

Exercise-1

Marked questions are recommended for Revision.

चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

PART - I : SUBJECTIVE QUESTIONS

भाग - I : विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

Section (A) : General Term & Coefficient of x^k in $(ax + b)^n$

खण्ड (A) : व्यापक पद एवं $(ax + b)^n$ में x^k का गुणांक

A-1. Expand the following :

निम्न का प्रसार करो :

$$(i) \left(\frac{2}{x} - \frac{x}{2} \right)^5, (x \neq 0) \quad (ii) \left(y^2 + \frac{2}{y} \right)^4, (y \neq 0)$$

Ans. (i) $\left(\frac{2}{x} \right)^5 - 5 \left(\frac{2}{x} \right)^3 + 10 \left(\frac{2}{x} \right) - 10 \left(\frac{x}{2} \right) + 5 \left(\frac{x}{2} \right)^3 - \left(\frac{x}{2} \right)^5$
(ii) $y^8 + 8y^5 + 24y^2 + \frac{32}{y} + \frac{16}{y^4}$

Sol. (i) $\left(\frac{2^5}{x} \right) - {}^5C_1 \left(\frac{2}{x} \right)^4 \left(\frac{x}{2} \right)^1 + {}^5C_2 \left(\frac{2}{x} \right)^3 \left(\frac{x}{2} \right)^2 - {}^5C_3 \left(\frac{2}{x} \right)^2 \left(\frac{x}{2} \right)^3 + {}^5C_4 \left(\frac{2}{x} \right)^1 \left(\frac{x}{2} \right)^4 - {}^5C_5 \left(\frac{x}{2} \right)^5$
 $= \left(\frac{2}{x} \right)^5 - 5 \left(\frac{2}{x} \right)^3 + 10 \left(\frac{2}{x} \right) - 10 \left(\frac{x}{2} \right) + 5 \left(\frac{x}{2} \right)^3 - \left(\frac{x}{2} \right)^5$
(ii) $(y^2)^4 + {}^4C_1 (y^2)^3 (2/y) + {}^4C_2 (y^2)^2 (2/y)^2 + {}^4C_3 (y^2) (2/y)^3 + {}^4C_4 (2/y)^4 = y^8 + 8y^5 + 24y^2 + \frac{32}{y} + \frac{16}{y^4}$

A-2. In the binomial expansion of $\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}} \right)^n$, the ratio of the 7th term from the beginning to the 7th term from the end is 1 : 6 ; find n.

$\left(\sqrt[3]{2} + \frac{1}{\sqrt[3]{3}} \right)^n$ के प्रसार में प्रारम्भ से 7वें पद और अंत से 7वें पद का अनुपात 1 : 6 है, तब n का मान ज्ञात करो।

Ans. n = 9

Sol. 7th term from beginning $T_7 = {}^nC_6 (2)^{\frac{n-6}{3}} \left(\frac{1}{3} \right)^2$

7th term from the end $T_{n-5} = {}^nC_{n-6} (2)^2 \left(\frac{1}{3} \right)^{\frac{n-6}{3}}$

$$\frac{T_7}{T_{n-5}} = \frac{1}{6} = \frac{2^{\left(\frac{n-6}{3}-2\right)}}{\left(\frac{1}{3}\right)^{\left(\frac{n-6}{3}-2\right)}} \Rightarrow \frac{1}{6} = (6)^{\frac{n-12}{3}} \Rightarrow \frac{n-12}{3} = -1 \Rightarrow n = 9$$

Hindi प्रारम्भ से 7वाँ पद $T_7 = {}^nC_6 (2)^{\frac{n-6}{3}} \left(\frac{1}{3} \right)^2$

$$\text{अन्त से } 7\text{वाँ पद } T_{n-5} = {}^nC_{n-6} (2)^2 \left(\frac{1}{3}\right)^{\frac{n-6}{3}}$$

$$\frac{T_7}{T_{n-5}} = \frac{1}{6} = \frac{2^{\left(\frac{n-6}{3}-2\right)}}{\left(\frac{1}{3}\right)^{\left(\frac{n-6}{3}-2\right)}} \Rightarrow \frac{1}{6} = (6)^{\frac{n-12}{3}} \Rightarrow \frac{n-12}{3} = -1 \Rightarrow n = 9$$

- A-3.** Find the degree of the polynomial $(x + (x^3 - 1)^{1/2})^5 + (x - (x^3 - 1)^{1/2})^5$.

बहुपद $(x + (x^3 - 1)^{1/2})^5 + (x - (x^3 - 1)^{1/2})^5$ की घात ज्ञात कीजिए।

Ans. 7

$$\begin{aligned} &= 2[x^5 + {}^5C_2 \cdot x^3(x^3 - 1)^{1/2}]^2 + {}^5C_4 \cdot x ((x^3 - 1)^{1/2})^4 \\ &= 2[x^5 + 10x^3(x^3 - 1) + {}^5C_4 \cdot x (x^3 - 1)^2] \\ &= 2[x^5 + 10x^6 - 10x^3 + 5x (x^6 - 2x^3 + 1)] \Rightarrow \text{degree is 7 (घात 7 है)} \end{aligned}$$

- A-4.** Find the coefficient of

$$(i) x^6y^3 \text{ in } (x+y)^9 \quad (ii) a^5 b^7 \text{ in } (a-2b)^{12}$$

गुणांक का मान ज्ञात करो—

$$(i) (x+y)^9 \text{ में } x^6y^3 \text{ का} \quad (ii) (a-2b)^{12} \text{ में } a^5 b^7 \text{ का}$$

$$\text{Ans. (i)} {}^9C_3 \quad \text{(ii)} -2^7 \cdot {}^{12}C_7$$

$$\text{Sol. (i)} (x+y)^9 = \sum_{r=0}^9 {}^9C_r x^{9-r} y^r \quad \therefore \text{co-efficient of } x^6y^3 = {}^9C_3$$

$$\text{(ii)} (a-2b)^{12} = \sum_{r=0}^{12} {}^{12}C_r a^{12-r} (-2b)^r \quad \therefore \text{Co-efficient of } a^5b^7 = {}^{12}C_7 (-2)^7$$

$$\text{Hindi. (i)} (x+y)^9 = \sum_{r=0}^9 {}^9C_r x^{9-r} y^r \quad \therefore x^6y^3 \text{ का गुणांक} = {}^9C_3$$

$$\text{(ii)} (a-2b)^{12} = \sum_{r=0}^{12} {}^{12}C_r a^{12-r} (-2b)^r \quad \therefore a^5b^7 \text{ का गुणांक} = {}^{12}C_7 (-2)^7$$

- A-5.** Find the co-efficient of x^7 in $\left(ax^2 + \frac{1}{bx}\right)^{11}$ and of x^{-7} in $\left(ax - \frac{1}{bx^2}\right)^{11}$ and find the relation between

'a' & 'b' so that these co-efficients are equal. (where $a, b \neq 0$).

$\left(ax^2 + \frac{1}{bx}\right)^{11}$ के प्रसार में x^7 का गुणांक और $\left(ax - \frac{1}{bx^2}\right)^{11}$ के प्रसार में x^{-7} का गुणांक ज्ञात करो। यदि ये गुणांक

परस्पर बराबर हो, तो 'a' एवं 'b' के बीच सम्बन्ध ज्ञात करो (जहाँ $a, b \neq 0$)

$$\text{Ans. } {}^{11}C_5 \frac{a^6}{b^5}, {}^{11}C_6 \frac{a^5}{b^6}, ab = 1$$

$$\text{Sol. Co-efficient of } x^7 \text{ in } \left(ax^2 + \frac{1}{bx}\right)^{11}$$

$$\text{General Term} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx}\right)^r = {}^{11}C_r a^{11-r} b^{-r} x^{22-3r}$$

$$\text{Put } 22 - 3r = 7$$

$$r = 5 \quad \therefore \text{Co-efficient of } x^7 = {}^{11}C_5 a^6 \cdot b^{-5}$$

$$\text{Co-efficient of } x^{-7} \text{ in } \left(ax - \frac{1}{bx^2} \right)^{11}$$

$$\text{General Term} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2} \right)^r = {}^{11}C_r a^{11-r} (b)^{-r} (-1)^r x^{11-3r}$$

$$\text{Put } 11 - 3r = -7 \Rightarrow r = 6 \quad \therefore \text{Co-efficient of } x^{-7} = {}^{11}C_6 a^5 b^{-6}$$

$$\text{Given that } {}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6} \Rightarrow ab = 1$$

Hindi. $\left(ax^2 + \frac{1}{bx} \right)^{11}$ के प्रसार में x^7 का गुणांक

$$\text{व्यापक पद} = {}^{11}C_r (ax^2)^{11-r} \left(\frac{1}{bx} \right)^r = {}^{11}C_r a^{11-r} b^{-r} x^{22-3r}$$

$$22 - 3r = 7 \text{ रखने पर}$$

$$r = 5 \quad \therefore x^7 \text{ का गुणांक} = {}^{11}C_5 a^6 \cdot b^{-5}$$

$$\left(ax - \frac{1}{bx^2} \right)^{11} \text{ के प्रसार में } x^{-7} \text{ का गुणांक}$$

$$\text{व्यापक पद} = {}^{11}C_r (ax)^{11-r} \left(-\frac{1}{bx^2} \right)^r = {}^{11}C_r a^{11-r} (b)^{-r} (-1)^r x^{11-3r}$$

$$11 - 3r = -7 \text{ रखने पर} \Rightarrow r = 6 \quad \therefore x^{-7} \text{ का गुणांक} = {}^{11}C_6 a^5 b^{-6}$$

$$\text{दिया गया है} \quad {}^{11}C_5 a^6 b^{-5} = {}^{11}C_6 a^5 b^{-6} \Rightarrow ab = 1$$

A-6. Find the term independent of 'x' in the expansion of the expression,

$$(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9.$$

$$(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \text{ के प्रसार में } x \text{ से स्वतंत्र पद ज्ञात कीजिए।}$$

$$\text{Ans} \quad \frac{17}{54}.$$

$$\begin{aligned} \text{Sol. Co-efficient of } x^0 \text{ in } (1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \\ &= \text{Co-efficient of } x^0 \text{ in } \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 + \text{Co-efficient of } x^{-1} \text{ in } \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \\ &\quad + 2 \text{ Co-efficient of } x^{-3} \text{ in } \left(\frac{3}{2}x^2 - \frac{1}{3x} \right)^9 \\ &= {}^9C_{r_1} \left(\frac{3}{2}x^2 \right)^{9-r_1} \left(-\frac{1}{3x} \right)^{r_1} + {}^9C_{r_2} \left(\frac{3}{2}x^2 \right)^{9-r_2} \left(-\frac{1}{3x} \right)^{r_2} + 2 {}^9C_{r_3} \left(\frac{3}{2}x^2 \right)^{9-r_3} \left(-\frac{1}{3x} \right)^{r_3} \\ &= {}^9C_6 \left(\frac{3}{2} \right)^3 \left(-\frac{1}{3} \right)^6 + 2 {}^9C_7 \left(\frac{3}{2} \right)^2 \left(-\frac{1}{3} \right)^7 = \frac{17}{54} \end{aligned}$$

Hindi. $(1 + x + 2x^3) \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$ के प्रसार में x^0 का गुणांक

$$\begin{aligned} & \Rightarrow \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 \text{ के प्रसार में } x^0 \text{ का गुणांक } + \left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 \text{ के प्रसार में } x^{-1} \text{ का गुणांक } + 2\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9 \text{ के} \\ & \text{प्रसार में } x^{-3} \text{ का गुणांक} \\ & = {}^9C_{r_1} \left(\frac{3}{2}x^2\right)^{9-r_1} \left(-\frac{1}{3x}\right)^{r_1} + {}^9C_{r_2} \left(\frac{3}{2}x^2\right)^{9-r_2} \left(-\frac{1}{3x}\right)^{r_2} + 2 \cdot {}^9C_{r_3} \left(\frac{3}{2}x^2\right)^{9-r_3} \left(-\frac{1}{3x}\right)^{r_3} \\ & = {}^9C_6 \left(\frac{3}{2}\right)^3 \left(-\frac{1}{3}\right)^6 + 2 \cdot {}^9C_7 \left(\frac{3}{2}\right)^2 \left(-\frac{1}{3}\right)^7 = \frac{17}{54} \end{aligned}$$

- A-7.**
- (i) Find the coefficient of x^5 in $(1 + 2x)^6(1 - x)^7$.
 - (ii) Find the coefficient of x^4 in $(1 + 2x)^4(2 - x)^5$
 - (i) $(1 + 2x)^6(1 - x)^7$ में x^5 का गुणांक ज्ञात कीजिए।
 - (ii) $(1 + 2x)^4(2 - x)^5$ में x^4 का गुणांक ज्ञात कीजिए।

Ans. (i) 171
(ii) -438

Sol. (i) $(1 + 2x)^6(1 - x)^7$
 $= (1 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_4(2x)^4 + {}^6C_5(2x)^5 + (2x)^6)(1 - x)^7$
 $= (1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6)(1 - x)^7$
 $= 1 \times \text{coeff of } x^5 + 12 \times \text{coeff. of } x^4 + 60 \times \text{coeff of } x^3 + 160 \times \text{coeff of } x^2 +$
 $240 \times \text{coeff of } x + 192 \times \text{constant term.}$
 $= 1 \times x^5 \text{ का गुणांक} + 12(x^4 \text{ का गुणांक}) + 60(x^3 \text{ का गुणांक}) + 160(x^2 \text{ का गुणांक})$
 $+ 240(x \text{ का गुणांक}) + 192 \times \text{अचर पद}$
 $= 1 \times (-1)^5 \cdot {}^7C_5 + 12 \times {}^7C_4 - 60 \times {}^7C_3 + 160 \times {}^7C_2 - 240 \times {}^7C_1 + 192 \times 1$
 $= -21 + 420 - 2100 + 3360 - 1680 + 192 = 171$

(ii) $(1 + 2x)^4(2 - x)^5$
 $[1 + {}^4C_1(2x) + {}^4C_2(2x)^2 + {}^4C_3(2x)^3 + {}^4C_4(2x)^4](2 - x)^5$
 $= (1 + 8x + 24x^2 + 32x^3 + 16x^4)(2 - x)^5$
coefficient of $x^4 = 1 \times \text{coefficient of } x^4 + 8 \times \text{coefficient of } x^3 + 24 \times \text{coefficient of } x^2 + 32 \times \text{coefficient of } x + 16 \times \text{constant term}$
 $x^4 \text{ का गुणांक} = 1 \times x^4 \text{ का गुणांक} + 8 \times x^3 \text{ का गुणांक} + 24 \times x^2 \text{ का गुणांक} + 32 \times x \text{ का गुणांक} + 16 \times \text{अचर पद}$
 $T_{r+1} = {}^5C_r \cdot 2^{5-r}(-x)^r = (-1)^r {}^5C_r 2^{5-r} x^r = 1 \times {}^5C_4 \times 2^1 - 8 \times {}^5C_3 \times 2^2 + 24 \times {}^5C_2 \times 2^3 - 32 \times {}^5C_1 \times 2^4 + 16 \times 2^5$
 $= 10 - 320 + 1920 - 2560 + 512 = -438$

A-8. In the expansion of $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in \mathbb{N}$, if the sum of the coefficients of x^5 and x^{10} is 0, then n is :

Ans. 15

यदि $\left(x^3 - \frac{1}{x^2}\right)^n$, $n \in \mathbb{N}$ के प्रसार में x^5 और x^{10} के गुणांकों का योग शून्य हो, तो n है—

Sol. $\left(x^3 - \frac{1}{x^2}\right)^n$

$$\text{General term} = \frac{n!}{n!(n-r)!} (-1)^{n-r} x^{5r-2n}$$

$$\text{If } 5r - 2n = 5, \text{ then } 5r = 2n + 5 \Rightarrow r = \frac{2n}{5} + 1$$

$$\text{If } 5r - 2n = 10, \text{ then } 5r = 2n + 10 \Rightarrow r = \frac{2n}{5} + 2$$

Let $n = 5k$

$$\begin{aligned} \text{Now } & \frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0 \\ \Rightarrow & \frac{1}{3k-1} - \frac{1}{2k+2} = 0 \quad \Rightarrow \quad k = 3 \quad \Rightarrow \quad n = 15 \end{aligned}$$

Hindi. $\left(x^3 - \frac{1}{x^2}\right)^n$

$$\text{व्यापक पद} = \frac{n!}{n!(n-r)!} (-1)^{n-r} x^{5r-2n}$$

$$\text{यदि } 5r - 2n = 5, \text{ तब } 5r = 2n + 5 \Rightarrow r = \frac{2n}{5} + 1$$

$$\text{यदि } 5r - 2n = 10, \text{ तब } 5r = 2n + 10 \Rightarrow r = \frac{2n}{5} + 2$$

माना $n = 5k$

$$\begin{aligned} \text{अब } & \frac{5k!}{(2k+1)!(3k-1)!} - \frac{5k!}{(2k+2)!(3k-2)!} = 0 \\ \Rightarrow & \frac{1}{3k-1} - \frac{1}{2k+2} = 0 \quad \Rightarrow \quad k = 3 \quad \Rightarrow \quad n = 15 \end{aligned}$$

Section (B) : Middle term, Remainder & Numerically/Algebraically Greatest terms

खण्ड (B) : मध्य पद, शेषफल और संख्यात्मक/बीजगणितीय महत्तम पद

- B-1.** Find the middle term(s) in the expansion of
निम्न के प्रसार में मध्य पद ज्ञात करो—

(i) $\left(\frac{x}{y} - \frac{y}{x}\right)^7$ (ii) $(1 - 2x + x^2)^n$

Ans. (i) $-\frac{35x}{y}, \frac{35y}{x}$ (ii) $(-1)^n \frac{(2n)!}{n! n!} x^n$

Sol. (i) $\left(\frac{x}{y} - \frac{y}{x}\right)^7$ T_4 & T_5 are the middle term

(ii) $(1 - 2x + x^2)^n = (x - 1)^{2n}$

$$T_{n+1} = {}^{2n}C_n (-1)^n x^n$$

Hindi. (i) $\left(\frac{x}{y} - \frac{y}{x}\right)^7$ के प्रसार में T_4 और T_5 मध्य पद हैं

(ii) $(1 - 2x + x^2)^n = (x - 1)^{2n}$

$$T_{n+1} = {}^{2n}C_n (-1)^n x^n$$

- B-2.** Prove that the co-efficient of the middle term in the expansion of $(1 + x)^{2n}$ is equal to the sum of the co-efficients of middle terms in the expansion of $(1 + x)^{2n-1}$.

सिद्ध करो कि $(1 + x)^{2n}$ के प्रसार में मध्य पद का गुणांक, $(1 + x)^{2n-1}$ के प्रसार में मध्य पदों के गुणांकों के योगफल के बराबर है।

Sol. Co-efficient of middle term $(1+x)^{2n} = {}^{2n}C_n$

$${}^{2n}C_n = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n$$

Hindi. $(1+x)^{2n}$ के प्रसार में मध्य पद का गुणांक $= {}^{2n}C_n$

$${}^{2n}C_n = {}^{2n-1}C_{n-1} + {}^{2n-1}C_n$$

B-3. (i) Find the remainder when 7^{98} is divided by 5

(ii) Using binomial theorem prove that $6^n - 5n$ always leaves the remainder 1 when divided by 25.

(iii) Find the last digit, last two digits and last three digits of the number $(27)^{27}$.

(i) यदि 7^{98} को 5 से विभाजित किया जाए, तो शेषफल ज्ञात करो।

(ii) द्विपद प्रमेय का उपयोग करते हुए सिद्ध कीजिए कि $6^n - 5n$ को 25 से विभाजित करने पर प्राप्त शेषफल सदैव 1 होता है।

(iii) $(27)^{27}$ का अन्तिम अंक, अन्तिम दो अंक व अन्तिम तीन अंक ज्ञात करो।

Ans. (i) 4

(iii) 3, 03, 803

Sol. (i) $7^{98} = (50-1)^{49} = {}^{49}C_0(50)^{49} - {}^{49}C_1(50)^{48} + \dots - {}^{49}C_{49}$ \Rightarrow Remainder शेषफल $= 5 - 1 = 4$

(ii) $6^n - 5n = (5+1)^n - 5n = 5^n + {}^nC_1 \cdot 5^{n-1} + \dots + {}^nC_{n-2} \cdot 5^2 + {}^nC_{n-1} \cdot 5 + 1 - 5n = 25\lambda + 1$

(iii) $(27)^{27} = 3^{81} = 3 \cdot (9)^{40}$

$$= 3(10-1)^{40} = 3(10^{40} - {}^{40}C_1 \cdot 10^{39} + \dots + {}^{40}C_{38} \cdot 10^2 - {}^{40}C_{39} \cdot 10 + 1) = 3(1000\lambda - 400 + 1)$$

Last 3 digits of this number = 803.

इस संख्या के अन्तिम 3 अंक = 803.

B-4. Which is larger : $(99^{50} + 100^{50})$ or $(101)^{50}$.

$(99^{50} + 100^{50})$ तथा $(101)^{50}$ में से कौनसा बड़ा है ?

Ans. 101^{50}

Sol. $(100+1)^{50} - 100^{50} - (100-1)^{50} = 2[{}^{50}C_1(100)^{49} + {}^{50}C_3(100)^{47} + \dots + {}^{50}C_{49}(100)] - (100)^{50} > 0$

$$(101)^{50} > (99^{50} + 100^{50})$$

B-5. (i) Find numerically greatest term(s) in the expansion of $(3-5x)^{15}$ when $x = \frac{1}{5}$

(ii) Which term is the numerically greatest term in the expansion of $(2x+5y)^{34}$, when $x = 3$ & $y = 2$?

(i) यदि $x = \frac{1}{5}$ तब $(3-5x)^{15}$ के प्रसार में महत्तम संख्यात्मक मान वाला(वाले) पद ज्ञात करो।

(ii) $(2x+5y)^{34}$ के विस्तार में संख्यात्मक महत्तम पद होगा जब $x = 3$ तथा $y = 2$?

Ans. (i) $T_4 = -455 \times 3^{12}$ and $T_5 = 455 \times 3^{12}$

(ii) 22

Sol (i) For numerically greatest term in $(x+a)^n$

$$r = \left[\frac{n+1}{1 + \left| \frac{x}{a} \right|} \right] = \left[\frac{15+1}{1 + \left| \frac{3}{1} \right|} \right] = 4$$

Since value of $\frac{n+1}{1 + \left| \frac{x}{a} \right|}$ is itself an integer. There are two terms, whose numerical values are greatest

T_4 and T_5

$$T_4 = {}^{15}C_3 (3)^{12} (-1)^3 = -455 \times 3^{12}$$

$$T_5 = {}^{15}C_4 (3)^{11} (-1)^4 = 455 \times 3^{12}$$

(i) $(x + a)^n$ के प्रसार में महत्तम संख्यात्मक मान वाले पद के लिए $r = \left[\frac{n+1}{1+\left| \frac{x}{a} \right|} \right] = \left[\frac{15+1}{1+\left| \frac{3}{1} \right|} \right] = 4$

चूंकि $\frac{n+1}{1+\left| \frac{x}{a} \right|}$ का मान एक पूर्णांक है अतः यहाँ दो पद हैं जिनका संख्यात्मक मान महत्तम है।

T_4 तथा T_5

$$T_4 = {}^{15}C_3 (3)^{12} (-1)^3 = -455 \times 3^{12}$$

$$T_5 = {}^{15}C_4 (3)^{11} (-1)^4 = 455 \times 3^{12}$$

(ii) For numerically greatest term महत्तम संख्यात्मक मान वाले पद के लिये $r = \left[\frac{n+1}{1+\left| \frac{x}{a} \right|} \right] = \left[\frac{34+1}{1+\left| \frac{6}{10} \right|} \right] \Rightarrow r = 21.$

B-6. Find the term in the expansion of $(2x - 5)^6$ which have

- | | |
|--|--------------------------------------|
| (i) Greatest binomial coefficient | (ii) Greatest numerical coefficient |
| (iii) Algebraically greatest coefficient | (iv) Algebraically least coefficient |

$(2x - 5)^6$ के प्रसार में वह पद ज्ञात करो जो रखता है

- | | |
|-------------------------------|-------------------------------|
| (i) महत्तम द्विपद गुणांक | (ii) महत्तम संख्यात्मक गुणांक |
| (iii) महत्तम बीजगणितीय गुणांक | (iv) न्यूनतम बीजगणितीय गुणांक |

Ans. (i) T_4 (ii) T_5, T_6 (iii) T_5 (iv) T_6

Sol. $(2x - 5)^6$

(i) Greatest binomial Co-efficient is of middle term $= T_{\frac{6+1}{2}} = T_4$

(ii) For greatest numerical term $r = \left[\frac{6+1}{1+\left| \frac{2}{5} \right|} \right] = \left[\frac{35}{7} \right] = 5$

Since $\frac{n+1}{1+\left| \frac{x}{a} \right|}$ itself is an integer.

$\therefore T_5$ and T_6 both terms have are greatest numerical value

(iii) The positive term of greatest numerical value is Algebraically greatest i.e. T_5 .

