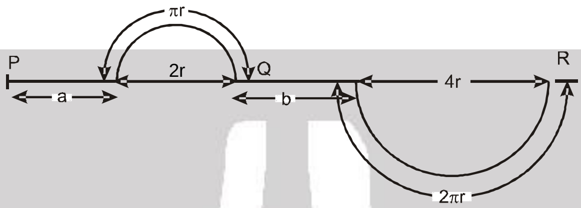
SOLUTIONS OF RECTILINEAR MOTION

EXERCISE–1
PART - I

Section (A)

A-1. a = 7m, b = 8m, $r = \frac{11}{\pi}\left[\pi = \frac{22}{7}\right]$



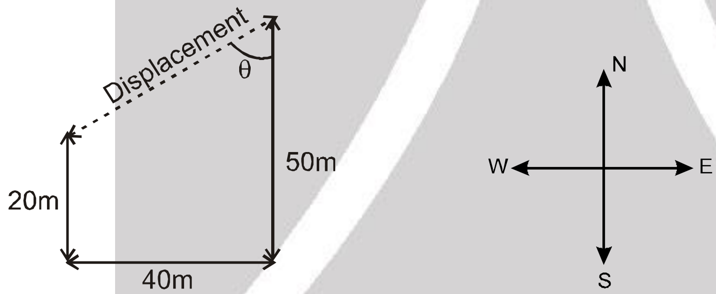
Distance travelled by the car from P to R

$= a + \pi r + b + 2\pi r = a + b + 3\pi r = 7 + 8 + 3\pi \times \frac{11}{\pi} =$ **48 m** **Ans**

Displacement of the car from P to R

$= a + 2r + b + 4r = a + b + 6r = 7 + 8 + 6 \times \frac{11}{\pi} = 15 + 6 \times \frac{11}{22} \times 7 =$ **36 m Ans**

A-2.



(a) Distance covered by the man to reach the field.

$= 50 + 40 + 20 =$ **110 m Ans**

(b) Displacement of man from his house to the field

$= \sqrt{(40)^2 + (30)^2} = \sqrt{1600 + 900} = \sqrt{2500}$ **= 50 m Ans**

Direction of displacement can be known by finding θ

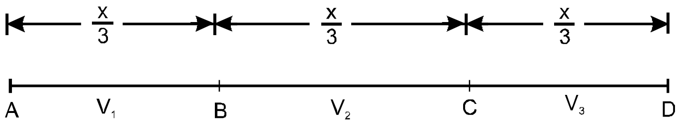
$\tan\theta = \frac{40}{30}$

$\Rightarrow \quad \theta = \tan^{-1}\left(\frac{4}{3}\right)$ West of South **Ans**

Section (B) :

B-1. Initial reading of meter = 12352 km
Final reading of meter = 12416 km
Time taken by car = 2 hr
(a) Distance covered by car
= (Final reading– Initial reading)
= (12416 – 12352) = 64 km
Average speed of car = 64/2 = 32 km/h **Ans**
(b) As the car returns to the initial point after whole journey,
hence, displacement of car = 0
Therefore, average velocity = 0 **Ans**

B-2.



Suppose the total distance covered by the particle i.e., AD = x
Particle covers first one – third distance AB with speed V_1
second one – third distance BC with speed V_2 and
third one – third distance CD with speed V_3.

Average speed of the particle = $\dfrac{\text{Total distance covered by the particle}}{\text{Total time taken by the particle}}$

$$= \frac{X}{\frac{X/3}{V_1} + \frac{X/3}{V_2} + \frac{X/3}{V_3}} = \frac{1}{\frac{1}{3V_1} + \frac{1}{3V_2} + \frac{1}{3V_3}} = \frac{3V_1V_2V_3}{V_1V_2 + V_2V_3 + V_3V_1}$$ **Ans**

Section (C)

C-1. The position of a body is given as $x = At + 4B\,t^3$

(a) $x = At + 4\,Bt^3$

$V = \dfrac{dx}{dt} = A + 12\,B\,t^2$, So, $a = \dfrac{dV}{dt} = 24\,Bt$ **Ans**

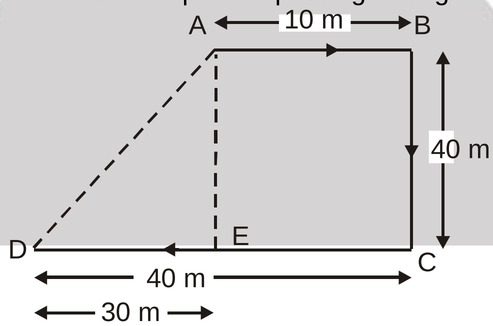
(b) At t = 5 s, $V = A + 12\,B\,(5)^2$,
i.e., $V = A + 300\,B$ **Ans**
At t = 5 s, a = 24 B (5) i.e., a = 120 B **Ans**

C-2. Maximum speed V = 18 km/h
= 18 × 5/18 = 5 m/s

Avg. acc. = $\dfrac{0 + V_{max}}{2} = \dfrac{0+5}{2}$,

So average acceleration = $\dfrac{5}{2}$ m/s^2 **Ans**

C-3 The particle starts from point A & reaches point D passing through B & C as shown in the figure.



Now, AE = 40 m & DE = 30 m

$\therefore$ Displacement = AD = $\sqrt{AE^2 + DE^2} = \sqrt{40^2 + 30^2} = 50$ m

Total time taken in the motion = $t_{AB} + t_{BC} + t_{CD}$

$$= 2 + \frac{40}{5} + 8 = 18 \text{ s}$$

Total distance travelled = AB + BC + CD = 10 + 40 + 40 = 90m

Average velocity = $\dfrac{\text{Displacement}}{\text{time}} = \dfrac{50}{18} = \dfrac{25}{9}$ m/s

Average speed = $\dfrac{\text{Distance}}{\text{time}} = \dfrac{90}{18} = 5$ m/s.

Section (D)

D-1. $u = 36\ km/h = 36 \times \frac{5}{18}\ m/s = 10\ m/s$

$V = 90\ km/h = 90 \times \frac{5}{18}\ m/s = 25\ m/s$

From the equation of motion;

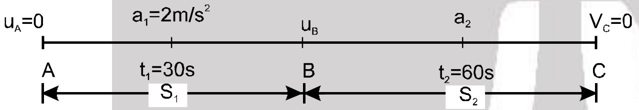
$V = u + at$ putting $t = 5s$

$25 = 10 + a(5)$, i.e., $a = \frac{25-10}{5} \Rightarrow a = 3\ m/s^2$ **Ans**

For distance travelled by the car in 5 sec, we use

$s = ut + \frac{1}{2}at^2 = 10 \times 5 + \frac{1}{2} \times 3\,(5)^2 = \frac{100+75}{2} = \frac{175}{2}$ i.e., = 87.5 m **Ans**

D-2.



(a) For motion from A to B :

$u_B = u_A + a_1t_1 = 0 + 2\,(30) = 60\ m/s$

Also, $S_1 = u_A\,t_1 + \frac{1}{2}a_1t_1^2 = 0 + \frac{1}{2}(2)(30)^2 \Rightarrow S_1 = 900\ m$

For motion from B to C :

$V_C = u_B - a_2\,t_2$; $0 = 60 - a_2\,(60)$;

i.e., $a_2 = \frac{60}{60} = 1\ m/s^2$, Also $V_C^2 = u_B^2 - 2a_2s_2$

$\Rightarrow (0)^2 = (60)^2 - 2(1)\,S_2$, i.e, $S_2 = \frac{60 \times 60}{2}$, i.e, $S_2 = 1800\ m$

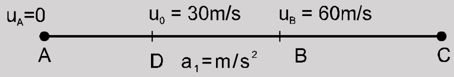
Now, total distance moved by the train

$S = S_1 + S_2 = 900 + 1800 \Rightarrow S = 2700\ m$ **Ans**

(b) Maximum speed attained by the train will be at the point B, as after this point train starts retarding

So, $V_{max} = V_B = 60\ m/s$ **Ans**

(c) There will be two positions at which the train will be at half the maximum speed for motion from A to B.



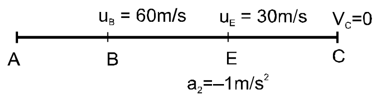
Let D be a point where $u_0 = \frac{u_B}{2} = \frac{60}{2}$; $u_D = 30\ m/s$

$u_D^2 = u_A^2 + 2a_1\,(AD)$

$\Rightarrow AD = \frac{u_D^2 - u_A^2}{2a_1} \Rightarrow AD = \frac{(30)^2 - (0)^2}{2 \times 2}$

$\Rightarrow AD = \frac{900}{4} \Rightarrow AD = 225\ m$ **Ans**

For motion from B to C



Let E be a point where $u_E = \frac{u_B}{2} = \frac{60}{2} = 30\ m/s$.

$u_E^2 = u_B^2 + 2a_2\,(BE) \Rightarrow (30)^2 = (60)^2 + 2\,(-1)\,BE$

$\Rightarrow BE = \frac{(60)^2 - (30)^2}{2} \Rightarrow BE = \frac{2700}{2}$

$\Rightarrow BE = 1350\ m$

Hence, the position of point from initial point (A)

$\Rightarrow AE = AB + BE = 900 + 1350 = 2250\ m$ $AE = 2.25\ km$ **Ans**

D-3. Given u = 72 km/h

$= 72 \times \frac{5}{18} = 20$ m/s & $a = -2$ m/s^2

(a) V = 0, s = ?

