SOLUTIONS OF PROJECTILE MOTION
EXERCISE-1
PART - I

SECTION (A)

A-1. $T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

$\frac{T_1}{T_2} = \frac{\sin\theta}{\sin(90-\theta)}$ = tan θ

{or $T_1 : T_2 = \tan\theta : 1$

A-2. $H = \frac{u^2\sin^2\theta}{2g}$, $H_1 = \frac{u^2\sin^2\theta}{2g}$, $H_2 = \frac{u^2\sin^2(90-\theta)}{2g}$

$\frac{H_1}{H_2} = \frac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$ $H_1 : H_2 = \tan^2\theta : 1$

A-3. Horizontal Range $R = \frac{u^2\sin 2\theta}{g}$

Vertical height $H = \frac{u^2\sin^2\theta}{2g}$

given R = H

So $\frac{u^2\sin 2\theta}{g} = \frac{u^2\sin^2\theta}{2g}$

$2\times 2\sin\theta\cos\theta = \sin^2\theta$

tan θ = 4

A-4. R same for θ_1 & θ_2

$\theta_2 = 90 - \theta_1$

$T = \frac{2u\sin\theta}{g}$ ∴ $T_1 = \frac{2u\sin\theta}{g}$; $T_2 = \frac{2u\sin(90-\theta)}{g}$

&, $R = \frac{u^2\sin 2\theta}{g}$; $T_1T_2 = \frac{2^2\times u^2\sin\theta\cos\theta}{g^2} = \frac{2}{g}\left(\frac{u^2\sin 2\theta}{g}\right)$

$T_1T_2 = \frac{2R}{g}$ **Ans**

A-5. R_{max} = 100 m (given) H_{max} = ? (for any θ)

$R_{max} = \frac{u^2\sin 90}{g} = 100 \Rightarrow u^2 = 1000$ (θ = 45º for maximum range)

∴ $(H)_{max} = \frac{u^2(\sin^2\theta)_{max}}{2g} = \frac{u^2}{2g}$ (θ = 90º for maximum height)

$= \frac{1000}{20}$

⇒ H_{max} = 50 m **Ans**

A-6. (1) $\theta = 45^o$ $u = 20$ m/s

$$T = \frac{2u_y}{g} = \frac{2 \times 20 \times \frac{1}{\sqrt{2}}}{10} = 2\sqrt{2}\ s \qquad u_x = 20 \times \frac{1}{\sqrt{2}}$$

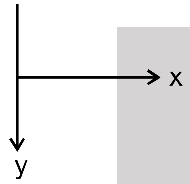
Now, $R = \left(20 \times \frac{1}{\sqrt{2}}\right) \times 2\sqrt{2} = 40$ m

$\Rightarrow$ The man should come (travel) 60 – 40 = 20 m

time $2\sqrt{2}$ s & vel = $\frac{20m}{2\sqrt{2}s} = 5\sqrt{2}$ m/s

SECTION (B) :

B-1. $u_x = 98$ m/s



(i) H = 490 m, g = 9.8 m/s^2, $u_y = 0$, $a_y = g = 9.8$ m/s^2

$s_y = u_y t + 1/2 a_y t^2$

$\therefore$ $490 = 0 + 1/2 \times 9.8\, t^2$, $100 = t^2$ $\Rightarrow$ $t = \pm 10$

Ignoring "–ve" value, as it gives time before the time of projection, we get t = 10 s **Ans**

(ii) Distance from the hill = $u_x \times T = 98 \times 10 = 980$ m **Ans**

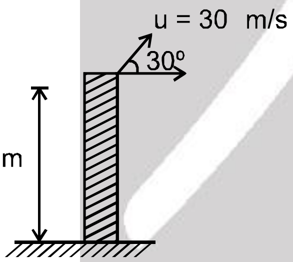
(iii) $V = \sqrt{V_x^2 + V_y^2}$ $V_x = u_x = 98$ m/s $V_y^2 = u_y^2 + 2a_y s_y$

$V_y^2 = 0 + 2 \times 9.8 \times 490$,

So $V = \sqrt{98^2 + 2 \times 9.8 \times 490}$,

$V = 98\sqrt{2}$ m/s. **Ans**

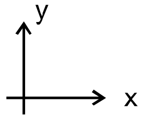
B-2.



$$H = \frac{u^2 \sin^2\theta}{2g} = \frac{30 \times 30 \times \frac{1}{2} \times \frac{1}{2}}{2 \times 10} = \frac{90}{8} = 11.25$$

$\therefore$ H from ground H = 50 + 11.25 = 61.25 m. **Ans**

(ii) $s_x = u_x T +\ a_x T^2$, $a_x = 0$ $\Rightarrow$ $s_x = u_x T$



To find T $s_y = u_y T + \frac{1}{2} a_y T^2$ Where, $s_y = -50$ = vertical displacement

T $s_y = u_y T + \frac{1}{2} a_y T^2$ $s_y = -50 =$

$u_y = u \sin 30^o = 15$ m/s , $a_y = -g = -10$ m/s^2

Substituting these values,

$-50 = 15T + \frac{1}{2}(-10)T^2$; or $T^2 - 3T - 10 = 0$; or, $T^2 - 5T + 2T - 10 = 0$;

or, $T(T-5) + 2(T-5) = 0$; or $(T-5)(T+2) = 0$; or, $T = 5$ or $T = -2$

$\Rightarrow$ T = 5 sec **Ans**

$s_x = u\cos\theta \cdot T = 30 \times \cos 30^\circ \times T = 30 \times \frac{\sqrt{3}}{2} \times 5 = 75\sqrt{3}$ m **Ans**

SECTION (C) :

C-1. $y = \sqrt{3}\,x - g\frac{x^2}{2}$, from the given (above) eq. with the standard equation of trajectory

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

we get $\sqrt{3} = \tan\theta \Rightarrow \theta = 60^\circ$

$u^2\cos^2\theta = 1$, Putting $\theta = 60^\circ$ we get $u^2 = \frac{1}{(1/2)^2} \Rightarrow u = 2$m/s.

Alternate Solution

$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$

In this eq. at t = 0, x = 0, y = 0 ; $a_x = 0$; $a_y = -g$

using these conditions in the given equation we get.

$$\frac{dy}{dx} = \sqrt{3} - \frac{1}{2}g2x\frac{dx}{dx}$$

To find θ, we now find $\tan\theta = \left.\frac{dy}{dx}\right]_{at\ t=0}$

$\tan\theta = \left.\frac{dy}{dx}\right]_{at\ t=0}$

$\therefore \left.\frac{dy}{dx}\right]_{t=0} = \sqrt{3} - 0$ $\{\because x = 0 \text{ at } t = 0\}\{\because t = 0\ x = 0\}$

$\tan\theta = \sqrt{3} \Rightarrow \theta = 60^\circ$ **Ans.**

$\frac{dy}{dt} = \sqrt{3}\frac{dx}{dt} - \frac{1}{2}g\left[2x\left(\frac{dx}{dt}\right)\right]$

$V_y = \sqrt{3}\ V_x - gx\,V_x$

At t = 0, x = 0, $V_y = u_y$ & $V_x = u_x$; $u_y = \sqrt{3}\ u_x$

$\frac{d^2y}{dt^2} = \sqrt{3}\frac{d^2x}{dt^2} - g\left[x\frac{d^2x}{dt^2} + \frac{dx}{dt}\times\frac{dx}{dt}\right]$ here $a_x = \frac{d^2x}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} = \sqrt{3}\times 0 - g\ [0 + V_x]^2 \Rightarrow a_y = -g\,V_x^2$

Now, $a_y = -g \Rightarrow V_x^2 = 1 \Rightarrow V_x = \pm 1$

$v_x = u_x + a_x t$, $a_x = 0 \Rightarrow v_x = u_x$

$\therefore u_x = \pm 1 \Rightarrow u_y = \sqrt{3}(\pm 1)$; $u_y = \pm\sqrt{3}$

$\therefore$ Speed = $u = \sqrt{u_x^2 + u_y^2} = \sqrt{(\pm 1)^2 + (\pm\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4}$; u = 2 m/s. **Ans**

C-2 $y = x\tan\theta\,(1 - x/R)$

$\Rightarrow \quad \frac{R}{4} = \frac{3R}{4}\tan\theta\left(1 - \frac{3R}{4R}\right) \qquad \Rightarrow \quad 1 = 3\tan\theta\,(1/4)$

$\Rightarrow \quad \tan\theta = 3/4 \qquad \Rightarrow \quad \theta = 53°$

C-3. Comparing $\vec{r} = at\hat{i} - bt^2\hat{j}$ with $\vec{r} = x\hat{i} + y\hat{j}$

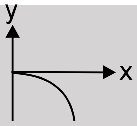
$\vec{r} = at\hat{i} - bt^2\hat{j} \quad \vec{r} = x\hat{i} + y\hat{j}$

we get $x = at$

& $y = -bt^2$

$\Rightarrow \quad y = -b\left(\frac{x}{a}\right)^2$ equation of trajectory

(i) $y = -\frac{bx^2}{a^2}$



Ans

(ii) $\vec{v} = a\hat{i} - 2bt\hat{j}$, acceleration = $-2b\hat{j}$,

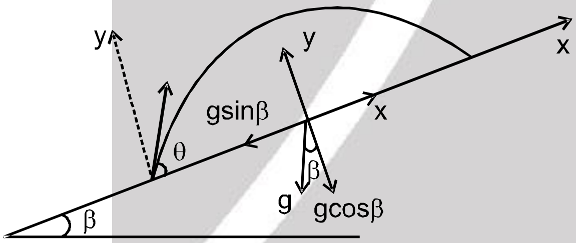
$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, |acceleration| = $2b$

$\vec{v} = a\hat{i} - 2bt\hat{j}, = -2b\hat{j}$,

$|\vec{v}| = \sqrt{a^2 + 4b^2t^2}$, $= 2b$

SECTION (D) :

D-1.



(a) a_x = x component of acceleration = $-g\sin\beta$

(b) y – component of acc^n = $a_y = -g\cos\beta$

(c) Let x – component of vel = V_x

(d) Let y – component of vel = V_y

(e) Let x-component of displacement = s_x

(f) Let s_y = y – component of displacement

PART - II

SECTION (A) :

A-1. $V = u + at$

V_y reduces then increases

$\Rightarrow$ V reduces then increase then increase ($\because$ V_x is constant V_x)

$\Rightarrow$ Speed first reduces then increases. So "A" is not correct

"A"

$KE = \frac{1}{2} mV^2 = \frac{m}{2}$ $(speed)^2$ $\Rightarrow$ "B" is not correct " B"

V_y = changes $\Rightarrow$ "C" is not correct. "C"

V_x = constt. since gravity is vertically down

V_x =

$\Rightarrow$ no component of acceleration along the horizontal direction.

