



SOLUTIONS OF MATHEMATICAL TOOLS

EXERCISE-1

PART - I

Section (A) :

A-1. $f(\pi/2) = \cos \pi/2 + \sin \pi/2 = 1$

A-2. $f(2) = 4 \times 2 + 3 = 11$

$f\{f(2)\} = f(11) = 4 \times 11 + 3 = 47$

A-3. $\tan 15 = \tan(45-30) = \frac{\tan 45 - \tan 30}{1 + \tan 45 \tan 30} = \frac{1 - \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1}$

$$= \frac{(\sqrt{3} - 1)^2}{2} = \frac{3 + 1 - 2\sqrt{3}}{2} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

A-4. $\cos^2 \theta = 1 - 2\sin^2 \theta$

$2\sin^2 \theta = 1 - \cos^2 \theta \Rightarrow \sin^2 \theta = \left(\frac{1 - \cos 2\theta}{2}\right)$

A-5. $\sin A \cdot [\sin A \cos B + \cos A \cdot \sin B]$
 $\sin^2 A \cdot \cos B + \sin A \cdot \cos A \cdot \sin B$
 $\sin^2 A \cdot \cos B + \frac{1}{2} \sin 2A \cdot \sin B$

A-6. $\therefore \cos\left(\frac{\pi}{2} + \theta\right) = -\sin \theta$ [cos in II quadrant gives negative value]

$\sin(\theta - \pi) = \sin[-(\pi - \theta)] = -\sin(\pi - \theta)$
 $= -\sin \theta$

$\sin(\pi + \theta) = -\sin \theta$ [sin function is -ve in III quadrant]

A-7. $x_1 = 8 \sin \theta$ and $x_2 = 6 \cos \theta$ then

$x_1 + x_2 = 10 \sin(\theta + 37^\circ)$

$(x_1 + x_2)_{\max} = 10 [\sin(\theta + 37^\circ)]_{\max} = 10 \times 1 = 10$

Section (B) :

B-1. $\frac{dy}{dx} = 2x + 1$

B-2. $\frac{dy}{dx} = \sec^2 x - \operatorname{cosec}^2 x$

B-3. $\frac{dy}{dx} = \cos x - \sin x$, $\frac{d^2y}{dx^2} = -\sin x - \cos x$

B-4. $\frac{dy}{dx} = \frac{1}{x} + e^x$, $\frac{d^2y}{dx^2} = -\frac{1}{x^2} + e^x$

Section (C) :

C-1. $\frac{d}{dx} e^x \ln x = \ln x \frac{de^x}{dx} + e^x \frac{d \ln x}{dx}$
 $e^x \ln x + \frac{e^x}{x}$

C-2. $\frac{d(\sin x \cos x)}{dx} = \sin x \frac{d(\cos x)}{dx} + \cos x \frac{d(\sin x)}{dx} = \cos^2 x - \sin^2 x$



**Section (D) :**

D-1. $y' = \frac{(3x-2)(2) - (2x+5)(3)}{(3x-2)^2} = \frac{-19}{(3x-2)^2}$

D-2. $\frac{dy}{dx} = \frac{d\left(\frac{\ln x}{x}\right)}{dx} = \frac{x \times \frac{1}{x} - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$

D-3. $\sec^2 x - \tan^2 x = 1$
 $\therefore \frac{d(1)}{dx} = 0$

D-4. (a) $\frac{d}{dx} uv = uv' + u'v$

$$= 5 \times 2 + (-1)(-3) = 13$$

(b) $\frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2} = \frac{(-1)(-3) - (5)(2)}{(-1)^2} = -7$

(c) $\frac{d}{dx} \left(\frac{v}{u} \right) = \frac{uv' - vu'}{u^2} = \frac{(5)(2) - (-1)(-3)}{(5)^2} = \frac{7}{25}$

(d) $\frac{d}{dx} (7v - 2u) = 7v' - 2u'$
 $= 7 \times 2 - 2(-3) = 20$

Section (E) :

E-1. $y = \sin 5x$

Let $5x = \theta$

$y = \sin \theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \cdot \frac{d\theta}{dx}$$

$$\frac{dy}{d\theta} = \cos \theta \quad \frac{d\theta}{dx} = 5$$

$$\therefore \frac{d\theta}{dx} = 5 \cos \theta \quad \theta = 5x$$

$$\therefore \frac{dy}{dx} = 5 \cos 5x$$

E-2. Let $u = (\omega x + \phi)$

$$\left(\frac{dy}{dx} \right) = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 2 \cos(\omega x + \phi) \times \omega$$

$$= 2\omega \cos(\omega x + \phi) \text{ Ans.}$$

E-3. $\frac{dy}{dx} = -27(4 - 3x)^8$

Section (F) :

F-1. $(x + y)^2 = 4$

$$2(x + y) \left(1 + \frac{dy}{dx} \right) = 0$$

$$\therefore x + y \neq 0 \quad \Rightarrow \quad 1 + \frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -1$$





F-2. $x^2y + xy^2 = 6$

$$x^2 \left[\frac{dy}{dx} \right] + y[+2x] + y^2 + x \left[2y \frac{dy}{dx} \right] = 0$$

$$\frac{dy}{dx} [x^2 + 2xy] + 2xy + y^2 = 0$$

$$\therefore \frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

Section (G) :

G-1. $\frac{dA}{dt} = \frac{d(\pi r^2)}{dt} = \frac{\pi d(r^2)}{dt} = \frac{2\pi r dr}{dt}$

G-2. $\frac{d}{dt} S = \frac{d(4\pi r^2)}{dt} = 8\pi r \frac{dr}{dt}$

Section (H) :

H-1. $x = -t^2 + 4t + 4$ (i)

for maxima

$$\frac{dx}{dt} = 0$$

$$-2t + 4 = 0$$

$$t = 2$$

from equation (i) $\frac{d^2x}{dt^2} = -2 < 0$

x has maximum value at $t = 2$ sec. maximum value of x

$$x_{\max} = -(2)^2 + 4(2) + 4 = 8$$

H-2. $y_{\max} = 39, y_{\min} = 38$

Section (I) :

I-1. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = 48(8x - 1)^2$$

I-2. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$3 \cos(3x + 1)$$

I-3. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$12x^3.$$

I-4. $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$

$$\frac{dy}{dx} = -\frac{1}{3} \sin \frac{x}{3}$$

PART - II

Section (A) :

A-1. $y = x^2 - 2x + 1$

$$\int y dx = \int (x^2 - 2x + 1) dx + c = \int x^2 dx - 2 \int x dx + \int dx + c$$

$$= \frac{x^3}{3} - x^2 + x + c$$





A-2. $y = (x)^{1/3} + x^{-1/3}$

$$\int y dx = \int x^{1/3} dx + \int x^{-1/3} dx$$

$$\frac{3x^{4/3}}{4} + \frac{3x^{2/3}}{2} + c$$

A-3. $y = \sec^2 x \quad \int y dx = \tan x + c$

A-4. $y = \operatorname{cosec}^2 x \quad \int y dx = -\cot x + c$

A-5. $y = \sec x \cdot \tan x = \int y dx \sec x + c$

A-6. $y = \frac{1}{3x} \quad \int y dx = \frac{1}{3} \int \frac{1}{x} dx + c = \frac{1}{3} \ln x + c$

Section (B) :

B-1. $\int x \sin(2x^2) dx \quad \text{Let } u = 2x^2 \quad du = 4x dx$

$$= \int \sin u \frac{du}{4}$$

$$= \frac{1}{4} \int \sin u du = -\frac{1}{4} \cos u + C$$

B-2. $\int \sec 2t \tan 2t dt$

Let $u = 2t \quad du = 2 dt$

$$= \int \sec u \tan u \frac{du}{2}$$

$$= \frac{1}{2} \sec u + C$$

$$= \frac{1}{2} \sec u + C$$

B-3. $\int \frac{3}{(2-x)^2} dx = 3 \int (2-x)^{-2} dx$

Let $u = 2-x \quad du = -dx$

$$3 \int u^{-2} (-dx)$$

$$= \frac{+3}{u} + C = \frac{3}{2-x} + C$$

B-4. $\int \sin(8z-5) dz$

Let $u = 8z-5 \quad du = 8 dz$

$$\int \sin u \frac{du}{8} = \frac{1}{8} (-\cos u) + C$$

$$= -\frac{1}{8} \cos(8z-5) + C$$

Section (C) :

C-1. $\frac{\pi}{2} \int_{-4}^{-1} d\theta = \frac{\pi}{2} [\theta]_{-4}^{-1} = \frac{\pi}{2} [(-1) - (-4)] = \frac{3\pi}{2}$

C-2. $\int_{\sqrt{2}}^{5\sqrt{2}} r dr = \left[\frac{r^2}{2} \right]_{\sqrt{2}}^{5\sqrt{2}} = \frac{(5\sqrt{2})^2}{2} - \frac{(\sqrt{2})^2}{2} = 25 - 1 = 24$

C-3. $\int_0^1 e^x dx = [e^x]_0^1 = e - 1$



**Section (D) :****D-1.** $y = 2x$

$$\text{Area} = \int_0^b y \, dx = \int_0^b 2x \, dx = \left[x^2 \right]_0^b = b^2 \text{ units}$$