From the equation of motion;

$V^2 = u^2 + 2as$

$(0)^2 = (20)^2 + 2\times(-2)$ S, i.e., 4s = 400, or S = 100 m **Ans**

(b) V = 0, t = ?

From the equation of motion;

V = u + at

$0 = 20 + (-2)t \Rightarrow 2t = 20 \Rightarrow t = 10$ s **Ans**

(c) Distance travelled during the first second

$[s_t = u + \frac{1}{2}(-2)(2 \times t - 1)]$

$S_1 = 20 + \frac{1}{2}(-2)(2 \times 1 - 1) \Rightarrow S_1 = 20 - 1$

$\Rightarrow S_1 = 19$ m **Ans**

Distance travelled during the third second

$S_3 = 20 + 1/2(-2)(2 \times 3 - 1)$; or $S_3 = 20 - 5$; or, $S_3 = 15$ m **Ans**

Alternatively :

as चूंकि $u = 72 \times 5/18 = 20$ m/s, $a = -2$ m/s^2.

(a) $v^2 = u^2 + 2as$

$(0)^2 = 400 + 2 \times -2 \times s$, s = 100m

(b) v = u + at , 0 = 20 – 2t t = 10 sec.

(c) $D_n = u + a/2\,(2n - 1)$

$\Rightarrow$ In First second

$D_1 = 20 - 2/2\,(2\times1 - 1) = 19$m

$\Rightarrow$ In Third second

$D_3 = 20 - 2/2\,(2 \times 3 - 1) = 15$m

D-4. Let h be the height of the tower and t be the total time taken by the ball to reach the ground.

Distance covered in t^{th} (last second) second = 15 m

$[s_t = u + 1/2\, g\,(2t - 1)]$

$0 + 1/2\, g\,(2t - 1) = 15$ or, $1/2\,(10)(2t - 1) = 15$;

or, $2t - 1 = 3$ or t = 2 sec

Now, height of the tower is given by

$h = ut + 1/2gt^2$; $h = 0 + 1/2\,(10)(2)^2$; ; i.e., h = 20 m **Ans**

D-5. (i) Maximum height reached by ball = 20 m.

So, taking upward direction as positive, $v^2 = u^2 + 2as$

So, $0 = u^2 - 2 \times 10 \times 20$

or u = 20 m/sec **Ans.**

Also time taken by ball = t = u/g = 20/10 = 2 sec. (for touching the plane)

(ii) Horizontal distance travelled by plane in this time $t = s = u_x t + 1/2\, a_x t^2$

where, u_x = initial velocity of plane, a_x = acceleration of plane.

So, $s = 0 \times 2 + 1/2 \times 2 \times 2^2 = 4$ m

(iii) Man catches the ball back 2 seconds after it touches the plane.

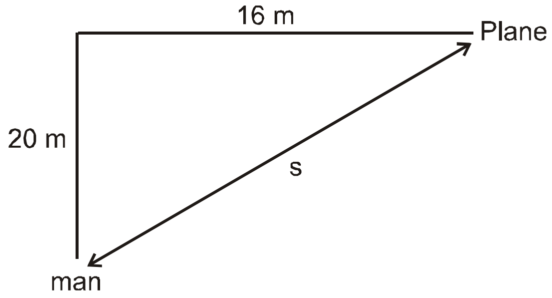
Velocity of plane when ball touches it

$\Rightarrow v_x = u_x + a_x t = 0 + 2 \times 2 = 4$ m/sec.

Now, acceleration of plane becomes : $a_x' = 4$ m/sec^2

so, s_x' = horizontal distance travelled by plane after touch with ball = $u_x' + 1/2\, a_x' t^2$

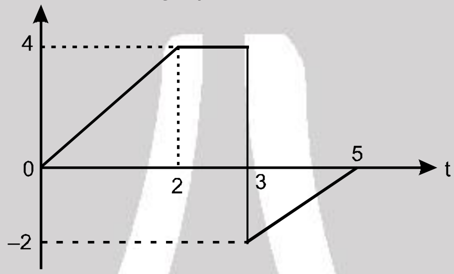
$= 4 \times 2 + 1/2 \times 4 \times 4 = 8 + 8 = 16$ m



Final distance between man and plane = s = $\sqrt{(20)^2 + (16)^2}$ = $\sqrt{656}$ m

Section (E)

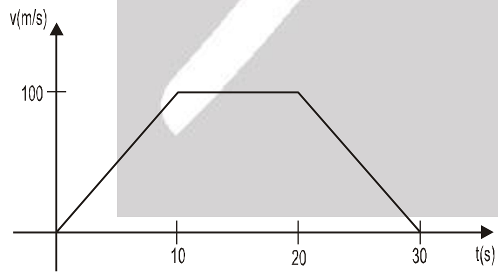
E-1. For a particle moving along x – axis, v – t graph is as shown.



Distance travelled by the particle = sum of areas under V – t graph
= 1/2 (3 + 1)4 + 1/2× 2 × 2 = 8 + 2 = 10 m
Displacement of the particle = area above t-axis – area below t-axis
= 1/2 (3 + 1) 4 – 1/2 × 2 × 2 = 8 – 2 = 6 m

Average velocity = $\dfrac{\text{Displacement}}{\text{time} - \text{interval}} = \dfrac{6}{5}$ = 1.2 m/s

E-2.



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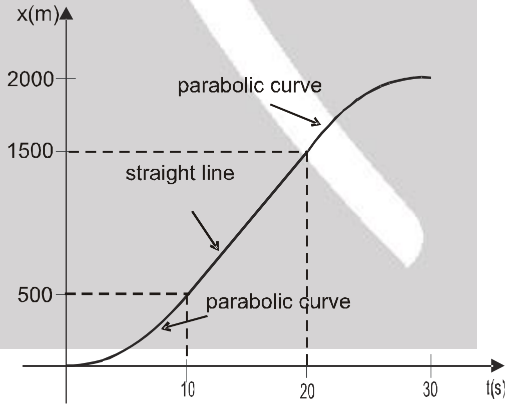
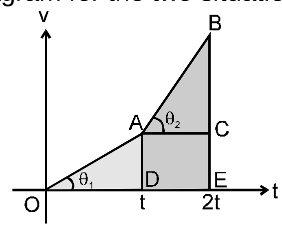


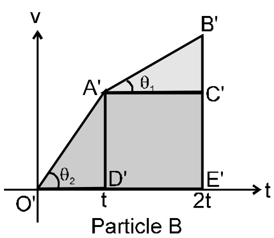
Distance travelled = area under V-t curve
= 2000 m

E-3. v - t diagram for the two situations is shown below



Particle A

$\tan\theta_1 = a$



Particle B

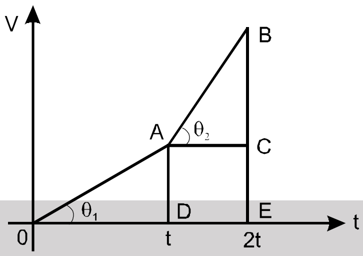
$\tan\theta_2 = 2a$

In v - t graph, distance travelled = area under the graph

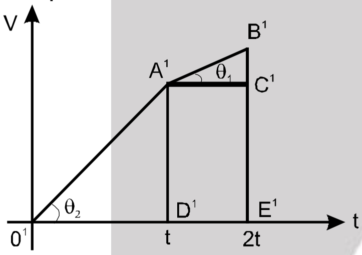
Area (AOD) = Area (A'B'C')
Area (ABC) = Area (O'A'D')
Area (ACED) < Area (A'C'E'D')
$\therefore$ particle B has covered larger distance.

Aleternate Solution:

For particle A :

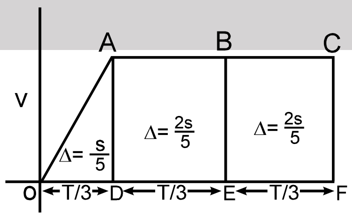


For v – t graph, slope = acceleration, Suppose the slope of OA, i.e., tan θ_1 = m ; hence, the slope of AB, i.e.,
tan θ_2 = 2m, AD = mt, BC = 2mt
Distance travelled by the particle A
S_A = 1/2 (t) (mt) + 1/2 (mt + 3mt)t
S_A = 2.5 mt
For particle B



slope of OA^1 tan θ_2 = 2m
slope of A^1B^1 tan θ_1 = m
AD = 2mt, BC = mt
Distance travelled by the particle B
S_B = 1/2 (t) (2mt) + 1/2 (2mt + 3mt) t
S_B = 3.5 mt
Therefore; $S_B > S_A$

E-4 Let the total time of race be T seconds and the distance be S = 100 m.
The velocity vs time graph is
Area of ΔOAD



Δ = s/5

$$\therefore \quad \frac{s}{5} = \frac{1}{2} a \left(\frac{T}{3}\right)^2 = \frac{1}{2} 8 \left(\frac{T}{3}\right)^2$$

or $T = 3\sqrt{5}$ m /s

PART - II

Section (A)

A-1. Dimension of hall, length of any side = 10 m = a (say) **(B) Ans**

Magnitude of displacement = Length of diagonal = $a\sqrt{3}$ = $10\sqrt{3}$ m

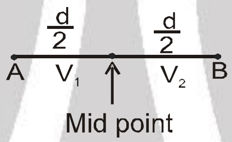
Section (B)

B-1. Suppose AB = x km

Average speed = $\dfrac{\text{Total distance covered}}{\text{Total time taken}}$

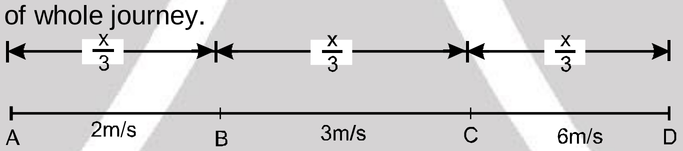
$= \dfrac{2x}{\dfrac{x}{20}+\dfrac{x}{30}} = \dfrac{2}{\dfrac{1}{20}+\dfrac{1}{30}} = \dfrac{20\times 60}{20+30}$ = 24 km/h = 24 kmh^{-1} **(B) Ans**

B-2. Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time interval}}$



$\Rightarrow \quad V = \dfrac{d}{\dfrac{d}{2v_1}+\dfrac{d}{2v_2}} = \dfrac{2V_1V_2}{V_1+V_2}$ **(A) Ans**

B-3. Let x be the length of whole journey.



Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time taken}}$

$= \dfrac{X}{\dfrac{x/3}{2}+\dfrac{x/3}{3}+\dfrac{x/3}{6}} = \dfrac{1}{\dfrac{1}{6}+\dfrac{1}{9}+\dfrac{1}{18}} = \dfrac{18}{3+2+1}$ = 3 m/s **(A) Ans**

B-4. Average speed = $\dfrac{\text{Total distance}}{\text{Total time taken}} = \dfrac{2\pi r}{62.8}$

$= \dfrac{2\times 3.14\times 100}{62.8}$ = 10m/s

Average velocity = $\dfrac{\text{Total displacement}}{\text{Total time taken}} = \dfrac{0}{62.8}$ = zero

Hence option (B) is correct.

Section (C)

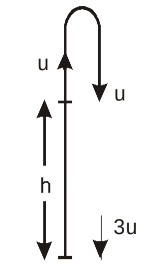
C-1. The displacement of a body is given as $2s = gt^2$

Differentiating both sides w.r.t. 't'

$\Rightarrow \quad 2\dfrac{ds}{dt} = 2\,gt \quad \Rightarrow \quad 2\,V = 2gt \quad \Rightarrow \quad V = gt$ **(A) Ans**

C-2. I mothod – Let downward direction is taken as +ve. Initial vel is –ve = – u (say)

$\therefore$ From the equation ; $v^2 - u^2 = 2as$ we get $(3u)^2 - (-u)^2 = 2hg$



$\Rightarrow$ $h = \dfrac{4u^2}{g}$ **"B" Ans.**

The stone is thrown vertically upward with an initial velocity u from the top of a tower it reaches the highest point and returns back and reaches the top of tower with the same velocity u vertically downward.

Now, from the equation, $V^2 = u^2 + 2gh$

$\Rightarrow$ $(3u)^2 = u^2 + 2\,gh$ $\Rightarrow$ $2gh = 9u^2 - u^2$ $\Rightarrow$ $h = \dfrac{8u^2}{2g}$ $\Rightarrow$ $h = \dfrac{4u^2}{g}$ **"B" Ans.**

C-3. u = 0,

Acceleration = a

t = n sec,

The velocity after n sec is

n sec

V = u + at

V = 0 + a(n)

V = an

a = V/n ....(i)

The displacement of the body in the last two seconds $[S = ut + 1/2\, at^2 = 1/2\, at^2]$

$S_2 = S_n - S_{n-2}$

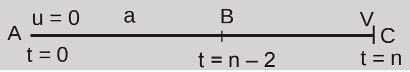
$= \dfrac{1}{2}an^2 - \dfrac{1}{2}a\,(n-2)^2 = \dfrac{1}{2}a\,[n^2 - (n-2)^2]$

$= \dfrac{1}{2}a\,[n^2 - n^2 - 4 + 4\,n]$

$S_2 = 2a(n-1)$

From equation (i) $S_2 = \dfrac{2V(n-1)}{n}$ **"A" Ans**

Aliter :



BC = ?

BC = AC – AB

$= [0 \times n + \dfrac{1}{2}an^2] - (0 \times (n-2) + \dfrac{1}{2}a\,(n-2)^2)$.

$BC = \dfrac{a}{2}\,[n^2 - n^2 - 4 + 4n] = \dfrac{4a}{2}\,[n-1]$

BC = 2 a (n – 1) ....(1)

For AC AC के लिए

V = u + at

V = 0 + an

a = V/n ......(2)

From (1) and (2) $BC = \dfrac{2V}{n}(n-1)$

Section (D)

D-1. $u = 0$, Let acceleration $= a$
Total time $t = 30$ s
X_1 = distance travelled in the first 10 s.

Using , $S = ut + \frac{1}{2}at^2$, we get

$X_1 = 0 + \frac{1}{2}a(10)^2$, i.e., $X_1 = 50a$

Similarly,
X_2 = distance travelled in the next 10 s

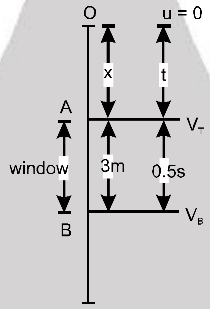
So, $X_2 = (0 + 10a)\,10 + \frac{1}{2}a(10)^2$

So, $X_2 = 100a + 50a$
or, $X_2 = 150a$
and, X_3 = distance travelled in the last 10 s

So, $X_3 = (10a + 10a)\,10 + \frac{1}{2}a(10)^2$

or या, $X_3 = 200a + 50a$
or, $X_3 = 250a$
Hence, $X_1 : X_2 : X_3 = 50a : 150a : 250a = 1 : 3 : 5$ **"C" Ans**

D-2. Let x be the distance of the top of window from the top of building and t be the time taken by the ball from the top of building to the top of window.



(i) Since, acceleration is constant = g

So, $S = \frac{u+v}{2}\,t$ (across the window)

$3 = \frac{v_T + v_B}{2}\,t \Rightarrow \quad 3 = \frac{v_T + v_B}{2}\,0.5$

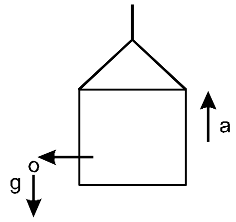
So, $v_T + v_B = 12$ m/sec.

Aliter :
For motion from O to A
$V_T^2 = u^2 + 2gx = (0)^2 + 2gx$
$V_T^2 = 2gx$ ....(i)
$V_T = u + gt = 0 + gt$
$V_T = gt$ ....(ii)
For motion from O to B
$V_B^2 = u^2 + 2g(x + 3)$
$V_B^2 = (0)^2 + 2g(x + 3)$
$V_B^2 = 2g(x + 3)$ ....(iii)
$V_B = u + g(t + 0.5)$
$V_B = 0 + g(t + 0.5)$
$V_B = g(t + 0.5)$ ....(iv)
From equations (ii) and (iv)
$V_B - V_T = g(0.5)$ ....(v)
From equations (i) and (iii)
$V_B^2 - V_T^2 = 2g(3)$ ....(vi)

From equations (v) and (vi)

$$\frac{V_B^2 - V_T^2}{V_B - V_T} = \frac{2g(3)}{g(0.5)} \Rightarrow \frac{(V_B - V_T)\ (V_B + V_T)}{(V_B - V_T)} = 12 \quad \Rightarrow \quad V_T + V_B = 12\ ms^{-1}$$ **(A) Ans**

D-3.



After the release of stone from the elevator going up with an acceleration a, stone will move freely under gravity (g), hence the acceleration of the stone will be g towards downwards.

"D" Ans

Aliter :

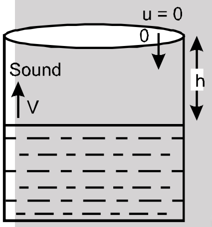
Acceleration of stone = g downward [free fall under gravity]

D-4. Initial velocity = u, acceleration = f = at

f = at $\quad$ dV/dt = at $\quad$ dV = at dt

Integrating both sides

$$\Rightarrow \int_u^v dV = \int_0^t at\,dt \quad \Rightarrow \quad V - u = \frac{at^2}{2} \quad \Rightarrow \quad V = u + \frac{at^2}{2}$$ **"B" Ans**



D-5.

Suppose, t_1 = time taken by stone to reach the level of water

t_2 = time taken by sound to reach the top of well

so, $\quad T = t_1 + t_2$

For t_1 : $u = 0$

$$h = ut + \frac{1}{2}gt^2 \qquad h = 0 + \frac{1}{2}gt_1^2 \qquad t_1 = \sqrt{\frac{2h}{g}}$$

For t_2 : As the velocity of sound is constant

$$h = Vt_2 \qquad \Rightarrow \qquad t_2 = \frac{h}{V}$$

Therefore, $\quad T = \sqrt{\frac{2h}{g}} + \frac{h}{V}$ $\qquad$ **"B" Ans**

Aliter :

T= Time taken by stone from top to level water. (T_1) + Time taken by sound from level water to top of the well. (T_2)

for downward journey of stone :

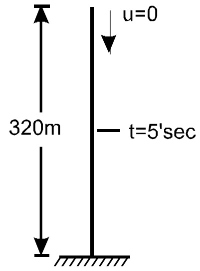
$$s = ut + \frac{1}{2}at^2 \quad \Rightarrow \quad h = 0 + \frac{1}{2}gT_1^2 \quad \Rightarrow \quad T_1 = \sqrt{\frac{2h}{g}}$$

for upward journey of sound, Time $\quad (T_2) = \frac{h}{v}$

$$\therefore \quad T = \sqrt{\frac{2h}{g}} + \frac{h}{v}$$

Hence option (B) correct.

