \geq 0.$

so, $D \geq 4f \quad \Rightarrow \quad D_{min} = 4f$

Lateral magnification in displacement method:

if m_1 and m_2 be two magnifications in two positions (In the displacement method)

$$m_1 = \frac{v_1}{u_1} = \frac{(D-a_1)}{-a_1} \qquad m_2 = \frac{v_2}{u_2} = \frac{D-a_2}{-a_2} = \frac{a_1}{-(D-a_1)}$$

So $m_1 m_2 = \frac{(D-a_1)}{-a_1} \times \frac{a_1}{-(D-a_1)} = 1.$

If image length are h_1 and h_2 in the two cases,

then $m_1 = -\frac{h_1}{H}$ $\Rightarrow$ $m_2 = -\frac{h_2}{H}$ $\Rightarrow$ $m_1 m_2 = 1$

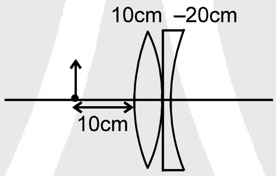
$\therefore \frac{h_1h_2}{H^2} = 1$ $\Rightarrow$ $h_1h_2 = H^2$ $\Rightarrow$ $H = \sqrt{h_1 \ h_2}$

14. COMBINATION OF LENSES

The equivalent focal length of thin lenses in contact is given by $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$ ...., where f_1, f_2, f_3 are focal lengths of individual lenses. If two lenses are separated by a distance d and the incident light rays are parallel to the common principal axis, then the combination behaves like a single lens of focal length given by the relation $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$ and the position of equivalent lens is $\frac{-dF}{f_1}$ with respect to 2nd lens.

Solved Example

Example 58. Find the lateral magnification produced by the combination of lenses shown in the figure.

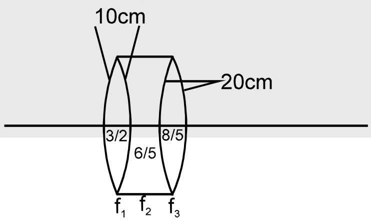


Solution : $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{10} - \frac{1}{20} = \frac{1}{20}$ $\Rightarrow$ $f = +20$

$\therefore \frac{1}{v} - \frac{1}{-10} = \frac{1}{20}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{20} - \frac{1}{10} = \frac{-1}{20} = -20$ cm

$\therefore m = \frac{-20}{-10} = 2$

Example 59. Find the focal length of equivalent system.



Solution : $\frac{1}{f_1} = \left(\frac{3}{2}-1\right)\left(\frac{1}{10}+\frac{1}{10}\right) = \frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$

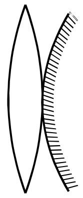
$\frac{1}{f_2} = \left(\frac{6}{5}-1\right)\left(\frac{-1}{10}-\frac{1}{20}\right) = \frac{1}{5} \times \left(\frac{-30}{10\times 20}\right) = \frac{-3}{100}$

$\frac{1}{f_3} = \left(\frac{8}{5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right) = \frac{3}{50}$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} = \frac{1}{10} + \frac{-3}{100} + \frac{3}{50}$ $f = \frac{100}{13}$ **Ans.**

15. COMBINATION OF LENS AND MIRROR

The combination of lens and mirror behaves like a mirror of focal length 'f' given by



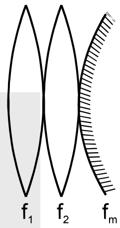
$$\frac{1}{f} = \frac{1}{F_m} - \frac{2}{F_\ell}$$

If lenses are more than one, 'f' is given by

$$\frac{1}{f} = \frac{1}{F_m} - 2\left(\sum \frac{1}{f_\ell}\right)$$

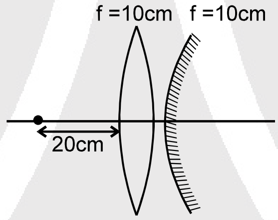
For the following figure, '

f' is given by $\frac{1}{f} = \frac{1}{F_m} - 2\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$



Solved Example

Example 60. Find the position of final image formed. (The gap shown in figure is of negligible width)



Solution : $\frac{1}{f_{eq}} = \frac{1}{10} - \frac{2}{10} = \frac{-1}{10}$

$\Rightarrow$ $f_{eq} = -10$ cm

$\frac{1}{v} + \frac{1}{-20} = \frac{1}{-10}$

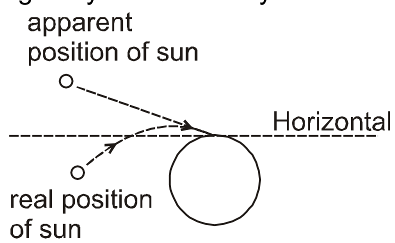
$\Rightarrow$ v = – 20 cm

Hence image will be formed on the object itself

Some interesting facts about light :

(1) THE SUN RISES BEFORE IT ACTUALLY RISES AND SETS AFTER IT ACTUALLY SETS :

The atmosphere is less and less dense as its height increase, and it is also known that the index of refraction decrease with a decrease in density. So, there is a decrease of the index of refraction with height. Due to this the light rays bend as they move in the earth's atmosphere



(2) THE SUN IS OVAL SHAPED AT THE TIME OF ITS RISE AND SET :

The rays diverging from the lower edge of the sun have to cover a greater thickness of air than the rays from the upper edge. Hence the former are refracted more than the latter, and so the vertical diameter of the sun appears to be a little shorter than the horizontal diameter which remains unchanged.

(3) THE STARS TWINKLE BUT NOT THE PLANETS :

The refractive index of atmosphere fluctuates by a small amount due to various reasons. This causes slight variation in bending of light due to which the apparent position of star also changes, producing the effect of twinkling.

(4) GLASS IS TRANSPARENT, BUT ITS POWDER IS WHITE :

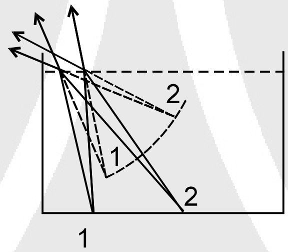
When powdered, light is reflected from the surface of innumerable small pieces of glass and so the powder appears white. Glass transmits most of the incident light and reflects very little hence it appears transparent.

(5) GREASED OR OILED PAPER IS TRANSPARENT, BUT PAPER IS WHITE :

The rough surface of paper diffusely reflects incident light and so it appears white. When oiled or greased, very little reflection takes place and most of the light is allowed to pass and hence it appears transparent.

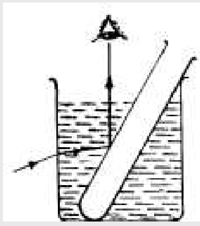
(6) AN EXTENDED WATER TANK APPEARS SHALLOW AT THE FAR END :

This is due to Total internal reflection



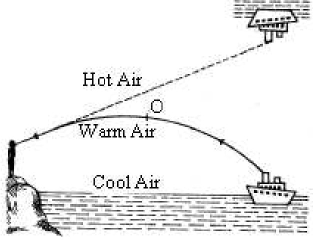
(7) A TEST TUBE OR A SMOKED BALL IMMERSED IN WATER PEARS SILVERY WHITE WHEN VIEWED FROM THE TOP :

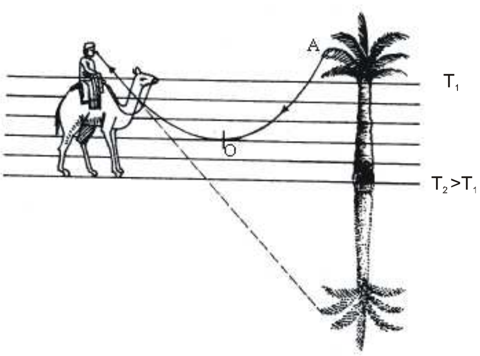
This is due to Total internal reflection



(8) SHIPS HANG INVERTED IN THE AIR IN COLD COUNTRIES AND TREES HANG INVERTED UNDERGROUND IN DESERTS:

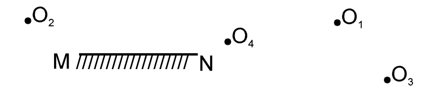
This is due to Total internal reflection



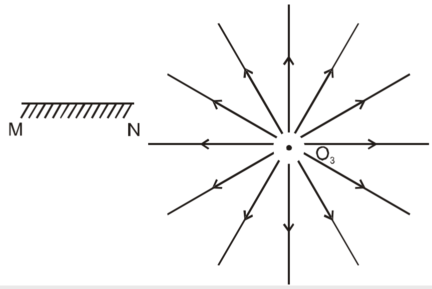


Solved Miscellaneous Problems

Problem 1. See the following figure. Which of the object(s) shown in figure will not form its image in the mirror.

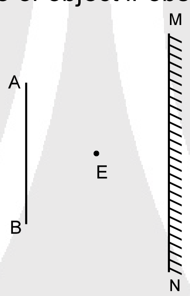


Solution :



No ray from O_3 is incident on reflecting surface of the mirror, so its image is not formed.

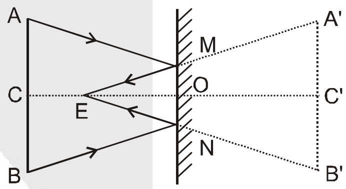
Problem 2. Figure shows an object AB and a plane mirror MN placed parallel to object. Indicate the mirror length required to see the image of object if observer's eye is at E.



Solution : Required length of mirror = MN.

Δ MNE & ΔA'B'E are similar

$$\frac{MN}{OE} = \frac{A'B'}{C'E} \Rightarrow MN = \frac{A'B'}{2} = \frac{AB}{2}.$$



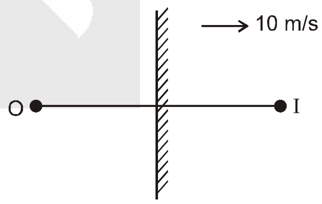
Problem 3. An object is kept fixed in front of a plane mirror which is moved by 10 m/s away from the object, find the velocity of the image.

