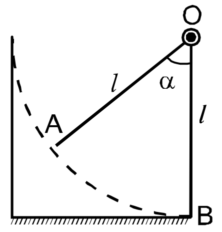
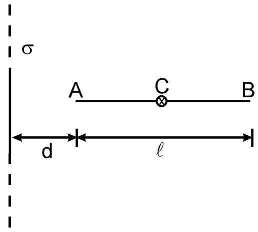
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. An electrometer consists of a fixed vertical metal bar OB at the top of which is attached a thin rod OA which gets deflected from the bar under the action of an electric charge (fig.). The rod can rotate in vertical plane about fixed horizontal axis passing through O. The reading is taken on a quadrant graduated in degrees. The length of the rod is ℓ and its mass is m. What will be the charge when the rod of such an electrometer is deflected through an angle α in equilibrium? Find the answer using the following two assumptions: The charge on the electrometer is equally distributed between the bar & the rod and the charges are concentrated at point A on the rod & at point B on the bar.



2. A uniform rod AB of mass m and length ℓ is hinged at its mid point C. The left half (AC) of the rod has linear charge density $-\lambda$ and the right half (CB) has $+\lambda$, where λ is constant. A large non conducting sheet of uniform surface charge density σ is also present near the rod. Initially, the rod is kept perpendicular to the sheet. The end A of the rod is initially at a distance d. Now the rod is rotated by a small angle in the plane of the paper and released. Prove that the rod will perform SHM and find its time period.

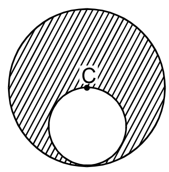


3. (i) Three equal charges q_0 each are placed at three corners of an equilateral triangle of side 'a'. Find out force acting on one of the charge due to other two charges?

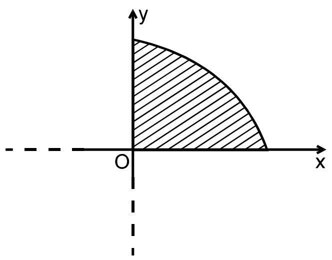
(ii) In the above question, if one of the charge is replaced by negative charge then find out force acting on it due to other charges ?

(iii) Repeat the part (i) if magnitude of each charge is doubled and side of triangle is reduced to half.

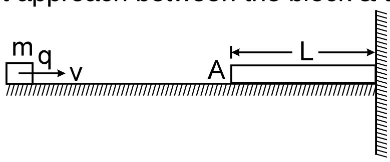
4. A solid sphere of radius 'R' is uniformly charged with charge density ρ in its volume. A spherical cavity of radius R/2 is made in the sphere as shown in the figure. Find the electric potential at the centre of the sphere.



5. A uniform surface charge of density σ is given to a quarter of a disc extending upto infinity in the first quadrant of x – y plane. The centre of the disc is at the origin O. Find the z – component of the electric field at the point (0, 0, z) and the potential difference between the points (0, 0, d) & (0, 0, 2d).



6. Figure shows a rod of length L which is uniformly charged with linear charge density λ kept on a smooth horizontal surface. Right end of rod is in contact with a vertical fixed wall.
A block of mass m and charge q is projected with a velocity v from a point very far from rod in the line of rod. Find the distance of closest approach between the block & the left end A of the rod.



7. A charged ball of mass 5.88×10^{-4} kg is suspended from two silk strings of equal lengths so that the strings are inclined at 90° with each other. Another ball carrying a charge which is equal in magnitude but opposite in sign of the first one is placed vertically below the first one at a distance of 4.2×10^{-2} m. Due to this, the tension in the strings is doubled. Determine the charge on the ball and the tension in the strings after electrostatic interaction. (g = 9.8 m/s^2)

8. A charge of 16×10^{-9} C is fixed at the origin of coordinates. A second charge of unknown magnitude is at x = 3m, y = 0 and a third charge of 12×10^{-9} C is at x = 6m, y = 0. What is the value of the unknown charge if the resultant field at x = 8m, y = 0 is 20.25 N/C directed towards positive x-axis?

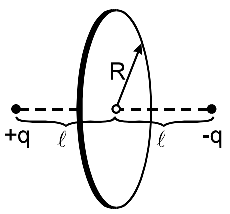
9. Two large conducting plates are placed parallel to each other with a separation of 2.00 cm between them. An electron starting from rest near one of the plates reaches the other plate in 2.00 microseconds. Find the surface charge density on the inner surfaces. Can you find out the charge density on outer surface?