$\Rightarrow$ "D" is correct. "D" **Ans**

A-2. In projectile motion Horizontal acceleration a_x = o & Vertical acceleration $a_y = g = 10 m/s^2$

a_x= o $a_y = g = 10m/s^2$

$a_x = 0$

a_y = 10 (down)

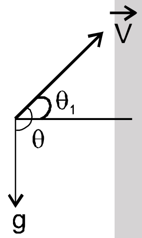
$\Rightarrow$ only "C" is correct "C" **Ans**

A-3. Acute Angle of Velocity with horizontal possible is – 90º to + 90º hence angle with g is 0° to 180º.

– 90º + 90º g 0° 180º

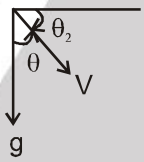
θ_1 is acute θ_1

$\Rightarrow$ $0º \le \theta_1 < 90º$ (during the upward journey of mass)



from fig $\theta = 90º + \theta_1$

or, $90º \le \theta < 180º$ ....(1)



During downward motion

$0º < \theta_2 < 90º$

$\theta = 90º – \theta_2$

$0º < \theta < 90º$ ....(2)

From eq. (1) and (2)

i.e., $0 < \theta < 90º$ U $90º \le \theta < 180º$

$\Rightarrow$ $0º < \theta < 180º$

"D" **Ans.**

A-4. Avg. vel. b/w A & B = $\frac{\vec{v}_1 + \vec{v}_2}{2}$ (Acceleration is constant = g)

Now, if $\vec{v}_1 = V_1 \hat{i} + V_1 \hat{j}$

Than $\vec{v}_2 = V_1 \hat{i} - V_1 \hat{j}$ (both A & B are at same lavel)

$\therefore$ $\vec{v}_{avg.} = V_1 \hat{i} = V \sin\theta$ (θ is from vertical) " B " **Ans.**

A-5. $y = ax^2$ .....(1)

given $V_x = c$

from (1) $\frac{dy}{dt} = 2a\,x.\ \frac{dx}{dt}$

$V_y = 2ax\,.c$ .....(2)

from (2) $\frac{dv_y}{dt} = 2ac.\ \frac{dx}{dt}$

$a_y = 2acV_x$

$a_y = 2ac^2$

$\vec{a}_y = 2ac^2\ \hat{j}$

A-6. Gravitational acceleration is constant near the surface of the earth.

A-7. At maximum height $v = u\cos\theta$

$$\frac{u}{2} = v \Rightarrow \frac{u}{2} = u\cos\theta \Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60°$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{u^2\sin(120°)}{g} = \frac{u^2\cos 30°}{g} = \frac{\sqrt{3}u^2}{2g}$$

A-8. $\vec{u}_x = 6\,\hat{i} + 8\,\hat{j}$

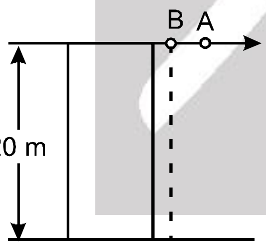
$\vec{u}_x = 6\hat{i}$

$u_y = 8\,\hat{j}$

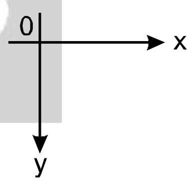
$$R = \frac{2u_xu_y}{g} = \frac{2\times6\times8}{10} = 9.6$$

SECTION (B) :

B-1.



$10\text{ m/s} = u_A$; $u_B = 0\text{ m/s}$ $u_A = 10\,\hat{i}$



On reaching the ground,

Both will have same vertical velocity $V_y^2 = u_y^2 + 2a_y\,s_y$

since $u_y = 0$ for both A & B

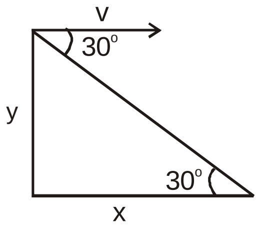
$a_y = g$ for both A & B

$s_y = 20$ m for both A & B

Thats why the time taken by both are same

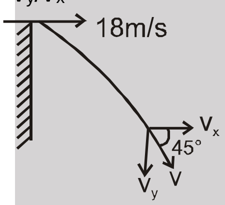
B-2. AC = $\frac{1}{2}gt^2$ = 45 m BC = $45\sqrt{3}$ m = u.t u = $\frac{45}{\sqrt{3}}$ = $15\sqrt{3}$ m/s.

Alter : Object is thrown horizontally so $u_x = v$ & $u_y = 0$
from Diagram`



$-y = u_y t - 1/2\ gt^2$; $y = 1/2 \times 10 \times (3)^2$
y = 45m ....(1)
& tan 30° = y/x= > y= $\sqrt{3}$ x ....(2)
& x = v t = 3v ....(3)
from equation (1), (2) & (3) `
$45\sqrt{3} = 3v$; $v = 15\sqrt{3}$ m/s

B-3. tan45° = v_y/v_x



⇒ $v_y = v_x$ = 18m/s **Ans.**

B-4. In 2 sec. horizontal distance travelled by bomb = 20 × 2 = 40 m.
In 2 sec. vertical distance travelled by bomb = 1/2 × 10 × 2^2 = 20 m.
In 2 sec. horizontal distance travelled by Hunter = 10 × 2 = 20 m.

Time remaining for bomb to hit ground = $\sqrt{\frac{2\times80}{10}} - 2 = 2$ sec.

Let V_x and V_y be the velocity components of bullet along horizontal and vertical direction.

Thus we use, $\frac{2V_y}{g} = 2 \Rightarrow V_Y = 10$ m/s and $\frac{20}{V_x - 20} = 2 \Rightarrow V_x = 30$ m/s

Thus velocity of firing is $V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{10}$ m/s.

2 sec. = 20 × 2 = 40 m.
2 sec. = 1/2 × 10 × 2^2 = 20 m.
2 sec. = 10 × 2 = 20 m.

$= \sqrt{\frac{2\times80}{10}} - 2 = 2$ sec.

V_x

$\frac{2V_y}{g} = 2 \Rightarrow V_Y = 10$ m/s $\frac{20}{V_x - 20} = 2 \Rightarrow V_x = 30$ m/s

$V = \sqrt{V_x^2 + V_y^2} = 10\sqrt{10}$ m/s.

SECTION (C) :

C-1. For (Y_{max}) $\Rightarrow$ $dY/dt = 0$

$\Rightarrow$ $\frac{d}{dt}(10\,t - t^2) = 10 - 2\,t$ $\Rightarrow$ $t = 5$

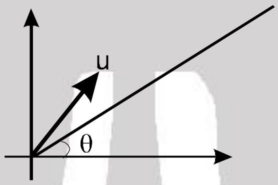
$\Rightarrow$ $Y_{max} = 10(5) - 5^2 = 25$ m

Ans "D"

SECTION (D) :

D-1. On the incline plane the maximum possible Range is

$R = \frac{V^2}{g(1+\sin\theta)}$

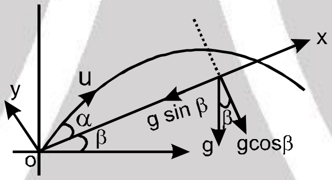


Range max = ?

Let $\theta = \beta$

And angle of projection from the inclined plane = α

= α



$u_y = u \sin\alpha$ $u_x = \cos\alpha$ $a_x = -g\sin\beta$ $a_y = -g\cos\beta$

Range = $s_x = u_xT + \frac{1}{2}a_xT^2$

(on the inclined plane) where $T = \frac{2u_y}{g_y}$

$\Rightarrow$ $s_x = (u\cos\alpha)\left[\frac{2u\sin\alpha}{g\cos\beta}\right] + \frac{1}{2}[-g\sin\beta]\left[\frac{2u\sin\alpha}{g\cos\beta}\right]^2$

$= \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos\alpha\cos\beta - \sin\alpha\sin\beta]$

$s_x = \frac{2u^2\sin\alpha}{g\cos^2\beta}[\cos(\alpha+\beta)] = \frac{u^2}{g\cos^2\beta}[2\sin\alpha\cos(\alpha+\beta)] = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta)+\sin(-\beta)]$

$s_x = \frac{u^2}{g\cos^2\beta}[\sin(2\alpha+\beta) - \sin\beta]$

Now s_x is max s_x

when $\sin(2\alpha + \beta)$ is max ($\because$ β = constt.)

$\sin(2\alpha + \beta)$ ($\because$ β = अचर)

$\Rightarrow$ $2\alpha + \beta = \pi/2$ $\Rightarrow$ $\alpha = \frac{\left(\frac{\pi}{2} - \beta\right)}{2}$

i.e., when ball is projected at the angle bisector of angle formed by inclined plane and dir. of net accelaration reversed.

& $(s_x)_{max} = \dfrac{u^2}{g\cos^2\beta}\left[1-\sin\beta\right] = \dfrac{u^2 \ (1-\sin\beta)}{g \ (1-\sin\beta) \ (1+\sin\beta)}$

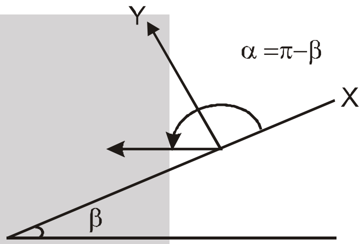
Max. Range on an inclined plane $= \dfrac{u^2}{g(1+\sin\beta)}$

Here $\beta = \theta$

$\Rightarrow \quad R_{max} = \dfrac{u^2}{g(1+\sin\theta)}$ **Ans** "B"

D-2.