Solution : $\vec{V}_{IM} = -\vec{V}_{OM}$

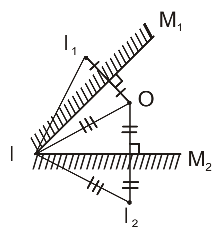
$\vec{V}_{I,G} - \vec{V}_{M,G} = -\vec{V}_{O,G} + \vec{V}_{M,G}$

$\Rightarrow \vec{V}_{M,G} = \frac{\vec{V}_{I,G} + \vec{V}_{O,G}}{2} = \frac{\vec{V}_{I,G}}{2}$ ($\because \vec{V}_{O,G} = 0$)

$\vec{V}_{I,G} = 10\hat{i}$ m/s $= 20\hat{i}$ m/s



Problem 4. Figure shows two inclined plane mirrors M_1 and M_2 and an object O. Its images formed in mirrors M_1 and M_2 individually are I_1 and I_2 respectively. Show that I_1 and I_2 and O lie on the circumference of a circle with centre at I. [This result can be extended to show that all the images will also lie on the same circle. Note that this result is independent of the angle of inclination of mirrors.]. I is the point of intersection of the mirrors.

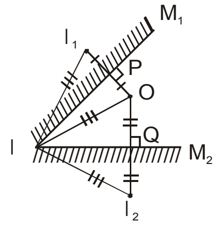


Solution : Clearly, ΔIOQ and $\Delta I I_2Q$ are congruent and ΔIOP and ΔII_1P are congruent

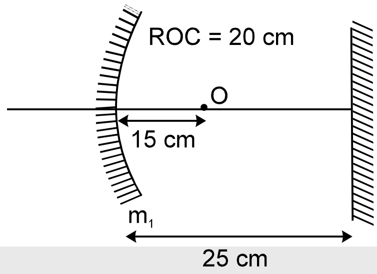
So, $II_1 = IO$ and $IO = II_2$

Hence, $II_1 = IO = II_2$

So, I_1 and I_2 and O lie on the circumference of a circle with centre I.



Problem 5. Find the position of final image after three successive reflections taking first reflection on m_1



Solution : **1st reflection at m_1**

u = – 15cm

f = –10 cm

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f} = \frac{-3+2}{30}=-\frac{1}{30}$$

v = – 30 cm

2nd reflection at plane mirror :

u = 5 cm

v = – 5 cm

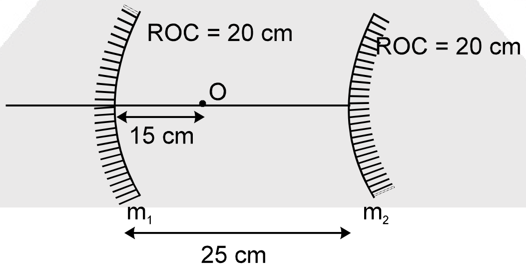
For III reflection on curved mirror again :

u = – 20 cm

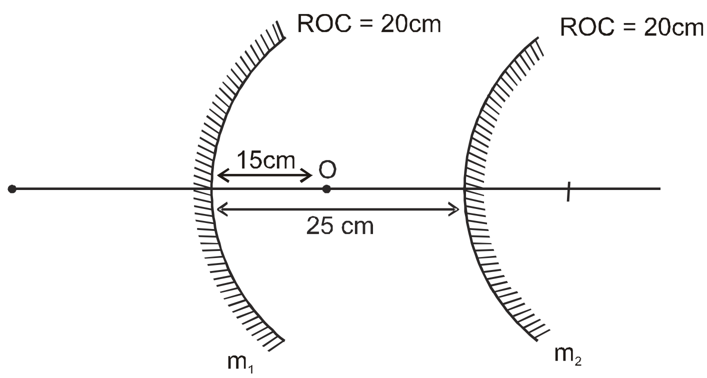
$$v = \frac{uf}{u-f} = \frac{(-20)\times(-10)}{-20+10} = \frac{200}{-10} = -20 \text{ cm}$$

Image is 20 cm right of m_1

Problem 6. Find the position of final image after three successive reflections taking first reflection on m_1.



Solution :



1st reflection at mirror m_1 : u = –15 cm, f = – 10cm

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

$$\therefore \quad v = \frac{uf}{u-f} = \frac{(-15)\times(-10)}{(-15)+10} = \frac{150}{-5} \text{ cm} = -30 \text{ cm.}$$

Thus, image is formed at a point 5 cm right of m_2 which will act as an object for the reflection at m_2

For 2nd reflection at m_2

u = 5 cm, f = 10 cm

$$v = \frac{uf}{u-f} = \frac{5\times10}{5-10} = \frac{50}{-5} = -10 \text{ cm.}$$

3rd reflection at m_1 again.

u = –15 cm f = – 10 cm

$$v = \frac{uf}{u-f} = \frac{-15\times(-10)}{(-15)+10} = -30 \text{ cm.}$$ **Ans.**

Image is formed at 30 cm right of m_1

Problem 7. A coin is placed 10 cm in front of a concave mirror. The mirror produces a real image that has diameter 4 times that of the coin. What is the image distance.

Solution : $m = \frac{d_2}{d_1} = -\frac{v}{u}$

$\Rightarrow \; -4 = -\frac{v}{u} \quad \Rightarrow \quad v = 4u$

= 4 × (–10) = – 40 cm

Problem 8. A small statue has a height of 1 cm and is placed in front of a spherical mirror. The image of the statue is inverted and is 0.5cm tall and located 10 cm in front of the mirror. Find the focal length and nature of the mirror.

Solution : We have $m = \frac{h_2}{h_1} = -\frac{0.5}{1} = -0.5$

v = – 10 cm (real image)

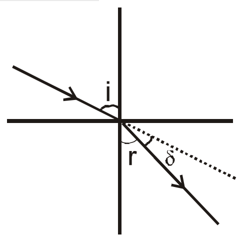
But $m = \frac{f-v}{f} \quad -0.5 = \frac{f+10}{f} \quad \Rightarrow \quad f = \frac{-20}{3}$ cm

so, concave mirror. **Ans.**

Problem 9. A light ray deviates by 30° (which is one third of the angle of incidence) when it gets refracted from vacuum to a medium. Find the refractive index of the medium.

Solution : $\delta = i - r$

$\Rightarrow \; \frac{i}{3} = i - r = 30^\circ \quad \Rightarrow \quad i = 90^\circ$

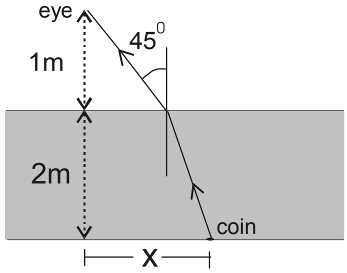


$\Rightarrow \; 2i = 3r$

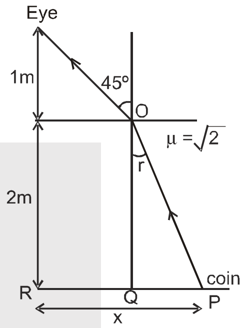
$\therefore \; r = \frac{2i}{3} = 60^\circ$

So, $\mu = \frac{\sin 90^\circ}{\sin 60^\circ} = \frac{1}{\sqrt{3}/2} = \frac{2}{\sqrt{3}}$ **Ans.**

Problem 10. A coin lies on the bottom of a lake 2m deep at a horizontal distance x from the spotlight (a source of thin parallel beam of light) situated 1 m above the surface of a liquid of refractive index $\mu = \sqrt{2}$ and height 2m. Find x.



Solution : $\sqrt{2} = \frac{\sin 45^o}{\sin r}$



$\Rightarrow$ $\sin r = \frac{1}{2}$

$\Rightarrow$ $r = 30^o$

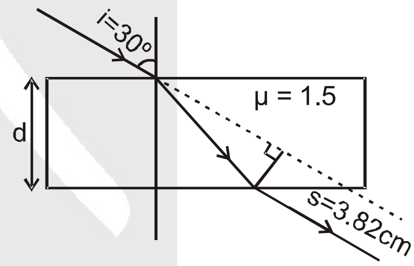
$x = RQ + QP = 1m + 2\tan 30° \ m$

$= \left(1 + \frac{2}{\sqrt{3}}\right) m$ **Ans.**

Problem 11. A ray of light falls at an angle of 30º onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 3.82 cm? [Given : $\sin^{-1}\left(\frac{1}{3}\right) = 19.5^o$; $\cos 19.5^o = 0.94$; $\sin 10.5^o = 0.18$]

Solution : Using $s = \frac{d\sin(i-r)}{\cos r}$

$\Rightarrow$ $d = \frac{3.82 \times \cos r}{\sin(30^o - r)}$ .....(1)



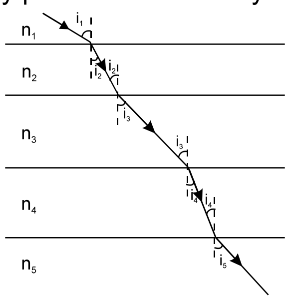
Also, $1.5 = \frac{\sin 30^o}{\sin r}$ $\Rightarrow$ $\sin r = \frac{1}{3}$

so, $r = 19.5^o$

So, $d = \frac{3.82 \times \cos 19.5^o}{\sin(30^o - 19.5^o)} = \frac{3.82 \times 0.94}{\sin 10.5^o}$

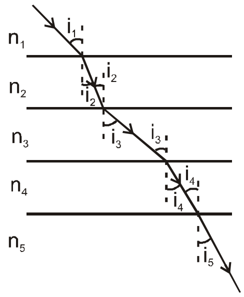
$= \frac{3.82 \times 0.94}{0.18} = 19.948 \text{ cm} \approx 0.2 \text{ m}$

Problem 12. A light passes through many parallel slabs one by one as shown in figure.



Prove that $n_1\sin i_1 = n_2\sin i_2 = n_3\sin i_3 = n_4\sin i_4$ =............**[Remember this].** Also prove that if $n_1 = n_4$ then light rays in medium n_1 and in medium n_4 are parallel.

