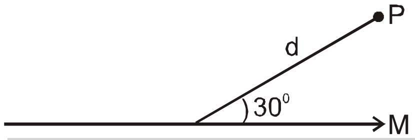
HINTS & SOLUTIONS OF EMF
EXERCISE-1
PART - I

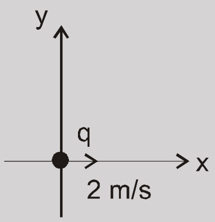
Section (A)

A-1.



$$B_P = \frac{\mu_0}{4\pi}\frac{M}{d^3}\sqrt{1+3\cos^2\theta} = \frac{\sqrt{13}}{2} \times 10^{-4}\ \text{wb/m}^2$$

A-2.



$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v}\times\vec{r})}{r^3}$$

(i) at (2, 0, 0) $\quad \vec{v} \parallel \vec{r}$ so $B = 0$ Ans.

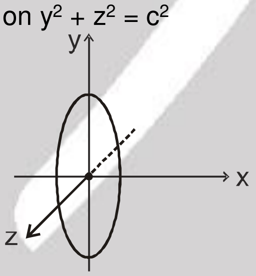
(ii) at (0, 2, 0) $\quad \vec{B} = \frac{\mu_0}{4\pi} q \frac{vr}{r^3}\hat{k} = (10^{-13})\hat{k}$ Ans.

(iii) at (0, 0, 2) $\quad \vec{B} = \frac{\mu_0}{4\pi} q \frac{vr}{r^3}(-\hat{j}) = (10^{-13})\ (-\hat{j})$ Ans.

(iv) at (2, 1, 2) $\quad \vec{r} = 2\hat{i} + \hat{j} + 2\hat{k}$

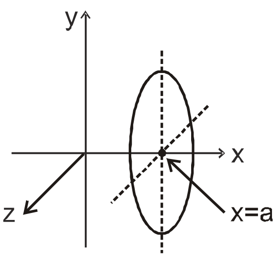
$$\vec{B} = \frac{\mu_0}{4\pi} q \frac{(\vec{v}\times\vec{r})}{r^3} = \frac{4}{27} \times 10^{-13}\,(-2\hat{j} + \hat{k})$$ Ans.

(v) on $y^2 + z^2 = c^2$



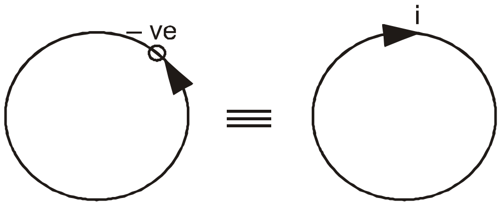
magnitude is constant
as $\vec{v}\times\vec{r}$ = vr at all points
but direction keeps on changing as direction of $\vec{v}\times\vec{r}$. (Yes / No) **Ans.**

(vi)



Still exactly in the similar fashion as in the privious problem. (Yes, No) **Ans.**

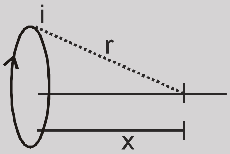
A-3.



(i) $i = qf = \frac{qv}{2\pi R}$

$B_{centre} = \frac{\mu_0 i}{2R} = \frac{\mu_0 qv}{4\pi R^2}$ Inwards.

(ii)



$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$

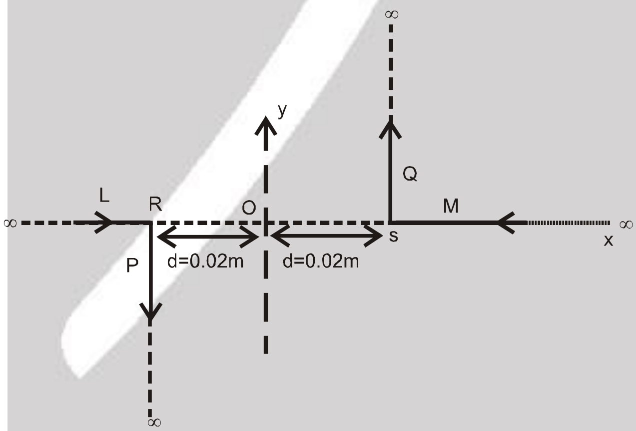
$r^2 = x^2 + R^2$

direction of $\vec{B}$ is along $\vec{v}\times\vec{r}$

Which keeps on changing as the particle revolves.

Section (B)

B-1



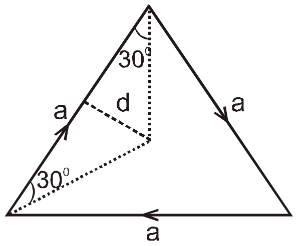
$B_{\text{due to L}} = B_{\text{due to M}} = 0$ at 0

$B_{\text{due to P}} = B_{\text{due to Q}} = \frac{\mu_0 i}{4\pi d}$ both out of paper

$B_{Net} = \frac{\mu_0 i}{2\pi d}$ out of paper

$= 10^{-4}$ wb/m^2 **Ans.**

B-2.



$\vec{B}_{Net} = 3\,(\vec{B}_{\text{due to one side}})$

$$B_{net} = 3 \times \frac{\mu_0 i}{4\pi d}\,[\cos 30^0 + \cos 30^0]$$

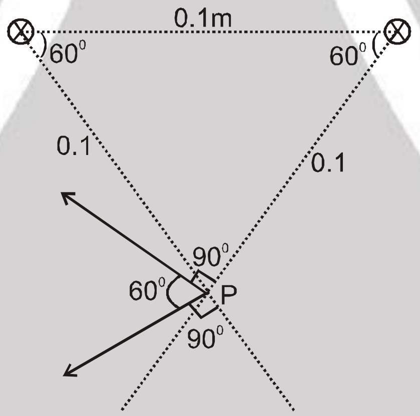
$$B_{net} = 3 \times 10^{-7} \times \frac{1}{\frac{a}{2}\tan 30^0}\;\; 2\cos 30^0 = 4 \times 10^{-5}\ \text{wb/m}^2$$ **Ans.**

B-3. (i) $B_1 = B_2 = \dfrac{\mu_0 i}{2\pi d}$

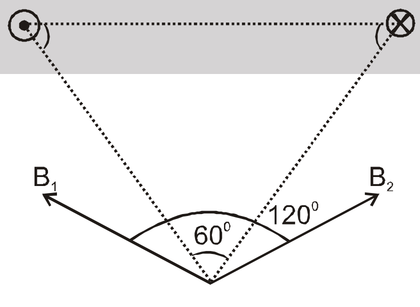
$B_{Net} = \sqrt{3}\, B_1$

$= \sqrt{3} \times \dfrac{2 \times 10^{-7} \times 10}{0.1}$

$= 2\sqrt{3} \times 10^{-5}$ tesla

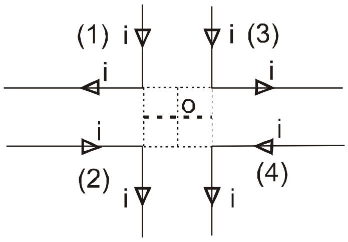


(ii)



$B_{Net} = B_1 = 2 \times 10^{-5}$ tesla

B-4.



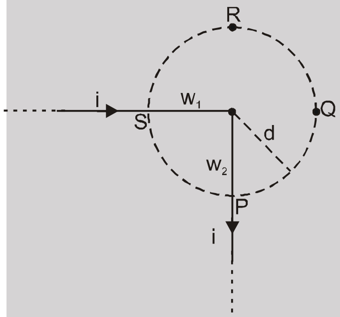
$B_{\text{due to}(1)} = B_{\text{due to}(2)} = B_{\text{due to}(3)} = B_{\text{due to}(4)}$

$B_{\text{due to}(1)}$ & $B_{\text{due to}(2)}$ are in same direction out of paper

$B_{\text{due to}(3)}$ & $B_{\text{due to}(4)}$ are into the paper

So $B_{Net} = 0$ Ans.

B-5.



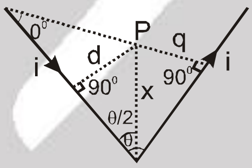
Magnetic field at Q is

$$B_Q = \frac{\mu_0 i}{4\pi d}$$

Now magnetic field at R is

$$B_R = \frac{\mu_0 i}{4\pi d}\sqrt{2}$$

B-6.



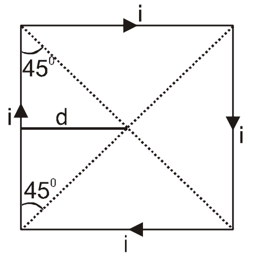
$$B = 2\frac{\mu_0 i}{4\pi d}\left[\cos\frac{\theta}{2} + \cos 0^0\right]$$

Out of the plane of paper

$$= 2\frac{\mu_0 i}{4\pi\, x \sin\frac{\theta}{2}}\left[1 + \cos\frac{\theta}{2}\right] = \frac{\mu_0 i}{2\pi x}\cot\frac{\theta}{4}$$

$\therefore \quad K = \frac{\mu_0 i}{2\pi x}$ **Ans.**

B-7.

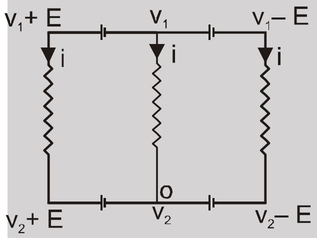


$$B = 4\frac{\mu_0 i}{4\pi d}\ [\cos 45^0 + \cos 45^0]$$

$$= 4\frac{\mu_0 i}{4\pi \frac{a}{2}}\ [2 \times \frac{1}{\sqrt{2}}]$$

$$= 2\sqrt{2}\ \frac{\mu_0 i}{\pi a}$$ into the plane of paper.

B-8.



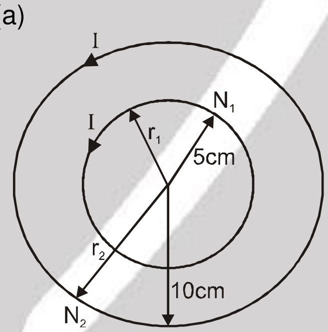
Junction rule at o gives

$i + i + i = 0$

So No current flows in any section. Hence B = 0 At pt. P

Section (C)

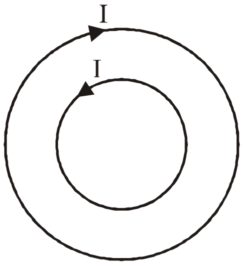
C-1. (i) (a)



$$B_{Net} = \frac{\mu_0 N_1 I}{2r_1} + \frac{\mu_0 N_2 I}{2r_2} = \frac{\mu_0 I}{2}\left[\frac{N_1}{r_1} + \frac{N_2}{r_2}\right]$$

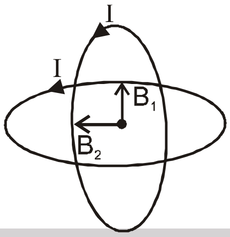
$$= \frac{4\pi \times 10^{-7} \times 1}{2}\left[\frac{20}{10^{-2}}\right] = 4\pi \times 10^{-4}\ \text{wb/m}^2$$ **Ans.**

(b)



$B_{Net} = \frac{\mu_0 N_1 I}{2r_1} - \frac{\mu_0 N_2 I}{2r_2} = 0$ **Ans.**

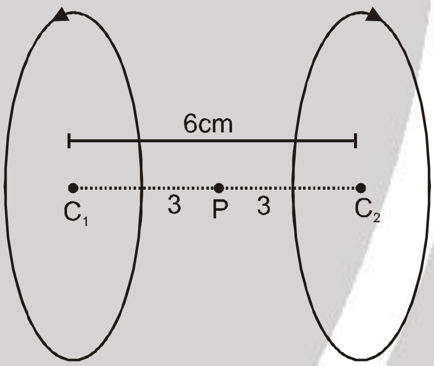
(ii)



$B_{Net} = \sqrt{B_1^{\ 2} + B_2^{\ 2}} = \sqrt{2}\ B_1$

$= \sqrt{2}\ \frac{\mu_0 N_1 I}{2r_1} = 2\sqrt{2}\ \pi \times 10^{-4}$ T **Ans.**

C-2.



(a) B at $C_1 = \frac{\mu_0 iN}{2\pi} - \frac{\mu_0 N i r^2}{2(r^2 + x^2)^{\frac{3}{2}}}$ x = 6cm.

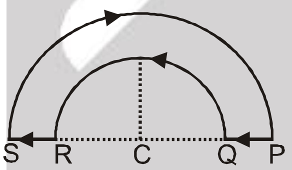
$= 13 \times 10^{-5}$ T

(b) $B_1 = \frac{\mu_0 N i r^2}{2(r^2 + x^2)^{\frac{3}{2}}}$ right, $B_2 = \frac{\mu_0 N i r^2}{2(r^2 + x^2)^{\frac{3}{2}}}$ left

So $B_{Net} = 0$

Section (D)

D-1.



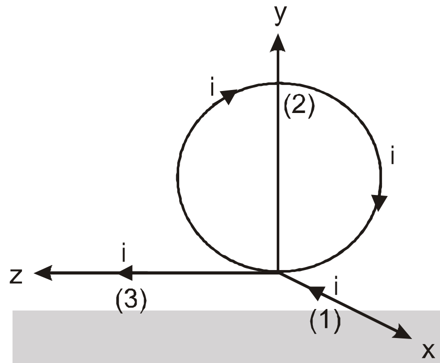
$B_{\text{due to st. part RS}} = B_{\text{due to st. part PQ}} = 0$ at 'c'

$B_{\text{due to curved part QR}} = \frac{\mu_0 I}{4R_1}$ out of paper

$B_{\text{due to curved part SP}} = \frac{\mu_0 I}{4R_2}$ into the paper

$B_{Net} = \frac{\mu_0 I}{4}\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$ **Ans.**

D-2.



$\vec{B}$ due to (1) $= \frac{\mu_0 i}{4\pi R}(-\hat{k})$

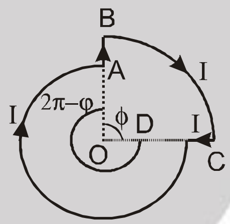
$\vec{B}$ due to (2) $= \frac{\mu_0 i}{2R}(-\hat{i})$

$\vec{B}$ due to (3) $= \frac{\mu_0 i}{4\pi R}(-\hat{i})$

$\vec{B}$ Net $= \frac{\mu_0 i}{4\pi R}\left[(-2\pi - 1)\hat{i} - 1\hat{k}\right]$

$|\vec{B}| = \frac{\mu_0 i}{4\pi R}\left[\sqrt{4\pi^2 + 1 + 4\pi + 1}\right] = \frac{\mu_0 i}{4\pi R}\left[\sqrt{4\pi^2 + 4\pi + 2}\right]k$ Ans.

D-3. (a)



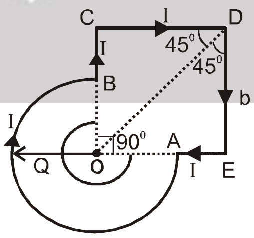
$B_{\text{due to st. part AB}} = 0 = B_{\text{due to st. part CD}}$

$B_{\text{due to curved part AD}} = \left(\frac{2\pi - \phi}{2\pi}\right)\left(\frac{\mu_0 i}{2a}\right)$ Into the plane of paper.

$B_{\text{due to curved part BC}} = \frac{\phi}{2\pi}\left(\frac{\mu_0 i}{2b}\right)$ Into the plane of paper.

$B_{\text{Net}} = \frac{\mu_0 i}{4\pi}\left[\frac{2\pi - \phi}{a} + \frac{\phi}{b}\right]$ Into the plane of paper. **Ans.**

(b)



$B_{\text{due to BC}} = B_{\text{due to EA}} = 0$

$B_{\text{due to curved part AB}} = \frac{\frac{3\pi}{2}}{2\pi}\,\frac{\mu_0 I}{2a}$

$= \frac{3}{8}\frac{\mu_0 I}{a}$ Into the plane of paper

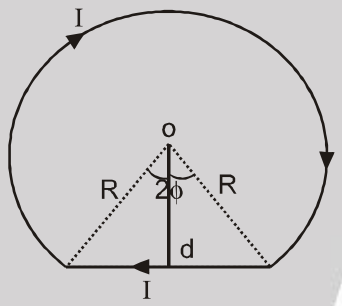
$B_{\text{due to CD}} = \frac{\mu_0 i}{4\pi b}$ [cos 90^0 + cos 45^0]

$= \frac{\mu_0 I}{4\sqrt{2}\pi b}$ Into the paper

$B_{\text{due to DE}} = \frac{\mu_0 I}{4\sqrt{2}\pi b}$ Into the plane of paper.

$\vec{B}_{\text{Net}} = \frac{\mu_0 I}{4\pi}\left[\frac{3\pi}{2a} + \frac{\sqrt{2}}{b}\right]$ Into the plane of paper **Ans.**

(c)



$B = B_{\text{due to st part}} + B_{\text{due to curved part}}$ both into the plane of paper

$= \left(\frac{2\pi - 2\phi}{2\pi}\right)\frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi d}$ [sin ϕ + sin ϕ]

$= \left(\frac{2\pi - 2\phi}{2\pi}\right)\frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R\cos\phi}$ [2 sinϕ]

$= \frac{\mu_0 i}{2\pi R}$ [$\pi - \phi$ + tanϕ]

$= 28\ \mu T$ **Ans.**

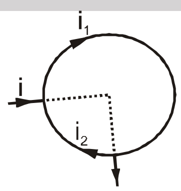
D-4. (a) $\vec{B} = \frac{\mu_0 I}{4\pi R}[-\hat{k}] + \frac{\mu_0 I}{4R}[-\hat{i}] + \frac{\mu_0 I}{4\pi R}[-\hat{k}] = \frac{\mu_0 I}{4\pi R}[-\pi\hat{i} - 2\hat{k}]$ Ans.

(b) $\vec{B} = \frac{\mu_0 I}{4\pi R}[-\hat{k}] + \frac{\mu_0 I}{4R}[-\hat{i}] + \frac{\mu_0 I}{4\pi R}[-\hat{i}] = \frac{\mu_0 I}{4R}\left[\left(1 + \frac{1}{\pi}\right)(-\hat{i}) - \left(\frac{1}{\pi}\right)\hat{k}\right]$ Ans.

(c) $\vec{B} = \frac{\mu_0 I}{4R}[-\hat{k}] + \frac{3\mu_0 I_1}{8R}[-\hat{i}] + \frac{1}{8}\frac{\mu_0 I_2}{R}[\hat{i}] + \frac{\mu_0 I}{4\pi R}[-\hat{j}]$

$i_1\left[\frac{3}{4}\pi\right] = i_2\left[\frac{1}{4}\pi\right]$

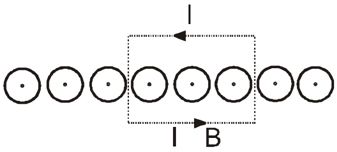
$i_2 = 3i_1$



So $\vec{B} = \frac{\mu_0 I}{4\pi R}[-\hat{j} - \hat{k}]$ **Ans.**

Section (E)

E-1.

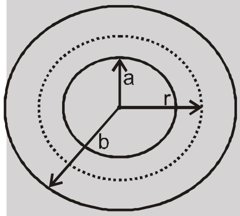


$\oint \vec{B}.d\ell = \mu_0 I$

$Bl + Bl = \mu_0 [n\ l\ i]$

$B = \frac{\mu_0 n i}{2}$ **Ans.**

E-2.



$\oint B.d\ell = \mu_0 I$

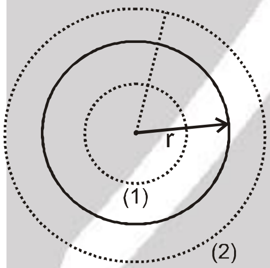
$B(2\pi r) = \mu_0 \left[\frac{i}{\pi(b^2 - a^2)} \pi(r^2 - a^2)\right]$

$B = \frac{\mu_0 i}{2\pi(b^2 - a^2)} \frac{(r^2 - a^2)}{r}$

for $a \to 0$

$B = \frac{\mu_0 i}{2\pi b^2} r$ Ans.

E-3.



(top view)

(a) loop (1)

$B = \left(2\pi \frac{r}{2}\right) \mu_0 (0)$

$B = 0$

(b) loop (2)

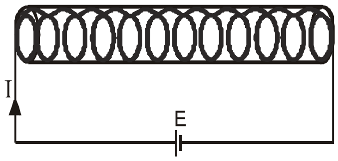
$B\left(2\pi \frac{5r}{4}\right) = \mu_0 i$

$B = \frac{2\mu_0 i}{5\pi r}$ Ans.

E-4. $B = \mu_0 n i$

$n = \frac{B}{\mu_0 i} = 2500$ turns/m. Ans.

E-5.



$E = IR$

where $B = \mu_0 ni$

$= \dfrac{B}{\mu_0 \frac{N}{\ell}} \times T \times N \times 2\pi r \qquad n = \dfrac{N}{\ell}$ and T in the resistance per unit length

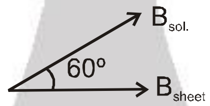
$= 1V$ Ans.

E-6 (a) $B_{Net} = 0$

$\Rightarrow \quad \mu_0 ni = \dfrac{\mu_0 k}{2}$

$i = \dfrac{k}{2n}$

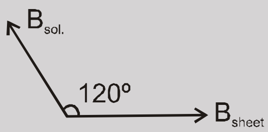
(b) $B_{Net} = \sqrt{\mu_0^2 n^2 i^2 + \dfrac{\mu_0^2 k^2}{4} + 2.\mu_0 ni.\dfrac{\mu_0 k}{2}\cos 60º}$



$= \dfrac{\mu_0 k}{2}\sqrt{3}$

and one more at an angle of 120º

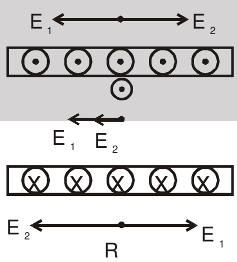
$B_{Net} = \sqrt{\mu_0^2 n^2 i^2 + \dfrac{\mu_0^2 k^2}{4} + 2.\mu_0 ni.\dfrac{\mu_0 k}{2}\cos 120º}$



$= \dfrac{\mu_0 K}{2}$

E-7. $E_P = E_1 - E_2 = 0$

$E_R = E_1 - E_2 = 0$



$E_Q = E_1 + E_2 = \dfrac{1}{2}\mu_0 k + \dfrac{1}{2}\mu_0 k$

$E_Q = \mu_0 k$ towards right.

Section (F) :

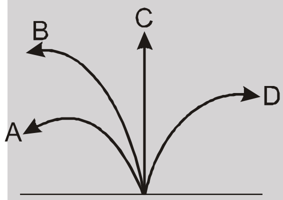
F-1. $qV = \frac{1}{2}mv^2$

$qvB = \frac{mv^2}{r}$

$\frac{q}{m} = \frac{v^2}{2V}$

$r = \frac{mv}{qB} = \frac{v}{B} \times \frac{2V}{v^2} = \frac{2V}{Bv} = 12\text{cm}$ Ans.

F-2.



C is for neutron as it will move undeflected
+ ve charge will be deflected left
– ve charge will be deflected right
So D is for electron

$r = \frac{mv}{qB}$

$\left(\frac{m}{q}\right)_\alpha > \left(\frac{m}{q}\right)_P$

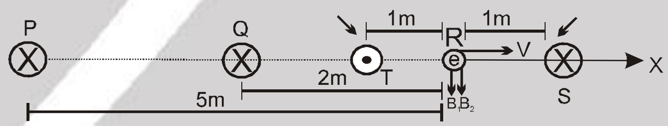
$r_\alpha > r_P$

B is for α particle.