D-2. $y = \frac{x}{2} + 1$

$$\begin{aligned} \int_0^b y \, dx &= \int_0^b \frac{x}{2} \, dx + \int_0^b 1 \, dx = \left[\frac{x^2}{4} \right]_0^b + \left[x \right]_0^b \\ &= \frac{b^2}{4} + b \end{aligned}$$

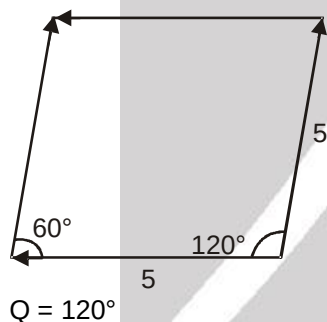
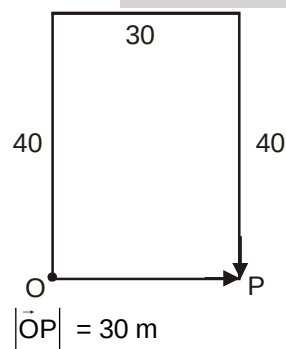
D-3. $\int_0^\pi y \, dx = \int_0^\pi \sin x \, dx = \left[-\cos x \right]_0^\pi$

$$= [-\cos \pi + \cos 0] = 2$$

D-4. $\int_0^\pi \sin^2 x \, dx = \int_0^\pi \frac{1 - \cos 2x}{2} \, dx$

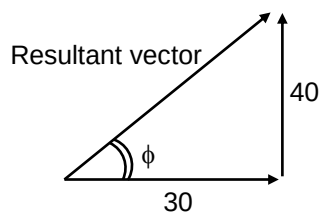
$$= \int_0^\pi \frac{1}{2} \, dx - \int_0^\pi \frac{\cos 2x}{2} \, dx$$

$$= \frac{\pi}{2} - \frac{1}{4} [\sin 2x]_0^\pi = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

PART - III**Section (A) :****A-1.** (i) 105° , (ii) 150° , (iii) 105° .**A-2.****A-3.** $\vec{V}_R = 5 \hat{j} \text{ m/s} = -5 \hat{j} \text{ m/s}$.**Section (B) :****B-1.**



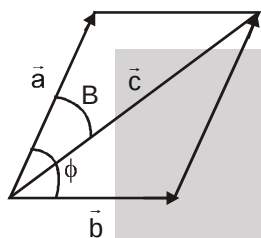
B-2. $\vec{A} = 30\hat{i}$
 $\vec{B} = 40\hat{j}$



$$\hat{i} = \vec{A} + \vec{B} = 30\hat{i} + 40\hat{j}$$

$$\tan \phi = 4/3 = 53^\circ$$

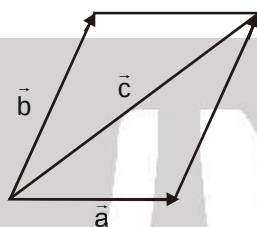
B-3.



$$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab \cos \theta}$$

$$|\vec{c}| = \sqrt{a^2 + b^2 + 2ab \cos \theta}$$

$$\therefore |\vec{c}^1| = |\vec{c}|$$

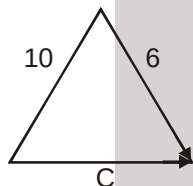


$$|\vec{a}^1| = |\vec{b}|$$

$$|\vec{b}^1| = |\vec{a}|$$

B-4. Can not be zero

B-5.



Using triangular inequality

$$C > 10 - 6$$

$$C < 10 + 6$$

$$\therefore 4 < c < 16$$

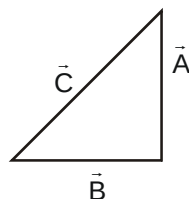
B-6. Resultant of vectors forming closed polygon is zero vector.

B-7. $|\vec{A} + \vec{B}| = \sqrt{A^2 + B^2 + 2AB \cos \theta}$

$\cos \theta$ is min. when $\theta = \pi$

B-9. $\vec{C} = \vec{A} + \vec{B}$

$$|\vec{A}| = 12 \quad |\vec{C}| = 13$$



$$|\vec{B}| = 5$$

They form a Pythagorean Triplet with $Q_{AB} = 90$



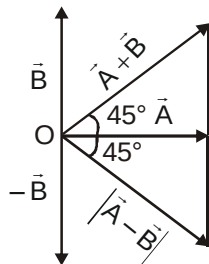


B-10. $\vec{P} + \vec{Q} = \vec{P} - \vec{Q}$

$$2\vec{Q} = 0$$

$$\Rightarrow \vec{Q} = 0$$

B-11.



Section (C) :

C-1. $\sqrt{(3)^2 + (2)^2 + (1)^2} = \sqrt{9+4+1} = \sqrt{14}$ unit.

C-2. $x = 25 \cos^2 10^\circ = -25 \cos 30^\circ$ $[210^\circ = 180^\circ + 30^\circ]$
 $y = 25 \sin^2 10^\circ = -25 \sin 30^\circ$

C-3. $|\vec{V}| = 60 \text{ km/h}$
 $|\vec{V}| = \sqrt{30^2 + x^2} = 60$

$$30^2 + x^2 = 60^2$$

$$x^2 = 3600 - 900$$

$$x^2 = 2700$$

$$x^2 = 900 \times 3$$

$$x = 30\sqrt{3}$$

C-4. $\vec{A} = 0.5\hat{i} + 0.8\hat{j} + c\hat{k}$ is a unit vector
 $0.5^2 + 0.8^2 + c^2 = 1^2$
 $\therefore c = \pm \frac{\sqrt{11}}{10}$

C-5. $\vec{A} = 2\hat{i} + 2\hat{j}$
 $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{2+2\sqrt{3}}{(2\sqrt{2})(2)}$
 $\vec{B} = \hat{i} + \sqrt{3}\hat{j}$
 $= \frac{2(1+\sqrt{3})}{4(\sqrt{2})} = \frac{1+\sqrt{3}}{2\sqrt{2}} \Rightarrow Q = 15^\circ$

C-6. $\vec{F} = 2\hat{i} - 3\hat{j} \text{ N}$

C-7. $A(1, 1, -1)$ $B(2, -3, 4)$
 $\vec{AB} = \vec{B} - \vec{A} = (2, -3, 4) - (1, -1, 1) = (1, -4, 5)$
 $\therefore \vec{AB} = \hat{i} - 4\hat{j} + 5\hat{k}$



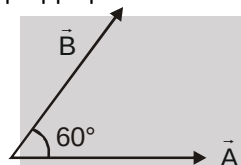
**Section (D) :**

D-1. $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ $\vec{B} = 2\hat{i} + \hat{j}$
 (a) $\vec{A} \cdot \vec{B} = (1)(2) + (1)(1) + (1)(0) = 2 + 1 = 3$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 1 & 0 \end{vmatrix} = \hat{i}(0-1) - \hat{j}(0-2) + \hat{k}(1-2)$$

$$\vec{A} \times \vec{B} = -\hat{i} + 2\hat{j} - \hat{k}$$

D-2. $|\vec{A}| = 4$ $|\vec{B}| = 3$
 $\theta = 60^\circ$
 $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 60^\circ = 4 \times 3 \times \frac{1}{2} = 6$



$$|\vec{A} \times \vec{B}| = |\vec{A}| |\vec{B}| \sin \theta = 4 \times 3 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$$

D-3. $\vec{A}$, $\vec{B}$ & $\vec{C}$ are non-zero vector
 $\vec{A} \cdot \vec{B} = 0$ & $\vec{A} \cdot \vec{C} = 0$
 $\vec{A}$ is $\perp$ to $\vec{B}$
 $\vec{A}$ is $\perp$ to $\vec{C}$ $\therefore \vec{B} \times \vec{C}$ is $\parallel$ to $\vec{A}$

D-4. $\vec{A} \cdot \vec{B} = 8$ $AB \cos \theta = 8$
 $|\vec{A} \times \vec{B}| = 8\sqrt{3}$ $AB \sin \theta = 8\sqrt{3}$
 $\tan \theta = \pm \sqrt{3}$ $\theta = 120^\circ$

**EXERCISE-2
PART - I**

1. $f(x) = \frac{x-1}{x+1}$ (i)

$$f\{f(x)\} = \frac{f(x)-1}{f(x)+1} = \frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = \frac{\frac{-2}{x+1}}{\frac{2x}{x+1}} = \frac{-1}{x}$$

2. $f(y) = f[f(x)] = \frac{2 \left[\frac{2x-3}{3x-2} \right] - 3}{3 \left[\frac{2x-3}{3x-2} \right] - 2} = x$

3. $120^\circ + \theta + \theta = 180^\circ$
 $\Rightarrow \theta = 30^\circ$
 $\frac{\sin 120}{10} = \frac{\sin \angle A}{a} = \frac{\sin \angle C}{c}$
 side $a =$ side $c = \frac{10\sqrt{3}}{3} \text{ m}$





5. (A) $\frac{3}{5} + \frac{4}{5} = \frac{4}{5} + \frac{3}{5}$ (B) $\frac{3}{5} - \frac{4}{5} = \frac{3}{5} - \frac{4}{5}$
 (C) $\frac{3}{4} + 1 \neq \frac{4}{3} - 1$ (D) $\frac{3}{4} \times \frac{4}{3} = 1$