Sol. $R = \dfrac{v^2}{g\cos^2\beta}\left[\sin(2\alpha+\beta)-\sin\beta\right]$



Putting $\beta = 45^o$ & $\alpha = \pi - \dfrac{\pi}{4} = \dfrac{3\pi}{4}$

$R = \dfrac{v^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\left(2\times\dfrac{3\pi}{4}+\dfrac{\pi}{4}\right)-\sin\dfrac{\pi}{4}\right]$

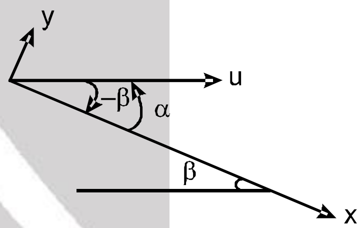
$s_x = \dfrac{v^2\times 2}{g}\left[-2\times\dfrac{1}{\sqrt{2}}\right] = -2\sqrt{2}\dfrac{v^2}{g}$

(–ve sign indicates that the displacement is in –ve x direction)

$\Rightarrow \quad \text{Range} = 2\sqrt{2}\dfrac{v^2}{g}$ **Ans** "D"

Alternate II method

$\beta = -\dfrac{\pi}{4}$ & $\alpha = \dfrac{\pi}{4}$



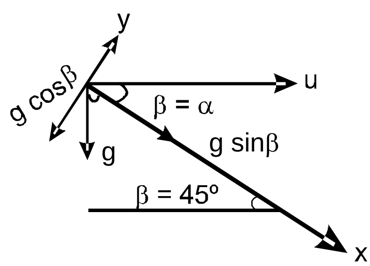
$R = \dfrac{u^2}{g \ \cos^2\beta}\left[\sin \ (2\alpha+\beta)-\sin\beta\right]$

$= \dfrac{u^2}{g\left(\frac{1}{\sqrt{2}}\right)^2}\left[\sin\dfrac{\pi}{4}-\sin\left(-\dfrac{\pi}{4}\right)\right]$

$R = \dfrac{2\sqrt{2}u^2}{g}$ (along +ve x die.) (+ve x)

III Method

$u_x = u\cos\beta$, $T = \left|\dfrac{2u\sin\beta}{g\cos\beta}\right|$, $a_x = g\sin\beta$; $a_y = -g\cos\beta$



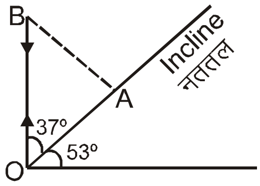
$s_x = u_x t + \dfrac{1}{2}a_x t^2 = (u\cos\beta)\left[\dfrac{2u\sin\beta}{g\cos\beta}\right] + \dfrac{1}{2}(g\sin\beta)\left(\dfrac{2u\sin\beta}{g\cos\beta}\right)^2$

Let $\alpha = \beta = 45^o$

So, $s_x = \dfrac{u^2 2}{g}\left(\dfrac{1}{\sqrt{2}}\right) + \dfrac{1}{2}g\left(\dfrac{1}{\sqrt{2}}\right)\dfrac{2.2u^2}{g^2} = \dfrac{2}{\sqrt{2}}\dfrac{u^2}{g}[1+1]$ $s_x = 2\sqrt{2}\dfrac{u^2}{g}$

Ans "D"

D-3. OB = $\frac{u^2}{2g}$ = 5m

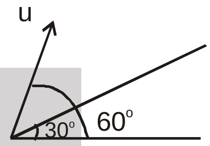


$\therefore$ AB = OB sin37º = 3m.

D-4.

u = 10m/s

Time of flight on the incline plane



T= $\frac{2u\sin\alpha}{g\cos\beta}$

given α =30º & β = 30º & u = $10\sqrt{3}$ m/s

T = $\frac{2\times10\sqrt{3}\sin30^o}{10\cos30^o}$

So, T= 2 sec .

D-5. H = $\frac{u_\perp^2}{2a_\perp}$

$a_\perp$ is same for all the three cases.

$H_A = \frac{(u\sin\alpha)^2}{2a_\perp}$, $H_B = \frac{u^2}{2a_\perp}$ and $H_C = \frac{(u\cos\alpha)^2}{2a_\perp}$ $\therefore$ $H_B = H_A + H_C$

PART - III

1. Time of flight T = $\frac{2u}{g\cos45^\circ}$ = $\frac{2u}{g\cos45^\circ} = \frac{2\sqrt{2}u}{g}$

$\therefore$ D $\rightarrow$ p

Velocity of stone is parallel to x-axis at half the time of flight.

$\therefore$ A $\rightarrow$ r

At the instant stone make 45° angle with x-axis its velocity is horizontal.

$\therefore$ The time is = $\frac{u\sin45^\circ}{g} = \frac{u}{\sqrt{2}g}$

$\therefore$ B $\rightarrow$ s

The time till its displacement along x-axis is half the range is = $\frac{1}{\sqrt{2}}T = \frac{2u}{g}$

$\therefore$ C $\rightarrow$ q

T = $\frac{2u}{g\cos45^\circ}$ = $\frac{2u}{g\cos45^\circ} = \frac{2\sqrt{2}u}{g}$ $\therefore$ D $\rightarrow$ p

$\therefore$ A $\rightarrow$ r

= $\frac{u\sin45^\circ}{g} = \frac{u}{\sqrt{2}g}$ $\therefore$ B $\rightarrow$ s

= $\frac{1}{\sqrt{2}}T = \frac{2u}{g}$ $\therefore$ C $\rightarrow$ q

2. Equation of path is given as $y = ax - bx^2$

Comparing it with standard equation of projectile;

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} \qquad \tan\theta = a,\ \frac{g}{2u^2\cos^2\theta} = b$$

Horizontal component of velocity = $u\cos\theta = \sqrt{\frac{g}{2b}}$

Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g} = \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$

Maximum height $H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g} = \frac{\left[\sqrt{\frac{g}{2b}}.a\right]^2}{2g} = \frac{a^2}{4b}$

Horizontal range $R = \frac{u^2\sin2\theta}{g} = \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$

$y = ax - bx^2$

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$\tan\theta = a,\ \frac{g}{2u^2\cos^2\theta} = b$$

$= u\cos\theta = \sqrt{\frac{g}{2b}}$

$$T = \frac{2u\sin\theta}{g} = \frac{2(u\cos\theta)\tan\theta}{g} = \frac{2\left(\sqrt{\frac{g}{2b}}\right)a}{g} = \sqrt{\frac{2a^2}{bg}}$$

$$H = \frac{u^2\sin^2\theta}{2g} = \frac{[u\cos\theta.\tan\theta]^2}{2g} = \frac{\left[\sqrt{\frac{g}{2b}}.a\right]^2}{2g} = \frac{a^2}{4b}$$

$$R = \frac{u^2\sin2\theta}{g} = \frac{2(u\sin\theta)(u\cos\theta)}{g} = \frac{2\left[\sqrt{\frac{g}{2b}}.a\right]\left[\sqrt{\frac{g}{2b}}\right]}{g} = \frac{a}{b}$$

EXERCISE-2
PART - I

1. $a_x = 2\ m/s^2$; $a_y = 0$

$u_x = 8$ m/s

$u_y = -15$ m/s.

$\vec{V} = V_x\hat{i} + V_y\hat{j}$

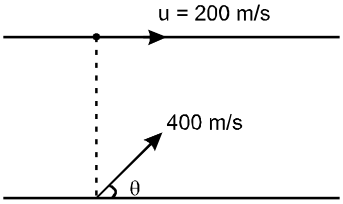
$V_y = u_y + a_y t$

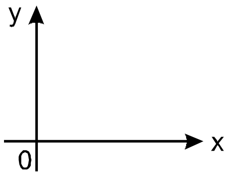
$\Rightarrow \quad V_y = -15$ m/s

$V_x = u_x + a_x t$

$V_x = 8 + 2t \qquad \Rightarrow \qquad V = [(8 + 2t)\hat{i} - 15\hat{j}]$ m/s. **Ans.**

2.





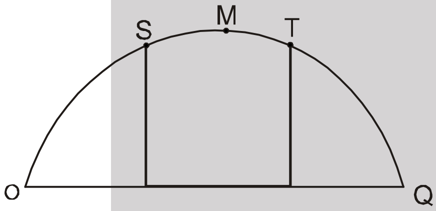
To hit, $400 \cos \theta = 200$

{$\because$ Both travel equal distances along horizontal, from their start and coordinates of x axis are same}

$\Rightarrow$ $\theta = 60^{o}$ **Ans.**

3. $\dfrac{R^2}{8h} + 2h = \dfrac{\left(\dfrac{u^2 2\sin\theta\cos\theta}{g}\right)^2}{\dfrac{8\times u^2\sin^2\theta}{2g}} + 2\,\dfrac{u^2\sin^2\theta}{2g} = \dfrac{u^2}{g}$ (max. horizontal Range)

4.



$t_{(OS)} = 1$ sec

$t_{(OT)} = 3$

or $t_{(ST)} = 1/2\ t_{(OT)} - t_{(OS)} = 3 - 1 = 2$ sec

$\therefore$ $t_{(SM)} = t_{(ST)} = 1$ sec.

$\therefore$ $t_{(OM)} = t_{(OS)} + t_{(SM)} = 1 + 1 = 2$sec.

$\therefore$ Time of flight = 2 × 2 = 4 sec. **Ans. "C"**

5. Let initial and final speeds of stone be u and v.

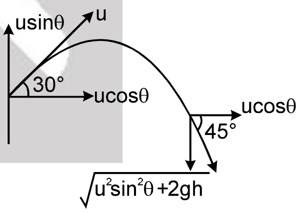
$\therefore$ $v^2 = u^2 - 2gh$ ....(1)

and $v \cos 30° = u \cos 60°$ ....(2)

solving 1 and 2 we get

$u = \sqrt{3gh}$

6.



Using $v = \sqrt{u^2 + 2gh}$

$v = \sqrt{u^2\sin^2\theta + 2gh}$ (vertical comp. when striking)

Now tan 45° = 1

$u \cos\theta = \sqrt{u^2\sin^2\theta + 2gh}$

$u^2\cos^2\theta = u^2\sin^2\theta + 2gh$ ......(1)

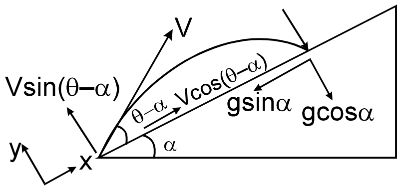
$u^2\left(\dfrac{3}{4} - \dfrac{1}{4}\right) = 2gh$

$u^2 = 4gh$

$u = 2\sqrt{gh}$; $\tan\theta = \dfrac{v_T}{v_H} = \dfrac{\sqrt{4gh.\dfrac{3}{4} + 2gh}}{2\sqrt{gh}\times\dfrac{1}{2}} = \dfrac{\sqrt{5gh}}{\sqrt{gh}} = \sqrt{5}$

7.

Applying equation of motion perpendicular to the incline for y = 0.



$0 = V\sin(\theta-\alpha)t + \frac{1}{2}(-g\cos\alpha)t^2$

$\Rightarrow \quad t = 0 \quad \& \quad \frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

At the moment of striking the plane, as velocity is perpendicular to the inclined plane hence component of velocity along incline must be zero.

