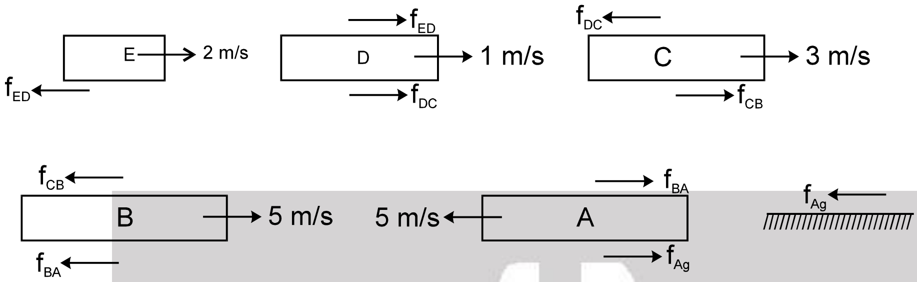
SOLUTIONS OF FRICTION

EXERCISE-1

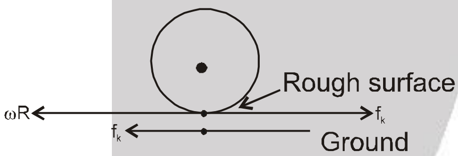
PART-I

A-1. Force along and opposite to the direction of motion is friction.

A-2.



A-3.



Kinetic friction is involved.

A-4. Direction of friction is such that it opposes the relative velocity between the contact surface.

A-5. Friction is kinetic because there is relative motion. Direction of friction is such that it opposes the relative velocity.

A-6. $a = -\mu mg/m = -\mu g = -1\ m/s^2$

$V_f^2 - V_i^2 = 2as, \qquad \therefore (V_f = 0, V_i = 5\ m/s)$

$\Rightarrow \qquad s = \dfrac{25}{2\times 1} = 12.5m.$

B-1. action-reaction force between M and vertical wall

$N = 0$ for $F\mu \le (M+m)g$

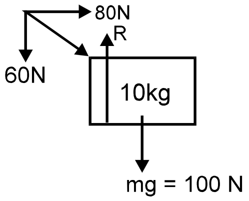
$N = F - \mu(M+m)g$ for $F > \mu(M+m)g$

Action-reaction force between m and M

$N = F - \mu mg$ for $F > \mu mg$

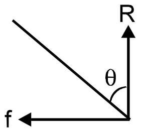
and $N = 0$ for $F < \mu mg$

B-2.



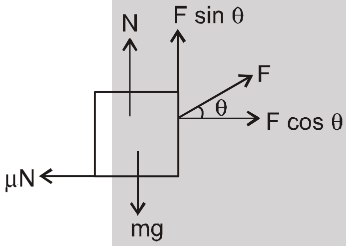
R = mg + 60 = 160 N

f = 80 N ($\because$ No sliding)



Angle of friction $\theta = \tan^{-1}\dfrac{f}{R} = \tan^{-1}\dfrac{80}{160} \Rightarrow \theta = \tan^{-1}\dfrac{1}{2}$

B-3.



$N = mg - F\sin\theta$

$F\cos\theta = \mu N = \mu\,[mg - F\sin\theta]$

$F = \dfrac{\mu\ mg}{\cos\theta + \mu\sin\theta}$

F is minimum when $\cos\theta + \mu\sin\theta$ is max

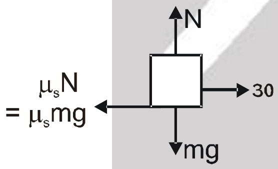
$\Rightarrow \dfrac{d}{d\theta}(\cos\theta + \mu\sin\theta) = 0 \quad \Rightarrow -\sin\theta + \mu\cos\theta = 0 \qquad \Rightarrow \mu = \tan\theta$

or $\quad \theta = \tan^{-1}\mu$

also $\cos\theta + \mu\sin\theta = \sqrt{1+\mu^2}$ for $\theta = \tan^{-1}\mu$

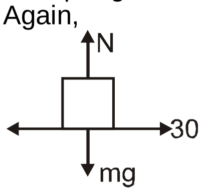
thus $F_{min} = \dfrac{\mu mg}{\sqrt{1+\mu^2}}$

C-1.



$30 = \mu_s mg \quad \Rightarrow \quad 30 = \mu_s \times 5 \times 10 \quad \Rightarrow \quad \mu_s = 0.6.$

Again,

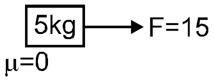


$\mu_k N$

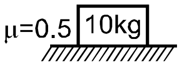
$\mu_k mg$

$S = \dfrac{1}{2}at^2$

$\Rightarrow \quad a = \dfrac{2S}{t^2} = \dfrac{2\times10}{25} = 0.8. \quad ; \qquad 30 - \mu_k mg = m \times 0.8 \quad \Rightarrow \quad \mu_k = \dfrac{30 - m\times0.8}{mg} = 0.52.$

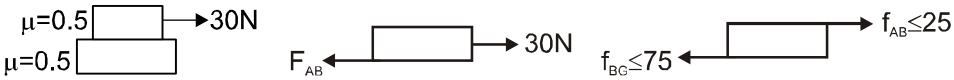
C-2.(i) 

$a_A = \frac{F}{m} = \frac{15}{5} = 3$



$a_B = \frac{0}{10} = 0$

$f_{AB} = 0$, $f_{BG} = 0$.

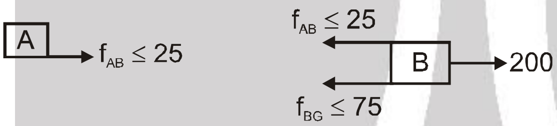
(ii) 

$f_{BG} \leq 75$

Since f_{AB} can't be greater than f_{BG} therefore acceleration of B will be zero.

and $\quad a_A = \frac{30-25}{5} = 1\text{m/sec}^2$

$f_{AB} = 25$ N, $f_{BG} = 25$ N.

(iii) 

$f_{AB} \leq 25 \quad \Rightarrow \quad a_A \leq \frac{25}{5} \quad \text{or} \quad a_A \leq 5$

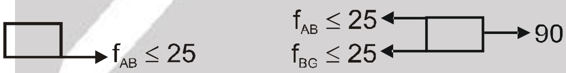
Let there is no sliding between A and B then common acceleration of A and B.

$= \frac{200-75}{15} = 8.33$

Since $a_A \leq 5 \quad \Rightarrow$ Hence, there will be sliding between A and B in that case.

$a_A = 5\text{ m/sec}^2$, $a_B = \frac{200-100}{10} = 10\text{ m/sec}^2$

$f_{AB} = 25$ N, $f_{BG} = 75$ N.

(iv) 

$a_A \leq 5$

Let A and B move together then common acceleration.

$= \frac{90-75}{15} = 1\text{m/sec}^2$

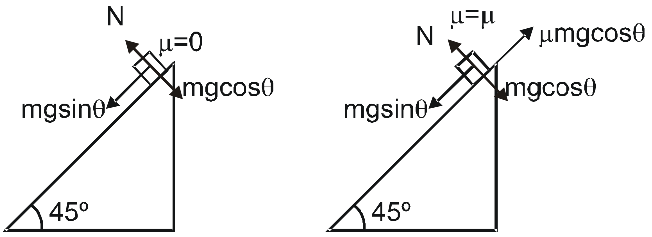
As common acceleration is less than a_A hence A and B will move together

$\therefore \; a_A = 1\text{m/sec}^2$, $a_B = 1\text{m/sec}^2$

$f_{AB} = m_A \times 1 = 5\text{N}$, $\qquad f_{BG} = 75$ N.

PART-II

A-1.



Let acceleration in I[st] case is a_1 and that in second case is a_2

Now , $\frac{1}{2}a_1t^2 = \frac{1}{2}a_2(2t)^2 \quad \Rightarrow \quad a_2 = \frac{a_1}{4}$ ....(i)

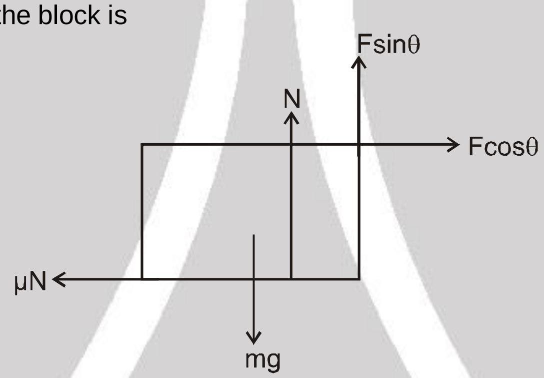
Clearly $a_1 = \frac{mg\sin\theta}{m} = g\sin\theta$ ....(ii)

and $a_2 = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = g\sin\theta - \mu g\cos\theta$ ....(iii)

From (i), (ii) and (iii),

we get $\mu = 0.75$.