6. $R^2 = 2A^2(1 + \cos\theta) = 2A^2\left(1 + 2\cos^2\frac{\theta}{2} - 1\right) = 2^2A^2\cos^2\frac{\theta}{2}$

$$R = 2A \cos \frac{\theta}{2}$$

7. $y = \ln x^2 + \sin x$

$$\frac{dy}{dx} = \frac{d(\ln x^2)}{dx} + \frac{d(\sin x)}{dx}$$

$$= \frac{1}{x^2} \frac{d}{dx}(x^2) + \cos x$$

$$= \frac{1}{x^2} \cdot 2x + \cos x$$

$$= \frac{2}{x} + \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d\left(\frac{2}{x}\right)}{dx} + \frac{d(\cos x)}{dx} = \frac{-2}{x^2} - \sin x$$

8. $\frac{dy}{dx} = \frac{d(x^{1/7})}{dx} + \frac{d(\tan x)}{dx} = \frac{x^{-6/7}}{7} + \sec^2 x$

$$\frac{d^2y}{dx^2} = \frac{1}{7} \frac{d(x^{-6/7})}{dx} + \frac{d(\sec^2 x)}{dx} = \frac{-6}{49} x^{-13/7} + 2 \sec x (\sec x \tan x)$$

$$= \frac{-6}{49} x^{-13/7} + 2 \tan x \sec^2 x$$

9. $y = \left(x + \frac{1}{x}\right) \left(x + \frac{1}{x} + 1\right)$

$$\frac{dy}{dx} = \left(x + \frac{1}{x}\right) \frac{d\left(x - \frac{1}{x} + 1\right)}{dx} + \left(x - \frac{1}{x} + 1\right) \frac{d\left(x + \frac{1}{x}\right)}{dx}$$

$$\left(x + \frac{1}{x}\right) + \left[\frac{dx}{dx} - \frac{d\left(\frac{1}{x}\right)}{dx} + \frac{d(1)}{dx}\right] + \left(x - \frac{1}{x} + 1\right) \left[\frac{dx}{dx} + \frac{d\left(\frac{1}{x}\right)}{dx}\right]$$

$$= \left(x + \frac{1}{x}\right) \left(1 + \frac{1}{x^2}\right) + \left(x - \frac{1}{x} + 1\right) \left(1 - \frac{1}{x^2}\right)$$

$$= x + \frac{1}{x} + \frac{1}{x} + \frac{1}{x^3} + x - \frac{1}{x} - \frac{1}{x} + \frac{1}{x^3} + 1 - \frac{1}{x^2} = \frac{2}{x^3} + 2x - \frac{1}{x^2} + 1$$

10. $r = (1 + \sec\theta) \sin\theta$

$$r = \sin\theta + \sec\theta \sin\theta$$

$$r = \sin\theta + \tan\theta$$

$$\frac{dr}{d\theta} = \cos\theta + \sec^2\theta$$





$$\begin{aligned}
 11. \quad \frac{dy}{dx} &= \frac{(1 + \cot x) \frac{d}{dx}(\cot x) - \cot x \frac{d}{dx}(1 + \cot x)}{(1 + \cot x)^2} \\
 &= \frac{(1 + \cot x)(-\operatorname{cosec}^2 x) - \cot x(-\operatorname{cosec}^2 x)}{(1 + \cot x)^2} = \frac{-\operatorname{cosec}^2 x}{(1 + \cot x)^2}
 \end{aligned}$$

$$\begin{aligned}
 12. \quad \frac{dy}{dx} &= \frac{\tan x \frac{d}{dx}(\ln x + e^x) - (\ln x + e^x) \frac{d}{dx} \tan x}{(\tan x)^2} \\
 &= \frac{\tan x \left(\frac{1}{x} + e^x \right) - (\ln x + e^x)(\sec^2 x)}{(\tan x)^2}
 \end{aligned}$$

$$\begin{aligned}
 13. \quad \frac{d}{dx}(\sin^3 x) + \frac{d}{dx}(\sin 3x) \\
 = \frac{d \sin^2 x}{d \sin x} \times \frac{d \sin x}{dx} + \frac{d \sin 3x}{d(3x)} \times \frac{d(3x)}{dx} = 3 \sin^2 x \cos x + \frac{\cos 3x}{3} \\
 = 3 \sin^2 x \cos x + 3 \cos 3x
 \end{aligned}$$

$$\begin{aligned}
 14. \quad \frac{dy}{dx} &= \frac{d}{dx} [\sin^2 (x^2 + 1)] \\
 &= \frac{d \sin^2 (x^2 + 1)}{d \sin (x^2 + 1)} \times \frac{d \sin (x^2 + 1)}{d (x^2 + 1)} \times \frac{d (x^2 + 1)}{dx} \\
 &= 4 x \sin (x^2 + 1) \cos (x^2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 15. \quad \frac{dq}{dr} &= \frac{d}{dr} (2r - r^2)^{1/2} \\
 &= \frac{d(2r - r^2)^{1/2}}{d(2r - r^2)} \times \frac{d(2r - r^2)}{dr} \\
 &= \frac{1}{2} (2r - r^2)^{-1/2} (2 - 2r) = \frac{1 - r}{\sqrt{2r - r^2}}
 \end{aligned}$$

$$\begin{aligned}
 16. \quad x^3 + y^3 &= 18xy \\
 \frac{d(x^3 + y^3)}{dx} &= \frac{d(18xy)}{dx} \\
 \frac{dx^3}{dx} + \frac{dy^3}{dx} &= 18x \cdot \frac{dy}{dx} + y \cdot \frac{d(18x)}{dx} \\
 3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 18x \cdot \frac{dy}{dx} + y \cdot 18 \\
 3x^2 + 3y^2 \cdot \frac{dy}{dx} &= 18x \cdot \frac{dy}{dx} + 18y \\
 3y^2 \frac{dy}{dx} - 18x \cdot \frac{dy}{dx} &= 18y - 3x^2 \\
 \frac{dy}{dx} (3y^2 - 18x) &= 18y - 3x^2 \\
 \frac{dy}{dx} &= \frac{(18y - 3x^2)}{(3y^2 - 18x)}
 \end{aligned}$$





17. $V = \pi r^2 h$.

$$\frac{dv}{dt} = \frac{d\pi r^2 h}{dt} = \pi \frac{dr^2 h}{dt}$$

$$= \pi \left\{ h \frac{dr^2}{dt} + r^2 \frac{dh}{dt} \right\}$$

$$= \pi \left\{ 2rh \frac{dr}{dt} + r^2 \frac{dh}{dt} \right\}$$

$$= \pi \{ 2rh \cdot 5 + r^2 \cdot 5 \} \left\{ \frac{dh}{dt} = 5 \text{ m/s} \ \& \ \frac{dr}{dt} = 5 \text{ m/s} \right\}$$

$$= \pi \{ 10rh + 5r^2 \}$$

18. $x + y = 60$

$$x = 60 - y$$

$$xy = (60 - y)y$$

$$f(y) = (60 - y)y$$

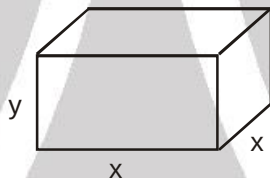
for maximum

$$f'(y) = 60 - 2y = 0$$

$$y = 30$$

$$\text{so } x = 30 \ \& \ y = 30$$

19. Let the dimensions of the tank be x and y area of the open tank $= x^2 + 4xy$.
Again x & y are related to surface area of this tank which is equal to 40 m^2



$$x^2 + 4xy = 40$$

$$y = \frac{40 - x^2}{4x}$$

$$\text{Volume } v = x^2 \left(\frac{40 - x^2}{4x} \right) = \frac{40x - x^3}{4}$$

$$\text{for maximum volume } v'(x) = \frac{40 - 3x^2}{4} = 0$$

$$x = \sqrt{\frac{40}{3}} \text{ and } v''(x) = -\frac{3x}{2}$$

$$v'' \sqrt{\frac{40}{3}} = -\frac{3}{2} \sqrt{\frac{40}{3}} < 0$$

$$\text{So volume is maximum at } x = \sqrt{\frac{40}{3}} \text{ m}$$

PART - II

1. $= \int (x^{-2} + x^{-3}) dx$

$$= \int x^{-2} dx + \int x^{-3} dx$$

$$= \frac{x^{-2+1}}{-2+1} + \frac{x^{-3+1}}{-3+1} + C$$

$$= -x^{-1} - \frac{1}{2} x^{-2} + C,$$





$$\begin{aligned}
 2. \quad \int (1 - \cot^2 x) dx &= \int 1 - (\operatorname{cosec}^2 x - 1) dx \\
 &= \int (2 - \operatorname{cosec}^2 x) dx \\
 &= \int 2 dx - \int \operatorname{cosec}^2 x dx \\
 &= 2x + \cot x + C
 \end{aligned}$$

$$\begin{aligned}
 3. \quad \int \cos \theta (\tan \theta + \sec \theta) d\theta \\
 &= \int \cos \theta \tan \theta d\theta + \int \cos \theta \sec \theta d\theta \\
 &= \int \cos \theta \frac{\sin \theta}{\cos \theta} d\theta + \int d\theta \\
 &= -\cos \theta + \theta + C
 \end{aligned}$$