$0 = v\cos(\theta-\alpha) + (-g\sin\alpha).\frac{2V\sin(\theta-\alpha)}{g\cos\alpha}$

$v\cos(\theta-\alpha) = \tan\alpha.\ 2V\sin(\theta-\alpha)$

$\cot(\theta-\alpha) = 2\tan\alpha$ **Ans. (D)**

8. Since time of flight depends only on vertical component of velocity and acceleration. Hence time of flight is

$T = \frac{2u_y}{g}$ where $u_x = u\cos\theta$ and $u_y = u\sin\theta$

$\therefore$ In horizontal (x) direction

$d = u_x t + \frac{1}{2}a_x t^2 = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) + \frac{1}{2}g\left(\frac{2u\sin\theta}{g}\right)^2$

$= \frac{2u^2}{g}(\sin\theta\cos\theta + \sin^2\theta)$

We want to maximise f(θ)

$f(\theta) = \cos\theta\sin\theta + \sin^2\theta$

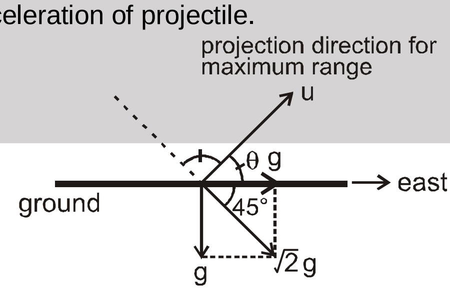
$\Rightarrow \quad f'\theta = -\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 0$

$\Rightarrow \quad \cos2\theta + \sin2\theta = 0 \quad \Rightarrow \quad \tan2\theta = -1$

or $\quad 2\theta = \frac{3\pi}{4} \quad$ or $\quad \theta = \frac{3\pi}{8} = 67.5°$

Alternate :

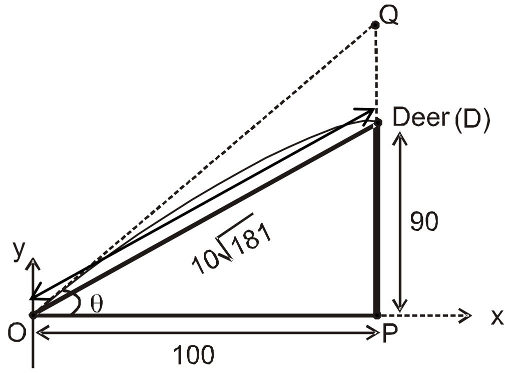
As shown in figure, the net acceleration of projectile makes on angle 45° with horizontal. For maximum range on horizontal plane, the angle of projection should be along angle bisector of horizontal and opposite direction of net acceleration of projectile.



$\therefore \quad \theta = \frac{135°}{2} = 67.5°$

PART - II

1. $OD = 10\sqrt{181}$ $\quad$ $PD = 90$



Let θ be angle of projection

we have $\quad$ $QD = PQ - PD$.

Also from equation of trajectory

$$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2}\sec^2\theta$$

Coordinate of deer $\equiv$ (100,90)

$$\therefore \quad 90 = 100\tan\theta - \frac{5(100)^2}{(100)^2}\sec^2\theta$$

or $\quad 90 = 100\tan\theta - 5(1 + \tan^2\theta)$

or $\quad \tan^2\theta - 20\tan\theta + 19 = 0$

or $\quad \tan^2\theta - 19\tan\theta - \tan\theta + 19 = 0$

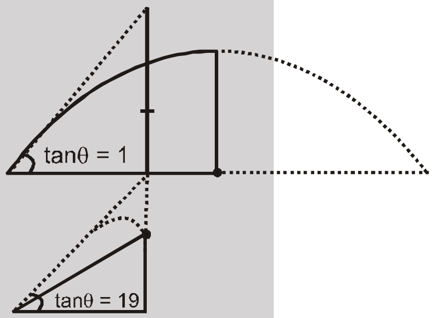
or $\quad \tan\theta(\tan\theta - 19) - 1(\tan\theta - 1) = 0$

or $\quad \tan\theta = 1$

$\quad \tan\theta = 19$

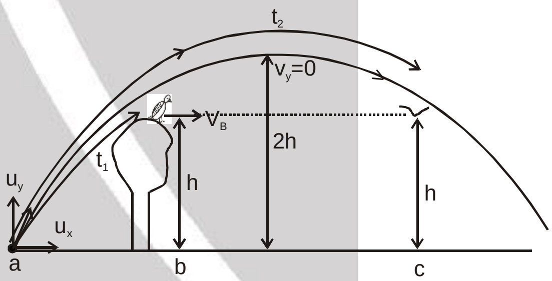
or $\quad \dfrac{PQ}{100} = 1 \quad$ for $\tan\theta = 1$

$\quad \dfrac{PQ}{100} = 19 \quad$ for $\tan\theta = 19$



The two roots signify the two possible trajectories for which the hit would be successful.

2. $ac = ab + bc$



$bc = ac - ab$

$V_Bt_2 = U_xt_2 - U_xt_1$

(V_B = velocity of bird)

$V_Bt_2 = U_x(t_2 - t_1)$ .... (1)

displacement in y direction in time 't'

$S_y = U_yt - 1/2\ gt^2$

$\therefore\ V_y^2 = U_y^2 - 2g(2h)$

$0 = U_y^2 - 2g(2h)$

$U_y = \sqrt{4gh}$

$$h = \sqrt{2g(2h)}\,t - \frac{1}{2}gt^2$$

on solving we get

$t_1 = \sqrt{h/g}\,(2 - \sqrt{2})$

$t_2 = \sqrt{h/g}\,(2 + \sqrt{2})$

On putting the value of t_1 and t_2 in equation (1)

we get $\quad \dfrac{U_x}{V_B} = \dfrac{\sqrt{2}+1}{2}$

Aliter

Time for stone to move from b to c = $2\sqrt{\dfrac{h}{g}}$

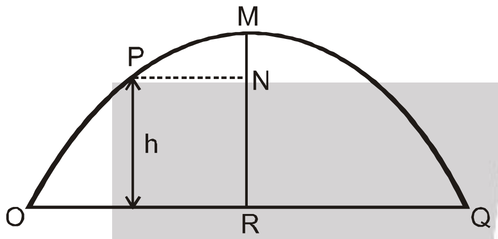
Time for bird to fly from b to c = $\sqrt{2\frac{h}{g}} + \sqrt{\frac{h}{g}}$

Therefore equating the distance bc from both the cases

$$V_B\left(\sqrt{\frac{2h}{g}} + \sqrt{\frac{h}{g}}\right) = U_x\left(2\sqrt{\frac{h}{g}}\right)$$

$$\frac{U_x}{V_B} = \frac{(\sqrt{2}+1)}{2}$$ **Ans.**

3.



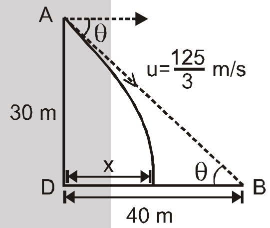
$t_{op} = 4$
$t_{PQ} = 5$ — $\therefore\ t_{OQ} = 4 + 5 = 9$

or tanθ = 9/2 = 4.5

RM – MN = h = 1/2 g [$(4.5)^2 - (.5)^2$]

= 1/2 × 9.8 × 20 = 98

4. In ΔABD, $\tan\theta = \frac{30}{40} = \frac{3}{4}$



Let time taken be 't' in x-direction

$x = u_x t$

$x = u\cos\theta\ t$

$x = \frac{125}{3} \times \frac{4}{5} t \Rightarrow x = \frac{100}{3}\ t$ ....(1)

In y-direction

$y = u_y t + \frac{1}{2} gt^2$

$30 = u\sin\theta\ t + \frac{1}{2} gt^2$

$30 = \frac{125}{3} \times \frac{3}{5} t + 5t^2$

$t^2 + 5t - 6 = 0$

$(t + 6)(t - 1) = 0$

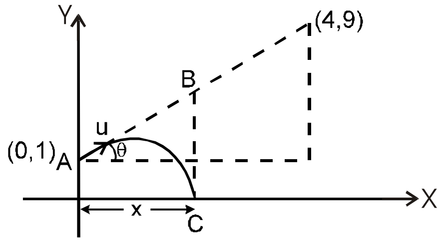
t = 1 sec. ....(2)

From (1) and (2)

x = 100/3

$\therefore$ Packet is short by a distance of $40 - \frac{100}{3} = \frac{20}{3}$ m **Ans.**

5. $\tan\theta = \frac{9-1}{4-0} = 2,$

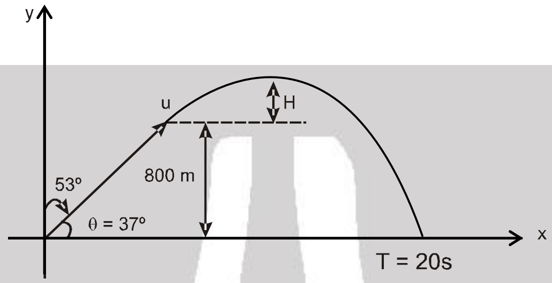


$y = u_y t + \frac{1}{2} a_y t^2$

now $-1 = u\sin\theta\,(1) - \frac{1}{2} g\,(1)^2$

$u\sin\theta = 4$ and $\sin\theta = \frac{2}{\sqrt{5}}$ $\Rightarrow u = 2\sqrt{5}$

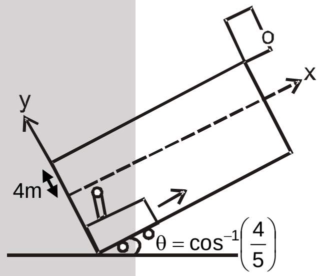
6.



$s_y = u_y t + \frac{1}{2} a_y t^2$ $800 = (-u\cos 53^o)\,T + \left(\frac{10}{2}\right) T^2 \Rightarrow$ $u = 100$ m/s

7. At t = 0

a_y be the vertical component of acc^n of the ball w.r.t ground.



$a_y = -g\cos\theta = -g \times \frac{4}{5}$

while crossing through loop the velocity is parallel to x-axis

$\therefore$ $V_y = 0$

y co-ordinate of loop = + 4

$V_y^2 - u_y^2 = 2a_y (y_f - y_i)$

$0 - u_y^2 = -2.\frac{4g}{5}(4-0)$

$u_y^2 = \frac{8g}{5}.4$

$u_y^2 = 8 \times 8$

$u_y = 8$ m/s

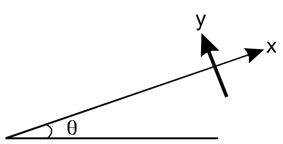
Time taken by the ball to reach the loop.