A-2. The normal reaction on the block is



$N = mg - F\sin\theta$

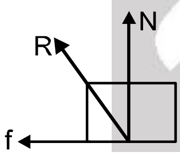
$\therefore$ Net force on block is

$F\cos\theta - \mu N = F\cos\theta - \mu mg + \mu F\sin\theta$

or acceleration of the block is

$$a = \frac{F(\cos\theta + \mu\sin\theta) - \mu mg}{m} = \frac{F}{m}(\cos\theta + \mu\sin\theta) - \mu g$$

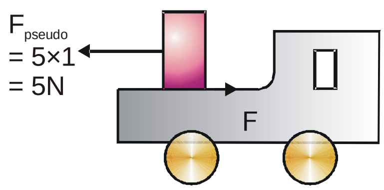
B-1. μ does not depend on normal reaction.



B-2. Acceleration of train will be from right to left.

$\Rightarrow$ Pseudo force will act on the box from left to right therefore friction will act from right to left.

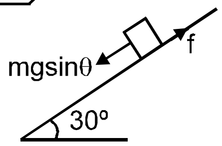
B-3. Solving from the frame of truck



(w.r.t. truck)

$f \le \mu mg = 6 \quad \Rightarrow \quad f = 5N.$

B-4.



since $\mu > \tan\theta$
the block will not slide therefore $f = mg \sin\theta$

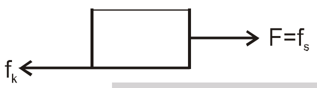
$= 2 \times 9.8 \times \frac{1}{2} = 9.8$ N.

B-5. Apply Newton's law for system along the string

$m_B g = \mu(m_A + m_C) \times g$

$\Rightarrow \quad m_C = \frac{m_B}{\mu} - m_A = \frac{5}{0.2} - 10 = 15$ kg

C-1.



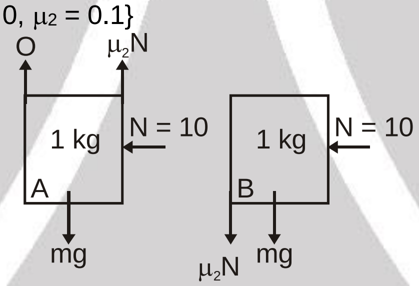
$a = \frac{f_s - f_k}{m} = \frac{(\mu_s - \mu_k)mg}{m} = (\mu_S - \mu_k)$ g

$= (0.5 - 0.4)10 = 1$ m/sec^2

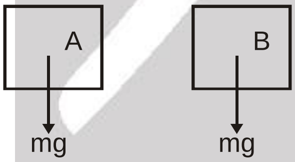
C-2. When F is less than $\mu_s$mg then tension in the string is zero.
When $\mu_s mg \le F < \mu_s 2mg$ then friction on block B is static.
If F is further increase friction on block B is kinetic.

PART – III

1. (i) FBD in (case (i)) $\{\mu_1 = 0, \mu_2 = 0.1\}$

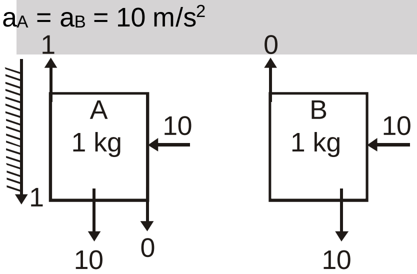


While friction's work is to oppose the relative motion and here if friction comes then relative motion will start and without friction there is no relative motion so both the block move together with same acceleration and friction will not come.



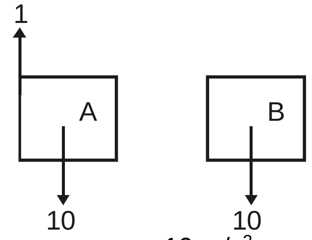
$\Rightarrow \quad a_A = a_B = 10$ m/s^2

(ii)



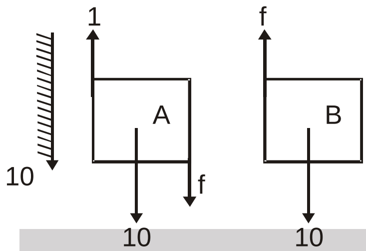
Friction between wall and the block opposes relative motion. Since wall is stationary so friction will try to stop block A also and maximum friction will act between wall and block while there is no friction between block.

Note : Friction between wall and block will oppose relative motion between wall and block only.

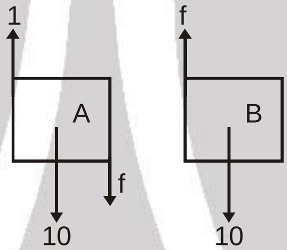


$a_A = 9\ m/s^2$; $a_B = 10\ m/s^2$

(iii)

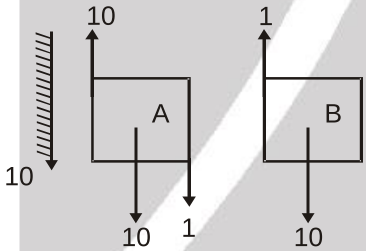


Maximum friction between wall and block will be 1N, but maximum friction available between block A and B is 10 N but if this will be there then relative motion will increase while friction is to oppose relative motion. So friction will come less than 10N so friction will be static.



by system $(20 - 1) = 2 \times a$ $\Rightarrow$ $a = \frac{19}{2} = 9.5\ m/s^2$

(iv)



$a_A = \frac{11-10}{1} = 1\ m/s^2$

$a_B = \frac{10-1}{1} = 9\ m/s^2$

2. The acceleration of two block system for all cases is $a = 2\ m/s^2$

In option (p) the net force on 2 kg block is frictional force

$\therefore$ Frictional force on 2 kg block is $f = 2 \times 2 = 4N$ towards right

In option (q) the net force on 4 kg block is frictional force

$\therefore$ Frictional force on 4 kg block is $f = 4 \times 2 = 8N$ towards right

In option (r) the net force on 2 kg block is $2 \times 2 = 4N$

$\therefore$ Friction force f on 2 kg block is towards left.

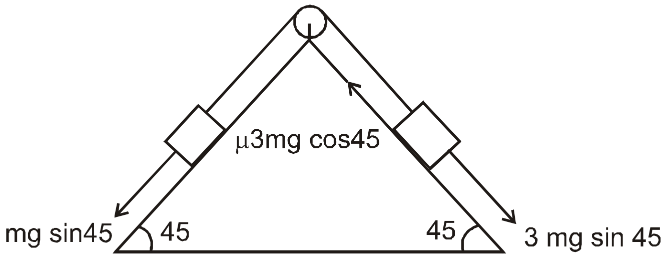
$\therefore$ $6 - f = 2 \times 2$ or $f = 2N$

In option (s) the net force on 2 kg block is $ma = 2 \times 2 = 4N$ towards right.

$\therefore$ Friction force on 2 kg block is 12N towards right.

EXERCISE-2
PART-I

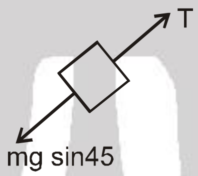
1.



Applying Newton's law for the system of m and 3m along the length of the string, we get
3mg sin45 – μ3mg cos 45 – mg sin45 = (3m + m)a

$\Rightarrow \quad \mu = \frac{2}{5}$ as $\quad a = \frac{g}{5\sqrt{2}}$

now making the F.B.D. of m we get

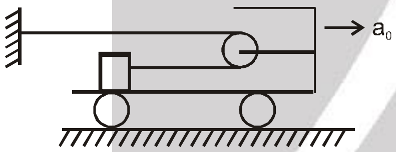


T – mg sin 45 = m a

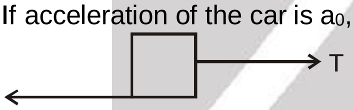
$\Rightarrow \quad T = \frac{mg}{5\sqrt{2}} + \frac{mg}{\sqrt{2}}$

$\quad T = \frac{6\ mg}{5\sqrt{2}}$

2.