F-3. $x = 3yz^2$

$[y] = \frac{[x]}{[z]^2} = \frac{Q^2M^{-1}L^{-2}T^2}{[MQ^{-1}T^{-1}]^2} = M^{-3}L^{-2}T^4Q^4 = M^{-3}L^{-2}T^8A^4$ Ans.

F-4.



(a) $B = B_1 + B_2 = \frac{\mu_0 i_1}{2\pi(5)} + \frac{\mu_0 i_2}{2\pi(2)} = \frac{\mu_0}{10\pi}(2.5) + \frac{\mu_0 I}{4\pi}$

$F = evB$

$B = \frac{F}{ev} = \frac{3.2 \times 10^{-20}}{1.6 \times 10^{-19} \times 4 \times 10^5} = 5 \times 10^{-7}$

Solving $I = 4A$ **Ans.**

(b) Net field at R $= 5 \times 10^{-7} \downarrow$

To produce zero net field , field due to third wire $= 5 \times 10^{-7} \uparrow$

$\frac{\mu_0 I}{2\pi r} = 5 \times 10^{-7}$ $\Rightarrow r = 1\text{m}$

Two possible positions :
1. Current into the plane of paper 1m from R at S pt.
2. Current out of the plane of paper 1m from R at T pt.

F-5. $\vec{F} = q(\vec{v} \times \vec{B})$

Let $\vec{v} = v_x\hat{i} + v_y\hat{j}$

$(4.0\,\hat{i} + 3.0\,\hat{j}) \times 10^{-10} = (0.5 \times 10^{-9})$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ V_x & V_y & 0 \\ 0 & 0 & 8\times10^{-3} \end{vmatrix}$$

$(4\,\hat{i} + 3\,\hat{j}) \times 10^{-1} = \hat{i}\,[8 \times 10^{-3}\,V_y] - \hat{j}\,[8 \times 10^{-3}\,V_x]\ 0.5$

So $V_y = 100$ m/s & $V_x = -75$ m/s

F-6. $\vec{F} = q(\vec{v} \times \vec{B})$

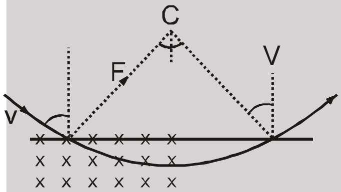
$\vec{F} \perp \vec{B} \Rightarrow \vec{a}$ is $\perp$ to $\vec{B}$

So $\vec{a}.\vec{B} = 0$

$7x - 21 = 0$

$x = 3.0$ Ans.

F 7.



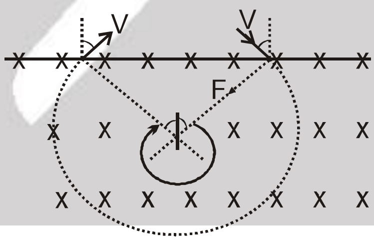
(a) $r = \dfrac{mv}{qB}$ **Ans.**

(b) Angle Subtended by the arc $= \dfrac{\pi}{2}$ **Ans.**

(c) $\theta = \omega t$

$\dfrac{\pi}{2} = \dfrac{qBt}{m} \Rightarrow t = \dfrac{\pi m}{2qB}$ Ans.

(d)

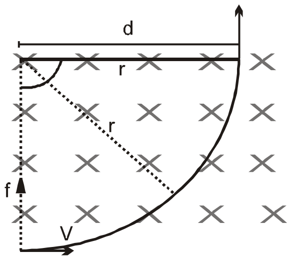


$r = \dfrac{mv}{qB}$

$\theta = \dfrac{3\pi}{2}$

$t = \dfrac{\theta}{\omega} = \dfrac{3\pi m}{2qB}$

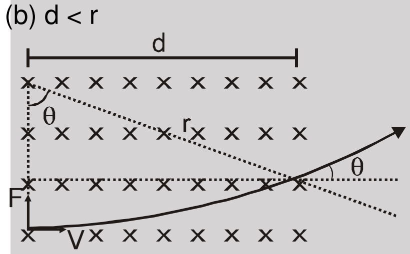
F 8.



$r = \dfrac{mv}{qB}$

(a) $d = r$ angle subtended = $\dfrac{\pi}{2}$

(b) $d < r$

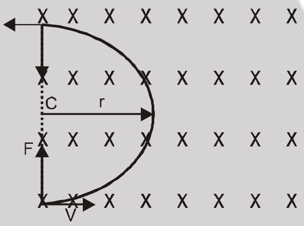


$d = \dfrac{r}{2}$

$\sin\theta = \dfrac{d}{r} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

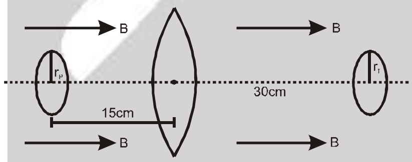
(c) $d > r$



$d = 3r$

$\theta = \pi$ radians

F-9.



$\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

$\dfrac{1}{v} + \dfrac{1}{15} = \dfrac{1}{10}$

V = 30cm

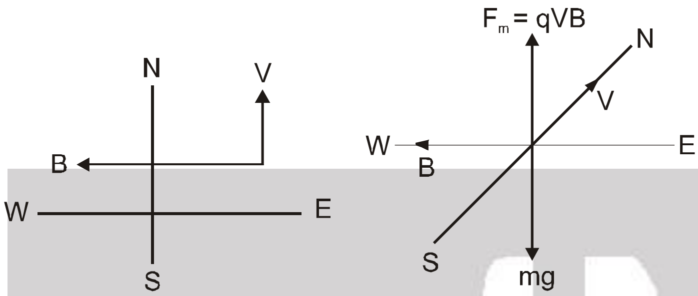
$r_{particle} = \dfrac{mv}{qB}$

$= \dfrac{2 \times 10^{-5} \times 4.8}{2 \times 10^{-3} \times 1.2}$ = 4cm

$\frac{r_I}{r_P} = \frac{V}{u} = 2$

$r_{Image} = 8cm$

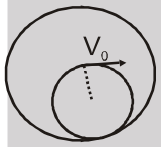
F-10.



$mg = qVB$

$V = \frac{mg}{qB} = 50$ m/s

F-11.



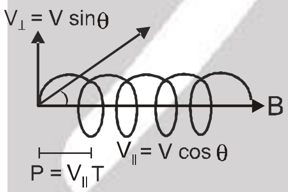
$R = \frac{mV_0}{qB} = \frac{r}{2}$

$= \frac{2mV_0}{qr} = B = \mu_0 ni$

$V_0 = \frac{\mu_0 niqr}{2m}$

Section (G) :

G-1.



$r = \frac{mV_\perp}{qB} = \frac{5 \times 10^{-12} \times 1 \times 10^{3} \times 0.9}{5 \times 10^{-6} \times 5 \times 10^{-3}} = 18cm$ **Ans.**

Diameter $= 36cm$

$T = \frac{2\pi m}{qB}$

$P = V\cos\theta \times \frac{2\pi m}{qB} = 4\pi\sqrt{19}$ cm **Ans.**

G-2. $r = \frac{mV_\perp}{qB}$

$V_\perp = \frac{qBr}{m}$

$= \frac{1.6 \times 10^{-19} \times 0.04 \times 0.05}{1.67 \times 10^{-27}} \approx 2 \times 10^5$ m/s Ans.

$P = V_{||} \frac{2\pi r}{V_\perp}$

$V_{||} = \frac{P \times V_\perp}{2\pi r}$

$= \frac{4}{\pi} \times 10^5$ m/s Ans.

G-3. $t = \frac{.1}{V\cos 60}$, $T = \frac{2\pi m}{qB}$

$nT = t$

$\frac{n2\pi mV}{qB} = 0.2$

$\frac{\pi n\sqrt{2mE}}{qB} = 0.1$

$B = \frac{\pi n\sqrt{2 \times 9 \times 10^{-31} \times 2 \times 10^3 \times 1.6 \times 10^{-19}}}{e \times 0.1}$

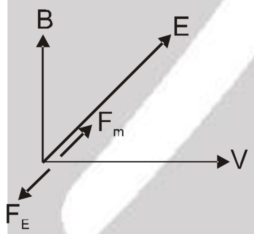
$B = \frac{\pi n}{0.1}\sqrt{\frac{36 \times 10^{-28}}{1.6 \times 10^{-19}}} = \frac{\pi n}{0.1}\sqrt{\frac{9 \times 10^{-8}}{4}}$

$B = 15\pi n \times 10^{-4}$ T

$B_{min} = 15\pi \times 10^{-4}$ T

Section (H)

H-1.



$F_m = F_E$

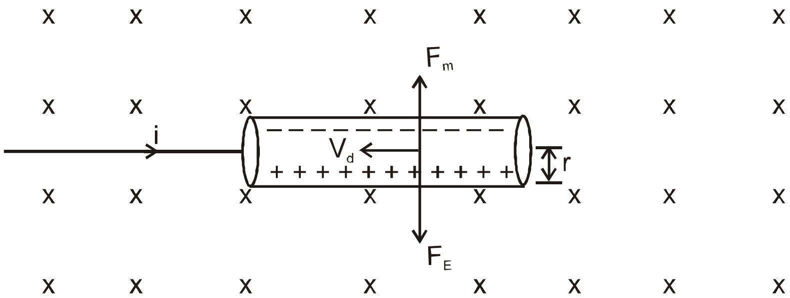
$eVB = eE$

$V = \frac{E}{B} = \frac{3.2 \times 10^4}{2 \times 10^{-3}}$

$= 16 \times 10^6$ m/s **Ans.**

$r = \frac{mV}{eB} = \frac{91}{20}$ cm **Ans.**

H-2.



$i = neAV_d$

$V_d = \frac{i}{neA} = \frac{i}{\pi r^2 ne}$

$F_m = eV_dB = \frac{iB}{nA}$ upward

$F_m = \frac{iB}{\pi r^2 n}$ upward

$F_m = F_E$

$eV_dB = eE$

$E = BV_d = \frac{Bi}{neA} = \frac{iB}{\pi r^2 ne}$

P.d.= $Ed = \frac{Bid}{neA} = \frac{2iB}{\pi rne}$

H-3. $r = \frac{mV}{qb}$

$\frac{q}{m} = \frac{V}{rB}$

As, $qE = qVB$

$E = VB$

$\frac{q}{m} = \frac{E}{rB^2} = \frac{5}{4} \times 10^5$ C/kg Ans.

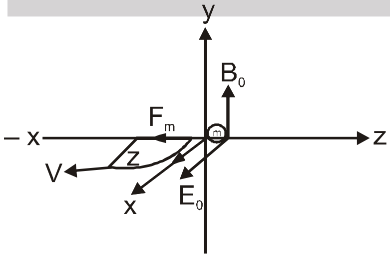
H-4. $V = \frac{E}{B}$

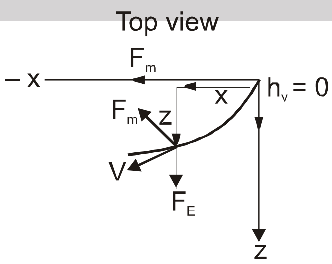
$r = \frac{mV}{qB}$

$B = \frac{mV}{rq} = \frac{1.6 \times 10^{-27} \times 2 \times 10^5}{4 \times 10^{-2} \times 1.6 \times 10^{-19}} = 5 \times 10^{-2}$ T Ans.

$E = VB = 5 \times 10^3$ N/C Ans.

H-5.





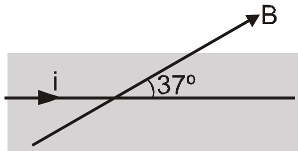
$W_E = \Delta k$

$qE_0x = \frac{1}{2}mV^2 - 0$

$V = \sqrt{\frac{2qE_0x}{m}}$ Ans.

Section (I)

I-1.

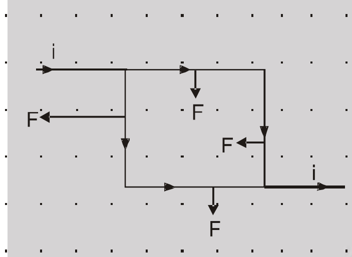


$\vec{F} = i\left(\vec{L} \times \vec{B}\right)$

$F = BiL \sin\theta$

$= 0.1 \times 10 \times 0.1 \times \frac{3}{5} = 6 \times 10^{-2}N$

I-2.



$F = B\frac{i}{2}l$

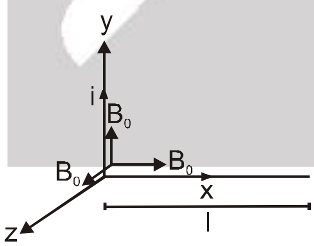
$= 0.1 \times 1 \times 0.1 = 1 \times 10^{-2}$ N Ans.

I-3. $F = BiL \qquad L = 2r$

$= Bi\,(2r)$

$= 1 \times 2 \times 4 \times 10^{-2} = 8 \times 10^{-2}$ N

I-4.

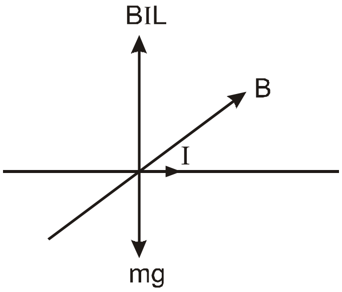


$\vec{F} = i\,[\vec{l} \times \vec{B}\,] = B_0il\,[\hat{j} \times (\hat{i} + \hat{j} + \hat{k})]$

$= B_0il\,[-\hat{k} + \hat{i}]$

$F = \sqrt{2}\,B_0il$ **Ans.**

I-5.



BIL $\sin\theta$ = mg

$$B = \frac{mg}{IL\sin\theta}$$

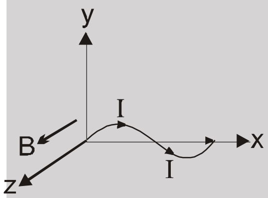
$$B_{min} = \frac{mg}{IL}$$

$$= \frac{10^{-4}\times 10}{10\times 0.2}$$

$$= 5\times 10^{-4}\ T$$

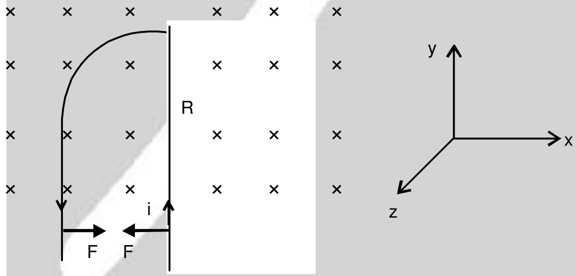
B should be horizontal $\perp^r$ to wire so that F = BIL is upward

I-6.



F = BiL = Biλ/2 **Ans.**

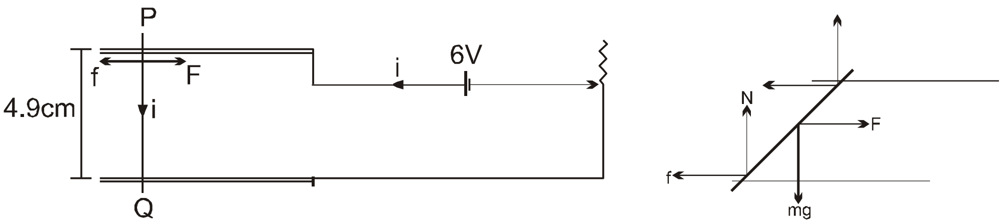
I-7.



Force due to straight portion cancle each other.

Force due to curved part = BiR $(-\hat{j})$ **Ans.**

I-8.



F = f_L

N = mg

BiL = μ mg

$$\mu = \frac{BIL}{mg} = \frac{0.8 \times \frac{6}{20} \times 4.9 \times 10^{-2}}{10 \times 10^{-3} \times 9.8} = 0.12$$ **Ans.**

I-9. The current carrying wires are electrically neutral. Hence the only interaction between wires is of attractive magnetic force.
But in parallel beams of electrons both the beams have negative charge.
Hence there is electrostatic repulsion and also magnetic attraction. Electrostatic repulsion has larger magnitude than magnetic attraction. Hence the beams repel.

I-10. $\vec{B} = B_0\left(1 - \frac{x}{\ell}\right)\hat{k}$

$\vec{F}_{AB}$ at x = 0

$B = B_0\hat{k}$

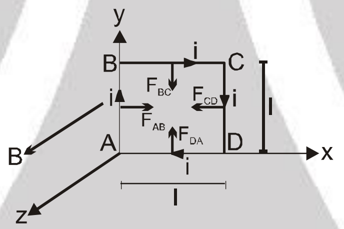
at x = ℓ B = 0

$\vec{F}_{AB} = B_0 i\ell\,\hat{i}$.

$\vec{F}_{CD} = 0$

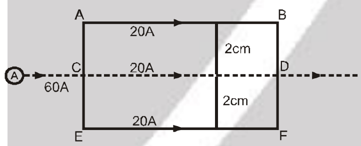
$\vec{F}_{BC} = -\vec{F}_{DA}$

So net force = $B_0 i\ell\,\hat{i}$



$F = B_0 i\ell\,\hat{i}$

I-11.



On AB :

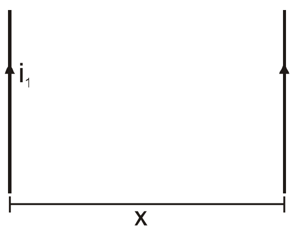
$$\frac{\mu_0 i_1 i_2}{2\pi(2cm)} + \frac{\mu_0 i_1 i_3}{2\pi(4cm)} = \frac{F}{l} = 2 \times 10^{-7} \times 400 \times 100\left(\frac{3}{4}\right)$$

$$\frac{F}{\ell} = 6 \times 10^{-3} \text{ N/m downward}$$

On CD :

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi(1cm)} - \frac{\mu_0 i_1 i_3}{2\pi(1cm)} = 0$$ Ans.

I-12.



$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi x}$$

$$\frac{dw}{\ell} = \frac{F}{\ell}dx = \frac{\mu_0 i_1 i_2}{2\pi x}dx$$

$$\frac{w}{\ell} = \frac{\mu_0 i_1 i_2}{2\pi}\int_{r_1}^{r_2}\frac{dx}{x} = \frac{\mu_0 i_1 i_2}{2\pi}\ln\left(\frac{r_2}{r_1}\right)$$ **Ans.**

I-13. Minimum required force to overcome friction

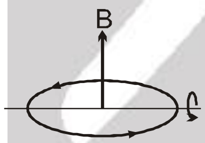
$$\frac{\mu m g}{\sqrt{1+\mu^2}} = Bi\ell\sin\theta$$

$$B = \frac{\mu m g}{i\ell\sqrt{1+\mu^2 \sin\theta}}$$

$$B_{min} = \frac{\mu m g}{i\ell\sqrt{1+\mu^2}}$$

Section (J)

J-1.



$U_i = -MB$

$U_f = +MB$

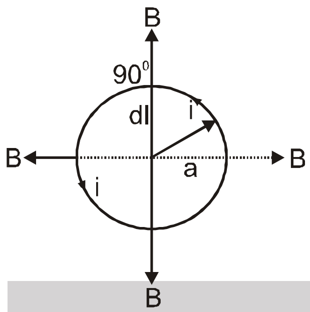
or $U_i = +MB$

$U_f = -MB$

$W = \Delta U = 2MB = 2 \times Ni\,\pi\,r^2 B$

$W = \pm 75\pi \times 10^{-3}$ J

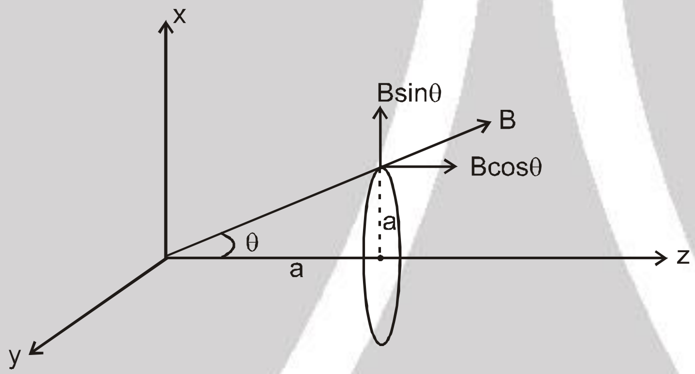
J-2.



(a) df = BidL

F = Bi (2πa) into the plane of paper.

(b)



$\vec{B} = B_0 \vec{e}_r$

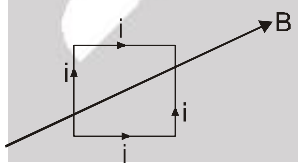
All force elements due to B_0 cosθ will be added up & due to B_0 sinθ will get cancled

$dF = B_0 \frac{1}{\sqrt{2}} \cos\theta \; i \; dl$

$dF = B_0 \times i \; dl$

$F = \frac{B_0 i}{\sqrt{2}} (2\pi a) = \sqrt{2} \; iB_0\pi a$ Ans.

J-3



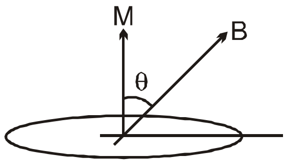
$\tau =$ MB

0.2 = iNL bB

$B = \frac{0.2}{100 \times 2 \times 5 \times 4 \times 10^{-4}}$

= 0.5 T

J-4



$\tau \quad = MB \sin\theta$

$\tau_{Max} \;= MB$

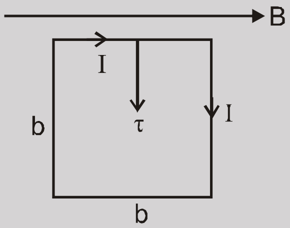
$= 50 \times \pi\,(0.04)^2 \times 2.5 \times 0.2$

$= 4\pi \times 10^{-2}$ N-m

$\sin\theta = \frac{1}{2}$ If $\tau = \frac{1}{2}\tau_{Max}$

i.e. angle between M & B is 30^0 OR angle between B and plane of coil = 60^0

J-5



In uniform magnetic field force acting on closed loop = zero

$\tau = MB$

$= i\ell^2 B$

$= 10 \times 0.1 \times 0.1 \times 0.2 = 2 \times 10^{-2}$ N-m

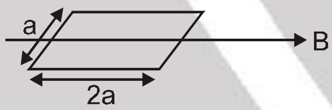
J-6. $\tau \;=\; MB \sin 30^0$

$= Ni\pi r^2 B \times$

$= 500 \times 1 \times \pi \times (0.01)^2 \times 0.4 \times$

$= \pi \times 10^{-2}$ N-m

J-7.