$$\begin{aligned}
 4. \quad \int 12 (y^4 + 4y^2 + 1)^2 (y^3 + 2y) dy \\
 \text{By Substituting } u = y^4 + 4y^2 + 1 \\
 du = dy = \frac{d(y^4 + 4y^2 + 1)}{dy} (4y^3 + 8y) dy = 4 (y^3 + 2y) dy \\
 \text{Then, } \int 12 (y^4 + 4y^2 + 1)^2 (y^3 + 2y) dy = \int 3u^2 du \\
 = 3 \int u^2 du = \frac{3u^3}{3} + C = (y^4 + 4y^2 + 1)^3 + C
 \end{aligned}$$

$$\begin{aligned}
 5. \quad \int \frac{dx}{\sqrt{5x+8}} \\
 \text{By Substituting } 5x + 8 = u, \\
 \frac{d(5x+8)}{dx} = \frac{du}{dx} \\
 5 = \frac{du}{dx} \\
 5dx = \left(\frac{du}{dx} \right) dx \\
 dx = \frac{du}{5} \\
 \text{then } \int \frac{du}{5\sqrt{u}} = \frac{1}{5} \int u^{-1/2} du = \frac{1}{5} \frac{u^{1/2}}{1/2} + C \\
 = \frac{2}{5} \sqrt{u} + C = \frac{2}{5} \sqrt{5x+8} + C
 \end{aligned}$$

$$\begin{aligned}
 6. \quad \text{Let } u = 3 - 2s \Rightarrow du = -2 ds \\
 \int \sqrt{u} \left(\frac{-du}{2} \right) = \frac{-1}{2 \times 3/2} u^{3/2} + C = \frac{-1}{3} (3 - 2s)^{3/2} + C
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \int \sec^2 (3x + 2) dx \\
 \text{Let } u = 3x + 2 \quad du = 3dx \\
 = \int \sec^2 u \frac{du}{3} \\
 = \frac{1}{3} \tan u + C = \frac{1}{3} \tan (3x + 2) + C
 \end{aligned}$$





$$8. \int \csc\left(\frac{v-\pi}{2}\right) \cot\left(\frac{v-\pi}{2}\right) dv$$

$$\text{Let } u = \frac{v-\pi}{2}$$

$$du = \frac{dv}{2}$$

$$= \int \csc u \cot u (2du)$$

$$= -2 \csc u + C$$

$$= -2 \csc\left(\frac{v-\pi}{2}\right) + C$$

$$9. \int \frac{6 \cos t}{(2 + \sin t)^3},$$

Substituting $2 + \sin t = u$

$$\cos t = \frac{du}{dt}$$

$$\cos t \, dt = du$$

$$\text{Then } \int \frac{6du}{u^3} = 6 \int \frac{du}{u^3}$$

$$= \frac{6u^{-3+1}}{-3+1} + C$$

$$= -3 (2 + \sin t)^{-2} + C$$

$$10. \left[\frac{\theta^2}{2} \right]_{\pi}^{2\pi} = \frac{4\pi^2}{2} - \frac{\pi^2}{2} = \frac{3\pi^2}{2} \quad \text{Ans.}$$

$$11. = \left[\frac{x^3}{3} \right]_0^{3\sqrt[3]{7}}$$

$$= \frac{((7)^{1/3})^3}{3} - 0 = \frac{7}{3}$$

$$12. \text{Substituting } x^2 = u,$$

$$\text{or } 2x \, dx = du$$

$$x \, dx = \frac{du}{2}.$$

Now, changing the limit for

$$x = 0, \quad u = 0$$

$$x = \sqrt{\pi}, \quad u = 2$$

$$\text{on } \int_{u=0}^{u=\pi} \sin u \, du = [-\cos u]_0^{\pi} = -\cos \pi + \cos 0 = 2$$

$$13. \left[\frac{1}{3} \ln(3x+2) \right]_0^1$$

$$= \frac{1}{3} (\ln 5 - \ln 2)$$

$$= \frac{1}{3} \ln \frac{5}{2} = \ln \left(\frac{5}{2} \right)^{1/3}$$





14. Area of the region between the given curve & x axis on the interval $[0, b]$

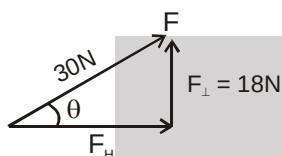
$$= \int_0^b 3x^2 dx$$

$$= \left[\frac{3x^3}{3} \right]_0^b = [x^3]_0^b = b^3 - 0 = b^3$$

PART - III

1. $\vec{B} = \lambda \vec{A} = -4 \times 3 \text{ N-E}$
 $= 12 \text{ S-W}$
 No it does not represent the same physical quantity.

2.



Given that $F_{\perp} = 18$

$$\text{From figure } F_H = \sqrt{F^2 - F_{\perp}^2} = \sqrt{30^2 - 18^2}$$

$$= \sqrt{576} = 24 \text{ N}$$

$$\text{From figure } \tan \theta = \frac{F_{\perp}}{F_H} = \frac{18}{24}$$

$$\tan \theta = 3/4 \Rightarrow \theta = 37^\circ$$

3. Let $\vec{P}$ & $\vec{Q}$ are two vector

If $\vec{Q}$ and $\vec{P}$ are collinear

$$P - Q = 10 \text{ unit} \quad \dots(1)$$

If $\vec{Q}$ and $\vec{P}$ are perpendicular

$$\sqrt{P^2 + Q^2} = 50 \text{ unit}$$

$$P^2 + Q^2 = 50^2 \quad \dots(2)$$

from equation (1)

$$(10 + Q)^2 + Q^2 = 50^2$$

$$2Q^2 + 20Q + 100 = 2500$$

$$2Q^2 + 20Q = 2400$$

$$Q^2 + 10Q - 1200 = 0$$

$$(Q + 40)(Q - 30) = 0$$

$$Q = 30$$

So, from equation (1)

$$P = 10 + Q$$

$$= 10 + 30 = 40 \text{ unit}$$

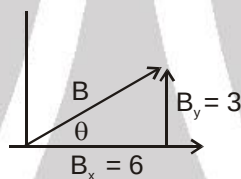
4. $\tan \alpha = \frac{\beta \sin \theta}{A + B \cos \theta}$
- $$\tan \alpha = \frac{6 \sin 90}{8 + 6 \cos 90} = \frac{1}{8} = \frac{3}{4}$$
- $$\tan \alpha = 3/4$$
- $$\alpha = 37^\circ \text{ Ans.}$$





$$\begin{aligned}
 5. \quad \vec{OA} &= r\hat{j} \\
 \vec{OC} &= r\hat{i} \\
 \vec{OB} &= r(\cos 45^\circ \hat{i} + \sin 45^\circ \hat{j}) \\
 \vec{OB} &= \frac{r}{\sqrt{2}} \hat{i} + \frac{r}{\sqrt{2}} \hat{j} \\
 \vec{R} &= \vec{OA} + \vec{OB} + \vec{OC} \\
 &= r\hat{j} + \frac{r}{\sqrt{2}} \hat{i} + \frac{r}{\sqrt{2}} \hat{j} + r\hat{i} \\
 |\vec{R}| &= \left| \left(r + \frac{r}{\sqrt{2}} \right) \hat{i} + \left(r + \frac{r}{\sqrt{2}} \right) \hat{j} \right| = r(1 + \sqrt{2})
 \end{aligned}$$

$$\begin{aligned}
 7. \quad \text{Given that} \\
 \vec{A} &= 4\hat{i} + 6\hat{j} \quad \dots\dots (1) \\
 \vec{A} + \vec{B} &= 10\hat{i} + 9\hat{j} \quad \dots\dots (2) \\
 \text{from equation (2)} \\
 \vec{B} &= 10\hat{i} + 9\hat{j} - \vec{A} \\
 &= 10\hat{i} + 9\hat{j} - (4\hat{i} + 6\hat{j}) = 6\hat{i} + 3\hat{j}
 \end{aligned}$$



$$\text{From figure } \tan \theta = \frac{B_y}{B_x} = \frac{3}{6} = \tan^{-1}\left(\frac{1}{2}\right)$$

9. Angle between two vectors can never be greater than 180°



on increasing the θ , the magnitude of resultant vectors decreases.

$$\begin{aligned}
 10. \quad \text{Initial velocity} &= 50\hat{j} \\
 \text{Final velocity} &= -50\hat{j} \\
 \text{1 change} &= 50\sqrt{2} \\
 &\text{along south west}
 \end{aligned}$$

11. Sum of any 3 sides should be greater than fourth side.

$$12. \quad a + b > |\vec{a} + \vec{b}| > a - b$$

$$\begin{aligned}
 13. \quad |\vec{A} + \vec{B}| &= |\vec{A}| = |\vec{B}| \\
 \text{Case - I Either } |\vec{A}| &= |\vec{B}| = 0 \text{ (zero vectors)} \\
 \text{Case - II } |\vec{A}| &= |\vec{B}| \neq 0 \\
 |\vec{A} + \vec{B}| &= A^2 + B^2 + AB \cos \theta \\
 |A| &= |B| \\
 &= 2A^2 + 2A^2 \cos \theta \\
 &= 2A^2 (1 + \cos \theta)
 \end{aligned}$$





$$= 2A (2\cos^2 \theta/2)$$

$$= 4A \cos^2 \theta/2$$

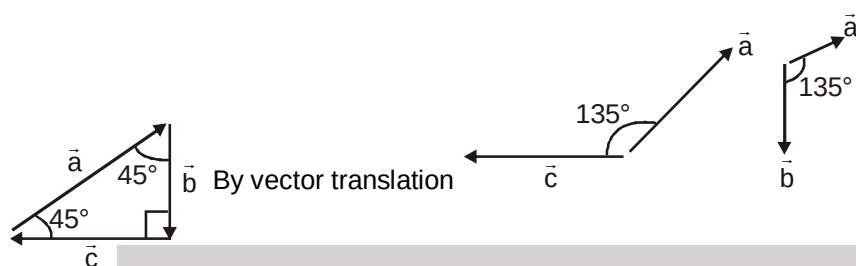
$$\text{Now } |A + B| = A$$

$$\cos \left| \frac{\theta}{2} \right| = \frac{1}{2}$$

$$\frac{\theta}{2} = 60^\circ$$

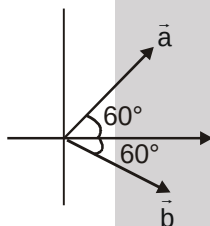
$$\theta = 120^\circ$$

14.