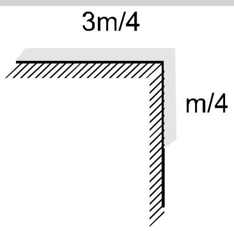


If acceleration of the car is a_0, acceleration of the block $2a_0 = 2 \times 2 = 4$ m/s^2 (→)



F = μN = 0.3 × 50 × 10 = 150
T – F = ma
$\Rightarrow$ T – 150 = 50 × 4
$\Rightarrow$ T = 350 N.

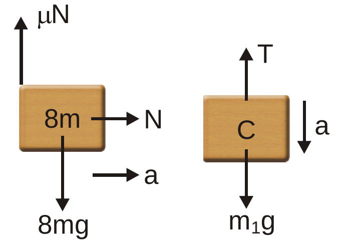
3. Apply system equation



$\frac{m}{4} g = \frac{3m}{4} g \times \mu$

$\Rightarrow \quad \mu = 1/3 = 0.33$

4. FBD of A



If the acceleration of 'C' is a

For block 'A' $\quad N = 8\,ma$ ....(1)

$8\text{ mg} - \mu N = 0$ ....(2)

and acceleration a can be written by the equation of system (A + B + C)

$m_1 g = (10\,m + m_1)\,a$ ....(3)

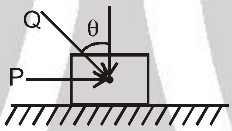
$$8\text{ mg} = \mu 8m\left(\frac{m_1 g}{10m + m_1}\right)$$

$$10\,m + m_1 = \mu\, m_1$$

$$10\,m = (\mu - 1)\, m_1 \qquad \Rightarrow m_1 = \frac{10m}{\mu - 1} \quad \textbf{Ans.}$$

5. $N = mg + Q\cos\theta$

$P + Q\sin\theta \le \mu N$



$$\Rightarrow \qquad \mu \ge \frac{P + Q\sin\theta}{N}, \qquad \mu \ge \frac{P + Q\sin\theta}{mg + Q\cos\theta}$$

6. For the sliding not to occur when

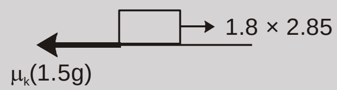
$\tan\theta \le \mu$

$$\tan\theta = \frac{dy}{dx} = \frac{2x}{a} = \frac{2\sqrt{ya}}{a} = 2\sqrt{\frac{y}{a}}$$

$$\therefore \qquad 2\sqrt{\frac{y}{a}} \le \mu \qquad \text{or} \qquad y \le \frac{a\mu^2}{4}$$

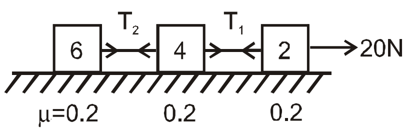
7._ $1.8\,t - \mu_k\,15 = 1.5\,(1.2\,t - 2.4)$

for t = 2.85 sec.



$\mu_k = 0.24$

8.



(i) Let the blocks do not move

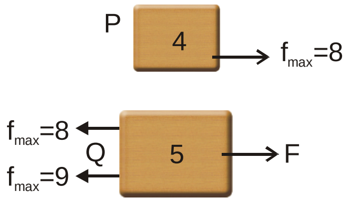
then $T_1 = 20 - 4$

$T_2 = T_1 - 8 = 20 - 4 - 8 = 8$ N

Since T_2 < max possible friction force for 6 kg block

hence it will be at rest and this assumption is right. Therefore tension in the string connecting 4kg and 6 kg block = 8N

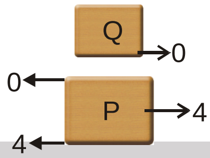
9. So block 'Q' is moving due to force while block 'P' due to friction.
Friction direction on both P + Q blocks as shown.



First block 'Q' will move and P will move with 'Q' so by FBD taking 'P' and 'Q' as system

$F - 9 = 0 \quad \Rightarrow F = 9$ N

When applied force is 4 N then FBD



4 kg block is moving due to friction and maximum friction force is 8 N.
So acceleration = 8/4 = 2 m/s^2 = a_{max}.
Slipping will start when Q has +ve acceleration equal to maximum acceleration of P i.e. 2 m/s^2.
$F - 17 = 5 \times 2 \quad \Rightarrow F = 27$ N.

10. **(A)** Limiting friction between A & B = 90 N
Limiting friction between B & C = 80 N
Limiting friction between C & ground = 60 N
Since limiting friction is least between C and ground, slipping will occur at first between C and ground.
This will occur when F = 60 N.

PART-II

1. $a_A = g\,[\sin 45 - \mu_A \cos 45] = \frac{8}{\sqrt{2}}$, $a_B = g\,[\sin 45 - \mu_B \cos 45] = \frac{7}{\sqrt{2}}$

$a_{AB} = a_A - a_B = g\,(\mu_B - \mu_A) \cos 45 = \frac{1}{\sqrt{2}}$, $s_{AB} = \sqrt{2}$

Now $s_{AB} = \frac{1}{2}\, a_{AB}\, t^2 \quad \Rightarrow \quad \sqrt{2} = \frac{1}{2} \times \frac{1}{\sqrt{2}} t^2 \quad \Rightarrow \quad t = 2$ sec.

2. $F = \sqrt{20^2 + 15^2} = 25$
$f_r = 0.5 \times 30 = 15$
$a = \frac{25 - 15}{2} = 5$ m/s^2.

3. $a_{block} = \frac{\mu mg}{m} = \mu g = 0.15 \times 10 = 1.5$
$a_T = 2$
$S_T - S_b = 5$

$\Rightarrow \quad \frac{1}{2} a_T t^2 - \frac{1}{2} a_B t^2 = 5$

$\Rightarrow \quad \frac{1}{2} t^2 [2 - 1.5] = 5$

$\Rightarrow \quad t^2 = 20$

$S_T = \frac{1}{2} a_T t^2$

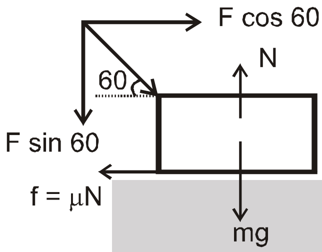
$= \frac{1}{2} \times 2 \times 20 = 20$ m.

4. $a_m = (mg \sin\theta - \frac{\mu}{2} mg \cos\theta) / m = g \sin\theta - \frac{\mu}{2} g \cos\theta$

$$a_M = \frac{Mg\sin\theta + \frac{\mu}{2}mg\cos\theta - \mu(M+m) \quad g\cos\theta}{M}$$

$$S_{mM} = \frac{1}{2} a_{mM} t^2 \Rightarrow t = \sqrt{\frac{2s_{mM}}{a_{mM}}} = \sqrt{\frac{2\ell}{a_m - a_M}} = \sqrt{\frac{4\ell M}{\mu g\cos\theta(M+m)}} = 2 \text{ sec}$$

5.



$$N = mg + F \sin 60 = \sqrt{3} \times 10 + \frac{F\sqrt{3}}{2} \quad \text{......(i)}$$

$$F \cos 60 = \mu N \quad \text{......(ii)}$$

$$\Rightarrow \quad \frac{F}{2} = \frac{1}{2\sqrt{3}} \times (10\sqrt{3} + \frac{F\sqrt{3}}{2})$$

$$\Rightarrow \quad \frac{F}{2} = 5 + \frac{F}{4} \Rightarrow \frac{F}{4} = 5 \Rightarrow F = 20 \text{ N}$$

6. The F.B.D. of A and B are

For sliding to start between A and B, the frictional $f = \mu N = \frac{1}{4} \times 2 \times 10 = 5 \text{ N} = f_{max}$

Applying Newton's second law to system of A + B

$$F = (m_A + m_B) a = 6a \quad \text{.....................(1)}$$

Applying Newton's second law to A

$$f = m_A a \Rightarrow a_{max} = \frac{f_{max}}{m_A} = \frac{5}{2} = 2.5 \text{ m/s}^2 \quad \text{.......................(2)}$$

from (1) and (2) $F_{min} = (m_A + m_B)\, 2.5 \text{ m/s}^2 = 6 \times 2.5 = 15$ N

Ans. $F_{min} = 15$ N

7. This problem can be solved in two steps :

Step I When the body is moving up on the inclined plane (shown in fig. A)

Here $N = mg \cos\alpha$

The acceleration of the body is

$$a_1 = -\left(\frac{mg\sin\alpha + \mu N}{m}\right) = -(g \sin\alpha + \mu g \cos\alpha)$$

Let body is projected with speed v_0 along the inclined plane (along the x-axis). After time t, body reaches at point P ($v = 0$) (as shown in fig.B).