$V_y - u_y = a_y t$

$0 - 8 = -\frac{4g}{5}t$

or t = 1 second

II method : $V_y = 0$ (given)

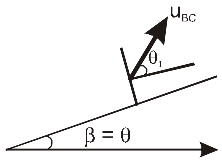


$V_y = u_y + a_y t$ ....(1)

$s_y = u_y t + \frac{1}{2} a_y t^2$ ...(2)

Two eq. two variable 'u_y' & 't'

to find 't' ; as shown below.



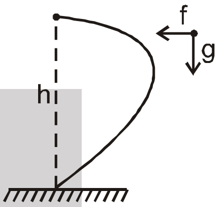
$V_y = u_y + a_y t$

$0 = u_{BC} \sin\theta_1 - g\cos\beta\, T.$ $\quad(\beta = \cos^{-1} 4/5 = 37^o)$

u_{BC} = vel. of ball wrt car. $\quad\Rightarrow\quad u_{BC}\sin\theta_1 = u_y = T. \times 10 \times 4/5 = 8T.$

$s_y = (8T)\,T + T^2 = 4,\ (s_y = 4\text{ m}) \Rightarrow$ $\quad T = 1s$ **Ans.**

8. Time taken to reach the ground is given by $h = \frac{1}{2}gt^2$ ....(1)



Since horizontal displacement in time t is zero

$\therefore \quad t = 2v/f$ ....(2)

$$h = \frac{2gv^2}{f^2}$$

9. At $t = 0$ $u_x = u\cos\theta$ and $u_y = u\sin\theta$

$\vec{u} = u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j}$

Let after time 't' the velocity of projectile be v if its intial velocity is u

At time t

$V_x = u\cos\theta,\quad V_y = u\sin\theta - gt$

$\vec{V} = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$

$u \perp v$

$\vec{u}\cdot\vec{v} = 0$

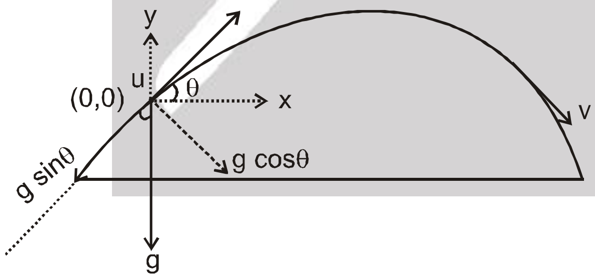
$(u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j})\,(u\cos\theta\,\hat{i} + u\sin\theta\,\hat{j})$

$u^2\cos^2\theta + (u\sin\theta)^2 - gt\,u\sin\theta = 0$

$u^2(\cos^2\theta + \sin^2\theta) = gtu\sin\theta$

$\frac{u}{g\sin\theta} = t$

ALTERNATE :



Now Let $\vec{u}$ be $\perp$ v after time t, then component of velocity along u becomes zero.

component of $\vec{g}$ along $\vec{u} = -g\sin\theta$

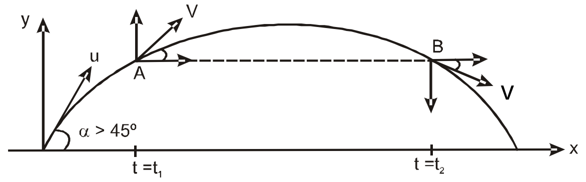
$= -g\sin\theta$

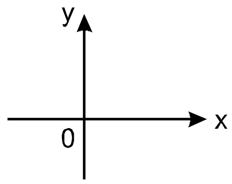
$\therefore \quad 0 = u - g\sin\theta\, t$

$t = u/g\sin\theta$

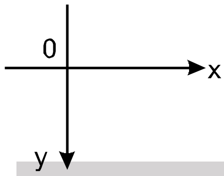
PART - III

1.





$V_x = V_y$ Now, we could have choosen coordinate axis as



$\Rightarrow$ Time at which $V_x = V_y$ is what we are solving

Now, $V_x = u\cos\alpha$

$V_y = u\sin\alpha - gt$

$\Rightarrow$ $u\cos\alpha = u\sin\alpha - gt$ $\{\because V_y = V_x\}$; at $t = t_1$ (say)

$\Rightarrow$ $t_1 = \frac{u}{g}(\sin\alpha - \cos\alpha)$ **"C" Ans**

Also when $V_y \equiv V_x$ {i.e., when we choose 'y' axis as – y} at $t = t_2$ (say)

$u\cos\alpha = -(u\sin\alpha - gt_2)$

$\Rightarrow$ $t_2 = \frac{u}{g}(\sin\alpha + \cos\alpha)$ **"B" Ans**

2. $x = 24 = u\cos\theta.t$

$\Rightarrow$ $t = \frac{24}{24\cos\theta} = \frac{1}{\cos\theta}$

$y = 14 = u\sin\theta t - \frac{1}{2}gt^2$

$\Rightarrow$ $14 = \frac{u\sin\theta}{\cos\theta} - \frac{5}{\cos^2\theta}$ $\Rightarrow$ $14 = u\tan\theta - 5\sec^2\theta$

$\Rightarrow$ $5\tan^2\theta - 24\tan\theta + 19 = 0$ $\Rightarrow$ $\tan\theta = 1, 19/5$. **Ans**

3. Since maximum heights are same, their time of flight should be same

$\therefore$ $T_1 = T_2$

Also, vertical components of initial velocity are same.

$\therefore$ Since range of 2 is greater than range of 1.

$\therefore$ Horizontal component of velocity of 2 > horizontal component of velocity of 1.

Hence, $u_2 > u_1$.

4. $R = \frac{u^2\sin 2\theta}{g}$

$\sqrt{3} = 20.\frac{2\sin\theta\cos\theta}{10}$

$\sin\theta\sqrt{1-\sin^2\theta} = \frac{\sqrt{3}}{4}$ $\Rightarrow$ $16\sin^4\theta - 16\sin^2\theta + 3 = 0$

$\sin^2\theta = \frac{1}{2} \pm \sqrt{\frac{1}{4} - \frac{3}{16}} \quad \Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}\ ;\ \frac{1}{2}$

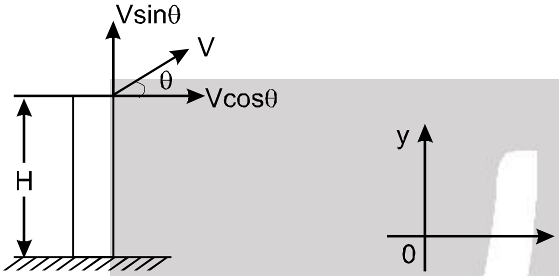
θ = 60º ; 30°

$H_{max} = \frac{u^2 \sin^2\theta}{2g} = 0.75\text{m} \ \& \ 0.25 \text{ m}$

$V_{min} = \sqrt{5}$ m/s, $\sqrt{15}$ m/s

$T = \frac{2u\sin\theta}{g} = \sqrt{\frac{3}{5}};\sqrt{\frac{1}{5}}$

5.



Let final vel be V_2

Now v_{2x} = horizontal component of velocity

$V_{2x} = V\cos\theta$ & $V^2_{2y} = (V\sin\theta)^2 + 2(-g)(-H)$

$\therefore$ $V^2_{2y} = V^2\sin^2\theta + 2gH$

$\Rightarrow$ $V^2_2 = V^2_{2x} + V^2_{2y}$

$= (V\cos\theta)^2 + [V^2\sin^2\theta + 2gH]$

$V^2_2 = V^2 + 2gH$

i.e., $V_2 = \sqrt{V^2 + 2gH}$

This magnitude of final velocity is independent of θ

$\Rightarrow$ all particles strike the ground with the same speed.

i.e., 'A' is correct.

In vertical motion

The highest velocity (initial) along the direction of displacement is possessed by particle (1). Hence particle (1) will reach the ground earliest. [Since a_y and s_y are same for all]

i.e., 'C' is correct **Ans** A & C

PART - IV

1. Velocity at P is completely horizontal i.e. $u\cos\theta = 20\cos30º = 20 \times \frac{\sqrt{3}}{2} = 10\sqrt{3}$ m/sec.

$\therefore v_{vertical}$ = 0 m/sec.

2. Assuming vertically downwards to be positive.

making equation along vertical direction (point A taken as reference)

$s = ut + \frac{1}{2}at^2$

$\therefore$ $20 = -20\sin\theta \times t + 1/2 \times 10 \times t^2$

$\therefore$ $20 = -20\sin30ºt + 5t^2$

$20 = -10t + 5t^2$

$\therefore$ $5t^2 - 10t - 20 = 0$ or $t^2 - 2t - 4 = 0$

$\therefore$ $t = \frac{2 \pm \sqrt{4+16}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$.

$\because$ at $(1 - \sqrt{5})$ sec the particle was at initial point on ground.

$\therefore$ accepted time = $(1 + \sqrt{5})$ sec

3. At point Q, x-component of velocity is zero. Hence, substituting in

$v_x = u_x + a_x t$

$0 = 10\sqrt{3} - 5\sqrt{3}t$

or $t = \frac{10\sqrt{3}}{5\sqrt{3}} = 2s$ **Ans.**

4. At point Q, $v = v_y = u_y + a_y t$

$\therefore$ $v = 0 - (5)(2) = -10$ m/s **Ans.**

Here, negative sign implies that velocity of particle at Q is along negative y direction.

5. PO = |displacement of particle along y-direction |

Here , $s_y = u_y t + \frac{1}{2} a_y t^2$

$= 0 - \frac{1}{2}(5)(2)^2 = -10$ m

$\therefore$ PO = 10 m

Therefore, h = PO sin 30° = $(10)\left(\frac{1}{2}\right)$ or h = 5 m Ans.

6. Distance OQ = displacement of particle along x-direction = s_x

Here, $s_x = u_x t + \frac{1}{2} a_x t^2$

$= (10\sqrt{3})(2) - \frac{1}{2}(5\sqrt{3})(2)^2 = 10\sqrt{3}$ m

or OQ $= 10\sqrt{3}$ m

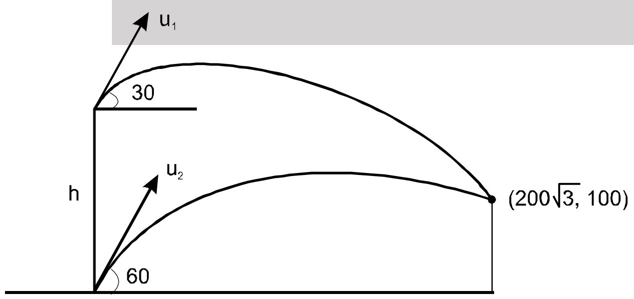
$\therefore$ $PQ = \sqrt{(PO)^2 + (OQ)^2} = \sqrt{(10)^2 + (10\sqrt{3})^2}$

$= \sqrt{100 + 300} = \sqrt{400}$

$\therefore$ PQ = 20 m **Ans.**

EXERCISE-3
PART - I

1.