10. A ball of mass 10^{-2} kg & having charge $+3 \times 10^{-6}$ C is tied at the end of a 1 m long thread. The other end of the string is fixed and a charge of -3×10^{-6} C is placed at this end. The ball can move in a circular orbit of radius 1 m in a vertical plane. Initially the ball is at the bottom. Find the minimum initial horizontal velocity of the ball so that it will be able to complete the full circle. [g = 10 m/s^2.] **[REE 1996, 5]**

11. A system consists of a thin charged wire ring of radius R and a very long uniformly charged thread oriented along the axis of the ring, with one of its ends coinciding with the centre of the ring. The total charge of the ring is equal to q. The charge of the thread (per unit length) is equal to. Find the interaction force between the ring and the thread.

12. A thin non-conducting ring of radius R has a linear charge density $\lambda = \lambda_0 \cos \varphi$, where λ_0 is a constant, φ is the azimuthal angle. Find the magnitude of the electric field strength
(a) At the centre of the ring.
(b) On the axis of the ring as a function of the distance x from its centre. Investigate the obtained function at x >> R.

13. Two point charges q and –q are separated by the distance 2ℓ (Figure). Find the flux of the electric field strength vector across a circle of radius R.



14. A system consists of a ball of radius R carrying a spherically symmetric charge and the surrounding space filled with a charge of volume density $\rho = \alpha/r$, where α is a constant, r is the distance from the centre of the ball. Find the ball's charge for which the magnitude of the electric field strength vector is independent of r outside the ball. How high is this strength ? The permittivities of the ball and the surrounding space are assumed to be equal to unity.

15. Find the electric field potential and strength at the centre of a hemisphere of radius R charged uniformly with the surface density σ.

16. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m and positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$. **[JEE 1999 (Mains), 5+5 /100]**

(i) Find the value of H if the particle just reaches the disc.
(ii) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

17. The potential difference between two large parallel plates is varied as v = at ; a is a positive constant and t is time. An electron starts from rest at t = 0 from the plate which is at lower potential. If the distance between the plates is L, mass of electron m and charge on electron -e then find the velocity of the electron when it reaches the other plate.

18. Four point charges +8μC, –1μC, –1μC and +8μC, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y – axis. A particle of mass 6×10^{-4} kg and of charge + 0.1 μC moves along the -x direction. Its speed at x = + ∞ is v_0. Find the least value of v_0 for which the particle will cross the origin. Find also the kinetic energy of the particle at the origin. Assume that space is gravity free. Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2 **[JEE 2000 (Mains),10/ 100]**

19. A small ball of mass 2×10^{-3} kg having a charge of 1 μC is suspended by a string of length 0.8 m. Another identical ball having the same charge is kept at the point of suspension. Determine the minimum horizontal velocity which should be imparted to the lower ball so that it can make complete revolution. (g = 10 m/s^2) **[JEE 2001 (Mains), 5/100 ; REE 1996]**

20. Three point charges Q, 2Q and 8Q are to be placed on a 10 cm long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge Q due to the other two charges? **[JEE 1987]**

21. A particle of charge - q and mass m moves in a circular orbit about a fixed charge + Q. Show that the "$r^3 \alpha T^2$" law, is satisfied, where r is the radius of orbit and T is time period.

22. The field potential in a certain region of space depends only on the x coordinate as $\varphi = -ax^3 + b$, where a and b are constants. Find the distribution of the space charge $\rho(x)$.

23. A conducting sphere S_1 of radius r is attached to an insulating handle. Another conducting sphere S_2 of radius R is mounted on an insulating stand. S_2 is initially uncharged. S_1 is given a charge Q, brought into contact with S_2 and removed. S_1 is then recharged such that the charge on it is again Q & it is again brought into contact with S_2 & removed. This procedure is repeated n times **[JEE 1998 Mains, 7+1/200]**

(a) Find the electrostatic energy of S_2 after n such contacts with S_1.
(b) What is the limiting value of this energy as $n \to \infty$?