Solution : We have, $\frac{\sin i_1}{\sin i_2} = \frac{n_2}{n_1}$



$\Rightarrow$ $n_1 \sin i_1 = n_2 \sin i_2$ ....(i)

Similarly $n_2 \sin i_2 = n_3 \sin i_3$

so on

so, $n_1 \sin i_1 = n_2 \sin i_2 = n_3 \sin i_3 = ...$

$n_1 \sin i_1 = n_4 \sin i_4 \Rightarrow$ $\sin i_1 = \sin i_4$ $(\because n_1 = n_2)$

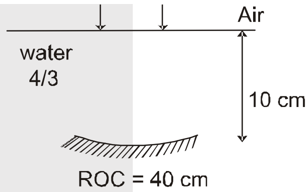
so, $i_1 = i_4$

Hence, light rays in medium n_1 and in medium n_4 are parallel.

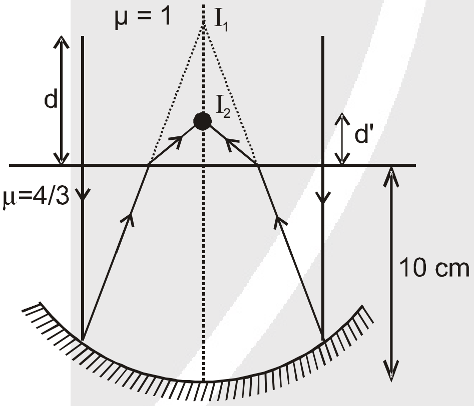
Problem 13. An object lies 90 cm in air above water surface .It is viewed from water nearly normally. Find the apparent height of the object.

Solution : $d' = \frac{d}{n_{rel}} = \frac{d}{n_i / n_r} = \frac{90 \ \ cm}{\frac{1}{4/3}} = \frac{90 \times 4}{3}$ cm = 120 cm **Ans.**

Problem 14. A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image.



Solution :



We have,

$u = -\infty$, $f = -20$ cm

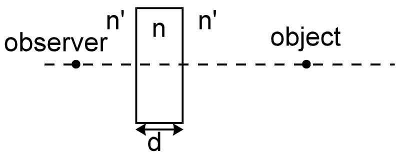
So, $v = -20$ cm

So, $d = 10$ cm

$\therefore$ $d' = \frac{d}{\mu_{rel}} = \frac{10cm}{4/3} = \frac{30}{4}$ cm = 7.5 cm **Ans.**

Problem 15. Prove that the shift in position of object due to parallel slab is given by shift = $d\left(1 - \frac{1}{n_{rel}}\right)$

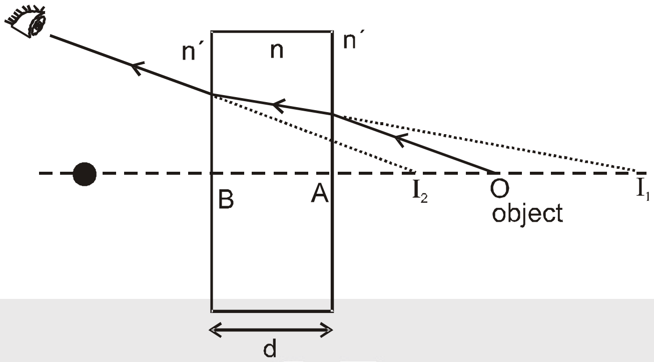
where $n_{rel} = \frac{n}{n'}$.



Solution : Because of the ray refraction at the first surface, the image of O is formed at I_1. For this refraction, the real depth is AO = x and apparent depth is AI_1.

Thus : $AI_1 = \frac{AO}{n_i/n_r} = \frac{AO}{n'/n} = \frac{n(AO)}{n'}$.

The point I_1 acts as the object for the refraction of second surface. Due to this refraction, the image of I_1 is formed at I_2. Thus,

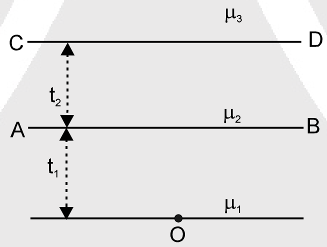


$BI_2 = \frac{(BI_1)}{(n/n')} = \frac{n'}{n}$ (BI_1) = n'/n $(AB + AI_1)$ = $\frac{n'}{n}\left[d + \frac{n}{n'}(AO)\right] = \frac{n'}{n}d + AO$.

Net shift = $OI_2 = BO - BI_2 = d + (AO) - \frac{n'}{n}d - AO$

$= d\left(1 - \frac{n'}{n}\right) = d\left(1 - \frac{1}{n_{rel}}\right)$ where $n_{rel} = \frac{n}{n'}$. **Ans.**

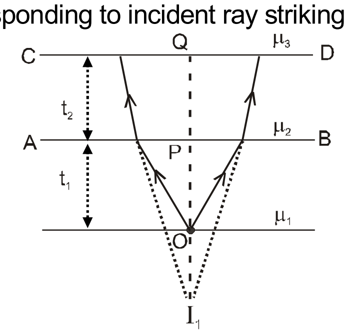
Problem 16. Find the apparent depth of object O below surface AB, seen by an observer in medium of refractive index μ_2



Solution : $d_{app.} = \frac{t_1}{\mu_1/\mu_2}$

Problem 17. In above question what is the depth of object corresponding to incident rays striking on surface CD in medium μ_2.

Solution : Depth of the object corresponding to incident ray striking on the surface CD in medium $\mu_2 = t_2 + PI_1$

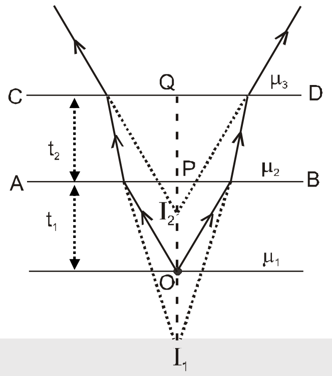


$= t_2 + \frac{t_1}{\mu_1/\mu_2}$

Problem 18. In above question if observer is in medium μ_3, what is the apparent depth of object seen below surface CD.

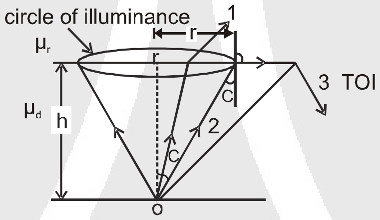
Solution : If the observer is in medium μ_3. Apparent depth below surface CD = QI_2.

$$= \sum \frac{t_i}{(n_{rel})_i} = \frac{t_2}{\mu_2/\mu_3} + \frac{t_1}{\mu_1/\mu_3}$$



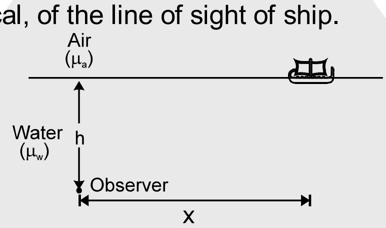
Problem 19. Find the radius of circle of illuminance, if a luminous object is placed at a distance h from the interface in denser medium.

Solution : $\tan C = \frac{r}{h}$. $\therefore$ $r = h \tan C$. But $C = \sin^{-1}\frac{1}{(\mu_d/\mu_r)}$

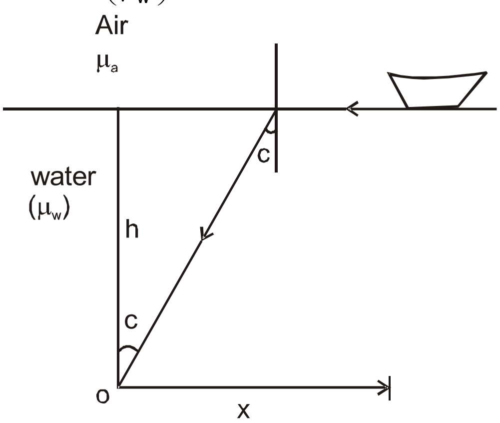


so, $r = h \tan\left[\sin^{-1}\frac{1}{(\mu_d/\mu_r)}\right] = h.\frac{\mu_r}{\sqrt{\mu_d^2 - \mu_r^2}}$

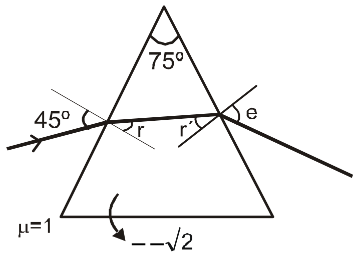
Problem 20. A ship is sailing in river. An observer is situated at a depth h in water (μ_w). If x >> h, find the angle made from vertical, of the line of sight of ship.



Solution : $C = \sin^{-1}\left(\frac{\mu_a}{\mu_w}\right)$



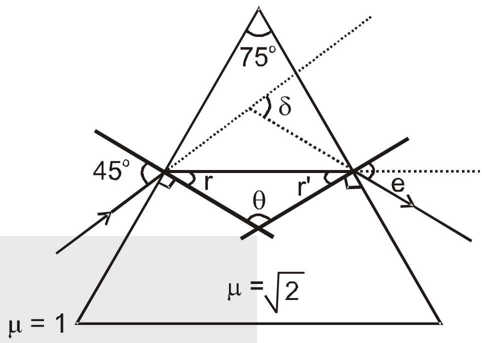
Problem 21. Find r, r′, e, δ for the case shown in figure.



Solution : Here $\theta = 180° - 75° = 105°$

$\sin 45° = \sqrt{2} \sin r$

$\therefore \quad r = \sin^{-1} \frac{1}{2} = 30°.$



$r' = 180° - (r + \theta) = 180° - 30° - 105° = 45°$

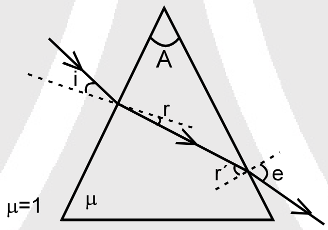
$\sin e = \sqrt{2} \sin r'$

$\therefore \quad \sin e = \sqrt{2} \times \sin 45° = 1$

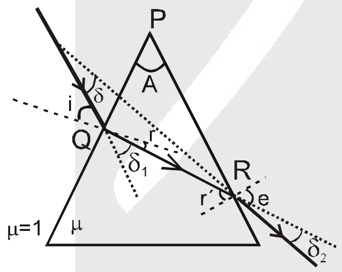
$\therefore \quad \boxed{e = 90°}$

So, $\delta = i + e - A = 45° + 90° - 75° = 60°$.