$L = 2\pi r$ $\quad$ $L = 6a \Rightarrow a = L/6$

$\tau = Bi\pi r^2$ $\quad$ $\tau = 2Bia^2$

$= \frac{iL^2B}{4\pi}$ $\quad$ As $i_{loop} > I_{square\ loop} = 2Bi\,\frac{L^2}{36} = \frac{BiL^2}{18}$

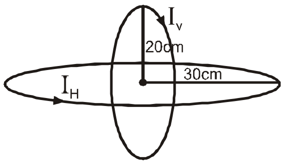
J-8. $i = Qf = \frac{Q\omega}{2\pi}$

$M = i \times \pi R^2 = \frac{Q\omega R^2}{2}$ **Ans.**

Section (K) :

K-1. As the field due to a current-carrying coil is along its axis, the vertical coil will produce horizontal field and horizontal coil vertical, i.e.,

$$\frac{\mu_0}{4\pi}\frac{2\pi N_V I_V}{R_V} = B_H \text{ and } \frac{\mu_0}{4\pi}\frac{2\pi N_H I_H}{R_H} = B_V$$



But as $\tan\phi = \frac{B_V}{B_H}$, $\quad B_V = B_H \tan\phi = \frac{B_H}{\sqrt{3}}$ $\qquad$ [as $\phi = (\pi/6)$]

and, $1\frac{A}{m} = 4\pi \times 10^{-7}\frac{Wb}{m^2}$

so, $10^{-7}\frac{2\pi\times 100\times I_v}{0.2} = 4\pi \times 10^{-7} \times 27.8$

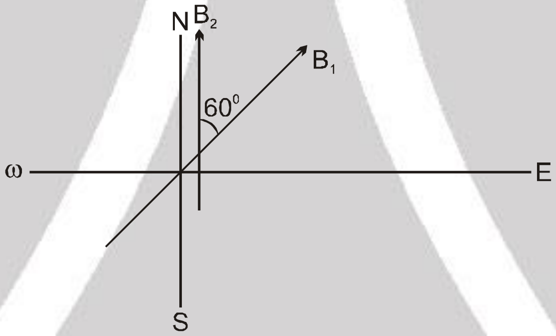
i.e., $\quad I_v = 1112 \times 10^{-4}$ A

and $10^{-7}\frac{2\pi\times 100\times I_H}{0.3} = 4\pi\times 10^{-7}\frac{27.8}{\sqrt{3}}$

i.e., $\quad I_H = 556\sqrt{3} \times 10^{-4}$ A

K-2. $B_R = \sqrt{B_1^2 + B_2^2 + 2B_1B_2\cos 60^0}$

$B_1 = 2\frac{\mu_0}{4\pi}\frac{M}{r^3}$



$= 2 \times 10^{-7}\frac{6}{(0.2)^3}$

$= 1.5 \times 10^{-4}$ T

$B_2 = 0.3 \times 10^{-4}$ T

$B_R = 10^{-4}\sqrt{(1.5)^2 + (0.3)^2 + 2\times 0.15\times 0.3\times\frac{1}{2}} = \sqrt{2.79}\times 10^{-4}$ **Ans.**

K-3. $L = N \times 2\pi r - 50 \times 2 \times \pi \times 0.1$

$A = \pi r^2 = \pi \times (0.1 \times 10^{-3})^2$

Resistance $R = \frac{\rho L}{A}$

$= \frac{2\times 10^{-8}\times 50\times 2\times\pi\times 0.1}{\pi\times 10^{-8}} = 20\Omega$

$i = \frac{\varepsilon}{R} = 1$ A

$B_H = \frac{\mu_0 N i}{2r}$

Section (L)

L-1. $B = \frac{\Phi}{A} = \frac{2.4 \times 10^{-5}\ Wb}{0.2 \times 10^{-4}\ m^2} = 1.2\ Wb/m^2 = 1.2\ N\ A^{-1}\ m^{-1}$.

The magnetising field (or magnetic intensity) H is 1600 Am^{-1}. Therefore, the magnetic permeability is given by -

$\mu = \frac{B}{H} = \frac{1.2\ NA^{-1}m^{-1}}{1600\ Am^{-1}} = 7.5 \times 10^{-4}\ N/A^2$.

Now, from the relation $\mu = \mu_0 (1 + \chi_m)$, the susceptibility is given by

$\chi_m = \frac{\mu}{\mu_0} - 1$.

We known that $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$.

$\therefore \quad \chi_m = \frac{7.5 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1 = 596$.

L-2. The magnetic field in the empty space enclosed by the windings of a toroid carrying a current i_0 is $\mu_0\ n\ i_0$ where n is the number of turns per unit length of the toroid and μ_0 is permeability of free space.
If the space is filled by a core of some material of permeability μ, then the field is given by
$B = \mu\ n\ i_0$
But $\mu = \mu_0 u_r$, where μ_r is the relative permeability of the core material. Thus,
$B = \mu_0\ u_r\ ni_0$
or $\mu_r = \frac{B}{\mu_0 n i_0}$

Here B = 2.5 T, i_0 = 0.6 A and $n = \frac{3000}{2\pi r}\ m^{-1}$, where r is the mean radius of the toroid ($r = \frac{11 + 12}{2}$ = 11.5 cm 11.5×10^{-2} m). Thus,

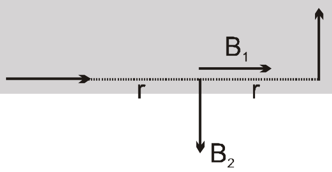
$\mu_r = \frac{2.5}{(4\pi \times 10^{-7}) \times (3000 / 2\pi \times 11.5 \times 10^{-2}) \times 0.6} = \frac{2.5 \times 11.5 \times 10^{-2}}{2 \times 10^{-7} \times 3000 \times 0.6}$

$\mu_r = 798.5$

PART - II

A-1. $B_1 = \frac{2\mu_0}{4\pi} \frac{M}{r^3}$ (As the dipole is short)

$= \frac{10^{-7} \times 1 \times 2}{(1)^3} = 2 \times 10^{-7}$ T

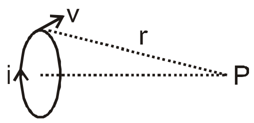


$B_2 = \frac{\mu_0}{4\pi} \frac{M}{r^3}$

$= 10^{-7}$ T

$B_{net} = \sqrt{5} \times 10^{-7}$ T (B) Ans.

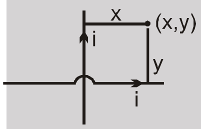
A-2. $\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$



Magnitude fixed but direction keeps on changing (A)

Section (B)

B-1. $\frac{\mu_0 i}{2\pi x} = \frac{\mu_0 i}{2\pi y}$



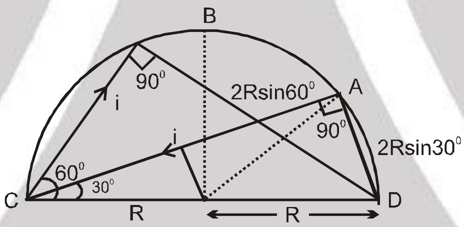
$y = x$ Ans. (A)

only in first and third quadrant the fields will be oppositely directed.

B-2. $B_{\text{due to AC}} = \frac{\mu_0 i}{4\pi\ 2R\sin 30^0}[\cos 30^0 + \cos 90^0]$

$= \frac{\mu_0 i\sqrt{3}}{8\pi R}$

$B_{\text{due to BC}} = \frac{\mu_0 i}{4\pi\ 2R\sin 60^0}[\cos 60^0 + \cos 90^0]$



$= \frac{\mu_0 i}{8\pi R\sqrt{3}}$

$B_{\text{Net}} = B_{\text{due to AC}} - B_{\text{due to BC}} = \frac{\mu_0 i}{4\pi R\sqrt{3}}$

B-3. $\vec{B}_{\text{due to first loop}} = 4\frac{\mu_0 i}{4\pi\frac{a}{2}}[\cos 45° + \cos 45°]$

$= \frac{2\sqrt{2}\mu_0 i}{\pi a}$

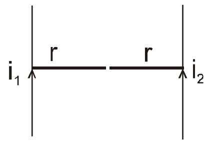
$\vec{B}_{\text{due to second loop}} = -\frac{4\mu_0 i}{4\pi\frac{2a}{2}}[\cos 45° + \cos 45°]$

$= \frac{-\sqrt{2}\mu_0 i}{\pi a}$

$\vec{B} = \frac{2\sqrt{2}\mu_0 i}{\pi a}[1 - \frac{1}{2} + \text{............}\infty]$

$= \frac{2\sqrt{2}\mu_0 i}{\pi a}\ell n\, 2$ **Ans. (C)**

B-4. $i_1 > i_2$

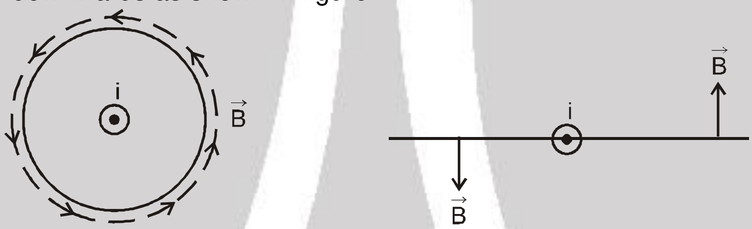


$$\frac{\mu_0}{2r}(i_1 - i_2) = 20$$

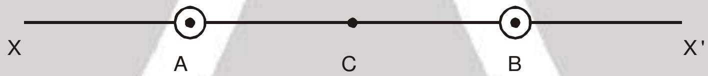
$$\frac{\mu_0}{2r}(i_1 + i_2) = 30$$

$$\frac{i_1 + i_2}{i_1 - i_2} = \frac{3}{2} \Rightarrow \frac{i_1}{i_2} = \frac{5}{1}$$ **Ans.(C)**

B-5. If the current flows out of the paper, the magnetic field at points to the right of the wire will be upwards & to the left will be downwards as shown in figure.



Now let us come to the problem.
Magnetic field at C = 0
Magnetic field in region BX' will be upwards (+ve) because all points lying in this region are to the right of both the wires.



Similary
Magnetic field in region AX will be downwards (-ve)
Magnetic field in region AC will be upwards (+ve) , because points are closer to A, compared to B.

Similarly Magnetic field in region BC will be downwards (-ve).

Graph (B) satisfies all these conditions. Therefore correct answer is (B).

B-6. H_1 = Magnetic field at M due to PQ + Magnetic field at M due to QR

But magnetic field at M due to QR = 0

Now H_2 = Magnetic field at M due to PQ (current I) + Magnetic field at M due to QS (current I/2)

+ Magnetic field at M due to QR

$$= H_1 + \frac{H_1}{2} + 0 = \frac{3}{2}H_1$$

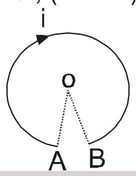
$$\therefore \frac{H_1}{H_2} = \frac{2}{3}$$

Magnetic field at any point lying on the current carrying straight conductor is zero.

Section (C)

C-1. $B = \frac{\mu_0 i}{4\pi R'}(2\pi - \theta)$

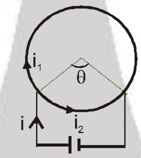
where ; $(2\pi - \theta)\,R' = 2\pi R$



$R' = \frac{2\pi R}{2\pi - \theta}$

$B = \frac{\mu_0 i}{2R}\left(\frac{2\pi - \theta}{2\pi}\right)^2$ **Ans. (A)**

C-2. $B_{\text{at centre}} = \frac{\mu_0 i_1}{2R}\frac{[2\pi - \theta]}{2\pi} - \frac{\mu_0 i_2}{2R}\left[\frac{\theta}{2\pi}\right]$



$i_1R_1 = i_2R_2 = \varepsilon$

$i_1\,(2\pi - \theta) = i_2(\theta)$

So $B_{\text{at centre}} = 0$ **Ans. (B)**

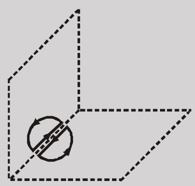
C-3. $B = \mu_0\mu_r ni$

$= 10^{-7} \times 4\pi \times 4000 \times 1000 \times 5$

$= 8\pi$ T

$= 25.12$ T Ans. (D)

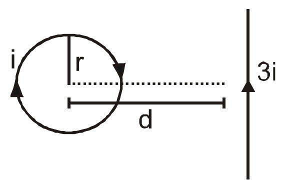
C-4. $\oint\limits_{ABCDA} \vec{B}.d\ell = \oint\limits_{ABCA} \vec{B}.d\ell + \oint\limits_{CDAC} \vec{B}.d\ell$



$= \mu_0\,(i_1 + i_3) + \mu_0\,(i_2 - i_3)$

$= \mu_0\,(i_1 + i_2)$ Ans. (D)

C-5. $B = \frac{\mu_0 i}{2r} = \frac{\mu_0 (3i)}{2\pi d}$



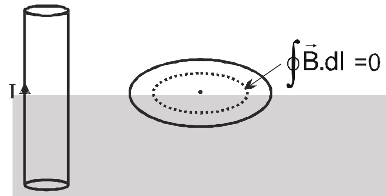
$d = \frac{3r}{\pi}$ Ans.

C-6. $n = \frac{1}{d} = \frac{2}{1 \times 10^{-3}}$ /m

$B = \mu_0 ni$

$= 4\pi \times 10^{-7} \times \frac{2}{10^{-3}} \times 2.5 = 2\pi \times 10^{-3}$T

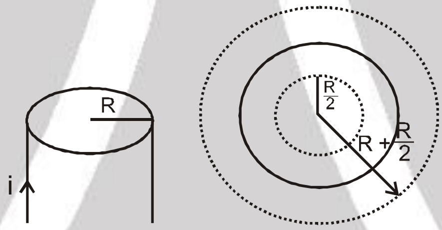
C-7.



$\oint \vec{B}.d\ell = 0$

$B = 0$ Ans. (B)

C-9. $B_{inside} = \frac{\mu_0 \frac{i}{\pi R^2} \times \frac{\pi R^2}{4}}{2\pi \frac{R}{2}} = \frac{\mu_0 i}{4\pi R}$



$B_{Outside} = \frac{\mu_0 i}{2\pi \frac{3R}{2}} = \frac{\mu_0 i}{3\pi R}$

Energy density $\propto B^2$

$\frac{\varepsilon_1}{\varepsilon_2} = \left[\frac{B_1}{B_2}\right]^2 = \frac{9}{16}$

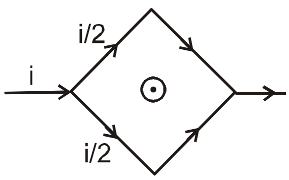
Section (D)

D-1. $F = qVB$

$F_{Min} = q_{Min}VB$ Ans. (B)

As from the given options proton has minimum charge.

D-2.



Magnetic field at centre is zero **Ans. (D)**

D-3. $qV = \frac{1}{2}mv^2$

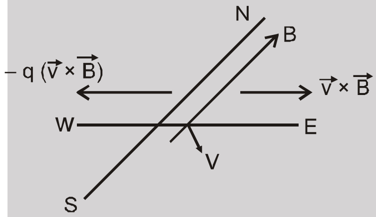
$$R = \frac{mv}{qB} = \frac{m\sqrt{\frac{2qV}{m}}}{qB} = \sqrt{\frac{2mV}{qB^2}}$$

$$\frac{R_1}{R_2} = \sqrt{\frac{m_1}{m_2}}$$

$$\frac{m_1}{m_2} = \left(\frac{R_1}{R_2}\right)^2$$

Ans. (C)

D-4.



F towards west

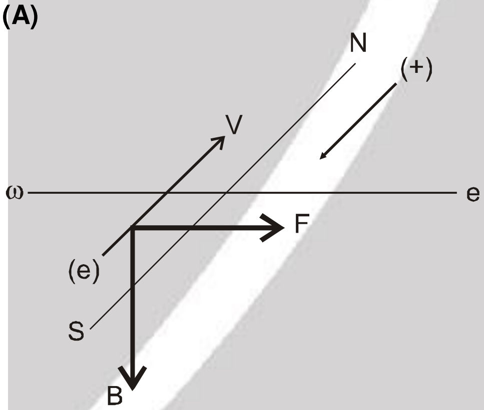
So particle will be deflected towards west **Ans. (B)**

D-5.

V || B

F = 0

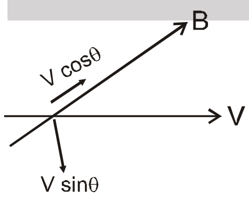
D-6. **(A)**



Electrostatic force between two change $q_1 = q_2 = 1.6 \times 10^{-19}$ coulomb is very stronger than the magnetic force. Hence, attracted.

Section (E)

E- 1.



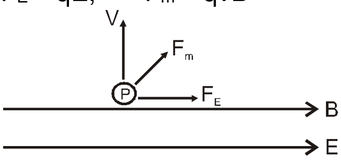
$$\frac{mv^2}{R} = qv\,(B\sin\theta)$$

$$R = \frac{mv}{qB\sin\theta}$$

Ans. (C)

Section (F)

F-1. $F_E = qE$, $F_m = qvB$



$R = \frac{mv}{qB}$

Pitch $p = V_{||}T$

$T = \frac{2\pi R}{V}$

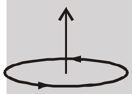
$V_{||} = 0 + \frac{qEt}{m}$ Ans. (D)

F-2. $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

If does not deflect then, resultant force must be zero.

Section (G)

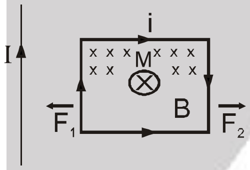
G-1.



In uniform magnetic filed force acting on a closed loop = 0.

Ans. (C)

G-2. $\vec{M} \times \vec{B} = 0$



$\tau = 0$

Loop will Not rotate

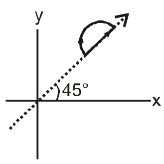
$F_1 > F_2$

So loop move towards the wire

Ans. (C)

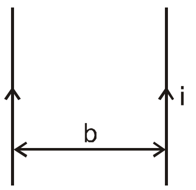
G-3. $\vec{B} = 3\hat{i} + 4\hat{j} + \hat{k}$

$\vec{l} = \frac{2}{\sqrt{2}}(\hat{i} + \hat{j})$



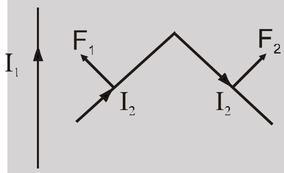
$\vec{F} = I(\vec{l} \times \vec{B}) = \sqrt{2}\,[(\hat{i} + \hat{j}) \times (3\hat{i} + 4\hat{j} + \hat{k})] = \sqrt{2}\,(\hat{i} - \hat{j} + \hat{k})$ Ans. (B)

G-4.



$F = \frac{\mu_0}{4\pi} \cdot \frac{2i^2}{b} = \frac{\mu_0 i^2}{2\pi b}$

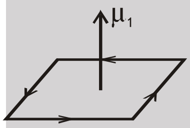
G-5.

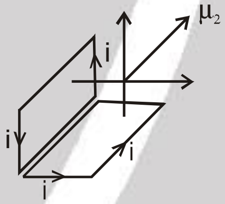


As ; $F_1 > F_2$
Resultent of F_1 & F_2 will be inclined towards F_1
Ans. (D)

G-6. $\mu_1 = L^2$

$\mu_2 = \sqrt{2} \times L \times L/2$

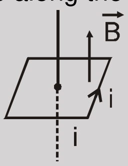




$\mu_2 = \frac{L^2}{\sqrt{2}}$

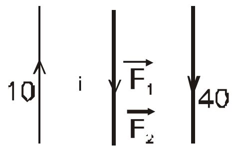
$\frac{\mu_1}{\mu_2} = \sqrt{2}$

G-7. Field produced by loop at the centre will be along the axis of the loop i.e. || to st. wire .



So $F = i(\vec{i} \times \vec{B}) = 0$

G-8. $F_1 = \frac{\mu_0 (10 \times i)}{2\pi l}$



$F_2 = \frac{\mu_0 (i \times 40)}{2\pi l}$

F_1 and F_2 both points in the same direction towards 40 A wire.

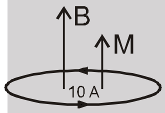
G-9. Vector sum $\vec{PQ} + \vec{QR} + \vec{RP} = 0$
Thus force on PQR = 0.

SECTION (H)

H-1. $U_i = -MB$
$U_f = MB$

$W = \Delta U = 2\ MB$
$= 2 \times 2.5 \times 0.2$
$= 1\ J$ Ans. (B)

H-2.

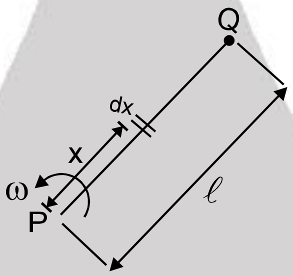


$\vec{\tau} = \vec{M} \times \vec{B} = 0$ Ans. (A)

H-3. Charge on the differential element dx, $dq = \frac{Q}{\ell}.dx$

equivalent current $di = f\,dq$
$\therefore$ magnetic moment of this element $d\mu = (di)\,NA$ (N = 1)

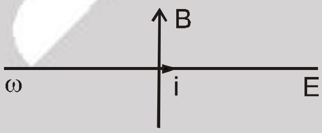
$d\mu = \left(\pi x^2\right) f \frac{Q}{\ell} dx$



$\Rightarrow \mu = \frac{\pi f Q}{\ell}\int_0^{\ell} x^2 dx$; $\mu = \pi fQ\,\ell^2$ ....**Ans.**

Section (I)

I-1.



$F = BiL = 10^{-4} \times 10 \times 1$

$= 10^{-3}$ N Ans. (C)

I-2. $B_H \tan\theta = \frac{\mu_0 N i}{2r}$

$i = \frac{0.34 \times 10^{-4} \times 2 \times .2}{4\pi \times 10^{-7} \times 20} = \frac{17}{10\pi}$ Ans. (A)

PART - III

1. The magnetic field is along negative y-direction in p,q,r, t
z-component of magnetic field is zero in all cases.

The magnetic field at P is $\frac{\mu_0}{4\pi}\frac{i}{d}$ for case (r)

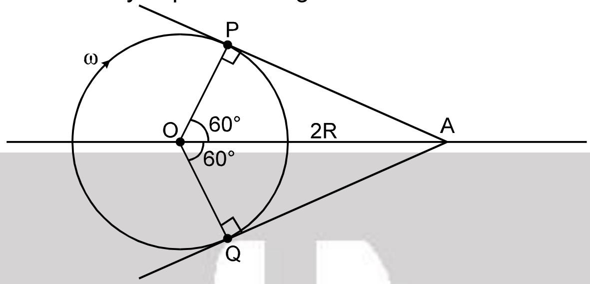
The magnetic field at P is less than $\frac{\mu_0}{2\pi}\frac{i}{d}$ for all cases.