24. The electric field strength depends only on the x and y coordinates according to the law

$$\vec{E} = a(x\hat{i} + y\hat{j})/(x^2 + y^2),$$

where a is a constant, $\hat{i}$ and $\hat{j}$ are the unit vectors of the x and y axes. Find the flux of the vector $\vec{E}$ through a sphere of radius R with its centre at the origin of coordinates.Using the above result, also calculate total charge enclosed by the sphere.

25. A positive charge is distributed in a spherical region with charge density $\rho = \rho_0 r$ for $r \leq R$ (where ρ_0 is a positive constant and r is the distance from centre). Find out electric potential and electric field at following locations.

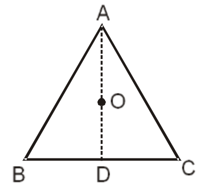
(a) At a distance r from centre inside the sphere. (b) At a distance r from centre outside the sphere.

26. Two point charges q and –2q are placed at a distance 6m apart on a horizontal plane (x–y plane). Find the locus of the zero potential points in the x–y plane.

27. Two metalic balls of radii R_1 and R_2 are kept in vacuum at a large distance compared to their radii. Find the ratio of the charges on the two balls for which electrostatic energy of the system is minimum. What is the potential difference between the two balls for this ratio ? Total charge of the balls is constant. Neglect the interaction energy. (Charge distribution on each ball is uniform)

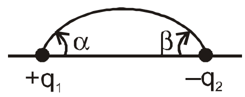
28. A ball of radius R carries a positive charge whose volume density depends only on the separation r from the ball's centre as $\rho = \rho_0 (1 – r/R)$, where ρ_0 is a constant. Assuming the perimittivities of the ball and the environment to be equal to unity, find :
(i) The magnitude of the electric field strength as a function of the distance r both inside and outside the ball;
(ii) The maximum intensity E_{max} and the corresponding distance r_m.

29. Consider an equilateral triangle ABC of side 2a in the plane of the paper as shown. The centroid of the triangle is O. Equal charges (Q) are fixed at the vertices A, B and C. In what follows consider all motion and situations to be confined the plane of the paper.

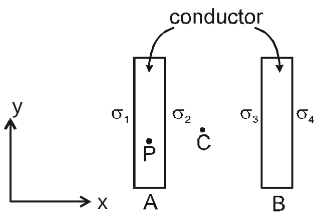


(a) A test charge (q), of same sign as Q, is placed on the median AD at a point at a distance δ below O. Obtain the force ($\vec{F}$) felt by the test charge.
(b) Assuming δ << a discuss the motion of the test charge when it is released.
(c) Obtain the force ($\vec{F}_D$) on this test charge if it is placed at the point D as shown in the figure.
(d) In the figure below mark the approximate locations of the equilibrium point(s) for this system. Justify your answer.
(e) Is the equilibrium at O stable or unstable if we displace the test charge in the direction of OP ? The line PQ is parallel to the base BC. Justify your answer.
(f) Consider a rectangle ABCD. Equal charges are fixed at the vertices A, B, C and D. O is the centroid. In the figure below mark the approximate locations of all the neutral points of the system for a test charge with same sign as the charges on the vertices. Dotted lines are drawn for the reference.
(g) How many neutral points are possible for a system in which N charges are placed at the N vertices of a regular N sided polygon ?

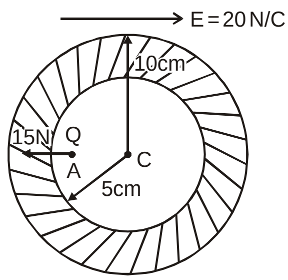
30. An electrostatic field line leaves at angle α from point charge q_1 and connects with point charge $–q_2$ at angle β (see figure).
Then find the relationship between α and β :



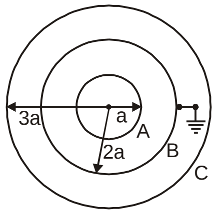
31. Figure shows two infinitely large conducting plates A and B. If electric field at C due to charge densities σ_1, σ_2, σ_3 and σ_4 is $E\hat{i}$, find σ_2 and σ_3 in terms of E. State whether this much information is sufficient to find σ_1 and σ_4 in terms of E. Derive a relation between σ_1 and σ_4.