Problem 22. For the case shown in figure prove the relations $r' - r = A$ and $\delta = |(i - e) + A|$ (do not try to remember these relations because the prism is normally not used in this way).



Solution :



In Δ PQR, $A + \angle PQR + \angle QRP = 180°$

$= A + r + 90° + 90° - r' = 180°$

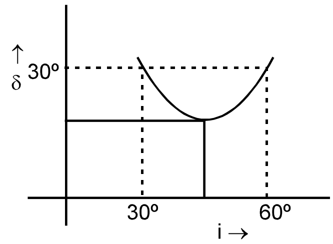
$\therefore \quad \boxed{r' - r = A}$

Deviation after I[st] refraction $\delta_1 = (i - r)$ (anticlock wise)

Deviation after II[nd] refraction $\delta_2 = (e - r')$ (clock wise)

Hence net deviation $\delta = \delta_1 - \delta_2 = (i - r) - (e - r') = i - e + A$

Problem 23. From the graph of angle of deviation δ versus angle of incidence i, find the prism angle



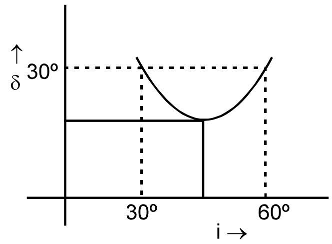
Solution : From the graph ;

$\delta = i + e - A$.

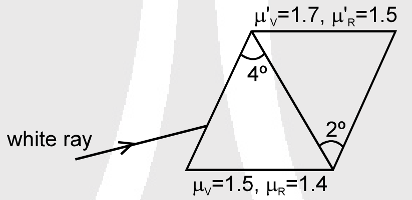
$30° = 30° + 60° - A$

$\therefore$ $\boxed{A = 60°}$

(use the result : If i and e are interchanged then we get same value of δ)



Problem 24. If two prisms are combined, as shown in figure, find the net angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution : Net angular dispersion = $(\delta_v - \delta_r) - (\delta'_v - \delta'_r)$

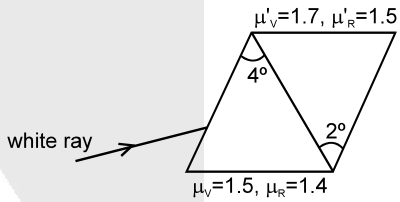
$= (\mu_v - \mu_r)\,A - (\mu'_v - \mu'_r)\,A'$

$= (1.5 - 1.4) \times 4° - (1.7 - 1.5) \times 2° = 0$

Angle of deviation

$$= \left(\frac{\mu_v + \mu_r}{2} - 1\right) A - \left(\frac{\mu'_v + \mu'_r}{2} - 1\right) A'$$

$$= \left(\frac{1.5 + 1.4}{2} - 1\right) \times 4° - \left(\frac{1.7 + 1.5}{2} - 1\right) \times 2° = 0.6°$$



Problem 25. The dispersive powers of crown and flint glasses are 0.03 and 0.05 respectively. The refractive indices for yellow light for these glasses are 1.517 and 1.621 respectively. It is desired to form an achromatic combination of prisms of crown and flint glasses which can produce a deviation of 1° in the yellow ray. Find the refracting angles of the two prisms needed.

Solution : $\omega_c = 0.03 = \dfrac{n_v - n_r}{n_y - 1}$

$\therefore$ $(n_v - n_r) = 0.03\,(1.5\,17 - 1) = 0.0155$

and, $\omega_f = 0.05 = \dfrac{n'_v - n'_r}{n'_y - 1}$

$\therefore$ $n'_v - n'_r = 0.05 \times (1.6\,21 - 1) = 0.031$

$\theta = (n_v - n_r)\,A - (n'_v - n'_r)\,A'$

$= 0.0155\,A - 0.031\,A'$ ......(1)

But $\delta_{net} = 1$

So, $(n_y - 1)\,A - (n'_y - 1)\,A' = 1$

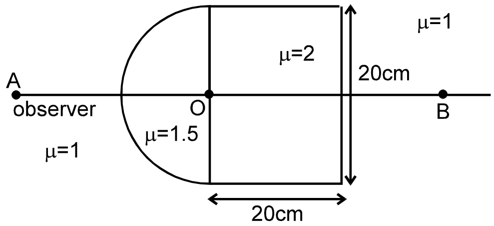
$= 0.517\,A - 0.621\,A' = 1$ ......(2)

$\therefore$ $A = 4.8°$ and $A' = 2.4°$

Problem 26. See the situation shown in figure

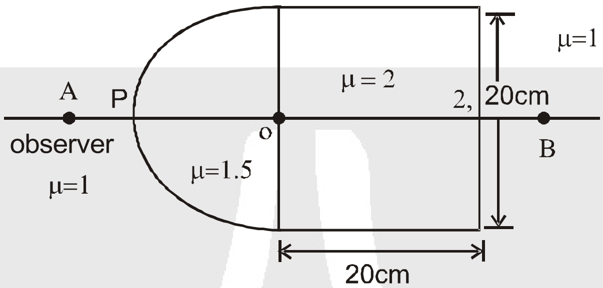
(1) Find the position of image as seen by observer A.

(2) Find the position of image as seen by observer B.



Solution : (i) As seen by observer A .

R = – 10 cm, u = – 10 cm.



So, $\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$, $\frac{1}{v} - \frac{1.5}{(-10)} = \frac{1-1.5}{(-10)}$

v = –10 cm (at O)

(ii) As seen by observer B

R = ∞

u = – 20 cm

$\frac{1}{v} - \frac{2.0}{-20} = \frac{1-2.0}{\infty}$

v = – 10 cm image will be formed 10 cm right of O.

Problem 27. Find the focal length of a double-convex lens with R_1 = 15 cm and R_2 = –25 cm. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{15} + \frac{1}{25}\right) = 0.5\left(\frac{10+6}{150}\right) = \frac{8}{150}$.

$f = \frac{150}{8} = 18.75$ cm

Problem 28. Find the focal length of a plano-convex lens with R_1 = 15 cm and R_2 = ∞. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{\infty}\right) = 0.5 \times \frac{1}{15}$

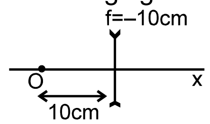
∴ f = 30 cm .

Problem 29. Find the focal length of a concavo-convex lens (positive meniscus) with R_1 = 15 cm and R_2 = 25 cm. The refractive index of the lens material n = 1.5.

Solution : $\frac{1}{f} = (1.5 - 1)\left(\frac{1}{15} - \frac{1}{25}\right) = 0.5\left(\frac{10-6}{150}\right)$.

$\therefore\ f = \frac{300}{4} = 75$ cm

Problem 30. Figure shows a point object and a diverging lens.



Find the final image formed.

Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = \frac{1}{-10} + \frac{1}{(-10)} = -\frac{2}{10}$ $\Rightarrow v = -5$ cm

Problem 31. An extended real object is placed perpendicular to the principal axis of a concave lens of focal length –10 cm, such that the image found is half the size of object.

(a) Find the object distance from the lens

(b) Find the image distance from the lens and draw the ray diagram

(c) Find the lateral magnification if object is moved by 1 mm along the principal axis towards the lens.

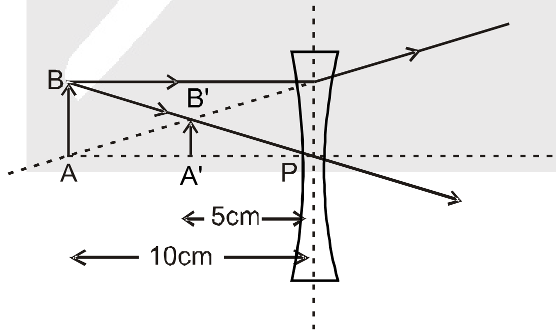
Solution : (a) We have, $f = -10$ cm.

$m = \frac{h_2}{h_1} = 0.5 = \frac{v}{u}$

So, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ gives.

$\Rightarrow \frac{1}{0.5u} - \frac{1}{u} = \frac{1}{-10}$ $\therefore\ u = -10$cm

(b)



$\frac{v}{u} = 0.5.$

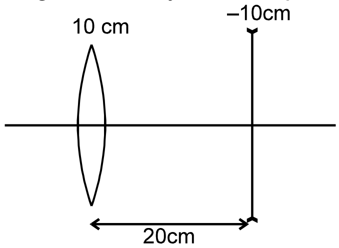
$\therefore\ v = 0.5 \times (-10)$ cm $= -5$ cm

Ray diagram :

(c) $m = \frac{f}{f+u} \Rightarrow dm = \frac{-f}{(f+u)^2}\,du = \frac{(+10)}{(-10-10)^2}\,(0.1) = 0.0025$ cm

so, final lateral magnification (m + dm) = 0.5025 cm **Ans.**

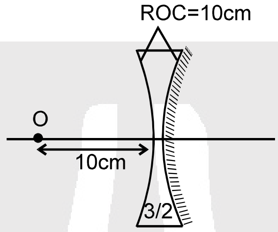
Problem 32. Find the equivalent focal length of the system for paraxial rays parallel to axis.



Solution : $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = \frac{1}{10} + \frac{1}{-10} - \frac{20}{10(-10)} = \frac{1}{5}$

$\Rightarrow$ f_{eq} = 5 cm

Problem 33. See the figure. Find the equivalent focal length of the combination shown in the figure and position of image.