15.

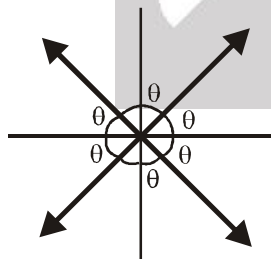
$$\therefore 90^\circ, 135^\circ, 135^\circ$$



Only horizontal along + x-axis

$$\Rightarrow 2\cos 60^\circ + 2\cos 60^\circ = 2$$

16.



Coplanar all in a single plane (xy plane)

$$6\theta = 360^\circ$$

$$\theta = 60^\circ$$

Check now every component cancels out net = 0



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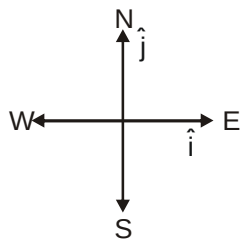
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ADVMT - 16



17.



$$\vec{A} \rightarrow -\hat{k}$$

$$\vec{B} \rightarrow +\hat{i}$$

$$\vec{A} \times \vec{B} = -\hat{k} \times \hat{i} = -\hat{j} \Rightarrow \text{south}$$

18. A vector is represented by sum of its component vectors.

$$\text{i.e., } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

so, from given options only option (A), (B), (C) are correct.

19. $a^2 + b^2 + 2ab \cos \theta = a^2 + 4b^2 - 4ab \cos \theta$

$$\text{or } \cos \theta = \frac{b}{2a} < 1$$

$$\therefore b < 2a$$

EXERCISE-3 PART - I

1. (A) $\int \sec x \tan x \, dx = \sec x + C$

(B) $\int \operatorname{cosec} kx \cot kx \, dx = \frac{-\operatorname{cosec} kx}{k} + C$

(C) $\int \operatorname{cosec}^2 kx \, dx = -\frac{\cot kx}{k} + C$

(D) $\int \cos kx \, dx = \frac{\sin kx}{k} + C$

2. (A) $|\vec{A} + \vec{B}| = A^2 + B^2 + 2AB \cos \theta$

$$A^2 = A^2 + A^2 + 2A^2 \cos \theta \quad \cos \theta = -\frac{1}{2}, \theta = 120^\circ$$

(B) $F_1 \sim F_2 \leq R \leq F_1 + F_2$

Here $F_1 \sim F_2 = 4$ and $F_1 + F_2 = 12$

(C) $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{0}{2\sqrt{2} \times 3} = 0 \quad \theta = 90^\circ$

(D) $\vec{A} + \vec{B} = 2\hat{i} + \hat{j} + 3\hat{k} \quad |\vec{A} + \vec{B}| = \sqrt{2^2 + 1 + 3^2} = \sqrt{14}$

PART - II

1. $x = t^3 - 3t^2 + 12t + 20$

$$v = \frac{dx}{dt} = 3t^2 - 6t + 12$$

$$t = 0 \Rightarrow v = 12 \text{ m/s}$$

2. $a = \frac{dv}{dt} = 6t - 6$

$$t = 0 \Rightarrow a = -6 \text{ m/s}^2$$





3. $a = 0 \Rightarrow t = 1 \text{ sec.}$
 $v = 3t^2 - 6t + 12 = 9 \text{ m/s}$
4. $F_{\text{net}} = \vec{F}_1 + \vec{F}_2 = 2\hat{i} + 5\hat{j} + 4\hat{k}$
5. $\cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_1| |\vec{F}_2|} = \left(\frac{3}{5\sqrt{2}} \right)$
6. $F_1 \cos\theta = \frac{\vec{F}_1 \cdot \vec{F}_2}{|\vec{F}_2|} = \frac{6}{5}$

PART - III

1. $|a| - |b| \leq |\vec{a} + \vec{b}| \leq |a| + |b|$
2. $R = \sqrt{A_x^2 + B_y^2} = \sqrt{g^2 + 6^2} = 10$
3. Based on theory
4. Based on theory
5. Angle between two vectors given by.
 $\cos\theta = \frac{\vec{A} \cdot \vec{B}}{AB}$
 $\text{so } \cos\theta = \frac{(\hat{i} + \hat{j}) \cdot \hat{k}}{|\hat{i} + \hat{j}| |\hat{k}|} = \frac{0}{\sqrt{2} \times 1} = 0$
 $\cos\theta = 0 \quad \theta = \frac{\pi}{2}$
6. Based on theory.

PART - IV

1. (i) if $f(x) = e^{-x}$
then $f'(x) = -e^{-x}$
so $f(x) = -f'(x)$
(ii) if $f(x) = e^x$ then $f'(x) = e^x$
so $f(x) = f'(x)$
(iii) Based on theory
(iv) Based on theory
(v) Based on theory

PART - V

1. $\vec{A} = A\hat{i} \quad \vec{B} = A\hat{j} \quad \vec{C} = A\hat{k}$
 $\vec{A} + \vec{B} + \vec{C} = A\hat{i} + A\hat{j} + A\hat{k}$
 $|\vec{A} + \vec{B} + \vec{C}| = \sqrt{A^2 + A^2 + A^2} = \sqrt{3} A$
2. Given $\vec{A} = 3\hat{i} + 4\hat{j} \quad \vec{B} = 7\hat{j} + 24\hat{j}$
Let $\vec{C} = |\vec{C}| \hat{C}$
Given that $|\vec{C}| = |\vec{B}| = \sqrt{7^2 + (24)^2} = 25$
and $\hat{C} = \hat{A} = \frac{3\hat{i} + 4\hat{j}}{5}$
 $\vec{C} = \frac{25 \times (3\hat{i} + 4\hat{j})}{5} \quad \vec{C} = 15\hat{i} + 20\hat{j}$





3. If $|\vec{A}\vec{B}|$

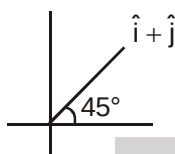
then angle between $\vec{A}$ and $\vec{B}$ is equal to zero

So, $\vec{A} \times \vec{B} = AB \sin \theta \hat{n} = 0$

4. Area of parallelogram = $|\vec{A} \times \vec{B}| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 2 & 0 & -4 \end{vmatrix} = |-8\hat{i} + 12\hat{j} - 4\hat{k}| = \sqrt{224}$

5. Perpendicular

6.



7. $\vec{A} = a\hat{i}$ and $\vec{B} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$

$$|\vec{A} + \vec{B}| = \sqrt{3} |\vec{A} - \vec{B}|$$

$$\sqrt{(a + a\cos\omega t)^2 + (a\sin\omega t)^2} = \sqrt{3} \sqrt{(a - a\cos\omega t)^2 + (a\sin\omega t)^2}$$

$$\Rightarrow 2\cos\frac{\omega t}{2} = \pm\sqrt{3} \times 2\sin\frac{\omega t}{2}$$

$$\tan\frac{\omega t}{2} = \pm\frac{1}{\sqrt{3}}$$

$$\frac{\omega t}{2} = n\pi \pm \frac{\pi}{6}$$

$$\frac{\pi}{12}t = n\pi \pm \frac{\pi}{6}$$

$$t = (12n \pm 2)s$$

$$= 2s, 10s, 14s, \dots$$

HIGH LEVEL PROBLEMS (HLP) PART - I

1. $g'(x) = \frac{(x+0.5) \frac{d}{dx}(x^2-4) - (x^2-4) \frac{d}{dx}(x+0.5)}{(x+0.5)^2}$

$$= \frac{(x+0.5)(2x) - (x^2-4)}{(x+0.5)^2}$$

$$= \frac{2x^2 + x - x^2 + 4}{(x+0.5)^2} = \frac{x^2 + x + 4}{(x+0.5)^2}$$

2. (a) $\frac{d}{dx}(uv) = uv' + vu'$

at $x = 1$ $= u(1)v'(1) + v(1)u'(1)$

$$= 2(-1) + 5(0)$$

$$= -2$$





$$(b) \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{vu' - uv'}{v^2}$$

$$\text{at } x = 1 \quad \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v(1) u'(1) - u(1)v'(1)}{[v(1)]^2}$$