(iv) The negative term of greatest numerical value is algebraically least i.e. T_6

Hindi. $(2x - 5)^6$

(i) महत्तम द्विपद गुणांक वाला पद मध्य पद होता है अतः $= T_{\frac{6+1}{2}} = T_4$

(ii) महत्तम संख्यात्मक मान वाला पद $r = \left[\frac{6+1}{1+\left| \frac{2}{5} \right|} \right] = \left[\frac{35}{7} \right] = 5$

चूंकि $\frac{n+1}{1+\left|\frac{x}{a}\right|}$ एक पूर्णांक है अतः

T_5 तथा T_6 दोनों पद महत्तम संख्यात्मक मान वाले पद होंगे

(iii) बीजगणितीय महत्तम मान वाला पद धनात्मक महत्तम संख्यात्मक मान वाले पद के बराबर होता है, अर्थात् T_5

(iv) बीजगणितीय न्यूनतम मान वाला पद ऋणात्मक महत्तम संख्यात्मक मान वाले पद के बराबर होता है, अर्थात् T_6

Section (C) : Summation of series, Variable upper index & Product of binomial coefficients

खण्ड (C) : श्रेणी का योग, चर ऊपरी सूचकांक एवं द्विपद गुणांको का गुणन

C-1. If $C_0, C_1, C_2, \dots, C_n$ are the binomial coefficients in the expansion of $(1+x)^n$ then prove that :

यदि $(1+x)^n, n \in \mathbf{N}$ के प्रसार में $C_0, C_1, C_2, \dots, C_n$ द्विपद गुणांक हैं, तो सिद्ध करो :

$$(i) \quad C_0 - \frac{C_1}{\sqrt{2}} + \frac{C_2}{2} - \frac{C_3}{2\sqrt{2}} \dots\dots\dots \text{upto } (n+1) \text{ terms equal to } \left(1 - \frac{1}{\sqrt{2}}\right)^n$$

$$(ii) \quad -C_1(3)^{n-1}(\sqrt{5})^1 + C_2(3)^{n-2}5 - C_3(3)^{n-3}(5\sqrt{5}) \dots\dots\dots \text{upto } (n) \text{ terms equal to } (3-\sqrt{5})^n - 3^n$$

$$(iii) \quad \frac{(3.2-1)}{2} C_1 + \frac{3^2 \cdot 2^2 - 1}{2^2} C_2 + \frac{3^3 \cdot 2^3 - 1}{2^3} C_3 + \dots\dots\dots + \frac{3^n \cdot 2^n - 1}{2^n} C_n = \frac{2^{3n} - 3^n}{2^n}$$

Sol. (i) Obvious

(ii) Obvious

$$(iii) \quad (C_0 + 3C_1 + 3^2 C_2 + \dots\dots\dots + 3^n C_n) - \left(C_0 + \frac{C_1}{2} + \frac{C_2}{2^2} + \dots\dots\dots + \frac{C_n}{2^n} \right)$$

$$= 4^n - \left(\frac{3}{2} \right)^n = 2^{2n} - \frac{3^n}{2^n} = \frac{2^{3n} - 3^n}{2^n}$$

C-2. If $C_0, C_1, C_2, \dots, C_n$ are the binomial coefficients in the expansion of $(1+x)^n$ then prove that :

यदि $(1+x)^n, n \in \mathbf{N}$ के प्रसार में $C_0, C_1, C_2, \dots, C_n$ द्विपद गुणांक हैं, तो सिद्ध करो :

$$(i) \quad \frac{C_1}{C_0} + 2 \frac{C_2}{C_1} + 3 \frac{C_3}{C_2} + \dots\dots\dots + n \frac{C_n}{C_{n-1}} = \frac{n(n+1)}{2}$$

$$(ii) \quad (C_0 + C_1)(C_1 + C_2)(C_2 + C_3)(C_3 + C_4)\dots\dots\dots(C_{n-1} + C_n) = \frac{C_0 C_1 C_2 \dots\dots\dots C_{n-1} (n+1)^n}{n!}.$$

$$(iii) \quad C_0 - 2C_1 + 3C_2 - 4C_3 + \dots + (-1)^n (n+1) C_n = 0$$

$$(iv) \quad 4C_0 + \frac{4^2}{2} \cdot C_1 + \frac{4^3}{3} C_2 + \dots\dots\dots + \frac{4^{n+1}}{n+1} C_n = \frac{5^{n+1} - 1}{n+1}$$

$$(v) \quad \frac{2^2 \cdot C_0}{1 \cdot 2} + \frac{2^3 \cdot C_1}{2 \cdot 3} + \frac{2^4 \cdot C_2}{3 \cdot 4} + \dots + \frac{2^{n+2} \cdot C_n}{(n+1) \cdot (n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1) \cdot (n+2)}$$

$$(vi) \quad 2 \cdot C_0 + \frac{2^2 \cdot C_1}{2} + \frac{2^3 \cdot C_2}{3} + \frac{2^4 \cdot C_3}{4} + \dots + \frac{2^{n+1} \cdot C_n}{n+1} = \frac{3^{n+1} - 1}{n+1}$$

Sol. (i) $\frac{C_1}{C_0} + 2 \frac{C_2}{C_1} + 3 \frac{C_3}{C_2} + \dots = 1 + 2 + 3 + \dots + (n-1) + n = \frac{n(n+1)}{2}$

(ii) $(C_0 + C_1)(C_1 + C_2) \dots (C_{n-1} + C_n) = ({}^{n+1}C_1)({}^{n+1}C_2) \dots ({}^{n+1}C_n) = ((n+1).C_0) \left(\frac{n+1}{2}.C_1\right) \dots \left(\frac{n+1}{n}.C_{n-1}\right)$

$$= \frac{(n+1)^n}{n!} C_0 C_1 \dots C_{n-1}$$

(iii) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $x(1+x)^n = C_0 x + C_1 x^2 + \dots + C_n x^{n+1}$

Differentiating w.r.t. x x के सापेक्ष अवकलन करने पर

$$(1+x)^n + n x (1+x)^{n-1} = C_0 + 2C_1 x + \dots + (n+1) C_n x^n$$

Putting $x = -1$ रखने पर

$$C_0 - 2C_1 + \dots + (-1)^n (n+1) C_n = 0$$

(iv) $S = \sum_{r=0}^n \frac{{}^n C_r}{r+1} 4^{r+1} = \frac{1}{n+1} \sum_{r=0}^n {}^{n+1} C_{r+1} 4^{r+1} = \frac{1}{n+1} [C_1 4 + C_2 4^2 + \dots + C_{n+1} 4^{n+1}] = \frac{1}{n+1} [5^{n+1} - 1]$

Aliter

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + \dots + {}^n C_n x^n$$

$$\frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = {}^n C_0 x + \frac{{}^n C_1 x^2}{2} + \dots + \frac{{}^n C_n x^{n+1}}{n+1}$$

put $x = 4$

$$\text{then } C_0 \cdot 4 + \frac{4^2}{2} C_1 + \dots + \frac{4^{n+1}}{n+1} C_n = \frac{5^{n+1} - 1}{n+1}$$

(iv) $S = \sum_{r=0}^n \frac{{}^n C_r}{r+1} 4^{r+1} = \frac{1}{n+1} \sum_{r=0}^n {}^{n+1} C_{r+1} 4^{r+1} = \frac{1}{n+1} [C_1 4 + C_2 4^2 + \dots + C_{n+1} 4^{n+1}]$

$$= \frac{1}{n+1} [5^{n+1} - 1]$$

वैकल्पिक :

$$(1+x)^n = {}^n C_0 + {}^n C_1 x + \dots + {}^n C_n x^n$$

$$\frac{(1+x)^{n+1}}{n+1} - \frac{1}{n+1} = {}^n C_0 x + \frac{{}^n C_1 x^2}{2} + \dots + \frac{{}^n C_n x^{n+1}}{n+1}$$

$x = 4$ रखने पर

$$\text{तब } C_0 \cdot 4 + \frac{4^2}{2} C_1 + \dots + \frac{4^{n+1}}{n+1} C_n = \frac{5^{n+1} - 1}{n+1}$$

(v) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$

Integrating from 0 to x

$$\frac{(1+x)^{n+1} - 1}{n+1} = C_0 x + \frac{C_1 x^2}{2} + \dots + \frac{C_n x^{n+1}}{n+1}$$

Again integrate 0 to x

$$\frac{(1+x)^{n+2} - 1}{(n+1)(n+2)} - \frac{x}{n+1} = \frac{C_0 x^2}{2} + \frac{C_1 x^3}{2.3} + \dots + \frac{C_n x^{n+1}}{(n+1)(n+2)}$$

put $x = 2$

$$\frac{2^2 \cdot C_0}{2} + \frac{2^3 \cdot C_1}{2 \cdot 3} + \dots + \frac{2^{n+2} \cdot C_n}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

(v) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$

0 से x तक समाकलन करने पर

$$\frac{(1+x)^{n+1} - 1}{n+1} = C_0 x + \frac{C_1 x^2}{2} + \dots + \frac{C_n x^{n+1}}{n+1}$$

पुनः 0 से x तक समाकलन करने पर

$$\frac{(1+x)^{n+2} - 1}{(n+1)(n+2)} - \frac{x}{n+1} = \frac{C_0 x^2}{2} + \frac{C_1 x^3}{2 \cdot 3} + \dots + \frac{C_n x^{n+1}}{(n+1)(n+2)}$$

x = 2 रखने पर

$$\frac{2^2 \cdot C_0}{2} + \frac{2^3 \cdot C_1}{2 \cdot 3} + \dots + \frac{2^{n+2} \cdot C_n}{(n+1)(n+2)} = \frac{3^{n+2} - 2n - 5}{(n+1)(n+2)}$$

(vi) $(1+x)^n = {}^n C_0 + {}^n C_1 x + \dots + {}^n C_n x^n$

$$\int_0^2 (1+x)^n dx = C_0 x + \frac{C_1 x^2}{2} + \dots + \frac{C_n x^{n+1}}{n+1} \Big|_0^2$$

$$\frac{3^{n+1} - 1}{n+1} = 2 \cdot C_0 + \frac{2^2 C_1}{2} + \dots + \frac{2^{n+1} C_n}{n+1}$$

C-3. Prove that सिद्ध करो

(i) ${}^n C_r + {}^{n-1} C_r + {}^{n-2} C_r + \dots + {}^r C_r = {}^{n+1} C_{r+1}$

(ii) ${}^{10} C_2 + {}^{11} C_2 + {}^{12} C_2 + \dots + {}^{19} C_2 = 1020$

Sol. (i) ${}^n C_r + {}^{n-1} C_r + \dots + {}^r C_r$ = Co-efficient of x^r in $(1+x)^n + (1+x)^{n-1} + \dots + (1+x)^r$

$$= \text{Co-efficient of } x^r \text{ in } (1+x)^r \left[\frac{(1+x)^{n-r+1} - 1}{x} \right] = \text{Co-efficient of } x^{r+1} \text{ in } (1+x)^{n+1} = {}^{n+1} C_{r+1}$$

Hindi. (i) ${}^n C_r + {}^{n-1} C_r + \dots + {}^r C_r = (1+x)^n + (1+x)^{n-1} + \dots + (1+x)^r$ के प्रसार में x^r का गुणांक

$$= (1+x)^r \left[\frac{(1+x)^{n-r+1} - 1}{x} \right] \text{ के प्रसार में } x^r \text{ का गुणांक}$$

$$= (1+x)^{n+1} \text{ के प्रसार में } x^r \text{ का गुणांक } = {}^{n+1} C_{r+1}$$

(ii) ${}^{20} C_3 - {}^{10} C_3 = \frac{20 \times 19 \times 18}{3 \times 2 \times 1} - \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120 = 1140 - 120 = 1020$

C-4. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$, prove that

यदि $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$, सिद्ध करो

(i) $C_0 C_3 + C_1 C_4 + \dots + C_{n-3} C_n = \frac{(2n)!}{(n+3)! (n-3)!}$

(ii) $C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n = \frac{(2n)!}{(n+r)! (n-r)!}$

(iii) $C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = 0$ or $(-1)^{n/2} C_{n/2}$ according as n is odd or even.

$C_0^2 - C_1^2 + C_2^2 - C_3^2 + \dots + (-1)^n C_n^2 = 0$ या $(-1)^{n/2} C_{n/2}$ यदि n विषम या सम है।

[Revision Planner]

- Sol.** (i) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $(x+1)^n = C_0 x^n + C_1 x^{n-1} + \dots + C_n$
 $C_0 C_3 + C_1 C_4 + \dots + C_{n-3} C_n$
 $= \text{Co-efficient of } x^{n-3} \text{ in } (1+x)^{2n} = {}^{2n}C_{n-3}$
- (ii) $C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n$
 $= \text{co-efficient of } x^{n-r} \text{ in } (1+x)^{2n}$
 $= {}^{2n}C_{n-r}$
- (iii) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $(x-1)^n = C_0 x^n - C_1 x^{n-1} + \dots + (-1)^n C_n$
 $C_0^2 - C_1^2 + \dots + (-1)^n C_n^2$
 $= \text{co-efficient of } x^n \text{ in } (x^2-1)^n = 0 \quad \text{if } n \text{ is odd}$
 $= {}^n C_{n/2} (-1)^{n/2} \quad \text{if } n \text{ is even}$
- Hindi.** (i) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $(x+1)^n = C_0 x^n + C_1 x^{n-1} + \dots + C_n$
 $C_0 C_3 + C_1 C_4 + \dots + C_{n-3} C_n$
 $= (1+x)^{2n} \text{ में } x^{n-3} \text{ का गुणांक} = {}^{2n}C_{n-3}$
- (ii) $C_0 C_r + C_1 C_{r+1} + \dots + C_{n-r} C_n$
 $= (1+x)^{2n} \text{ में } x^{n-r} \text{ का गुणांक}$
 $= {}^{2n}C_{n-r}$
- (iii) $(1+x)^n = C_0 + C_1 x + \dots + C_n x^n$
 $(x-1)^n = C_0 x^n - C_1 x^{n-1} + \dots + (-1)^n C_n$
 $C_0^2 - C_1^2 + \dots + (-1)^n C_n^2$
 $= (x^2-1)^n \text{ में } x^n \text{ का गुणांक} = 0 \quad \text{यदि } n \text{ विषम है।}$
 $= {}^n C_{n/2} (-1)^{n/2} \quad \text{यदि } n \text{ सम है।}$

Section (D) : Negative & fractional index, Multinomial theorem**खण्ड (D) : ऋणात्मक व भिन्न घातांक, बहुपदीय प्रमेय****D-1.** Find the co-efficient of x^6 in the expansion of $(1-2x)^{-5/2}$. $(1-2x)^{-5/2}$ के प्रसार में x^6 का गुणांक ज्ञात कीजिए।

Ans. $\frac{15015}{16}$

Sol. In the expansion of $(1-2x)^{-5/2}$

$$T_{r+1} = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)\dots\left(\frac{5}{2}+r-1\right)}{r!} \cdot (-1)^r (2)^r x^r$$

$$\therefore \text{Coefficient of } x^6 = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)\left(\frac{5}{2}+3\right)\left(\frac{5}{2}+4\right)\left(\frac{5}{2}+5\right)}{6!} = \frac{15015}{16}$$

Hindi. $(1-2x)^{-5/2}$ के प्रसार में

$$T_{r+1} = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)\dots\left(\frac{5}{2}+r-1\right)}{r!} \cdot (-1)^r (2)^r x^r$$

$$\therefore x^6 \text{ का गुणांक} = \frac{\frac{5}{2}\left(\frac{5}{2}+1\right)\left(\frac{5}{2}+2\right)\left(\frac{5}{2}+3\right)\left(\frac{5}{2}+4\right)\left(\frac{5}{2}+5\right)}{6!} = \frac{15015}{16}$$

D-2. (i) Find the coefficient of x^{12} in $\frac{4+2x-x^2}{(1+x)^3}$

(ii) Find the coefficient of x^{100} in $\frac{3-5x}{(1-x)^2}$

(i) $\frac{4+2x-x^2}{(1+x)^3}$ में x^{12} का गुणांक ज्ञात कीजिए।

(ii) $\frac{3-5x}{(1-x)^2}$ में x^{100} का गुणांक ज्ञात कीजिए।

Ans. (i) 142

(ii) -197

Sol. (i) $(4+2x-x^2)(1+x)^{-3}$
 $= 4 \times \text{coeff of } x^{12} + 2 \times \text{coeff of } x^{11} - 1 \times \text{coeff of } x^{10}$
 $= 4(x^{12} \text{ का गुणांक}) + 2(x^{11} \text{ का गुणांक}) - 1(x^{10} \text{ का गुणांक})$

$$\begin{aligned} & \ln(1+x)^{-3} \\ T_{r+1} &= (-1)^r {}^{3+r-1}C_r x^r \\ &= (-1)^r {}^{r+2}C_r x^r \\ &= 4 \times {}^{14}C_{12} - 2 \times {}^{13}C_{11} - {}^{12}C_{10} \\ &= 4 \times \frac{14 \times 13}{2} - 2 \times \frac{13 \times 12}{2} - \frac{12 \times 11}{2} \\ &= 364 - 156 - 66 = 142 \end{aligned}$$

(ii) $(3-5x)(1-x)^{-2}$
 $= 3 \times \text{coeff of } x^{100} - 5 \times \text{coeff of } x^{99}$
 $= 3(x^{100} \text{ का गुणांक}) - 5(x^{99} \text{ का गुणांक})$
 $\ln(1-x)^{-2}$
 $T_{r+1} = {}^{2+r-1}C_r x^r = {}^{r+1}C_r x^r$
 $= 3 \times {}^{101}C_{100} - 5 \times {}^{100}C_{99}$
 $= 3 \times 101 - 5 \times 100$
 $= -197$

D-3. Assuming 'x' to be so small that x^2 and higher powers of 'x' can be neglected, show that,

$$\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}}$$
 is approximately equal to, $1 - \frac{305}{96}x$.

यदि 'x' का मान इतना अल्प है कि x^2 और 'x' की उच्च घातों को नगण्य माना जा सकता है तो प्रदर्शित कीजिए कि

$$\frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}}$$
 का मान लगभग $1 - \frac{305}{96}x$ है।

Sol.
$$\begin{aligned} \frac{\left(1+\frac{3}{4}x\right)^{-4}(16-3x)^{1/2}}{(8+x)^{2/3}} &= \frac{(1-3x) \cdot 4\left(1-\frac{3}{32}x\right)}{4\left(1+\frac{2x}{24}\right)} = \left(1-3x-\frac{3}{32}x\right)\left(1-\frac{x}{12}\right) \\ &= 1 - \frac{x}{12} - 3x - \frac{3}{32}x = 1 - \frac{305}{96}x \end{aligned}$$

- D-4.** (i) Find the coefficient of $a^5 b^4 c^7$ in the expansion of $(bc + ca + ab)^8$.
(ii) Sum of coefficients of odd powers of x in expansion of $(9x^2 + x - 8)^6$

Ans. (i) 280 (ii) 2⁵

Sol. (i) $(bc + ca + ab)^8$

$$\frac{8!}{r_1! r_2! r_3!} (bc)^{r_1} (ca)^{r_2} (ab)^{r_3}$$

$$\left. \begin{array}{l} r_2 + r_3 = 5 \\ r_1 + r_3 = 4 \\ r_2 + r_1 = 7 \end{array} \right\} \Rightarrow r_2 = 4, r_1 = 3, r_3 = 1$$

$$\text{or या } \frac{8!}{4!3!1!} = 280$$

(ii) $(9x^2 + x - 8)^6 = a_0 + a_1x + a_2x^2 + \dots + a_{12}x^{12}$

$$2^6 = a_0 + a_1 + \dots + a_{12} \quad (x = 1)$$

$$0 = a_0 - a_1 + \dots + a_{12} \quad (x = -1)$$

$$\Rightarrow a_1 + a_3 + \dots + a_{11} = 2^5$$

- D-5.** Find the coefficient of x^7 in $(1 - 2x + x^3)^5$.

$(1 - 2x + x^3)^5$ में x^7 का गुणांक ज्ञात कीजिए।

Ans. 20

Sol. Co-efficient of x^7 in $(1 - 2x + x^3)^5$ के प्रसार में x^7 का गुणांक है

$$= \frac{n!}{r_1! r_2! r_3!} (1)^{r_1} (-2x)^{r_2} (x^3)^{r_3} = r_2 + 3r_3 = 7 \text{ & } r_1, r_2, r_3 \leq 5$$

(i) $r_2 = 4, r_3 = 1, r_1 = 0$

(ii) $r_2 = 1, r_3 = 2, r_1 = 2 = \frac{5!}{4!1!} (2)^4 + \frac{5!}{2!2!1!} \times (-2)^1 = 20$

PART - II : ONLY ONE OPTION CORRECT TYPE

भाग - II : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

Section (A) : General Term & Coefficient of x^k in $(ax + b)^n$

खण्ड (A) : व्यापक पद एवं $(ax + b)^n$ में x^k का गुणांक

A-1. The $(m + 1)^{\text{th}}$ term of $\left(\frac{x}{y} + \frac{y}{x}\right)^{2m+1}$ is:

(A) independent of x

(B) a constant

(C*) depends on the ratio x/y and m

(D) none of these

$$\left(\frac{x}{y} + \frac{y}{x}\right)^{2m+1} \text{ का } (m + 1)\text{वाँ पद}$$

(A) x पर निर्भर नहीं है।

(B) अचर है।

(C) अनुपात x/y और m पर निर्भर है।

(D) इनमें से कोई नहीं

Sol. ${}^{2m+1}C_m \left(\frac{x}{y}\right)^{m+1} \left(\frac{y}{x}\right)^m = {}^{2m+1}C_m \left(\frac{x}{y}\right)$

Dependent upon the ratio $\frac{x}{y}$ and m. अनुपात x/y और m पर निर्भर है।

- A-2.** The total number of distinct terms in the expansion of, $(x + a)^{100} + (x - a)^{100}$ after simplification is :

(A) 50 (B) 202 (C*) 51 (D) none of these

$(x + a)^{100} + (x - a)^{100}$ के प्रसार में सरल करने के बाद मिन्न-मिन्न पदों की कुल संख्या है :

(A) 50 (B) 202 (C) 51 (D) इनमें से कोई नहीं

Sol. $(x + a)^{100} + (x - a)^{100}$

$$= 2 \left({}^{100}C_0 x^{100} + {}^{100}C_2 x^{98}a^2 + \dots + {}^{100}C_{100} a^{100} \right)$$

Number of terms = 51 terms (पदों की संख्या = 51 पद)

- A-3.** The value of, $\frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 81 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64}$ is :

(A*) 1 (B) 2 (C) 3 (D) none

$$\frac{18^3 + 7^3 + 3 \cdot 18 \cdot 7 \cdot 25}{3^6 + 6 \cdot 243 \cdot 2 + 15 \cdot 81 \cdot 4 + 20 \cdot 27 \cdot 8 + 15 \cdot 9 \cdot 16 + 6 \cdot 3 \cdot 32 + 64} \text{ का मान है } -$$

(A) 1 (B) 2 (C) 3 (D) इनमें से कोई नहीं

Sol. $\frac{(18+7)^3}{(3+2)^6} = \frac{25^3}{5^6} = 1$

- A-4.** In the expansion of $\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}} \right)^{15}$ the 11th term is a:

(A) positive integer (B*) positive irrational number
(C) negative integer (D) negative irrational number.

$$\left(3 - \sqrt{\frac{17}{4} + 3\sqrt{2}} \right)^{15} \text{ के प्रसार में 11वाँ पद है } -$$

(A) धनात्मक पूर्णांक (B) धनात्मक अपरिमेय संख्या
(C) ऋणात्मक पूर्णांक (D) ऋणात्मक अपरिमेय संख्या

Sol. $T_{11} = {}^{15}C_{10} (3)^5 \left(-\sqrt{\frac{17}{4} + 3\sqrt{2}} \right)^{10} = {}^{15}C_{10} (3)^5 \left(\frac{17}{4} + 3\sqrt{2} \right)^5 = \text{a positive irrational number}$

Hindi $T_{11} = {}^{15}C_{10} (3)^5 \left(-\sqrt{\frac{17}{4} + 3\sqrt{2}} \right)^{10} = {}^{15}C_{10} (3)^5 \left(\frac{17}{4} + 3\sqrt{2} \right)^5 = \text{धनात्मक अपरिमेय संख्या}$

- A-5.** If the second term of the expansion $\left[a^{1/13} + \frac{a}{\sqrt{a^{-1}}} \right]^n$ is $14a^{5/2}$, then the value of $\frac{{}^nC_3}{{}^nC_2}$ is:

यदि $\left[a^{1/13} + \frac{a}{\sqrt{a^{-1}}} \right]^n$ के प्रसार में द्वितीय पद $14a^{5/2}$ है, तो $\frac{{}^nC_3}{{}^nC_2}$ का मान है—

(A*) 4 (B) 3 (C) 12 (D) 6

Sol. $T_2 = {}^nC_1 (a^{1/13})^{n-1} (a^{3/2}) = 14a^{5/2} \Rightarrow n = 14 \therefore \frac{{}^nC_3}{{}^nC_2} = 4$

- A-6.** In the expansion of $(7^{1/3} + 11^{1/9})^{6561}$, the number of terms free from radicals is:

$(7^{1/3} + 11^{1/9})^{6561}$ के प्रसार में करणी चिन्ह (radical sign) से रहित पदों की संख्या है —

(A*) 730 (B) 729 (C) 725 (D) 750

Sol. $T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/9})^r$

Here r should be multiple of 9
 $r = 0, 9, 18, \dots, 6561$
 Number of terms = 730

Hindi $T_{r+1} = {}^{6561}C_r (7)^{\frac{6561-r}{3}} (11^{1/r})^r$
 यहाँ r, 9 का गुणज होना चाहिए
 $r = 0, 9, 18, \dots, 6561$
 पद्धति की संख्या = 730

Sol. $T_{2m+1} \Rightarrow^{10} C_{2m}$ $T_{4m+5} \Rightarrow^{10} C_{4m+4}$] equal बराबर है।

$$2m + 4m + 4 = 10 \Rightarrow 6m + 4 = 10$$

$$m = 1$$

- A-8.** The co-efficient of x in the expansion of $(1 - 2x^3 + 3x^5) \left(1 + \frac{1}{x}\right)^8$ is :

$(1 - 2x^3 + 3x^5) \left(1 + \frac{1}{x}\right)^8$ के प्रसार में x का गुणांक है—

Sol.
$$(1 - 2x^3 + 3x^5) \left(1 + \frac{1}{x}\right)^8$$

$$\text{Co-efficient of } x \text{ (} x \text{ का गुणांक) } = -2 \cdot {}^8C_2 + 3 \cdot {}^8C_4 = 154$$

Sol. $(x^{1/3} - x^{-1/2})^{15}$

$$T_{r+1} = {}^{15}C_r x^{\left(\frac{15-r}{3}\right)} (-x^{-1/2})^r$$

$$\text{For constant term } \frac{15-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 6$$

$$\text{Co-efficient of } x^0 = {}^{15}\text{C}_6 = 5 \text{ m} \quad \Rightarrow \quad m = 1001$$

Hindi. $(x^{1/3} - x^{-1/2})^{15}$

$$T_{r+1} = {}^{15}C_r x^{\left(\frac{15-r}{3}\right)} (-x^{-1/2})^r$$

$$\text{अचर पद के लिये } \frac{15-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 6$$

$$x^0 \text{ का गुणांक} = {}^{15}\text{C}_6 = 5 \text{ m} \quad \Rightarrow \quad m = 1001$$

Sol. $\sum_{r=1}^{10} r \cdot \frac{nC_r}{nC_{r-1}} = \sum_{r=1}^{10} n-r+1 = (n+1) \times 10 - \frac{10 \times 11}{2} = 10n - 45$

B-6. $\sum_{r=0}^{n-1} \frac{nC_r}{nC_r + nC_{r+1}} =$

(A*) $\frac{n}{2}$

(B) $\frac{n+1}{2}$

(C) $(n+1) \frac{n}{2}$

(D) $\frac{n(n-1)}{2(n+1)}$

Sol. $\sum_{r=0}^{n-1} \frac{nC_r}{nC_r + nC_{r+1}} = \sum_{r=0}^{n-1} \frac{r+1}{n+1} = \frac{1}{n+1} [1+2+\dots+n] = \frac{1}{n+1} \times \frac{n(n+1)}{2} = \frac{n}{2}$

B-7. Find numerically greatest term in the expansion of $(2 + 3x)^9$, when $x = 3/2$.