Solution : For the concave lens $\frac{1}{f} = (n - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$= \left(\frac{3}{2} - 1\right)\left(\frac{1}{-10} - \frac{1}{10}\right) = -\frac{1}{2} \times \frac{2}{10} = \frac{1}{10}$

And, $f_m = \frac{R}{2} = \frac{10}{2}$ = 5 cm

$\therefore \frac{1}{f_{eq}} = \frac{1}{f_m} - 2\frac{1}{f} = \frac{1}{5} + 2 \times \frac{1}{10} = \frac{2}{5}$ $\boxed{f_{eq} = 2.5\text{cm}}$ **Ans.**

Optical Instruments

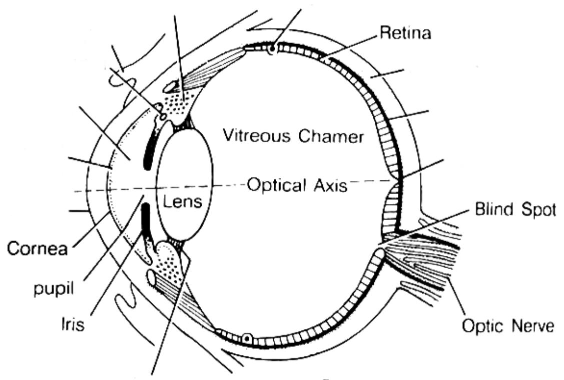
1 Human Eye

1.1 Structure of Eye

Light enters the eye through a curved front surface, the cornea. It passes through the pupil which is the central hole in the iris. The size of the pupil can change under control of muscles. The light is further focussed by the eye-lens on the retina. The retina is a film of nerve fibres covering the curved back surface of the eye. The retina contains rods and cones which sense light intensity and colour, respectively, and transmit electrical signals via the optic nerve to the brain which finally processes this information. The shape (curvature) and therefore the focal length of the lens can be modified somewhat by the ciliary muscles. For example, when the muscle is relaxed, the focal length is about 2.5 cm and (for a normal eye) objects at infinity are in sharp focus on the retinas.

When the object is brought closer to the eye, in order to maintain the same image-lens distance (2.5 cm), the focal length of the eye-lens becomes shorter by the action of the ciliary muscles. This property of the eye in called **accommodation**.

If the object is too close to the eye, the lens cannot curve enough to focus the image on to the retina, and the image is blurred.



Human Eye

The closest distance for which the lens can focus light on the retina is called the **least distance of distinct vision or the near point.** The standard value (for normal vision) taken here is 25 cm (the near point is given the symbol D.)

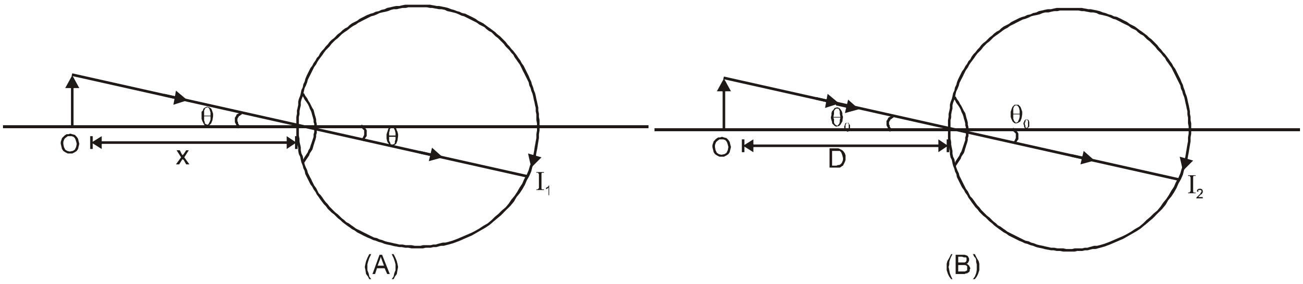
When the image is situated at infinity the ciliary muscles are least strained to focus the final image on the retina, this situation is known as **normal adjustment**.

1.2 Regarding Eye:

1. In eye convex eye-lens forms real inverted and diminished image at the retina by changing its convexity (the distance between eye lens and retina is fixed)

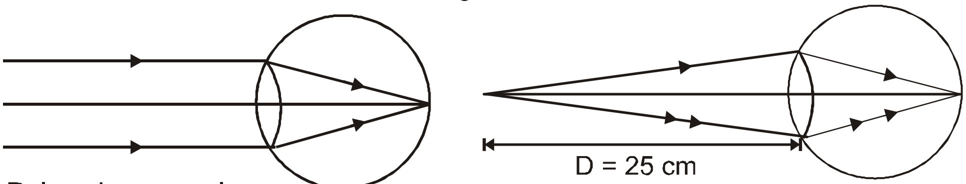
2. The human eye is most sensitive to yellow green light having wavelength 5550 Å and least to violet (4000Å) and red (7000 Å)

3. The size of an object as perceived by eye depends on its visual-angle when object is distant its visual angle θ and hence image I_1 at retina is small (it will appear small) and as it is brought near to the eye its visual angle θ_0 and hence size of image I_2 will increase.



4. The far and near point for normal eye are usually taken to be infinity and 25 cm respectively i.e.., normal eye can see very distant object clearly but near objects only if they are at distance greater than 25 cm from the eye. The ability of eye to see objects from infinite distance to 25 cm from it is called **Power of accommodation.**

5. If object is at infinity i.e. parallel beam of light enters the eye is least strained and said to be relaxed or unstrained. However, if the object is at least distance of distinct vision (L.D.D.V) i.e., D (=25 cm) eye is under maximum strain and visual angle is maximum.



Relaxed or normal eye (A) Maximum strained eye (B)

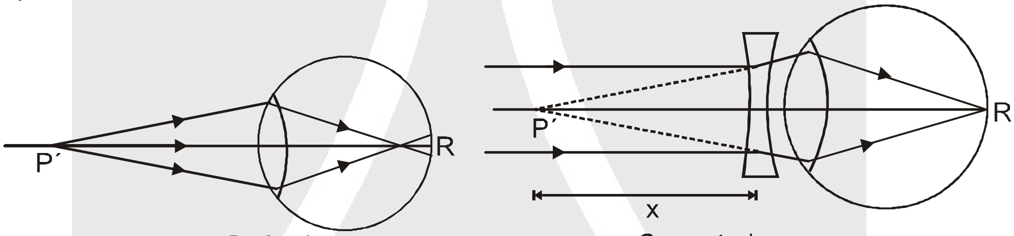
6. The limit of resolution of eye is one minute i.e. two object will not be visible distinctly to the eye if the angle subtended by them on the eye is lesser than one minute.

7. The persistence of vision is (1/10) sec i.e., If time interval between two consecutive light pulses is lesser than 0.1 sec eye cannot distinguish them separately. This fact is taken into account in motion pictures.

1.3 Defects of vision

1. Myopia [or short-sightendness or near - sightendness]

In it distant objects are not clearly visible. The far point for a myopic eye is much nearer than infinity.



Defective eye (A) Corrected eye (B)

If P´ is far point for a myopic eye, then the image of an object placed at the point P´ will be formed on the retina as shown in the figure (A).

The myopic eye will get cured against this defect, if it is able to see the objects at infinity clearly. In order to correct the eye for this defect, a concave lens of suitable focal length is placed close to the eye, so that the parallel ray of light from point P´ of the myopic eye as shown in figure (B).

If x is the distance of the far point from the eye, then for the concave lens placed before the eye:

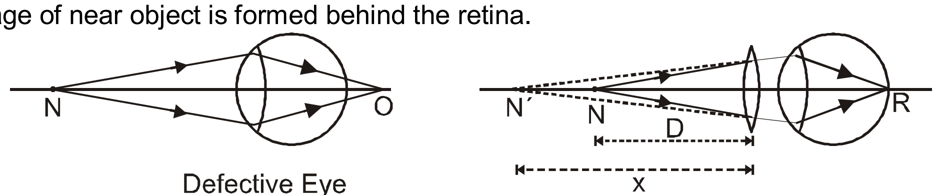
$u = \infty$ and $v = -x$

solving, $f = -x$

Thus, myopic eye is cured against the defect by using a concave lens of focal length equal to the distance of its far point from the eye.

2 Hypermetropia [Or Long-sightendness or far-sightendness]

In it near object are not clearly visible i.e., Near Point is at a distance greater than 25 cm and hence image of near object is formed behind the retina.



Defective Eye (A) Corrected Eye (B)

In case of a hypermetropic eye, when the object lies at the point N (at the near point for a normal eye), its image is formed behind the retina as shown in figure (A).

The near point N´ for hypermetropic eye is farther than N, the near point for a normal eye.

Such defect will get cured, if the eye can see an object clearly, when place at the near point N for the normal eye. To correct this defect, a convex lens of suitable focal length is placed close to the eye so that the rays of light form an object placed at the point N after refraction through the lens appear to come from the near point N´ of the hypermetropic eye as shown in figure (B).

Let x be the distance of the near point N´ from the eye and D, the least distance of distinct vision i.e. the distance of near point N for the normal eye. Then, for the convex lens placed before the eye,

$u = -D$

and $v = -x$

If f is the focal length of the required convex lens, then from the lens formula, we have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{-x} - \frac{1}{-D} = \frac{1}{f}$$

$$f = \frac{xD}{x-D} \qquad \text{...(1)}$$

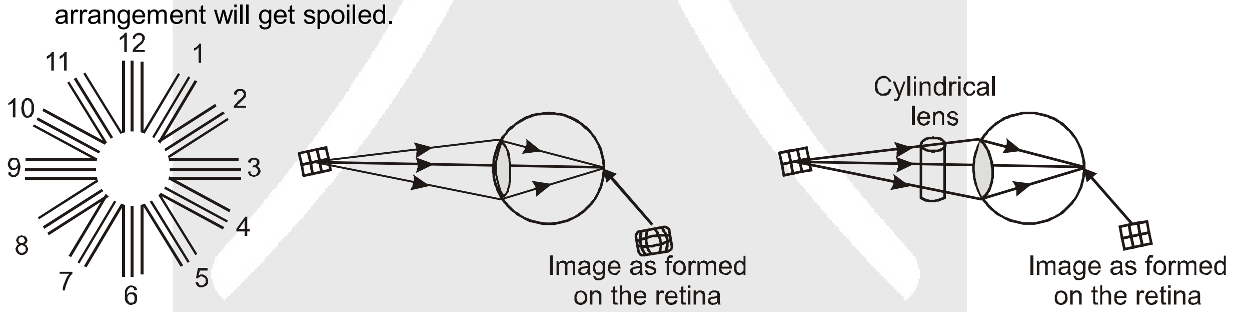
Thus, an eye suffering from hypermetropia can be cured against the defect by using a convex lens of focal length given by equation (1).