2. The Force on a magnetic dipole placed in uniform magnetic field is zero. Hence option p is common to all the four situations. Torque on magnetic dipole is $\vec{\tau} = \vec{\mu} \times \vec{B}$ and potential energy of dipole in external magnetic $U = -\vec{\mu} \cdot \vec{B}$

(A) Since $\theta = 0$, therefore $\tau=0$
(B) Since $\theta = \pi/2$, therefore $\tau=\mu B$
(C) Since θ is acute, torque is non zero and less than μB in magnitude
(D) Since $\theta = \pi$, therefore $\tau=0$ and $U=\mu B$
Magnitic dipole will be in stable equilibrium only when $\theta = 0$

EXERCISE-2
PART - I

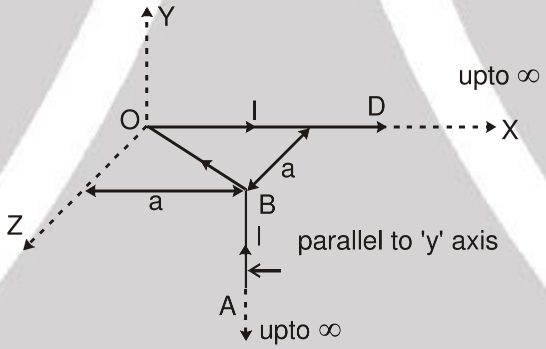
1. Point A shall record zero magnetic field (due to α-particle) when the α-particle is at position P and Q as shown in figure. The time taken by α-particle to go from P to Q is



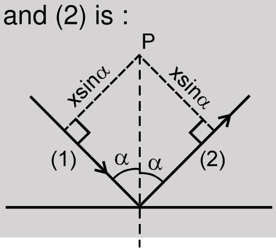
$$t = \frac{1}{3}\frac{2\pi}{\omega} \qquad \text{or} \qquad \omega = \frac{2\pi}{3t}$$

2. $\vec{B} = \frac{\mu_0}{4\pi} q \frac{\vec{v} \times \vec{r}}{r^3}$ and $\vec{E} = \frac{1}{4\pi \epsilon_0} \frac{q\vec{r}}{r^3}$ $\therefore$ $\vec{B} = \mu_0 \epsilon_0 (\vec{v} \times \vec{E}) = \frac{\vec{v} \times \vec{E}}{c^2}$

3. $B_{OD} = 0$
$B_{OB} = 0$
$B_{AB} = \frac{\mu_0 I}{4\pi a\sqrt{2}} [\cos 45°(-\hat{i}) + \cos 45°\hat{k}] = \frac{\mu_0 I}{8\pi a}(-\hat{i} + \hat{k})$



4. Magnetic field at 'P' due to wires (1) and (2) is :



$B_1 = \frac{\mu_0 I}{2\pi(x \sin\alpha)} + \frac{\mu_0 I}{2\pi(x \sin\alpha)} = \frac{2\mu_0 I}{2\pi(x \sin\alpha)}$ (outside the paper).

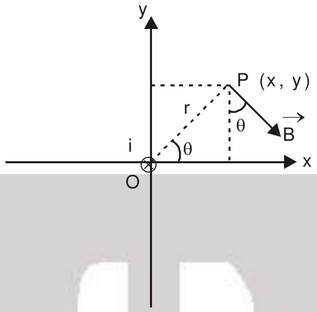
Now if a current of $\frac{2I}{\sin\alpha}$ is flowing in the third wire then the magnetic field due to the same will be :

$B_2 = \frac{\mu_0}{2\pi x}\left(\frac{2I}{\sin\alpha}\right)$, which will cancel B_1 if it is inside paper which is possible if the current $\frac{2I}{\sin\alpha}$ in the third wire is from right to left.

5. Magnetic field at P is $\vec{B}$, perpendicular to OP in the direction shown in figure.

So, $\vec{B} = B \sin\theta\,\hat{i} - B\cos\theta\,\hat{j}$

Here $B = \frac{\mu_0}{2\pi}\frac{I}{r}$

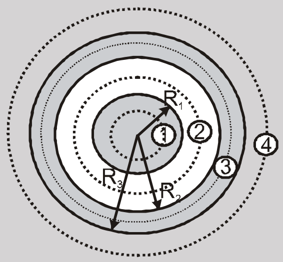


$\sin\theta = \frac{y}{r}$ and $\cos\theta = \frac{x}{r}$

$\therefore \quad \vec{B} = \frac{\mu_0 I}{2\pi}\cdot\frac{1}{r^2}(y\hat{i} - x\hat{j}) = \frac{\mu_0 I(y\hat{i} - x\hat{j})}{2\pi(x^2 + y^2)}$ (as $r^2 = x^2 + y^2$)

6. loop (1)

$$B = \frac{\mu_0 \frac{i}{\pi R_1^2} \times \pi r^2}{2\pi r} = \frac{\mu_0 i}{2\pi R_1^2} r \qquad B \propto r$$



loop (2)

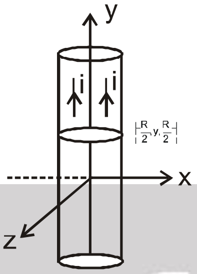
$$B = \frac{\mu_0 i}{2\pi r} \qquad B \propto \frac{1}{r}$$

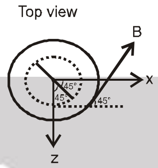
loop (3)

$$B = \frac{\mu_0 \left(i - \frac{i}{R_3^2 - R_2^2}\left[r^2 - R_2^2\right]\right)}{2\pi r} = \frac{\mu_0 (R_3^2 - r^2)}{2\pi r(R_3^2 - R_2^2)}$$

loop (4) $B = \frac{\mu_0 (i - i)}{2\pi r} = 0$ Ans. (C)

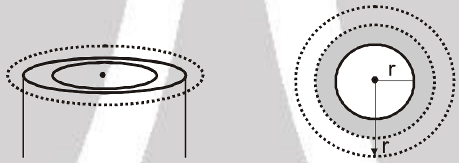
7. $$B = \frac{\frac{\mu_0 i}{\pi R^2}\times \pi\left(\frac{R}{\sqrt{2}}\right)^2}{2\pi\ \sqrt{2}\,\frac{R}{2}}\ [\cos 45°\,\hat{i}\ - \cos 45°\,\hat{k}\ ]$$





$$= \frac{\mu_0 i^2}{4\pi R}(\hat{i}\ -\ \hat{k}\ )$$ Ans. (A)

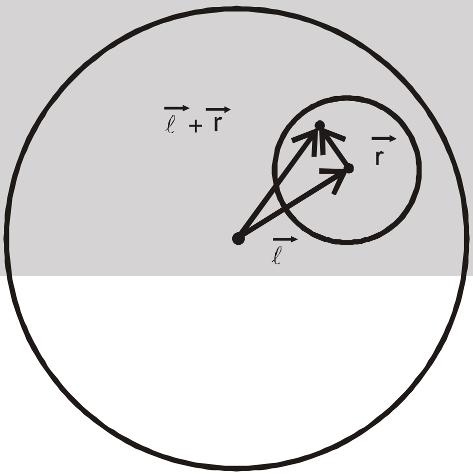
8. $$\oint \vec{B}.d\vec{\ell} = \mu_0 \frac{i}{\pi R^2}\times \pi r^2$$



$$= \frac{\mu_0 i r^2}{R^2}$$ Ans. (B)

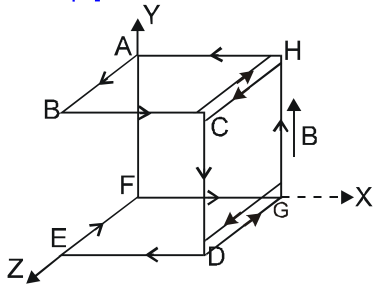
$$\oint \vec{B}.d\vec{\ell} = \mu_0 i$$

9. $$B = \frac{\mu_0}{2}[\vec{J}\times(\vec{\ell}+\vec{r})] + \frac{\mu_0}{2}[-\vec{J}\times\vec{r}]$$



$$\vec{B} = \frac{\mu_0}{2}[\vec{J}\times\vec{\ell}]$$

10. The torque of system = Torque on loop [AFGH + BCPE + ABEF]



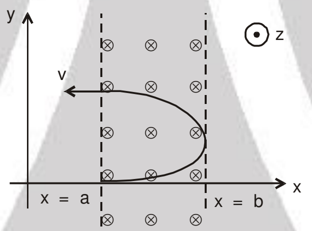
= ISB $(-\hat{i})$ + ISB$(\hat{i})$+ ISB$\hat{k}$ (I = current, S = area of loop, B = magnetic field.

= I S B$\hat{k}$

= 1 × 1 × 2$\hat{k}$ = 2$\hat{k}$ units

11. $F = q(\vec{V}\times\vec{B}) = Q\left[v\hat{i}\times\left[\frac{\mu_0 I}{4\pi R}(\hat{i}+\hat{j})\right]\right] = \frac{Qv.\mu_0 I}{4\pi R}(\hat{k})$.

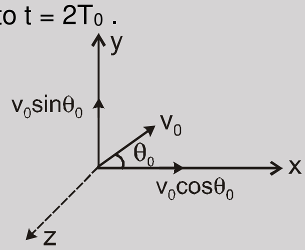
12. If (b – a) ≥ r
(r = radius of circular path of particle)
The particle can not enter the region x > b.



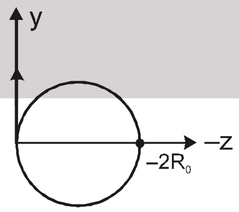
So, to enter in the region x > b, r > (b – a)

or $\frac{mv}{Bq}$ > (b – a) or $v > \frac{q(b-a)B}{m}$

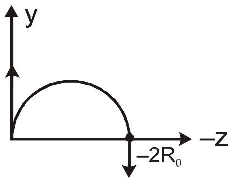
13. As the magnetic field is along the x-axis, the magnetic force will be along (–) z-axis from t = 0 to t = T_0 and along (+)ve z-axis from t = T_0 to t = $2T_0$.



For t = 0 to t = T_0 :



At t = $\frac{T_0}{2}$;



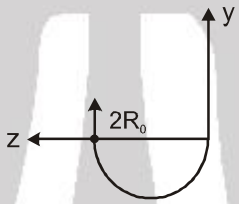
x-coordinate = $\frac{(V_0 \cos\theta)T_0}{2} = \frac{P_0}{2}$ (Since pitch = $P_0 = (V_0\cos\theta)T_0$)

y-coordinate = 0 (from figure)

and z-coordinate = $-2R_0$ (from figure)

Hence (A) is correct.

Similarly at t = $\frac{3T_0}{2}$;

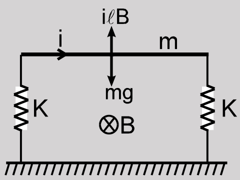


coordinates are $\left(\frac{3P_0}{2}, 0, 2R_0\right)$ Hence (B) is correct.

Note : z -coordinate will be + $2R_0$, because from t = T_0 to t = $2T_0$, direction of $\vec{B}$ changes.
As the charge will perform circular motion about x-axis, the two extremes from x-axis are $2R_0$ from each other.
Hence (C) is also correct.
Hence only (D) is incorrect.

14. The force on the rod due to magnetic field and gravity is

$i\ell B - mg$ (upwards)

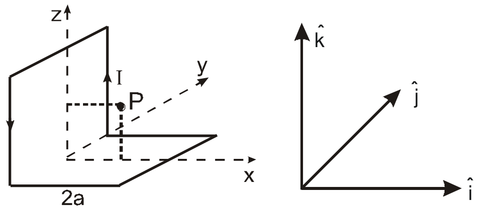


Hence the extension in the springs is $\frac{i\ell B - mg}{2k}$ (Note that effective spring constant is 2k)

Therefore the length of the spring is $\ell_0 + \frac{i\ell B - mg}{2k}$

15. The magnetic field at P(a, 0) due to the loop is equal to the vector sum of the magnetic field produced by loops ABCDA and AFEBA as shown in the figure.

Magnetic field due to loop ABCDA will be along $\hat{i}$ and due to loop AFEBA, along $\hat{k}$. Magnitude of magnetic field due to both the loops will be equal. Therefore, direction of resultant magnetic field at P will be $\frac{1}{\sqrt{2}}(\hat{i} + \hat{k})$



This is a common practice, when by assuming equal current in opposite directions in an imaginary wire (here AB), loops are completed and solution becomes easy.

16. Loss in potential energy = gain in kinetic energy
$(-MB\cos 90^0) - (-MB\cos 0^0) = KE$
$= MB = KE = \pi R^2 IB = KE.$

17. $\tau_{max} = NiAB$

$\tau_{max} = 300 \,.\, 10 \times 10^{-3} \times \dfrac{\pi 4 \times 10^{-4}}{4} \times 5 \times 10^{-2} = 4.7 \times 10^{-5}$

18. $B = \dfrac{\mu_0 Ir}{2} = \dfrac{\mu_0 Ir}{2\pi a^2}$

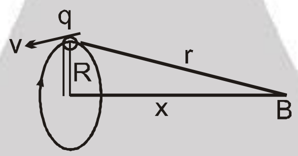
19. $\dfrac{N\mu_0 I}{2r} = B_E \Rightarrow I = \dfrac{B_E 2r}{N\mu_0}$

$I = \dfrac{7 \times 10^{-5} \times 2 \times 5 \times 10^{-2}}{10^2\left(4 \times \dfrac{22}{7} \times 10^{-7}\right)} = 5.57 \times 10^{-2} = 55.7 \times 10^{-3}A = 55.7\ mA$

20. $U = -\vec{M} \cdot \vec{B}$
U is max when $\theta = 180°$

PART - II

1. $\vec{B} = \dfrac{\mu_0}{4\pi} q\left(\dfrac{\vec{v} \times \vec{r}}{r^3}\right)$



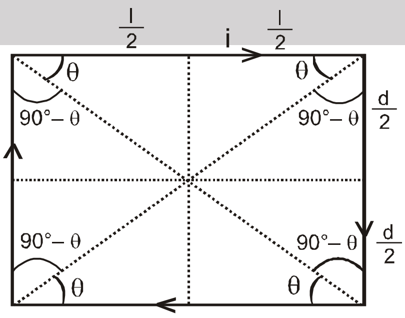
$B = \dfrac{\mu_0}{4\pi} \dfrac{qv}{(x^2 + R^2)}$

$= \dfrac{10^{-7} \times 1 \times 0.6 \times 10^4 \pi}{1}$

$= 6\pi \times 10^{-4}\ T$

2. (a) $B = \dfrac{\mu_0 i}{4\pi \dfrac{d}{2}}\ 2\ (\cos\theta + \cos\theta) + 2\dfrac{\mu_0 i}{4\pi \dfrac{l}{2}}\ (\sin\theta + \sin\theta)$

$= \dfrac{2\mu_0 i}{\pi}\left[\dfrac{\cos\theta}{d} + \dfrac{\sin\theta}{l}\right]$



$$= \frac{2\mu_0 i}{\pi}\left[\frac{l}{d\sqrt{l^2+d^2}} + \frac{d}{l\sqrt{l^2+d^2}}\right]$$

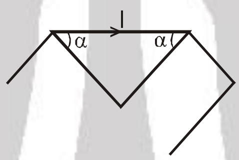
$$= \frac{2\mu_0 i}{\pi l d}\sqrt{l^2+d^2}$$

If $\ell >> d$

then $\ell^2 + d^2 \approx \ell^2$

$$B = \frac{2\mu_0 i}{\pi d}$$

3. $B = n \times \dfrac{\mu_0 i\, 2\cos\alpha}{4\pi \dfrac{l}{2}\tan\alpha} = \dfrac{n\mu_0 i \cos\alpha}{\pi l \tan\alpha}$

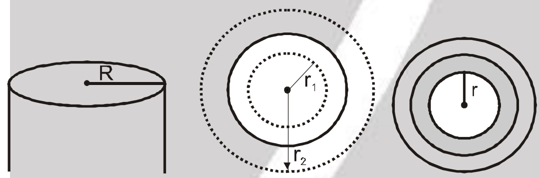


$\alpha = \dfrac{\pi}{2} - \dfrac{\pi}{n}$

$$= \frac{\mu_0 i n^2 \sin\frac{\pi}{n}\tan\frac{\pi}{n}}{\pi L}$$

Section (E)

4.



$B_1 = \dfrac{\mu_0 i_1}{2\pi r_1}$

$B_1 = \dfrac{\mu_0 b r_1^2}{3}$ $\quad\left(\because\ i_1 = \int_0^{r_1} di = \int_0^{r_1}(br)(2\pi r\,dr) = \dfrac{2\pi b r_1^3}{3}\right)$

5. $i = \dfrac{\Delta q}{\Delta t} = \dfrac{50 \times 10^6 (20-18)}{2} = 50\ \mu C/sec$

$B = \mu_0 ni = 4\pi \times 10^{-7} \times 8000 \times 50 \times 10^{-6}$

$= 16\pi \times 10^{-8}$ T

Alternate Sol.

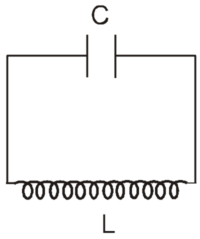
Let q_0 is intial charge on capacitor & q is charge supplied at t = t.

$i = i_0 \sin\omega t$

$\Rightarrow$ $(q_0 - q) = q_0 \cos\omega t$.

$$\frac{q_0}{C} \times \frac{90}{100} = \frac{q_0 \cos\omega t}{C}$$

$$\cos\omega t = \frac{9}{10}$$



$\cos 2\omega = \frac{9}{10}$

$$i_{av} = \frac{i_0 \int_0^{2\,sec} \sin \omega t\, dt}{2} = \frac{i_0}{2\omega}\left[\cos \omega t\right]_0^2$$

$$= \frac{i_0}{2\omega}\left[1 - \cos 2\omega\right] = \frac{i_0}{2\omega} \times \frac{1}{10} = \frac{i_0}{20\omega}\ .$$

$$B_{av} = \mu_0 n i_{av}$$

$$= 4\pi \times 10^{-7} \times 8000 \times \frac{i_0}{20\omega}$$

$$= 4\pi \times 10^{-7} \times 8000 \times \frac{q_0}{20}$$

$$= \frac{4\pi \times 10^{-7} \times 8000 \times 50 \times 10^{-6} \times 20}{20}$$

$$B_{av} = 16\pi \times 10^{-8}\ T$$

SECTION (F)

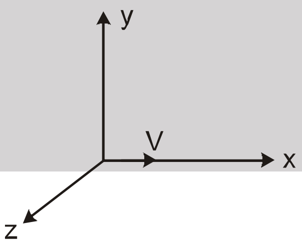
6. Pitch $= \frac{2\pi m v \cos\theta}{qB}$ $\quad$ $eV = \frac{1}{2}mv^2$

$$= \frac{2\pi\sqrt{2meV}}{eB} \qquad (\cos\theta \approx 1)$$

pitch $= \sqrt{\frac{8\pi^2 mV}{eB^2}}$

$\therefore$ Distance from point of divergence $= \sqrt{\frac{32\pi^2 mV}{eB^2}}$

7. Magnetic force is in $\hat{j}$ direction and electric field is in $-\hat{j}$ direction



Resultant force = qVB – qE

= q(1.28 × 10^6 × 8 × 10^{-2} – 102.4 × 10^3)

= 0

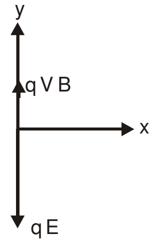
R = 100 m

Charge will move only in x direction.

x = V × t = 1.28 × 10^6 × 5 × 10^{-6} = 6.4 m

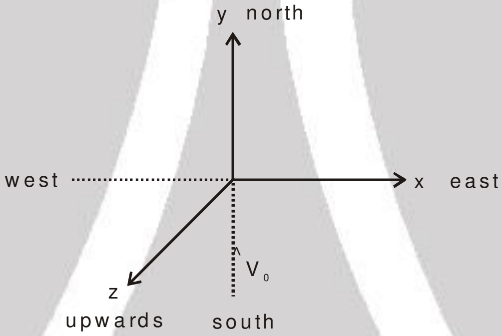
Now electric fields is switched off

8. When $V = 50$ m/s $\quad\quad qVB = qE \Rightarrow B = \frac{E}{50}$



When $V = 100$ m/s $\quad\quad qVB - qE = 2 \times 10^{-19}$
$E = 1.25$ N/C

9. $\vec{F}_E = ma_0$ towards east
$+eE = ma_0$
$E = \frac{ma_0}{e}$ towards east.



Net acceleration $= 3a_0$,
acc due to electric force $= a_0$ (along east)
acc due to megnetic force $= 2a_0$ (along east)

$$\vec{F}_B = 2ma_0\,\hat{i}$$
$$qV_0B\sin\theta = 2ma_0$$
$$B = \frac{2ma_0}{qV_0\sin\theta}$$
$$B_{min} = \frac{2ma_0}{qV_0}$$ q means +e.

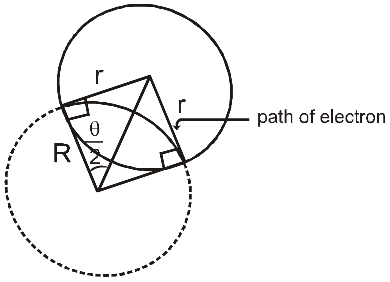
For magnetic force along west with $\theta = 90°$
Direction of magnetic field must be along $+z$. (upward)

10. $R = \frac{mV}{eB}$

$\tan\frac{\theta}{2} = \frac{r}{R}$

$\frac{\theta}{2} = \tan^{-1}\left(\frac{r}{R}\right)$

$\theta = 2\tan^{-1}\left(\frac{r}{R}\right)$



$$t = \frac{2\pi m}{eB} \times \frac{\theta}{2\pi}$$

$$= \frac{2m}{eB}\tan^{-1}\left(\frac{reB}{mV}\right)$$

Section (H)

11. At equilibrium - (1)

$$mg = \frac{\mu_0 i_1 i_2}{2\pi d} \quad .... (1)$$

For a short displacement x

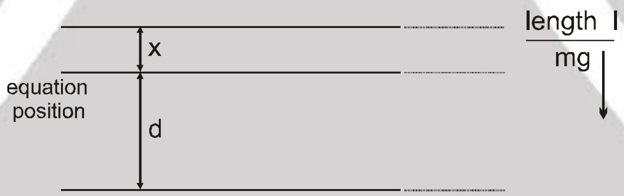
$$F_{mg} = \frac{\mu_0 i_1 i_2 l}{2\pi(d+x)} - mg$$

$$= \frac{mgd}{d+x} - mg = mg\left[\frac{d}{d+x} - 1\right]$$

$$= mg\left[\frac{d-d-x}{d+x}\right]$$

$$F_{mg} = \frac{-mgx}{d+x}$$

$$a = \frac{-g}{d}x \quad \text{For very small x.}$$



$$T = 2\pi\sqrt{\frac{d}{g}}$$

$$T = 2\pi\sqrt{\frac{.01}{g}}$$

$$= \frac{2\pi \times 0.1}{\pi}$$

$$T = 0.2 = \frac{1}{5}\text{ sec.}$$

12. Magnetic force on Rod = iBl (Leftward)
Gravitational force on Rod = mg (Downward)

$$\alpha_{Rod} = \frac{3iBL.\frac{L}{2}}{mL^2} = \frac{3iB}{2m}$$

$$a_{CM} = \frac{3iB}{2m} \times \frac{L}{2} = \frac{3iBL}{4m}$$

$$iBL - F_{horizontal} = ma_{CM}$$

$$F_{horizontal} = iBL - \frac{3iBL}{4} = \frac{iBL}{4} \text{ (right side)}$$

$$F_{Vertical} = mg \text{ (upward)}$$

13. Field due to strip $B = \frac{\mu_0 j_0}{\pi} \tan^{-1}\left(\frac{d}{2h}\right)$ towards x axis

Force on unit length dl of wire.

$$d\vec{F} = i\,[dl\,\hat{j} \times B\,\hat{i}\,]$$

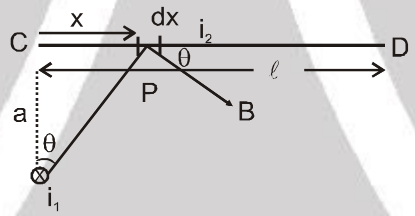
$$\Rightarrow \quad \frac{d\vec{F}}{dl} = \frac{\mu_0 j_0 i}{\pi} \tan^{-1}\left(\frac{d}{2h}\right)(-\hat{k})$$

14. Field at point P

$$B = \frac{\mu_0 i_1}{2\pi\sqrt{a^2 + x^2}}$$

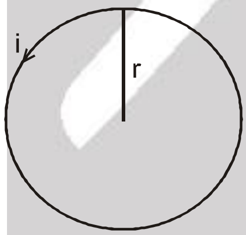
Force on differential element, $dF = \frac{\mu_0 i_1 i_2\, dx \sin\theta}{2\pi\sqrt{a^2 + x^2}}$