$(2 + 3x)^9$ के प्रसार में $x = 3/2$ के लिए महत्तम संख्यात्मक मान वाला पद है –

(A*) ${}^9C_6 \cdot 2^9 \cdot (3/2)^{12}$ (B) ${}^9C_3 \cdot 2^9 \cdot (3/2)^6$ (C) ${}^9C_5 \cdot 2^9 \cdot (3/2)^{10}$ (D) ${}^9C_4 \cdot 2^9 \cdot (3/2)^8$

Sol. For numerically greatest term $r = \left[\frac{n+1}{1 + \left| \frac{x}{a} \right|} \right] = \left[\frac{9+1}{1 + \left| \frac{4}{9} \right|} \right] \Rightarrow r = 6$

Numerically greatest term $T_{r+1} = {}^9C_6 (2)^3 \left(\frac{9}{2} \right)^6$

Hindi. महत्तम संख्यात्मक मान वाले पद के लिये $r = \left[\frac{n+1}{1 + \left| \frac{x}{a} \right|} \right] = \left[\frac{9+1}{1 + \left| \frac{4}{9} \right|} \right] \Rightarrow r = 6$

महत्तम संख्यात्मक पद $T_{r+1} = {}^9C_6 (2)^3 \left(\frac{9}{2} \right)^6$

B-8. The greatest integer less than or equal to $(\sqrt{2} + 1)^6$ is

$(\sqrt{2} + 1)^6$ से कम या बराबर महत्तम पूर्णांक है –

(A) 196 (B*) 197 (C) 198 (D) 199

Sol. T_{22} is the numerically greatest term. T_{22} संख्यात्मक महत्तम पद है।

$(\sqrt{2} + 1)^6 = I + f$

$(\sqrt{2} - 1)^6 = f'$

$2[{}^6C_0 + {}^6C_2 \cdot 2 + {}^6C_4 \cdot (2)^2 + \dots] = I + f + f'$

$f + f' = 1$ or $f' = 1 - f$

$I = 2[{}^6C_0 + {}^6C_2 \cdot 2 + {}^6C_4 \cdot 4 + {}^6C_6 \cdot 8] - 1$

$I = 2[1 + 30 + 60 + 8] - 1 = 197$



Section (C) : Summation of series, Variable upper index & Product of binomial coefficients

खण्ड (C) : श्रेणी का योग, चर ऊपरी सूचकांक एवं द्विपद गुणांकों का गुणन

C-1. $\frac{^{11}C_0}{1} + \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \dots + \frac{^{11}C_{10}}{11} =$

(A) $\frac{2^{11}-1}{11}$

(B*) $\frac{2^{11}-1}{6}$

(C) $\frac{3^{11}-1}{11}$

(D) $\frac{3^{11}-1}{6}$

Sol. $\frac{^{11}C_0}{1} + \frac{^{11}C_1}{2} + \frac{^{11}C_2}{3} + \dots + \frac{^{11}C_{10}}{11}$

$$= \left[\frac{12}{1} \cdot {}^{11}C_0 + \frac{12}{2} \cdot {}^{11}C_1 + \frac{12}{3} \cdot {}^{11}C_2 + \dots + \frac{12}{11} \cdot {}^{11}C_{10} \right] = \frac{1}{12} \left[{}^{12}C_1 + {}^{12}C_2 + {}^{12}C_3 + \dots + {}^{12}C_{11} \right]$$

$$= \frac{1}{12} (2^{12} - 2) = \frac{2^{11}-1}{6}$$

C-2. The value of $\frac{C_0}{1.3} - \frac{C_1}{2.3} + \frac{C_2}{3.3} - \frac{C_3}{4.3} + \dots + (-1)^n \frac{C_n}{(n+1) \cdot 3}$ is :

(A) $\frac{3}{n+1}$

(B) $\frac{n+1}{3}$

(C*) $\frac{1}{3(n+1)}$

(D) none of these

$$\frac{C_0}{1.3} - \frac{C_1}{2.3} + \frac{C_2}{3.3} - \frac{C_3}{4.3} + \dots + (-1)^n \frac{C_n}{(n+1) \cdot 3} \text{ का मान होगा} -$$

(A) $\frac{3}{n+1}$

(B) $\frac{n+1}{3}$

(C*) $\frac{1}{3(n+1)}$

(D) इनमें से कोई नहीं

Sol. $\int_0^1 (1-x)^n dx = \int_0^1 \left(C_0 - C_1 x + C_2 x^2 - C_3 x^3 + \dots + (-1)^n C_n x^n \right) dx$

$$\Rightarrow \frac{1}{n+1} = \left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \dots + (-1)^n \frac{C_n}{n+1} \right]$$

$$\Rightarrow \frac{1}{3} \left(\frac{1}{n+1} \right) = \frac{1}{3} \left[C_0 - \frac{C_1}{2} + \frac{C_2}{3} - \dots + (-1)^n \frac{C_n}{n+1} \right]$$

C-3. The value of the expression ${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$ is equal to :

व्यंजक ${}^{47}C_4 + \sum_{j=1}^5 {}^{52-j}C_3$ का मान बराबर है—

(A) ${}^{47}C_5$

(B) ${}^{52}C_5$

(C*) ${}^{52}C_4$

(D) ${}^{49}C_4$

Sol. ${}^{47}C_4 + {}^{51}C_3 + {}^{50}C_3 + {}^{49}C_3 + {}^{48}C_3 + {}^{47}C_3 = {}^{52}C_4$

C-4. The value of $\binom{50}{0} \binom{50}{1} + \binom{50}{1} \binom{50}{2} + \dots + \binom{50}{49} \binom{50}{50}$ is, where ${}^nC_r = \binom{n}{r}$

$$\binom{50}{0} \binom{50}{1} + \binom{50}{1} \binom{50}{2} + \dots + \binom{50}{49} \binom{50}{50} \text{ का मान होगा, जहाँ } {}^nC_r =$$

(A) $\binom{100}{50}$

(B*) $\binom{100}{51}$

(C) $\binom{50}{25}$

(D) $\binom{50}{25}^2$

Sol. ${}^{50}C_0 \times {}^{50}C_1 + {}^{50}C_1 \times {}^{50}C_2 + \dots + {}^{50}C_{49} \times {}^{50}C_{50}$

(A) 2^{10}

(B) 2

(C) 2^{20}

(D) इनमें से कोई नहीं

Sol. $(1+x)^{10} = a_0 + a_1 + a_2 x^2 + \dots + a_{10} x^{10}$

Put $x = i$,

$$(1+i)^{10} = a_0 - a_2 + a_4 + \dots + a_{10} + i(a_1 - a_3 + \dots + a_9)$$

$$a_0 - a_2 + a_4 + \dots + a_{10} = \text{real part of } (1+i)^{10} = 2^5 \cos 10\pi/4$$

$$a_1 - a_3 + \dots = \text{imaginary part of } (1+i)^{10} = 2^5 \sin 10\pi/4 \quad \dots(2)$$

$$(1)^2 + (2)^2 = 2^{10}$$

Hindi. $(1+x)^{10} = a_0 + a_1 + a_2 x^2 + \dots + a_{10} x^{10}$

 $x = i$ रखने पर

$$(1+i)^{10} = a_0 - a_2 + a_4 + \dots + a_{10} + i(a_1 - a_3 + \dots + a_9)$$

$$a_0 - a_2 + a_4 + \dots + a_{10} = (1+i)^{10} \text{ का वास्तविक भाग} = 2^5 \cos 10\pi/4$$

$$a_1 - a_3 + \dots = (1+i)^{10} \text{ का काल्पनिक भाग} = 2^5 \sin 10\pi/4 \quad \dots(2)$$

$$(1)^2 + (2)^2 = 2^{10}$$

PART - III : MATCH THE COLUMN

भाग - III : कॉलम को सुमेलित कीजिए (MATCH THE COLUMN)

1. Column - I

- (A) If $(r+1)^{\text{th}}$ term is the first negative term in the expansion of $(1+x)^{7/2}$, then the value of r (where $0 < x < 1$) is
- (B) If the sum of the co-efficients in the expansion of $(1+2x)^n$ is 6561, and T_r is the greatest term in the expansion for $x = 1/2$ then r is
- (C) ${}^n C_r$ is divisible by n , ($1 < r < n$) if n is
- (D) The coefficient of x^4 in the expression $(1+2x+3x^2+4x^3+\dots \text{ up to } \infty)^{1/2}$ is c , ($c \in \mathbb{N}$), then $c+1$ (where $|x| < 1$) is

Ans. (A) $\rightarrow (q, s)$, (B) $\rightarrow (q, s)$, (C) $\rightarrow (s)$, (D) $\rightarrow (p, s)$

स्तरम् - I

- (A) यदि $(1+x)^{7/2}$ के प्रसार में $(r+1)$ वाँ पद प्रथम ऋणात्मक पद है, तो r का मान है – (जहाँ $0 < x < 1$)
- (B) यदि $(1+2x)^n$ के प्रसार में गुणांकों का योगफल 6561, और $x = 1/2$ के लिए T_r अधिकतम पद है तब r है।
- (C) ${}^n C_r$, ($1 < r < n$), n से विभाजित होगा यदि n है,
- (D) व्यंजक $(1+2x+3x^2+4x^3+\dots \text{ अनन्त पदों तक})^{1/2}$ में x^4 का गुणांक c , ($c \in \mathbb{N}$) है, तो $c+1$ है – (जहाँ $|x| < 1$)

Column - II

- (p) divisible by 2
- (q) divisible by 5
- (r) divisible by 10
- (s) a prime number

स्तरम् - II

- (p) 2 से विभाजित है
- (q) 5 से विभाजित है
- (r) 10 से विभाजित है
- (s) एक अभाज्य संख्या

Sol. (A) We have, $T_{r+1} = \frac{\frac{7}{2} \left(\frac{7}{2}-1\right) \left(\frac{7}{2}-2\right) \dots \left(\frac{7}{2}-r+1\right) x^r}{r!}$

This will be the first negative term when $\frac{7}{2} - r + 1 < 0$ i.e. $r > \frac{9}{2}$

Hence $r = 5$.

(B) $3^n = 6561$ (put $x = 1$) $\Rightarrow n = 8$

$$\frac{T_{r+1}}{T_r} = \frac{8-r+1}{r} \geq 1 \Rightarrow 8-r+1 \geq r \Rightarrow r \leq \frac{9}{2} \Rightarrow r = 4 \quad (5^{\text{th}} \text{ term is greatest})$$

(C) Obviously a prime number.

(D) We have : $(1 + 2x + 3x^2 + 4x^3 + \dots)^{1/2}$

$$= [(1-x)^{-2}]^{1/2} = (1-x)^{-1} = 1 + x + x^2 + \dots + x^n + \dots$$

Hence, coefficient of $x^4 = 1 \quad \therefore c = 1$, hence $c + 1 = 2$

$$\text{चूंकि } T_{r+1} = \frac{\frac{7}{2} \left(\frac{7}{2}-1\right) \left(\frac{7}{2}-2\right) \dots \left(\frac{7}{2}-r+1\right) x^r}{r!}$$

यह प्रथम ऋणात्मक पद होगा यदि $\frac{7}{2} - r + 1 < 0$ i.e. $r > \frac{9}{2}$

अतः $r = 5$.

(B) $3^n = 6561$ ($x = 1$ रखने पर) $\Rightarrow n = 8$

$$\frac{T_{r+1}}{T_r} = \frac{8-r+1}{r} \geq 1 \Rightarrow 8-r+1 \geq r \Rightarrow r \leq \Rightarrow r = 4 \quad (5^{\text{th}} \text{ पद महत्तम है।})$$

(C) स्पष्टतया: एक अभाज्य संख्या।

(D) $(1 + 2x + 3x^2 + 4x^3 + \dots)^{1/2}$

$$= [(1-x)^{-2}]^{1/2} = (1-x)^{-1} = 1 + x + x^2 + \dots + x^n + \dots$$

अतः x^4 का गुणांक = 1 $\therefore c = 1$, अतः $c + 1 = 2$

Hindi

(A) चूंकि $T_{r+1} = \frac{\frac{7}{2} \left(\frac{7}{2}-1\right) \left(\frac{7}{2}-2\right) \dots \left(\frac{7}{2}-r+1\right) x^r}{r!}$

यह प्रथम ऋणात्मक पद होगा यदि $\frac{7}{2} - r + 1 < 0$ i.e. $r > \frac{9}{2}$

अतः $r = 5$.

(B) $3^n = 6561$ ($x = 1$ रखने पर) $\Rightarrow n = 8$

$$\frac{T_{r+1}}{T_r} = \frac{8-r+1}{r} \geq 1 \Rightarrow 8-r+1 \geq r \Rightarrow r \leq \Rightarrow r = 4 \quad (5^{\text{th}} \text{ पद महत्तम है।})$$

(C) स्पष्टतया: एक अभाज्य संख्या।

(D) $(1 + 2x + 3x^2 + 4x^3 + \dots)^{1/2}$

$$= [(1-x)^{-2}]^{1/2} = (1-x)^{-1} = 1 + x + x^2 + \dots + x^n + \dots$$

अतः x^4 का गुणांक = 1 $\therefore c = 1$, अतः $c + 1 = 2$



Exercise-2

- Marked questions are recommended for Revision.
- चिन्हित प्रश्न दोहराने योग्य प्रश्न है।

PART - I : ONLY ONE OPTION CORRECT TYPE

भाग-1 : केवल एक सही विकल्प प्रकार (ONLY ONE OPTION CORRECT TYPE)

1. In the expansion of

$$\left(3\sqrt{\frac{a}{b}} + 3\sqrt{\frac{b}{\sqrt{a}}} \right)^{21}, \text{ the term containing same powers of } a \text{ & } b \text{ is}$$

- (A) 11th term (B*) 13th term (C) 12th term (D) 6th term

$\left(3\sqrt{\frac{a}{b}} + 3\sqrt{\frac{b}{\sqrt{a}}}\right)^{21}$, के विस्तार में a और b की समान घातों का पद है –

$$\text{Sol. } T_{r+1} = {}^{21}C_r \left(\frac{a}{b}\right)^{\frac{21-r}{3}} \left(\frac{b}{\sqrt{a}}\right)^{\frac{r}{3}}$$

$$= {}^{21} C_r \cdot a^{\frac{21-r}{3}} b^{\frac{r}{3}}$$

$$= {}^{21}C_r \cdot a^{\frac{42-3r}{6}} \cdot b^{\frac{2r-21}{3}}$$

$$=^{21} C_r \cdot a^{\frac{14-r}{2}} \cdot b^{\frac{2r-21}{3}}$$

$$\frac{14-r}{2} = \frac{2r-21}{3}$$

$$42 - 3r = 4r - 42$$

$$2r = 84$$

$r = 12 \Rightarrow T_{13}$ term (T_{13} वा पद)

- 2.** Consider the following statements :

S₁ : Number of dissimilar terms in the expansion of $(1 + x + x^2 + x^3)^n$ is $3n + 1$

S₂ : $(1 + x)(1 + x + x^2)(1 + x + x^2 + x^3) \dots (1 + x + x^2 + \dots + x^{100})$ when written in the ascending power of x then the highest exponent of x is 5000.

$$S_3 : \sum_{k=1}^{n-r} {}^n C_r = {}^n C_{r+1}$$

$$\mathbf{S_4 :} \quad \text{If } (1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}, \text{ then } a_0 + a_2 + a_4 + \dots + a_{2n} = \frac{3^n - 1}{2}$$

State, in order, whether S_1 , S_2 , S_3 , S_4 are true or false

माना कि निम्न कथन है -

S₁ : $(1 + x + x^2 + x^3)^n$ के प्रसार में असमान पदों की संख्या $3n + 1$ है।

S₂: $(1+x)(1+x+x^2)(1+x+x^2+x^3)\dots(1+x+x^2+\dots+x^{100})$ को यदि x की बढ़ती हुई घातों के क्रम में लिखा जाता है, तो x की अधिकतम घात 5000 होगी –

$$\mathbf{S}_3 : \sum_{k=1}^{n-r} {}^n C_r = {}^n C_{r+1}$$

$$\mathbf{S}_4 : \text{यदि } (1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}, \text{ तो } a_0 + a_2 + a_4 + \dots + a_{2n} = \frac{3^n - 1}{2}$$

S_1, S_2, S_3, S_4 के सत्य (T) या असत्य (F) होने का सही क्रम है –

(A*) TFTF

(B) TTTT

(C) FFFF

(D) FTFT

Sol. S_1 : Number of dissimilar terms will be same as in $(1+x)^{3n}$ i.e. $3n+1$

$$\mathbf{S}_2 : (1+x)(1+x+x^2)\dots + (1+x+\dots+x^{100}) \\ \text{Highest exponent of } x = 1+2+\dots+100 \\ = 5050$$

$$\mathbf{Hindi.} (1+x)(1+x+x^2)\dots + (1+x+\dots+x^{100}) \\ x \text{ की अधिकतम घात} = 1+2+\dots+100 \\ = 5050$$

$$\mathbf{S}_3 : \sum_{k=1}^{n-r} {}^n C_r = {}^n C_y \Rightarrow {}^n C_y = {}^{n-1} C_r + {}^{n-2} C_r + \dots + {}^n C_r \\ \Rightarrow {}^n C_y = \text{co-efficient of } x^r \text{ in } ((1+x)^r + \dots + (1+x)^{n-1}) \\ = \text{co-efficient of } x^r \text{ in } (1+x)^r \left[\frac{(1+x)^{n-r} - 1}{x} \right] = \text{co-efficient of } x^{r+1} \text{ in } (1+x)^n = {}^n C_{r+1}$$

$$\mathbf{Hindi} \sum_{k=1}^{n-r} {}^n C_r = {}^n C_y \Rightarrow {}^n C_y = {}^{n-1} C_r + {}^{n-2} C_r + \dots + {}^n C_r \\ \Rightarrow {}^n C_y = ((1+x)^r + \dots + (1+x)^{n-1}) \text{ में } x^r \text{ का गुणांक} \\ = (1+x)^r \left[\frac{(1+x)^{n-r} - 1}{x} \right] \text{ में } x^r \text{ का गुणांक} = (1+x)^n \text{ में } x^{r+1} \text{ का गुणांक} = {}^n C_{r+1}$$

$$\mathbf{S}_4 : (1+x+x^2)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n} \\ \text{put } x = 1 \quad \text{रखने पर} \\ 3^n = a_0 + a_1 + a_2 + \dots + a_{2n} \quad \dots(i) \\ x = -1 \\ 1 = a_0 - a_1 + a_2 + \dots + a_{2n} \quad \dots(ii) \\ \text{adding (i) & (ii)} \quad (i) \text{ तथा (ii) को जोड़ने पर} \\ \frac{3^n + 1}{2} = a_0 + a_2 + \dots + a_{2n}.$$

3. If $\frac{{}^n C_r + 4{}^n C_{r+1} + 6{}^n C_{r+2} + 4{}^n C_{r+3} + {}^n C_{r+4}}{{}^n C_r + 3{}^n C_{r+1} + 3{}^n C_{r+2} + {}^n C_{r+3}} = \frac{n+k}{r+k}$ then the value of k is :

$$\text{यदि } \frac{{}^n C_r + 4{}^n C_{r+1} + 6{}^n C_{r+2} + 4{}^n C_{r+3} + {}^n C_{r+4}}{{}^n C_r + 3{}^n C_{r+1} + 3{}^n C_{r+2} + {}^n C_{r+3}} = \frac{n+k}{r+k} \text{ हो तो } k \text{ का मान ज्ञात कीजिए।}$$

(A) 1

(B) 2

(C*) 4

(D) 5

$$\mathbf{Sol.} \text{ Numerator} = {}^n C_r + {}^n C_{r+1} + 3({}^n C_{r+1} + {}^n C_{r+2}) + 3({}^n C_{r+2} + {}^n C_{r+3}) + {}^n C_{r+3} + {}^n C_{r+4} \\ = {}^{n+1} C_{r+1} + 3 {}^{n+1} C_{r+2} + 3 {}^{n+1} C_{r+3} + {}^{n+1} C_{r+4} \\ = {}^{n+1} C_{r+1} + {}^{n+1} C_{r+2} + 2({}^{n+1} C_{r+2} + {}^{n+1} C_{r+3}) + {}^{n+1} C_{r+3} + {}^{n+1} C_{r+4} \\ = {}^{n+2} C_{r+2} + 2 {}^{n+2} C_{r+3} + {}^{n+2} C_{r+4}$$



$$\begin{aligned}
 &= {}^{n+2}C_{r+2} + {}^{n+2}C_{r+3} + {}^{n+2}C_{r+4} + {}^{n+2}C_{r+3} \\
 &= {}^{n+3}C_{r+3} + {}^{n+3}C_{r+4} = {}^{n+4}C_{r+4} \\
 \text{Denominator} &= {}^nC_r + {}^nC_{r+1} + 2({}^nC_{r+1} + {}^nC_{r+2}) + ({}^nC_{r+2} + {}^nC_{r+3}) \\
 &= {}^{n+1}C_{r+1} + 2{}^{n+1}C_{r+2} + {}^{n+1}C_{r+3} \\
 &= {}^{n+1}C_{r+1} + {}^{n+1}C_{r+2} + {}^{n+1}C_{r+2} + {}^{n+1}C_{r+3} \\
 &= {}^{n+2}C_{r+2} + {}^{n+2}C_{r+3} = {}^{n+3}C_{r+3}
 \end{aligned}$$

$$\therefore \text{The expression is equal to : } \frac{{}^{n+4}C_{r+4}}{{}^{n+3}C_{r+3}} = \frac{(n+4)!(r+3)!(n-r)!}{(r+4)!(n-r)!(n+3)!} = \frac{n+4}{r+4}$$

$$\therefore k = 4$$

4. The co-efficient of x^5 in the expansion of $(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$ is :

$(1+x)^{21} + (1+x)^{22} + \dots + (1+x)^{30}$ के प्रसार में x^5 का गुणांक है :

- (A) 5C_5 (B) 9C_5 (C*) ${}^{31}C_6 - {}^{21}C_6$ (D) ${}^{30}C_5 + {}^{20}C_5$

Sol. $(1+x)^{21} [1 + (1+x) + \dots + (1+x)^9] = (1+x)^{21} \left[\frac{(1+x)^{10} - 1}{x} \right] = \frac{(1+x)^{31} - (1+x)^{21}}{x}$

Coefficient of $x^5 = {}^{31}C_6 - {}^{21}C_6$

x^5 का गुणांक $= {}^{31}C_6 - {}^{21}C_6$

5. The coefficient of x^{52} in the expansion $\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ is :

$\sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} \cdot 2^m$ के प्रसार में x^{52} का गुणांक है –

- (A) ${}^{100}C_{47}$ (B*) ${}^{100}C_{48}$ (C) $-{}^{100}C_{52}$ (D) $-{}^{100}C_{100}$

Sol. $S = \sum_{m=0}^{100} {}^{100}C_m (x-3)^{100-m} 2^m$

$$S = {}^{100}C_0 (x-3)^{100} + {}^{100}C_1 (x-3)^{99} \cdot 2 + \dots + {}^{100}C_{100} \cdot 2^{100}$$

$$S = (2 + (x-3))^{100} = (x-1)^{100}$$

Co-efficient of $x^{52} = {}^{100}C_{52} = {}^{100}C_{48}$

x^{52} का गुणांक $= {}^{100}C_{52} = {}^{100}C_{48}$

6. The sum of the coefficients of all the integral powers of x in the expansion of $(1+2\sqrt{x})^{40}$ is :

$(1+2\sqrt{x})^{40}$ के प्रसार में x की सभी पूर्णांक घातों के गुणांकों का योगफल है –

- (A) $3^{40} + 1$ (B) $3^{40} - 1$ (C) $\frac{1}{2} (3^{40} - 1)$ (D*) $\frac{1}{2} (3^{40} + 1)$

Sol. $(1+2\sqrt{x})^{40} = {}^{40}C_0 + {}^{40}C_1 2\sqrt{x} + \dots + {}^{40}C_{40} (2\sqrt{x})^{40}$

$$(1-2\sqrt{x})^{40} = {}^{40}C_0 - {}^{40}C_1 2\sqrt{x} + \dots + {}^{40}C_{40} (2\sqrt{x})^{40}$$

$$(1+2\sqrt{x})^{40} + (1-2\sqrt{x})^{40} = 2[{}^{40}C_0 + {}^{40}C_2 (2\sqrt{x})^2 + \dots + {}^{40}C_{40} (2\sqrt{x})^{40}]$$

Putting $x = 1$ रखने पर

$${}^{40}C_0 + {}^{40}C_2 (2)^2 + \dots + {}^{40}C_{40} (2)^{40} = \frac{3^{40} + 1}{2}$$

7. $\sum_{r=0}^n (-1)^r {}^nC_r \cdot \frac{(1+r\ln 10)}{(1+\ell n 10^n)^r} =$

$$\sum_{r=0}^n (-1)^r \binom{n}{r} \cdot \frac{(1+r\ln 10)}{(1+\ell\ln 10^n)^r} \text{ का मान है—}$$

$$\begin{aligned}
 \text{Sol.} & \sum_{r=0}^n (-1)^r \cdot {}^nC_r \cdot \frac{1}{(1+n\ell\ln 10)^r} + \sum_{r=0}^n (-1)^r r \cdot {}^nC_r \cdot \frac{\ell n 10}{(1+n\ell\ln 10)^r} \\
 & = \left(1 - \frac{1}{1+n\ell\ln 10} \right)^n + n\ell n 10 \sum_{r=1}^n (-1)^{r-n-1} C_{r-1} \frac{1}{(1+n\ell\ln 10)^r} \\
 & = \left(\frac{n\ell 10}{1+n\ell 10} \right)^n - \frac{n\ell 10}{1+\ell 10} \sum_{r=1}^n (-1)^{r-1-n-1} C_{r-1} \left(\frac{1}{n\ell\ln 10} \right)^{r-1} \\
 & = \left(\frac{n\ell 10}{1+n\ell 10} \right)^n - \frac{n\ell 10}{1+n\ell 10} \times \left(1 - \frac{1}{1+n\ell 10} \right)^{n-1} \\
 & = \left(\frac{n\ell 10}{1+\ell 10} \right)^n - \frac{n\ell 10}{1+\ell n\ell 10} \times \frac{(n\ell 10)^{n-1}}{(1+n\ell 10)^{n-1}} = 0
 \end{aligned}$$