3 Presbyopia

In this both near and far objects are not clearly visible i.e., far point is lesser than infinity and near point greater than 25 cm. It is an old age disease as at old age ciliary muscles lose their elasticity and so can not change the focal length of eye-lens effectively and hence eye loses its power of accommodation.

4 Astigmatism

In it due to imperfect spherical nature of eye-lens, the focal length of eye lens is two orthogonal directions becomes different and so eye cannot see object in two orthogonal directions clearly simultaneously. This defect is directional and is remedied by using cylindrical lens in particular direction. If in the spectacle of a person suffering from astigmatism, the lens is slightly rotated the arrangement will get spoiled.



Example 1. A person cannot see objects clearly beyond 50 cm. What should be the power of corrective lens used ?

Solution : $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

for correcting for point $u = -\infty$, $v = -50$ cm

$$-\frac{1}{50} + \frac{1}{\infty} = \frac{1}{f}$$

f = –50 cm

$$P = \frac{1}{f} = \frac{1}{-0.5} = -2D$$

Example 2. A certain myopic person has a far point of 150 cm. (a) What power a corrective lens must have to allow him to see distant objects clearly ? (b) If he is able to read a book at 25 cm, while wearing the glasses, is his near point less than 25 cm ?

Solution : (a) Here, the distance of the far point, x = 150 cm

The defect can be corrected by using concave lens of focal length,

f = –x = –150 cm = –1.5 m

The power of the lens is given by

$$P = \frac{1}{f} = \frac{1}{-1.5} = 0.67\text{ D}$$

(b) here, u = –25 cm ; f = –150 cm

From the lens equation, we have

$$v = \frac{uf}{u+f} = \frac{(-25)\times(-150)}{(-25)+(-150)} = 21.43\text{ cm}$$

Therefore, the near point will be at a distance of 21.43 cm i.e. less than 25cm.

Optical instruments used primarily to assist the eye in viewing an object.

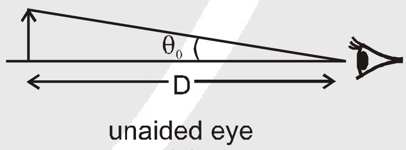
1. Microscope

It is an optical instrument used to increase the visual angle of neat objects which are too small to be seen by naked eye.

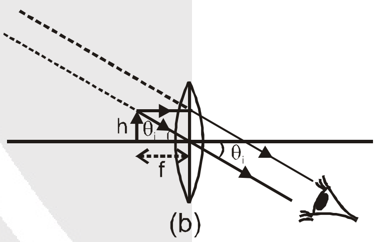
1.1 Simple Microscope

The normal human eye can focus a sharp image of an object on the retina if the object is located anywhere from infinity to a certain point called the **near point (D)**. If you move the object closer to the eye than the near point, the perceived retinal image becomes fuzzy. For an average viewer the near point, D = 25 cm from the eye. When the object is at the eye's near point, its image on the retina is as large as it can be and still be in focus.

The apparent size of an object is determined by the size of its image on the retina. If the eye is unaided, this size depends on the angle θ_o subtended by the object at the eye, called its **angular size** as shown in figure (a).



(a)



(b)

To look closely at a small object, such as an insect or a crystal, you bring it close to your eye, making the subtended angle and the retinal image as large as possible. But your eye cannot focus sharply on objects that are closer than the near point, so the angular size of an object is greatest (that is, it subtends the largest possible viewing angle) when it is placed at the near point.

A converging lens can be used to form a virtual image that is larger and farther from the eye than the object itself, as shown in figure (b). Then the object can be moved closer to the eye, and the angular size of the image may be substantially larger than the angular size of the object at 25 cm without the lens. A lens used in this way is called a **simple microscope,** otherwise known as a magnifying glass.

The usefulness of the magnifier is given by its angular magnification.

(i) **If the image is formed at infinity (normal adjustment):** The virtual image is most comfortable to view when it is placed at infinity, so that the ciliary muscle of the eye is relaxed; this means that the object is placed at the focal point of the magnifier. In this case we find angular magnification.

Angular magnification or magnifying power (M) is defined as the ratio of the angle subtended by the image (situated at infinity) at the eye to the angle subtended by the object seen directly at the eye when situated at near point D.

In figure (a) the object is at the near point, where it subtends an angle θ_o at the eye.

$$\theta_o \approx \tan\theta_o \approx \frac{h}{D} \qquad ...(1)$$

In figure (b) a magnifier in front of the eye forms an image at infinity, and the angle subtended at the magnifier is θ_i

$$\theta_i \approx \tan\theta_i \approx \frac{h}{f} \qquad ...(2)$$

Angular magnification, $M = \frac{\theta_i}{\theta_o} = \frac{D}{f}$ ...(3)

(ii) **If the image is at formed at near point, D:** The linear magnification 'm', for the image formed at the near point D, by a simple microscope can be obtained by using the relation

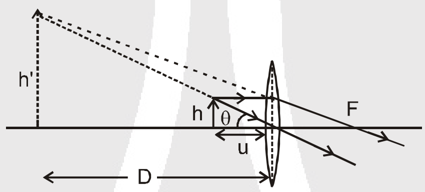
$$m = \frac{h'}{h} = \frac{v}{u} = \left(1 - \frac{v}{f}\right) \qquad ...(4)$$

h is the size of the object and h' is the size of the image

$$m = \left(1 + \frac{D}{f}\right) \qquad ...(5) \qquad (v = -D)$$

Angular magnification or magnifying power(M) is defined as the ratio of the angle subtended by the image at the eye to the angle subtended by the object seen directly at the eye when both lie at near point D.

In figure (c) a magnifier in front of the eye forms an image at D, and the angle subtended at the magnifier is θ_i



(c)

Example 3. A man with normal near point (25 cm) reads a book with small print using a magnifying thin convex lens of focal length 5 cm. (a) What is the closest and farthest distance at which he can read the book when viewing through the magnifying glass? (b) What is the maximum and minimum magnifying power possible using the above simple microscope?

Solution : (a) As for normal eye far and near point are and 25 cm respectively, so for magnifier $v_{max} = -\infty$ and $v_{min} = -25\,cm$. However, for a lens as

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \qquad \text{i.e.,} \qquad u = \frac{f}{\left(\frac{f}{v}\right) - 1}$$

So u will be minimum when $v_{min} = -25\,cm$

i.e., $(u)_{min} = \frac{5}{\left(\frac{-5}{25}\right) - 1} = -\frac{25}{6} = -4.17\,cm$

And u will be maximum when $v_{max} = \infty$

So, the closest and farthest distance of the book from the magnifier (or eye) for clear viewing are 4.17 cm and 5 cm respectively.

(b) An in case of simple magnifier $MP = \left(\frac{D}{u}\right)$. So MP will be minimum when u_{max} = 5 cm

i.e. $(MP)_{min} = \frac{-25}{-5} = 5 \qquad \left[= \frac{D}{f}\right]$

And MP will be maximum when $u_{min} = \left(\frac{25}{6}\right)$cm i.e., $(MP)_{max} = \frac{-25}{-\left(\frac{25}{6}\right)} = 6\left[= 1 + \frac{D}{f}\right]$

1.2 Compound Microscope

When we need greater magnification than what we can get with a simple magnifier, the instrument that we usually use is a compound microscope.

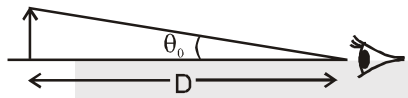
The essential parts of a compound microscope are two convex lenses of different focal length placed coaxially. These lenses are referred to as:

(a) Objective lens or objective: It is a lens of small aperture and small focal length placed facing the object.

(b) Eye piece: It is a lens of large aperture and small focal length placed facing the object.

The object O to be viewed is placed just beyond the first focal point of the **objective** lens that forms a real and enlarged image I′ as shown in figure. In a properly designed instrument this image lies just inside the first focal point of the **eyepiece**. The eyepiece acts as a simple magnifier, and forms a final virtual image of I. The position of may be anywhere between the near and far points of the eye.

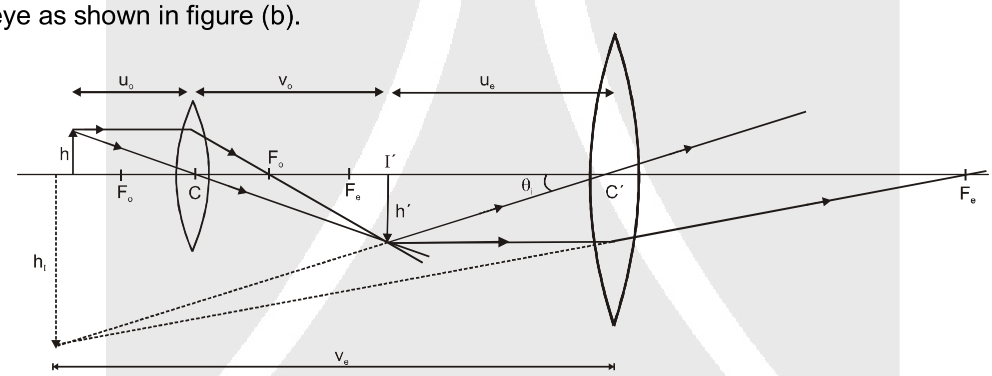
In figure (a) the object is at the near point, where it subtends an angle θ_o at the eye.



unaided eye
(a)

$$\theta_o \approx \tan\theta_o \approx \frac{h}{D} \qquad ...(1)$$

(i) When image is formed at near point, D: Let θ_i be the angle subtended by the final image at the eye as shown in figure (b).



(b)

Angular magnification or magnifying power (M) is defined as the ratio of the angle subtended by the final image at the eye to the angle subtended by the object seen directly at the eye when both lie at near point D.