Net force $F = \frac{\mu_0 i_1 i_2}{4\pi} \int_0^{\ell} \frac{2x}{a^2 + x^2}\, dx$



$$= \frac{\mu_0 i_1 i_2}{4\pi} \ell n\left(\frac{a^2 + \ell^2}{a^2}\right)$$

15. $i = qf = \frac{q\omega}{2\pi}$

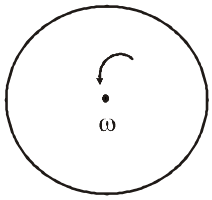


$M = \pi r^2 i$

$= \pi r^2 \frac{q\omega}{2\pi}$

$= \frac{q\omega r^2}{2}$

16. $\dfrac{M}{\dfrac{mr^2\omega}{2}} = \dfrac{q}{2m}$

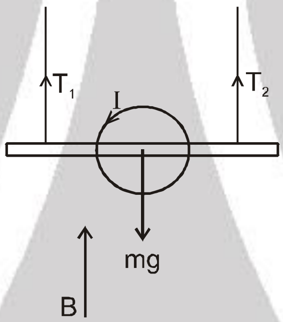


$M = \dfrac{qr^2\omega}{4}$

17. $T_0 = \dfrac{mg}{2}$

$T_1 + T_2 = mg = 2T_0$

$T_2\dfrac{\ell}{2} + \pi b^2 iB = T_1\dfrac{\ell}{2}$ (applying equation of torque about centre of mass)

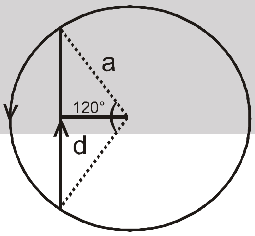


$T_1 = T_0 + \dfrac{\pi b^2 iB}{\ell}$

$T_2 = T_0 - \dfrac{\pi b^2 iB}{\ell}$

18. $\dfrac{\mu_0 i}{2a} \times \dfrac{\dfrac{2\pi}{3}}{2\pi} = B_{Arc}$ (out)

$\dfrac{\mu_0 i}{4\pi\dfrac{a}{2}}[2\cos 30°] = B_{st.}$ (in)



$B_{Net} = \dfrac{\mu_0 i\sqrt{3}}{2\pi a} - \dfrac{\mu_0 i}{6a}$

$F = qvB_{Net}$

$= \dfrac{\mu_0 i}{6a}\left(\dfrac{3\sqrt{3}}{\pi} - 1\right)qv\,\hat{i}$ **Ans.**

Area of the loop = $\frac{1}{2}a^2\left(\frac{2\pi}{3}\right) - \frac{1}{2} \times 2a\sin 60° \times 9\cos 60°$

$= \frac{\pi a^2}{3} - \frac{\sqrt{3}a^2}{4}$

$\vec{\tau} = BiA\,\hat{j} = Bia^2\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]\hat{j}$

19. $dB = \frac{\mu_0 n i}{2x}$

$= \frac{\mu_0\left[\frac{N}{b-a}dx\right]i}{2x}$

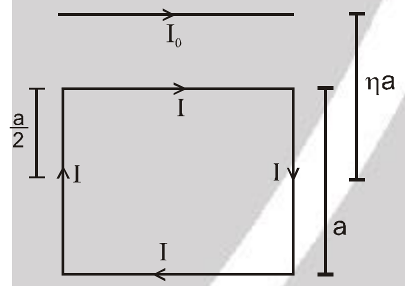
$dB = \frac{\mu_0 N i}{2(b-a)}\frac{dx}{x}$

$B = \frac{\mu_0 N i}{2(b-a)}\ln\left[\frac{b}{a}\right] = \frac{4\pi \times 10^{-7} \times 100 \times 8 \times 10^{-3}}{2\,(50 \times 10^{-3})} \times 2.303 \times 0.3010 = 7\ \mu T$

$dM = \frac{N\,dx}{b-a} \times i\pi x^2$

$M = \frac{N}{b-a}\ i\pi\frac{\left[b^3 - a^3\right]}{3} = 15\ mA\text{-}m^2$ **Ans.**

20.

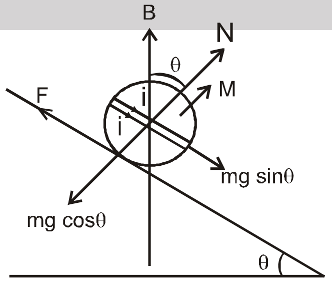


$F = \frac{\mu_0 I I_0 a}{2\pi\left[\eta a - \frac{a}{2}\right]} - \frac{\mu_0 I I_0 a}{2\pi\left[\eta a + \frac{a}{2}\right]}$

$= \frac{\mu_0 I I_0}{\pi}\left[\frac{1}{2\eta - 1} - \frac{1}{2\eta + 1}\right] = \frac{2\mu_0 I I_0}{\pi\left[4\eta^2 - 1\right]}$

21. $M = L \times 2R \times i$

$MB\sin\theta = mg\sin\theta\ R$

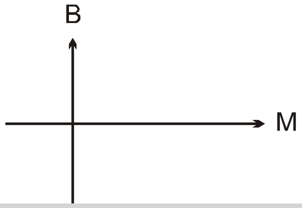


$i = \frac{mg}{2BNl} = 2.5\ A$

PART - III

1. $\vec{\tau} = \vec{M} \times \vec{B}$

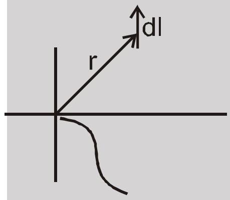
$\vec{F} = m\vec{B}$



couple of force may give torque but not force is zero.
m = pole strength depending on angle
$\tau = 0$ $\tau \neq 0$, & $F = 0$ $F \neq 0$
(ABC) **Ans.**

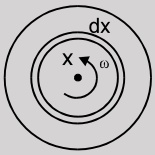
2. $B = \frac{\mu_0}{4\pi} i \left(\frac{\vec{r} \times dl}{r^3}\right)$

$B = \frac{\mu_0}{4\pi} - i\left(\frac{dl \times \vec{r}}{r^3}\right)$



3. Consider a ring of radius x and thickness dx.

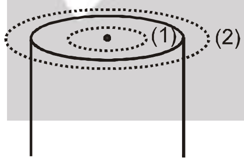
Equivalent current in this ring = $\frac{\omega}{2\pi}$ × charge on ring = $\frac{\omega}{2\pi}$ × $(2\pi x\ dx)$ $\frac{Q}{\pi R^2}$



dB (due to this ring) = $\frac{\mu_0}{2x}\left(\frac{\omega}{2\pi}\frac{2xQ}{R^2}dx\right)$ $\therefore$ $B = \int_0^R \frac{\mu_0\omega}{2\pi}\frac{Q}{R^2}$ dx.

$= \frac{\mu_0\omega\theta}{2\pi R^2} . R = \frac{\mu_0\omega\theta}{2\pi R}$.

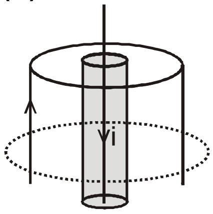
4.



Loop (1)
$B(2\pi r) = 0$
$B = 0$
Loop (2)
$B(2\pi r) = \mu_0 i$

$B \propto \frac{1}{r}$ **Ans. (B, C, D)**

5. **(A)**



$B\,(2\pi r) = \mu_0\,(i - i) = 0$
$B = 0$

6. $W_E + W_B = \Delta k$

$\Rightarrow \quad qE\,(2a) = \frac{1}{2} m(2v)^2 - \frac{1}{2} mv^2$

$= \frac{3}{2} mv^2$

$E = \frac{3}{4} \frac{mv^2}{qa}$

At P Rate of work done by E = $qEv = \frac{3}{4} \frac{mv^3}{a}$

At Q Rate of work done by E = $qE\,(2v)\cos 90° = 0$
At Q Rate of work done by B = 0

7. $\vec{V}$ constant in direction and may be in magnetude

$\vec{a} = 0$

$q\vec{E} + q(\vec{V} \times \vec{B}) = 0$

I[st] posibility

$\vec{E} = 0 \quad \& \quad \vec{B} = 0 \qquad \longrightarrow V$

II[nd] posibility

$\vec{E} = 0 \quad \& \quad \vec{V} \parallel \vec{B} \quad$ i.e. $\vec{B} \neq 0$

III[rd] possibility $\qquad \longrightarrow V$

$\longrightarrow E \quad \vec{E} \parallel \vec{V}$ & $B = 0$

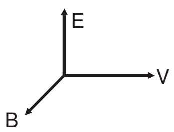
IV[th] posibility

$\longrightarrow B$

$\longrightarrow V \quad \vec{E} \parallel \vec{V} \parallel \vec{B}$

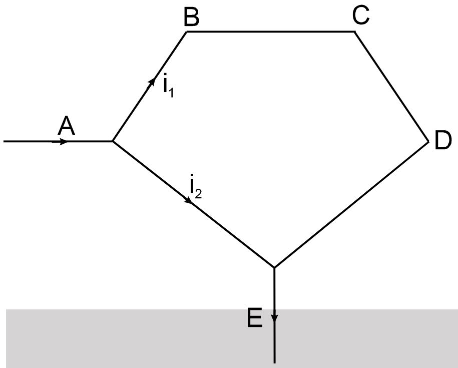
$\longrightarrow E \quad \vec{V} \times \vec{B} = 0$

V[th] posibility



$q\vec{E} = -q(\vec{V} \times \vec{B})$

8.



$B_{AB} = B_{BC} = B_{CD} = B_{DE}$ $\qquad \frac{i_1}{i_2} = \frac{1}{4}$ Because from V = IR

from above results $B_{ABCDE} = -B_{AE}$ $\qquad$ So $\quad B_{centre} = 0$

$B_{AB} = B_{BC} = B_{CD} = B_{DE\ A}$ $\qquad \frac{i_1}{i_2} = \frac{1}{4}$ V = IR

$B_{ABCDE} = -B_{AE}$ $\quad B_{centre} = 0$

9.

	H^+	He^+	O^{2+}
$\frac{q}{m}$	$\frac{1}{1}$	$\frac{1}{4}$	$\frac{2}{16}$
$\frac{q}{\sqrt{m}}$	1	$\frac{1}{2}$	$\frac{2}{4}$

$R = \frac{mv}{qB}$

$= \frac{\sqrt{2km}}{qB}$

$= \frac{\sqrt{m}}{q}\frac{\sqrt{2k}}{B}$

$R_{He^+} = R_{O^{2+}}$

$R \propto \frac{1}{\frac{q}{\sqrt{m}}}$

$R_{H^+} : R_{He^+} : R_{O^{2+}} = 1 : 2 : 2$ $\qquad$ **Ans.** (A, C)

10. $R = \frac{mv}{eB} = \frac{P}{eB}$

Energy gained = 0

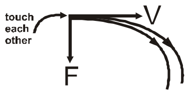
As $W_B = 0$

$F_C = \frac{mv^2}{r}$

$= evB = \frac{ePB}{m}$ $\qquad$ **Ans. (C, D)**

11. $R = \frac{mv}{qB}$

More q means less R



$\left(\frac{R_1}{R_2}\right) = \left(\frac{q_2}{q_1}\right)$ **Ans. (B, D)**

14. For given condition :

Magnitude of $B_{solienoid}$ = Magnitude of B_{loop}

$\mu_0 ni = \frac{\mu_0 I}{2R}$ here $n = \frac{\text{Total no. of turn}}{\text{Total length}} = \frac{1300}{0.65}$

$i = \frac{I}{2R} \times \frac{1}{n} = \frac{8 \times 0.65}{2 \times 0.02 \times 1300 \times 10^{-2}} = 10$ A.

For given condition :

Total magnetic field at the centre of loop

$= |B_{loop}| + |B_{solenoid}|$ $\because$ $|B_{loop}| = ||B_{solenoid}|$

$= 2|B_{loop}| = 2 \times \frac{\mu_0 I}{2R}$

$= \frac{2 \times 4\pi \times 10^{-7} \times 8}{2 \times 0.02 \times 10^{-2}} = 16\,\pi \times 10^{-3}$ T.

15. Field due to each plate $= \frac{1}{2}\mu_0 K = 2\mu T$

At A, fields add up, being in the same directions whereas at B, cancel out due to opposite directions.

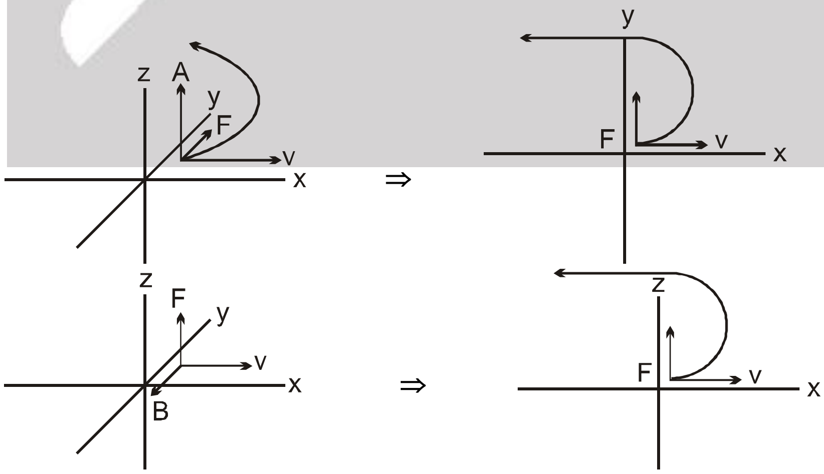
16. $F = \frac{1}{4\pi \in_0} \cdot \frac{q_1 q_2}{r^2}$

$\therefore [\varepsilon_0] = \frac{[q_1][q_2]}{[F][r^2]} = \frac{[IT]^2}{[MLT^{-2}][L^2]} = [M^{2} L^{-3} T^{4} I^{2}]$

Speed of light, $c = \frac{1}{\sqrt{\in_0 \mu_0}}$

$\therefore$ $[\mu_0] = \frac{1}{[\in_0][c]^2} = \frac{1}{[M^{-1}L^{-3}T^4I^2][LT^{-1}]^2} = [MLT^{-2}I^{-2}]$

17.



18. $T = 2\pi\sqrt{\dfrac{I}{MB}}$

$I\uparrow, T\uparrow$ | $B\uparrow, T\uparrow$ | $I = \dfrac{m\ell^2}{12}$

So correct options are (A, D)

PART - IV

1. When charge is accelerated by electric field it gains energy for first time $KE_1 = \dfrac{qV}{2}$

for second time $KE_2 = \dfrac{3}{2}qV$

for third time $KE_3 = \dfrac{5}{2}qV$

hence the ratio of radii are

$r_1 : r_2 : r_3 : ....\ : : \dfrac{\sqrt{2mqv}}{qB} : \dfrac{\sqrt{2m3qv}}{qB} : ....$

$r_1 : r_2 : r_3\ ....\ : : \sqrt{1} : \sqrt{3} : \sqrt{5}\ ....$

2. In one full cycle it gets accelerated two times so change in KE = 2 qV.

3. $f = \dfrac{qB}{2\pi m} \quad \Rightarrow 10^6 = \dfrac{10^6 B}{2\pi} \quad \Rightarrow 2\pi T.$

4. Distance travelled by particle in one time period :

$\pi(r_1 + r_2) : \pi(r_3 + r_4) : \pi(r_5 + r_6)\ ....$

$: : \dfrac{\sqrt{2mqv}}{qB} + \dfrac{\sqrt{2m3qv}}{qB} : \dfrac{\sqrt{2m5qv}}{qB} + \dfrac{\sqrt{2m7qv}}{qB} : \dfrac{\sqrt{2m9qv}}{qB} + \dfrac{\sqrt{2m11qv}}{qB}\ ....$

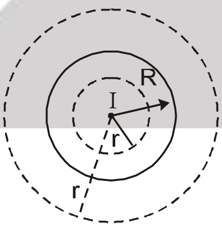
$S_1 : S_2 : S_3\ ....\ : : (\sqrt{1} + \sqrt{3}) : (\sqrt{5} + \sqrt{7}) : (\sqrt{9} + \sqrt{11})$

5. Frequency of A.C. depends on charge and mass only so it can be tuned by magnetic field only.

6. Inside the cylinder

$B.2\pi r = \mu_0 . \dfrac{I}{\pi R^2}\pi r^2$

$B = \dfrac{\mu_0 I}{2\pi R^2}.r$ ....(1)



Outside the cylinder

$B.2\pi r = \mu_0 I$

$\therefore \quad B = \dfrac{\mu_0 I}{2\pi r}$ ....(2)

Inside cylinder B α r and outside B α 1/r

So at the surface nature of magnetic field changes.

Hence clear from graph, wire 'c' has greatest radius.

7. Magnitude of magnetic field is maximum at the surface of wire 'a'.

8. Inside the wire

$B(r) = \frac{\mu_0}{2\pi} \cdot \frac{I}{R^2} \cdot r = \frac{\mu_0 J r}{2}$; $\frac{dB}{dr} = \frac{\mu_0 J}{2}$

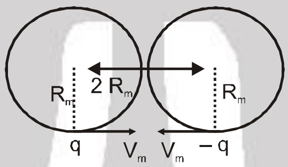
i.e. slope $\propto J$

$\propto$ current density

It can be seen that slope of curve for wire a is greater than wire C.

9. (a) $d = 2R_m$

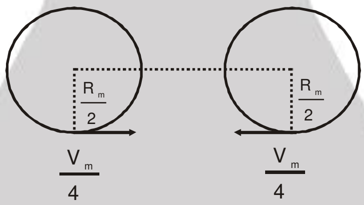
$d = 2\frac{mV_m}{qB}$



$V_m = \frac{qBd}{2m}$

10. $R = \frac{m\frac{V_m}{4}}{qB} = \frac{R_m}{4}$

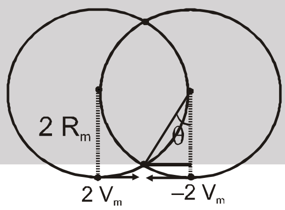
min distance $= d - \frac{d}{4} = \frac{3d}{4}$



11. max distance $= d + \frac{d}{4} = \frac{5d}{4}$

12. $4R_m \sin\theta = d$; $2d\sin\theta = d$

$\sin\theta = \frac{1}{2}, \theta = \frac{\pi}{6}$; $\omega = \frac{qB}{m}$



$t = \frac{\pi/6}{\omega} = \frac{\pi m}{6qB}$

13. After collision

net charge = 0

net mass = 2 kg.

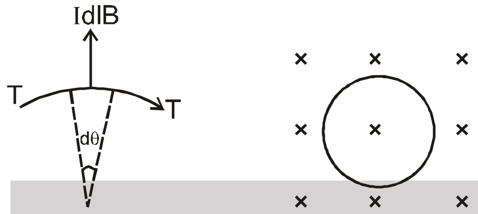
net force = 0

Motion will be along straight line with uniform velocity.

EXERCISE-3

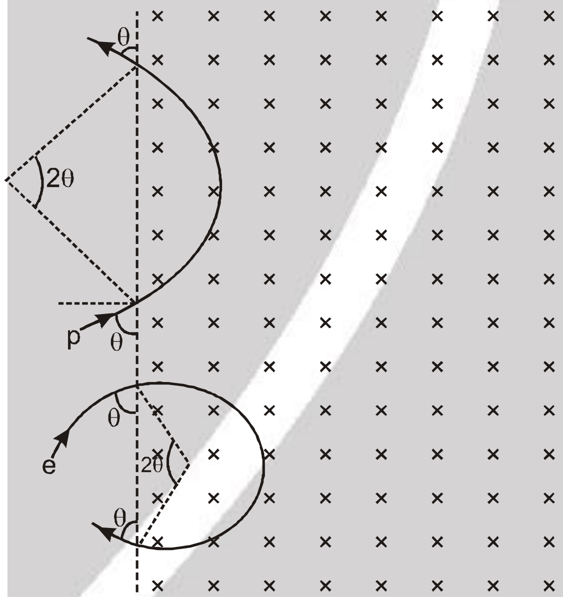
PART - I

1.



$$2T\sin\left(\frac{d\theta}{2}\right) = IdlB \quad \Rightarrow \quad 2T\frac{d\theta}{2} = IRdlB \quad \Rightarrow \quad T = BIR = \frac{BIL}{2\pi}.$$

2.



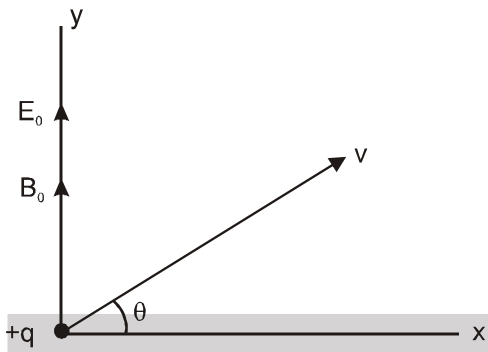
$$t_p = \frac{2\theta \times R_P}{v} = \frac{2\theta \times m_p v}{eBv} = \frac{2\theta m_p}{eB}$$

$$t_e = \frac{(2\pi - 2\theta) \times R_e}{v} = \frac{(2\pi - 2\theta)m_e v}{eBv} = \frac{(2\pi - 2\theta)m_e}{eB}$$

$\because t_e \neq t_p$

3. $$B = \int \frac{\mu_0 dNi}{2x} = \int \frac{\mu_0\left(\frac{N}{b-a}dx\right)i}{2x} = \frac{\mu_0 Ni}{2(b-a)}\ell n\frac{b}{a}$$

4.



If θ = 0° then due to magnetic force path is circular but due to force qE_0 (↑) q will have accelerated motion along y–axis. So combined path of q will be a helical path with variable pitch so (A) and (B) are wrong.

If θ = 10° then due to $v\cos\theta$, path is circular and due to qE_0 and $v\sin\theta$**,** q has accelerated motion along y-axis so combined path is a helical path with variable pitch (C) is correct.