Let OP = s

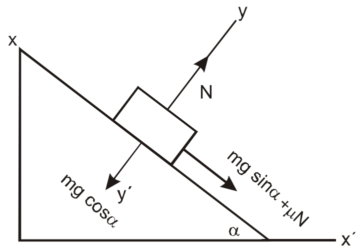


Fig. A

$$s = \left(\frac{\nu_0 + \nu}{2}\right) t_1 = \frac{\nu_0}{2}\, t_1$$

$$a_1 = \frac{0 - \nu_0}{t_1} \qquad \therefore\ \nu_0 = -\, a_1 t_1$$

$$\therefore \qquad s = -\,\frac{a_1 t_1^2}{2}$$

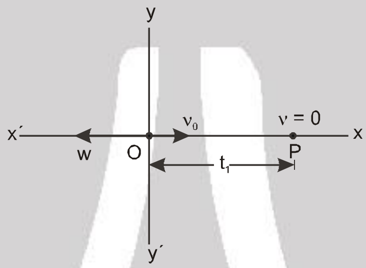


Fig. B

$$\therefore \qquad t_1 = \sqrt{-\frac{2s}{a_1}}$$

$$= \sqrt{\frac{2s}{g\sin\alpha + \mu g\cos\alpha}} \qquad ....(1)$$

Step II : When the body starts to return from point P, the acceleration is

$$a_2 = \frac{mg\sin\alpha - \mu mg\cos\alpha}{m} \quad \text{(In fig.C)}$$

$$= g \sin \alpha - \mu g \cos \alpha$$

$$\therefore \qquad s = \frac{1}{2}\, a_2 t_2^2$$

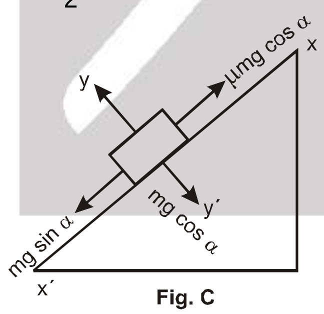


Fig. C

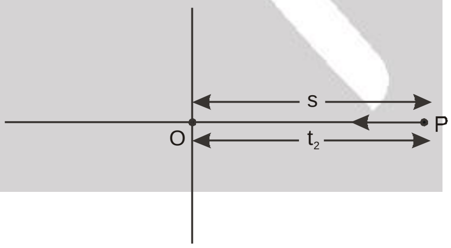


Fig. D

$$\therefore \qquad t_2 = \sqrt{\frac{2s}{a_2}} = \sqrt{\frac{2s}{g\sin\alpha - \mu g\cos\alpha}} \qquad ....(ii)$$

According to the problem,

$t_2 = \eta t_1$,

Putting the value of t_1 and t_2 from equations (i) and (ii), we get

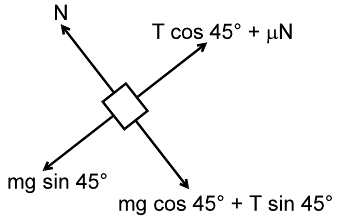
$$\mu = \left[\frac{(\eta^2 - 1)}{(\eta^2 + 1)}\right] \tan \alpha = 0.16 = k$$

than 100 k = 16 Ans.

PART-III

1. The free body diagram of the block is

N is the normal reaction exerted by inclined plane on the block.



Applying Newton's second law to the block along and normal to the incline.

$mg \sin 45° = T \cos 45° + \mu N$ .... (1)

$N = mg \cos 45° + T \sin 45°$ .... (2)

On solving we get

$\mu = 1/2$

so any value of μ greater than 0.5 is answer

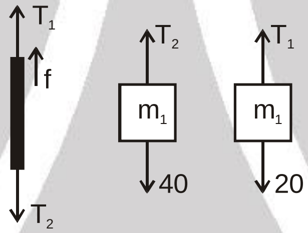
2. Applying NLM on the part that moves through slit.

$T_2 - f - T_1 = 0$

For 4 kg mass $40 - T_2 = 4a$

For 2 kg mass $T_1 - 20 = 2a$

On solving $10 = 6a$

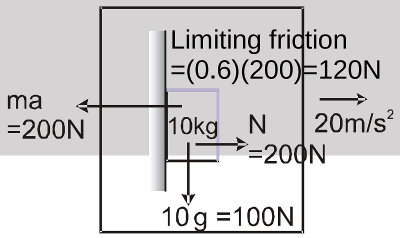


$a = \frac{5}{3}$ m/s^2

Force exerted on 2kg mass by string = $T_1 = \frac{70}{3}$ N.

Tension in the string will not be same throughout, due to the friction force exerted by the slit.

3. The breaking force is insufficient, so the block will not slide.



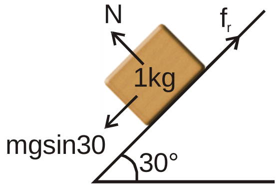
So friction force = 100 N

and acceleration will be 20 m/sec^2 only

Net contact force on the block = $\sqrt{(200)^2 + (100)^2} = 100\sqrt{5}$ N

All mechanical interactions are electromagnetic at microscopic level.

4. Block is moving upwards due to friction



f_r – mg sin 30 = ma

⇒ f_r – 1 × 10 × 1/2 = 1 × 1

⇒ f_r = 6 N

Contact force is the resultant of N and f_r = $\sqrt{N^2 + f_r^2}$ = $\sqrt{(mg\cos 30)^2 + (6)^2}$ = 10.5 N

5. Suppose blocks A and B move together. Applying NLM on C, A + B, and D

60 – T = 6a

T – 18 – T' = 9a

T' – 10 = 1a

On solving a = 2 m/s^2

To check slipping between A and B, we have to find friction force in this case. If it is less than limiting static friction, then there will be no slipping between A and B.

Applying NLM on A.

T – f = 6(2)

as T = 48 N

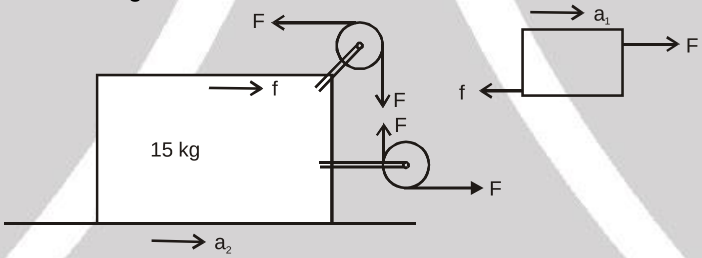
f = 36 N

and f_s = 42 N hence A and B move together.

and T' = 12 N.

PART-IV

1. First, let us check upto what value of F, both blocks move together. Till friction becomes limiting, they will be moving together. Using the FBDs



10 kg block will not slip over the 15 kg block till acceleration of 15 kg block becomes maximum as it is created only by friction force exerted by 10 kg block on it

$a_1 > a_{2(max)}$

$\frac{F - f}{10} = \frac{f}{15}$ for limiting condition as f maximum is 60 N.

F = 100 N.

Therefore for F = 80 N, both will move together.

Their combined acceleration, by applying NLM using both as system F = 25a

a = $\frac{80}{25}$ = 3.2 m/s^2

2. If F = 120 N, then there will be slipping, so using FBDs of both (friction will be 60 N)

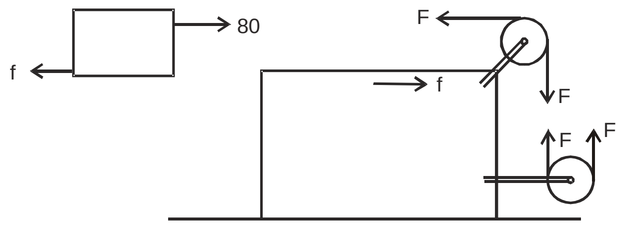
For 10 kg block

120 – 60 = 10 a ⇒ a = 6 m/s^2

For 15 kg block

60 = 15a ⇒ a = 4 m/s^2

3. In case 80 N force is applied vertically, then



For 10 kg block 80 – 60 = 10a

$a = 2\ m/s^2$

For 15 kg block in horizontal direction.