The angular magnification produced is,

$$M = \frac{\theta_i}{\theta_o} \approx \frac{\tan\theta_i}{\tan\theta_o} \qquad ...(2)$$

$$\theta_i \approx \tan\theta_i = \frac{h_i}{v_e} = \frac{h_i}{D} \qquad (\because v_e = D \text{ in magnitude})$$

$$\theta_o \approx \tan\theta_o = \frac{h}{D}$$

$$M = \frac{h_i}{h} \qquad ...(3)$$

Linear magnification, $\quad m = \frac{h_i}{h} = m_o \times m_e \qquad ...(4)$

$M = m_o m_e$ (from eq. (3) and eq. (4))

where m_o = linear magnification produced by objective lens = $\frac{v_o}{u_o}$ ...(5)

m_e = linear magnification produced by eye piece = $\frac{v_e}{u_e}$

using lens formula for eye piece, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$

$$m_e = \frac{v_e}{u_e} = 1 - \frac{v_e}{f_e} = 1 + \frac{D}{f_e} \qquad \text{...(6)} \qquad (\because v_e = -D)$$

From equations (3), (4) and (6) we have

$$M = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right), \qquad \text{...(7)}$$

In practice, the focal length of objective lens is very small and the object is just placed outside the focus of the objective lens,

$u_o = -f_o$

Since the focal length of the eye lens is also small, the distance of image I' from objective lens is nearly equal to the length of the microscope tube, L

$v_o = L$

substituting in equation (7),

$$M = -\frac{L}{f_o}\left(1 + \frac{D}{f_e}\right)$$

This equation shows that a compound microscope will have high magnifying power, if the objective lens and the eye piece both have small focal length. The negative sign shows that final image will be inverted w.r.t. object.

(ii) **When image is formed at infinity**: The magnifying power of compound microscope is given by

$M = m_o \times m_e$

Magnification produced by objective lens, $m_o = \frac{v_o}{u_o}$

The eye lens produces the final image at infinity. Then,

$m_e = \frac{D}{f_e}$ (as discussed in case of simple microscope)

Therefore, $M = \frac{v_o}{u_o}\frac{D}{f_e}$,

$$M = -\frac{L}{f_o}\frac{D}{f_e}$$

Example 4. The focal length of the objective and eyepiece of a microscope are 2 cm and 5 cm respectively and the distance between them is 20 cm. Find the distance of object from the objective, when the final image seen by the eye is 25 cm from the eyepiece. Also find the magnifying power.

Solution : Given f_0 = 2 cm, f_e = 5 cm

$|v_o| + |u_e| = 20$ cm

$\therefore \quad v_e = -25$ cm

From lens formula $\frac{1}{f_e} = \frac{1}{v_o} - \frac{1}{u_e}$

$$\frac{1}{u} = \frac{1}{v_e} - \frac{1}{f_e} = -\frac{1}{25} - \frac{1}{5}$$

$$\therefore \quad u_e = -\frac{25}{6} \text{ cm}$$

Distance of real image from objective

$v_o = 20 - |u_e| = 20 - \frac{25}{6} = \frac{120-25}{6} = \frac{95}{6}$ cm

Now $\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{u_o}$

given $\frac{1}{u_o} = \frac{1}{v_o} - \frac{1}{f_o} = \frac{1}{(95/6)} - \frac{1}{2}$ i.e., $\frac{1}{u_o} = \frac{6}{95} - \frac{1}{2} = \frac{12-95}{190} = -\frac{83}{190}$

$\therefore$ $u_o = -\frac{190}{83} = -2.3$ cm

Magnifying power $M = -\frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) = -\frac{95/6}{(190/83)}\left(1+\frac{25}{3}\right) = -41.5$

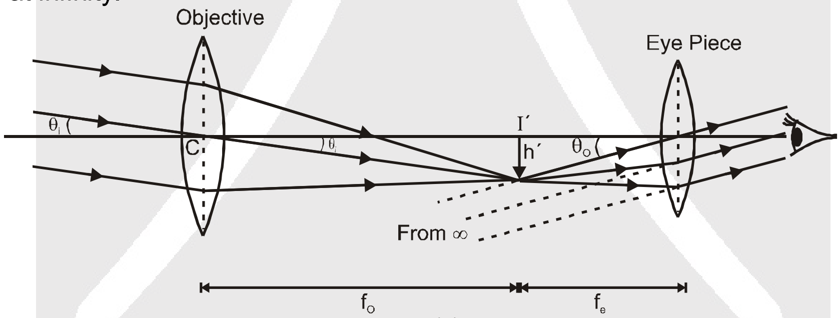
2. Telescope

2.1 Astronomical Telescope

It is an optical instrument used to increase the visual angle of distant large objects such as a star a planet or a cliff etc. Astronomical telescope consists of two converging lens. The one facing the object is called objective and has large focal length and aperture. Other lens is called eye piece. It has small aperture and is of small focal length. The distance between the two lenses is adjustable.
The objective forms a real and inverted image at its focal plane of the distant object. The distance of the eye piece is adjusted, till the final image is formed at the near point, D. In case, the position of the eye piece is so adjusted that final image is formed at infinity, the telescope is said to be in normal adjustment.

(i) When the image is formed at infinity (Normal adjustment)

When a parallel beam of light rays from a distant object falls on objective, its real and inverted image I´ is formed on the other side of the objective and at a distance f_o. If the position of the eye piece is adjusted, so that the image I´ lies at its focus, then the final highly magnified image will be formed at infinity.



(a)

Angular magnification or magnifying power (M) here is defined as the ratio of the angle subtended by the final image at the eye as seen through the telescope to the angle subtended by the object, seen directly at the eye when both the object and the image lie at infinity.

$M = \frac{\theta_i}{\theta_o} = \frac{\tan\theta_i}{\tan\theta_o}$ (for small angle $\tan\theta \approx \theta$)

From figure (a), $\tan\theta_i = \frac{h'}{CI'}$, $\tan\theta_o = \frac{h'}{C'I'}$

$M = \frac{CI'}{C'I'} = -\frac{f_o}{f_e}$ ($CI' = f_o$, $C'I' = -f_e$)

If f_o is large and f_e is small, the magnification will be high. In normal adjustment the length of tube

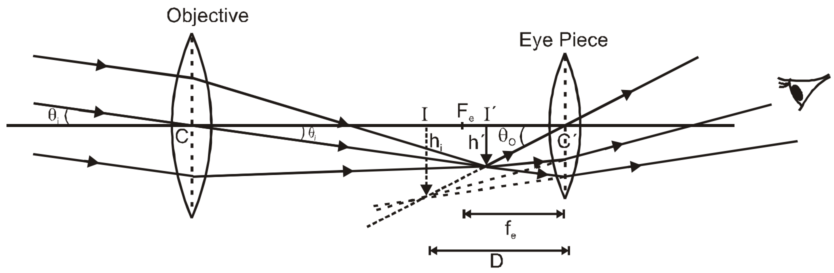
$L = (f_0 + u_e)$

(ii) If the final image is formed at D (near point)

When a parallel beam of light rays from a distant object falls on objective, its real and inverted image I´ is formed on the other side of the objective and at a distance f_o. If the position of the eye piece is adjusted, so that the final image I is formed at near point D.

Angular magnification or magnifying power (M) here is defined as the ratio of the angle subtended by the final image formed at near point at the eye to the angle subtended by the object lying at infinity seen directly at the eye.

$$M = \frac{\theta_i}{\theta_o} = \frac{\tan\theta_i}{\tan\theta_o} \qquad \text{(for small angle } \tan\theta \approx \theta\text{)}$$



(b)

From figure (b), $\tan\theta_o = \frac{h'}{CI'}$, $\tan\theta_i = \frac{h'}{C'I'}$

$$M = \frac{CI'}{C'I'} = -\frac{f_o}{u_e}$$

For eye lens, $\frac{1}{v_e} - \frac{1}{u_e} = \frac{1}{f_e}$,

$$\frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} \qquad (\because\ u_e = -u_e,\ v_e = -D)$$

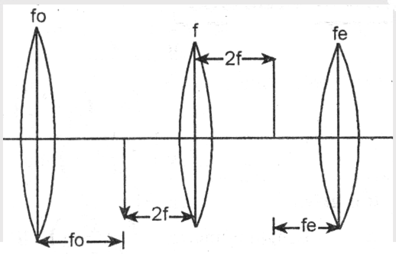
$$M = -\frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$$

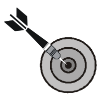
If f_o is large and f_e is small, the magnification will be high.

2.2 Terrestrial Telescope

Uses a third lens in between objective and eyepieces so as to form final image erect. This lens simply invert the image formed by objective without affecting the magnification.

Length of tube $L = f_0 + f_e + 4f$





Example 5. A telescope consists of two convex lens of focal length 16 cm and 2 cm. What is angular magnification of telescope for relaxed eye? What is the separation between the lenses? If object subtends an angle of 0.5º on the eye, what will be angle subtended by its image ?

Solution : Angular magnification

$$M = \frac{\alpha}{\beta} = \frac{F}{f} = \frac{16}{2} = 8 \text{ cm}$$

Separation between lenses = F + f = 16 + 2 = 18 cm

Here $\alpha = 0.5^o$

$\therefore$ Angular subtended by image

$\beta = M\,\alpha = 8 \times 0.5^o = 4^o$

Example 6. The magnifying power of the telescope if found to be 9 and the separation between the lenses is 20 cm for relaxed eye. What are the focal lengths of component lenses ?