$u_1 \cos 30^o = u_2 \cos 60^o$ (strike simultaneously)

$\sqrt{3}\, u_1 = u_2$

$100 = 200\sqrt{3} \tan 60^o - \frac{1}{2} \times \frac{g(200\sqrt{3})^2}{u_2^2 \cos^2 60^o}$ $\Rightarrow$ $u_2 = 40\sqrt{3}$ m/s from eq (1) and (2)

$u_1 = \frac{u_2}{\sqrt{3}}$ $\quad u_1 = 40$ m/s

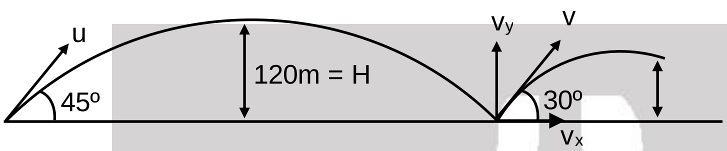
$x = u_2 \cos 60^o \times T \quad 200\sqrt{3} = 40\sqrt{3} \times \frac{1}{2} \times T \quad \Rightarrow \quad T = 10$ sec

$\Rightarrow \quad (h - 100) = 200\sqrt{3}\ \tan 30^o - \frac{1}{2} g \frac{(200\sqrt{3})^2}{u_1^2 \cos^2 30^o}$

Putting $g = 10$ m/sec^2

& $\quad u_1 = 40$ m/sec $\quad h = 400$ m

2.



$K_2 = \frac{1}{2} mu^2 \qquad \frac{v_y}{v_x} = \tan 30^o = \frac{1}{\sqrt{3}}$

$H = \frac{u^2 \sin^2 45^o}{2g} \qquad \sqrt{3}\, v_y = v_x.$

$= \frac{u^2}{4g} = 120m \qquad K_f = \frac{1}{2} mv^2 = \frac{1}{2} m(v_x^2 + v_y^2)$

$K_f = \frac{1}{2} K_i$

$\Rightarrow \quad \frac{1}{2} m(v_x^2 + v_y^2) = \frac{1}{2} \times \frac{1}{2} mu^2$

$\Rightarrow \quad v_x^2 + v_y^2 = \frac{u^2}{2} \quad \Rightarrow \quad$ using $u_x = \sqrt{3}\, u_y$

$\Rightarrow \quad 3u_y^2 + u_y^2 = \frac{u^2}{2} \quad \Rightarrow \quad u_y^2 = \frac{u^2}{8}$

$h = \frac{u_y^2}{2g} = \frac{u^2}{16g} = \frac{1}{4}\left(\frac{u^2}{4g}\right) = \frac{H}{4} = \frac{120}{4} = 30$ m

3. For first projectile

$< V > = \frac{R}{T} = U_x = v_1$

For journey

$$< V >_{1-n} = \frac{R_1 + R_2 + ... + R_n}{T_1 + T_2 + ... + T_n} = \frac{\frac{2u_{x_1}u_{y_1}}{g} + \frac{2u_{x_2}u_{y_2}}{g} + ... + \frac{2u_{x_n}u_{y_n}}{g}}{\frac{2u_{y_1}}{g} + \frac{2u_{y_2}}{g} + ... \frac{2u_{y_n}}{g}}$$

$$U_x\left[\frac{1+\frac{1}{\alpha^2}+\frac{1}{\alpha^4}+.....\frac{1}{\alpha^{2n}}}{1+\frac{1}{\alpha}+\frac{1}{\alpha^2}+....+\frac{1}{\alpha^n}}\right] = 0.8\,v_1 \Rightarrow \frac{v_0\left[\frac{1}{1-\frac{1}{\alpha^2}}\right]}{\left[\frac{1}{1-\frac{1}{\alpha}}\right]} = 0.8v_1$$

$\Rightarrow \quad \frac{\alpha}{1+\alpha} = 0.8 \quad \Rightarrow \quad \alpha = 4$

PART - II

1. $\vec{v} = \vec{u} + \vec{a}\,t$

$= (3\hat{i}+4\hat{j}) + (0.4\hat{i}+0.3\hat{j}) \times 10 = 7\,i + 7\,j \; |\vec{v}| = 7\sqrt{2}$

2. $\frac{d\vec{r}}{dt} = K(y\hat{i}+x\hat{j})$

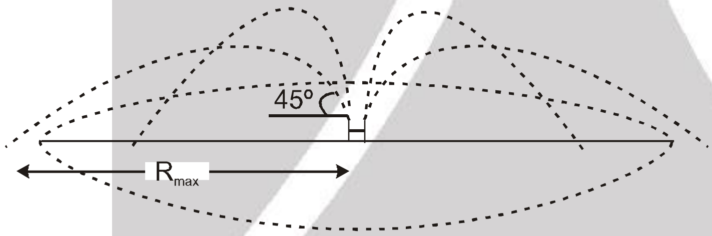
$\Rightarrow \quad dx/dt = y, \qquad dy/dt = x$

So, $dy/dx = x/y$

$\int y\,dy = \int x\,dx \quad ; \qquad \frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + \text{constant}$

3.



$R_{max} = \frac{v^2}{g}\sin 2\theta = \frac{v^2}{g}$

area $= \pi R^2$

$= \pi\frac{v^4}{g^2}$ **Ans.**

4. $h_{max} = \frac{u^2}{2g} = 10$

$u^2 = 200 \qquad ....(1)$

$R_{max} = \frac{u^2}{g} = 20m$

5. $\vec{v} = \hat{i} + 2\hat{j}$

$\Rightarrow \quad x = t \qquad ...(i)$

$y = 2t - 1/2\,(10t^2) \qquad ...(ii)$

From (i) and (ii)

$y = 2x - 5x^2$

6. On inclined plane (range) $R = \dfrac{2u^2 \sin\alpha \cos(\alpha+\beta)}{g\cos^2\beta}$

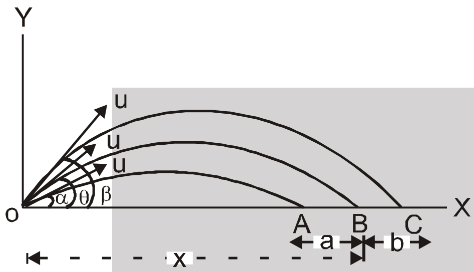
Where $\alpha = 15^o$, $\beta = 30^o$, u = 2 m/s

On solving we get

$R = \dfrac{4}{5}\left(\dfrac{1}{\sqrt{3}} - \dfrac{1}{3}\right) \approx 20$ cm

HIGH LEVEL PROBLEMS (HLP)

1.



$OA = x - a = \dfrac{u^2 \sin 2\alpha}{g}$ ...(1)

$OC = x + b = \dfrac{u^2 \sin 2\beta}{g}$ ...(2)

$OB = x = \dfrac{u^2 \sin 2\theta}{g}$ ...(3)

From eqs. (1) and (2)

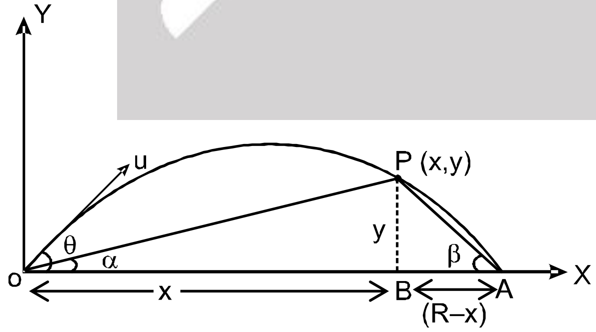
$x\,(b + a) = \left(\dfrac{b\sin 2\alpha + a\sin 2\beta}{g}\right)u^2$

Substituting the value of x from eq. (3), we get

$\dfrac{u^2 \sin 2\theta}{g}\,(b + a) = \left(\dfrac{b\sin 2\alpha + a\sin 2\beta}{g}\right)u^2$

Solving this equation, we will get θ.

2. The situation is shown in the fig.



From fig $\tan\alpha + \tan\beta = \dfrac{y}{x} + \dfrac{y}{(R-x)}$

where R is the range.

$\therefore \tan\alpha + \tan\beta = \dfrac{y(R-x) + xy}{x(R-x)}$

or $\quad \tan\alpha + \tan\beta = \frac{y}{x} \times \frac{R}{(R-x)}$ ....(1)

but $y = x\tan\theta\left(1 - \frac{x}{R}\right)$

or $\quad \tan\theta = \frac{y}{x} \times \frac{R}{(R-x)}$ ....(2)

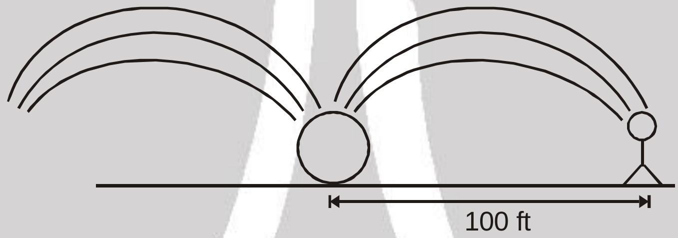
From equations (1) and (2), we have

$\tan\theta = \tan\alpha + \tan\beta.$

3. According to given problem u = 80 f / s

Range = $\frac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \frac{100 \times 32}{(80)^2} = 1/2$



$\theta = 15°$ For same Range $\theta = 15°, 75°$

Thus there will be two time of flight

$T_1 = \frac{2u\sin 15°}{g} = \frac{2 \times 80 \times \sin 15°}{32}$ (minimum time)

$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

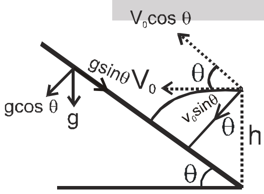
$T_2 = \frac{2u\sin 75°}{g} = \frac{2 \times 8 \times \sin 75°}{32}$ (maximum time)

$\sin 75° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Danger time = Maximum time – Minimum time = $(T_2 - T_1)$

$= \frac{2 \times 80}{32}[\sin 75° - \sin 15°] = \frac{2 \times 80}{32}\left[\frac{\sqrt{3}+1}{2\sqrt{2}} - \frac{\sqrt{3}-1}{2\sqrt{2}}\right] = \frac{5}{\sqrt{2}}$ sec.