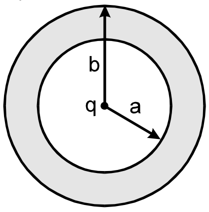
32. In a neutral conducting hollow sphere of inner and outer radii 5 cm and 10 cm respectively, a point charge Q = 1 C is placed at point A, that is 3 cm from the centre C of the hollow sphere. An external uniform electric field of magnitude 20 N/C is also applied. Net electric force on this charge is 15 N, away from the centre of the sphere as shown. Then find :
(a) Force due to external electric field on the outer surface of the shell.
(b) Net force on shell.
(c) Net force on point charge due to shell.



33. Figure shows a system of three concentric metal shells A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then find the final charge on the shell B



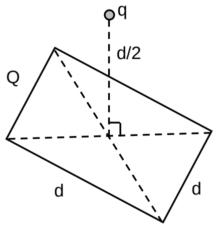
34. A point charge q is brought slowly from infinity and is placed at the centre of a conducting neutral spherical shell of inner radius a and outer radius b, then find work done by external agent :



35. Consider two solid dielectric spheres of radius a, separated by a distance R(R >>a). One of the spheres has a charge q, and the other is neutral (see figure.) We double the distance between two sphere. How much charge should reside on the first sphere now so that the force between the spheres remains the same ?

36. A triangle is made from thin insulating rods of different lengths, and the rods are uniformly charged, i.e. the linear charge density on each rod is uniform and the same for all three rods. Find a particular point in the plane of the triangle at which the electric field strength is zero.

37. A square of side d, made from a thin insulating plate, is uniformly charged and carries a total charge of Q. A point charge q is placed on the symmetrical normal axis of the square at a distance d/2 from the plate. How large is the force acting on the point charge?



38. A point charge q is located between two mutually perpendicular conducting half-planes. Its distance from each half-plan is equal to ℓ. Find the modulus of the vector of the force acting on the charge.

39. A point charge q is located at a distance ℓ from an infinite conducting plane. Determine the surface density of charges induced on the plane as a function of separation r from the base of the perpendicular drawn to the plane from the charge.

40. A very long straight thread is oriented at right angles to an infinite conducting plane; its end is separated from the plane by a distance ℓ. The thread carries a uniform charge of linear density λ. Suppose the point O is the trace of the thread on the plane. Find the surface density of the induced charge on the plane

(a) At the point O (b) As a function of a distance r from the point O.

High Level Problems (HLP)

1. $q = 4\ell\sqrt{4\pi\epsilon_0\, mg\sin\left(\frac{\alpha}{2}\right)}\ \sin\frac{\alpha}{2}$

2. $T = 2\pi\sqrt{\frac{2m\epsilon_0}{3\lambda\sigma}}$

3. (i) $\frac{\sqrt{3}Kq_0^2}{a^2}$, away from the charges along perpendicular bisector of line joining remaining two charges.

(ii) $\frac{\sqrt{3}Kq_0^2}{a^2}$; towards the charges along perpendicular bisector of line joining remaining two charges.

(iii) $\frac{16\sqrt{3}Kq_0^2}{a^2}$; away from the charges along angle bisector.

4. $V = \frac{5\rho R^2}{12\epsilon_0}$

5. $\frac{\sigma}{8\epsilon_0}, \frac{\sigma}{8\epsilon_0}|d|$

6. $r = \frac{L}{\left(e^{\frac{2\pi\epsilon_0 mv^2}{\lambda q}} - 1\right)}$

7. 3.36×10^{-8}C, 8.15×10^{-3} N

8. -25×10^{-9} C

9. 0.505×10^{-12} C/m^2, No

10. $u = \sqrt{58.1} = 7.6$ m/s

11. $F = \frac{q\lambda}{4\pi\varepsilon_0 R}$

12. (a) $\frac{\lambda}{4\varepsilon_0 R}$ (b) $E = \frac{\lambda_0 R^2}{4\varepsilon_0 (x^2 + R^2)^{3/2}}$.

For x >> R this strength $E \approx \frac{p}{4\pi\varepsilon_0 x^3}$, where $p = \pi R^2\lambda_0$.

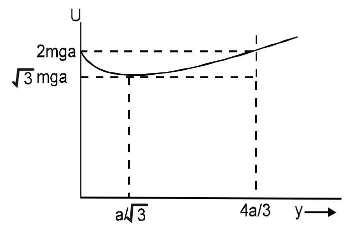
13. $|\phi| = \frac{q}{\varepsilon_0}\left(1 - \frac{1}{\sqrt{1+(R/\ell)^2}}\right)$. The sign of ϕ depends on how the direction of the normal to the circle is chosen.