Solution : Magnification M = $\frac{F}{f}$

Separation between lenses

d = F + f

Given $\frac{F}{f}$ = 9 i.e., F = 9f ......(1)

and F + f = 20 ......(2)

Putting value of F from (1) in (2), we get

$9f + f = 20 \Rightarrow 10f = 20 \Rightarrow f = \frac{20}{10} = 2cm$

$\therefore$ F = 9f = 9 × 2 = 18 cm

$\therefore$ F = 18 cm, f = 2 cm

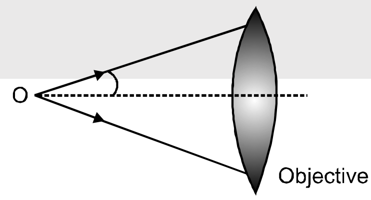
Comparison between Compound - Microscope & Astronomical - Telescope

S.No.	Compound - Microscope	Astronomical - Telescope
1.	It is used to increase visual angle of near tiny object.	It is used to increase visual angle of distant large objects.
2.	In it field and eye lens both are convergent, of short focal length and aperture. and	In it field lens is of large focal length and aperture while eye lens of short focal length aperture and both are convergent.
3.	Final image is inverted. virtual and enlarged and at a distance D to ∞ from the eye.	Final image in inverted, virtual and enlarged at a distance D to ∞ from the eye
4.	MP does not change appreciably if field and eye lens are interchanged [MP ~ $(LD/f_0 f_e)$]	MP becomes $(1/m^2)$ times of its initial value if field and eye-lenses are interchanged as MP ~$[f_0/f_e]$
5.	MP is increased by decreasing the focal of length of both the lenses viz. find and length eye lens.	MP is increased by increasing the focal length field of field lens (and decreasing the focal of eye lens.)
6.	RP is increased by decreasing the wavelength of light used.	RP is increased by increasing the aperture of objective.

RESOLVING POWER (R.P.)

(1) Microscope : In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RPO)

R.L. = $\frac{\lambda}{2\mu\sin\theta}$ and R.P. = $\frac{2\mu\sin\theta}{\lambda} \Rightarrow$ R.P. $\propto \frac{1}{\lambda}$



λ = Wavelength of light used to illuminate the object

μ = Refractive index of the medium between object and objective.

θ = Half angle of the cone of light from the point object, $\mu \sin\theta$ = Numerical aperture.

(2) Telescope : Smallest angular separations ($d\theta$) between two distant object, whose images are separated in the telescope is called resolving limit. So resolving limit $d\theta = \frac{1.22\lambda}{a}$ and resolving power

(RP) = $\frac{1}{d\theta} = \frac{a}{1.22\lambda} \Rightarrow$ R.P. $\propto \frac{1}{\lambda}$ where a = aperture of objective.

Example 7 Find the half angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

Solution. Here $\sin\theta = \frac{\lambda}{a}$

Where θ is half angular width of the central maximum.

A = 12×10^{-5} cm, λ = 6000Å = 6×10^{-5}cm.

∴ $\sin\theta = \frac{\lambda}{a} = \frac{6\times10^{-5}}{12\times10^{-5}} = 0.50$

or θ = 30°

Example 8 In Fraunhofer diffraction due to a narrow slit a screen is placed 2 m away from the lens to obtain the pattern. If the slit width is 0.2 mm and the first minima lie 5 mm on either side of the central maximum, find the wavelength of light.

Solution. In the case of Fraunhofer diffraction at a narrow rectangular aperature,

a sin θ = nλ

n = 1

∴ a sin θ = λ

$\sin\theta = \frac{x}{D}$

∴ $\frac{ax}{D} = \lambda$

$\lambda = \frac{ax}{D}$

Here a = 0.2 mm = 00.2cm

x = 5 mm = 0.5 cm

D = 2m = 200 cm

∴ $\lambda = \frac{0.02\times0.5}{200}$

λ = 5×10^{-5} cm

L = 5000Å

Example 9. Light of wavelength 6000Å is incident on a slit of width 0.30 nm. The screen is placed 2m from the slit. Find : (a) the position of the first dark fringe and (b) the width of the central bright fringe.

Solution. The first dark fringe is on either side of the central bright fringe.

Here, n = ±1, D = 2m

λ = 6000Å = 6×10^{-7}m

$\sin\theta = \frac{x}{D}$

a = 0.30 mm = 3×10^{-4} m

a sin θ = nλ

(a) $x = \frac{n\lambda D}{a}$

$x = \pm\left[\frac{1\times6\times10^{-7}\times2}{3\times10^{-4}}\right]$

x = $\pm4 \times 10^{-3}$m

The positive and negative signs correspond to the dark fringes on either side of the central bright fringe.

(b) The width of the central bright fringe,

y = 2x

= $2 \times 4 \times 10^{-3}$

= 8×10^{-3} m = 8 mm

Example 10. A signal slit of width 0.14 mm is illuminated normally by monochromatic light and diffraction bands are observed on a screen 2m away. If the centre of the second dark band is 1.6 cm from the middle of the central bright band, deduce the wavelength of light used.

Solution. In the case of Fraunhofer diffraction at a narrow rectangular slit,

$a \sin \theta = n\lambda$

Here θ gives the directions of the minimum

$n = 2$, $\lambda = ?$

$a = 0.14$ mm $= 0.14 \times 10^{-3}$ m

$D = 2$ m

$x = 1.6$ cm $= 1.6 \times 10^{-2}$ m

$\sin \theta = \frac{x}{D} = \frac{n\lambda}{a}$ $\therefore$ $\lambda = \frac{xa}{nD}$

$= \frac{1.6 \times 10^{-2} \times 0.14 \times 10^{-3}}{2.2} = 5.6 \times 10^{-7}$ m $= 5600$Å

Example 11. A screen is placed 2m away from a narrow slit which is illuminated with light of wavelength 6000Å. If the first minimum lies 50 mm on either side of the central maximum, calculate the slit width.

Solution. In the case of Fraunhofer diffraction at a narrow slit,

$a \sin \theta = n\lambda$

$\sin \theta = \frac{x}{D}$ $\therefore$ $\frac{ax}{D} = n\lambda$

Here width of the slit = a = ?

$x = 5$ mm $= 5 \times 10^{-3}$ m

$D = 2$m

$\lambda = 6000$Å $= 6 \times 10^{-7}$m

$n = 1$

$a = \left(\frac{n\lambda D}{x}\right)$

$a = \left(\frac{1 \times 6 \times 10^{-7} \times 2}{5 \times 10^{-3}}\right)$

$a = 2.4 \times 10^{-4}$ m

$a = 0.24$ mm

Example 12. Find the angular width of the central bright maximum in the Fraunhofer diffraction pattern of a slit of width 12×10^{-5} cm when the slit is illuminated by monochromatic light of wavelength 6000Å.

Solution. Here $\sin\theta = \frac{\lambda}{a}$

where θ is the half angular width of the central maximum

$a = 12 \times 10^{-5}$ cm $= 12 \times 10^{-7}$ m

$\lambda = 6000$Å $= 6 \times 10^{-7}$ m

$\sin\theta = \frac{6 \times 10^{-7}}{12 \times 10^{-7}} = 0.5$

$\theta = 30°$

Angular width of the central maximum.

$2\theta = 60°$

Example 13. Diffraction pattern of a signal slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between the first dark and the next bright fringe from the axis. Wave length = 4890Å.

Solution. For minimum intensity

$a \sin \theta_n = n\lambda$

$\sin \theta_n = \frac{x_1}{f'} \Rightarrow n = 1$

$\frac{x_1}{f} = \frac{\lambda}{a}$

Here λ = 4890Å = 4890 × 10^{-10} m

a = 0.5 cm = 5 × 10^{-3} m

f = 40 cm = 0.4 m

$x_1 = \frac{f\lambda}{a}$

$x_1 =$

$x_1 = 3.912 \times 10^{-5}$ m

For secondary maximum

$a \sin \theta_n = \frac{0.4 \times 4890 \times 10^{-10}}{5 \times 10^{-3}}$

For the first secondary maximum

n = 1

$\sin \theta_n = \frac{(2n+1)\lambda}{2}$

$\frac{x_2}{f} = \frac{3\lambda}{2a}$

$x_2 = \frac{3\lambda f}{2a}$

$x_2 = \frac{3 \times 4890 \times 10^{-10} \times 0.4}{2 \times 5 \times 10^{-5}}$

$x_2 = 5.868 \times 10^{-5}$ m

Difference,

$x_2 - x_1 = 5.868 \times 10^{-5} - 3.912 \times 10^{-5}$

$= 1.956 \times 10^{-5}$ m

$= 1.596 \times 10^{-2}$ mm

Example 14. Find the separation of two points on the moon that can be resolved by a 500 cm telescope. The distance of the moon is 3.8 × 10^5 km. The eye is most sensitive to light of wavelength 5500 Å.

Solution. The limit of resolution of a telescope is given by

$d\theta = \frac{1.22\lambda}{a}$

Here λ = 5500 × 10^{-8}cm, a = 500 cm

$\therefore \quad d\theta = \frac{1.22 \times 5500 \times 10^{-8}}{500}$

$\therefore \quad d\theta = 13.42 \times 10^{-8}$ radian

Let the distance between the two point be x

$\therefore \quad d\theta = \frac{x}{R}$

Here R = 3.8 × 10^{10} cm

x = R.dθ

= 3.8 × 10^{10} × 13.42 × 10^{-8}

= 50.996 × 10^2 cm = 50.996 meters

Example 15. Calculate the aperture of the objective of a telescope which may by used to resolve stars separated by 4.88×10^{-6} radian for light of wavelength 6000Å

Solution. Here λ = 6000Å = 6×10^{-5} cm, $\theta = 4.88 \times 10^{-6}$ radian

D = ?

$$\theta = \frac{1.22\lambda}{\theta}$$

or $$D = \frac{1.22\lambda}{\theta} = \frac{1.22 \times 6 \times 10^{-5}}{4.88 \times 10^{-8}} = 15\text{cm}$$

Example 16. Two pin holes 1.5 mm apart are placed in front of a source of light of wavelength 5.5×10^{-5} cm and seen through a telescope with its objectives stopped down to a diameter of 0.4 cm. Find the maximum distance from the telescope at which the pin holes can be resolved.

Solution. Here, $\lambda = 5.5 \times 10^{-5}$ cm

a = 0.4 cm

$$d\theta = \frac{1.22\lambda}{a}$$

Also $$d\theta = \frac{x}{d}$$

x = 1.5 mm = 0.15 cm

$$\therefore \quad \frac{x}{d} = \frac{1.22\lambda}{a}$$

$$d = \frac{xa}{1.22\lambda}$$

$$d = \frac{0.15 \times 0.4}{1.22 \times 5.5 \times 10^{-5}}\text{cm}$$

d = 894.2 cm = 8.942 m