D-6.



Let t be the time taken by the superman to reach the student for saving the students life just before reaching the ground. Hence, the time taken by the student to reach the ground = (t + 5) s

For motion of student

u = 0, h = 320 m, g = 10 m/s^2

From equation, $h = ut + \frac{1}{2}gt^2$,

i.e., $320 = 0 + \frac{1}{2}(10)(t + 5)^2$

i.e., $(t + 5)^2 = 64$; or $t + 5 = 8$; i.e., $t = 3$ sec

For motion of superman

Let initial velocity u = V, h = 320 m, g = 10m/s^2

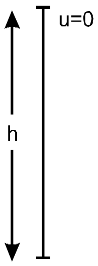
from equation $h = ut + \frac{1}{2}gt^2$

i.e., $320 = V(3) + \frac{1}{2}(10)(3)^3$,

i.e., 320 = 3 V + 45,

or 3V = 320 – 45, or $V = \frac{275}{3}$ m/s **"B" Ans**

D-7.



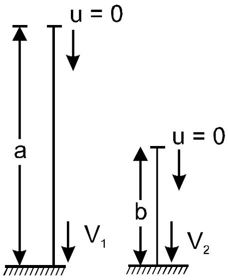
In the above problem, if height of the skyscraper is such that student covers the full height within 5 sec then superman will be unable to save him.

u = 0, t = 5 sec, g = 10 m/s^2

Hence; from equation h = ut + 1/2 gt^2,

or h = 0 + 1/2 (10) (5)^2, i.e, h = 125 m **"C" Ans**

D-8.



From the equation,

$V^2 = u^2 + 2gh$

$V_1^2 = 0 + 2ga$

$V_1^2 = 2ga$ ....(i)

$V_2^2 = 2gb$ ....(ii)

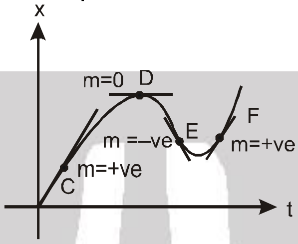
From the equations (i) and (ii)

we get $\frac{V_1^2}{V_2^2} = \frac{2ga}{2gb} = \frac{a}{b}$ i.e., $\frac{V_1}{V_2} = \frac{\sqrt{a}}{\sqrt{b}}$ **(B) Ans**

$\therefore$ option (B) is correct

Section (E)

E-1. The slope of position–time (x–t) graph at any point shows the instantaneous velocity at that point.
The slope of given x – t graph at different point can be shown as



Obviously the slope is negative at the point E as the angle made by tangent with +ve X–axis is obtuse, hence the instantaneous velocity of the particle is negative at the point E i.e., **"C" Ans**

Aliter : As Instantaneous velocity is negative where slope of x–t curve is negative .

At. point C = slope is positive

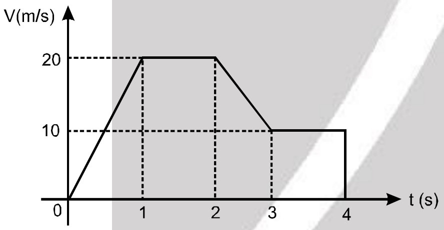
At. point D = slope is zero

At. point E = slope is negative

At. point F = slope is positive

Hence, option (C) is correct

E-2.



The distance travelled by the particle in 4s

= Sum of areas under V–t graph

= 1/2 × 1 × 20 + 1× 20 + 1/2 (20 + 10) × 1 + 1 × 10 = 55 m

E-3. u = 0, a = Constant = k (let)

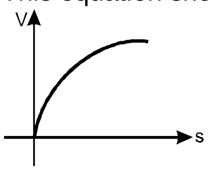
From equation of motion;

$V^2 = u^2 + 2as$

$V^2 = (0)^2 + 2ks$

$V^2 = 2\ ks$

This equation shows a parabola with S-axis as its axis. Hence, its graph can be shown as



i.e., **"B" Ans**

E-4. As the slope of displacement - time (x – t) graph shows the velocity, the ratio of velocities of two particles A and B is given by

$\frac{V_A}{V_B} = \frac{\tan\theta_A}{\tan\theta_B} = \frac{\tan 30^o}{\tan 60^o} = \frac{1}{\sqrt{3}\times\sqrt{3}} = \frac{1}{3}$ i.e., **"D" Ans**

E-5. $V_{t=3} - V_{t=0}$ = area under a – t curve

$\therefore$ $V_{t=3} = 10.5$ m/s

PART - III

1. From graph (a) $\Rightarrow$ v = kx
where k is positive constant

Acceleration = $v\frac{dv}{dx}$ = kx . k = k^2x

$\therefore$ Acceleration is non uniform and directly proportional to x.

$\therefore$ a $\rightarrow$ Q, S

From graph (b) $\Rightarrow$ v^2 = kx.

Differentiating both sides with respect to x.

$2v\frac{dv}{dx} = k$ or $v\frac{dv}{dx} = \frac{k}{2}$

Hence acceleration is uniform.

$\therefore$ b $\rightarrow$ P

From graph (c) $\Rightarrow$ v = kt

Acceleration = dv/dt = k

Hence acceleration is uniform $\Rightarrow$ c $\rightarrow$ P

From graph (d) $\Rightarrow$ v = kt^2

Acceleration = dv/dt = 2kt

Hence acceleration is non uniform and directly proportional to t.

$\therefore$ d $\rightarrow$ Q, R

EXERCISE–2
PART - I

1. **(A)** Relation between position x and time t is given as

$x^2 + 2x = t$

Differentiating both sides w.r.t 't'

$2x\frac{dx}{dt} + 2\frac{dx}{dt} = 1$, i.e., 2 (x + 1) V = 1 $\Rightarrow$ $V = \frac{1}{2(x+1)}$

Again differentiation both sides w.r.t 't'

$\Rightarrow \frac{dV}{dt} = \frac{-1}{2(x+1)^2}\left(\frac{dx}{dt}\right)$ $\Rightarrow$ $a = \frac{-V}{2(x+1)^2}$ $\Rightarrow$ $a = \frac{-1}{4(x+1)^3}$

Hence, the retardation of the particle is $\frac{1}{4(x+1)^3}$

[Note : As v and a are oppositely directed, so particle is retarding]

Aliter : $x^2 + 2x = t$

By Differentiation w. r. t. time

2x v + 2v = 1

or $xv + v = \frac{1}{2}$ ....(1) $v = \frac{1}{2}$ (x + 1)

or Again differentiate eq ....(1)

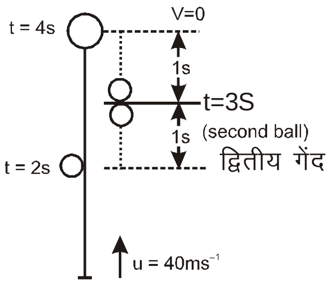
we have

$x\frac{dv}{dt} + v\frac{dx}{dt} + \frac{dv}{dt} = 0$

x a + v^2 + a = 0

$a = -\frac{v^2}{(x+1)}$ $a = -\frac{1}{4(x+1)^3}$

2.



$u = 40$ m/s , $g = 10$ m/s^2

Let t be time taken by the first ball to reach the highest point.

$V = u - gt$ $0 = 40 - 10\,t$ $t = 4$ s

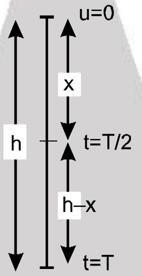
From figure second ball will collide with first ball after 3 second, therefore the height of collision point = height gained by the second ball in 3 sec

$= 40\,(3) - 1/2\,(10)\,(3)^2$

$= 120 - 45 = 75$ m **"B" Ans**

3. $u = 0,\ t = T\ ;\ h = ut + \frac{1}{2}gt^2\ ;\ h = \frac{1}{2}gT^2$

$h = \frac{1}{2}gT^2$ ...(i)



Let x be the distance covered by the body in t = T/2

$x = 0 + \frac{1}{2}g\,(T/2)^2$

$x = \frac{1}{8}gT^2$ ...(ii)

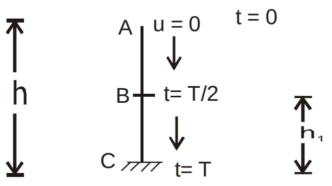
From equations (i) and (ii)

$\frac{h}{x} = \frac{1/2\ \ gT^2}{1/8\ \ gT^2}$ $\frac{h}{x} = \frac{4}{1}$ $\Rightarrow$ $x = \frac{h}{4}$

Therefore height of that point from ground

$= h - x = h - \frac{h}{4} = \frac{3h}{4}$ **"C" Ans**

Aliter :



Let at $t = \frac{T}{2}$ body is at point B.