If θ = 90° then F_B = 0 and due to qE_0 motion is accelerated along y-axis. (D)

5. $B_1 = \frac{\mu_0 J_a}{2} - \frac{\mu_0 J_a}{12}$

$= \left(\frac{\mu_0 Ja}{2}\right)\left(1 - \frac{1}{6}\right) = \frac{5}{6}\left(\frac{\mu_0 Ja}{2}\right) = \frac{5\mu_0 aJ}{12} = \frac{N}{12}\mu_0 aJ$

N = 5

6. Area = $a^2 + 4 \times \frac{\pi\left(\frac{a}{2}\right)^2}{2}$

$= a^2 + \frac{\pi a^2}{2}$

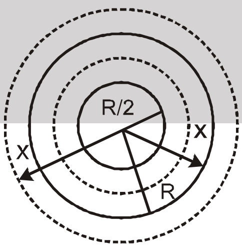
$A = \left(1 + \frac{\pi}{2}\right) a^2 I\hat{k}$

7. **Case-I** $x < \frac{R}{2}$

$|B| = 0$

Case-II $\frac{R}{2} \le x < R$

$\int \vec{B}.d\vec{\ell} = \mu_0 I$



$|B|\, 2\pi x = \mu_0 \left[\pi x^2 - \pi\left(\frac{R}{2}\right)^2\right] J$

$|B| = \frac{\mu_0 J}{2x}\left(x^2 - \frac{R^2}{4}\right)$

Case-III $x \geq R$

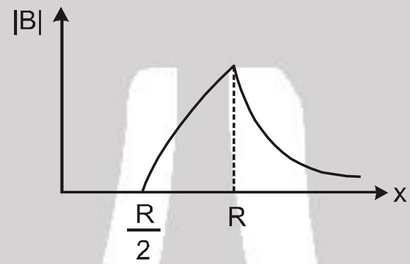
$\int \vec{B}.d\vec{\ell} = \mu_0 I$

$|B|\, 2\pi x = \mu_0 \left[\pi R^2 - \pi\left(\frac{R}{2}\right)^2\right] J$

$|B| = \frac{\mu_0 J}{2x}\,\frac{3}{2} R^2$

$|B| = \frac{3\mu_0 J R^2}{8x}$

so



8. Component of final velocity of particle is in positive y direction.
Centre of circle is present on positive y axis. so magnetic field is present in negative z-direction
Angle of deviation is 30° because

$\tan\theta = \frac{v_y}{v_x} = \frac{1}{\sqrt{3}}$

$\theta = \frac{\pi}{6}$

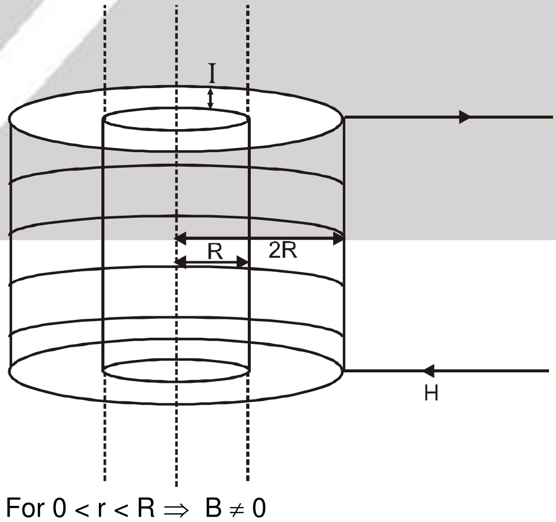
$\omega t = \theta$

$\theta = \frac{QB}{M} t$

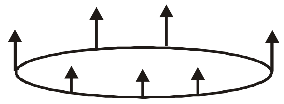
$B = \frac{M\theta}{Qt}$

$B = \left(\frac{50M\pi}{3Q}\right)$

9.



(A) For $0 < r < R \Rightarrow B \neq 0$



(D) For $r > 2R \Rightarrow B \neq 0$

10. $B_2 = \frac{\mu_0 I}{2\pi x_1} + \frac{\mu_0 I}{2\pi(x - x_1)}$ (opposite)

$B_1 = \frac{\mu_0 I}{2\pi x_1} - \frac{\mu_0 I}{2\pi(x - x_1)}$ (same)

Case-1 : When current is in the same direction

$B = B_1 = \frac{3\mu_0 I}{2\pi x_0} - \frac{3\mu_0 I}{4\pi x_0} = \frac{3\mu_0 I}{4\pi x_0}$

$R_1 = \frac{mv}{qB_1}$

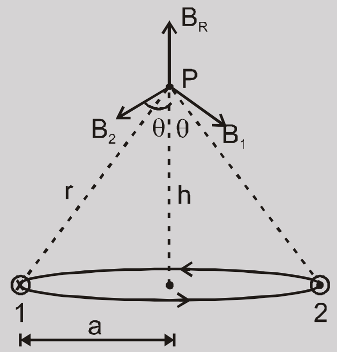
Case-2 : When current is in oposite direction

$B = B_2 = \frac{9\mu_0 I}{4\pi x_0}$

$R_2 = \frac{mv}{qB_2}$

$\frac{R_1}{R_2} = \frac{B_2}{B_1} = \frac{9}{3} = 3$

11. $\vec{B}_R = \vec{B}$ due to ring



$\vec{B}_1 = \vec{B}$ due to wire-1

$\vec{B}_2 = \vec{B}$ due to wire-2

In magnitudes $B_1 = B_2 = \frac{\mu_0 I}{2\pi r}$

Resultant of B_1 and $B_2 = 2B_1 \cos\theta = \frac{\mu_0 I a}{\pi r^2}$

$B_R = \frac{2\mu_0 I \pi a^2}{4\pi r^3}$

For zero magnetic field at P

$$\frac{\mu_0 I a}{\pi r^2} = \frac{2\mu_0 I \pi a^2}{4\pi r^3}$$

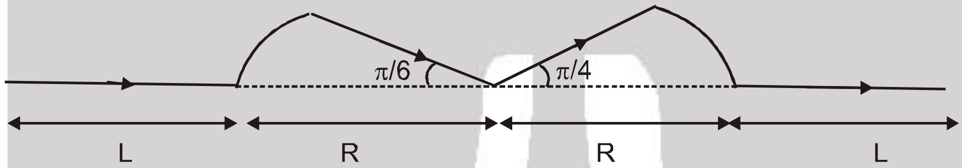
$\Rightarrow$ $h \approx 1.2a$

12. Magnetic field at mid point of two wires $= \frac{\mu_0 I}{\pi d} \otimes$

Magnetic moment of loop = $I\pi a^2$

Torque on loop = M B sin 150°

$$= \frac{\mu_0 I^2 a^2}{2d}$$



13.

$\vec{F} = i\left(\vec{\ell} \times \vec{B}\right)$ $= i\{2(L + R)\ \hat{i}\ \times \vec{B}\}$

If $\vec{B}$ is along $\hat{z}$

$\vec{F}$ = [i 2(L + R)B] $\left(-\hat{j}\right)$

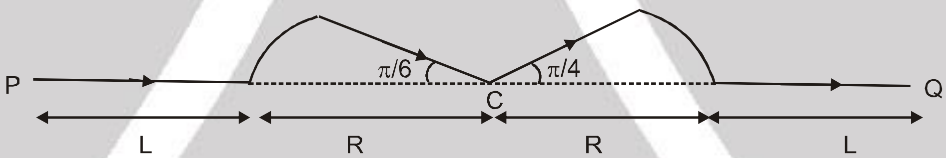
If $\vec{B}$ is along $\hat{x}$

$\vec{F} = 0$

If $\vec{B}$ is along $\hat{y}$

$\vec{F} = i\left\{2(L + R)B)\hat{k}\right\}$

Alternate solution



$d\vec{F} = i\left(d\vec{\ell} \times \vec{B}\right)$

In unfirom magnetic field

$\int d\vec{F} = \int i\left(d\vec{\ell} \times \vec{B}\right) = i\left(\int d\vec{\ell} \times \vec{B}\right)$

$\Rightarrow$ $\vec{F} = i\left(\overrightarrow{PQ} \times \vec{B}\right)$

(A) F = [i 2(L + R)B] = 2iB (L + R)
(B) F = 0
(C) F = [i 2(L + R)B] = 2iB (L + R)
(D) F = [i 2(L + R)B] = 2iB (L + R)

14. $q\,v\,B = \frac{q(V_m - V_k)}{w}$ v velocity of electrons

$V_m - V_k = w\,v\,B$.

I = neAv = ne(wd)v

$$wv = \frac{I}{ned}$$

$$V_m - V_k = \frac{I}{ned}B$$

(A) $w_1 = w_2$, $d_1 = 2d_2$ $\Rightarrow$ $V_2 = 2V_1$

(D) $w_1 = 2w_2$, $d_1 = d_2$ $\Rightarrow$ $V_1 = V_2$

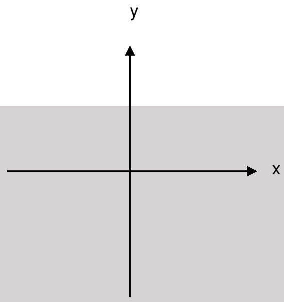
15. $V_M - V_K = \frac{IB}{ned}$

(A) $n_1 = 2n_2$; $B_1 = B_2$ $\Rightarrow$ $V_2 = 2V_1$ (correct)

(C) $B_1 = 2B_2$, $n_1 = n_2$

$V_1 = 2V_2$ $\Rightarrow$ $V_2 = 0.5$ V, correct

16.



For constant velocity

$\vec{F} = q\vec{E} + q(\vec{V} \times \vec{B}) = 0$

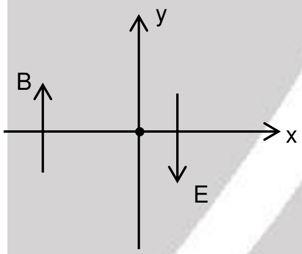
$\vec{E} = -(\vec{V} \times \vec{B})$

Ans. (C)

$-E_0\hat{x} = -\left[\frac{E_0}{B_0}\hat{y} \times B_0\hat{z}\right]$

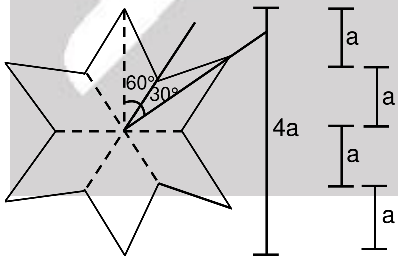
17. For helix with axis along positive z–direction magnetic field should be along z–direction.

18.



Force due to Electric field is along –y axis and force due to $\vec{B}$ is zero.

19.



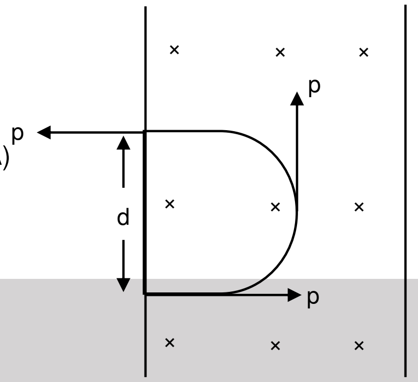
Total Magnetic Field

at centre = 12 times magnetic field due to one wire

B = $\frac{12\mu_0 I}{4\pi a}$ [sin60° – sin30°] = $\frac{\mu_0 I}{4\pi a} \times 12\left[\frac{\sqrt{3}}{2} - \frac{1}{2}\right]$

$\Rightarrow B = \frac{\mu_0 I}{4\pi a} \times 6\left(\sqrt{3} - 1\right)$

20.

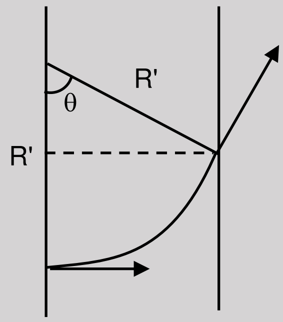
(A) 

$$\left|\Delta \vec{p}\right| = \sqrt{2}p$$

(B) $R' = \frac{mv}{QB}$

$d = 2R' = \frac{2mv}{QB}$ $\quad\quad d \propto m$

(C) $R'(1-\cos\theta) = R$



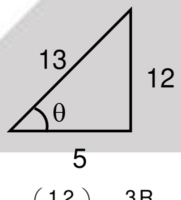
$R'\sin\theta = \frac{3R}{2}$

$$\frac{\sin\theta}{1-\cos\theta} = \frac{3}{2}$$

$$\frac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}} = \frac{3}{2}$$

$\cot\frac{\theta}{2} = \frac{3}{2} \quad \Rightarrow \quad \tan\frac{\theta}{2} = \frac{2}{3}$

$\Rightarrow \quad \tan\theta = \frac{2\left(\frac{2}{3}\right)}{1-\frac{4}{9}} = \frac{\frac{4}{3}}{\frac{5}{9}} = \frac{4}{3}\times\frac{9}{5} = \frac{12}{5}$



$\sin\theta = \frac{12}{13}$

$R'\left(\frac{12}{13}\right) = \frac{3R}{2}$; $R' = \frac{13R}{8} = \frac{P}{QB}$; $B = \frac{8P}{13QR}$

(D) $\frac{P}{QB} < \frac{3R}{2}$

$B > \frac{2P}{3QR}$

21. Magnetic field due to ring at origin

$= \frac{\mu_0 \times I \times R^2}{2 \times 8R^3}\left(-\hat{K}\right) = \frac{\mu_0 I}{16R}\left(-\hat{K}\right)$

Magnetic field at origin due to wires

$= \left(\frac{\mu_0 I_1}{2\pi R} - \frac{\mu_0 I_2}{2\pi R}\right)\hat{K}$

Here I_1 and I_2 will be substituted with sign

If $I_1 = I_2$ then $\vec{B}_0 = \frac{\mu_0 I}{16R}\left(-\hat{K}\right)$

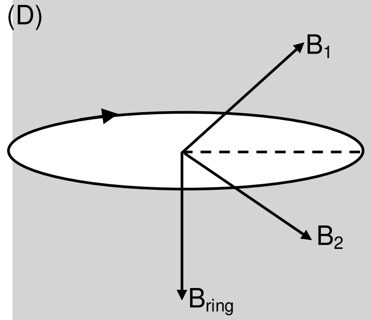
It can be zero

If $I_1 < 0, I_2 > 0$

then $\vec{B}_0 = -\left[\frac{\mu_0\left(I_1 + I_2\right)}{2\pi R} + \frac{\mu . I}{16R}\right]\hat{K}$

It can not be zero

(D)



$B_1 = B_2$

magnetic field along z-axis is only due to ring

$B = \frac{\mu_0 i}{2R}$ in –z direction

22. If average speed is considered along x-axis

$R_1 = \frac{mv_0}{qB_1},\ R_2 = \frac{mv_0}{qB_2} = \frac{mv_0}{4qB_1}$

$R_1 > R_2$

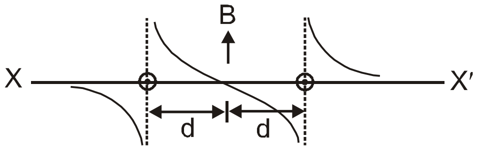
distance along x-axis $\Delta x = 2(R_1 + R_2) = \frac{5mv_0}{2qB_1}$

Total time $= \frac{\pi m}{qB_1} + \frac{\pi m}{qB_2} = \frac{\pi m}{qB_1} + \frac{\pi m}{4qB_1} = \frac{5\pi m}{4qB_1}$

Magnitude of average speed $= \frac{\frac{5mv_0}{2qB_1}}{\frac{5\pi m}{4qB_1}} = 2\ m/s$

PART - II

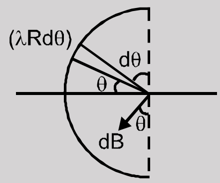
1.



Towards left of both wires direction of B is downward and at mid point between two wires, magnetic field is zero

2. $v = \frac{I}{\pi R}$

$dB = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{R}$ $\qquad\qquad$ $I = \lambda R\, d\theta$



$\therefore \quad B = \int_{-\pi/2}^{\pi/2} dB\cos\theta$

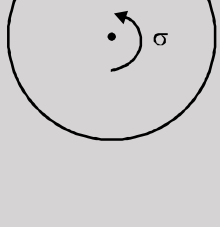
$= \frac{\mu_0\lambda}{2\pi}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta$

$= \frac{\mu_0\lambda}{\pi} = \frac{\mu_0 I}{\pi^2 R}$ **Ans.**

3. $\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$

$= F_y = 7q\,\hat{j}$.

4. $\frac{q}{2M} = \frac{\text{Magnetic dipole moment}}{\text{Angular momentum}}$



$\therefore$ Magnetic dipole moment(M)

$M = \frac{q}{2M}\cdot\left(\frac{MR^2}{2}\right)\cdot\omega$

$= \frac{1}{4}\sigma.\pi R^4\omega.$

5. $dB = \frac{\mu_0(dq)}{2r}\left(\frac{\omega}{2\pi}\right)$

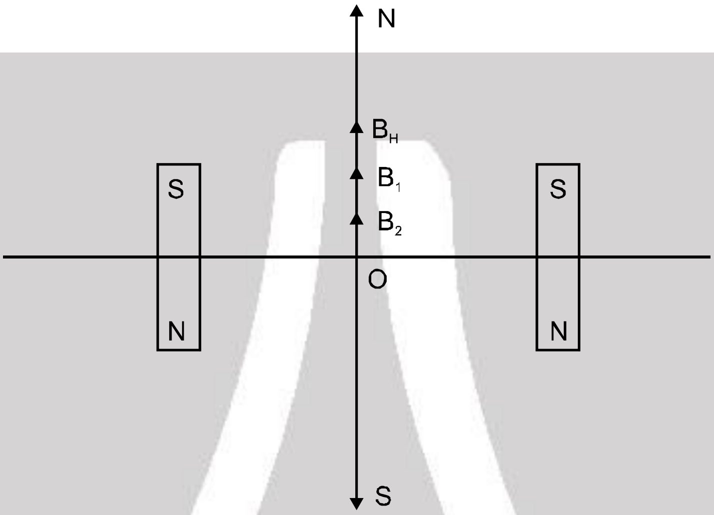
$B = \int dB = \frac{\mu_0\omega}{4\pi}\cdot\frac{Q}{\pi R^2}2\pi\int_0^R \frac{r\,dr}{r}$

$$B = \frac{\mu_0 \omega Q}{2\pi R^2}.R$$

$$B = \frac{\mu_0 \omega Q}{2\pi R}$$

$$B \propto \frac{1}{R}$$

6. $B_{net} = B_1 + B_2 + B_H$



$$B_{net} = \frac{\mu_0}{4\pi}\frac{(M_1 + M_2)}{r^3} + B_H$$

$$= \frac{10^{-7}(1.2 + 1)}{(0.1)^3} + 3.6 \times 10^{-5} = 2.56 \times 10^{-4}\ \text{wb/m}^2$$

Ans. (2)

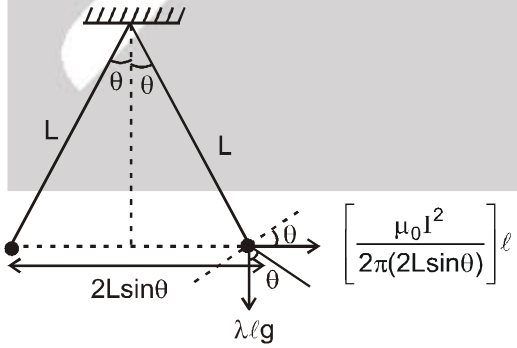
7. $F_{ext.} = B(x)\ IL$

$$P = \frac{1}{t}\int_0^2 F_{ext.} \cdot dx = \frac{1}{t}\int_0^2 B(x) IL dx = \frac{1}{5 \times 10^{-3}}\int_0^2 3 \times 10^{-4} e^{-0.2x} \times 10 \times 3 dx$$

$$= 9\ [1 - e^{-0.4}]$$

$$= 9\left[1 - \frac{1}{e^{0.4}}\right] = 2.96$$

9.

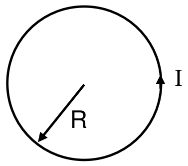


$$\lambda \ell g \sin\theta = \frac{\mu_0 I^2}{2\pi(2L \sin\theta)} \ell \cos\theta$$

$$2\sin\theta \sqrt{\frac{\lambda g \pi L}{\mu_0 \cos\theta}} = I.$$

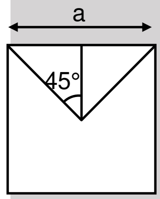
10. For stable equilibrium angle should be zero and for unstable equilibrium angle between $\vec{M}$ and $\vec{B}$ should be π.

11.



Magnetic field at centre of circle

$B_A = \frac{\mu_0 I}{2R} = \frac{\mu_0 I \pi}{\ell}$ [Also $\ell = 2\pi R$]



Magnetic field at centre = $\frac{4\mu_0 I}{4\pi \frac{a}{2}}(2 \sin 45°)$

$= \frac{16\mu_0 I}{\sqrt{2}\pi\ell}$ [Also $4a = \ell$]

Now $\frac{B_A}{B_B} = \frac{\pi^2}{8\sqrt{2}}$

12. Since area of hysterics curve of (B) is smaller it is suitable for electromagnet and transformer.

13. $M = 6.7 \times 10^{-2}$ A – m^2

$I = 7.5 \times 10^{-6}$ kgm^2

$T = 2\pi\sqrt{\frac{I}{MB}}$

$= 2\pi\sqrt{\frac{7.5 \times 10^{-6}}{6.7 \times 10^{-2} \times 10^{-2}}}$

$= 2\pi\sqrt{\frac{7.5}{6.7} \times 10^{-2}}$

$= 2\pi \times 10^{-1}\sqrt{\frac{75}{67}}$

t = 10T

$= 2\pi\sqrt{\frac{75}{67}}$ = 6.65 sec.

14. For circular path in magnetic field.

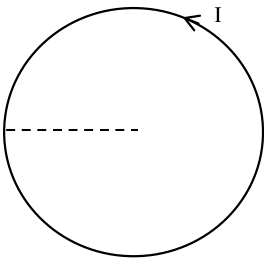
$r = \frac{\sqrt{2mE}}{qB}$ E = kinetic energy

So

	e	p	α
m	1/1836	1	4
q	–e	+e	2e

$r_p = r_\alpha > r_e$

15.



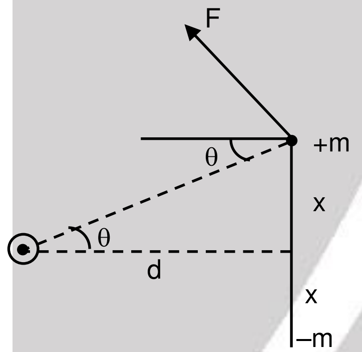
Dipole moment (m) = $I\pi R^2$

Magnetic induction (B) = $\frac{\mu_0 I}{2R}$

$B \alpha \frac{1}{\sqrt{m}}$

$\frac{B_1}{B_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{2}$

16.



Force one pole

$F = m \times \frac{\mu_0 I}{2\pi\sqrt{d^2 + x^2}}$

Total force = 2 F sinθ.

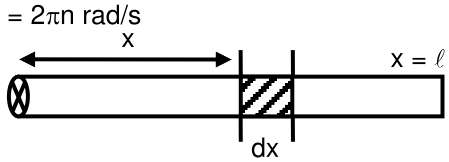
$= 2 \times \frac{\mu_0 I m}{2\pi\sqrt{d^2 + a^2}} \times \frac{x}{\sqrt{d^2 + a^2}}$

$= \frac{\mu_0 I \ mx}{\pi(d^2 + a^2)}$

m × 2 = M = I × πa^2 (magnetic moment)

Total force = $\frac{\mu_0 I \ a^2}{2(d^2 + a^2)} \Rightarrow = \frac{\mu_0 I \ a^2}{2d^2}$ (As d >>a)

17. $\omega = 2\pi n$ rad/s



$dQ = \rho.\ dx$

$= \frac{\rho_0}{\ell} x dx$

$dI = \frac{dQ\ .\omega}{2\pi}$

$dM = dI \times A$

$\int dM = \int_0^{\ell} \frac{\omega}{2\pi} . \frac{\rho_0}{\ell} x . \pi x^2 dx$

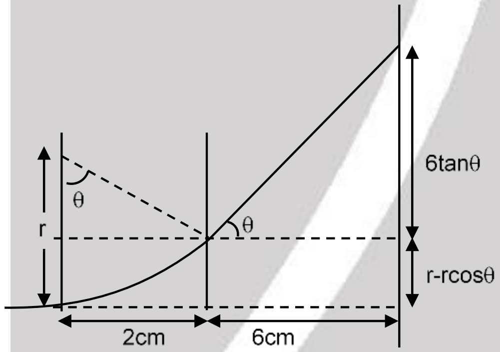
$M = \frac{\pi n \rho_0 \ell^3}{4}$

18. $I = xH$

$\frac{20 \times 10^{-6}}{10^{-6}} = x\ 60 \times 10^3$

$x = 1/3 \times 10^{-3} = 3.3 \times 10^{-4}$

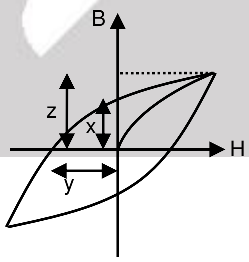
19.