8. The coefficient of the term independent of x in the expansion of $\left(\frac{x+1}{\frac{2}{x^3} - \frac{1}{x^3} + 1} - \frac{x-1}{x - x^{\frac{1}{2}}} \right)^{10}$ is :

$$\left(\frac{\frac{x+1}{2} - \frac{x-1}{x-x^2}}{x^{\frac{1}{3}} - x^{\frac{1}{3}} + 1} \right)^{10} \text{ के प्रसार में } x \text{ से स्वतंत्र पद का गुणांक है -}$$

$$\text{Sol. } \left(\frac{\frac{x+1}{2} - \frac{x-1}{1}}{\frac{1}{x^3 - x^3} + 1} \right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}} \right)^{10}$$

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} \left(-\frac{1}{\sqrt{x}} \right)^r$$

$$\text{For independent term } \frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$$

Coefficient of the term independent of x = ${}^{10}C_4$

$$\text{Hindi. } \left(\frac{\frac{x+1}{\frac{2}{x^3} - \frac{1}{x^3} + 1} - \frac{x-1}{\frac{1}{x-x^2}}} \right)^{10} = \left(x^{1/3} + 1 - 1 - \frac{1}{\sqrt{x}} \right)^{10}$$

$$T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} \left(-\frac{1}{\sqrt{x}}\right)^r$$

$$\text{स्वतन्त्र पद हेतु } \frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow r = 4$$

x से स्वतन्त्र पद का गुणांक = ${}^{10}C_4$

- 9.** Coefficient of x^{n-1} in the expansion of, $(x+3)^n + (x+3)^{n-1}(x+2) + (x+3)^{n-2}(x+2)^2 + \dots + (x+2)^n$ is :

$(x+3)^n + (x+3)^{n-1}(x+2) + (x+3)^{n-2}(x+2)^2 + \dots + (x+2)^n$ के प्रसार में x^{n-1} का गुणांक है -

- (A) $n+1C_2(3)$ (B) $n-1C_2(5)$ (C*) $n+1C_2(5)$ (D) $nC_2(5)$

$$\text{Sol. } (x+3)^n + (x+3)^{n-1}(x+2) + \dots + (x+2)^n = (x+3)^n \left(\frac{1 - \left(\frac{x+2}{x+3} \right)^{n+1}}{1 - \frac{x+2}{x+3}} \right) = [(x+3)^{n+1} - (x+2)^{n+1}]$$

$$\text{Coefficient of } x^{n-1} \text{ का गुणांक} = {}^{n+1}C_{n-1} (3)^2 - {}^{n+1}C_{n-1} \times 4$$

- 10.** Let $f(n) = 10^n + 3 \cdot 4^{n+2} + 5$, $n \in \mathbb{N}$. The greatest value of the integer which divides $f(n)$ for all n is :

माना $f(n) = 10^n + 3 \cdot 4^{n+2} + 5$, $n \in \mathbb{N}$ तब वह अधिकतम पूर्णांक जो $f(n)$ को n के प्रत्येक मान के लिए विभाजित करता है—

Sol. $f(n) = 10^n + 3 \cdot 4^{n+2} + 5$

put n = 1

- (B) 9

- (C) 3

- (D) इनमें से कोई नहीं

$$f(1) = 10 + 192 + 5 =$$

$$f(1) = 10 + 192 + 5 = 207 \text{ यह } ;$$

... (.) 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

If $(1+x)^n = \sum_{r=0}^n a_r x^r$ and $b = 1 + \frac{a_r}{r}$ and $\prod_{r=1}^n b = \frac{(1+n)!}{n!}$

- 11.** If $(1 + x)^n = \sum_{r=0}^n a_r x^r$ and $b_r = 1 + \frac{a_r}{a_{r-1}}$ and $\prod_{r=1}^n b_r = \frac{(101)^{100}}{100!}$, then n equals to :

यदि $(1+x)^n = \sum_{r=0}^n a_r x^r$ और $b_r = 1 + \frac{a_r}{a_{r-1}}$ और $\prod_{r=1}^n b_r = \frac{(101)^{100}}{100!}$, तो n बराबर है :

$$\text{Sol. } (1+x)^n = \sum_{r=0}^n a_r x^r = a_0 + a_1 x + \dots + a_n x^n$$

$$b_r = 1 + \frac{a_r}{a_{r-1}} = 1 + \frac{n-r+1}{r} = \frac{n+1}{r}$$

$$\prod_{r=1}^n b_r = b_1 b_2 \dots b_n = \frac{(n+1)^n}{1 \cdot 2 \cdot 3 \dots n} = \frac{(101)^{100}}{100!} \Rightarrow n = 100$$

- 12.** Number of rational terms in the expansion of $(1 + \sqrt{2} + \sqrt{5})^6$ is :

$(1+\sqrt{2}+\sqrt{5})^6$ के विस्तार में परिमेय पदों की संख्या है –

Sol. General term is = व्यापक पद

$$= \frac{6!}{r_1! r_2! r_3!} (1)^{r_1} (\sqrt{2})^{r_2} (\sqrt{5})^{r_3} = \frac{6!}{r_1! r_2! r_3!} (2)^{\frac{r_2}{2}} (2)^{\frac{r_3}{2}}$$

where जहाँ $r_1 + r_2 + r_3 = 6$

r_1	r_2	r_3	r_1	r_2	r_3
2	4	0	2	4	0
4	2	0	4	2	0
0	4	2	0	4	2
2	2	2	2	2	2
4	0	2	4	0	2
0	2	4	0	2	4
2	0	4	2	0	4
0	0	6	0	0	6
0	6	0	0	6	0
6	0	0	6	0	0
<u>10 terms are possible</u>			<u>10 पद संभव हैं।</u>		

13. If $S = {}^{404}C_4 - {}^4C_1 \cdot {}^{303}C_4 + {}^4C_2 \cdot {}^{202}C_4 - {}^4C_3 \cdot {}^{101}C_4 = (101)^k$ then k equals to :

यदि $S = {}^{404}C_4 - {}^4C_1 \cdot {}^{303}C_4 + {}^4C_2 \cdot {}^{202}C_4 - {}^4C_3 \cdot {}^{101}C_4 = (101)^k$ तब k का मान है—

Sol. $S = \text{coeff. of } x^4 \text{ in } S = x^4$ में का गुणांक

$$\begin{aligned}
 &= \left[{}^4C_0 \cdot \left((1+x)^{101} \right)^4 - {}^4C_1 \left((1+x)^{101} \right)^3 + {}^4C_2 \left((1+x)^{101} \right)^2 - {}^4C_3 \left((1+x)^{101} \right)^1 + {}^4C_4 \right] - {}^4C_4 \\
 &= \left((1+x)^{101} - 1 \right)^4 - 1 \\
 &= (1 + {}^{101}C_1 x + {}^{101}C_2 x^2 + {}^{101}C_3 x^3 + {}^{101}C_4 x^4 + \dots - 1)^4 - 1 \\
 &= x^4 \left({}^{101}C_1 + {}^{101}C_2 x + {}^{101}C_3 x^2 + \dots \right)^4 - 1 \\
 &= x^4 \left(101 + ({}^{101}C_2 x + {}^{101}C_3 x^2 + \dots) \right)^4 - 1 \\
 &= 101^4 \\
 k &= 4
 \end{aligned}$$

10 - 3

$$14. \quad {}^{10}C_0^2 - {}^{10}C_1^2 + {}^{10}C_2^2 - \dots - \left({}^{10}C_9\right)^2 + \left({}^{10}C_{10}\right)^2 =$$

$$\text{Sol. } (1+x)^{10} = {}^{10} C_0 + {}^{10} C_1 x + {}^{10} C_2 x^2 + \dots + {}^{10} C_9 x^9 + {}^{10} C_{10} x^{10}$$

$$(x-1)^{10} = {}^{10}C_0 x^{10} - {}^{10}C_1 x^9 + {}^{10}C_2 x^8 - \dots - {}^{10}C_9 x + {}^{10}C_{10}$$

15. The sum $\sum_{r=0}^n (r+1) C_r^2$ is equal to :

योगफल $\sum_{r=0}^n (r+1) C_r^2$ बराबर है -

- $$(A^*) \frac{(n+2)(2n-1)!}{n!(n-1)!} (B) \frac{(n+2)(2n+1)!}{n!(n-1)!} (C) \frac{(n+2)(2n+1)!}{n!(n+1)!} (D) \frac{(n+2)(2n-1)!}{n!(n+1)!}$$

$$\text{Sol.} \quad \because \quad (1+x)^n = C_0 + C_1x + \dots + C_n x^n$$

Multiply by x & then differentiate

(A*) 4

(B) 6

(C) 8 (D) None of these इनमें से कोई नहीं

$$\text{Sol. } \sum_{r=0}^{n-1} \left(\frac{{}^n C_r}{{}^n C_r + {}^n C_{r+1}} \right)^3 = \frac{4}{5}$$

$$\Rightarrow \sum_{r=0}^{n-1} \left(\frac{\frac{n!}{r!(n-r)!}}{\frac{(n+1)!}{(r+1)!(n-r)!}} \right)^3 = \frac{4}{5} \Rightarrow \sum_{r=0}^{n-1} \left(\frac{r+1}{n+1} \right)^3 = \frac{4}{5}$$

$$\Rightarrow \frac{1}{(n+1)^3} (1^3 + 2^3 + 3^3 + \dots + n^3) = 4/5$$

$$\Rightarrow \frac{1}{(n+1)^3} \times \frac{n^2(n+1)^2}{4} = \frac{4}{5} \Rightarrow 5n^2 - 16n - 16 = 0 \Rightarrow n = 4$$

20. The number of terms in the expansion of $\left(x^2 + 1 + \frac{1}{x^2}\right)^n$, $n \in \mathbb{N}$, is :

$$\left(x^2 + 1 + \frac{1}{x^2}\right)^n, n \in \mathbb{N} \text{ के प्रसार में पदों की संख्या है -}$$

(A) 2n

(B) 3n

(C*) 2n + 1

(D) 3n + 1

$$\text{Sol. } \left(\left(x + \frac{1}{x} \right)^2 - 1 \right)^n = {}^n C_0 \left(x + \frac{1}{x} \right)^{2n} - {}^n C_1 \left(x + \frac{1}{x} \right)^{2n-2} + \dots + {}^n C_n (-1)^n$$

Total number of terms = 2n + 1 कुल पदों की संख्या = 2n + 1

21. Suppose

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0$$

{[BT-BC]-M-305}

holds for some positive integer n. Then $\sum_{k=0}^n \frac{{}^n C_k}{k+1}$ equals.

मानाकि

किसी धनात्मक पूर्णांक n के लिए

$$\det \begin{bmatrix} \sum_{k=0}^n k & \sum_{k=0}^n {}^n C_k k^2 \\ \sum_{k=0}^n {}^n C_k k & \sum_{k=0}^n {}^n C_k 3^k \end{bmatrix} = 0, \text{ तो } \sum_{k=0}^n \frac{{}^n C_k}{k+1} \text{ का मान है -}$$

[JEE(Advanced) 2019, Paper-2 , (4, -1)/62]

Ans. (6.20)

[Binomial Theorem_M]

Sol.
$$\left| \begin{array}{cc} \frac{n(n+1)}{2} & n2^{n-1} + n(n-1)2^{n-2} \\ n2^{n-1} & 4^n \end{array} \right| = 0$$

$$\frac{n(n+1)}{2} - \frac{n^2}{4} - \frac{n^2(n-1)}{8} = 0$$

$$n = 0 \quad \text{or} \quad 4(n+1) - 2n - n(n-1) = 0$$

$$4n + 4 - 2n - n^2 + n = 0$$

$$3n - n^2 + 4 = 0 \Rightarrow n^2 - 3n - 4 = 0$$

$$(n-4)(n+1) = 0$$

$$n = 4$$

$$\sum_{r=0}^4 \frac{{}^4C_r}{r+1} = \sum_{r=0}^4 \frac{{}^5C_{r+1}}{5} = \frac{2^5 - 1}{5} = \frac{31}{5} = 6.20$$

PART-II: NUMERICAL VALUE QUESTIONS

भाग-II : संख्यात्मक प्रश्न (NUMERICAL VALUE QUESTIONS)

INSTRUCTION :

- ❖ The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto two digit.
- ❖ If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.

निर्देश :

- ❖ इस खण्ड में प्रत्येक प्रश्न का उत्तर संख्यात्मक मान के रूप में है जिसमें दो पूर्णांक अंक तथा दो अंक दशमलव के बाद में है।
- ❖ यदि संख्यात्मक मान में दो से अधिक दशमलव स्थान है, तो संख्यात्मक मान को दशमलव के दो स्थानों तक ट्रंकेट/राउंड ऑफ (truncate/round-off) करें।

1. If $\frac{1}{1!10!} + \frac{1}{2!9!} + \frac{1}{3!8!} + \dots + \frac{1}{10!1!} = \frac{(2^{10}-1)}{k \cdot 10!}$ then find the value of k.

यदि $\frac{1}{1!10!} + \frac{1}{2!9!} + \frac{1}{3!8!} + \dots + \frac{1}{10!1!} = \frac{(2^{10}-1)}{k \cdot 10!}$ तब k का मान ज्ञात कीजिए।

Ans. 05.50

Sol. $\frac{1}{11!} \left[\frac{11!}{1!10!} + \frac{11!}{2!9!} + \frac{11!}{3!8!} + \dots + \frac{11!}{1!10!} \right] = \frac{1}{11!} [{}^{11}C_1 + {}^{11}C_2 + \dots + {}^{11}C_{10}]$

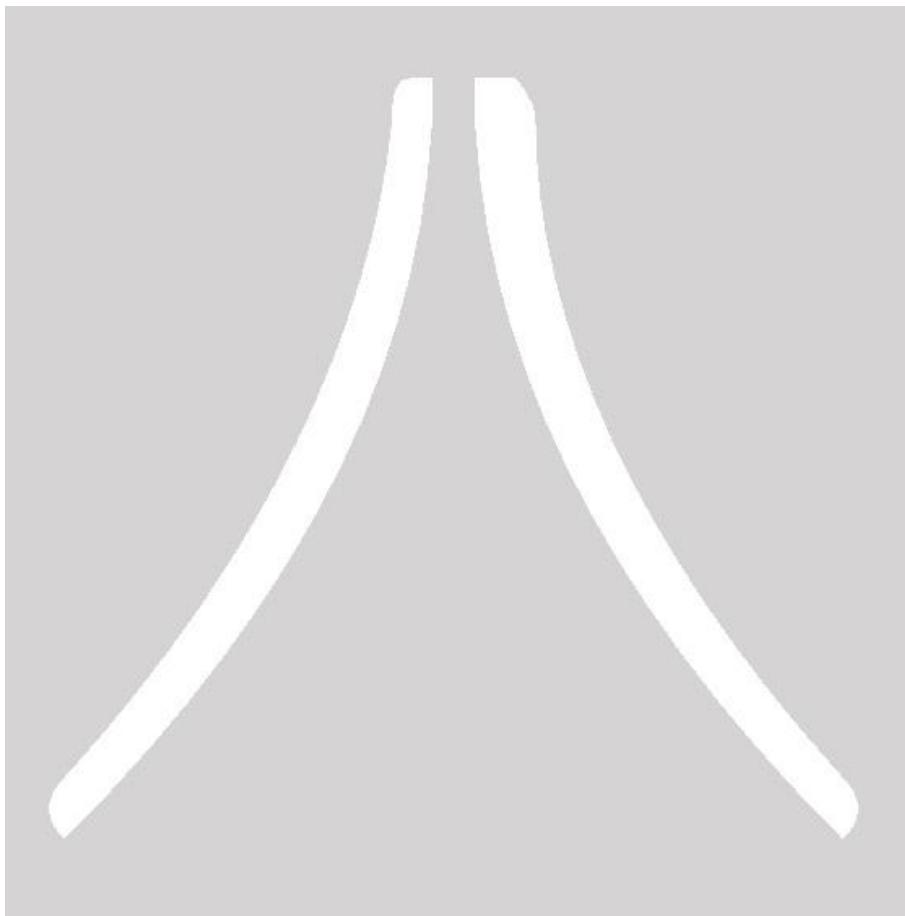
$$= \frac{1}{11!} [2^{11} - 2] = \frac{2}{11!} [2^{10} - 1] \Rightarrow k = 11$$

2. If the 6th term in the expansion of $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x \right]^8$ is 5600, then $x =$

यदि $\left[\frac{1}{x^{8/3}} + x^2 \log_{10} x \right]^8$ के प्रसार में छठवां पद 5600 है, तो $x =$

Ans. 10.00

$$\text{Sol. } T_6 = {}^8C_5 \left(\frac{1}{x^{8/3}} \right)^3 (x^2 \log_{10} x)^5 = 5600 \Rightarrow \frac{1}{x^8} x^{10} (\log_{10} x)^5 = 100 \Rightarrow x = 10$$



- 3.** The number of values of 'x' for which the fourth term in the expansion,

$$\left(5^{\frac{2}{5}} \log_5 \sqrt[4]{x+44} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1}+7}}} \right)^8 \text{ is } 336, \text{ is :}$$

'x' के मानों की संख्या होगी जिसके लिए व्यंजक $\left(5^{\frac{2}{5}} \log_5 \sqrt{4^x + 44} + \frac{1}{5^{\log_5 \sqrt[3]{2^{x-1} + 7}}} \right)^8$ में चौथा पद 336 है –

Ans. 02.00

$$\begin{aligned}
 \text{Sol. } T_4 &= {}^8C_3 \left(\frac{\frac{1}{5} \log_5(4^x + 44)}{5^5} \right)^5 \left(\frac{1}{\frac{1}{5^3} \log_5(2^{x-1} + 7)} \right)^3 \\
 &\Rightarrow {}^8C_3 (4^x + 44) \left(\frac{1}{2^{x-1} + 7} \right) = 336 \\
 \Rightarrow \frac{4^x + 44}{2^{x-1} + 7} &= 6 \quad \Rightarrow \quad 4^x + 44 = 3 \cdot 2^x + 42 \quad \Rightarrow \quad (2^x)^2 - 3 \cdot 2^x + 2 = 0 \\
 \Rightarrow (2^x - 1)(2^x - 2) &= 0 \quad \Rightarrow \quad x = 0 \text{ & } 1
 \end{aligned}$$

4. If second, third and fourth terms in the expansion of $(x + a)^n$ are 240, 720 and 1080 respectively, then ratio of last term and first term is.

यदि $(x + a)^n$ के विस्तार में दुसरा, तीसरा और चौथा पद क्रमशः 240, 720 तथा 1080 हैं तब अन्तिम पद तथा प्रथम पद का अनुपात होगा –

Ans. 07.59

Sol. $T_2 = {}^nC_1 (x)^{n-1} \cdot a = 240$ (i)

$$T_3 = {}^nC_2 (x)^{n-2} a^2 = 720 \quad \dots\dots(ii)$$

$$T_4 = {}^nC_3 (x)^{n-3} a^3 = 1080 \quad \dots\dots \text{(iii)}$$

From (i) and (ii) (i) तथा (ii) से

$$\text{Here } \frac{n C_1(x)^{n-1} a}{n C_2 x^{n-2} a^2} = \frac{2x}{(n-1)a} = \frac{240}{720} = \frac{1}{3} \Rightarrow 6x = (n-1)a$$

From (ii) and (iii) (ii) तथा (iii) से

$$9x = 2(n - 2) a$$

$$\text{On dividing भाग देने पर } \frac{3}{2} = \frac{2(n-2)}{(n-1)} \Rightarrow 3n - 3 = 4n - 8 \Rightarrow n = 5$$

$$\text{hence } \frac{T_6}{T_1} = \frac{\frac{5}{5}C_5(x^0)(a)^5}{\frac{5}{5}C_0x^5a^0} = \left(\frac{a}{x}\right)^5 = \left(\frac{3}{2}\right)^5 = \mathbf{07.59}$$

5. Let the co-efficients of x^n in $(1+x)^{2n}$ & $(1+x)^{2n-1}$ be P & Q respectively, then $\left(\frac{P+Q}{P}\right)^4 =$

मानाकि $(1+x)^{2n}$ एवं $(1+x)^{2n-1}$ के प्रसार में x^n के गुणांक क्रमशः P एवं Q है, तो $\left(\frac{P+Q}{P}\right)^4 =$

Ans. 05.06

Sol. $P = {}^{2n}C_n$ and तथा $Q = {}^{2n-1}C_n$ $\Rightarrow \frac{P}{Q} = 2$; $\left(1 + \frac{Q}{P}\right)^4 = \left(1 + \frac{1}{2}\right)^4 = \frac{81}{16}$

6. In the expansion of $\left(3^{\frac{-x}{4}} + 3^{\frac{5x}{4}}\right)^n$, the sum of the binomial coefficients is 256 and four times the term with greatest binomial coefficient exceeds the square of the third term by $21n$, then find x .

$\left(3^{\frac{-x}{4}} + 3^{\frac{5x}{4}}\right)^n$ के विस्तार में द्विपद गुणाकों का योग 256 है और अधिकतम द्विपद गुणांक का चार गुना, तीसरे पद के वर्ग से $21n$, अधिक है तब x का मान ज्ञात कीजिए।

Ans. 00.50

Sol. $2^n = 256 = 2^8$

$n = 8$

$$\left(3^{\frac{-x}{4}} + 3^{\frac{5x}{4}}\right)^8$$

$4 T_5 = T_3^2 + 21n$

$$4 \times {}^8C_4 \times 3^{\frac{-x \times 4}{4}} \times 3^{\frac{5x \times 4}{4}} = \left({}^8C_2 \times 3^{\frac{-x \times 6}{4}} \times 3^{\frac{5x \times 2}{4}} \right)^2 + 21n$$

$1120 \times 3^{\frac{x}{4}} = (28 \times 3^x)^2 + 21n$

$1120 \times 3^{\frac{x}{4}} = 28^2 \times 3^{2x} + 21 \times 8$

$$\Rightarrow x = \frac{1}{2}$$

7. If $\sum_{k=1}^{19} \frac{(-2)^k}{k!(19-k)!} = \frac{1}{k 18!}$ then find k .

यदि $\sum_{k=1}^{19} \frac{(-2)^k}{k!(19-k)!} = \frac{1}{k 18!}$ हो तो k का मान ज्ञात कीजिए।

Ans. 09.50

Sol. $\frac{1}{19!} \sum_{k=1}^{19} (-2)^k \cdot {}^{19}C_k$

$$= \frac{1}{19!} \sum_{k=1}^{19} (-1)^k \cdot 2^k \cdot {}^{19}C_k$$

$$= \frac{1}{19!} \left[-{}^{19}C_1 \cdot 2 + {}^{19}C_2 \cdot 2^2 - {}^{19}C_3 \cdot 2^3 + \dots - 2^{19} \cdot {}^{19}C_{19} \right]$$

$$= \frac{1}{19!} \left[{}^{19}C_0 - {}^{19}C_1 + {}^{19}C_2 \cdot 2^2 - \dots - 2^{19} \cdot {}^{19}C_{19} - 1 \right]$$

$$= \frac{1}{19!} \left((1-2)^{19} - 1 \right) = \frac{-2}{19!}$$

8. The value of p , for which coefficient of x^{50} in the expression

$(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1001x^{1000}$ is equal to ${}^{1002}C_p$, is :

व्यंजक $(1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1001x^{1000}$ में x^{50} का गुणांक ${}^{1002}C_p$ है, तो p का मान है

Ans. 50.00

Sol. Co-efficient of x^{50} (x^{50} का गुणांक)

$$S = (1+x)^{1000} + 2x(1+x)^{999} + 3x^2(1+x)^{998} + \dots + 1001x^{1000} \quad \dots(i)$$

$$\frac{xS}{1+x} = x(1+x)^{999} + 2x^2(1+x)^{998} \dots + 1000x^{1000} + \frac{1001 x^{1001}}{(1+x)} \dots \text{(ii)}$$

$$\frac{S}{1+x} = (1+x)^{1000} + x(1+x)^{999} + \dots + x^{1000} - \frac{1001 x^{1001}}{1+x}$$

$$\Rightarrow \frac{S}{1+x} = (1+x)^{1000} \left[\frac{1 - \left(\frac{x}{1+x} \right)^{1001}}{1 - \frac{x}{1+x}} \right] - \frac{1001 x^{1001}}{(1+x)}$$

$$\Rightarrow S = (1+x)^{1002} - x^{1001}(1+x) - 1001 x^{1001}$$

Co-efficient of $x^{50} = {}^{1002}C_{50}$ (x^{50} का गुणांक = ${}^{1002}C_{50}$)

9. If {x} denotes the fractional part of 'x', and $\left\{ \frac{3^{1001}}{82} \right\} = \frac{1}{\lambda}$ then value of λ is

यदि {x}, 'x' के मिन्नात्मक भाग को प्रदर्शित करता है, तथा $\left\{ \frac{3^{1001}}{82} \right\} = \frac{1}{\lambda}$ हो तो λ का मान होगा –

Ans. 27.33

$$\text{Sol. } \left\{ \frac{3^{1001}}{82} \right\} = \left\{ \frac{3 \cdot (82-1)^{250}}{82} \right\} = \left\{ \frac{3 \cdot [{}^{250}C_0 (82)^{250} + {}^{250}C_1 (82)^{249} (-1) + \dots + {}^{250}C_{250}]}{82} \right\} = \frac{3}{82}$$

10. The index 'n' of the binomial $\left(\frac{x}{5} + \frac{2}{5} \right)^n$ if the only 9th term of the expansion has numerically the greatest coefficient ($n \in \mathbb{N}$) then find $\frac{T_9}{T_8}$ (where T_r denote coefficient of rth term from beginning in the expansion)

यदि $\left(\frac{x}{5} + \frac{2}{5} \right)^n$, ($n \in \mathbb{N}$) के प्रसार में केवल 9वाँ पद संख्यात्मक रूप से महत्तम गुणांक वाला पद हो, तो $\frac{T_9}{T_8}$ का मान