For AC

$s = ut + 1/2\,at^2$

$-h = -\frac{1}{2}\,g\,T^2$

For AB

$s = ut + 1/2\,at^2$

$-(h - h_1) = -\frac{1}{2}g\left(\frac{T}{2}\right)^2$

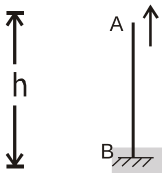
$h = g\frac{T^2}{2}$ ....(1) $\qquad h - h_1 = g\frac{T^2}{2\times 4}$ ....(2)

From (1) and (2) , we have

$h - h_1 = h/4$

$h - \frac{h}{4} = h_1$ or $h_1 = \frac{3h}{4}$ from the ground

4.



For AB

$s = ut + \frac{1}{2}at^2.$

$-h = vt - \frac{g}{2}t^2$

$\frac{g}{2}t^2 - vt - h = o$

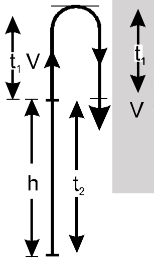
$$t = \frac{v \pm \sqrt{v^2 + 4\times\frac{g}{2}\times h}}{2\times\frac{g}{2}}$$

$$t = \frac{v \pm \sqrt{v^2 + 2gh}}{g}$$

$t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$, $\quad t = \frac{v}{g}\left[1-\sqrt{1+\frac{2gh}{v^2}}\right]$ [as time cannot be negative so we neglect it]

$\therefore \qquad t = \frac{v}{g}\left[1+\sqrt{1+\frac{2gh}{v^2}}\right]$

Aliter



Let t_1 be the time taken by ball from top of tower to the highest point then it will take again t_1 time to return back to the top of tower Let t_2 be the time taken be ball from top of tower to the ground.

For t_1 : From equation

$V = u - gt$ i.e., $0 = V - gt_1$ or, $t_1 = V/g$

For t_2 : From equation

$h = ut + \frac{1}{2}gt^2 \quad h = Vt^2 + \frac{1}{2}gt_2^2$; or, $gt_2^2 + 2Vt_2 - 2h = 0$, or, $t_2 = \frac{-2V \pm \sqrt{4V^2 + 8gh}}{2g}$

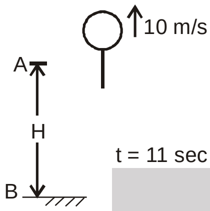
Taking (+) sign only (as we are interested in time projection i.e., t = 0) $\quad t_2 = \frac{-V + \sqrt{V^2 + 2gh}}{g}$

Note that, –ve time indicate time before the projection.

Hence, the time after which the ball strikes ground T = $2t_1 + t_2$ $\Rightarrow T = \frac{2V}{g} + \frac{-V+\sqrt{V^2+2gh}}{g}$

$T = \frac{V+\sqrt{V^2+2gh}}{g}$ $\Rightarrow$ $T = \frac{V}{g}\left[1+\sqrt{1+\frac{2gh}{V^2}}\right]$

5.



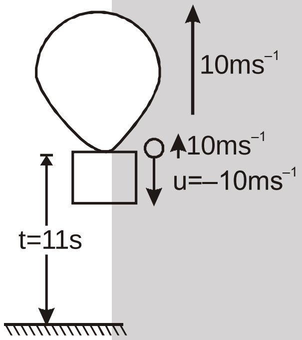
As $s = ut + at^2$

$-H = 10 \times 11 - 5 \times (11)^2$

$-H = 110 - 605$

$H = 495$ m

Aliter :



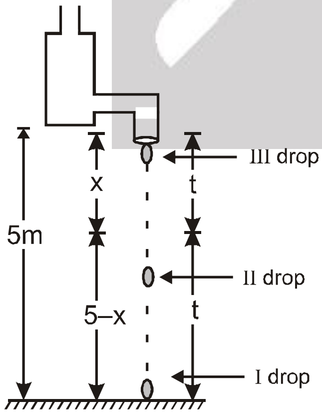
At the time of release, velocity of stone will be same as that of balloon, hence

$u = -10\ ms^{-1}$, $t = 11$ s

$h = ut + \frac{1}{2}gt^2$

$= (-10) \times 11 + \frac{1}{2}(10)(11)^2 = -110 + 605 = 495$ m **"A" Ans**

6.



Let t be the time interval between two successive drops. For the first drop :

From equation, $h = ut + \frac{1}{2}gt^2$

$5 = 0 + \frac{1}{2}g(2t)^2$ $\Rightarrow$ $5 = \frac{1}{2}g(2t)^2$ ....(i)

For the second drop :

From equation , $h = ut + \frac{1}{2}gt^2$

$x = 0 + \frac{1}{2}gt^2 \quad \Rightarrow \quad x = \frac{1}{2}gt^2$ ....(ii)

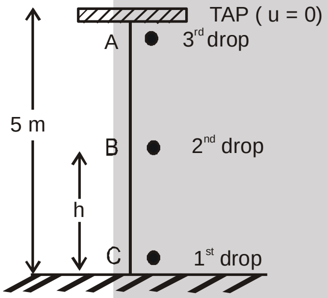
From the equations (i) and (ii);

$\frac{5}{x} = \frac{\frac{1}{2}g(2t)^2}{\frac{1}{2}g\,2t^2} \Rightarrow \frac{5}{x} = \frac{4}{1} \Rightarrow \quad x = \frac{5}{4} = 1.25$ m

The distance of the second drop from the ground

$= 5 - x = 5 - 1.25 = 3.75 = \frac{15}{4}$ m

Aliter :



Let the time interval b/w two consecutive drops be t.

$\therefore$ Time b/w 1st and 3rd drop = 2t.

For AC

$s = ut + \frac{1}{2}at^2$.

$-5 = 0 + \frac{1}{2} x - 10 \text{ x } (2\,t)^2 \qquad \frac{1}{2} = t^2, \quad t = \frac{1}{2}$ sec.

$\therefore$ height of second drop.

$s = ut + \frac{1}{2}at^2 \qquad -(5 - h) = 0 + \frac{1}{2} x - 10 \times \frac{1}{4}$

$5 - h = \frac{10}{8} \qquad h = 5 - \frac{5}{4} = 3.75 \text{ m} = \frac{15}{4}$ m

7. (A) the given x – t graph has 5 points at which the slope of tangent is zero i.e, velocity becomes zero 5 times.

As we know that particle is at rest when its position does not change withe time. Clearly, from x–t graph, particle is at rest 5 times

$\therefore$ option (A) is correct .

(B) Slope is not zero at t = 0.

$\therefore$ option (B) is incorrect .

(C) Velocity is positive, when slope of x-t curve is positive. Slope changes from positive to negative and negative to zero.

$\therefore$ option (C) is incorrect

(D) Average velocity $= \frac{\text{Total Displacement}}{\text{Total Time taken}}$.

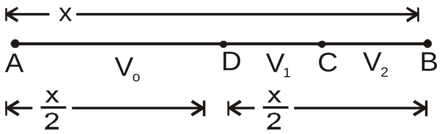
Total Displacement is positive

$\therefore$ Average velocity = positive

$\therefore$ option (D) is incorrect .

PART - II

1.



Let T is the time to cover DB.

$\therefore$ Time in DC = CB = $\frac{T}{2}$.

As mean velocity = $\frac{\text{Total Displacement (T.D)}}{\text{Total time taken.(TTT)}}$

T.D = x

T.T.T = $T_{AD} + T_{DC} + T_{CB} = \frac{x}{2V_o} + \frac{T}{2} + \frac{T}{2} = \frac{x}{2V_O} + T$ ......(1)

Now , BD = DC + CB

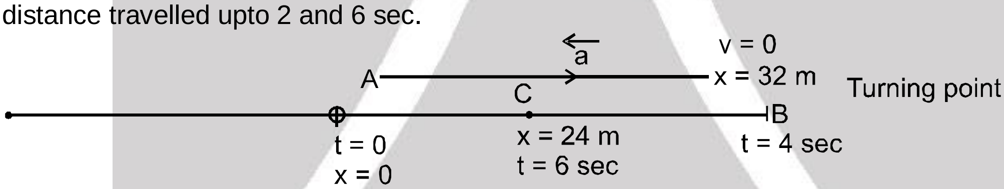
or $\frac{x}{2} = \frac{V_1T}{2} + \frac{V_2T}{2}$ or $x = T(V_1 + V_2)$. or $T = \frac{x}{V_1 + V_2}$. ....(2)

From (1) and (2)

T.T.T = $\frac{x}{2V_o} + \frac{x}{V_1 + V_2}$.

or Mean velocity = $\frac{x}{\frac{x}{2V_o} + \frac{x}{V_1 + V_2}} = \frac{2V_o(V_1 + V_2)}{(2V_o + V_1 + V_2)}$

2. distance travelled upto 2 and 6 sec.



As $x = 16t - 2t^2$

At t = 0, x = 0

Now, V = 16 – 4t = 0 [$a = -4\ m/s^2$]

t = 4 sec.

At t = 4 sec, x = 16 × 4 – 2 × 16 = 32 m

Now, At t = 6 sec, x = 16 × 6 – 2 × 36 = 96 –72 = 24 m

$\therefore$ Distance upto 2 sec. = Displacemnent in 2 sec = 24 m.

[As turning point is at t = 4 sec]

and distance in 6 sec = AB + BC = 32 + (32 – 24) = 32 + 8 = 40 m.