4.



Parallel to plane

$0 = V_0\cos\theta - g\sin\theta \times t$

$t = \frac{V_0\cos\theta}{g\sin\theta}$ ...... (1)

Perpendicular to plane

$h\cos\theta = V_0 \sin\theta\, t + \frac{1}{2}(g\cos\theta)t^2$

$h\cos\theta = V_0 \sin\theta\left(\frac{V_0\cos\theta}{g\sin\theta}\right) + (\frac{1}{2}g\cos\theta)\left(\frac{V_0\cos\theta}{g\sin\theta}\right)^2$

$h\cos\theta = V_0^2\frac{\cos\theta}{g} + \frac{V_0^2\cos\theta\cot^2\theta}{2g}$

$h = \frac{V_0^2}{g} + \frac{V_0^2\cot^2\theta}{2g}$

$2gh = (2+\cot^2\theta)V_0^2$

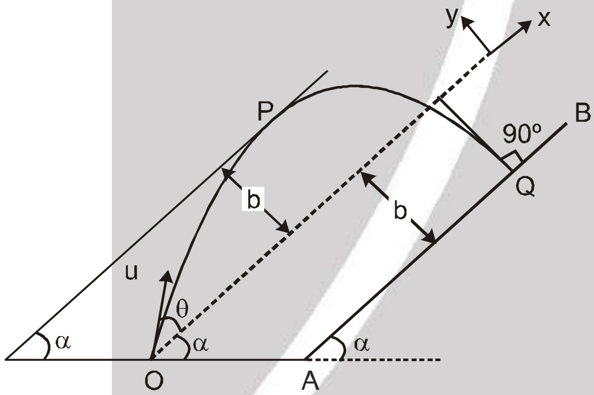
$V_0 = \sqrt{\frac{2gh}{2+\cot^2\theta}}$

5. Consider the motion of the particle from O to P.
The velocity v_y at P is zero.

$v_y^2 = u_y^2 + 2a_ys_y$

$\therefore \quad 0 = (u\sin\theta)^2 - 2(g\cos\alpha)\,b$

or $\quad b = \frac{u^2\sin^2\theta}{2g\cos\alpha}$ ....(i)



Now, consider the motion of the particle from O to Q.
The particle strikes the point Q at 90º to AB, i.e., its velocity along x-direction is zero.
Using $\quad v_x = u_x + a_x t$, we have

$0 = u\cos\theta - (g\sin\alpha)t$

or $\quad t = \frac{u\cos\theta}{g\sin\alpha}$ ...(ii)

For motion in y-direction, $s_y = u_y t + \frac{1}{2}a_yt^2$

or $\quad -b = u\sin\theta\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2$ ...(iii)

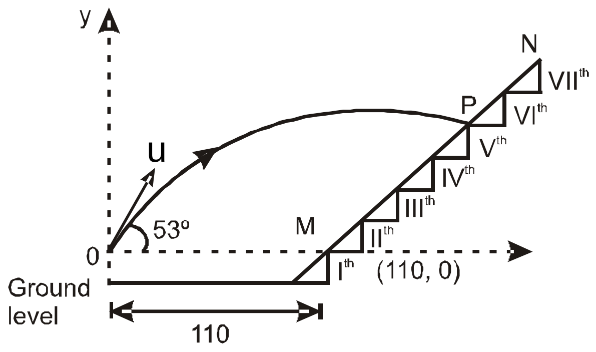
From Eqs. (i) and (iii)

or $\quad -\frac{u^2\sin^2\theta}{2g\cos\alpha} = \frac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \frac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$

or $\quad -\frac{\sin^2\theta}{2\cos\alpha} = \frac{\sin\theta\cos\theta}{\sin\alpha} - \frac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$

Solving, we get $\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. Equation of ball,



$$y = x \tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Substituting the values,

$y = 1.33\,x - 0.011\,3x^2$ .....(1)

Slope of line MN is 1 and it passes through point (110 m, 0). Hence the equation of this line can be written as,

$y = x - 110$ ....(2)

Point of intersection of two curves is say P. Solving (1) and (2) we get positive value of y equal to 4.5 m.

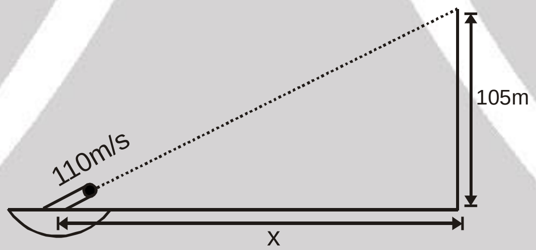
i.e., $y_p = 4.5$

Height of one step 1 m. Hence, the ball will collide somewhere between

y = 4 m and y = 5 m. Which comes out to be 6th step.

7. $y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$$105 = x\tan\theta - \frac{5}{(110)^2}x^2(1+\tan^2\theta)$$



$$\frac{5x^2}{(110)^2}\tan^2\theta - x\tan\theta + \left[105 + \frac{5x^2}{(110)^2}\right] = 0 \qquad (b^2 - 4ac > 0)$$

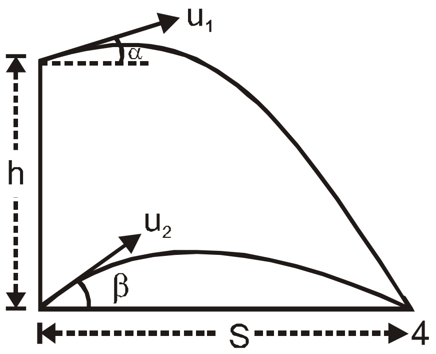
$$x^2 - 4 \times \frac{5x^2}{110^2}\left(105 + \frac{5x^2}{110^2}\right) > 0$$

$$1 - 20 \times \frac{105}{110^2} - \frac{100x^2}{(110)^4} > 0$$

On solving we get

x = 1100 m.

8.



$-h = (u_1 \sin\alpha)\, t - \frac{1}{2} g t^2$

$0 = (u_2 \sin\beta)\, t - \frac{1}{2} g t^2$

$\therefore \quad (u_1 \sin\alpha)t + h = (u_2 \sin\beta)t$

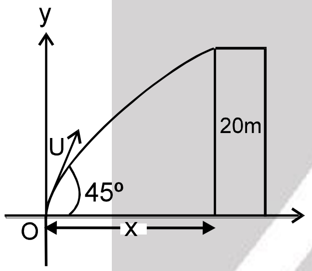
But $\quad t = \frac{s}{u_1 \cos\alpha} = \frac{s}{u_2 \cos\beta}$

$(u_1 \sin\alpha)\left(\frac{s}{u_1 \cos\alpha}\right) + h = (u_2 \sin\beta)\left(\frac{s}{u_2 \cos\beta}\right)$

$h + s\tan\alpha = s\tan\beta$

$h = s(\tan\beta - \tan\alpha)$.

9.



Let us assume that person throws ball from distance x. Assuming point of projection as origin

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

$20 = x\tan 45^o - \frac{1}{2}\frac{10x^2}{u^2\cos^2 45}$

$u^2 = \frac{10x^2}{x-20}$ ....(1)

For ‘u’ to be minimum du/dx = 0

On differentiating w.r.t. ‘x’

$2u\frac{du}{dx} = \frac{10\times 2x\ (x-20) - 10x^2\times 1}{(x-20)^2} = 0$

du/dx = 0

$\Rightarrow \quad 20x(x-20) - 10x^2 = 0$

$20x^2 - 10x^2 - 400x = 0$

$10x(x-40) = 0$

$x = 40$

For x greater than 40, slope is positive & x less than 40, slope is negative
So at x = 40 There is a minima
Required minimum velocity from equation (1)

$u^2_{min} = \dfrac{10 \times 40^2}{40 - 20}$

$u_{min} = \sqrt{800}$

$\mathbf{u_{min} = 20\sqrt{2}\ m/s}$

10. Along Horizontal direction

$x = v_0 \cos 53^o\, t = v_0 \cos 37^o\, (t - t_0)$

$\dfrac{3}{5}\, t = \dfrac{4}{5}\,(t - t_0) \quad \Rightarrow \quad 3t = 4\,(t - t_0)$ ....(1)

Vertical direction ;

$y = v_0 \sin 53^o\, t - \dfrac{1}{2} g t^2 = v_0 \sin 37^o\,(t - t_0) - \dfrac{1}{2} g\,(t - t_0)^2$ ....(2)

$v_0 \times \dfrac{4}{5}\, t - 5t^2 = v_0 \times \dfrac{3}{5} \times \dfrac{3t}{4} - \dfrac{10}{2} \times \dfrac{9t^2}{16}$

$\dfrac{v_0 t}{5}\left(4 - \dfrac{9}{4}\right) = 5\, t^2 \left(1 - \dfrac{9}{16}\right)$

$\dfrac{250}{5} \times \dfrac{7}{4} = 5t \times \dfrac{7}{16} \quad \Rightarrow \quad t = 40 \quad \text{so} \quad t_0 = 10 \text{ sec}$

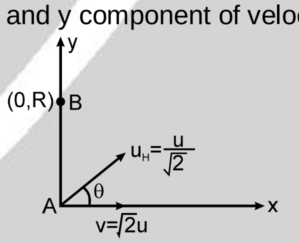
11. Let the speed of shell be u and the speed of wind be v.
The time of flight T remains unchanged due to windstorm

$T = \dfrac{\sqrt{2}\, u}{g}$ ....(1)

Horizontal component of velocity of shell in absence of air

$u_H = \dfrac{u}{\sqrt{2}}$ ....(2)

Hence the net x and y component of velocity of shell (see figure) are



$u_x = \sqrt{2}\, u + \dfrac{u}{\sqrt{2}} \cos\theta$ ....(3a)

$u_y = \dfrac{u}{\sqrt{2}} \sin\theta$ ....(3b)

$\therefore$ The x and y coordinate of point P where shell lands is

$x = u_x T = \left(\sqrt{2}\, u + \dfrac{u}{\sqrt{2}} \cos\theta\right) \dfrac{\sqrt{2}u}{g} = 2R + R\cos\theta$ ....(4a)

$y = u_y T = \left(\dfrac{u}{\sqrt{2}} \sin\theta\right) \dfrac{\sqrt{2}u}{g} = R\sin\theta$ ....(4b)

$\therefore$ The distance S between B and P is given by

$S^2 = (x - 0)^2 + (y - R)^2 = (2R + R\cos\theta)^2 + (R\sin\theta - R)^2$

$= R^2 [6 + 4\cos\theta - 2\sin\theta]$

$= R^2 \left[6 + \sqrt{20}\left(\frac{4\cos\theta}{\sqrt{20}} - \frac{2\sin\theta}{\sqrt{20}}\right)\right]$

$\therefore \quad S_{minimum} = R\sqrt{6-\sqrt{20}}$

$= R\sqrt{6-2\sqrt{5}}$ or $R(\sqrt{5}-1)$ **Ans.**

Alternate : Circle in fig. (1) represents locus of all points where shell lands on the ground in absence of windstorm.