$\sin\theta = \frac{x}{r}$ and $r = \frac{mv}{qB} = \frac{\sqrt{2mk}}{qB}$

so $d = r - r\cos\theta + 6\tan\theta \approx 12.87$ cm

20.

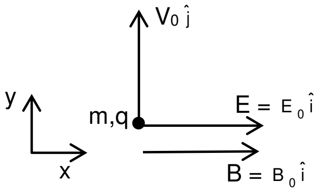


x = retentivity

y = coercivity

z = saturation magnetization

21.

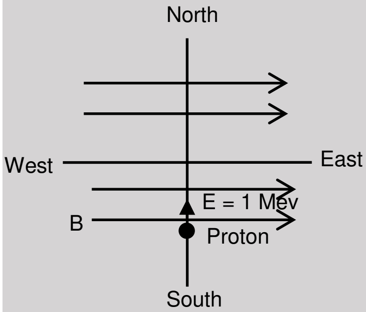


As $\vec{v} = v_0\hat{j}$ (magnitude of velocity does not change in y–z plane)

$(2v_0)^2 = v_0^2 + v_x^2$; $v_x = \sqrt{3}v_0$

$\therefore \quad \sqrt{3}v_0 = 0 + \frac{qE}{m}t$; $t = \frac{mv_0\sqrt{3}}{qE}$

22.



$\because$ K.E. $= 1.6 \times 10^{-13} = \frac{1}{2} \times 1.6 \times 10^{-27}\, v^2$

$v = \sqrt{2} \times 10^7$ m/s

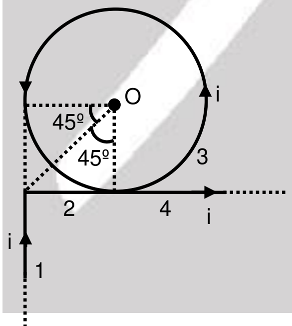
$\therefore$ Bqv = ma

$B = \frac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$

$= 0.71 \times 10^{-3}$ T

So, 0.71 mT

23.



$$\vec{B}_0 = (\vec{B}_0)_1 + (\vec{B}_0)_2 + (\vec{B}_0)_3 + (\vec{B}_0)_4$$

$$\frac{\mu_0 i}{4\pi R}[\sin 90º - \sin 45º] \otimes + \frac{\mu_0 i}{2R} \odot + \frac{\mu_0 i}{4\pi R}(\sin 45º + \sin 90º) \odot$$

$$= \frac{-\mu_0 i}{4\pi R}\left[1 - \frac{1}{\sqrt{2}}\right] + \frac{\mu_0 i}{2R} + \frac{\mu_0 i}{4\pi R}\left[\frac{1}{\sqrt{2}} + 1\right] \odot$$

$$= \frac{\mu_0 i}{4\pi R}\left[-1 + \frac{1}{\sqrt{2}} + 2\pi + \frac{1}{\sqrt{2}} + 1\right] \odot = \frac{\mu_0 i}{4\pi R}\left[\sqrt{2} + 2\pi\right] \odot = \frac{\mu_0 i}{2\pi R}\left[\frac{1}{\sqrt{2}} + \pi\right] \odot$$

24. (A) by work energy theorem

$$W_{mag} + W_{ele} = \frac{1}{2}m(2v)^2 - \frac{1}{2}m(v)^2$$

$$0 + qE_0 2a = \frac{3}{2}mv^2$$

$$E_0 = \frac{3}{4}\frac{mv^2}{qa}$$

(B) Rate of work done at A = power of electric force

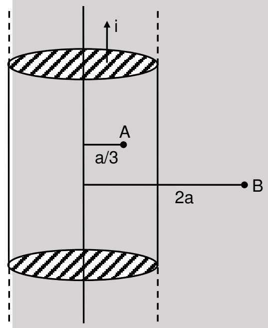
$$= qE_0V$$

$$= \frac{3}{4}\frac{mv^3}{a}$$

(C) at Q, $\frac{dw}{dt} = 0$ for both forces

(D) $\Delta\vec{L} = (-m\,2v\,2a\hat{k}) - (-mva\;\hat{k}) \Rightarrow \left|\Delta\vec{L}\right| = 3mva$

25.



$$B_A = \frac{\mu_0 ir}{2\pi a^2} = \frac{\mu_0 i\frac{a}{3}}{2\pi a^2} = \frac{\mu_0 i}{\pi a^2}\frac{a}{6} = \frac{\mu_0 i}{6\pi a}$$

$$B_B = \frac{\mu_0 i}{2\pi(2a)} \Rightarrow \frac{B_A}{B_B} = \frac{4}{6} = \frac{2}{3}$$

26. $R_{max} = \frac{R}{2} = \frac{mv_{max}}{e\mu_0 in}$

$$V_{max} = \frac{Re\;\mu_0 in}{2m}$$

27. $\tau = MB\sin\theta = I\alpha$

$$\pi R^2 I\,B\theta = \frac{mR^2}{2}\alpha$$

$$\omega = \sqrt{\frac{2\pi IB}{m}} = \frac{2\pi}{T}$$

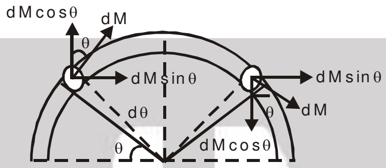
$$T = \sqrt{\frac{2\pi m}{IB}}$$

HIGH LEVEL PROBLEMS [HLP]

SUBJECTIVE QUESTIONS

1. $M = \int_{\theta=0}^{\pi} dM \sin\theta$

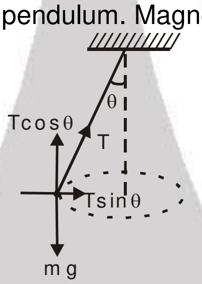
$= \int_{\theta=0}^{\pi} (dN).i.A.\sin\theta = \int_{0}^{\pi} \frac{N}{\pi}.i.\left(\frac{\pi d^2}{4}\right)\sin\theta d\theta$



$M = \frac{1}{2}N.i.d^2$

$M = \frac{1}{2}$ (Amp. m^2)

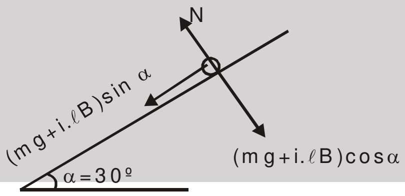
2. Charge ball move in the form of conical pendulum. Magnetic force act along radial direction.



$T\sin\theta = mr\omega^2 \pm q.(r\omega).B$
$T\cos\theta = mg$
$\tan\theta = \frac{(m\omega^2 \pm q\omega B)\,r}{mg}$

$$\frac{r}{\sqrt{r^2+L^2}} = \frac{(m\omega^2 \pm q\omega B)r}{mg} \Rightarrow \quad r = \left[L^2 - \left\{\frac{mg}{m\omega^2 \pm q\omega B}\right\}^2\right]^{1/2}.$$

3. $N = (mg + i.\ell B)\cos\alpha$
$F_{max} = (mg + i.\ell.B)\sin\alpha + \mu N$



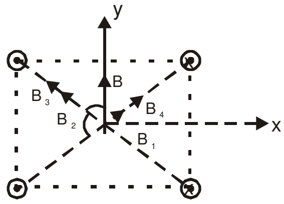
$F_{max} = \frac{3}{4}\left(1 + \frac{\sqrt{3}}{4}\right)$ N

$F_{min} = (mg + i.\ell B)\sin\alpha - \mu N$

$F_{min} = \frac{3}{4}\left(1 - \frac{\sqrt{3}}{10}\right)$N

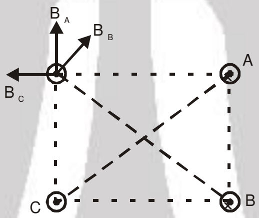
4. (a) $B = 4B_1 \cos 45^\circ$

$$= 4 \times \frac{\mu_0}{2\pi} \, , \, \frac{I}{a\sqrt{2}} \times \frac{1}{\sqrt{2}}$$



$B = \frac{\mu_0 I}{\pi a}$ along y–axis.

(b) Magnetic field at point D :



$$\vec{B}_A = \frac{\mu_0}{2\pi}\left(\frac{I}{2a}\right)(\hat{j})$$

$$\vec{B}_B = \frac{\mu_0}{2\pi}\left(\frac{I}{2\sqrt{2}\ a}\right)\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$$

$$\vec{B}_C = \frac{\mu_0}{2\pi}\left(\frac{I}{2a}\right)\ (-\hat{i})$$

Net magnetic field at D :

$$\vec{B} = \vec{B}_A + \vec{B}_B + \vec{B}_C = \frac{\mu_0 I}{4\pi a}\left(\frac{\hat{i}}{2} - \frac{3\hat{j}}{2}\right)$$

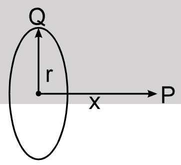
Force per meter acting on wire at point D :

$$\vec{F} = I\left[(+\hat{k}) \times \vec{B}\right] = \frac{\mu_0 I^2}{8\pi a}(-3\hat{i} - \hat{j})$$

5. Electric field at P is

$$E = \frac{Qx}{4\pi \epsilon_0 (x^2 + r^2)^{3/2}}$$

Magnetic field at P is $B = \frac{\mu_0}{4\pi}\frac{2\pi i r^2}{(x^2+r^2)^{3/2}} = \frac{\mu_0}{4\pi}\frac{2\pi Q f r^2}{(x^2+r^2)^{3/2}}$



f = frequency of revolution.

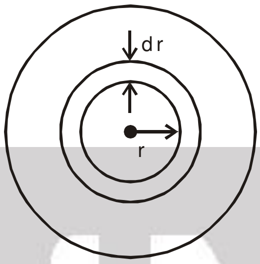
Electric energy density = $\frac{1}{2}\varepsilon_0 E^2$; Magnetic energy density $\frac{B^2}{2\mu_0}$

$$\frac{\text{Electric energy density}}{\text{magnetic energy density}} = \frac{\frac{1}{2}\varepsilon_0 E^2}{\frac{B^2}{2\mu_0}} = \frac{x^2}{4\pi^2 \epsilon_0 \mu_0 f^2 r^4} = \frac{x^2 c^2}{4\pi^2 f^2 r^4} = \frac{9}{\pi^2} \times 10^{10} = 9 \times 10^9 \text{ J}$$

6. Current in the element = $J(2\pi r \,.\, dr)$

Current enclosed by Amperian loop of radius $\frac{a}{2}$

$$I = \int_0^{a/2} \frac{J_0 r}{a} \,.\, 2\pi r \,.\, dr = \frac{2\pi J_0}{3a}\left(\frac{a}{2}\right)^3 = \frac{\pi J_0 a^2}{12}$$



Applying Ampere's law

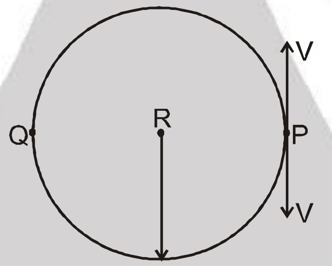
$$B \,.\, 2\pi \,.\, \frac{a}{2} = \mu_0 \,.\, \frac{\pi J_0 a^2}{12} \Rightarrow B = \frac{\mu_0 J_0 a}{12}$$

On putting values

$B = 10\ \mu T$

7. Since, total charge is zero initially thus the two particle will be of opposite charges. Initially the neutral particle is at rest, so both will have same speed. As both particle move in opposite directions, magnetic force on them will be in the same direction and of same magnitude.

Using $R = \frac{mV}{qB}$, both will be moving in the circle of same radius. So they will meet at point Q. i.e. diametrically opposite to starting point P. So time taken will be



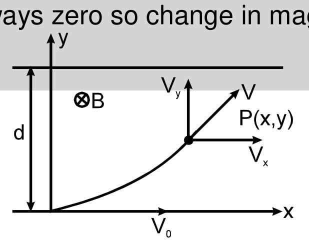
$t = \frac{\pi R}{V} = \frac{\pi m}{qB}$ .... **Ans.**

8. Let at time t particle be at point P (x, y) and its velocity be

$$\vec{V} = \left(V_x \hat{i} + V_y \hat{j}\right).$$

$$\left|\vec{V}\right| = \left|\vec{V}_0\right| \Rightarrow V_0^2 = V_x^2 + V_y^2 .$$

(work done by magnetic field is always zero so change in magnitude of velocity)



Then, magnetic force on the particle at point P is

$$.\ \vec{F} = q\left(V_x \hat{i} + V_y \hat{j}\right) B_0 \ \Rightarrow \left(1 + \frac{y}{d}\right)\left(-\hat{k}\right) \qquad -q\,B_0\left[1 + \frac{y}{d}\right] dy = m dv_x$$

Now when the particle will be coming out of the at that point y = d. Let the velocity in x-direction be V_x then integrating we get,

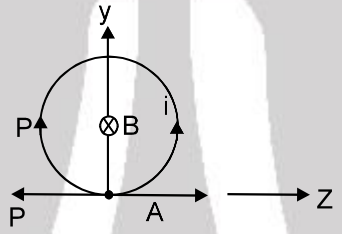
$$\int_{v_0}^{v_x} dv_x = -\frac{qB_0}{m}\int_0^d \left[1+\frac{y}{d}\right]dy = -\frac{qB_0}{m}\left[d+\frac{d^2}{2d}\right] = -\frac{3qB_0d}{2m}$$

so $\quad V_x = V_0 - \frac{3qB_0d}{2m}$ **...Ans.**

Now $\quad \Rightarrow \quad V_y = \sqrt{V_0^2 - V_x^2}$

$\Rightarrow \quad V_y = \sqrt{V_0^2 - \left(V_0 - \frac{3qB_0}{2m}d\right)^2}$ **...Ans.**

9. A and P will have the same momentum in magnitude and they will move in opposite directions. They will move in the circle of same radius and the same centre but in opposite directions. If they meet after time t then



$\omega_A t + \omega_P t = 2\pi$

$\Rightarrow \quad t = \frac{2\pi}{\omega_A + \omega_P} = \frac{2\pi}{\frac{2e.B}{4m} + \frac{2eB}{(A-4)m}}$

$t = \frac{4(A-4)m\pi}{eBA}$; $\theta_A = \omega_A t = \frac{2eB}{4m} \times \frac{4m(A-4)}{eBA}$

$= \frac{2(A-4)\pi}{A} = \frac{48}{25}\pi \Rightarrow n = 48$

10. As the particle enters the region of magnetic field, it moves in a circular path of radius

$R = \frac{mv}{qB} = 1$ m whose centre is at O. and $\omega = \frac{qB}{m} = 1$ rad/sec.

we assume d to be sufficiently large so that the particle emerges out of region of magnetic field at Q figure - (a).

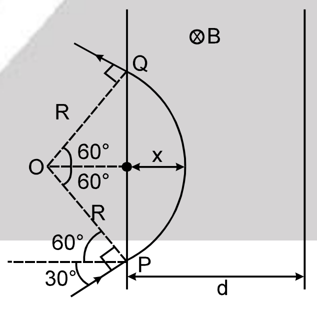


figure - a

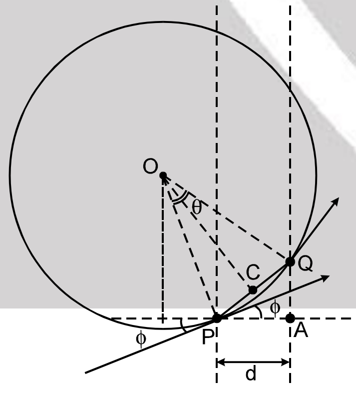


figure - b

$\therefore \quad x = R - R\cos 60° = 0.5 > d$

$\therefore \quad$ The charge will cross the field and emerge from the right side.

$\therefore \quad$ The trajectory of the particle in the region of magnetic field is as shown in figure - b

In the figure (b) PQ is the chord and OC is $\perp$ bisector of line PQ. Q is the point from where the particle emerges out. We can see from the geometry that $\angle APQ = \phi + \frac{\theta}{2}$

$\therefore$ $PQ = d \sec\left(\phi + \frac{\theta}{2}\right) = 2R \sin\frac{\theta}{2}$

$\Rightarrow$ $d = 2R \sin\frac{\theta}{2} \cos\left(\phi + \frac{\theta}{2}\right) = R\left[\sin\left(\frac{\theta}{2} + \phi + \frac{\theta}{2}\right) + \sin(-\phi)\right]$

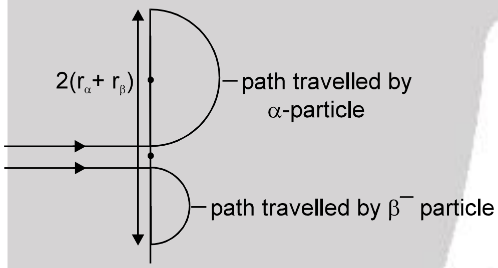
$\Rightarrow$ $d = R[\sin(\theta + \phi) - \sin\phi]$ $\Rightarrow$ $\sin(\theta + \phi) = \frac{d}{R} + \sin\theta = 0.7$

$\Rightarrow$ $\theta + \phi = 45°$ $\Rightarrow$ $\theta = 15°$

$\therefore$ Now $\omega t = \theta$

$\Rightarrow$ $1 \times t = \frac{15 \times \pi}{180}$ $\Rightarrow$ $t = \frac{\pi}{12}$ sec. **Ans.**

11.



$\frac{mv^2}{r} = qVB$ $\Rightarrow$ $r = \frac{mv}{qB} = \frac{\sqrt{2mT}}{qB}$

$\therefore$ Separation between α & β particle

$= 2(r_\alpha + r_\beta) = 2\left\{\frac{\sqrt{2m_\alpha T}}{(2e)B} + \frac{\sqrt{2m_e T}}{eB}\right\} = \frac{2\sqrt{2T}}{eB}\left\{\frac{\sqrt{m_\alpha}}{2} + \sqrt{m_e}\right\}$

12. $\vec{F} = q\vec{v} \times \vec{B}$, Let $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$\vec{F}_1 = -e(1\hat{i}) \times \{B_x\hat{i} + B_y\hat{j} + B_z\hat{k}\} = -e\hat{j}$

$\Rightarrow$ $eB_y\hat{k} - eB_z\hat{j} = e\hat{j}$

$\Rightarrow$ $B_y = 0$; $B_z = -1$

$\vec{F}_2 = e(\hat{i} - \hat{j}) \times \{B_x\hat{i} + 1\hat{k}\} = -e(\hat{i} + \hat{j})$

$\Rightarrow$ $e(-\hat{j} + B_x\hat{k} - \hat{i}) = -e(\hat{i} + \hat{j})$

$\Rightarrow$ $B_x = 0$

$\Rightarrow$ $\vec{B} = -1\hat{k} = -\hat{k}$ wb/m^2

Now, $\vec{v}_3 = \vec{v}_1 \times \vec{v}_2 = 1\hat{i} \times (\hat{i} - \hat{j}) = -\hat{k}$

Now, $\vec{F} = e\vec{v}_3 \times \vec{B} = e(-\hat{k} \times \hat{k}) = 0$

13. $B_{sol} = \mu_0 \frac{N}{L} i$ where N is total no. of turns & L is length of the solenoid.

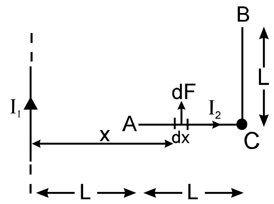
$T = \frac{2\pi m}{qB}$ and pitch = $V_{||} T$

No. of revolution = $\frac{L}{pitch} = \frac{L.qB}{V_{||}.2\pi m}$

using $B = \mu_0 \frac{N}{L} . i \Rightarrow \frac{\mu_0 .Ni}{V_{||}.2\pi} . \frac{q}{m}$

using values $\Rightarrow$ $\frac{4\pi \times 10^{-7} \times 8000 \times 4 \times \sqrt{3} \times 10^{11}}{400 \cdot \sqrt{3} \times 2\pi} = 16 \times 10^5$

14.



Torque on element dx of current carrying wire AC about C is

$d\tau = (dF).\ (2L - x)$ in clockwise sense.

$dF = (I_2\, dx)\ \dfrac{\mu_0}{2\pi}\ \dfrac{I_1}{x}$

$\therefore$ Net torque on wire AC about C is

$$\tau_1 = \int_L^{2L} \frac{\mu_0}{2\pi}\ \frac{I_1 I_2 (2L - x)}{x}\ dx = \frac{\mu_0}{2\pi}\ I_1 I_2\, L\ (\ell n\, 4 - 1) = \frac{0.4 \mu_0 I_1 I_2 L}{2\pi} \text{ (clockwise direction)}$$

Magnetic field at each point on segment BC due to infinite wire is uniform.

$\therefore$ Net torque or wire BC about C is

$$\tau_2 = \left(\frac{\mu_0}{2\pi}\ \frac{I_1}{2L}\right) I_2\, L \times \frac{L}{2} = \frac{\mu_0}{8\pi}\ I_1\, I_2\, L \text{ (anticlockwise direction)}$$

$\because$ $\tau_1 > \tau_2 \Rightarrow$ net torque $\tau = \tau_1 - \tau_2 = \dfrac{\mu_0}{8\pi}\ (0.6)\ I_1 I_2\, L$ (clockwise direction)

moment of inertia of L shaped rod about C is $I = \dfrac{mL^2}{3}$

angular acceleration $\alpha = \dfrac{\tau}{I} = \dfrac{0.6 \mu_0 I_1 I_2 L}{8\pi} \times \dfrac{3}{mL^2} = \dfrac{9 \mu_0 I_1 I_2}{40 \pi m L}$

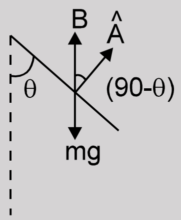
15. Applying Energy conservation, initially, kinetic energy = 0
gravitational P.E. = 0 (say) & Magnetic P.E. = μB

where, μ = magnetic moment of the loop = i. $\left(\dfrac{\sqrt{3}\, a^2}{4}\right)$

Finally when the loop becomes horizontal, Kinetic energy = 0

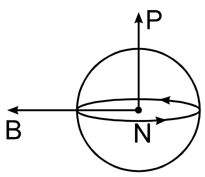
gravitational P.E.= $\left(\dfrac{a}{\sqrt{3}}\right)$ mg(because mg acts on the centre of mass)

magnetic P.E. = 0



$\Rightarrow \quad 0 + 0 + \mu B = 0 + \dfrac{mga}{\sqrt{3}} + 0 \Rightarrow \quad B = \dfrac{mga}{\sqrt{3}\ \mu} = \dfrac{4mg}{3ia}$

16.