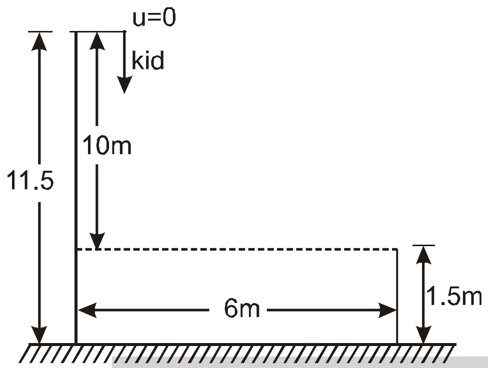
Aliter :

The distance travelled upto 6s

$= \left|x\right|_{t=4} + \left|\int_4^6 V dt\right| = \left|16(4) - 2(4)^2\right| + \left|\int_4^6 (16 - 4t)dt\right|$

$= 64 - 32 + \left|[16t - 2t^2]_4^6\right| = 32 + \left|32 - 40\right| = 32 + 8 = 40$ m **Ans**

3.



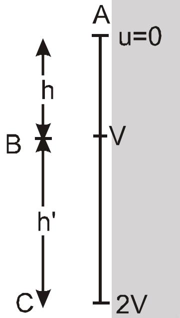
Let a be the acceleration of the youngman.
As the youngman catches the kid at the arms height (1.5 m) then the time taken by kid to fall through 10 m will be same as the time taken by the youngman to run 6 m on horizontal ground.
For motion of kid.
$u = 0$, $g = 10\ m/s^2$, $h = 10$ m
For motion of kid.
$u = 0$, $g = 10\ m/s^2$, $h = 10$ m

From the equation $h = ut + \frac{1}{2}gt^2 \Rightarrow 10 = 0 + \frac{1}{2}(10)t^2$

For motion of youngman

$6 = 0 + \frac{1}{2}at^2$ substitute value of t ; a = 6 m/s^2.

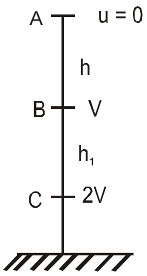
4.



For motion from A to B $u = 0$
From equation $V^2 = u^2 + 2gh$ $V^2 = (0)^2 + 2gh$ $V^2 = 2gh$ ...(i)
For motion from B to C $u = V$
From equation $V^2 = u^2 + 2gh$ $(2V)^2 = V^2 + 2gh'$ $4V^2 = V^2 + 2gh' \Rightarrow 3V^2 = 2gh'$ ...(ii)

From equations (i) and (ii) $\frac{V^2}{3V^2} = \frac{2gh}{2gh'} \Rightarrow h' = 3h$ **(C) Ans**

Aliter :

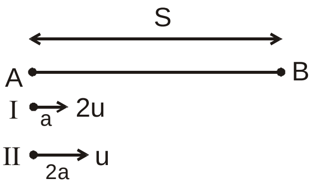


Let h_1 = distance it has to travel down further to double its velocity i.e., 2V

For AB
$V^2 = 2gh$ ......(1)

For BC
$(2v)^2 = (v)^2 + 2gh_1$
$3V^2 = 2gh_1$
$\therefore\ 3h = h_1$

5.



Suppose at point B (displacement S) II particle overtakes particle I

For I particle $S = (2u)\,t + \frac{1}{2} a t^2$ .... (1)

For II particle

$S = u\,t + \frac{1}{2}(2a)\,t^2$ ....(2)

$\therefore\ 2ut + \frac{1}{2} a t^2 = ut + \frac{1}{2}(2a)\,t^2$

$ut = \frac{1}{2} a t^2$

$t = \frac{2u}{a}$

Putting this value in equation (1) we get

$S = 2u \times \frac{2u}{a} + \frac{1}{2} \times a \times \left(\frac{2u}{a}\right)^2$

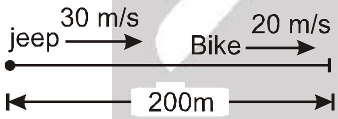
$= \frac{4u^2}{a} + \frac{2u^2}{a} = \frac{6u^2}{a}$

6. Let a be the retardation produced by resistive force, t_a and t_d be the time of ascent and time of descent respectively.
If the particle rises upto a height h

then $h = \frac{1}{2}(g + a)\,t_a^2$ and $h = \frac{1}{2}(g - a)\,t_d^2$

$\therefore\ \frac{t_a}{t_d} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$ **Ans.** $\sqrt{\frac{2}{3}}$

7.



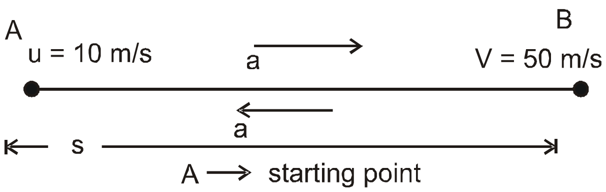
$200 = 10\,(t) + \frac{1}{2}(2)t^2$

$t^2 + 10\,t - 200 = 0$

t = 10 seconds

Distance = 200 + 200 = 400 m **Ans.**

8.



For AB

$V^2 = u^2 + 2as$

2400 = 2as or as = 1200 (1)

Now, for BA

$V_A^2 = (50)^2 + 2\,(-a)\,(-s)$

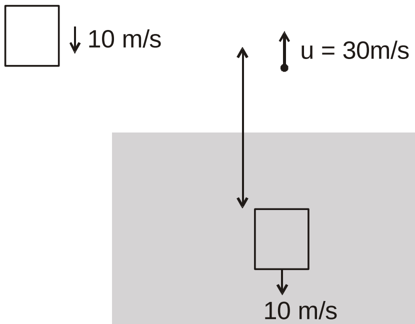
$V_A^2 = 2500 + 2 \times 1200$

$V_A = \sqrt{4900}$

$V_A = 70$ m/s

$\therefore$ velocity of particle when it reaches the starting point is 70 m/s.

9.



Let the time be t after which the thrown stone hits the lift at a depth d below the top of shaft

$d = ut + \frac{1}{2}gt^2$

$d = -30\,t + \frac{1}{2}\,(10)\,t^2$ .....(1)

for lift

$d = 40 + 10\,t$ .....(2)

(1) = (2)

$-30\,t + 5\,t^2 = 40 + 10\,t$

$5t^2 - 40\,t - 40 = 0$

$t^2 - 8t - 8 = 0$

$t = \frac{8 \pm \sqrt{64+32}}{2} = \frac{8 \pm \sqrt{96}}{2} = \frac{8 \pm 4\sqrt{6}}{2}$

$t = 4 + 2\sqrt{6}$

Net time after to hit the lift start desending

$= 4 + 4 + 2\sqrt{6} = 8 + 2\sqrt{6}$ sec

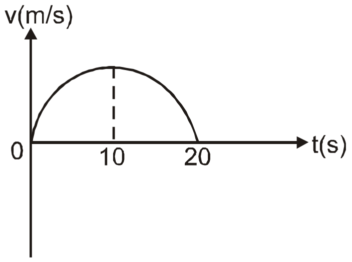
Putting value of t in equation (2)

$d = 40 + (4 + 2\sqrt{6})\,10$

$= 40 + 40 + 20\sqrt{6} = 129$ m

PART - III

1.



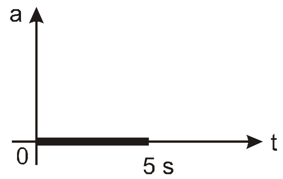
$a_{Avg} = \frac{\Delta\vec{v}}{t} = \frac{0}{20} = 0$

From 0 to 20 time interval velocity of particle doesn't change it's direction.

Area under v–t curve is not zero.

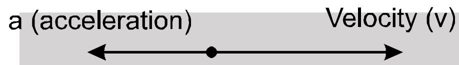
As the magnitude of area under v – t graph from t = 0 to 10 is same as from t = 10 to 20, hence the average speed in both the intervals will be same. **'D" is correct i.e., A & D Ans**

2.

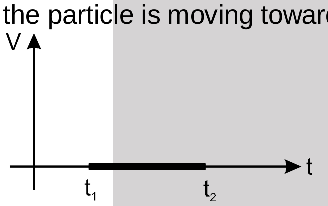


If the acceleration a is zero from t = 0 to 5 s, then speed is constant from t = 0 to 5s and as the speed is zero at t = 0. Hence speed is zero from t = 0 to t = 5 s.
If the speed is zero for a time interval from t = 0 to t = 5 s, as the speed is constant in this interval hence the acceleration is also zero in this interval.
Because zero speed = object is not moving = velocity = constant (= 0) $\Rightarrow$ acceleration = 0

3.



If the velocity (u) and acceleration (a) have opposite directions, then velocity (v) will decrease, therefore the object is slowing down.
If the position (x) and velocity (u) have opposite sign the position (x) reduces to become zero. Hence the particle is moving towards the origin.



If $\vec{a} \cdot \vec{v} > 0$ speed will increase.
If velocity V = 0, $t_1 < t < t_2$

Hence; acceleration $a = \frac{\Delta V}{\Delta t} = 0$; $t_1 < t < t_2$

Therefore if the velocity is zero for a time interval, the acceleration is zero at any instant within the time interval. **(D) is correct**

[acc, $a = \frac{dv}{dt} \Rightarrow v = u + at$]

Now, v = 0 $\Rightarrow$ a = 0 $\Rightarrow$ a = – u/t $\Rightarrow$ acceleration may not be zero when vel. 'V' = 0, 'c' is incorrect.