ज्ञात कीजिए (जहाँ T_r प्रसार में प्रारम्भ से rवें पद के गुणांक को प्रदर्शित करता है)

Ans. 01.25

$$\text{Sol. For } T_9 \text{ to be the numerically greatest term, } r = \left[\frac{n+1}{1 + \left| \frac{x}{a} \right|} \right] = \left[\frac{n+1}{1 + \left| \frac{1}{2} \right|} \right] = 8$$

$$\Rightarrow 8 < \frac{2(n+1)}{3} < 9 \Rightarrow 11 < n < 12.5 \Rightarrow n = 12$$

$$\text{then } \frac{T_9}{T_8} = \frac{{}^{12}C_8 \left(\frac{1}{5} \right)^4 \left(\frac{2}{5} \right)^8}{{}^{12}C_7 \left(\frac{1}{5} \right)^5 \left(\frac{2}{5} \right)^7} = \frac{5}{4}$$

$$\text{Hindi. } \text{चूँकि } T_9 \text{ संख्यात्मक रूप से महत्तम पद है अतः } r = \left[\frac{n+1}{1 + \left| \frac{x}{a} \right|} \right] = \left[\frac{n+1}{1 + \left| \frac{1}{2} \right|} \right] = 8 \Rightarrow 8 < \frac{2(n+1)}{3} < 9$$

$$\Rightarrow 11 < n < 12.5 \Rightarrow n = 12$$

$$\text{तब } \frac{T_9}{T_8} = \frac{^{12}C_8 \left(\frac{1}{5}\right)^4 \left(\frac{2}{5}\right)^8}{^{12}C_7 \left(\frac{1}{5}\right)^5 \left(\frac{2}{5}\right)^7} = \frac{5}{4}$$

11. Sum of square of all possible values of 'r' satisfying the equation,

$$^{39}C_{3r-1} - ^{39}C_{r^2} = ^{39}C_{r^2-1} - ^{39}C_{3r} \text{ is :}$$

समीकरण $^{39}C_{3r-1} - ^{39}C_{r^2} = ^{39}C_{r^2-1} - ^{39}C_{3r}$ को संतुष्ट करने वाले 'r' के सभी संभावित मानों के वर्गों का योग होगा –

Ans. 34.00

$$^{39}C_{3r-1} - ^{39}C_{r^2} = ^{39}C_{r^2-1} - ^{39}C_{3r}$$

$$\Rightarrow ^{39}C_{3r-1} + ^{39}C_{3r} = ^{39}C_{r^2-1} + ^{39}C_{r^2}$$

$$\Rightarrow ^{40}C_{3r} = ^{40}C_{r^2} \Rightarrow \quad (i) \ r^2 = 3r \Rightarrow r = 0, 3$$

$$(ii) r^2 + 3r = 40 \Rightarrow r = 5, -8$$

possible values are 3,5

12. Find the value of

$$^6C_0 \cdot ^{12}C_6 - ^6C_1 \cdot ^{11}C_6 + ^6C_2 \cdot ^{10}C_6 - ^6C_3 \cdot ^9C_6 + ^6C_4 \cdot ^8C_6 - ^6C_5 \cdot ^7C_6 + ^6C_6 \cdot ^6C_6$$

Ans. 01.00

Sol. Coeff of $x^6 : ^6C_0 \cdot (1+x)^{12} - ^6C_1(1+x)^{11} + ^6C_2(1+x)^{10} - ^6C_3(1+x)^9$

$$+ ^6C_4(1+x)^8 - ^6C_5(1+x)^7 + ^6C_6(1+x)^6$$

$$= (1+x)^{12} \left[^6C_0 - ^6C_1 \left(\frac{1}{1+x} \right) + ^6C_2 \left(\frac{1}{1+x} \right)^2 - ^6C_3 \left(\frac{1}{1+x} \right)^3 + ^6C_4 \left(\frac{1}{1+x} \right)^4 - ^6C_5 \left(\frac{1}{1+x} \right)^5 + ^6C_6 \left(\frac{1}{1+x} \right)^6 \right]$$

$$= (1+x)^{12} \cdot (1 - \frac{1}{1+x})^6$$

$$= (1+x)^6 \cdot x^6$$

$$= 1 \times \text{coeff of } x^6 = 1$$

13. If n is a positive integer & $C_k = {}^nC_k$, find the value of $\left(\sum_{k=1}^n \frac{k^3}{n(n+1)^2 \cdot (n+2)} \left(\frac{C_k}{C_{k-1}} \right)^2 \right)$ is :

$$\text{यदि } n \text{ एक धनात्मक पूर्णांक है तथा } C_k = {}^nC_k \text{ तब } \left(\sum_{k=1}^n \frac{k^3}{n(n+1)^2 \cdot (n+2)} \left(\frac{C_k}{C_{k-1}} \right)^2 \right) \text{ का मान है--}$$

Ans. 00.08

$$\begin{aligned} \sum_{k=1}^n k^3 \left(\frac{n-k+1}{k} \right)^2 &= \sum_{k=1}^n k(n-k+1)^2 = \sum_{k=1}^n (n^2k + k^3 + k - 2nk^2 + 2nk - 2k^2) \\ &= \frac{(n+1)^2 \cdot n(n+1)}{2} + \left[\frac{n(n+1)}{2} \right]^2 - \frac{2(n+1)n(n+1)(2n+1)}{6} = \frac{n(n+1)^2(n+2)}{12} \end{aligned}$$

14. The value of the expression $\left(\sum_{r=0}^{10} {}^{10}C_r\right) \left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k}\right)$ is :

व्यंजक $\left(\sum_{r=0}^{10} {}^{10}C_r\right) \left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k}\right)$ का मान है—

Ans. 01.00

$$\text{Sol. } \left(\sum_{r=0}^{10} {}^{10}C_r\right) \left(\sum_{k=0}^{10} (-1)^k \frac{{}^{10}C_k}{2^k}\right) = ({}^{10}C_0 + \dots + {}^{10}C_{10}) \left({}^{10}C_0 - \frac{{}^{10}C_1}{2} + \frac{{}^{10}C_2}{2^2} - \dots + \frac{{}^{10}C_{10}}{2^{10}}\right) = 2^{10} \times \left(1 - \frac{1}{2}\right)^{10} = 1$$

15. The value of λ if $\sum_{m=97}^{100} {}^{100}C_m \cdot {}^mC_{97} = \lambda {}^{99}C_{96}$ is :

λ का मान होगा यदि $\sum_{m=97}^{100} {}^{100}C_m \cdot {}^mC_{97} = \lambda {}^{99}C_{96}$ है।

Ans. 08.24 or 08.25

$$\text{Sol. } \sum_{m=p}^n {}^nC_m \cdot {}^mC_p = \sum_{m=p}^n \frac{n!}{m!(n-m)!} \times \frac{m!}{p!(m-p)!} = \sum_{m=p}^n {}^nC_p \cdot {}^{n-p}C_{m-p}$$

$$= {}^nC_p [{}^{n-p}C_0 + {}^{n-p}C_1 + \dots + {}^{n-p}C_{n-p}] = {}^nC_p 2^{n-p}; \text{ where } n = 100 \text{ and } p = 97.$$

जहाँ $n = 100$ तथा $p = 97$

16. If $(1 + x + x^2 + \dots + x^p)^n = a_0 + a_1x + a_2x^2 + \dots + a_{np}x^{np}$, then the value of :

$$\frac{1}{p(p+1)^7} [a_1 + 2a_2 + 3a_3 + \dots + 7p a_{7p}] \text{ is :}$$

यदि $(1 + x + x^2 + \dots + x^p)^n = a_0 + a_1x + a_2x^2 + \dots + a_{np}x^{np}$, हो, तो $\frac{1}{p(p+1)^7} [a_1 + 2a_2 + 3a_3 + \dots + 7p a_{7p}]$ का मान है—

Ans. 03.50

$$\text{Sol. } (1 + x + x^2 + \dots + x^p)^n = a_0 + a_1x + \dots + a_{np}x^{np}$$

Differentiating both side w.r.t. x के सापेक्ष अवकलन करने पर

$$n(1 + x + x^2 + \dots + x^p)^{n-1}(1 + 2x + \dots + px^{p-1}) = a_1 + 2a_2x + \dots + np a_{np}x^{np-1}$$

Now put $x = 1$ रखने पर

$$a_1 + 2a_2 + \dots + np a_{np} = n(p+1)^{n-1}(1 + 2 + \dots + p) = \frac{n(p+1)^n \cdot p}{2}, \text{ where } n = \frac{7}{2}$$

17. If $({}^{2n}C_1)^2 + 2 \cdot ({}^{2n}C_2)^2 + 3 \cdot ({}^{2n}C_3)^2 + \dots + 2n \cdot ({}^{2n}C_{2n})^2 = 18 \cdot {}^{4n-1}C_{2n-1}$, then n is :

यदि $({}^{2n}C_1)^2 + 2 \cdot ({}^{2n}C_2)^2 + 3 \cdot ({}^{2n}C_3)^2 + \dots + 2n \cdot ({}^{2n}C_{2n})^2 = 18 \cdot {}^{4n-1}C_{2n-1}$, तब n है —

Ans. 09.00

$$\text{Sol. } \because (1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1x + \dots + {}^{2n}C_{2n}x^{2n}$$

differentiating it

$$2n(1+x)^{2n-1} = {}^{2n}C_1 + 2 \cdot {}^{2n}C_2x + \dots + 2n {}^{2n}C_{2n}x^{2n-1}$$

$$\text{Again } (x+1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} + \dots + {}^{2n}C_{2n}$$

Required expression = coefficient of x^{2n-1} in $2n(1+x)^{4n-1}$

$$= 2n \cdot {}^{4n-1}C_{2n-1}$$

$$\text{Hindi } \because (1+x)^{2n} = {}^{2n}C_0 + {}^{2n}C_1x + \dots + {}^{2n}C_{2n}x^{2n}$$

अवकलन करने पर

$$2n(1+x)^{2n-1} = {}^2nC_1 + 2 \cdot {}^2nC_2 x + \dots + 2n {}^2nC_{2n} x^{2n-1}$$

पुनः $(x+1)^{2n} = {}^2nC_0 x^{2n} + {}^2nC_1 x^{2n-1} + {}^2nC_2 x^{2n-2} + \dots + {}^2nC_{2n}$

अभीष्ट व्यंजक = $2n (1+x)^{4n-1}$ में x^{2n-1} का गुणांक
 $= 2n \cdot {}^4nC_{2n-1}$

18. If $\sum_{r=0}^n \frac{2r+3}{r+1} \cdot {}^nC_r = \frac{(2n+3k)2^n - 1}{n+1}$ then 'k' is

यदि $\sum_{r=0}^n \frac{2r+3}{r+1} \cdot {}^nC_r = \frac{(2n+3k)2^n - 1}{n+1}$ है तब 'k' का मान है

Ans. 01.33

Sol.
$$\begin{aligned} \sum_{r=0}^n \frac{2r+3}{r+1} \cdot {}^nC_r &= \sum_{r=0}^n 2 \cdot {}^nC_r + \sum_{r=0}^n \frac{1}{r+1} \cdot {}^nC_r = 2 \cdot 2^2 + \frac{1}{n+1} \cdot \sum_{r=0}^n {}^{n+1}C_{r+1} \\ &= 2^{n+1} + \frac{1}{n+1} \cdot (2^{n+1} - 1) = \frac{(n+2) \cdot 2^{n+1} - 1}{n+1} \end{aligned}$$

19. If $\sum_{r=0}^n \frac{(-1)^r \cdot C_r}{(r+1)(r+2)(r+3)} = \frac{a}{(n+b)}$, then a + b is

यदि $\sum_{r=0}^n \frac{(-1)^r \cdot C_r}{(r+1)(r+2)(r+3)} = \frac{a}{(n+b)}$ है, तब a + b का मान है

Ans. 03.50

Sol.
$$\begin{aligned} \sum_{r=0}^n \frac{(-1)^r \cdot C_r}{(r+1)(r+2)(r+3)} &= \frac{-1}{(n+1)(n+2)(n+3)} \\ &\frac{-1}{(n+1)(n+2)(n+3)} \left[(1-1)^{n+3} - \left\{ {}^{n+3}C_0 (-1)^0 + {}^{n+3}C_1 (-1)^1 + {}^{n+3}C_2 (-1)^2 \right\} \right] \\ &\frac{1}{(n+1)(n+2)(n+3)} (n+2) \times \frac{(n+1)}{2} = \frac{1}{2(n+3)} \end{aligned}$$

20. $\sum_{k=1}^{3n} {}^6nC_{2k-1} (-3)^k$ is equal to :

$\sum_{k=1}^{3n} {}^6nC_{2k-1} (-3)^k$ बराबर है

Ans. 00.00

Sol. $S = \sum_{k=1}^{3n} {}^6nC_{2k-1} (-3)^k \Rightarrow S = {}^6nC_1 (-3) + {}^6nC_3 (-3)^2 + \dots + {}^6nC_{6n-1} (-3)^{3n}$

$$\Rightarrow S = (\sqrt{3} i) \sum_{k=1}^{3n} {}^6nC_{2k-1} (\sqrt{3} i)^{2k-1} \Rightarrow S = (\sqrt{3} i) \left[\frac{(\sqrt{3} i)^{6n} - (1-\sqrt{3} i)^{6n}}{2} \right] = 0$$

21. If x is very large as compare to y , then the value of k in $\sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} = 1 + \frac{ky^2}{x^2}$

यदि x, y की तुलना में बहुत बड़ा हो, तो $\sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} = 1 + \frac{ky^2}{x^2}$ में k का मान है

Ans. 00.50

$$\text{Sol. } \sqrt{\frac{x}{x+y}} \sqrt{\frac{x}{x-y}} = \left(\frac{1}{1 + \frac{y}{x}} \right)^{1/2} \left(\frac{1}{1 - \frac{y}{x}} \right)^{1/2} = \left(1 - \frac{y^2}{x^2} \right)^{-1/2} = 1 + \frac{1}{2} \cdot \frac{y^2}{x^2} \Rightarrow k = 2$$

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

भाग - III : एक या एक से अधिक सही विकल्प प्रकार

1. In the expansion of $\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$

(A*) the number of irrational terms is 19 (B*) middle term is irrational
 (C*) the number of rational terms is 2 (D*) 9th term is rational

$$\left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$$
 के प्रसार में

(A) अपरिमेय पदों की संख्या 19 है।
 (C) परिमेय पदों की संख्या 2 है।

(B) मध्य पद अपरिमेय है।
 (D) 9वाँ पद परिमेय है।

$$\text{Sol. } \left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$$

$$T_{r+1} = {}^{20}C_r (4^{1/3})^{20-r} (6^{-1/4})^r$$

For rational terms

$$20 - r = 3k \text{ & } r = 4p, \text{ where } k, p \in I$$

$$\therefore \text{no. of rational terms} = 2$$

$$\Rightarrow r = 20 \text{ & } r = 8$$

$$\therefore \text{no. of irrational terms} = 19$$

$$\text{Hindi } \left(\sqrt[3]{4} + \frac{1}{\sqrt[4]{6}}\right)^{20}$$

$$T_{r+1} = {}^{20}C_r (4^{1/3})^{20-r} (6^{-1/4})^r$$

परिमेय पदों हेतु

$$20 - r = 3k \text{ तथा } r = 4p, \text{ जहाँ } k, p \in I$$

$$\Rightarrow r = 20 \text{ तथा } r = 8$$

$$\therefore \text{परिमेय पदों की संख्या} = 2 \quad \therefore \text{अपरिमेय पदों की संख्या} = 19$$

2. The coefficient of x^4 in $\left(\frac{1+x}{1-x}\right)^2, |x| < 1$, is

$$\left(\frac{1+x}{1-x}\right)^2, |x| < 1 \text{ में } x^4 \text{ का गुणांक है—}$$

(A) 4 (B) -4 (C*) $10 + {}^4C_2$ (D*) 16

$$\text{Sol. } (1+x)^2 (1-x)^{-2} = (1+x^2 + 2x) (1-x)^{-2}$$

$$\text{Co-efficient of } x^4 \text{ का गुणांक} = {}^5C_4 + {}^3C_2 + 2 {}^4C_3 = 16$$

3. $7^9 + 9^7$ is divisible by :

(A*) 16 (B) 24 (C*) 64 (D) 72
 $7^9 + 9^7$ विभाजित है —

- Sol.** (A) 16 से (B) 24 से (C) 64 से (D) 72 से
 $7^9 + 9^7 = (8 - 1)^9 + (8 + 1)^7 = {}^9C_0(8)^9 - {}^9C_1(8)^8 + {}^9C_2(8)^7 \dots + {}^9C_8(8) - {}^9C_9 + {}^7C_0(8)^7 + \dots + {}^7C_6(8) + {}^7C_7$
 This is divisible by 64 & 16 यह 64 और 16 से भाजित है

4. The sum of the series $\sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r (a-r)$ is equal to :

- (A*) 5 if $a = 5$ (B) -5 if $a = 5$ (C*) -5 if $a = -5$ (D) 5 if $a = -5$

श्रेणी $\sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r (a-r)$ का योगफल बराबर है

- (A*) 5 यदि $a = 5$ (B) -5 यदि $a = 5$ (C*) -5 यदि $a = -5$ (D) 5 यदि $a = -5$

Sol. $= a \sum_{r=1}^n (-1)^{r-1} \cdot {}^n C_r - \sum_{r=1}^n r \cdot {}^n C_r (-1)^{r-1} = a[{}^n C_1 - {}^n C_2 + {}^n C_3 \dots + (-1)^{n-1} \cdot {}^n C_n] - n \sum_{r=1}^n (-1)^{r-1} \cdot {}^{n-1} C_{r-1}$
 $= a(1) - n[{}^{n-1} C_0 - {}^{n-1} C_1 + \dots + (-1)^{(n-1)} \cdot {}^{n-1} C_{n-1}] = a - n(0) = a$

5. Let $a_n = \frac{1000^n}{n!}$ for $n \in N$, then a_n is greatest, when

मानाकि $n \in N$ के लिए $a_n = \frac{1000^n}{n!}$ हो, तो a_n महत्तम होगा, यदि

- (A) $n = 997$ (B) $n = 998$ (C*) $n = 999$ (D*) $n = 1000$

Sol. $a_n = \frac{(1000)(1000)\dots(1000)}{1.2\dots n}$

$a_{999} = a_{1000}$

a_n is maximum for $n = 999$ and $n = 1000$

Hindi $a_n = \frac{(1000)(1000)\dots(1000)}{1.2\dots n}$

$a_{999} = a_{1000}$

a_n , $n = 999$ तथा $n = 1000$ के लिए अधिकतम है।

6. ${}^n C_0 - 2.3 {}^n C_1 + 3.3^2 {}^n C_2 - 4.3^3 {}^n C_3 + \dots + (-1)^n (n+1) {}^n C_n 3^n$ is equal to

- (A*) $2^n \left(\frac{3n}{2} + 1 \right)$ if n is even (B) $2^n \left(n + \frac{3}{2} \right)$ if n is even

- (C*) $-2^n \left(\frac{3n}{2} + 1 \right)$ if n is odd (D) $2^n \left(n + \frac{3}{2} \right)$ if n is odd

${}^n C_0 - 2.3 {}^n C_1 + 3.3^2 {}^n C_2 - 4.3^3 {}^n C_3 + \dots + (-1)^n (n+1) {}^n C_n 3^n$ का मान बराबर है

- (A*) $2^n \left(\frac{3n}{2} + 1 \right)$ यदि n सम है। (B) $2^n \left(n + \frac{3}{2} \right)$ यदि n सम है।

- (C*) $-2^n \left(\frac{3n}{2} + 1 \right)$ यदि n विषम है। (D) $2^n \left(n + \frac{3}{2} \right)$ यदि n विषम है।

Sol. $(1+x)^n = {}^n C_0 + {}^n C_1 x + {}^n C_2 x^2 + \dots + {}^n C_n x^n$

Multiply it by x

$$x(1+x)^n = {}^n C_0 x + {}^n C_1 x^2 + {}^n C_2 x^3 + \dots + {}^n C_n x^{n+1}$$

Differentiate w.r. to x and put $x = -3$

$$n x (1+x)^{n-1} + (1+x)^n = {}^n C_0 + 2^n C_1 x + 3 {}^n C_2 x^2 + 4 {}^n C_3 x^3 + \dots + (n+1) {}^n C_n x^n$$

So answer, $-3n (-2)^{n-1} + (-2)^n$

$$= (-2)^n \left(1 + \frac{3n}{2}\right) = (-1)^n 2^n \left(\frac{3n}{2} + 1\right)$$

Hindi $(1+x)^n = {}^nC_0 + {}^nC_1 x + {}^nC_2 x^2 + \dots + {}^nC_n x^n$

x से गुणा करने पर

$x(1+x)^n = {}^nC_0 x + {}^nC_1 x^2 + {}^nC_2 x^3 + \dots + {}^nC_n x^{n+1}$

x के सापेक्ष अवकलन करके $x = -3$ रखने पर

$n x (1+x)^{n-1} + (1+x)^n = {}^nC_0 + 2^n C_1 x + 3 {}^nC_2 x^2 + 4 {}^nC_3 x^3 + \dots + (n+1) {}^nC_n x^n$

$$\text{अतः उत्तर } -3n (-2)^{n-1} + (-2)^n = (-2)^n \left(1 + \frac{3n}{2}\right) = (-1)^n 2^n \left(\frac{3n}{2} + 1\right)$$

7. Element in set of values of r for which, ${}^{18}C_{r-2} + 2. {}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ is :

r के मानों की संख्या होगी जिसके लिए ${}^{18}C_{r-2} + 2. {}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ है

Sol. ${}^{18}C_{r-2} + 2. {}^{18}C_{r-1} + {}^{18}C_r \geq {}^{20}C_{13}$ (A*) 9 (B) 5 (C*) 7 (D*) 10

or या ${}^{19}C_{r-1} + {}^{19}C_r \geq {}^{20}C_{13}$

or या ${}^{20}C_r \geq {}^{20}C_{13}$

$r = 7, 8, 9, 10, 11, 12, 13$

8. The expansion of $(3x+2)^{-1/2}$ is valid in ascending powers of x , if x lies in the interval.

$(3x+2)^{-1/2}$ का प्रसार x की बढ़ती हुई घातों के लिए वैध है यदि x अन्तराल में स्थित होगा।

(A*) $(0, 2/3)$ (B) $(-3/2, 3/2)$
(C*) $(-2/3, 2/3)$ (D) $(-\infty, -3/2) \cup (3/2, \infty)$

Sol. $(3x+2)^{-1/2}$ has infinite expansion when $\left|\frac{3x}{2}\right| < 1 \Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

Hindi. $(3x+2)^{-1/2}$ का प्रसार अनन्त पदों तक होगा यदि $\left|\frac{3x}{2}\right| < 1 \Rightarrow x \in \left(-\frac{2}{3}, \frac{2}{3}\right)$

9. If $(1+2x+3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$, then :

यदि $(1+2x+3x^2)^{10} = a_0 + a_1x + a_2x^2 + \dots + a_{20}x^{20}$ हो, तो

(A*) $a_1 = 20$ (B*) $a_2 = 210$
(C*) $a_4 = 8085$ (D) $a_{20} = 2^2 \cdot 3^7 \cdot 7$

Sol. General term व्यापक पद = $\frac{10!}{r_1! r_2! r_3!} (1)^{r_1} (2x)^{r_2} (3x^2)^{r_3}$

$a_1 = \text{Coeff. of } x$ (x का गुणांक)

$$r_2 + 2r_3 = 1 \Rightarrow r_2 = 1, r_1 = 9, r_3 = 0 \quad \therefore a_1 = \frac{10!}{1! 9!} (2)^1 = 20$$

$a_2 = \text{Coeff. of } x^2$ (x^2 का गुणांक)

$$r_2 + 2r_3 = 2 \Rightarrow r_2 = 2, r_1 = 8, r_3 = 0 \\ r_2 = 0, r_1 = 9, r_3 = 1$$

$$a_2 = \frac{10!}{2! 8!} = (2)^2 + \frac{10!}{9! 1!} (3) = 210$$

$a_4 = \text{coeff. of } x^4$ (x^4 का गुणांक)

$$r_2 + 2r_3 = 4 \Rightarrow r_2 = 4, r_1 = 6, r_3 = 0 \\ r_2 = 2, r_1 = 7, r_3 = 1 \\ r_2 = 0, r_1 = 8, r_3 = 2$$

$$a_4 = \frac{10!}{4! 6!} (2)^4 + \frac{10!}{2! 7! 1!} (2)^2 (3) + \frac{10!}{8! 2!} (3)^2 = 8085$$

$$a_{20} = 3^{10}$$

10. In the expansion of $(x + y + z)^{25}$

- (A*) every term is of the form ${}^{25}C_r \cdot {}^rC_k \cdot x^{25-r} \cdot y^{r-k} \cdot z^k$ (B*) the coefficient of $x^8 y^9 z^9$ is 0
 (C) the number of terms is 325 (D) none of these
 $(x + y + z)^{25}$ के प्रसार में
 (A) प्रत्येक पद ${}^{25}C_r \cdot {}^rC_k \cdot x^{25-r} \cdot y^{r-k} \cdot z^k$ रूप में होगा। (B) $x^8 y^9 z^9$ का गुणांक 0 है।
 (C) पदों की संख्या 325 है। (D) इनमें से कोई नहीं

Sol. $(x + y + z)^{25}$

$$\text{General term व्यापक पद} = \frac{25!}{r_1! r_2! r_3!} x^{r_1} y^{r_2} z^{r_3}$$

Putting $r_3 = k$, $r_2 = r - k$ and तथा $r_1 = 25 - r$ रखने पर

$$= \frac{25!}{(25-r)!(r-k)!(k)!} \times \frac{r!}{r!} \times x^{25-r} y^{r-k} z^k = {}^{25}C_r \cdot {}^rC_k \cdot x^{25-r} y^{r-k} z^k$$

$$r_1 + r_2 + r_3 = 25$$

$$\therefore \text{coefficient of } x^8 y^9 z^9 \text{ is } 0 \quad \therefore \quad x^8 y^9 z^9 \text{ का गुणांक} = 0$$

11. If $(1 + x + 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + \dots + a_{40} x^{40}$, then $a_0 + a_2 + a_4 + \dots + a_{38}$ is equal to :

यदि $(1 + x + 2x^2)^{20} = a_0 + a_1 x + a_2 x^2 + \dots + a_{40} x^{40}$ हो, तो $a_0 + a_2 + a_4 + \dots + a_{38}$ बराबर है –

- (A) $2^{19} (2^{30} + 1)$ (B*) $2^{19} (2^{20} - 1)$ (C*) $2^{39} - 2^{19}$ (D) $2^{39} + 2^{19}$
Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1 x + \dots + a_{40} x^{40}$

$$x = 1, \text{ then } a_0 + a_1 + \dots + a_{40} = 4^{20}$$

$$x = -1, \text{ then } a_0 - a_1 + a_2 - \dots + a_{40} = 2^{20}$$

$$2^{20} + 2^{40} = 2[a_0 + a_2 + \dots + a_{38} + a_{40}]$$

$$\Rightarrow a_0 + a_2 + \dots + a_{38} = 2^{19} + 2^{39} - 2^{20} = 2^{19} (2^{20} - 1) \quad \therefore a_{40} = a^{20}$$

Hindi. $(1 + x + 2x^2)^{20} = a_0 + a_1 x + \dots + a_{40} x^{40}$

$$x = 1, \text{ तो } a_0 + a_1 + \dots + a_{40} = 4^{20}$$

$$x = -1, \text{ तो } a_0 - a_1 + a_2 - \dots + a_{40} = 2^{20}$$

$$2^{20} + 2^{40} = 2[a_0 + a_2 + \dots + a_{38} + a_{40}]$$

$$\Rightarrow a_0 + a_2 + \dots + a_{38} = 2^{19} + 2^{39} - 2^{20} = 2^{19} (2^{20} - 1) \quad \therefore a_{40} = a^{20}$$

12. $n^n \left(\frac{n+1}{2}\right)^{2n}$ is ($n \in \mathbb{N}$)

(A) Less than $\left(\frac{n+1}{2}\right)^3$

(B*) Greater than or equal to $\left(\frac{n+1}{2}\right)^3$

(C) Less than $(n!)^3$

(D*) Greater than or equal to $(n!)^3$.