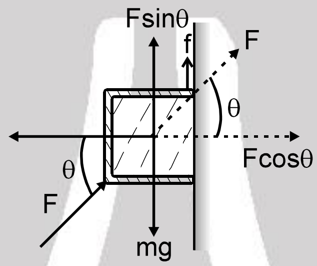
F – f = 15a

$a = 4/3\ m/s^2$, towards left.

4. $F \sin\theta + f = mg$

and $F\cos\theta = N$

for minimum ; $f = \mu N = \mu F\cos\theta$



$\therefore F_{min.} = \dfrac{mg}{\sin\theta + \mu\cos\theta}$

5. As f = 0 $\therefore F \sin\theta = mg$

$F = \dfrac{mg}{\sin\theta}$

6. If $F < F_{min}$; the block slides downward

7 to 8. (a) $\mu_2 > \mu_1$ therefore both will more together

and $a = \dfrac{(m_1 + m_2)g\sin\theta - \mu_1 m_1 g\cos\theta - \mu_2 m_2 g\cos\theta}{m_1 + m_2}$

$= g \sin\theta - \dfrac{g(\mu_1 m_1 + \mu_2 m_2)}{m_1 + m_2} \cos\theta$

$5 - \dfrac{4}{\sqrt{3}}$ (= 2.7 m/s^2) for both

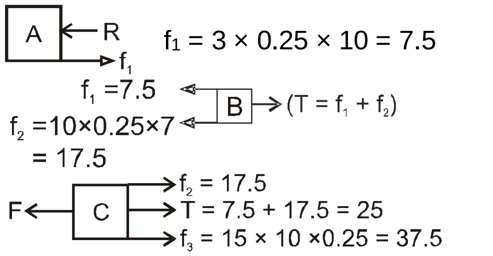
(b) $\mu_2 < \mu_1$ therefore ` $a_2 > a_1$

$a_2 = g \sin\theta - \mu_2 g \cos\theta$

$= 10 \times \dfrac{1}{2} - 0.2\times 10 \times \dfrac{\sqrt{3}}{2} = 3.2\ m/sec^2$

$a_1 = g \sin\theta - \mu_1 g \cos\theta = 5 - 1.5\sqrt{3} = 2.4\ m/sec^2$

9 to 10.

(1) 

$F = 17.5 + 25 + 37.5 = 80$ N

(2) If $F = 200$ then $a_B = a_C$

$\Rightarrow$ $T - f_1 - f_2 = m_B a$ ....(1)

$F - T - f_2 - f_3 = m_C a$ ....(2)

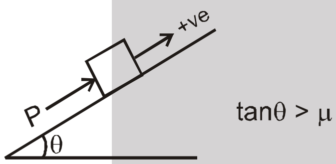
from equation (1) and (2)

$F - f_1 - 2f_2 - f_3 = (m_B + m_C)a$

$= a = \dfrac{F - f_1 - 2f_2 - f_3}{m_B + m_C} = \dfrac{200 - 7.5 - 35 - 37.5}{12}$ 10 m/sec^2

EXERCISE-3
PART-I

1.

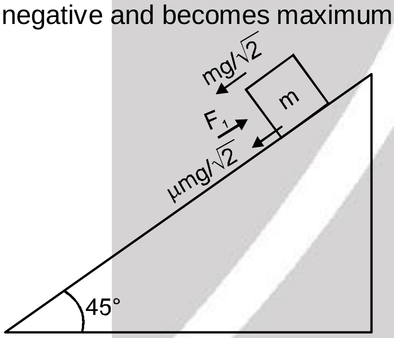


$\tan\theta > \mu$

$P_1 = mg\sin\theta - \mu mg\cos\theta$

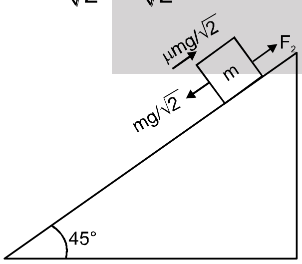
$P_2 = mg\sin\theta + \mu mg\cos\theta$

Initially block has tendency to slide down and as $\tan\theta > \mu$, maximum friction $\mu mg\cos\theta$ will act in positive direction. When magnitude P is increased from P_1 to P_2, friction reverse its direction from positive to negative and becomes maximum i.e. $\mu mg\cos\theta$ in opposite direction.



2.

$F_1 = \dfrac{mg}{\sqrt{2}} + \dfrac{\mu mg}{\sqrt{2}}$

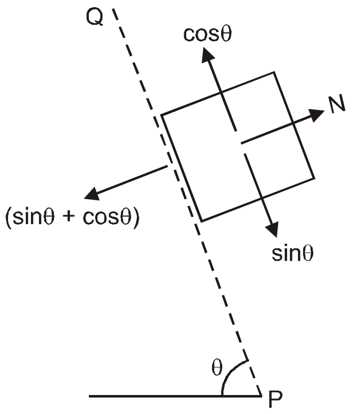


$F_2 = \dfrac{mg}{\sqrt{2}} - \dfrac{\mu mg}{\sqrt{2}}$

$F_1 = 3F_2$; $1 + \mu = 3 - 3\mu$

$4\mu = 2$; $\mu = 1/2$; $k = 10\mu$; $k = 5$ **Ans.**

3.



f = 0, If sinθ = cosθ ⇒ θ = 45°
f towards Q, sinθ > cosθ ⇒ θ > 45°
f towards P, sinθ < cosθ ⇒ θ < 45°

4. Block will not slip if $(m_1 + m_2)\, g \sin\theta \le \mu\, m_2\, g \cos\theta$

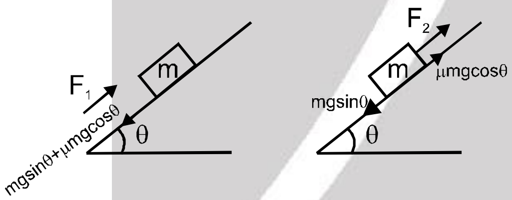
$3 \sin\theta \le \left(\frac{3}{10}\right) (2) \cos\theta$

$\tan\theta \le 1/5$ ⇒ $\boxed{\theta \le 11.5°}$

(P) θ = 5° friction is static $f = (m_1 + m_2)g \sin\theta$
(Q) θ = 10° friction is static $f = (m_1 + m_2)g \sin\theta$
(R) θ = 15° friction is kinetic $f = \mu m_2 g \cos\theta$
(S) θ = 20° friction is kinetic $f = \mu m_2 g \cos\theta$

PART-II

1.

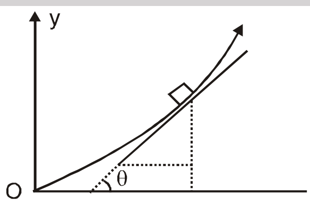


$F_1 = mg \sin\theta + \mu mg \cos\theta$; $F_2 = mg \sin\theta - \mu mg \cos\theta$

$$\frac{F_1}{F_2} = \frac{\sin\theta + \mu\cos\theta}{\sin\theta - \mu\cos\theta}$$

$$\frac{\tan\theta + \mu}{\tan\theta - \mu} = \frac{2\mu + \mu}{2\mu - \mu} = \frac{3\mu}{\mu} = 3.$$

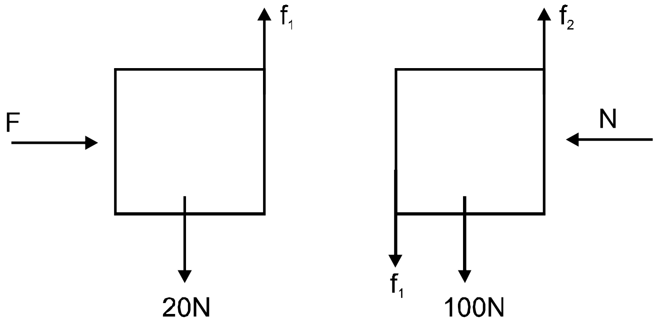
2. dy/dx = tanθ = μ in limiting case



$$\frac{dy}{dx} = \frac{3x^2}{6} = \frac{1}{2} \quad \Rightarrow \quad x = \pm 1$$

So y = 1/6

3.