14. $q = 2\pi\alpha R^2$, $E = \frac{1}{2}\frac{\alpha}{\varepsilon_0}$

15. $V = \frac{\sigma R}{2\epsilon_0}$, $E = \frac{\sigma}{4\epsilon_0}$

16. (i) H = 4 a/3

(ii) $U(y) = 2mg\left[\sqrt{y^2 + a^2} - y\right] + mgy$; at equilibrium $\frac{dU}{dy} = 0 \Rightarrow y = \frac{a}{\sqrt{3}}$



17. $V = \left(\frac{9}{2}\frac{eaL}{m}\right)^{1/3}$

18. $v_0 = 3$ m/s; K.E. at origin $(27 - 10\sqrt{6}) \times 10^{-4}$ J $= 2.5 \times 10^{-4}$ J

19. $\sqrt{\frac{275}{8}} = 5.86$ m/s

20. Q at a distance of 10/3 cm from 2Q between 2Q and 8Q, E = 0.

22. $6a\varepsilon_0 x$ **23.** (a) $U_2 = \frac{a^2Q^2}{8\pi\epsilon_0 R}\left(\frac{1-a^n}{1-a}\right)^2$ where $a = \frac{R}{r+R}$ (b) $U_2\ (n \to \infty) = \frac{RQ^2}{8\pi\epsilon_0 r^2}$

24. $\phi = 4\pi Ra$, $Q = 4\pi Ra\epsilon_0$ **25.** (a) $\vec{E} = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$; $V = \frac{\rho_0[4R^3 - r^3]}{12\varepsilon_0}$ (b) $\vec{E} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$; $V = \frac{\rho_0 R^4}{4\varepsilon_0 r}$

26. Locus is a circle (equation depends on choice of coordinate system) **27.** $\frac{Q_1}{Q_2} = \frac{R_1}{R_2}$; 0

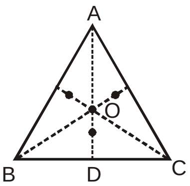
28. (i) $E = \frac{\rho_0 r}{3\ \varepsilon_0}\left(1 - \frac{3r}{4R}\right)$ for r < R, $E = \frac{\rho_0 R^3}{12\varepsilon_0 r^2}$ for r > R (ii) $E_{max} = \frac{1}{9}\frac{\rho_0 R}{\varepsilon_0}$ for $r_m = \frac{2}{3}R$.

29. (a) $\vec{F} = \frac{2KQq\left(\frac{a}{\sqrt{3}} - \delta\right)}{\left(a^2 + \left(\frac{a}{\sqrt{3}} - \delta\right)^2\right)^{3/2}} - \frac{KQq}{\left(\frac{2a}{\sqrt{3}} + \delta\right)^2}$ Here $K = 1/4\pi\varepsilon_0$ and direction is upward (towards A)

(b) Using binomial approximation, $\vec{F} = KQq\frac{9\sqrt{3}}{16}\frac{\delta}{a^3}$ (upward) which is linear in δ. Hence charge will oscillate simple harmonically about O when released.

(c) $\vec{F}_D = \frac{KQq}{3a^2}$ (downward)

(d) For small δ force on the test charge is upwards while for large δ (eg. at D) force is downwards. So there is a neutral point between O and D. By symmetry there will be neutral points on other medians also. In figure x. Below all possible (4) neutral points are shown by •.

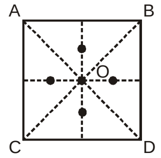


(e) Let the distance along P be x and O to be at (0, 0). Electric potential of a test charge along OP can be written as

$$V(x) = \frac{KQ}{\sqrt{x^2 + (4/3)}} + \frac{KQ}{\sqrt{(x+1)^2 + (1/3)}} + \frac{KQ}{\sqrt{(x-1)^2 + (1/3)}} \approx KQ\sqrt{\frac{3}{4}}\left(3 + \frac{9}{16}x^2\right)$$

We can see that $V(x) \propto x^2$, hence it is a stable equilibrium.

(f) Equilibrium points are indicated by •.