$n^n \left(\frac{n+1}{2}\right)^{2n}$ है – ($n \in \mathbb{N}$)

(A) $\left(\frac{n+1}{2}\right)^3$ से छोटा

(B*) $\left(\frac{n+1}{2}\right)^3$ से बड़ा या बराबर

(C) $(n!)^3$ से छोटा

(D*) $(n!)^3$ से बड़ा या बराबर

Sol. $n^n \left(\frac{n+1}{2}\right)^{2n} = \left(\frac{\left(n(n+1)\right)^2}{n}\right)^n = \left(\frac{1^3 + 2^3 + \dots + n^3}{n}\right)^n ; \frac{1^3 + 2^3 + \dots + n^3}{n} \geq \sqrt[n]{(n!)^3}$

13. If recursion polynomials $P_k(x)$ are defined as $P_1(x) = (x - 2)^2$, $P_2(x) = ((x - 2)^2 - 2)^2$
 $P_3(x) = ((x - 2)^2 - 2)^2 - 2^2$ (In general $P_k(x) = (P_{k-1}(x) - 2)^2$, then the constant term in $P_k(x)$ is
(A*) 4 (B) 2 (C) 16 (D*) a perfect square
यदि व्यंजक $P_k(x)$ इस तरह से परिभाषित है कि $P_1(x) = (x - 2)^2$, $P_2(x) = ((x - 2)^2 - 2)^2$
 $P_3(x) = ((x - 2)^2 - 2)^2 - 2^2$ (व्यापक रूप से $P_k(x) = (P_{k-1}(x) - 2)^2$), तो $P_k(x)$ में अचर पद है—
(A*) 4 (B) 2 (C) 16 (D*) एक पूर्ण वर्ग

Sol. Constant term in $P_1(x)$ is 4

If the constant term in $P_k(x)$ is also 4, then

$$P_k(x) = 4 + a_1x + a_2x^2 + \dots$$

$$\text{and } P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + \dots + 2)^2$$

Hindi $P_1(x)$ में नियत पद 4 है।

$P_k(x)$ में भी नियत पद 4 है, तो

$$P_k(x) = 4 + a_1x + a_2x^2 + \dots$$

$$\text{और } P_{k+1}(x) = (P_k(x) - 2)^2 = (a_1x + a_2x^2 + \dots + 2)^2$$

PART - IV : COMPREHENSION

भाग - IV : अनुच्छेद (COMPREHENSION)

Comprehension # 1 (Q. No. 1 to 3)

Consider, sum of the series $\sum_{0 \leq i < j \leq n} f(i)f(j)$

In the given summation, i and j are not independent.

In the sum of series $\sum_{i=1}^n \sum_{j=1}^n f(i)f(j) = \sum_{i=1}^n \left(f(i) \left(\sum_{j=1}^n f(j) \right) \right)$ i and j are independent. In this summation,

three types of terms occur, those when $i < j$, $i > j$ and $i = j$.

Also, sum of terms when $i < j$ is equal to the sum of the terms when $i > j$ if $f(i)$ and $f(j)$ are symmetrical.

So, in that case

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n f(i)f(j) &= \sum_{0 \leq i < j \leq n} f(i)f(j) \\ &\quad + \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ &= 2 \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ &\Rightarrow \sum_{0 \leq i < j \leq n} f(i)f(j) = \frac{\sum_{i=0}^n \sum_{j=0}^n f(i)f(j) - \sum_{i=j} f(i)f(j)}{2} \end{aligned}$$

When $f(i)$ and $f(j)$ are not symmetrical, we find the sum by listing all the terms.

अनुच्छेद # 1 (Q. No. 1 to 3)

माना कि श्रेणियों का योगफल $\sum_{0 \leq i < j \leq n} f(i)f(j)$ सूत्र से दिया जाता है जहाँ i तथा j स्वतंत्र नहीं हैं।

श्रेणियों के योगफल $\sum_{i=1}^n \sum_{j=1}^n f(i)f(j) = \sum_{i=1}^n \left(f(i) \left(\sum_{j=1}^n f(j) \right) \right)$ में i व j स्वतंत्र हैं। इस योगफल में तीन प्रकार के पद होते हैं।

जिनमें $i < j$, $i > j$ तथा $i = j$ तथा जब $i < j$ के लिए पदों का योगफल, $i > j$ के लिए पदों के योगफल के बराबर हैं। यदि $f(i) = f(j)$ तथा $f(j) = f(i)$ समित हैं। इस स्थिति में

$$\begin{aligned} \sum_{i=1}^n \sum_{j=1}^n f(i)f(j) &= \sum_{0 \leq i < j \leq n} f(i)f(j) \\ &\quad + \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ &= 2 \sum_{0 \leq i < j \leq n} f(i)f(j) + \sum_{i=j} f(i)f(j) \\ \Rightarrow \sum_{0 \leq i < j \leq n} f(i)f(j) &= \frac{\sum_{i=0}^n \sum_{j=0}^n f(i)f(j) - \sum_{i=j} f(i)f(j)}{2} \end{aligned}$$

जब $f(i) = f(j)$ समित नहीं है। तब हम सभी पदों का योगफल ज्ञात करते हैं।

1. $\sum_{0 \leq i < j \leq n} {}^n C_i \cdot {}^n C_j$ is equal to

$\sum_{0 \leq i < j \leq n} {}^n C_i \cdot {}^n C_j$ का मान बराबर है—

- (A*) $\frac{2^{2n} - {}^n C_n}{2}$ (B) $\frac{2^{2n} + {}^n C_n}{2}$ (C) $\frac{2^{2n} - {}^n C_n}{2}$ (D) $\frac{2^{2n} + {}^n C_n}{2}$

- Sol. $\sum_{0 \leq i < j \leq n} {}^n C_i \cdot {}^n C_j$

$$= \frac{\left(\sum_{i=0}^n \sum_{j=0}^n {}^n C_i \cdot {}^n C_j \right) - \sum_{i=0}^n ({}^n C_i)^2}{2} = \frac{\left(\sum_{i=0}^n {}^n C_i \cdot 2^n \right) - \sum_{i=0}^n ({}^n C_i)^2}{2} = \frac{2^n 2^n - \sum_{i=0}^n ({}^n C_i)^2}{2} = \frac{2^{2n} - {}^n C_n}{2}$$

2. Let ${}^0 C_0 = 1$, then $\sum_{m=0}^n \sum_{p=0}^m {}^n C_m \cdot {}^m C_p$ is equal to

माना ${}^0 C_0 = 1$, तब $\sum_{m=0}^n \sum_{p=0}^m {}^n C_m \cdot {}^m C_p$ का मान बराबर है—

- (A) $2^n - 1$ (B*) 3^n (C) $3^n - 1$ (D) 2^n

- Sol. $\sum_{m=0}^n \sum_{p=0}^m {}^n C_m \cdot {}^m C_p = \sum_{m=0}^n {}^n C_m \left(\sum_{p=0}^m {}^m C_p \right) = \sum_{m=0}^n {}^n C_m (2^m) = 3^n$

3. $\sum_{0 \leq i < j \leq n} ({}^n C_i + {}^n C_j)$

- (A*) $(n+2)2^n$ (B) $(n+1)2^n$ (C) $(n-1)2^n$ (D) $(n+1)2^{n-1}$

$$\begin{aligned}
 \text{Sol. } \sum_{0 \leq i \leq j \leq n} ({}^n C_i + {}^n C_j) &= \frac{\left(\sum_{i=0}^n \sum_{j=0}^n ({}^n C_i + {}^n C_j) \right) - \sum_{i=0}^n 2^n C_i}{2} = \frac{\left(\sum_{i=0}^n \left(\sum_{j=0}^n {}^n C_i + \sum_{j=0}^n {}^n C_j \right) \right) - 2 \times 2^n}{2} \\
 &= \frac{\left(\sum_{i=0}^n \left({}^n C_i + \sum_{j=0}^n 1 + 2^n \right) \right) - 2^{n+1}}{2} = \frac{\left(\sum_{i=0}^n ({}^n C_i (n+1) + 2^n) \right) - 2^{n+1}}{2} \\
 &= \frac{(n+1) \sum_{i=1}^n {}^n C_i + 2^n \sum_{i=0}^n 1 - 2^{n+1}}{2} = \frac{(n+1)2^n + 2^n(n+1) - 2^{n+1}}{2} \\
 &= (n+1)2^n - 2^n = n2^n
 \end{aligned}$$

Comprehension # 2 (Q. No. 4 to 6)

अनुच्छेद # 2 (Q. No. 4 to 6)

[Revision Planner]

Let P be a product given by $P = (x + a_1)(x + a_2) \dots (x + a_n)$

and Let $S_1 = a_1 + a_2 + \dots + a_n = \sum_{i=1}^n a_i$, $S_2 = \sum_{i < j} a_i a_j$, $S_3 = \sum_{i < j < k} a_i a_j a_k$ and so on,

then it can be shown that

$$P = x^n + S_1 x^{n-1} + S_2 x^{n-2} + \dots + S_n.$$

यदि एक गुणनफल P इस प्रकार है, $P = (x + a_1)(x + a_2) \dots (x + a_n)$

तथा माना $\circ S_1 = a_1 + a_2 + \dots + a_n = \sum_{i=1}^n a_i$, $S_2 = \sum_{i < j} a_i a_j$, $S_3 = \sum_{i < j < k} a_i a_j a_k$ इसी प्रकार आगे,

तो

निम्न को सिद्ध किया जा सकता है –

$$P = x^n + S_1 x^{n-1} + S_2 x^{n-2} + \dots + S_n.$$

- 4.** The coefficient of x^8 in the expression $(2+x)^2(3+x)^3(4+x)^4$ must be

व्यंजक $(2+x)^2(3+x)^3(4+x)^4$ में x^8 का गुणांक है –

$$(A) 26 \quad (B) 27 \quad (C) 28 \quad (D^*) 29$$

$$\begin{aligned}
 \text{Sol. } \text{The expression } (2+x)^2(3+x)^3(4+x)^4 &= (x+2)(x+2)(x+3)(x+3)(x+4)(x+4)(x+4) \\
 &= x^9 + (2+2+3+3+4+4+4+4)x^8 + \dots
 \end{aligned}$$

\Rightarrow Co-efficient of $x^8 = 29$

- Hindi.** व्यंजक $(2+x)^2(3+x)^3(4+x)^4 = (x+2)(x+2)(x+3)(x+3)(x+4)(x+4)(x+4)(x+4)$

$$= x^9 + (2+2+3+3+4+4+4+4)x^8 + \dots$$

$\Rightarrow x^8$ का गुणांक = 29

- 5.** The coefficient of x^{203} in the expression $(x-1)(x^2-2)(x^3-3) \dots (x^{20}-20)$ must be

व्यंजक $(x-1)(x^2-2)(x^3-3) \dots (x^{20}-20)$ में x^{203} का गुणांक है –

$$(A) 11 \quad (B) 12 \quad (C^*) 13 \quad (D) 15$$

$$\text{Sol. Expression} = x \cdot x^2 \cdot x^3 \dots x^{20} \left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \dots \left(1 - \frac{20}{x^{20}}\right)$$

$$\text{Let } E = \left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \dots \left(1 - \frac{20}{x^{20}}\right)$$

Now Co-efficient of x^{203} in original expression

\Rightarrow Co-efficient of x^{-7} in E.
But

$$E = 1 - \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \dots \right) + \left(\frac{1}{x} \cdot \frac{6}{x^6} + \frac{2}{x^2} \cdot \frac{5}{x^5} + \frac{3}{x^3} \cdot \frac{4}{x^4} + \dots \right) - \left(\frac{1}{x} \cdot \frac{2}{x^2} \cdot \frac{4}{x^4} + \dots \right)$$

= Co-efficient of $x^{-7} = -7 + 6 + 10 + 12 - 8 = 13$

Hindi. व्यंजक = $x \cdot x^2 \cdot x^3 \cdots \cdots \cdots x^{20} \left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \cdots \cdots \cdots \left(1 - \frac{20}{x^{20}}\right)$

माना $E = \left(1 - \frac{1}{x}\right) \left(1 - \frac{2}{x^2}\right) \left(1 - \frac{3}{x^3}\right) \cdots \cdots \cdots \left(1 - \frac{20}{x^{20}}\right)$

अब मूल व्यंजक में x^{203} का गुणांक = E में x^{-7} का गुणांक परन्तु

$$E = 1 - \left(\frac{1}{x} + \frac{2}{x^2} + \frac{3}{x^3} + \dots \right) + \left(\frac{1}{x} \cdot \frac{6}{x^6} + \frac{2}{x^2} \cdot \frac{5}{x^5} + \frac{3}{x^3} \cdot \frac{4}{x^4} + \dots \right) - \left(\frac{1}{x} \cdot \frac{2}{x^2} \cdot \frac{4}{x^4} + \dots \right)$$

$$\Rightarrow x^{-7} \text{ का गुणांक} = -7 + 6 + 10 + 12 - 8 = 13$$

6. The coefficient of x^{98} in the expression of $(x - 1)(x - 2) \dots (x - 100)$ must be

- (A) $1^2 + 2^2 + 3^2 + \dots + 100^2$
 (B) $(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)$
 (C*) $\frac{1}{2} [(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)]$

(D) None of these

(B) None of these
 व्यंजक $(x - 1)(x - 2)$ $(x - 100)$ में x^{98} का गणांक है—

- (A) $1^2 + 2^2 + 3^2 + \dots + 100^2$
 (B) $(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)$

(D) $\frac{1}{2}[(1 + 2 + 3 +$

- Sol.** The Co-efficient of $x^{98} = (1.2 + 2.3 + \dots + 99.100)$
 = Sum of product of first 100 natural numbers taken two at a time

$$= \frac{1}{2} [(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)]$$

Hindi. x^{98} का गुणांक = $(1.2 + 2.3 + \dots + 99.100)$
 = दो - दो एक साथ लेने पर प्रथम 100 प्राकृत संख्याओं के गुणनफलों का योगफल
 $= \frac{1}{2} [(1 + 2 + 3 + \dots + 100)^2 - (1^2 + 2^2 + 3^2 + \dots + 100^2)]$

Comprehension # 3 (Q.No. 7 to 9)

where I & f are its integral and fractional parts respectively.

It means $0 < f < 1$

$$\text{Now, } 0 < 7 - 4\sqrt{3} < 1 \Rightarrow 0 < (7 - 4\sqrt{3})^n < 1$$

$$\text{Let } (7 - 4\sqrt{3})^n = f' = {}^nC_0 \cdot 7^n - {}^nC_1 \cdot 7^{n-1} \cdot (4\sqrt{3})^1 + \dots \quad \dots \dots \dots \text{ (ii)}$$

$$\Rightarrow 0 < f' < 1$$

Adding (i) and (ii) (so that irrational terms cancelled out)

$$I + f + f' = (7 + 4\sqrt{3})^n + (7 - 4\sqrt{3})^n \\ = 2 [{}^nC_0 7^n + {}^nC_2 7^{n-2} (4\sqrt{3})^2 + \dots]$$

$I + f + f' = \text{even integer} \Rightarrow (f + f' \text{ must be an integer})$

$$0 < f + f' < 2 \quad \Rightarrow \quad f + f' = 1$$

with help of above analysis answer the following questions

अनुच्छेद # 3 (Q.No. 7 to 9)

$$\text{माना } (7 + 4\sqrt{3})^n = I + f = {}^nC_0 \cdot 7^n + {}^nC_1 \cdot 7^{n-1} \cdot (4\sqrt{3})^1 + \dots \dots \dots \quad \text{(i)}$$

जहाँ I तथा f इसके पूर्णक व मिन्नात्मक भाग हैं

अर्थात् $0 < f < 1$

$$\text{अब, } 0 < 7 - 4\sqrt{3} < 1 \Rightarrow 0 < (7 - 4\sqrt{3})^n < 1$$

$$\text{मानाकि } (7 - 4\sqrt{3})^n = f' = {}^nC_0 \cdot 7^n - {}^nC_1 \cdot 7^{n-1} \cdot (4\sqrt{3})^1 + \dots \dots \dots \quad \text{(ii)}$$

$$\Rightarrow 0 < f' < 1$$

(i) व (ii) का योग करने पर इससे अपरिमेय पद निरस्त हो जायेगें

$$I + f + f' = (7 + 4\sqrt{3})^n + (7 - 4\sqrt{3})^n$$

$$= 2 [{}^nC_0 \cdot 7^n + {}^nC_2 \cdot 7^{n-2} (4\sqrt{3})^2 + \dots \dots \dots]$$

$I + f + f'$ = समपूर्णक $\Rightarrow (f + f')$ एक पूर्णक होना चाहिए

$$0 < f + f' < 2 \Rightarrow f + f' = 1$$

उपरोक्त विश्लेषण के आधार पर निम्न प्रश्नों के उत्तर दीजिए।

7. If $(3\sqrt{3} + 5)^n = p + f$, where p is an integer and f is a proper fraction, then find the value of

$$(3\sqrt{3} - 5)^n, n \in \mathbb{N}, \text{ is}$$

(A*) $1 - f$, if n is even (B) f , if n is even (C) $1 - f$, if n is odd (D*) f , if n is odd

यदि $(3\sqrt{3} + 5)^n = p + f$, जहाँ p पूर्णक है और f मिन्नात्मक भाग है, तो $(3\sqrt{3} - 5)^n, n \in \mathbb{N}$ का मान है।

(A*) $1 - f$, यदि n सम है।

(B) f , यदि n सम है।

(C) $1 - f$, यदि n विषम है।

(D*) f , यदि n विषम है।

$$p + f = (3\sqrt{3} + 5)^n = {}^nC_0 (3\sqrt{3})^n 5^0 + {}^nC_1 (3\sqrt{3})^{n-1} 5^1 + \dots$$

$$f' = (3\sqrt{3} - 5)^n = {}^nC_0 (3\sqrt{3})^n 5^0 - {}^nC_1 (3\sqrt{3})^{n-1} 5^1 + \dots$$

$$p + f + f' = 2 [{}^nC_0 (3\sqrt{3})^n + {}^nC_2 (3\sqrt{3})^{n-2} 5^2 + \dots]$$

$$\Rightarrow p + f + f' = \text{even integer सम पूर्णक} \quad (\text{if } n \text{ is even}) \quad (\text{यदि } n \text{ सम है})$$

$$\Rightarrow f + f' = 1 \Rightarrow f' = 1 - f$$

$$p + f - f' = 2 [{}^nC_1 (3\sqrt{3})^{n-1} (5) + {}^nC_3 (3\sqrt{3})^{n-3} 5^3 + \dots] \quad (\text{if } n \text{ is odd}) \quad (\text{यदि } n \text{ विषम है})$$

$$\Rightarrow f - f' = 0 \Rightarrow f' = f$$

8. If $(9 + \sqrt{80})^n = I + f$, where I, n are integers and $0 < f < 1$, then :

(A*) I is an odd integer

(B) I is an even integer

$$(C*) (I + f)(1 - f) = 1$$

$$(D*) 1 - f = (9 - \sqrt{80})^n$$

यदि $(9 + \sqrt{80})^n = I + f$ जहाँ I, n पूर्णक हैं और $0 < f < 1$, तो –

(A) I एक विषम पूर्णक है।

(B) I एक सम पूर्णक है।

$$(C) (I + f)(1 - f) = 1$$

$$(D) 1 - f = (9 - \sqrt{80})^n$$

$$(9 + \sqrt{80})^n = I + f$$

$$(9 - \sqrt{80})^n = f'$$

$$2[{}^nC_0 (9)^n + {}^nC_2 (9)^{n-2} (\sqrt{80})^2 + \dots] = I + f + f'$$

Hindi coefficient of $t^{24} = {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$
 $(1+t^2)^{12} (1+t^{12}+t^{24}+t^{36}) = (1+t^{12}+t^{24}) (1+t^2)^{12}$
 t^{24} का गुणांक $= {}^{12}C_{12} + {}^{12}C_6 + {}^{12}C_0 = {}^{12}C_6 + 2$

2. Prove that $2^k \binom{n}{0} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n}{2} \binom{n-2}{k-2} - \dots + (-1)^k \binom{n}{k} \binom{n-k}{0} = \binom{n}{k}$.

[IIT-JEE-2003, Main, (2, 0), 60]

सिद्ध कीजिए कि $2^k \binom{n}{0} - 2^{k-1} \binom{n}{1} \binom{n-1}{k-1} + 2^{k-2} \binom{n}{2} \binom{n-2}{k-2} - \dots + (-1)^k \binom{n}{k} \binom{n-k}{0} = \binom{n}{k}$.

Sol. $S = 2^k {}^n C_0 - 2^{k-1} {}^n C_1 {}^{n-1} C_{k-1} + 2^{k-2} {}^n C_2 {}^{n-2} C_{k-2} + \dots$

$$\Rightarrow S = \sum_{r=0}^k (-1)^r {}^n C_r {}^{n-r} C_{k-r} \cdot 2^{k-r} \Rightarrow S = \sum_{r=0}^k (-1)^r \frac{n!}{r!(n-r)!} \times \frac{(n-r)!!}{(n-k)!(k-r)!} \\ = \sum_{r=0}^k (-1)^r 2^{k-r} \frac{n!}{k! (n-k)!} \times \frac{k!}{r! (k-r)!} = 2^k {}^n C_k \left(1 - \frac{1}{2}\right)^k = {}^n C_k$$

3. If ${}^{(n-1)}C_r = (k^2 - 3) {}^n C_{r+1}$, then an interval in which k lies is [IIT-JEE-2004, Scr, (3, -1), 84]

यदि ${}^{(n-1)}C_r = (k^2 - 3) {}^n C_{r+1}$ हो, तो k के मान का अन्तराल है – [Scr, (3, -1), 84]

- (A) $(2, \infty)$ (B) $(-\infty, -2)$ (C) $[-\sqrt{3}, \sqrt{3}]$ (D*) $(\sqrt{3}, 2]$

Sol. ${}^{(n-1)}C_r = (k^2 - 3) {}^n C_{r+1}$

or या ${}^{(n-1)}C_{n-(r+1)} = (k^2 - 3) {}^n C_{n-(r+1)}$

$$1 \geq k^2 - 3 > 0 \Rightarrow k \in [-2, -\sqrt{3}] \cup (\sqrt{3}, 2]$$

4. The value of

[IIT-JEE-2005, Scr, (3, -1), 84]

$$\binom{30}{0} \binom{30}{10} - \binom{30}{1} \binom{30}{11} + \binom{30}{2} \binom{30}{12} - \dots + \binom{30}{20} \binom{30}{30} \text{ is :}$$

- (A) $\binom{60}{20}$ (B*) $\binom{30}{10}$ (C) $\binom{30}{15}$ (D) None of these

$$\binom{30}{0} \binom{30}{10} - \binom{30}{1} \binom{30}{11} + \binom{30}{2} \binom{30}{12} - \dots + \binom{30}{20} \binom{30}{30} \text{ का मान है – } [Scr, (3, -1), 84]$$

- (A) $\binom{60}{20}$ (B) $\binom{30}{10}$ (C) $\binom{30}{15}$ (D) इनमें से कोई नहीं

Sol. $S = {}^{30}C_0 - {}^{30}C_{20} - {}^{30}C_1 + {}^{30}C_2 - {}^{30}C_{18} \dots$

$S = \text{Co-efficient of } x^{20} \text{ in } (1-x)^{30} (1+x)^{30}$

$S = \text{Co-efficient of } x^{20} \text{ in } (1-x^2)^{30} = {}^{30}C_{10}$

Hindi. $S = {}^{30}C_0 - {}^{30}C_{20} - {}^{30}C_1 + {}^{30}C_2 - {}^{30}C_{18} \dots$

$S = (1-x)^{30} (1+x)^{30}$ में x^{20} का गुणांक

$S = (1-x^2)^{30}$ में x^{20} का गुणांक $= {}^{30}C_{10}$

5. For $r = 0, 1, \dots, 10$, let A_r, B_r and C_r denote, respectively, the coefficient of x^r in the expansions of

$(1+x)^{10}, (1+x)^{20}$ and $(1+x)^{30}$. Then $\sum_{r=1}^{10} A_r (B_{10} B_r - C_{10} A_r)$ is equal to

माना कि $r = 0, 1, \dots, 10$ के लिए A_r, B_r तथा C_r क्रमशः $(1+x)^{10}, (1+x)^{20}$ तथा $(1+x)^{30}$ के प्रसार में x^r के गुणांक हैं।

तो $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ का मान निम्न है

[IIT-JEE 2010, Paper-2, (5, -2)/79]