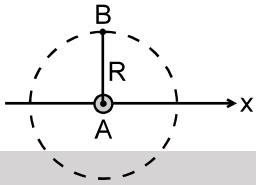


fig.1

Let the speed of shell be 'u' and the speed of wind be $v = \sqrt{2}\,u$. Let T be the time of flight, which remains unaltered even when the windstorm blows. Since R is the maximum range angle of projection is 45º with the horizontal.

Then $R = \frac{u}{\sqrt{2}}\,T$ ....(1)

As a result of flow of wind along x-axis, there is an additional shift (Δx) of the shell along x-axis in time of flight.

$\Delta x = vT = \sqrt{2}\,uT = 2R.$

Hence locus of all points where shell lands on ground shifts along x-axis by 2R as shown in fig. (2).

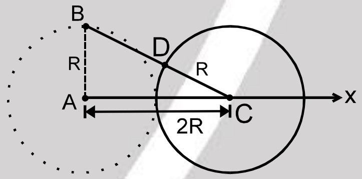


fig.2

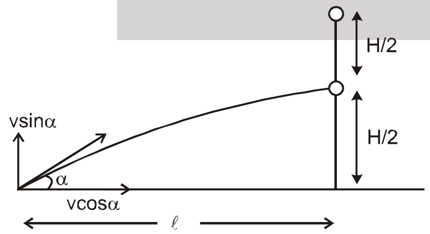
From the fig (2).

$BC = \sqrt{R^2 + (2R)^2} = \sqrt{5R^2} = \sqrt{5}\,R$

Hence the minimum required distance is

BD = BC – DC **=** $\sqrt{5}$ R – R **= ($\sqrt{5}$ – 1) R Ans.**

12.



Let time taken in collison is t. Then $\frac{H}{2} = \frac{1}{2}gt^2$ ...(i)

also for projetile motion

$\ell = (v\cos\alpha)t$ ...(ii)

and $\frac{H}{2} = (v\sin\alpha)t - \frac{1}{2}gt^2$ ...(iii) from (i) and (iii)

$$\frac{H}{2} = vt\sin\alpha - \frac{H}{2}$$

$\Rightarrow$ H = vt sinα ...(iv)

from (ii) & (iv)

$$\frac{H}{\ell} = \tan\alpha$$

$$v = \frac{H}{t\sin\alpha} = \frac{H}{\sqrt{\frac{H}{g}}} \qquad (\because t = \sqrt{\frac{H}{g}} \text{ from (i)})$$

and $\sin\alpha = \frac{H}{\sqrt{\ell^2 + H^2}}$

$$\text{So } V = \sqrt{\frac{\ell^2 + H^2}{H/g}} = \sqrt{\frac{g}{H}(\ell^2 + H^2)} = \sqrt{\frac{gH^2}{H}\left(\frac{\ell^2}{H^2} + 1\right)} = \sqrt{gH\left(1 + \frac{\ell^2}{H^2}\right)}$$

13. $u = 5\sqrt{3}$ m/s.

$\therefore$ $u\cos 60° = \frac{5\sqrt{3}}{2}$ m/s

and तथा $u\sin 60^0 = 5\sqrt{3} \times \frac{\sqrt{3}}{2}$ = 7.5 m/s

Since the horizontal displacement of both the shots are equal , the second should be fired early because its horizontal component of velocity u cos60^0 is less than the other's which is u or $5\sqrt{3}$ m/s.

Now let first shot takes t_1 time to reach the point P and the second t_2. Then –

$x = (u\cos 60^o)\, t_2 = u.\, t_1$

or $x = \left(\frac{5\sqrt{3}}{2}\right) t_2 = 5\sqrt{3}\, t_1$ ....(1)

or $t_2 = 2t_1$ ....(2)

and $h = \frac{1}{2} g\, t_1^2 = \frac{1}{2} g t_2^2 - (7.5)\, t_2$

Taking g = 10 m/s^2 लेने पर

$h = 5t_2^2 - 7.5\, t_2 = 5t_1^2$ ....(3)

Substituting $t_2 = 2t_1$ in equation (3), we get

$5(2t_1)^2 - 7.5\,(2\,t_1) = 5t_1^2$

or $5t_1^2 = 5t_1$

t_1 = 0 and 1s

Hence t_1 = 1s and

$t_2 = 2t_1 = 2$s

$x = 5\sqrt{3}\, t_1 = 5\sqrt{3}$ m (From equation I)

and $h = 5\, t_1^2 = 5\,(1)^2 = 5$ m (From equation 3)

$\therefore$ y = 10 – h = (10 – 5) = 5 m

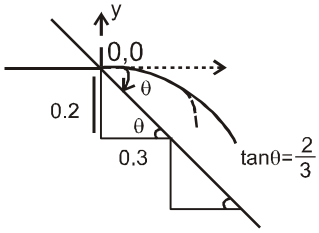
Hence

(i) Time interval between the firings = $t_2 - t_1$ = (2 – 1) s

Δ = 1s

(ii) Coordinates of point P = (x, y) = $5\sqrt{3}$ m. 5 m

14. We have the point of projection as (0, 0)
We have the equation of straight line (as shown in fig.)



$y = x \tan\theta$

$y = \frac{-2}{3} \times$ ....(1)

Also the equation of trajectory for horizantal projection

$y = \frac{-1}{2} g \frac{x^2}{u^2}$ ....(2)

from (1) and (2)

$\frac{1}{2} g \frac{x^2}{u^2} = \frac{2}{3} x$

or $x = \frac{2}{3} \times \frac{u^2}{5} = \frac{2}{3} \times \frac{4.5 \times 4.5}{5} = 3 \times 0.9$

If no. of steps be n then $n \times 0.3 = 3 \times 0.9$
n = 9

15. $x = y^2 + 2y + 2$

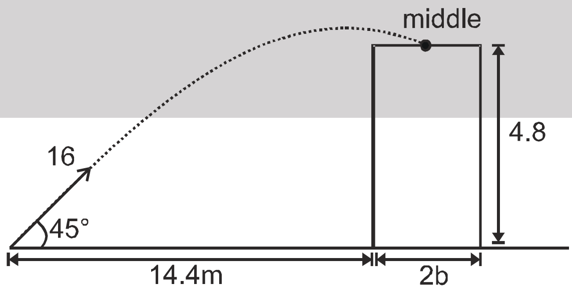
$\frac{dx}{dt} = 2y\frac{dy}{dt} + 2\frac{dy}{dt} + 0$

$\frac{d^2x}{dt^2} = 2\left(\frac{dy}{dt}\right)^2 + 2y\frac{d^2y}{dt^2} + 2\frac{d^2y}{dt^2}$

$\frac{d^2y}{dt^2} = 0.$ ($\frac{dy}{dt}$ =5 m/s)

$\frac{d^2x}{dt^2} = 2\,(5^2) + 0 + 0 = 50$ m/s^2. **Ans. "A"**

16. Equation of Trajectory



$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$

4.8 = (14.4 + b) × 1 – 1/2
on solving this equation
we get 2b = 9.6 m ⇒ b = 4.8 m
Now to find angle of projection for projectile having speed $10\sqrt{3}$ m/s.

$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$ $\qquad$ [$x = 2b + 14.4 \Rightarrow 9.6 + 14.4 = 24$ m]

$4.8 = 24\tan\theta - \frac{1}{2}\frac{10(24)^2\sec^2\theta}{(10\sqrt{3})^2}$

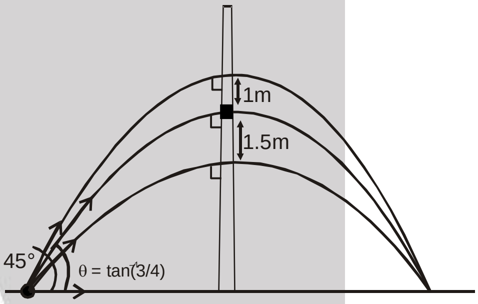
$4.8 = 24\tan\theta - \frac{24\times 4}{10}(1+\tan^2\theta)$

$4\tan^2\theta - 10\tan\theta + 6 = 0$

$\tan\theta = \frac{3}{2}, 1$

$\theta = \tan^{-1}\frac{3}{2}, \theta = 45°$

17. $H = \frac{u^2\sin^2\theta}{2g}$, $R = \frac{u^2\sin 2\theta}{g}$



so, $\frac{H}{R} = \left(\frac{\tan\theta}{4}\right)$ ......(1)

$\frac{H+1}{R} = \left(\frac{\tan 45°}{4}\right) = \frac{1}{4}$ ....(2)

$\frac{H-1.5}{R} = \frac{\tan\left[\tan^{-1}(3/4)\right]}{4}$

$\frac{H-1.5}{R} = \frac{3/4}{4} = \frac{3}{16}$ ....(3)

$\frac{H+1}{H-1.5} = \frac{4}{3} \Rightarrow \frac{10}{R} = \frac{1}{4} \Rightarrow$ **R = 40 m**

$3H + 3 = 4H - 6$

$H = 9$ m

$\frac{9}{40} = \frac{\tan\theta}{4}$

$\tan\theta = \frac{9}{10} \qquad \Rightarrow \theta = \tan^{-1}\left(\frac{9}{10}\right)$

$R = 40 \qquad \tan\theta = 9/10$

$\frac{u^2\sin 2\theta}{g} = 40 \qquad \sin\theta = \frac{9}{\sqrt{181}}$

Using $R = \frac{u^2.2\sin\theta\cos\theta}{g}$

$\frac{u^2 2\left(\frac{9}{\sqrt{181}}\right)\left(\frac{10}{\sqrt{181}}\right)}{10} = 40$

$u^2 = \frac{3620}{9} \Rightarrow u = \frac{\sqrt{3620}}{3}$ m/s