$$= \frac{5(0) - (2)(-1)}{(5)^2} = \frac{2}{25}$$

$$(c) \frac{d}{dx} \left(\frac{v}{u} \right) = \frac{uv' - vu'}{u^2} = \frac{uv'(1) - v(1)u'(1)}{[u(1)]^2}$$

$$(d) \frac{d}{dx} (7v - 2u) = 7v' - 2u'$$

$$\text{at } x = 1$$

$$= 7v'(1) - 2u'(1)$$

$$7(-1) - 2(0) = -7$$

$$3. \quad S = \frac{1 + \operatorname{cosec} t}{1 - \operatorname{cosec} t}$$

$$\frac{ds}{dt} = \frac{(1 - \operatorname{cosec} t)(-\operatorname{cosec} t \cot t) - (1 + \operatorname{cosec} t)(+\operatorname{cosec} t \cot t)}{(1 - \operatorname{cosec} t)^2}$$

$$= \frac{-2 \operatorname{cosec} t \cot t}{(1 - \operatorname{cosec} t)^2}$$

$$4. \quad S = \frac{\sin t}{1 - \cos t}$$

$$\frac{ds}{dt} = \frac{(1 - \cos t) \cos t - \sin t (\sin t)}{(1 - \cos t)^2}$$

$$= \frac{\cos t - (\cos^2 t + \sin^2 t)}{(1 - \cos t)^2} = \frac{\cos t - 1}{(1 - \cos t)^2} = \frac{1}{\cos t - 1}$$

$$5. \quad \text{With } u = \sin x, y = u^3 : \frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx} = 3u^2 \cos x = 3 \sin^2 x (\cos x)$$

$$6. \quad \text{With } u = \cos x$$

$$y = 5u^{-4}$$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

$$= 5(-4)u^{-5}(-\sin x)$$

$$\frac{dy}{dx} = 20 \sin x \cos^{-5} x$$

$$7. \quad r = (\csc \theta + \cot \theta)^{-1}$$

$$r = \frac{1}{\csc \theta + \cot \theta}$$

$$\frac{dr}{d\theta} = \frac{(\cot \theta + \operatorname{cosec} \theta)(0) - 1(-\operatorname{cosec}^2 \theta - \operatorname{cosec} \theta \cot \theta)}{(\cot \theta + \operatorname{cosec} \theta)^2}$$

$$\frac{dr}{d\theta} = \frac{\operatorname{cosec} \theta}{\cot \theta + \operatorname{cosec} \theta}$$



8. $r = -(\sec\theta + \tan\theta)^{-1}$

$$r = \frac{-1}{\sec\theta + \tan\theta}$$

$$\frac{dr}{d\theta} = \frac{1}{(\sec\theta + \tan\theta)^2} = \frac{\sec\theta}{\sec\theta + \tan\theta}$$

PART - II

1. (a) $\int \csc x \cot x dx$

$$= -\csc x + C$$

(b) $\int -\csc 5x \cot 5x dx$

$$= \frac{\csc 5x}{5} + C$$

(c) $\int -\pi \csc \frac{\pi}{2} x \cot \frac{\pi}{2} x dx$

$$= \frac{+\pi \csc \frac{\pi}{2} x}{\pi/2} = 2 \csc \frac{\pi}{2} x + C$$

2. $\int (1 + 2\cos x)^2 dx$

$$= \int (1 + 4\cos^2 x + 4\cos x) dx$$

$$= \int 1 dx + \int 4\cos x dx + \int 4\cos^2 x dx$$

$$= x + 4\sin x + \int 4\left(\frac{\cos 2x + 1}{2}\right) dx$$

$$= x + 4\sin x + \int 2\cos 2x + \int 2 dx$$

$$= x + 4\sin x + \frac{2\sin 2x}{2} + 2x$$

$$= 3x + 4\sin x + \sin 2x + C$$

3. $\int \frac{x^{1/2}}{2} dx + \int 2x^{-1/2} dx$

$$= \frac{1}{2} \frac{x^{1/2+1}}{(1/2+1)} + 2 \frac{x^{-1/2+1}}{-\frac{1}{2}+1} + c = \frac{1}{3} x^{3/2} + 4x^{1/2} + c$$

4. $\int 8y dy - \int \frac{2}{y^{1/4}} dy$

$$\frac{8y^2}{2} - \frac{2 y^{-1/4+1}}{-\frac{1}{4}+1} + c$$

$$4y^2 - \frac{8}{3} y^{3/4} + c$$

5. $\int [(2x - (2x)x^{-3})] dx$

$$= \int 2x dx - \int 2x^{-2} dx$$

$$= \frac{2x^2}{2} - \frac{2x^{-2+1}}{-2+1} = x^2 + \frac{2}{x} + c$$





$$10. \quad \int 4 \sec x \tan x \, dx - 2 \int \sec^2 x \, dx$$

$$= 4 \sec x - 2 \tan x + c$$

$$11. \quad \int \frac{1}{2} \csc^2 x \, dx - \frac{1}{2} \int \csc x \cot x \, dx$$

$$= -\frac{1}{2} \cot x + \frac{1}{2} \csc x + c$$

$$12. \quad \int \sin 2x \, dx - \int \csc^2 x \, dx$$

$$= -\frac{\cos 2x}{2} + \cot x + c$$

$$13. \quad \int 2 \cos 2x \, dx - \int 3 \sin 3x \, dx$$

$$\frac{2 \sin 2x}{2} - 3 \left(\frac{-\cos 3x}{3} \right) + c$$

$$\sin 2x + \cos 3x + c$$

$$14. \quad \int 4 \sin^2 y \, dy$$

$$\therefore \sin^2 y = \frac{1 - \cos 2y}{2}$$

$$\int 4 \left(\frac{1 - \cos 2y}{2} \right) dy$$

$$= \int 2 \, dy - \int 2 \cos 2y \, dy$$

$$= 2y - \frac{2 \sin 2y}{2}$$

$$= 2y - \sin 2y + c$$

$$15. \quad \int \frac{\csc \theta}{\csc \theta - \sin \theta} d\theta$$

$$= \int \frac{1}{\frac{1}{\sin \theta} - \sin \theta} d\theta$$

$$= \int \frac{1}{1 - \sin^2 \theta} d\theta = \int \sec^2 \theta \cdot d\theta = \tan \theta + c$$

$$16. \quad \int \frac{1}{\sqrt{5s+4}} ds$$

$$\text{Let } u = 5s + 4$$

$$du = 5 \, ds$$

$$ds = \frac{du}{5}$$

$$= \frac{1}{5} \int u^{-1/2} du = \frac{1}{5} \frac{u^{-1/2+1}}{-1/2+1} + c$$

$$= \frac{2}{5} \sqrt{u} + c = \frac{2}{5} \sqrt{5s+4} + c$$





17. $\int 3y \sqrt{7-3y^2} dy$

Let $7-3y^2 = t^2$

diff. w.r.t. y both side

$$-6y dy = 2t dt$$

$$= 3y dy = -t dt$$

$$= \int -t^2 dt$$

$$= -\frac{t^3}{3} + c$$

$$= -\frac{(7-3y^2)^{3/2}}{3} + c$$

18. $\int \sin^5 \frac{x}{3} \cos \frac{x}{3} dx$

Let $\sin \frac{x}{3} = t$

$$\cos \frac{x}{3} \frac{dx}{3} = dt$$

$$\cos \frac{x}{3} dx = 3 dt$$

$$= \int t^5 3 dt$$

$$= 3 \frac{t^6}{6} + c = \frac{\sin^6 \frac{x}{3}}{2} + c$$

19. $\int \tan^7 \left(\frac{x}{2} \right) \sec^2 \frac{x}{2} dx$

Let $\tan \frac{x}{2} = t$

$$\sec^2 \frac{x}{2} \cdot \frac{dx}{2} = dt$$

$$\sec^2 \frac{x}{2} dx = 2 dt$$

$$\int t^7 2 dt = 2 \frac{t^8}{8} + c = \frac{\tan^8 x/2}{4} + c$$

20. $\int r^2 \left(\frac{r^3}{18} - 1 \right)^5 dr$

Let $\frac{r^3}{18} - 1 = t$

$$\frac{3r^2}{18} dr = dt$$

$$r^2 dr = 6 dt$$

$$= \int t^5 6 dt$$

$$= 6 \frac{t^6}{6} + c = \left(\frac{r^3}{18} - 1 \right)^6 + c$$





21. $\int r^4 \left(7 - \frac{r^5}{10} \right)^3 dr$

Let $7 - \frac{r^5}{10} = t$

$-\frac{5r^4}{10} dr = dt$

$r^4 dr = -2 dt$

$\int t^3 (-2dt)$

$= -2 \frac{t^4}{4} + c$

$= -\frac{\left(7 - \frac{r^5}{10} \right)^4}{2} + c$

22. $\int x^{\frac{1}{3}} \sin(x^{\frac{4}{3}} - 8) dx$

Let $x^{\frac{4}{3}} - 8 = t$

$\frac{4}{3} x^{\frac{1}{3}} dx = dt$

$x^{\frac{1}{3}} dx = \frac{3}{4} dt$

$\int \frac{3}{4} \sin t dt$

$= -\frac{3}{4} \cos t + c$

$= -\frac{3}{4} \cos (x^{\frac{4}{3}} - 8) + c$

23. $\int \csc \left(\frac{v-\pi}{2} \right) \cot \left(\frac{v-\pi}{2} \right) dv$