(A) $B_{10} - C_{10}$

(B) $A_{10}(B_{10}^2 - C_{10}A_{10})$

(C) 0

(D*) $C_{10} - B_{10}$

Sol. $B_{10} \sum_{r=1}^{10} A_r B_r - C_{10} \sum_{r=1}^{10} (A_r)^2 = {}^{20}B_{10}({}^{30}C_{20} - 1) - {}^{30}C_{10}({}^{20}C_{10} - 1) = {}^{30}C_{10} - {}^{20}C_{10} = C_{10} - B_{10}$

6. The coefficients of three consecutive terms of $(1+x)^{n+5}$ are in the ratio 5 : 10 : 14. Then $n = (1+x)^{n+5}$ के तीन क्रमागत पदों के गुणांक 5 : 10 : 14 के अनुपात में हैं, तब $n =$

Ans. 6

[JEE (Advanced) 2013, Paper-1, (4, -1)/60]

Sol. ${}^{n+5}C_{r-1} : {}^{n+5}C_r : {}^{n+5}C_{r+1} = 5 : 10 : 14$

$$\Rightarrow \frac{{}^{n+5}C_r}{{}^{n+5}C_{r-1}} = \frac{10}{5} \quad \& \quad \frac{{}^{n+5}C_{r+1}}{{}^{n+5}C_r} = \frac{14}{10}$$

$$\Rightarrow \frac{(n+5)-r+1}{r} = 2 \quad \& \quad \frac{(n+5)-(r+1)+1}{r+1} = \frac{7}{5}$$

$$\Rightarrow \frac{n+6}{r} = 3 \quad \& \quad \frac{n+6}{r+1} = \frac{12}{5} \Rightarrow 3r = \frac{12}{5}(r+1) \Rightarrow r = 4$$

$$\therefore n+6 = 12 \Rightarrow n = 6$$

7. Coefficient of x^{11} in the expansion of $(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ is

[JEE (Advanced) 2014, Paper-2, (3, -1)/60]

$(1+x^2)^4(1+x^3)^7(1+x^4)^{12}$ विस्तार में (expansion) x^{11} का गुणांक (coefficient) है—

[JEE (Advanced) 2014, Paper-2, (3, -1)/60]

Ans. (A) 1051
(C) 1113

(B) 1106

(C) 1113
(D) 1120

Sol. Coefficient of $x^{11} \equiv \frac{(1+x^2)^4(1+x^3)^7(1+x^4)^{12}(1-x^2)^4}{(1-x^2)^4}$

$$\begin{aligned} \text{Coefficient of } x^{11} &\equiv (1-x^8)^4(1+x^4)^8(1+x^3)^7(1-x^2)^{-4} \\ &= (1-4x^8)(1+x^4)^8(7x^3 + 35x^9)(1-x^2)^{-4} \\ &= (7x^3 + 35x^9 - 28x^{11})(1+x^4)^8(1-x^2)^{-4} \end{aligned}$$

$$\begin{aligned} \text{Coefficient of } x^8 &= (7x + 35x^6 - 28x^8)(1+8x^4+28x^8)(1-x^2)^{-4} \\ &= (7+35x^6-28x^8+56x^4+196x^8)(1-x^2)^{-4} \end{aligned}$$

$$\begin{aligned} \text{Coefficient of } t^4 &\equiv (7+56t^2+35t^3+168t^4)(1-t)^{-4} \\ &= 7 \cdot {}^7C_3 + 56 \cdot {}^5C_3 + 35 \cdot {}^4C_3 + 168 \\ &= 245 + 700 + 168 = 1113. \end{aligned}$$

Hindi $\frac{(1+x^2)^4(1+x^3)^7(1+x^4)^{12}(1-x^2)^4}{(1-x^2)^4}$ में x^{11} का गुणांक

$$\begin{aligned} 1-x^8)^4(1+x^4)^8(1+x^3)^7(1-x^2)^{-4} \text{ में } x^{11} \text{ का गुणांक} \\ &= (1-4x^8)(1+x^4)^8(7x^3 + 35x^9)(1-x^2)^{-4} \\ &= (7x^3 + 35x^9 - 28x^{11})(1+x^4)^8(1-x^2)^{-4} \\ (7x + 35x^6 - 28x^8)(1+8x^4+28x^8)(1-x^2)^{-4} \text{ में } x^8 \text{ का गुणांक} \\ &= (7+35x^6-28x^8+56x^4+196x^8)(1-x^2)^{-4} \\ (7+56t^2+35t^3+168t^4)(1-t)^{-4} \text{ में } t^4 \text{ का गुणांक} \\ &= 7 \cdot {}^7C_3 + 56 \cdot {}^5C_3 + 35 \cdot {}^4C_3 + 168 \\ &= 245 + 700 + 168 = 1113. \end{aligned}$$

Alternative : वैकल्पिक हल

$$2x + 3y + 4z = 11$$

$$(x, y, z) = (0, 1, 2) {}^4C_0 \times {}^7C_1 \times {}^{12}C_2$$

$$(1, 3, 0) {}^4C_1 \times {}^7C_3$$

$$(2, 1, 1) {}^4C_2 \times {}^7C_1 \times {}^{12}C_1$$

$$(4, 1, 0) {}^7C_1$$

$$\text{coefficient of } x^{11} \text{ का गुणांक} = 66 \times 7 + 35 \times 4 + 42 \times 12 + 7$$

$$= 1113. \text{ Ans.}$$

8. The coefficient of x^9 in the expansion of $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ is (Moderate)
 $(1+x)(1+x^2)(1+x^3)\dots(1+x^{100})$ के विस्तार में x^9 के गुणांक का मान है
[JEE (Advanced) 2015, P-2 (4, 0) / 80]

Ans. 8

Sol. 9 = (0, 9), (1, 8), (2, 7), (3, 6), (4, 5) # 5 cases
9 = (1, 2, 6), (1, 3, 5), (2, 3, 4) # 3 cases
total = 8

9. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+mx)^{50}$ is $(3n+1) {}^{51}C_3$ for some positive integer n . Then the value of n is

[JEE (Advanced) 2016, Paper-1, (3, 0)/62]

माना कि m ऐसा न्यूनतम धनात्मक पूर्णांक (smallest positive integer) है कि

$(1+x)^2 + (1+x)^3 + \dots + (1+x)^{49} + (1+mx)^{50}$ के विस्तार में x^2 का गुणांक $(3n+1) {}^{51}C_3$ किसी धनात्मक पूर्णांक n के लिए है। तब n का मान है—

Ans. 5

Sol. Coeff. x^2 का गुणांक

$${}^2C_2 + {}^3C_2 + {}^4C_2 + \dots + {}^{49}C_2 + {}^{50}C_2 m^2 = (3n+1) {}^{51}C_3$$

$${}^3C_3 + {}^3C_2 + {}^4C_2 + \dots + {}^{49}C_2 + {}^{50}C_2 m^2 = (3n+1) {}^{51}C_3$$

$${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r \Rightarrow {}^{50}C_3 + {}^{50}C_2 \cdot m^2 = (3n+1) {}^{51}C_3$$

$${}^{50}C_3 + {}^{50}C_2 + (m^2 - 1) {}^{50}C_2 = 3n \cdot \frac{51}{3} \cdot {}^{50}C_2 + {}^{51}C_3 \Rightarrow {}^{51}C_3 + (m^2 - 1) {}^{50}C_2 = 51n \cdot {}^{50}C_2 + {}^{51}C_3$$

$$m^2 - 1 = 51n \Rightarrow m^2 = 51n + 1$$

min value of m^2 for $51n + 1$ is integer for $n = 5$ (51n + 1 के पूर्णांक होने के लिए m^2 का न्यूनतम मान $n = 5$)

10. Let $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \dots + 10({}^{10}C_{10})^2$ where ${}^{10}C_r, r \in \{1, 2, \dots, 10\}$ denote binomial coefficients. Then the value of $\frac{1}{1430} X$ is _____ . [JEE (Advanced) 2018, Paper-1, (3, 0)/60]

माना कि $X = ({}^{10}C_1)^2 + 2({}^{10}C_2)^2 + 3({}^{10}C_3)^2 + \dots + 10({}^{10}C_{10})^2$, जहाँ ${}^{10}C_r, r \in \{1, 2, \dots, 10\}$, द्विपद गुणांकों (binomial coefficients) को दर्शाते हैं। तब $\frac{1}{1430} X$ का मान है _____।

Ans. (646)

$$\text{Sol. } X = \sum_{r=1}^{10} r \cdot {}^{10}C_r \cdot {}^{10}C_r = 10 \cdot \sum_{r=1}^{10} {}^9C_{r-1} \cdot {}^{10}C_{10-r} = 10 \cdot {}^{19}C_9$$

$$\text{Now अब } \frac{X}{1430} = \frac{10 \cdot {}^{19}C_9}{1430} = \frac{{}^{19}C_9}{143} = \frac{{}^{19}C_9}{11 \times 13} = \frac{19 \cdot 17 \cdot 16}{8} = 19 \times 34 = 646$$

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

भाग - II : JEE (MAIN) / AIEEE (पिछले वर्षों) के प्रश्न

1. Let $S_1 = \sum_{j=1}^{10} j (j-1) {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j {}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2 {}^{10}C_j$. [AIEEE 2009, (4, -1), 144]

Statement -1 : $S_3 = 55 \times 2^9$.

Statement -2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement -1 is true, Statement-2 is true ; Statement -2 is not a correct explanation for Statement -1.

(2*) Statement-1 is true, Statement-2 is false.

(3) Statement -1 is false, Statement -2 is true.

(4) Statement -1 is true, Statement -2 is true; Statement-2 is a correct explanation for Statement-1.

माना $S_1 = \sum_{j=1}^{10} j (j-1) {}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j {}^{10}C_j$ तथा $S_3 = \sum_{j=1}^{10} j^2 {}^{10}C_j$.

प्रकथन -1 : $S_3 = 55 \times 2^9$.

प्रकथन -2 : $S_1 = 90 \times 2^8$ तथा $S_2 = 10 \times 2^8$.

(1) प्रकथन-1 सत्य है, प्रकथन-2 सत्य है ; प्रकथन-2, प्रकथन-1 की सही व्याख्या नहीं है।

(2*) प्रकथन-1 सत्य है, प्रकथन-2 मिथ्या है।

(3) प्रकथन-1 मिथ्या है, प्रकथन-2 सत्य है।

(4) प्रकथन-1 सत्य है, प्रकथन-2 सत्य है ; प्रकथन-2, प्रकथन-1 की सही व्याख्या है।

Sol. $S_1 = \sum_{j=1}^{10} j(j-1) \cdot \frac{10(10-1)}{j(j-1)} {}^8C_{j-2}$

$$\Rightarrow S_1 = 9 \times 10 \sum_{j=2}^{10} {}^8C_{j-2} \Rightarrow S_1 = 90 \cdot 2^8$$

$$S_2 = \sum_{j=1}^{10} j \cdot \frac{10}{j} {}^9C_{j-1} = 10 \cdot 2^9$$

$$S_3 = \sum_{j=1}^{10} (j(j-1) + j) {}^{10}C_j = \sum_{j=1}^{10} j(j-1) {}^{10}C_j + \sum_{j=1}^{10} j {}^{10}C_j = 90 \sum_{j=2}^{10} {}^8C_{j-2} + 10 \sum_{j=1}^{10} {}^9C_{j-1}$$

$$= 90 \times 2^8 + 10 \times \sum_{j=2}^{10} {}^8C_{j-2} 2^9 = (45 + 10) \cdot 2^9 = (45 + 10) \cdot 2^9 = 55 \cdot 2^9$$

so statement-1 is true and statement 2 is false.

इसलिए कथन-1 सत्य है तथा कथन - 2 असत्य है।

Hence correct option is (2)

अतः सही विकल्प (2) है।

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is :

[AIEEE 2011, (4, -1), 120]

$(1 - x - x^2 + x^3)^6$ के प्रसार में x^7 का गुणांक है :

(1) 144 (2) -132 (3*) -144 (4) 132

Sol. (3)

$$(1 - x - x^2 + x^3)^6$$

$$(1 - x)^6 (1 - x^2)^6$$

$$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6) ({}^6C_0 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 + \dots + {}^6C_6 x^{12})$$

$$\text{Now coefficient of } x^7 = {}^6C_1 {}^6C_3 - {}^6C_3 {}^6C_2 + {}^6C_5 {}^6C_1$$

$$= 6 \times 20 - 20 \times 15 + 36$$

$$= 120 - 300 + 36$$

$$= 156 - 300 = -144 \quad \text{Ans.}$$

Hindi $(1 - x - x^2 + x^3)^6$

$$(1 - x)^6 (1 - x^2)^6$$

$$({}^6C_0 - {}^6C_1 x^1 + {}^6C_2 x^2 - {}^6C_3 x^3 + {}^6C_4 x^4 - {}^6C_5 x^5 + {}^6C_6 x^6) ({}^6C_0 - {}^6C_1 x^2 + {}^6C_2 x^4 - {}^6C_3 x^6 + {}^6C_4 x^8 + \dots + {}^6C_6 x^{12})$$

$$x^7 \text{ का गुणांक} = {}^6C_1 {}^6C_3 - {}^6C_3 {}^6C_2 + {}^6C_5 {}^6C_1$$

$$= 6 \times 20 - 20 \times 15 + 36$$

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$$= 156 - 30$$

$$\begin{aligned}
 &= 120 - 300 + 36 \\
 &= 156 - 300 \\
 &= -144 \text{ Ans.}
 \end{aligned}$$

3. If n is a positive integer, then $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$ is : [AIEEE 2012, (4, -1), 120]

- (1*) an irrational number
 (2) an odd positive integer
 (3) an even positive integer
 (4) a rational number other than positive integers

यदि n एक धनपूर्णांक है, तो $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$:

- (1*) एक अपरिमेय संख्या है।
 (2) एक विषम धनपूर्णांक है।
 (3) एक सम धनपूर्णांक है।
 (4) धनपूर्णांकों को छोड़ कर एक परिमेय संख्या है।

Sol. Ans. (1)

$$\begin{aligned}
 &(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n} \\
 &= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \dots] \\
 &= \text{which is an irrational number}
 \end{aligned}$$

Hindi. $(\sqrt{3} + 1)^{2n} - (\sqrt{3} - 1)^{2n}$

$$\begin{aligned}
 &= 2[{}^{2n}C_1(\sqrt{3})^{2n-1} + {}^{2n}C_3(\sqrt{3})^{2n-3} + {}^{2n}C_5(\sqrt{3})^{2n-5} + \dots] \\
 &= \text{जो कि एक अपरिमेय संख्या है।}
 \end{aligned}$$

4. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10}$ is : [AIEEE - 2013, (4, -1), 120]

$$\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}} \right)^{10} \text{ के प्रसार में } x \text{ से स्वतंत्र पद है :} \quad [\text{AIEEE - 2013, (4, -1), 120}]$$

- (1) 4
 (2) 120
 (3)
 (4) 310

$$\begin{aligned}
 &\left((x^{1/3} + 1) - \left(\frac{\sqrt{x} + 1}{\sqrt{x}} \right) \right)^{10} \\
 &T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r \\
 &\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0 \\
 &\Rightarrow r = 4 \\
 &T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210
 \end{aligned}$$

5. If the coefficients of x^3 and x^4 in the expansion of $(1 + ax + bx^2)(1 - 2x)^{18}$ in powers of x are both zero, then (a, b) is equal to [Binomial Theorem] [JEE(Main)] 2014, (4, -1), 120]

यदि $(1 + ax + bx^2)(1 - 2x)^{18}$ के x की घातों में प्रसार में x^3 तथा x^4 दोनों के गुणांक शून्य हैं, तो (a, b) बराबर है—

[Binomial Theorem] [JEE(Main)] 2014, (4, -1), 120]

- (1) $\left(14, \frac{272}{3} \right)$
 (2*) $\left(16, \frac{272}{3} \right)$
 (3) $\left(16, \frac{251}{3} \right)$
 (4) $\left(14, \frac{251}{3} \right)$

Sol. Ans. (2)

$$\begin{aligned}
 &(1 + ax + bx^2)(1 - 2x)^{18} \\
 &\text{coeff of } x^3 \text{ का गुणांक} = {}^{18}C_3(-2)^3 + a(-2)^2 \cdot {}^{18}C_2 + b(-2) \cdot {}^{18}C_1 = 0 \\
 &\text{coeff of } x^4 \text{ का गुणांक} = {}^{18}C_4(-2)^4 + a(-2)^3 \cdot {}^{18}C_3 + b(-2)^2 \cdot {}^{18}C_2 = 0
 \end{aligned}$$

$$\Rightarrow 51a - 3b = 544 \text{ and } 32a - 3b = 240$$

Subtracting we get घटाने पर प्राप्त होता है $a = 16$

$$\Rightarrow b = \frac{272}{3}$$

6. The sum of coefficients of integral powers of x in the binomial expansion of $(1 - 2\sqrt{x})^{50}$ is $(1 - 2\sqrt{x})^{50}$ के द्विपद प्रसार में x की पूर्णांकीय घातों के गुणांकों का योग है :[JEE(Main) 2015, (4, - 1), 120]

- (1) $\frac{1}{2} (3^{50} + 1)$ (2) $\frac{1}{2} (3^{50})$ (3) $\frac{1}{2} (3^{50} - 1)$ (4) $\frac{1}{2} (2^{50} + 1)$

Ans. (1)

$$(1 - 2\sqrt{x})^{50} = C_0 - C_1 (2\sqrt{x}) + C_2 (2\sqrt{x})^2 + \dots + C_{50} (2\sqrt{x})^{50}$$

$$(1 + 2\sqrt{x})^{50} = C_0 + C_1 (2\sqrt{x}) + C_2 (2\sqrt{x})^2 + \dots + C_{50} (2\sqrt{x})^{50}$$

Put $x = 1$ रखने पर

$$\therefore \frac{1+3^{50}}{2} = C_0 + C_2 (2)^2 + \dots$$

7. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is [JEE(Main) 2016, (4, - 1), 120]

यदि $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n$, $x \neq 0$ के प्रसार में पदों की संख्या 28 है, तो इस प्रसार में आने वाले सभी पदों के गुणांकों का योग है—

- (1) 2187 (2) 243 (3) 729 (4) 64

Ans. (3) or Bonus

Sol. Theoretically the number of terms are $2N + 1$ (i.e. odd) But As the number of terms being odd hence considering that number clubbing of terms is done hence the solutions follows :

$$\text{Number of terms} = {}^{n+2}C = 28 \quad \therefore n = 6$$

$$\text{sum of coefficient} = 3^n = 3^6 = 729$$

put $x = 1$

सिद्धान्तः पदों की संख्या $2N + 1$ है (अर्थात् विषम) परन्तु जैसा कि पदों की संख्या विषम है अतः पदों के मिश्रण के अनुसार हल लिखने पर।

$$\text{पदों की संख्या} = {}^{n+2}C = 28 \quad \therefore n = 6$$

$$\text{गुणांकों का योग} = 3^n = 3^6 = 729$$

$x = 1$ रखने पर

8. The value of $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$ is $({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \dots + ({}^{21}C_{10} - {}^{10}C_{10})$ का मान है—

[JEE(Main) 2017, (4, - 1), 120]

- (1) $2^{21} - 2^{11}$ (2) $2^{21} - 2^{10}$ (3) $2^{20} - 2^9$ (4) $2^{20} - 2^{10}$

Ans. (4)

$$({}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \dots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10}) = S_1 - S_2$$

$$S_1 = {}^{21}C_1 + {}^{21}C_2 + {}^{21}C_3 + \dots + {}^{21}C_{10}$$

$$S_1 = \frac{1}{2} ({}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{20}) = \frac{1}{2} ({}^{21}C_0 + {}^{21}C_1 + {}^{21}C_2 + \dots + {}^{21}C_{20} + {}^{21}C_{21} - 2)$$

$$S_1 = 2^{20} - 1$$

$$S_2 = ({}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + \dots + {}^{10}C_{10}) = 2^{10} - 1$$

$$\text{Therefore इसलिए, } S_1 - S_2 = 2^{20} - 2^{10}$$

9. The sum of the co-efficients of all odd degree terms in the expansion of $(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, ($x > 1$) is : [JEE(Main) 2018, (4, - 1), 120]

$(x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5$, ($x > 1$) के प्रसार में सभी विषम घातों वाले पदों के गुणांकों का योग है :

- Sol. (1) 1 (2) 2 (3) -1 (4) 0

$$\begin{aligned} & (x + \sqrt{x^3 - 1})^5 + (x - \sqrt{x^3 - 1})^5 \\ &= (T_1 + T_2 + T_3 + T_4 + T_5 + T_6) + (T_1 - T_2 + T_3 - T_4 + T_5 - T_6) \end{aligned}$$

$$\begin{aligned} &= 2(T_1 + T_3 + T_5) \\ &= 2(5C_0(x)^5 + 5C_2(x)^3 (\sqrt{x^3 - 1})^2 + 5C_4(x)^1 (\sqrt{x^3 - 1})^4) \end{aligned}$$

$$\begin{aligned} &= 2(x^5 + 10x^3(x^3 - 1) + 5x(x^6 + 1 - 2x^3)) \\ &= 2(x^5 + 10x^6 - 10x^3 + 5x^7 + 5x - 10x^4) \\ &= 2(5x^7 + 10x^6 + x^5 - 10x^4 - 10x^3 + 5x) \end{aligned}$$

sum of odd degree terms विषम घात के पदों का योगफल = $10 + 2 - 20 + 10 = 2$

10. If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to :

[JEE(Main) 2019, Online (09-01-19), P-1 (4, - 1), 120]

यदि संख्या $\frac{2^{403}}{15}$ का भिन्नात्मक भाग (fractional part) $\frac{k}{15}$ है तो k बराबर है—

- Ans. (1) 14 (2) 8 (3) 6 (4) 4

Sol. $\left\{ \frac{2^{203}}{15} \right\}$

$$8 \cdot 2^{200} \Rightarrow 8 \cdot 16^{50} = 8(1 + 15)^{50} = 8(1 + 15)$$

hence remainder is 8.

(अतः शेषफल 8 है।)

11. If $\sum_{i=1}^{20} \left(\frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} \right)^3 = \frac{k}{21}$, then k equals : [JEE(Main) 2019, Online (10-01-19), P-1 (4, - 1), 120]

यदि $\sum_{i=1}^{20} \left(\frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} \right)^3 = \frac{k}{21}$, तो k बराबर है:

- Ans. (1) 50 (2) 400 (3) 200 (4) 100

Sol. $\sum_{i=1}^{20} \left(\frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} \right)^3$

$$\text{Now अब } \frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} = \frac{^{20}C_{i-1}}{^{21}C_i} = \frac{i}{21}$$

Let given sum be S, so माना दिया गया योग S है, तब

$$S = \sum_{i=1}^{20} \frac{(i)^3}{21^3} = \frac{1}{(21)^3} \left(\frac{20 \cdot 21}{2} \right)^2 = \frac{100}{21}$$

Given दिया गया है कि $S = \frac{k}{21} \Rightarrow k = 100$

12. If $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K \left({}^{50}C_{25} \right)$, then K is equal to :

यदि $\sum_{r=0}^{25} \left\{ {}^{50}C_r \cdot {}^{50-r}C_{25-r} \right\} = K \left({}^{50}C_{25} \right)$ है, तो K बराबर है : [JEE(Main) 2019, Online (10-01-19), P-2 (4, - 1),

120]

- (1) 2^{25} (2) $2^{25} - 1$ (3) $(25)^2$ (4) 2^{24}

Ans.

$$\begin{aligned}
 \text{Sol. } & \sum_{r=0}^{25} {}^{50}C_r {}^{50-r}C_{25-r} \\
 &= \sum_{r=0}^{25} \frac{50!}{r!(50-r)!} \frac{(50-r)!}{(25-r)!(25)!} \\
 &= \sum_{r=0}^{25} \frac{50!}{r!(25-r)!(25)!} 25! \\
 &= {}^{50}C_{25} \sum_{r=0}^{25} {}^{25}C_r = {}^{50}C_{25} \times 2^{25} = k({}^{50}C_{25})
 \end{aligned}$$

$$\Rightarrow k = 2^{25}$$

13. Let $S_n = 1 + q + q^2 + \dots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$.

where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \dots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$ then α is equal to

माना $S_n = 1 + q + q^2 + \dots + q^n$ तथा $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$. जहाँ q एक वास्तविक संख्या है तथा $q \neq 1$ यदि ${}^{101}C_1 + {}^{101}C_2 \cdot S_1 + \dots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$ तो α बराबर है -

[JEE(Main) 2019, Online (11-01-19),P-2 (4, - 1), 120]

(1) 200

(2) 2^{99}

(3) 2^{100}

(4) 202

Ans. (3)

$$\begin{aligned}
 \text{Sol. } & \sum_{r=1}^{101} {}^{101}C_r S_{r-1} = \sum_{r=1}^{101} {}^{101}C_r \frac{q^r - 1}{q - 1} = \frac{1}{q - 1} \left(\sum_{r=1}^{101} {}^{101}C_r q^r - \sum_{r=1}^{101} {}^{101}C_r \right) = \frac{1}{q - 1} ((1+q)^{101} - 1 - 2^{101} + 1) \\
 & \Rightarrow \alpha \frac{\left(\frac{q+1}{2}\right)^{101} - 1}{\frac{q+1}{2} - 1} = \frac{1}{q - 1} ((1+q)^{101} - 2^{101}) \\
 & \Rightarrow \frac{\alpha}{2^{100}} \left(\frac{(1+q)^{101} - 2^{101}}{q - 1} \right) = \frac{1}{q - 1} ((1+q)^{101} - 2^{101}) \\
 & \Rightarrow \text{Hence अतः } \alpha = 2^{100}
 \end{aligned}$$



High Level Problems (HLP)

SUBJECTIVE QUESTIONS

विषयात्मक प्रश्न (SUBJECTIVE QUESTIONS)

1. Find the coefficient of x^{49} in

$$\left(x + \frac{C_1}{C_0}\right) \left(x + 2^2 \frac{C_2}{C_1}\right) \left(x + 3^2 \frac{C_3}{C_2}\right) \dots \dots \left(x + 50^2 \frac{C_{50}}{C_{49}}\right) \text{ where } C_r = {}^{50}C_r$$

Ans. 22100

$\left(x + \frac{C_1}{C_0}\right) \left(x + 2^2 \frac{C_2}{C_1}\right) \left(x + 3^2 \frac{C_3}{C_2}\right) \dots \dots \left(x + 50^2 \frac{C_{50}}{C_{49}}\right)$ में x^{49} का गुणांक ज्ञात कीजिए। (जहाँ $C_r = {}^{50}C_r$)