(g) N + 1

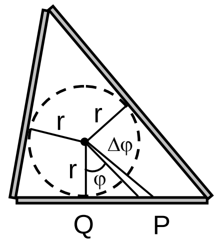
30. $q_1 \sin^2\frac{\alpha}{2} = q_2 \sin^2\frac{\beta}{2}$ **31.** $\sigma_1 = \sigma_4$, $\sigma_2 = \epsilon_0 E$, $\sigma_3 = -\epsilon_0 E$

32. (a) 20 N right hand side (b) 35 N right hand side (c) 35 N left hand side

33. $-\frac{8Q}{11}$ **34.** $\frac{kq^2}{2b} - \frac{kq^2}{2a}$ **35.** $q' = 4\sqrt{2}q$

36. We are going to prove that the electric field strength is zero at the socalled incentre, the centre of the triangle's inscribed circle (which has radius r in the figure)

Let us consider a small length of rod at position P on one of the sides of the triangle; let it subtend an angle $\Delta\varphi$ at the incentre (see figure). Its distance from the incentre is $r/\cos\varphi$. Its small length Δx can be found by noting that P is a distance $x = r\tan\varphi$ along the rod from the fixed point Q and so $\Delta x = (r\,\Delta\varphi)/(\cos^2\varphi)$.



Consequently the charge it carries is$\Delta q = \dfrac{\lambda r \Delta\varphi}{\cos^2\varphi}$

where λ is the linear charge density on the rods. The magnitude of the elementary contribution of this small piece to the electric field at the incentre is

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q \cos^2\varphi}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r \Delta\varphi}{r^2}$$

It can be seen from this result that the same electric field (in both magnitude and direction) would be produced by an arc of the inscribed circle that subtends $\Delta\varphi$ at the circle's centre and carries the same linear charge density λ as the rod.
Summing up the contributions of the small arc pieces correspondingto all threesides of the triangle, we will, because of the circular symmetry, obtain zero net field. It follows that the electric field strength produced by the charged sides of the triangle is also zero at the incentre.

37. According to Newton's third law, the insulating plate acts on the point charge with a force of the same magnitude (but opposite direction) as the point charge does on the plate. We calculate the magnitude of this latter force.
Divide the plate (notionally) into small pieces, and denote the area of the ith piece by ΔA_i. Because of the uniform charge distribution, the charge on this small piece is

$$\Delta Q_i = \frac{Q}{d^2}\Delta A_i$$

and so the electric force acting on it is Fi = Ei_Qi, where Ei is the magnitude of the electric field produced by the point charge q at the position of the small piece.
The force acting on the insulating plate, as a whole, can be calculated as the vector sum of the forces acting on the individual pieces of the plate. Because of the axial symmetry, the net force is erpendicular to the plate, and so it is sufficient to sum the perpendicular components of the forces :

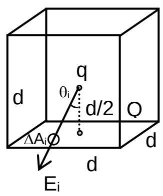
$$F = \sum_i F_i \cos\theta_i = \sum_i E_i \frac{Q}{d^2}\Delta A_i \cos\theta_i = \frac{Q}{d^2}\sum_i E_i \Delta A_i \cos\theta_i$$

where θ_i is the angle between the normal to the plate and the line that connects the point charge to the ith piece of it.
The sum in the given expression is nothing other than the electric flux through the square sheet produced by the point charge q :

$$\psi = \sum_i E_i \Delta A_i \cos\theta_i$$

and can be evaluated as follows.
Let us imagine that a cube of edge d is constructed symmetrically around the point charge (see figure). Then, the distance of the point charge from each side of the cube is just d/2. According to Gauss's law, the total electric flux passing through the six sides of the cube is q/ε_0 and so the flux through a single side is one-sixth of this :

$$\psi = \frac{q}{6\varepsilon_0}$$



Using this and our previous observations, we calculate the magnitude of the force acting on the point charge due to the presence of the charged insulating plate as

$$F = \frac{Qq}{6e_0 d^2}$$

38. $F = \dfrac{(2\sqrt{2}-1)q^2}{32\pi\varepsilon_0\ell^2}$

39. $\sigma = -\dfrac{q\ell}{2\pi(\ell^2+r^2)^{3/2}}$, $q_{ind} = -q$

40. (a) $\sigma = \dfrac{\lambda}{2\pi\ell}$; (b) $\sigma(r) = \dfrac{\lambda}{2\pi\sqrt{\ell^2+r^2}}$