Let $\frac{v-\pi}{2} = t \Rightarrow dv = 2dt$

$\int \csc t \cot t \cdot 2dt = -2 \csc t$

$= -2 \csc \left(\frac{v-\pi}{2} \right) + c$

24. $\int \sqrt{\cot y} \csc^2 y dy$

Let $\cot y = t$

$-\csc^2 y dy = dt$

$-\int \sqrt{t} dt$

$= -\frac{2t^{3/2}}{3} + c$

$= -\frac{2}{3} (\cot y)^{3/2} + c$



25. $\int \frac{\sec z \tan z}{\sqrt{\sec z}} dz$

Let $\sec z = t$
 $\sec z \tan z dz = dt$

$$\int \frac{1}{\sqrt{t}} dt$$

$$= 2\sqrt{t} + c$$

$$= 2\sqrt{\sec z} + c$$

26. $\int \frac{1}{t^2} \cos\left(\frac{1}{t} - 1\right) dt$

Let $\frac{1}{t} - 1 = x$

$$= -\frac{1}{t^2} dt = dx$$

$$\int \cos x (-dx)$$

$$= -\sin x + c$$

$$= -\sin\left(\frac{1}{t} - 1\right) + c$$

27. $\int_{1/2}^{3/2} (-2x + 4) dx$

$$= \left[-x^2 + 4x \right]_{1/2}^{3/2}$$

$$= \left[-\left(\frac{3}{2}\right)^2 + 4\left(\frac{3}{2}\right) \right] - \left[-\frac{1}{4} + 4 \times \frac{1}{2} \right]$$

$$= 2 \text{ square units}$$

28. $\int_{-2}^1 |x| dx$

$$\int_{-2}^0 -x dx + \int_0^1 x dx = \left[-\frac{x^2}{2} \right]_{-2}^0 + \left[\frac{x^2}{2} \right]_0^1$$

$$= 2 + 1/2 = 2.5 \text{ square units}$$

29. $\int_0^{\pi/2} \theta^2 d\theta \Rightarrow \left[\frac{\theta^3}{3} \right]_0^{\pi/2} \Rightarrow \frac{\pi^3}{24}$

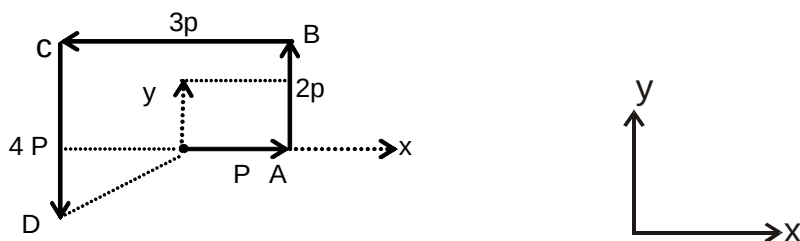
30. $\int_0^{3b} x^2 dx \Rightarrow \left[\frac{x^3}{3} \right]_0^{3b} = 9b^3$





PART - III

1.



Let O be the origin & unit vector along x axis is $\hat{i}$ & unit vector along y axis is $\hat{j}$

So, $\vec{OA} = P\hat{i}$

$$\vec{AB} = 2P\hat{j}$$

$$\vec{BC} = -3P\hat{i}$$

$$\vec{CD} = -4P\hat{j}$$

Resultant force

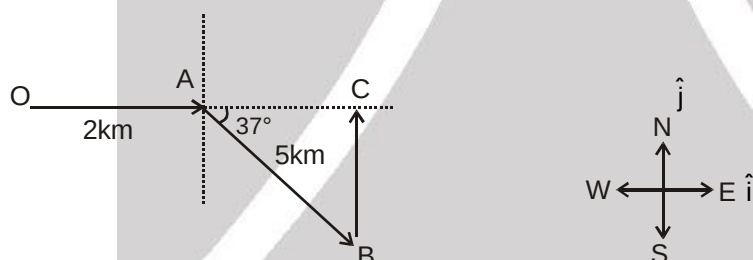
$$\vec{R} = \vec{OA} + \vec{AB} + \vec{BC} + \vec{CD}$$

$$= P\hat{i} + 2P\hat{j} - 3P\hat{i} - 4P\hat{j}$$

$$\vec{R} = -2P\hat{i} - 2P\hat{j}$$

$$|\vec{R}| = 2\sqrt{2} P \quad \text{Ans.}$$

2.



Final displacement of boat = $6\hat{i}$, from figure

$$\vec{OA} = 2\hat{i}$$

$$\vec{BC} = 5\cos 37^\circ \hat{i} - 5\sin 37^\circ \hat{j} = 4\hat{i} - 3\hat{j}$$

from figure

$$\vec{OC} = \vec{OA} + \vec{AB} + \vec{BC}$$

$$\vec{BC} = \vec{OC} - \vec{OA} - \vec{AB}$$

$$= 6\hat{i} - 2\hat{i} - 4\hat{i} + 3\hat{j}$$

$$\vec{BC} = 3\hat{j}$$

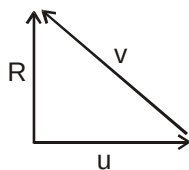
$$= 3 \text{ km in north}$$





3. Given $\vec{R} = \vec{u} + \vec{v}$ and $\vec{R} \perp \vec{u}$

$$\text{and } |\vec{R}| = \frac{|\vec{v}|}{2}$$



$$\text{Let } |\vec{v}| = x \text{ then } |\vec{R}| = x/2$$

$$\text{from figure } u^2 = v^2 - R^2 = x^2 - \frac{x^2}{4} \Rightarrow u = \frac{\sqrt{3}}{2} x$$

we know that

$$R^2 = u^2 + v^2 + 2uv\cos\theta$$

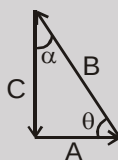
$$\frac{x^2}{4} = \frac{3}{4}x^2 + x^2 + 2\left(\frac{\sqrt{3}}{2}x\right)(x)\cos\theta$$

$$\Rightarrow \cos\theta = -\frac{\sqrt{3}}{2}$$

$$\theta = 150^\circ$$

4. Given that $|\vec{A}| = 5\text{N}$, $|\vec{B}| = 13\text{N}$ & $|\vec{C}| = 12\text{N}$

$$\text{for angle between } \vec{A} \text{ \& } \vec{B}, \text{ from figure } \sin\alpha = \frac{5}{13}$$



$$\alpha = 23^\circ$$

$$\text{so angle } \theta = 90 - 23$$

$$\text{so angle between } \vec{A} \text{ \& } \vec{B} = 180 - \theta = 180 - 90 + 23 = 90 + 23 = 113^\circ$$

5. From given condition

$$|\vec{A} + \vec{B}| = |\vec{A}| = |\vec{B}| = x$$

$$A^2 + B^2 + 2AB\cos\theta = A^2$$

$$x^2 + x^2 + 2x^2\cos\theta = x^2$$

$$\cos\theta = -1/2$$

$$\theta = 120^\circ$$

$$|\vec{A} - \vec{B}| = |\vec{A}|$$

$$A^2 + B^2 - 2AB\cos\theta' = A^2$$

$$x^2 + x^2 - 2x^2\cos\theta' = x^2$$

$$\cos\theta' = 1/2$$

$$\theta' = 60^\circ$$





6. Let θ be the angle between $\vec{P}$ and $\vec{Q}$. Then.

$$R^2 = |\vec{P} + \vec{Q}|^2 = P^2 + Q^2 + 2PQ \cos \theta \quad \dots(i)$$

If $\vec{Q}$ is doubled, $\vec{R}$ is doubled. That means, the magnitude of resultant of $2\vec{Q}$ and $\vec{P}$ is $2R$

$$(2R)^2 = P^2 + (2Q)^2 + 2P(2Q) \cos \theta$$

This yields, $4R^2 = P^2 + 4Q^2 + 4PQ \cos \theta \quad \dots(ii)$

When $\vec{Q}$ is reversed, $\vec{R}$ is doubled. Hence, the magnitude of resultant of $\vec{P}$ and $(-\vec{Q})$ is $2R$.