Assuming both the blocks are stationary

N = F

f_1 = 20N

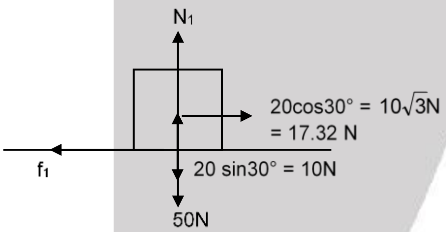
f_2 = 100+20 = 120N

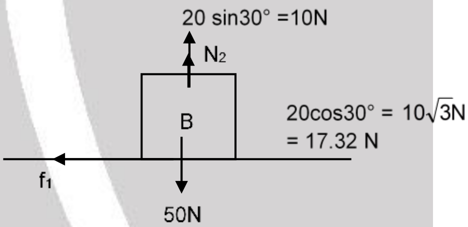
4. $\mu(m + m_2) = m_1$

$m + m_2 = \dfrac{m_1}{\mu} \quad \Rightarrow \quad m = \dfrac{m_1}{\mu} - m_2$

$m = \dfrac{5}{0.15} - 10 = 23.33\text{kg}$

5.





$N_1 = 50 + 10 = 60\text{N}$

$F_{Lim} = 0.2 \times 60 = 12\text{N}$

$F_{applied} > 12\text{N}$

$\therefore \quad f_1 = 12\text{N}$

$\therefore \quad a_1 = \dfrac{17.32 - 12}{5}$

$= \dfrac{5.32}{5}$

$N_2 = 50 - 10 = 40\text{N}$

$F_{Lim} = 0.2 \times 40 = 8\text{N}$

$F_{applied} > 8\text{N}$

$\therefore f_1 = 8\text{N}$

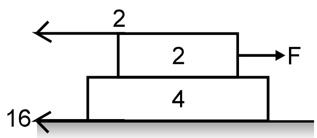
$a_2 = \dfrac{17.32 - 8}{5}$

$\therefore \quad a_1 - a_2 = -4/5$

$\therefore \quad |a_1 - a_2| = 4/5 = 0.8 \text{ m/s}^2$

HIGH LEVEL PROBLEMS (HLP)

1. for uppermost block $a_{max.} = 1 \text{ m/s}^2$

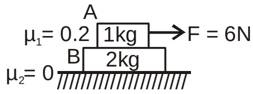


for lowermost block $a_{max} = \dfrac{(24 - 16)}{4} = 2 \text{ m/s}^2$

Hence sliding between middle and lower block will start only after sliding between middle and upper block has already started.

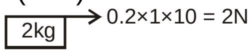
for middle block $F - 18 = 6 \times 2 \quad \Rightarrow F = 30 \text{ N}$

2. **Case - I** : For the lower block



$a_{max} = \frac{2}{2} = 1 m/s^2$

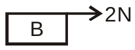
and common possible acceleration = $\frac{6}{(1+2)} = 2\ m/s^2$



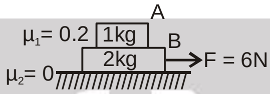
Hence, blocks move with different accelerations.

$a_A = \frac{6-2}{1} = 4\ m/s^2$

$a_B = \frac{2}{2} = 1\ m/s^2$



Case II : When the force is acting on the lower block maximum possible acceleration of A



$= \frac{2}{1} = 2\ m/s^2$

and common acceleration of the two blocks $= \frac{6}{(1+2)} = 2\ m/s^2$

Hence, both blocks move with common acceleration of $a_A = a_B = 2\ m/s^2$

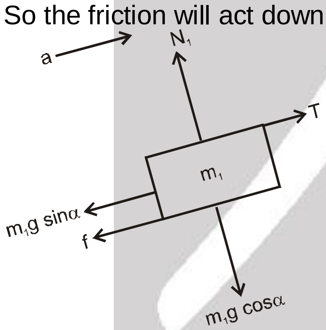
3. For checking the direction of friction, let us assume there is no friction. Then net force acting on the system along the string in vertically downward direction is given by

$m_2g - m_1g \sin \alpha = (m_1 + m_2)\ a$

$\eta m_1g - m_1g/2 = (m_1 + \eta m_1)a$

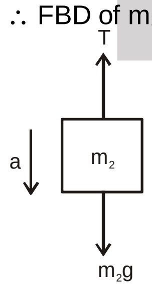
$a = \frac{\eta - \frac{1}{2}}{\eta + 1} g \quad \Rightarrow \quad a > 0.$

So the friction will act down the incline FBD of m_1 gives : –



$T - f - m_1g \sin \alpha = m_1a. \Rightarrow T - k\ m_1g \cos \alpha - m_1g\ \text{sing}\ \alpha = m_1a$ ——(i)

$\therefore$ FBD of m_2



$m_2g - T = m_2a$ ——(ii)

from (i) and (ii)

$a = \frac{g(\eta - \sin\alpha - k\cos\alpha)}{(\eta+1)}$

Putting $\eta = 2/3$, $\alpha = 30^o$ and k = 0.1 ; a = 0.05 g (downward for m_2)

4. Acceleration of mass at distance x

$$a = g(\sin\theta - \mu_0 x \cos\theta)$$

Speed is maximum, when a = 0

$g(\sin\theta - \mu_0 x \cos\theta) = 0$

$x = \frac{\tan\theta}{\mu_0}$

for maximum speed

$a = g(\sin\theta - \mu_0 x \cos\theta)$

$\frac{vdv}{dx} = g(\sin\theta - \mu_0 x \cos\theta)$

$vdv = (g\sin\theta - \mu_0 gx \cos\theta)dx$

Integrating both sides

$$\int_0^{v_{max}} vdv = \int_0^{\tan\theta/\mu_0} (g\sin\theta - \mu_0 gx\cos\theta)\ dx$$

$$\frac{v_{max}^2}{2} = \left[gx\sin\theta - \frac{\mu_0 gx^2\cos\theta}{2}\right]_0^{\frac{\tan\theta}{\mu_0}}$$

After Applying limits

$$v_{max} = \sqrt{\frac{g\sin\theta\tan\theta}{\mu_0}}$$

5. The acceleration of block is

$$w = \frac{mg\sin\alpha - kmg\cos\alpha}{m}$$

or $w = g\sin\alpha - kg\cos\alpha$

Let $AB = s \therefore \cos\alpha = \frac{\ell}{s}$

$\therefore$ $s = \ell\sec\alpha$

$\therefore$ According to kinematics equation

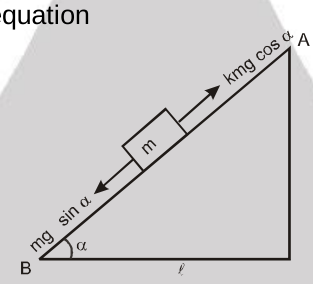


Fig. A

$$s = \frac{1}{2}wt^2$$

$\therefore$ $t = \sqrt{\frac{2s}{w}} = \sqrt{\frac{2\ell\sec\alpha}{g\sin\alpha - kg\cos\alpha}}$

or $t^2 = \frac{2\ell\sec\alpha}{g(\sin\alpha - k\cos\alpha)} = \frac{2\ell}{g(\sin\alpha\cos\alpha - k\cos^2\alpha)}$

For being t_0 minimum

$\sin\alpha\cos\alpha - k\cos^2\alpha$ is maximum

$\frac{d}{d\alpha}(\sin\alpha\cos\alpha - k\cos^2\alpha) = 0$

$\cos^2\alpha - \sin^2\alpha + 2k\cos\alpha\sin\alpha = 0$

$\cos 2\alpha + k\sin 2\alpha = 0$

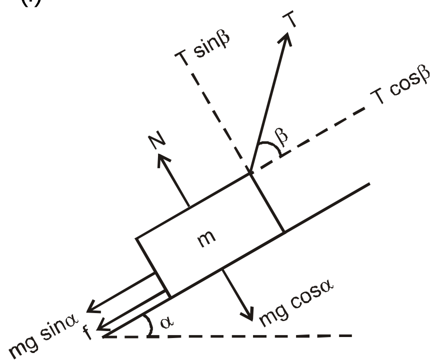
$\tan 2\alpha = \frac{-1}{k} \quad \Rightarrow \quad \alpha = \frac{1}{2}\tan^{-1}\left(\frac{-1}{k}\right)$