4. $s \propto t^2$

$\therefore$ $s = ct^2$ where c = constant

(i) $v = \frac{ds}{dt} = 2\,ct$

$\therefore$ $v \propto t$

(ii) $a = \frac{dv}{dt} = 2c$

so, a = constant.

5. $y = u\,(t - 2) + a(t - 2)^2$

Velocity of particle at time t

$$\frac{dy}{dt} = u + 2a\,(t - 2)$$

Velocity at t = 0 $\frac{dy}{dt} = u - 4a$

acceleration of particle

$$\frac{d^2y}{dt^2} = 2a$$

$y_{t=2} = 0$

So correct answer is (C) and (D).

PART - IV

1 to 4.

(1) $\langle \vec{v} \rangle = \frac{x_f - x_i}{\Delta t} = \frac{-100-100}{20} = -10 m/s$

(2) **(C)** $\langle \vec{a} \rangle = \frac{v_f - v_i}{\Delta t} = \frac{\tan\theta_2 - \tan\theta_1}{20} = 0$ (since $\theta_2 = \theta_1$)

(3) during first 10 sec, speed decreases

$\therefore$ acceleration is opposite to the velocity

$\therefore$ acceleration is in $\hat{i}$

(4) **(C)** during first 10 sec., the slope of x-t curve decreases in negative direction

$\therefore$ Motion is retarded.

t = 0 to t = 10 s

Ans. (1) – 10m/s (2) 0 (3) $\hat{i}$ (4) t = 0 to t = 10 s

5. $x = 2(t - t^2)$

velocity = $\frac{dx}{dt} = 2 - 4t$

acceleration = $\frac{d^2x}{dt^2} = -4$ $\Rightarrow$ (C) is correct.

6. velocity = $\frac{dx}{dt} = 2 - 4t$ $\upsilon = 0$ $\Rightarrow$ $t = \frac{1}{2}$

After $t = \frac{1}{2}$ sec., particle moves to left

Position at $t = \frac{1}{2}$ sec $x = 2\left(\frac{1}{2} - \frac{1}{4}\right) = 2 \times \frac{1}{4} = \frac{1}{2}$ m. (C) is correct

7. (C) is correct

8. u = 0 at $t = \frac{1}{2}$ s

$\therefore$ position at $t = \frac{1}{2}$ s $\Rightarrow x = \frac{1}{2}$

position at t = 1 s $\Rightarrow x = 0$

$\therefore$ distance moved = $\left|\frac{1}{2} - 0\right| + \left|1 - \frac{1}{2}\right|$

= 1 m **Ans.**

EXERCISE–3
PART - I

1. Distance travelled in t[th] second is,

$s_t = u + at - \frac{1}{2}a$; $u + \frac{a}{2}(2t - 1)$

Given : u = 0

$\therefore$ $\frac{s_n}{s_{n+1}} = \frac{an - \frac{1}{2}a}{a(n+1) - \frac{1}{2}a} = \frac{2n-1}{2n+1}$

Hence, the correct option is (B).

2. Area under acceleration-time graph gives the change in velocity.

Hence, $v_{max} = \frac{1}{2} \times 10 \times 11 = 55$ m/s

Therefore, the correct option is (C)

PART - II

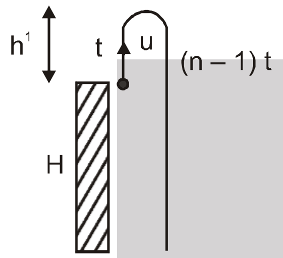
1. $\int_{6.25}^{0} \frac{dv}{\sqrt{v}} = -2.5\int_{0}^{t} dt$

$\left|2\sqrt{\upsilon}\right|_{6.25}^{0} = -2.5\ t$

$2.\sqrt{6.25} = 2.5\ t$

t = 2 sec. **Ans.**

2.



$t = u/g$ .....(1)

$h^1 = \frac{u^2}{2g}$ .....(2)

$h^1 + H = \frac{1}{2} g\ (n-1)^2\ t^2$

$\frac{u^2}{2g} + H = \frac{1}{2} g\ (n-1)^2 \frac{u^2}{g^2}$

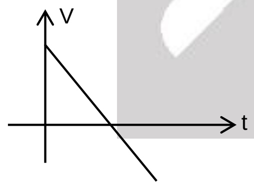
$H = \frac{(n-1)^2 u^2}{2g} - \frac{u^2}{2g} \Rightarrow H = \frac{u^2}{2g}\ [n^2 - 2n]$

3. a = –g = constant

dv/dt = constant

slop of V – t curve is

constant & –ve



4. As in distance vs time graph slope is equal to speed In the given graph slope increase initially which is incorrect

5. $v = bx^{1/2}$

$\frac{dx}{dt} = bx^{1/2}$

$\int_{0}^{x} \frac{dx}{x^{1/2}} = \int_{0}^{t} b\,dt$

$2\sqrt{x} = bt$

$x = \frac{b^2t^2}{4} \Rightarrow v = \frac{dx}{dt} = \frac{b^2t}{2}$

6. Area = $\left(\frac{1}{2}\times2\times2\right)$ + (2 × 2) + (1 × 3)

Displacement = 2 + 4 + 3 = 9m

7. $x^2 = at^2 + 2bt + c$

$2xv = 2at + 2b$

$xv = at + b$

$v^2 + ax = a$

$ax = a - \left(\frac{at+b}{x}\right)^2$

$a = \frac{a(at^2+2bt+c)-(at+b)^2}{x^3}$

$a = \frac{ac-b^2}{x^3}$

$a \propto x^{-3}$

8. $S_y = u_y t + \frac{1}{2}a_y t^2$

$32 = 0 + \frac{1}{2}\times 4t^2 \quad \Rightarrow \quad t = 4$ sec

$S_x = u_x t + \frac{1}{2}a_x t^2$

$= 3 \times 4 + \frac{1}{2} \times 6 \times 16$

= 60 m.

HIGH LEVEL PROBLEMS
SUBJECTIVE QUESTIONS

1. Velocity of car on highway = v

Velocity of car on field = v/η

Let CD = x and AD = b

$T = t_{AC} + t_{CB} = \frac{b-x}{v} + \frac{\sqrt{\ell^2+x^2}}{(v/\eta)}$

$\frac{dT}{dx} = 0 \quad \Rightarrow \quad -\frac{1}{v} + \frac{\eta}{v}\left(\frac{2x}{2\sqrt{\ell^2+x^2}}\right) = 0$

$\Rightarrow \quad x = \frac{\ell}{\sqrt{\eta^2-1}}$

2. (a) $V = \alpha\sqrt{x}$

$\frac{dx}{dt} = \alpha\sqrt{x} \quad \Rightarrow \quad \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt$

$\therefore \quad x = \frac{\alpha^2 t^2}{4}$

$\therefore \quad \frac{dx}{dt} = V = \frac{\alpha^2 t}{2}$

Also $a = \frac{dv}{dt} = \frac{\alpha^2}{2}$.

(b) Let t_0 be the time taken to cover the first s metre

$\therefore \quad s = \frac{\alpha^2 t_0^2}{4} \quad \Rightarrow \quad t_0 = \frac{2\sqrt{s}}{\alpha}$

$\therefore \quad < v > = \frac{\int_0^{t_0} v\, dt}{\int_0^{t_0} dt}$

$\therefore \quad < v > = \frac{\int_0^{t_0} \frac{\alpha^2 t}{2}\, dt}{t_0}$

$= \frac{\alpha^2}{2} \cdot \frac{1}{2} t_0$

$= \frac{\alpha^2}{4} \cdot \frac{2\sqrt{s}}{\alpha} = \frac{\alpha\sqrt{s}}{2}$.

Aliter:

$v = \alpha\sqrt{x}$, $\quad v^2 = \alpha^2 x$

$= 2\frac{\alpha^2}{2} x$

Comparing with $v^2 = 2\,a\,x$

$\therefore \quad a = \frac{\alpha^2}{2}$ and $< v > = \frac{S}{t} = \frac{S}{\sqrt{2S/a}} = \frac{S\sqrt{a}}{\sqrt{2S}} = \sqrt{\frac{S^2}{2S}\frac{\alpha^2}{2}} = \frac{\alpha}{2}\sqrt{S}$.

3. **(a)** From graph, obviously engine stopped at its highest velocity i.e., 190 ft/s. **Ans**

(b) The engine burned upto the instant it reached to its maximum velocity. Hence it burned for 2s. **Ans**

(c) The rocket reached its highest point for the time upto which the velocity is positive. Hence, from graph, rocket reached its highest point in 8 s.

$y_{max} \Rightarrow dy/dt = 0$

$\Rightarrow$ Velocity in y direction = v_y = 0 m/s.

(d) When the parachute opened up, the velocity of rocket starts increasing. Hence, at t = 10.85 (from graph), parachute was opened up. At that moment the velocity of the rocket falling down was 90 ft/s.

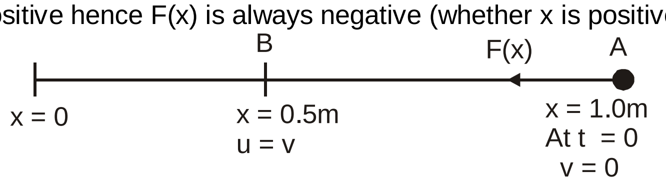
(e) The rocket starts falling when its velocity becomes negative. From the graph hence time taken by rocket to fall before the parachute opened will be (10.8 – 8) s = 2.8 s.