Then, $(2R)^2 = P^2 + Q^2 + 2PQ \cos (180^\circ - \theta).$

This yields $4R^2 = P^2 + Q^2 - 2PQ \cos \theta \quad \dots(iii)$

Eq. (i) – Eq. (iii) yields $PQ \cos \theta = \frac{-3R^2}{4} \quad \dots(iv)$

Eq. (i) + Eq. (iii) yields $P^2 + Q^2 = \frac{5R^2}{2} \quad \dots(v)$

Eq. (ii) + Eq (iv) yields $P^2 + 4Q^2 = 7R^2 \quad \dots(vi)$

Solving Eq. (v) and Eq. (vi) we obtain $Q = \frac{\sqrt{3}}{2} R$ and $P = R$

Hence $P : Q : R = \sqrt{2} : \sqrt{3} : \sqrt{2}$ **Ans.**

7. It is relevant from the vector diagram that each vector deviates from its neighbour by an angle of 60° . We bring the tails of each vector to a point (origin) and observe that $\vec{A}_1$ and $\vec{A}_4$ are equal and opposite; and $\vec{A}_2$ and $\vec{A}_5$ are equal and opposite.



$$\vec{A}_1 + \vec{A}_4 = \vec{0} \text{ and } \vec{A}_2 + \vec{A}_5 = \vec{0}$$

Hence $\vec{R} = \vec{A}_1 + \vec{A}_2 + \vec{A}_3 + \vec{A}_4 + \vec{A}_5$

$$= \vec{A}_3 = (|\vec{A}_3| \cos 60^\circ) (-\hat{i}) + (|\vec{A}_3| \sin 60^\circ) \hat{j}$$

Substituting $|\vec{A}_3| = 1$, we have, $\vec{A}_3 = \frac{1}{2}(-\hat{i} + \sqrt{3}\hat{j})$

8. The forces exerted by the ants are given as,

$$\vec{F}_1 = 3 (\cos 30^\circ \hat{i} + \sin 30^\circ \hat{j})$$

$$\vec{F}_2 = -1 \hat{i}$$

$$\vec{F}_3 = 2 \cos 45^\circ \hat{i} + 2 \sin 45^\circ (-\hat{j})$$

and $(= F) = x \hat{i} + y \hat{j}$, say.

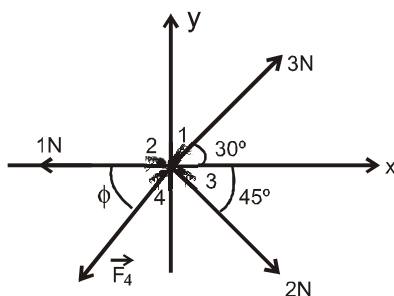
Since the grain is in equilibrium, it experiences a net force

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

Substituting the values of the above forces, we have

$$\hat{i} \left(\frac{3\sqrt{3}}{2} - 1 + \sqrt{2} + x \right) + \left(\frac{3}{2} - \sqrt{2} + y \right) \hat{j} = 0$$





Comparing the coefficients of $\hat{i}$ and $\hat{j}$, we have

$$x = -\left(\frac{3\sqrt{3}}{2} + \sqrt{2} - 1\right) \text{ and } y = -\left(\frac{3}{2} - \sqrt{2}\right)$$

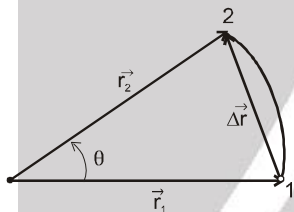
Hence the force exerted by the 4th ant has the magnitude

$$F_4 = \sqrt{x^2 + y^2} = \sqrt{\left(\frac{3\sqrt{3}}{2} + \sqrt{2} - 1\right)^2 + \left(\frac{3}{2} - \sqrt{2}\right)^2}$$

and directed at an angle $\phi = \tan^{-1}\left[\frac{3 - 2\sqrt{2}}{3\sqrt{3} + 2\sqrt{2} - 2}\right]$ with -ve x-direction, as shown in the figure.

9. When the insect moves from position 1 to position 2, the displacement $\vec{s}$ = change in position vector $\Delta\vec{r}$. Since $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$, the magnitude of the displacement is

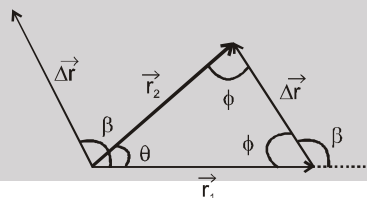
$$|\Delta\vec{r}| = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos \theta}$$



Since the insect moves in a circular path,

$$r_1 = r_2 = R$$

Hence $|\Delta\vec{r}| = 2R \sin \theta/2$. The direction of $\Delta\vec{r}$ is given as $\beta = \frac{\pi}{2} + \frac{\theta}{2}$, as shown in the figure.



10. Let us draw a straight line PQ which is given by

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

where

$$\vec{OQ} = \vec{r} \text{ and } \vec{OP} = \vec{r}_0 \text{ (given)}$$

This gives

$$\vec{PQ} = \vec{r} - \vec{r}_0$$

Since

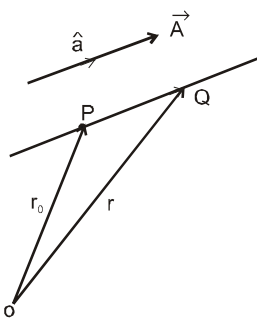
$$\vec{PQ} \parallel \vec{A} \text{ (given)}$$

$$\vec{PQ} = PQ \cdot \hat{a}, \text{ where } \hat{a} = \frac{\vec{A}}{A}$$





Hence $\vec{r} - \vec{r}_0 = \frac{PQ}{A} \hat{a}$ This yields



$\vec{r} = \vec{r}_0 + \frac{PQ}{A} \hat{a}$. Then by putting $\frac{PQ}{A} = n$, we obtain $\vec{r} = \vec{r}_0 + n\hat{a}$

11. $\vec{A} + \vec{B} = \vec{R}$, say,

Since $|\vec{R}|^2 = \vec{R} \cdot \vec{R} = (\vec{A} + \vec{B}) \cdot (\vec{A} + \vec{B})$

expanding the product, we have

$$|\vec{R}|^2 = \vec{A} \cdot \vec{A} + \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} + \vec{B} \cdot \vec{B}$$

Substituting $\vec{A} \cdot \vec{A} = A^2$, $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$ and $\vec{B} \cdot \vec{B} = |\vec{B}|^2$, we have

$$|\vec{R}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2\vec{A} \cdot \vec{B}$$

Hence $|\vec{R}| = |\vec{A} + \vec{B}| = \sqrt{|\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta}$

12. Differentiating both sides of $\vec{A} = t\hat{i} - \sin\pi t\hat{j} + t^2\hat{k}$

$$\begin{aligned} \text{we have } \frac{d\vec{A}}{dt} &= \frac{d}{dt}(t\hat{i} - \sin\pi t\hat{j} + t^2\hat{k}) \\ &= \frac{d}{dt}(t)\hat{i} - \frac{d}{dt}(\sin\pi t)\hat{j} + \frac{d}{dt}(t^2)\hat{k} \\ &= \hat{i} - \pi\cos\pi t\hat{j} + 2t\hat{k} \end{aligned}$$

$$\begin{aligned} \text{Putting } t = 1, \text{ we have } \left. \frac{d\vec{A}}{dt} \right|_{t=1} &= \hat{i} - (\pi\cos\pi)\hat{j} + 2(1)\hat{k} \\ &= \hat{i} + \pi\hat{j} + 2\hat{k} \end{aligned}$$

13. We have

Position : $s = 5 \cos t$

$$\text{Velocity : } v = \frac{ds}{dt} = \frac{d}{dt}(5 \cos t) = 5 \frac{d}{dt}(\cos t) = -5 \sin t$$

$$\text{Acceleration : } a = \frac{dv}{dt} = \frac{d}{dt}(-5 \sin t) = -5 \frac{d}{dt}(\sin t) = -5 \cos t$$





14. (a) volume v of water remains constant

$$v = \pi (4R^2) h - \pi R^2 x$$

$$0 = 16\pi R^2 \frac{dh}{dt} - \pi R^2 \frac{dx}{dt} = 0$$

$$\frac{dh}{dt} = \frac{1}{16} \frac{v}{dt}$$

- (b) Area of wet surface ℓ

$$A = \pi R^2 + 2\pi R \frac{d\ell}{dt}$$

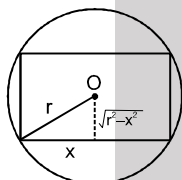
ℓ is length inside water

$$\frac{dA}{dt} = 0 + 2\pi R \frac{d\ell}{dt} \quad \therefore \quad \frac{d\ell}{dt} = \frac{dx}{dt} + \frac{dh}{dt}$$

$$\frac{dA}{dt} = 2\pi R \left[v + \frac{v}{16} \right] = \frac{17\pi R v}{8}$$

Ans. (a) $\frac{dh}{dt} = \frac{v}{15}$ (b) $\frac{32\pi R v}{15}$

- 15.



Let A = area of rectangle

$$A = 8 \times \frac{1}{2} \times x \times \sqrt{r^2 - x^2} = 4x \sqrt{r^2 - x^2}$$

For maxima :

$$\frac{dA}{dx} = 0, \quad \frac{dA}{dx} = 4 \left(\frac{x(-2x)}{2\sqrt{r^2 - x^2}} + \sqrt{r^2 - x^2} \right)$$

$$0 = \frac{4(r^2 - 2x^2)}{\sqrt{r^2 - x^2}}$$

or $x = \frac{r}{\sqrt{2}}$

For $x = \frac{r}{\sqrt{2}}, \quad \frac{d^2A}{dx^2} < 0$

$\therefore$ Area is maximum for $x = \frac{r}{\sqrt{2}}$

and $A_{\max} = \frac{4r}{\sqrt{2}} \sqrt{r^2 - \left(\frac{r}{\sqrt{2}}\right)^2}$

$$A_{\max} = 2r^2.$$