After putting the values, $\alpha = 49^\circ$

6. For limiting friction

N + T sin β = mg cos α

⇒ N = mg cos α – T sin β ——(i)



and, Tcos β = mg sin α + f

= mg sin α + k (mg cos α – T sin β) (from (i))

⇒ T (cos β + k sin β) = mg sin α + k mg cos α

$$\Rightarrow T = \frac{mg\ (\sin\alpha + k\cos\alpha)}{(\cos\beta + k\sin\beta)}$$

for minimum T , cos β + k sin β should be maximum

$$\frac{dy}{d\beta} = -\sin\beta + k\cos\beta = 0$$

⇒ tan β = k

$$\therefore \quad T_{min} = \frac{mg(\sin\alpha + k\cos\alpha)}{\left(\frac{1}{\sqrt{1+k^2}} + \frac{k^2}{\sqrt{1+k^2}}\right)} = \frac{mg(\sin\alpha + k\cos\alpha)}{\sqrt{1+k^2}}$$

7. **Step -I :** Draw force diagram separately : In fig B, P is a point on the string

From fig. A,

$mg - f = mw_1$

$\therefore \quad w_1 = \frac{mg - f}{m}$ .....(i)

From fig.B,,

T = f .....(ii)

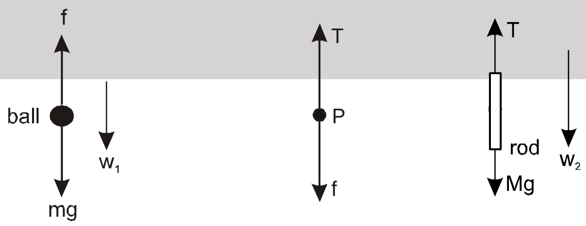


Fig. A Fig.B Fig.C

From fig.C

$Mg - T = Mw_2$

$Mg - f = Mw_2$

$\therefore \quad w_2 = \frac{Mg - f}{M}$ ......(iii)

Step II : Apply kinematic relation :

$s_{rel} = u_{rel}\,t\ \frac{1}{2}\ w_{rel}\,t^2$ (Shown in fig.D)

Here $s_{rel} = \ell$, $u_{rel} = 0$

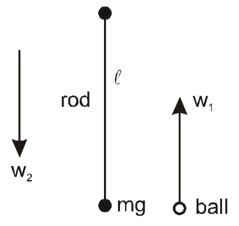


Fig. D

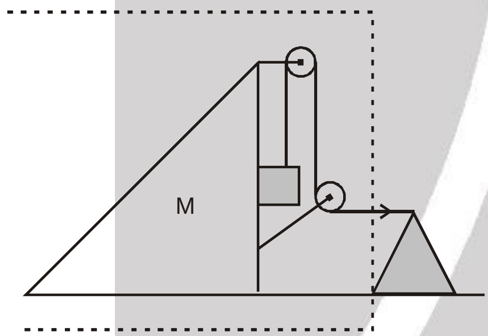
$w_{rel} = w_2 - w_1$

$\therefore \quad \ell = \frac{1}{2}(w_2 - w_1)\,t^2 \qquad \therefore \quad t = \sqrt{\frac{2\ell}{(w_2 - w_1)}}$

Putting the value of w_1 and w_2,

$$f = \frac{2\ell Mm}{(M-m)t^2}$$

8.

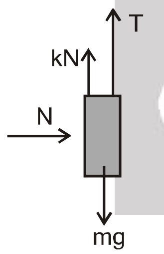


Considering the system as marked in the diagram

$\therefore \quad T = (M + m)a.$

a is the common acceleration of the two masses in horizontal direction.
Now taking the body m as system.

F.B.D. for the body will be



$\therefore \quad N = ma$ (m has acceleration a in horizontal direction)

Also $mg - kN - T = ma$ (m has acceleration a downward w.r.t. wedge because of the constraint)

or $a = \frac{mg}{M + 2m + Km}$; $\vec{a}_{bg} = \vec{a}_{bw} + \vec{a}_{wg}$

$$|\vec{a}_{bg}| = \sqrt{a^2 + a^2} = \sqrt{2}a = \frac{\sqrt{2}mg}{M + 2m + Km} = \frac{\sqrt{2}g}{2 + K + M/m}$$

9. If acceleration in bar is zero, then the body (1) will slip on bar rightward and the body (2) moves downward. To prevent slipping, net force on each body should be zero in the frame of bar (non-inertial reference frame)

In fig. A, $T = mw + kmg$ ....(i)

In fig. B, $T + kN = mg$ ....(ii)

$N = mw$ ....(iii)

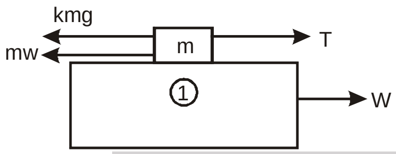


Fig. A

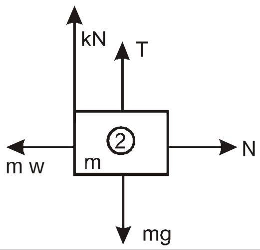


Fig. B

From (ii) and (iii), we get

$$T + kmw = mg$$

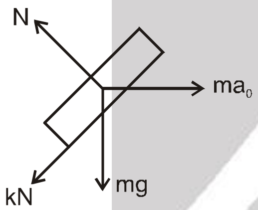
$$T = mg - kmw \quad ....(iv)$$

From (i) and (iv), we get

$$w = \frac{g(1-k)}{(1+k)} \quad ....(v)$$

Since, relative acceleration of body with respect to bar (w_{rel}) is zero : So, the value of w in eqn.(v) is minimum value of w.

10.



(a_0 is acceleration of the wedge leftward)

As a_0 increases the component of ma_0 up the up the incline increases and friction attains its max value.

Writing the force equation along the incline and perpendicular to the incline.

$$ma_0 \cos\alpha - mg \sin\alpha - kN = 0 \quad ....(i)$$

$$mg \cos\alpha + ma_0 \sin\alpha = N \quad ....(ii)$$

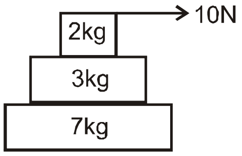
From equation (i) and (ii),

$$a_0 \cos\alpha - g \sin\alpha = kg \cos\alpha + K\, a_0 \sin\alpha$$

$$a_0 (\cos\alpha - k \sin\alpha) = g\{k \cos\alpha + \sin\alpha\}$$

$$a_0 = \frac{g\{k\cos\alpha + \sin\alpha\}}{\{\cos\alpha - k\sin\alpha\}}$$

11. (i) Assuming there is no slipping anywhere and the common acceleration of the three blocks be a



Writing the force equation F = ma

10 = 12a or a = 5/6.

Now $f_{max.}$ between 2 kg & 3kg is

$\mu_1 N = 0.2 \times 20 = 4N$.

For 2 kg block F.B.D. will be:

$F - f = ma = 2 \times 5/6$

$10 - f = 10/6$

$10 - 10/6 = f$

$50/6 = f > f_{max}$

∴ There will be slipping between 2 kg and 3 kg block. Now considering the slipping the new equation would be

$F - f_{max} = ma_1$

$10 - 4 = 2a_1$ or $a_1 = 3\ ms^{-2}$

Now lets take 3 kg & 7 kg as system and writing the force equation.

$f_{max} = 10\ a_0$.

or $4 = 10\ a_0 \Rightarrow a_0 = 0.4\ ms^{-2}$

To check the required friction between 3 kg and 7 kg block.