Torque on the (coil + sphere) due to flow of charge through coil is

$\left|\vec{p} \times \vec{B}\right| =$ (where $\vec{p}$ is the dipole moment of the coil and $\vec{B}$ is the geomagnetic field)

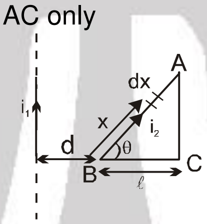
$= i\,N\,\pi\,r^2\;\;B = I\dfrac{d\omega}{dt}$

$\therefore \qquad d\omega = \dfrac{N\pi r^2 B}{I}\,i\,dt$ or $\qquad \omega = \dfrac{N\pi r^2 B}{\frac{2}{3}m r^2}\displaystyle\int_0^{\Delta t} i\,dt \;=\; \dfrac{3N\pi BQ}{2M}$ **Ans.**

[Ans: $\omega = \dfrac{3}{2}\dfrac{BN\pi Q}{M}$ = 2.7 π × 10^{-2} rad/s.]

17. $(2\,\mu_0 - \mu_0) + (-\mu_0 + 2\,\mu_0) + (-2\,\mu_0 + 4\,\mu_0) + (4\,\mu_0 - 2\,\mu_0) = \mu_0 I \;\Rightarrow\; I = 6\,A$

18. Force of interaction will act an AB and AC only



$$F_{AB} = \int_0^{\ell/\cos\theta} \frac{\mu_0 i_1 i_2 \sin\theta}{2\pi(d + x\cos\theta)}\,dx$$

$= \dfrac{\mu_0 i_1 i_2 \tan\theta}{2\pi}\,\ell n\,\dfrac{d+\ell}{d}$ towards left

$F_{AC} = \dfrac{\mu_0 i_1 i_2 \tan\theta}{2\pi}\,\dfrac{\ell}{d+\ell}$ towards right

$F_{net} = F_{AB} - F_{AC}$

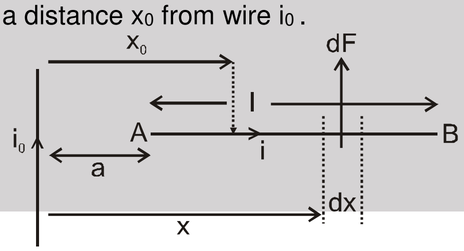
$F_{net} = \dfrac{\mu_0 i_1 i_2 \tan\theta}{2\pi}\left[\ell n\,\dfrac{d+\ell}{d} - \dfrac{\ell}{d+\ell}\right]$ towards left.

19. Force on differential length dF $= i\,.\,\dfrac{\mu_0 i_0}{2\pi}\,dx$

Net force F $= \dfrac{\mu_0 i_0 i}{2\pi}\displaystyle\int_a^{\ell+a}\dfrac{dx}{x}$

$= \dfrac{\mu_0 i_0 i}{2\pi}\,\ell n\left(\dfrac{\ell + a}{a}\right)$

Torque about point of application = 0
Let point of application is at a distance x_0 from wire i_0.



$d\tau = \dfrac{\mu_0 i_0 i\,(x - x_0)}{2\pi\;x}\,dx$

$\tau = \dfrac{\mu_0 i_0 i}{2\pi}\displaystyle\int_a^{\ell+a}\left(1 - \dfrac{x_0}{x}\right)dx$

$\Rightarrow \qquad \left[x - x_0\,\ell n\,x\right]_a^{\ell+a} = 0$

$\Rightarrow \qquad \ell + a - x_0\,\ell n\,(\ell + a) - a + x_0\,\ell n\,a = 0$

$$x_0\, \ell n \left(\frac{\ell + a}{a}\right) = \ell$$

$$x_0 = \frac{\ell}{\ell n\left(1+\frac{\ell}{a}\right)}$$

20. $\vec{v} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$

$\vec{v}_x = \frac{qE}{m}t$, $v_y = v_0 \cos\left(\frac{qB}{m}t\right)$

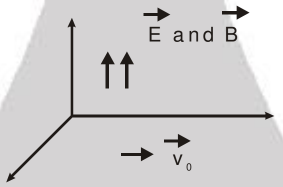
$v_z = -v_0 \sin\left(\frac{qB}{m}t\right)$

$i = \frac{\vec{E}}{|E|} = \frac{\vec{B}}{|B|}$

$\hat{j} = \frac{\vec{v}_0}{|\vec{v}_0|}$, $\hat{k} = \frac{-\vec{v}_0 \times \vec{B}}{|\vec{v}_0 \times B|} = \frac{\vec{v}_0 \times \vec{E}}{|\vec{v}_0 \times E|}$

Sol. (II) $\hat{j} = \frac{\vec{E}}{E}$ or $\frac{\vec{B}}{B}$: $\hat{i} = \frac{\vec{v}_0}{v_0}$

$\hat{k} = \frac{\vec{v}_0\vec{B}}{v_0B}$



Force due to electric field will be along y-axis. Magnetic force will not affect the motion of charged particle in the direction of electric field (or y-axis) So

$a_y = \frac{F_e}{m} = \frac{qE}{m}$ = constant. therefore, $v_y = a_y t = \frac{qE}{m} . t$ ......(1)

The charged particle under the action of magnetic field describes a circle in x-z plane (perpendicular to $\vec{B}$) with

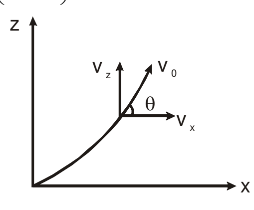
$T = \frac{2\pi m}{Bq}$ or $\omega = \frac{2\pi}{T} = \frac{qE}{m}$

Initially (t = 0) velocity was along x-axis. Therefore, magnetic force ($\vec{F}$ m) will be along positive z-axis [$\vec{F}$ m $= q(\vec{v}_0 \times \vec{B})$]. Let it makes an angle θ with x-axis at time t, then

$\theta = \omega t$

$\therefore$ $v_x = v_0 \cos \omega t = v_0 \cos\left(\frac{qB}{m}t\right)$ and ....(2)

$v_z = v_0\left(\frac{qB}{m}t\right) \sin \omega t = v_0 \sin\left(\frac{qB}{m}t\right)$ .....(3)



From (1), (2) and (3)

$\therefore \qquad \vec{v} = v_x + v_y \hat{j} + v_z \hat{k}$

$\therefore \qquad \vec{v} = v_0 \cos\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0}{v_0}\right) + \frac{qB}{m}t\left(\frac{\vec{E}}{E}\right) + v_0 \sin\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0 \times \vec{B}}{v_0 B}\right)$

or $\qquad \vec{v} = \cos\left(\frac{qB}{m}t\right)(\vec{v}_0) + \left(\frac{q}{m}t\right)(\vec{E}) + \sin\left(\frac{qB}{m}t\right)\left(\frac{\vec{v}_0 \times \vec{B}}{B}\right)$ **Ans.**

$\Rightarrow$ The path of the particle will be a halix of increasing pitch. The axis of the helix will be along y-axis.

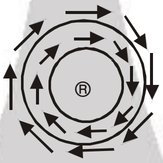
21. **(a)** Given i = 10A, r_1 = 0.08 m and r_2 = 0.12m Straight portions i.e., CD etc. will produce zero magnetic field at the centre.Rest eight arcs will produce the magnetic field at the centre in the same direction i.e.,perpendicular to the paper putwards to vertically upwards and its magnitude is

$$B = B_{\text{inner arcs}} + B_{\text{outer arcs}}$$

$$= \frac{1}{2}\left\{\frac{\mu_0 i}{2r_1}\right\} + \frac{1}{2}\left\{\frac{\mu_0 i}{2r_2}\right\}$$

$$= \left\{\frac{\mu_0 i}{4\pi}\right\}(\pi\, i)\left(\frac{r_1 + r_2}{r_1 r_2}\right)$$

Substituting the values, we have



$$B = \frac{(10^{-7})(3.14)(10)(0.08 + 0.12)}{(0.08 \times 0.12)} \text{ Tesla}$$

B = 6.54 × 10^{-5} T (Vertically upward or outward normal to the paper) **Ans.**

Force on AC

(b) Force on circular portions of the circuit i.e.AC etc.due to the wire at the centre will be zero because magnetic field due to the central wire at these arcs will be tangential ($\theta = 180^0$) as shown.

Force on CD

Current in central wire is also i = 10 A.

Magnetic field at P due to central wire,

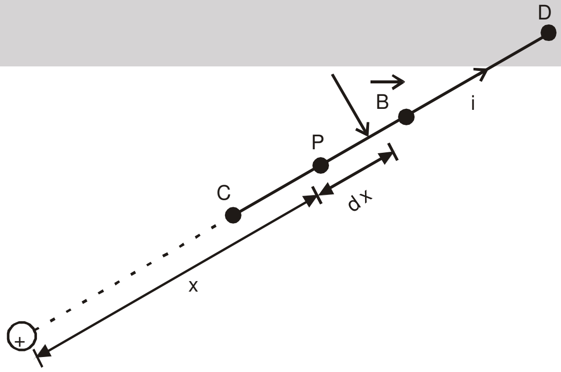
$$B = \frac{\mu_0}{2\pi} \cdot \frac{i}{x}$$

$\therefore$ Magnetic force on element dx due to this magnetic field

$$dF = (i)\left(\frac{\mu_0}{2\pi} \cdot \frac{i}{x}\right).dx = \left(\frac{\mu_0}{2\pi}\right) i^2 \frac{dx}{x} \qquad (F = i/B \sin 90^0)$$

Therefore,net force on CD is

$$F = \int_{x=r_1}^{x=r_2} dF = \frac{\mu_0 i^2}{2\pi} - \int_{0.08}^{0.12} \frac{dx}{x} = \frac{\mu_0}{2\pi} i^2 \ln\left(\frac{3}{2}\right)$$



Substituting the values, F = (2×10^{-7}) $(10)^2$ ln (1.5) or $F = 8.1 \times 10^{-6}$ N (inwards)

Force on wire at the centre

Net magnetic field at the centre due to the circuit is in vertical direction and current in the wire in centre is also in vertical direction.Therefore, net force on the wire at the centre will be zero.($\theta = 180^0$). Hence

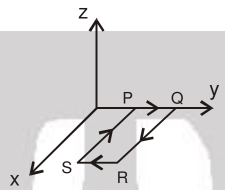
(i) Force acting on the wire at the centre is zero.

(ii) Force on arc AC = 0. **Ans.**

(iii) Force on segment CD is 8.1×10^{-6} N (inwards).

22. (a) and (c)

Let the direction of current in wire PQ is from P to Q and its magnitude be I.



The magnetic moment of the given loop is :

$$\vec{M} = -\,Iab\hat{k}$$

Torque on the loop due to magnetic forces is :

$$\vec{\tau}_1 = \vec{M} \times \vec{B} = (-\,Iab\hat{k}) \times \{(3\hat{i} + 4\hat{k})B_0\} = -\,3IabB_0\hat{j}$$

Torque of weight of the loop about axis PQ is :

$$\vec{\tau}_2 = \vec{r} \times \vec{F} = \left(\frac{a}{2}\hat{i}\right) \times (-mg\hat{k}) = \frac{mga}{2}\hat{j}$$

We see that when the current in the wire PQ is from P to Q, $\vec{\tau}_1$ and $\vec{\tau}_2$ are in opposite direction so they can cancel each other and the loop may remain in equilibrium. So the direction of current I in the wire PQ is from P to Q.Further for equilibrium of the loop :

$$|\vec{\tau}_1| = |\vec{\tau}_2|$$

or $3IabB_0 = \dfrac{mga}{2}$

$\mathbf{I} = \dfrac{mg}{6bB_0}$ **Ans.**

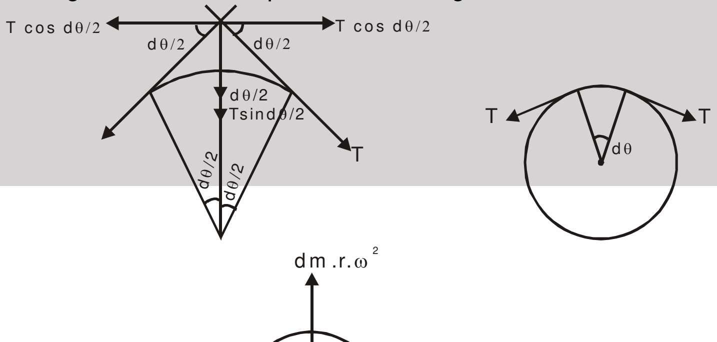
(b) Magnetic force on wire RS is :

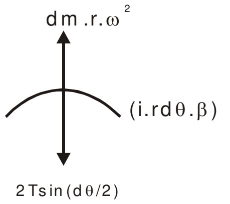
$$\vec{F} = I(\vec{\ell} \times \vec{B})$$

$$= I\,[(-b\hat{j}) \times \{(3\hat{i} + 4\hat{k})B_0\}]$$

or $\vec{F} = \mathbf{IbB_0\,(3\hat{k} - 4\hat{i})}$ **Ans.**

23. Net ampere force acting on a closed loop in uniform magnetic field is zero.





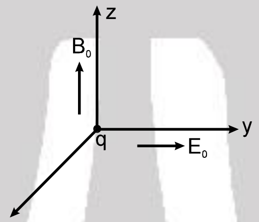
$2T\sin = \dfrac{d\theta}{2}\ (dm.r.\omega^2) + r.d\theta.iB$

$$T.d\theta = \frac{m}{2\pi r}.rd\theta.r.\omega^2 + r.d\theta.i.B$$

$$T = \frac{r}{2\pi}\ (m\omega_0^2 + 2\pi i.B)\ .$$

24. $$m\frac{dv}{dt} = qE_0\hat{j} + q\left[v_x\hat{i} + v_y\hat{j}\right] \times B_0\hat{k}$$

$$m\frac{dv_y}{dt}\hat{j} + m\frac{dv_x}{dt}\hat{i} = [qE_0 - qv_xB_0]\ \hat{j} + qv_yB_0\hat{i}$$



$$m\ \frac{dv_y}{dt} = [qE_0 - qv_xB_0] \qquad .....(1)$$

$$m\ \frac{dv_x}{dt} = qv_yB_0 \qquad .....(2)$$

From (1) $v_x = \left[qE_0 - m\frac{dv_y}{dt}\right]\frac{1}{qB_0}$

From (2) $\frac{m}{qB_0}\frac{d}{dt}\left[qE_0 - m\frac{dv_y}{dt}\right] = qv_yB_0$

$$-\frac{d^2v_y}{dt^2} = \frac{q^2v_yB_0^2}{m^2} \qquad \text{or} \qquad \frac{d^2v_y}{dt^2} + \frac{q^2v_yB_0^2}{m^2} = 0$$

Solution of above equation :

$v_y = A\sin(\omega t + \phi)$ .....(3)

where $\omega = \frac{qB_0}{m}$ at $t = 0$, $v_y = 0$, $\phi = 0$ $v_y = A\sin\omega t$

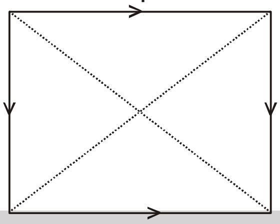
at $t = 0$, $a = \frac{qE_0}{m}$ $a = \frac{dv_y}{dt} = A\omega\cos\omega t$ $\frac{qE_0}{m} = A \times \frac{qB_0}{m} \Rightarrow A = \frac{E_0}{B_0}$

This equation (3) $v_y = \frac{E_0}{B_0}\sin\omega t\ \frac{dy}{dt} = \frac{E_0}{B_0}\ \sin\omega t \Rightarrow y = \left[-\frac{E_0}{B_0}\cos\omega t\right]_0^t$

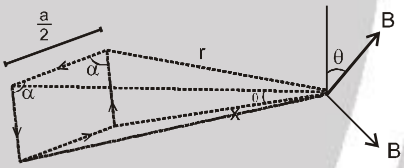
$$y = \frac{E_0m}{B_0 \times qB_0}\ [1 - \cos\omega t] \ \Rightarrow y = \frac{E_0m}{qB_0^2}\left[1 - \cos\frac{qB_0}{m}t\right]$$

25. (a) $B_R = 4\ B\sin\theta$

$= 4 \times \frac{\mu_0 i}{4\pi r \sin\alpha}\ [\cos\alpha + \cos\alpha] \times \sin\theta$



$$= \frac{2\mu_0 i}{\pi\left[x^2 + \frac{a^2}{4}\right]^{\frac{1}{2}}} \times \frac{\frac{a}{2}}{\left[x^2 + \frac{a^2}{2}\right]^{\frac{1}{2}}} \times \frac{\frac{a}{2}}{\left[x^2 + \frac{a^2}{4}\right]^{\frac{1}{2}}} = \frac{4\mu_0 a^2 i}{\pi\left[4x^2 + a^2\right]\left[4x^2 + 2a^2\right]^{\frac{1}{2}}}$$



(b) $x = 0$

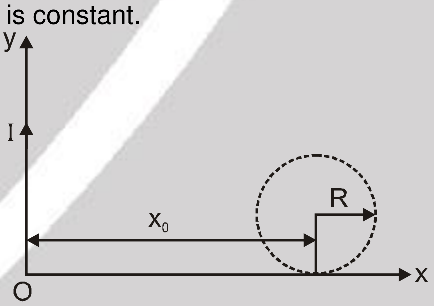
$B_R = \frac{4\mu_0 a^2 i}{\pi (a^2)(\sqrt{2}\,a)}$

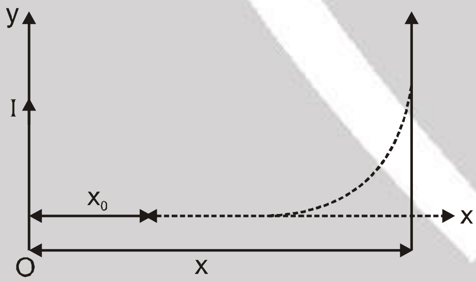
$= 2\sqrt{2}\,\frac{\mu_0 i}{\pi a}$

(c) $x >> a$

$B = \frac{4\mu_0 a^2 i}{\pi\ 4x^2(2 - x)} = \frac{\mu_0 a^2 i}{2\pi x^3} = \frac{\mu_0 M}{2\pi x^3}$

26. Since the magnetic field is not uniform, the particle doesn't follows circular path but the speed (v) of the particle is constant.





Here the magnetic field set-up by the straight current is along the negative z-axis, the initial velocity of the particle is along x-axis and the force F is in the x-y plane.
The force at time t after starting from point P is
F = q(v × B)

or $F = q\left[(v_x\hat{i} + v_y\hat{j}) \times \left(\frac{\mu_0 I}{2\pi x}(-k)\right)\right]$

$= \frac{\mu_0 qI}{2\pi x}(-v_y\hat{i} + v_x\hat{j})$

So, $F_x = \frac{-\mu_0}{2\pi}\frac{qIv_y}{x}$ $\therefore$ $a_x = \frac{-\mu_0 qIv_y}{2\pi m x}$

or $\dfrac{v_x dv_x}{dx} = \dfrac{\mu_0 q I v_y}{2\pi m x}$ ......(i)

But $v_x^2 + v_y^2 = v^2$

$\therefore$ $2v_x dv_x + 2v_y dv_y = 0$

or $v_x dv_x = -v_y dv_y$ ......(ii)

From Eqs. (i) and (ii)

$$\frac{2\pi m}{\mu_0 q I} = dv_y = \frac{dx}{x}$$

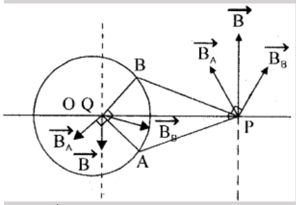
or $\dfrac{2\pi m}{\mu_0 q I}\displaystyle\int_0^v dv_y = \int_{x_0}^{x}\frac{dx}{x}$ or $\dfrac{2\pi m}{\mu_0 q I} v = \ell n \dfrac{x}{x_0}$

$\therefore$ $X = x_0 e^{2\pi m v/\mu_0 q I}$

27. $\dfrac{M}{\frac{2}{5} m r^2 \omega} = \dfrac{q}{2m}$

$M = \dfrac{1}{5} q\omega r^2$

28. (a) Consider a thin shell which is equivalent to a set of parallel long wires put around a circle. Consider two wires A and B which are equi-distant from OQP.



Let $\vec{B}_A$ be the magnetic field due to wire A and $\vec{B}_B$ be the magnetic field due to wire B. According to the Biot Savart law, both magnetic fields are equal in magnitude and their projections on the line OQP are also equal, but in opposite directions by geometry.

The resultant magnetic field

$$\vec{B} = \vec{B}_A + \vec{B}_B$$

is always perpendicular to the line OQP and therefore always tangential to the circle through the point of observation.

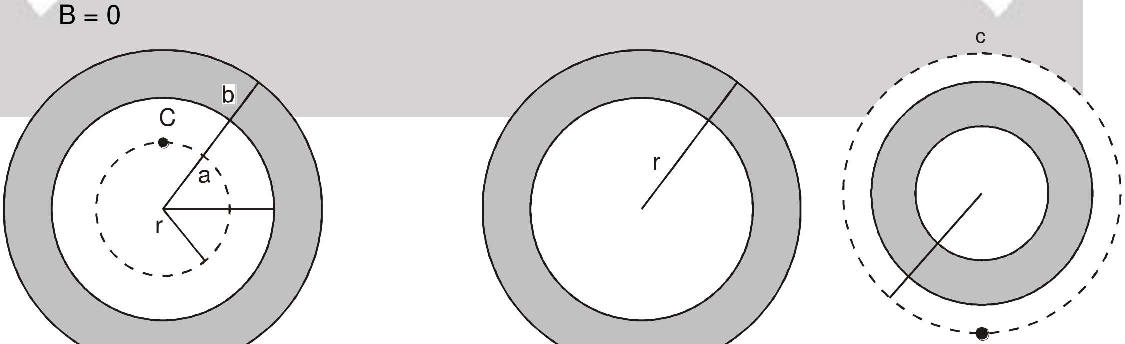
(b)

(i) $r \le a$: Using Ampere's law

$$\oint_c \vec{B}.d\vec{l} = \mu_0 i$$

$$B 2\pi r = 0$$

$B = 0$



(i) (ii) (iii)

(ii) $a \le r \le b$

$$B2\pi r = \mu_0 I \frac{\pi(r^2 - a^2)}{\pi(b^2 - a^2)}$$

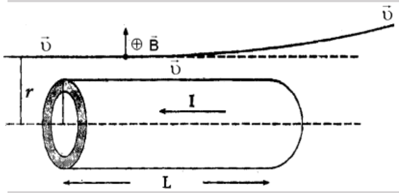
$$B = \frac{\mu_0 I(r^2 - a^2)}{2\pi r(b^2 - a^2)}$$

(iii) $r \geq b$:

$$B2\pi r = \mu_0 I$$

$$B = \frac{\mu_0 I}{2\pi r}$$

(c)



$$\vec{F} = q(\vec{\upsilon} \times \vec{B}) \;;\; F_r = qvB = qv\frac{\mu_0 I}{2\pi r}$$

Impulse :

$$\int F_r dt = \frac{q\upsilon\mu_0 I}{2\pi}\int \frac{dt}{r} = \frac{q\upsilon\mu_0 I}{2\pi}\int \frac{dx}{r\upsilon} = \frac{q\mu_0 I}{2\pi}\int \frac{dx}{r} = \frac{q\mu_0 I}{2\pi}\frac{L}{r}$$

Change of momentum along radial direction :

$$P_r = \int F_r dt = \frac{q\mu_0 IL}{2\pi r}$$

Deflection :

$$\theta \approx \frac{P_r}{P} = \frac{q\mu_0 IL}{2\pi r m\upsilon} = \frac{\mu_0 IqL}{2\pi\, m\upsilon}\cdot\frac{1}{r}$$