(f) Rocket's acceleration was greatest when the slope of tangent in V – t graph was maximum. As t = 2 sec, the tangent is vertical i.e, slope is infinity hence the rocket's acceleration was greatest at t = 2 s.

(g) The acceleration is constant when V – t graph is linear. Hence, the acceleration was constant between 2 and 10.8 s. Its value is given by slope = $-\frac{190}{8-2}$ = – 32 ft/s^2 (nearest to integer) **Ans**

4. **(a)** $F(x) = \frac{-k}{2x^2}$

k and x^2 both are positive hence F(x) is always negative (whether x is positive or negative .)



$mv\frac{dv}{dx} = -\frac{k}{2x^2}$

$m\int_0^v v\,dv = \frac{-k}{2}\int_1^{0.5}\frac{1}{x^2}\, dx$

$$m\left[\frac{v^2}{2}\right]_0^v = \frac{-k}{2}\left[\frac{-1}{x}\right]_1^{0.5}$$

$v^2 = 1$

$v = \pm 1$

but v is along –ve x direction so $v = -1\ \hat{i}$

(b) $m\int_0^v dv = \frac{-k}{2}\int_1^x \frac{1}{x^2}\ dx$

$$v^2 = \left[\frac{1}{x} - \frac{1}{1}\right]$$

$$v^2 = \frac{1-x}{x}$$

$$v = \sqrt{\frac{1-x}{x}}$$

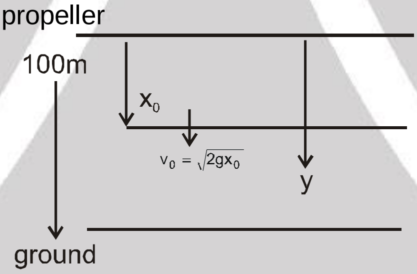
but $\quad v = -\left(\frac{dx}{dt}\right) = \sqrt{\frac{1-x}{x}}$

$\therefore \quad \sqrt{\frac{x}{1-x}}\,dx = -\,dt$

or $\quad \int_1^{0.25}\sqrt{\frac{x}{1-x}}\,dx = -\int_0^t dt$

Solving this, we get t = 1.48s

5. After switching on parachuite propeller



$$v\frac{dv}{dy} = -2v$$

$$\int_{\sqrt{2gx_0}}^{0} dv = -2\int_{x_0}^{100} dy$$

$\sqrt{2gx_0} = 2(100 - x_0)$

$x_0^2 - 205x_0 + 10000 = 0$

$x_0 = 80$m

$\Rightarrow \quad$ time of free fall $t = \sqrt{\frac{2(80)}{10}} = 4$ sec

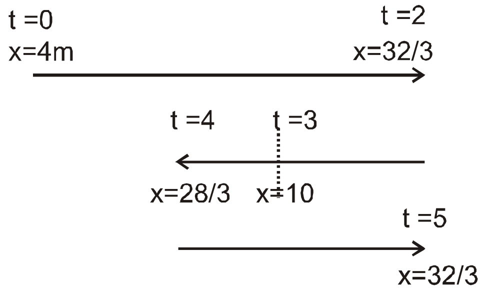
6. $x = t^3/3 - 3t^2 + 8t + 4$

$v = t^2 - 6t + 8 = (t-2)(t-4)$

$a = 2(t-3)$

	0–2	2–3	3–4	4–
V	+	–	–	+
a	–	–	+	+

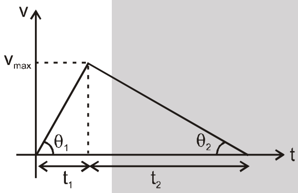
$$S_1 = \left(\frac{32}{3} - 4\right) + \left(\frac{32}{3} - \frac{28}{3}\right) + \left(\frac{32}{3} - \frac{28}{3}\right) = \frac{20}{3} + \frac{8}{3} = \frac{28}{3}\ \text{m}.$$



$S_2 = \left(\frac{32}{3} - 4\right) + \left(10 - \frac{28}{3}\right) = \frac{20}{3} + \frac{2}{3} = \frac{22}{3}$ m

$\frac{s_1}{s_2} = \frac{28}{22} = \frac{14}{11}$

7.



here, v_{max} = v is the maximum velocity which can be achieved for the given path

from I[st] part, $\tan\theta_1 = 10 = \frac{v}{t_1} \Rightarrow t_1 = \frac{v}{10}$

from II[nd] part, $\tan\theta_2 = 5 = \frac{v}{t_2} \Rightarrow t_2 = \frac{v}{5}$

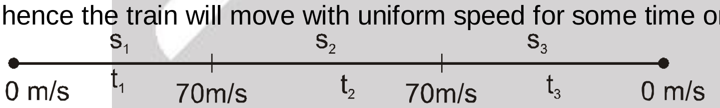
now, area under the graph is equal to total displacement

so, $\frac{1}{2}v\left[t_1 + t_2\right] = 1000$

$\frac{1}{2}v\left[\frac{v}{10} + \frac{v}{5}\right] = 1000$

so, $v_{max} = v = \frac{100\sqrt{2}}{\sqrt{3}}$ m/s = 81.6 m/s (approx)

The maximum speed is 70 m/s which is lesser than maximum possible speed v, hence the train will move with uniform speed for some time on the path.



The motion of train will be as shown

Let I[st] part of path has length s_1

then, by $v^2 = u^2 + 2as$, we get

$70^2 = 0^2 + 2 \times 10 \times s_1$, so $s_1 = 245$ m

Similarly by III[rd] equation of motion

$0^2 = 70^2 - 2 \times 5 \times s_3$, so $s_3 = 490$ m

Hence, $s_2 = 1000 - (490 + 245) = 265$ m

for part 1 of the path, time taken = t_1

from v = u + at, we get

$70 = 0 + 10\,t_1$ so, $t_1 = 7$ seconds

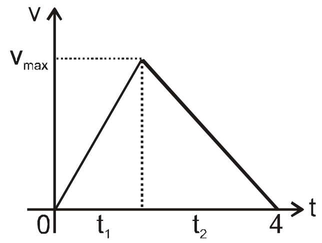
for part 2 of the path, time taken = $t_2 = \frac{s_2}{70} = \frac{265}{70} = \frac{53}{14}$ seconds

for 3rd part of the path, $0 = 70 - 5 \times t_3$

so, $t_3 = 14$ seconds.

Total time taken = $t_1 + t_2 + t_3 = 7 + \frac{53}{14} + 14 = \frac{347}{14}$ seconds

8.



Area of v-t curve is displacment which is equal to 2

$$\frac{1}{2} \times v_{max} \times 4 = 2$$

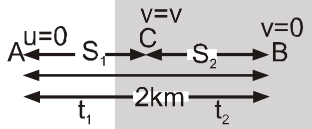
$v_{max} = 1$

Also $\quad t_1 + t_2 = 4$

$$\frac{v_{max}}{x} + \frac{v_{max}}{y} = 4$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} = 4$$

Alter :



Given, $\quad S_1 + S_2 = 2 \quad$ ....(i)

$\quad t_1 + t_2 = 4 \quad$ ....(ii)

For motion from A to C:

From, $\quad V = u + at$

$V = 0 + xt,$

$t_1 = V/x$

From $\quad V^2 = u^2 + 2as$

$V^2 = 0 + 2xS_1$

$$S_1 = \frac{V^2}{2x}$$

Similarly for motion from C to B

$t_2 = V/y$

$S_2 = V^2/2y$

From eqn.(i)

$$\frac{V^2}{2x} + \frac{V^2}{2y} = 2$$

$$\frac{V^2}{2}\left(\frac{1}{x} + \frac{1}{y}\right) = 2 \qquad \text{................(iii)}$$

From eqn. (ii)

$$V\left(\frac{1}{x} + \frac{1}{y}\right) = 4 \qquad \text{................(iv)}$$

Solving (iii) & (iv) we get,

$$\frac{1}{x} + \frac{1}{y} = 4$$

9. (a) $h_A = \frac{1}{2}g\left(\frac{t_0}{2}\right)^2$

$h_A = \frac{gt_0^2}{8}$

(b) $h = ut + gt^2$ ...(i)

$h = -u(t + t_0) + \frac{1}{2}g(t + t_0)^2$ ....(ii)

(i) × $(t + t_0)$ and (ii) × t

$h(t + t_0) = u(t)(t + t_0) + \frac{1}{2}gt^2(t + t_0)$

$h(t) = -u(t + t_0)(t) + \frac{1}{2}g(t + t_0)^2 t$

$h(2t + t_0)\frac{1}{2} = gt(t + t_0)(2t + t_0)$

$h_T = \frac{1}{2}gt(t + t_0)$

Ans. (a) $h_A = \frac{gt_0}{8}$ (b) $h_T = \frac{1}{2}gt(t + t_0)$

10. $a = v\frac{dv}{dx} = cx + d$

Let at x = 0 v = u

$\therefore \int_u^v v\,dv = \int_0^x (cx + d)\,dx$

or $v^2 = cx^2 + 2dx + u^2$

v shall be linear function of x if $cx^2 + 2dx + u^2$ is perfect square

$\therefore \sqrt{\frac{d^2}{c}} = 3$