F on 7 kg block $F = 7 \times 0.4 = 2.8$ N

f_{max} between 7 kg and 3 kg = $0.3\ N_2 = 0.3 \times 50 = 15$ N

$2.8 < f_{max}$

hence there is no slipping between the two blocks

(ii) Now if force is applied on 3 kg block

Assuming there is no slipping anywhere and the common acceleration of the three blocks is

$\frac{5}{6}\ ms^{-2}$

Now, if all system is going with common acceleration $\frac{5}{6} ms^{-2}$

for a = $\frac{5}{6} ms^{-2}$ required friction force between 2 kg and 3 kg block $= m_1 a = 2 \times \frac{5}{6} = \frac{5}{3} N < f_{max}$

so there is no slipping

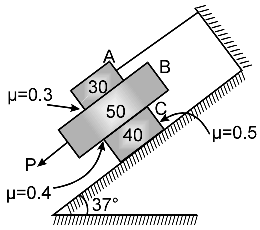
Same for 7 kg block, required friction is $= m_3 a = 7 \times \frac{5}{6} = \frac{35}{6} N < f_{max}$

so there is no slipping

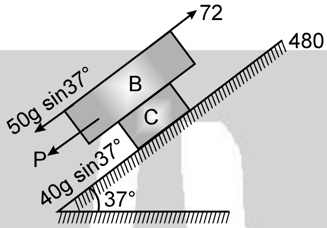
$\therefore a_1 = a_2 = a_3 = \frac{5}{6} ms^{-2}$

(iii) Same as (b

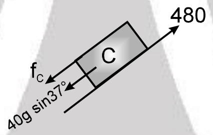
12. $f_{ABmax} = 0.3 \times 30g \cos 37°$



$f_{ABmax} = 0.3 \times 30g \cos 37° = 72$ N
$f_{BCmax} = 0.4 \times 80g \cos 37° = 256$ N
$f_{Cmax} = 0.5 \times 120g \cos 37° = 480$ N
when block 'B' is pulled two cases are possible
(1) B & C both moves together and there is just slipping at A & B contact and at C and wedge contact
(2) A & C remains stationary only 'B' tends to move downwards.
Taking case (1) Let B & C moves together and there is just slipping between A & B and between wedge and C

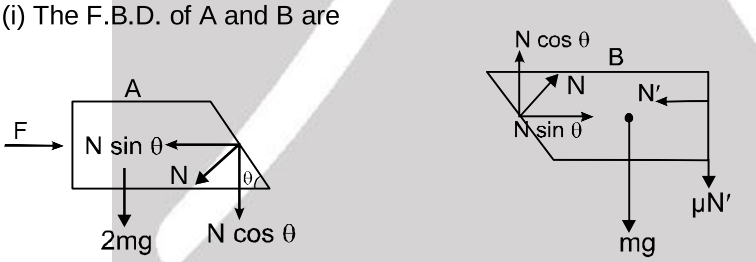


$\Rightarrow \quad P + 40\,g \sin 37° + 50\,g \sin 37° - 72 - 480 = 0$
$\Rightarrow \quad P = 12$ N
Checking friction between B & C
$f_{BC} + 40\,g \sin 37° - 480 = 0$



$f_{BC} = 240$
Which is within limit so case is correct.
So max. force for which there will be no slipping $P_{max} = 12$ N the at this force both B & C tends to move together.

13. (i) The F.B.D. of A and B are



For A to be in equilibrium ; $F = N \sin\theta$ ....(1)
For B to just lift off ; $N \cos\theta = mg + \mu_s N'$ ....(2)
For horizontal equilibrium of B ; $N' = N \sin\theta$ ....(3)
From (2) and (3)

$N(\cos\theta - \mu_s \sin\theta) = mg$ or $N = \left(\frac{4}{5} - \frac{2}{3} \times \frac{3}{5}\right) mg$ or $N = \frac{5}{2}\,mg$ ....(4)

From equation (1) $F = N \times 3/5$ $\Rightarrow$ $\therefore$ $F = 3/2\,mg$
(ii) The acceleration of the block A be a and B be b

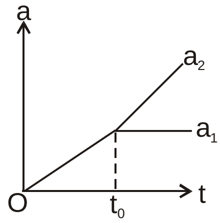
$F - N \sin\theta = 2ma$ ....(1)
$N \cos\theta - mg - \mu_k N' = mb$ ....(2)
$N' = N \sin\theta$ ....(3)

From constraint =
$a \sin\theta = b \cos\theta$ ....(4)

Solving (1), (2), (3) and (4) we get $\Rightarrow$ $b = \frac{3g}{22}$

14. When $t \le t_0$, the accelerations $a_1 = a_2 = kt / (m_1 + m_2)$; when $t \ge t_0$

$a_1 = \mu g m_2 / m_1$, $a_2 = (kt - \mu m_2 g) / m_2$. Here $t_0 = \frac{\mu g(m_1 + m_2)}{k} \times \frac{m_2}{m_1}$

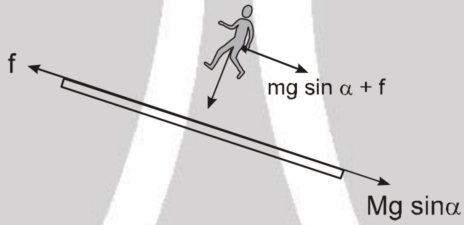


When $t \le t_0$, there is no slipping occurs and the accelerations $a_1 = a_2 = kt / (m_1 + m_2)$;

when $t \ge t_0$ maximum friction exert between plank and rod

$a_1 = \mu g m_2 / m_1$, $a_2 = (kt - \mu m_2 g) / m_2$. Here $t_0 = \frac{\mu g(m_1 + m_2)}{k} \times \frac{m_2}{m_1}$

15. F.B.D. of man and plank are



For plank be at rest, applying Newton's second law to plank along the incline

$Mg \sin \alpha = f$ ....(1)

and applying Newton's second law to man along the incline.

$mg \sin \alpha + f = ma$ ....(2)

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

Alternate Solution : :

If the friction force is taken up the incline on man, then application of Newton's second law to man and plank along incline yields.

$f + Mg \sin \alpha = 0$ ....(1)

$mg \sin \alpha - f = ma$ ....(2)

Solving (1) and (2)

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

Alternate Solution :

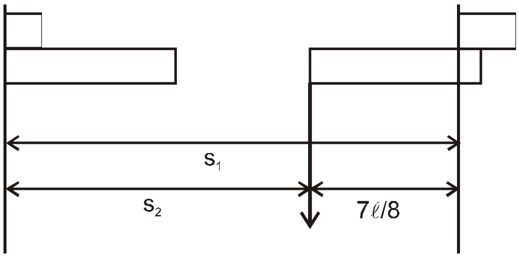
Application of Newton's seconds law to system of man + plank along the incline yields

$mg \sin \alpha + Mg \sin \alpha = ma$

$a = g \sin \alpha \left(1 + \frac{M}{m}\right)$ down the incline

16. $a_1 = \dfrac{mg - \mu_k mg}{2m} = \dfrac{g}{2}(1 - \mu_k)$

$a_2 = \dfrac{\mu_k mg}{4m} = \dfrac{\mu_k g}{4}$



$s_1 = \dfrac{1}{2} a_1 t^2$

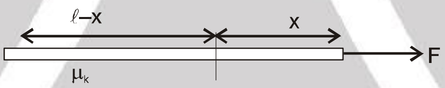
$s_2 = \dfrac{1}{2} a_2 t^2$

$s_1 - s_2 = \dfrac{7\ell}{8} \quad \Rightarrow \quad \dfrac{1}{2}\dfrac{g}{2}(1 - \mu_k)t^2 - \dfrac{1}{2}\dfrac{g}{4}\mu_k t^2 = \dfrac{7\ell}{8}$

$\Rightarrow \quad t^2 = \dfrac{7\ell}{2g(1-\mu_k) - g\mu_k} = \dfrac{7\ell}{g(2 - 3\mu_k)}$

$s_2 = \dfrac{1}{2} a_2 t^2 = \dfrac{1}{2} \times \dfrac{\mu_k g}{4} \times \dfrac{7\ell}{g(2-3\mu_k)} = \dfrac{7\ell\mu_k}{8(2-3\mu_k)}$

17. Considering the forces on the chain for the given situation we have



$F - \mu_k(\ell - x)\rho g = \ell\rho a$

$\dfrac{F}{\rho\ell} - \dfrac{\mu_k(\ell - x)g}{\ell} = \dfrac{dv}{dx}.v.$

$\int_0^\ell \dfrac{F}{\rho\ell}dx - \int_0^\ell \dfrac{\mu_k(\ell - x)}{\ell} g\,dx = \int_0^v dv\,v$

$\dfrac{F}{\rho\ \ell}x\Big|_0^\ell - g\mu_k\left(x - \dfrac{x^2}{2\ell}\right)\Big|_0^\ell = \dfrac{v^2}{2}\Big|_0^v$

$\dfrac{F}{\rho} - g\mu_k\dfrac{\ell}{2} = \dfrac{v^2}{2}$

$\sqrt{\dfrac{2F}{\rho} - \mu_k g\ell} = v = 4\text{ m/s}$