Sol. Coeff. of $x^{49} = \left(\frac{C_1}{C_0} + 2^2 \frac{C_2}{C_1} + 3^2 \frac{C_3}{C_2} + \dots \dots + 50^2 \frac{C_{50}}{C_{49}}\right) = \sum_{r=1}^{50} r^2 \frac{{}^{50}C_r}{{}^{50}C_{r-1}} = \sum_{r=1}^{50} r^2 \left(\frac{50-r+1}{r}\right)$

$$= \sum_{r=1}^{50} r(51-r) = \frac{51 \times 50 \times 51}{2} - \frac{50 \times 51 \times 101}{6} = 22100$$

2. The expression, $\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\frac{2}{\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}}\right)^6$ is a polynomial of degree

व्यंजक $\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\frac{2}{\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}}\right)^6$ किस घात का एक बहुपद है—

Ans. 6

Sol. $\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\frac{2}{\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}}\right)^6$

$$= \left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\sqrt{2x^2 + 1} - \sqrt{2x^2 - 1}\right)^6$$

$$= 2 \left({}^6C_0 (2x^2 + 1)^3 + {}^6C_2 (2x^2 + 1)^2 (2x^2 - 1) + {}^6C_4 (2x^2 + 1)(2x^2 - 1)^2 + {}^6C_6 (2x^2 - 1)^3\right)$$

clearly '6'

Hindi $\left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\frac{2}{\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}}\right)^6$

$$= \left(\sqrt{2x^2 + 1} + \sqrt{2x^2 - 1}\right)^6 + \left(\sqrt{2x^2 + 1} - \sqrt{2x^2 - 1}\right)^6$$

$$= 2 \left({}^6C_0 (2x^2 + 1)^3 + {}^6C_2 (2x^2 + 1)^2 (2x^2 - 1) + {}^6C_4 (2x^2 + 1)(2x^2 - 1)^2 + {}^6C_6 (2x^2 - 1)^3\right)$$

स्पष्टतया '6'

3. Find the co-efficient of x^5 in the expansion of $(1 + x^2)^5 (1 + x)^4$.

$(1 + x^2)^5 (1 + x)^4$ के विस्तार में x^5 का गुणांक ज्ञात कीजिए।

Ans. 60

Sol. Co-efficient of x^5 in $(1 + x^2)^5 (1+x)^4 = {}^4C_1 \cdot {}^5C_2 + {}^4C_3 \cdot {}^5C_1 = 40 + 20 = 60$

Hindi $(1 + x^2)^5 (1+x)^4$ में x^5 का गुणांक $= {}^4C_1 \cdot {}^5C_2 + {}^4C_3 \cdot {}^5C_1 = 40 + 20 = 60$

4. Prove that the co-efficient of x^{15} in $(1 + x + x^3 + x^4)^n$ is $\sum_{r=0}^5 {}^n C_{15-3r} {}^n C_r$.

सिद्ध कीजिए कि $(1 + x + x^3 + x^4)^n$ के विस्तार में x^{15} का गुणांक $\sum_{r=0}^5 {}^n C_{15-3r} {}^n C_r$ है

Sol. $(1 + x + x^3 + x^4)^n = [(1 + x)(1 + x^3)]^n = (1 + x)^n (1 + x^3)^n$

power of x in $(1 + x^3)^n$ expansion is multiple of 3

so possible cases to get x^{15} are :-

$$\begin{array}{lll} (9n) (1+x)^n & 9n(1+x^3)^n \\ \downarrow & \downarrow \\ {}^n C_{15} x^{15} & {}^n C_0 (x^3)^0 & = {}^n C_{15} \cdot {}^n C_0 x^{15} \\ {}^n C_{12} x^{12} & {}^n C_1 (x^3)^1 & = {}^n C_{12} \cdot {}^n C_1 x^{15} \\ {}^n C_{12} x^9 & {}^n C_2 (x^3)^2 & = {}^n C_9 \cdot {}^n C_2 x^{15} \\ {}^n C_6 x^6 & {}^n C_3 (x^3)^3 & = {}^n C_6 \cdot {}^n C_3 x^{15} \\ {}^n C_3 x^3 & {}^n C_4 (x^3)^4 & = {}^n C_3 \cdot {}^n C_4 x^{15} \\ {}^n C_0 x^0 & {}^n C_5 (x^3)^5 & = {}^n C_0 \cdot {}^n C_5 x^{15} \end{array}$$

∴ coefficient of x^{15} is

$$= {}^n C_{15} \cdot {}^n C_0 + {}^n C_{12} \cdot {}^n C_1 + {}^n C_9 \cdot {}^n C_2 + {}^n C_6 \cdot {}^n C_3 + {}^n C_3 \cdot {}^n C_4 + {}^n C_0 \cdot {}^n C_5$$

$$= \sum_{r=0}^5 {}^n C_{15-3r} \cdot {}^n C_r \text{ hence proved}$$

Hindi $(1 + x + x^3 + x^4)^n = [(1 + x)(1 + x^3)]^n = (1 + x)^n (1 + x^3)^n$

$(1 + x^3)^n$ में x की घात, 3 का गुणज है।

x^{15} की संभाविय स्थिति

$$\begin{array}{lll} (9n) (1+x)^n & 9n(1+x^3)^n \\ \downarrow & \downarrow \\ {}^n C_{15} x^{15} & {}^n C_0 (x^3)^0 & = {}^n C_{15} \cdot {}^n C_0 x^{15} \\ {}^n C_{12} x^{12} & {}^n C_1 (x^3)^1 & = {}^n C_{12} \cdot {}^n C_1 x^{15} \\ {}^n C_{12} x^9 & {}^n C_2 (x^3)^2 & = {}^n C_9 \cdot {}^n C_2 x^{15} \\ {}^n C_6 x^6 & {}^n C_3 (x^3)^3 & = {}^n C_6 \cdot {}^n C_3 x^{15} \\ {}^n C_3 x^3 & {}^n C_4 (x^3)^4 & = {}^n C_3 \cdot {}^n C_4 x^{15} \\ {}^n C_0 x^0 & {}^n C_5 (x^3)^5 & = {}^n C_0 \cdot {}^n C_5 x^{15} \end{array}$$

∴ x^{15} का गुणांक है।

$$= {}^n C_{15} \cdot {}^n C_0 + {}^n C_{12} \cdot {}^n C_1 + {}^n C_9 \cdot {}^n C_2 + {}^n C_6 \cdot {}^n C_3 + {}^n C_3 \cdot {}^n C_4 + {}^n C_0 \cdot {}^n C_5$$

$$= \sum_{r=0}^5 {}^n C_{15-3r} \cdot {}^n C_r \quad \text{अतः सिद्ध हुआ}$$

5. If n is even natural and coefficient of x^r in the expansion of $\frac{(1+x)^n}{1-x}$ is 2^n , ($|x| < 1$), then prove that $r \geq n$

यदि n सम प्राकृत संख्या है तथा $\frac{(1+x)^n}{1-x}$ के विस्तार में x^r का गुणांक 2^n ($|x| < 1$) है तब सिद्ध कीजिए $r \geq n$

Sol. $y = (1-x)^{-1} (1+x)^n$

$$y = (1+x + x^2 + \dots + \infty) (1+x)^n$$

$$y = (1+x)^n + x(1+x)^n + x^2(1+x)^n + \dots$$

Co-efficient of x^r =

$${}^n C_r + {}^n C_{r-1} + \dots + {}^n C_0 = 2^n$$

$r \geq n$ (As ${}^n C_{n+1} = 0$)

Hindi $y = (1-x)^{-1} (1+x)^n$

$y = (1+x + x^2 + \dots \infty) (1+x)^n$

$y = (1+x)^n + x(1+x)^n + x^2(1+x)^n + \dots$

x^r का गुणांक =

${}^n C_r + {}^n C_{r-1} + \dots + {}^n C_0 = 2^n$

$r \geq n$ (As ${}^n C_{n+1} = 0$)

6. Find the coefficient of x^n in polynomial $(x + {}^{2n+1}C_0)(x + {}^{2n+1}C_1) \dots (x + {}^{2n+1}C_n)$.

बहुपद $(x + {}^{2n+1}C_0)(x + {}^{2n+1}C_1) \dots (x + {}^{2n+1}C_n)$ में x^n का गुणांक ज्ञात कीजिए।

Ans. 2^{2n}

Sol. Co-efficient of $x^n = {}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n = 2^{2n}$

Hindi x^n का गुणांक $= {}^{2n+1}C_0 + {}^{2n+1}C_1 + \dots + {}^{2n+1}C_n = 2^{2n}$

7. Find the value of $\sum_{r=1}^n \left(\sum_{p=0}^{r-1} {}^n C_r {}^r C_p 2^p \right)$.

$\sum_{r=1}^n \left(\sum_{p=0}^{r-1} {}^n C_r {}^r C_p 2^p \right)$ का मान ज्ञात कीजिए।

Ans. $4^n - 3^n$

$$\begin{aligned} \text{Sol. } \sum_{r=1}^n \left(\sum_{p=0}^{r-1} {}^n C_r {}^r C_p 2^p \right) &= \sum_{r=1}^n {}^n C_r \sum_{p=0}^{r-1} {}^r C_p \cdot 2^p = \sum_{r=1}^n {}^n C_r [{}^r C_0 + {}^r C_1 \cdot 2 + \dots + {}^r C_{r-1} \cdot 2^{r-1}] \\ &= \sum_{r=1}^n {}^n C_r (3^r - 2^r) = 4^n - 3^n \end{aligned}$$

Comprehension (Q-8 to Q.10)

अनुच्छेद

For $k, n \in \mathbb{N}$, we define

$B(k, n) = 1.2.3 \dots k + 2.3.4 \dots (k+1) + \dots + n(n+1) \dots (n+k-1)$, $S_0(n) = n$ and $S_k(n) = 1^k + 2^k + \dots + n^k$.

To obtain value $B(k, n)$, we rewrite $B(k, n)$ as follows

$$\begin{aligned} B(k, n) &= k! \left[{}^k C_k + {}^{k+1} C_k + {}^{k+2} C_k + \dots + {}^{n+k-1} C_k \right] = k! \left({}^{n+k} C_{k+1} \right) \\ &= \frac{n(n+1) \dots (n+k)}{k+1} \end{aligned}$$

$$\text{where } {}^n C_k = \frac{n!}{k! (n-k)!}$$

$k, n \in \mathbb{N}$ के लिए परिभाषित किया जाता है कि

$B(k, n) = 1.2.3 \dots k + 2.3.4 \dots (k+1) + \dots + n(n+1) \dots (n+k-1)$, $S_0(n) = n$ एवं $S_k(n) = 1^k + 2^k + \dots + n^k$.

$B(k, n)$ का मान ज्ञात करने के लिए $B(k, n)$ को निम्न प्रकार पुनः लिखने पर

$$\begin{aligned} B(k, n) &= k! \left[{}^k C_k + {}^{k+1} C_k + {}^{k+2} C_k + \dots + {}^{n+k-1} C_k \right] = k! \left({}^{n+k} C_{k+1} \right) \\ &= \frac{n(n+1) \dots (n+k)}{k+1} \text{ जहाँ } {}^n C_k = \frac{n!}{k! (n-k)!} \end{aligned}$$

8. Prove that $S_2(n) + S_1(n) = B(2, n)$

सिद्ध कीजिए $S_2(n) + S_1(n) = B(2, n)$

$$\text{Sol. } S_2(n) + S_1(n) = \sum n^2 + \sum n$$

$$= \sum n(n+1)$$

$$= 1.2 + 2.3 + 3.4 + \dots + n(n+1)$$

$$= B(2, n)$$

9. Prove that सिद्ध कीजिए $S_3(n) + 3S_2(n) = B(3, n) - 2B(1, n)$

$$\text{Sol. } S_3(n) + 3S_2(n) + 2S_1(n) - 2S_1(n)$$

$$= \sum n^3 + 3\sum n^2 + 2\sum n - 2\sum n$$

$$= \sum n(n+1)(n+2) - 2\sum n$$

$$= B(3, n) - 2B(1, n)$$

10. If $(1+x)^p = 1 + {}^pC_1 x + {}^pC_2 x^2 + \dots + {}^pC_p x^p$, $p \in \mathbb{N}$, then show that ${}^{k+1}C_1 S_k(n) + {}^{k+1}C_2 S_{k-1}(n) + \dots + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k+1} S_0(n) = (n+1)^{k+1} - 1$

यदि $(1+x)^p = 1 + {}^pC_1 x + {}^pC_2 x^2 + \dots + {}^pC_p x^p$, $p \in \mathbb{N}$ तब दर्शाइये कि ${}^{k+1}C_1 S_k(n) + {}^{k+1}C_2 S_{k-1}(n) + \dots + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k+1} S_0(n) = (n+1)^{k+1} - 1$

$$\text{Sol. } (1+x)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k x + {}^{k+1}C_{k-1} x^2 + {}^{k+1}C_{k-2} x^3 + \dots + {}^{k+1}C_0 x^{k+1}$$

Put $x = 1, 2, \dots, n$ रखने पर

$$2^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 1 + {}^{k+1}C_{k-1} \cdot 1^2 + {}^{k+1}C_{k-2} \cdot 1^3 + \dots + {}^{k+1}C_0 \cdot 1^{k+1}$$

$$3^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 2 + {}^{k+1}C_{k-1} \cdot 2^2 + \dots + {}^{k+1}C_0 \cdot 2^{k+1}$$

⋮

$$(1+n)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot n + {}^{k+1}C_{k-1} \cdot n^2 + \dots + {}^{k+1}C_0 \cdot n^{k+1}$$

$$2^{k+1} + 3^{k+1} + \dots + (1+n)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k-1} S_2(n) + \dots + {}^{k+1}C_0 S_{(k+1)}(n)$$

$$2^{k+1} + 3^{k+1} + \dots + (n+1)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + \dots + {}^{k+1}C_1 S_k(n) + 1^{k+1} + 2^{k+1} + 3^{k+1} + \dots + n^{k+1}$$

So $(n+1)^{k+1} - 1$

$$\text{Hindi } (1+x)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k x + {}^{k+1}C_{k-1} x^2 + {}^{k+1}C_{k-2} x^3 + \dots + {}^{k+1}C_0 x^{k+1}$$

Put $x = 1, 2, \dots, n$ रखने पर

$$2^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 1 + {}^{k+1}C_{k-1} \cdot 1^2 + {}^{k+1}C_{k-2} \cdot 1^3 + \dots + {}^{k+1}C_0 \cdot 1^{k+1}$$

$$3^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot 2 + {}^{k+1}C_{k-1} \cdot 2^2 + \dots + {}^{k+1}C_0 \cdot 2^{k+1}$$

⋮

$$(1+n)^{k+1} = {}^{k+1}C_{k+1} + {}^{k+1}C_k \cdot n + {}^{k+1}C_{k-1} \cdot n^2 + \dots + {}^{k+1}C_0 \cdot n^{k+1}$$

$$2^{k+1} + 3^{k+1} + \dots + (1+n)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + {}^{k+1}C_{k-1} S_2(n) + \dots + {}^{k+1}C_0 S_{(k+1)}(n)$$

$$2^{k+1} + 3^{k+1} + \dots + (n+1)^{k+1} = {}^{k+1}C_{k+1} S_0(n) + {}^{k+1}C_k S_1(n) + \dots + {}^{k+1}C_1 S_k(n) + 1^{k+1} + 2^{k+1} + 3^{k+1} + \dots + n^{k+1}$$

अतः $(n+1)^{k+1} - 1$

11. Show that $25^n - 20^n - 8^n + 3^n$, $n \in \mathbb{I}^+$ is divisible by 85.

प्रदर्शित कीजिए कि $25^n - 20^n - 8^n + 3^n$, $n \in \mathbb{I}^+$, 85 से भाज्य है।

$$\text{Sol. } 85 = 17 \times 5 \quad (\text{Both are prime number})$$

$$25^n = (20+5)^n$$

$$\text{and } 8^n = (5+3)^n$$

So clearly $(20+5)^n - 20^n - (5+3)^n + 3^n$ is divisible by 5

Also $(17+8)^n - 8^n - (17+3)^n + 3^n$ is divisible by 17

So expression is divisible by 85

Hindi $85 = 17 \times 5$ (दोनों अभाज्य संख्याएँ हैं।)

$$25^n = (20 + 5)^n \quad \text{तथा} \quad 8^n = (5 + 3)^n$$

अतः स्पष्टतया $(20 + 5)^n - 20^n - (5 + 3)^n + 3^n$, 5 से भाज्य है

पुनः $(17 + 8)^n - 8^n - (17 + 3)^n + 3^n$, 17 से भाज्य है अतः व्यंजक 85 से भाज्य है।

12. Prove that ${}^nC_1 \cdot {}^nC_2 \cdot {}^nC_3 \cdot \dots \cdot {}^nC_n \leq \left(\frac{2^n}{n+1} \right)^{n+1} C_2$.

सिद्ध कीजिए कि ${}^nC_1 \cdot {}^nC_2 \cdot {}^nC_3 \cdot \dots \cdot {}^nC_n \leq \left(\frac{2^n}{n+1} \right)^{n+1} C_2$.

Sol. A.M. \geq G.M

$$\frac{{}^nC_1 + 2 \cdot {}^nC_2 + 3 \cdot {}^nC_3 + \dots + n \cdot {}^nC_n}{1+2+3+\dots+n} \geq \sqrt[n]{{}^nC_1 \cdot {}^nC_2 \cdot \dots \cdot {}^nC_n}$$

$$= \frac{n \cdot 2^{n-1} \cdot 2}{n(n+1)} \geq \sqrt[n]{{}^nC_1 \cdot {}^nC_2 \cdot \dots \cdot {}^nC_n}$$

$${}^nC_1 \cdot {}^nC_2 \cdot \dots \cdot {}^nC_n \leq \left(\frac{2^n}{n+1} \right)^{n(n+1)/2} \quad \left(\text{Also } \frac{n(n+1)}{2} = {}^{n+1} C_2 \right)$$

13. If p is nearly equal to q and n > 1, show that $\frac{(n+1) \cdot p + (n-1)q}{(n-1)p + (n+1)q} = \left(\frac{p}{q} \right)^{1/n}$. Hence find the approximate

value of $\left(\frac{99}{101} \right)^{1/6}$.

यदि p, q के लगभग बराबर हैं तथा n > 1, प्रदर्शित कीजिए कि $\frac{(n+1) \cdot p + (n-1)q}{(n-1)p + (n+1)q} = \left(\frac{p}{q} \right)^{1/n}$. इसकी सहायता से

$$\left(\frac{99}{101} \right)^{1/6} \text{ का निकटतम मान ज्ञात कीजिए।}$$

Ans. $\frac{1198}{1202}$

Sol. Let p = q + h (say), where h is small that its square and higher powers may be neglected. then -

$$\frac{(n+1) \cdot p + (n-1) \cdot q}{(n-1)p + (n+1) \cdot q} = \frac{(n+1) \cdot (q+h) + (n-1) \cdot q}{(n-1) \cdot (q+h) + (n+1) \cdot q} \quad (\because p = q + h)$$

$$= \frac{2nq + (n+1) \cdot h}{2nq + (n-1) \cdot h} = \left(1 + \left(\frac{n+1}{2nq} \right) \cdot h \right) \left(1 + \left(\frac{n-1}{2nq} \right) \cdot h \right)^{-1} = 1 + \frac{h}{nq} = \left(1 + \frac{h}{q} \right)^{1/n} = \left(\frac{p}{q} \right)^{1/n}$$

put p = 99, q = 101 and n = 6

$$\frac{(6+1) \times 99 + (6-1) \times 101}{(6-1) \times 99 + (6+1) \times 101} = \left(\frac{99}{101} \right)^{1/6} = \frac{1198}{1202}$$

Hindi मानाकि p = q + h (माना), जहाँ h इतना छोटा है कि इसके वर्ग तथा अन्य बड़ी घातों को नगण्य मान सकते हैं, तो

$$\frac{(n+1) \cdot p + (n-1) \cdot q}{(n-1)p + (n+1) \cdot q} = \frac{(n+1) \cdot (q+h) + (n-1) \cdot q}{(n-1) \cdot (q+h) + (n+1) \cdot q} \quad (\because p = q + h)$$

$$= \frac{2nq + (n+1) \cdot h}{2nq + (n-1) \cdot h} = \left(1 + \left(\frac{n+1}{2nq} \right) \cdot h \right) \left(1 + \left(\frac{n-1}{2nq} \right) \cdot h \right)^{-1} = 1 + \frac{h}{nq} = \left(1 + \frac{h}{q} \right)^{1/n} = \left(\frac{p}{q} \right)^{1/n}$$

p = 99, q = 101 तथा n = 6 रखने पर

$$\frac{(6+1) \times 99 + (6-1) \times 101}{(6-1) \times 99 + (6+1) \times 101} = \left(\frac{99}{101}\right)^{1/6} = \frac{1198}{1202}$$

14. If $(18x^2 + 12x + 4)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, then prove that

$$a_r = 2^n 3^r \left({}^{2n}C_r + {}^nC_1 {}^{2n-2}C_r + {}^nC_2 {}^{2n-4}C_r + \dots \right)$$

यदि $(18x^2 + 12x + 4)^n = a_0 + a_1x + a_2x^2 + \dots + a_{2n}x^{2n}$, तब सिद्ध कीजिए

$$a_r = 2^n 3^r \left({}^{2n}C_r + {}^nC_1 {}^{2n-2}C_r + {}^nC_2 {}^{2n-4}C_r + \dots \right)$$

Sol. a_r is the coefficient of x^r in R.H.S.

$$\begin{aligned} (18x^2 + 12x + 4)^n &= 2^n (1+(1+3x)^2)^n \\ &= 2^n \left({}^nC_0 (1+3x)^{2n} + {}^nC_1 (1+3x)^{2n-2} + {}^nC_2 (1+3x)^{2n-4} + \dots \right) \\ &= 2^n \left({}^nC_0 3^r {}^{2n}C_r + {}^nC_1 3^r {}^{2n-2}C_r + {}^nC_2 3^r {}^{2n-4}C_r + \dots \right) \end{aligned}$$

Hindi. a_r , R.H.S. में x^r का गुणांक है।

$$\begin{aligned} (18x^2 + 12x + 4)^n &= 2^n (1+(1+3x)^2)^n \\ &= 2^n \left({}^nC_0 (1+3x)^{2n} + {}^nC_1 (1+3x)^{2n-2} + {}^nC_2 (1+3x)^{2n-4} + \dots \right) \\ &= 2^n \left({}^nC_0 3^r {}^{2n}C_r + {}^nC_1 3^r {}^{2n-2}C_r + {}^nC_2 3^r {}^{2n-4}C_r + \dots \right) \end{aligned}$$

15. Prove that $1^2 \cdot C_0 + 2^2 \cdot C_1 + 3^2 \cdot C_2 + 4^2 \cdot C_3 + \dots + (n+1)^2 C_n = 2^{n-2} (n+1) (n+4)$.

Sol. $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$

multiply by x and then differentiate

$$(1+x)^n + x \cdot n (1+x)^{n-1} = C_0 + 2C_1x + 3C_2x^2 + \dots + (n+1) \cdot C_n x^n$$

again multiply by x and then differentiate

$$(1+x)^n + nx (1+x)^{n-1} + 2nx (1+x)^{n-2} + n(n-1)x^2 (1+x)^{n-2} = C_0 + 2^2 C_1x + 3^2 C_2x^2 + \dots + (n+1)^2 C_n x^n$$

put $x = 1$

$$\text{then } S = 2^n + n \cdot 2^{n-1} + 2n \cdot 2^{n-2} + n(n-1) 2^{n-2}$$

$$= 2^{n-2} [4 + 2n + 4n + n^2 - n]$$

$$= 2^{n-2} (n+1)(n+4)$$

Hindi. $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$

x से गुणा करने के बाद अवकलन करने पर

$$(1+x)^n + x \cdot n (1+x)^{n-1} = C_0 + 2C_1x + 3C_2x^2 + \dots + (n+1) \cdot C_n x^n$$

दुबारा x से गुणा करके अवकलन करने पर

$$(1+x)^n + nx (1+x)^{n-1} + 2nx (1+x)^{n-2} + n(n-1)x^2 (1+x)^{n-2} = C_0 + 2^2 C_1x + 3^2 C_2x^2 + \dots + (n+1)^2 C_n x^n$$

$x = 1$ रखने पर

$$\text{तब } S = 2^n + n \cdot 2^{n-1} + 2n \cdot 2^{n-2} + n(n-1) 2^{n-2}$$

$$= 2^{n-2} [4 + 2n + 4n + n^2 - n]$$

$$= 2^{n-2} (n+1)(n+4).$$

16. If $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$, find the value of, $a_0 + a_1 + a_2 + \dots + a_n$.

$$(1-x)^{-n} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots, a_0 + a_1 + a_2 + \dots + a_n$$
 का मान ज्ञात करो—

Ans. $\frac{(2n)!}{(n!)^2}$

Sol. $(1-x)^{-n} = a_0 + a_1x + a_2x^2 + \dots + a_n x^n = \sum_{r=0}^n {}^{n+r-1}C_r x^r$

$$\begin{aligned}
 a_0 + a_1 + \dots + a_n &= {}^{n-1}C_0 + {}^nC_1 + {}^{n+1}C_2 + \dots + {}^{2n-1}C_n \\
 &= {}^{n-1}C_{n-1} + {}^nC_{n-1} + {}^{n+1}C_{n-1} + \dots + {}^{2n-1}C_{n-1} \\
 &= {}^nC_n + {}^nC_{n-1} + {}^{n+1}C_{n-1} + \dots + {}^{2n-1}C_{n-1} = {}^{2n}C_n \quad \{{}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r\}
 \end{aligned}$$

17. Find the remainder when $32^{32^{32}}$ is divided by 7.

$32^{32^{32}}$ को 7 से भाग देने पर शेषफल ज्ञात करो।

Ans. 4

Sol. $32^{32} = (2^5)^{32} = 2^{160}$

$$(3 - 1)^{160} = 3\lambda + 1$$

$$32^{32^{32}} = (2^5)^{(3\lambda + 1)} = 2^{(15\lambda + 3) + 2} = 4 \cdot (2^3)^{(5\lambda + 1)} = 4(7 + 1)^{\beta} = 4(7\mu + 1)$$

∴ remainder is 4

Hindi. $32^{32} = (2^5)^{32} = 2^{160}$

$$(3 - 1)^{160} = 3\lambda + 1$$

$$32^{32^{32}} = (2^5)^{(3\lambda + 1)} = 2^{(15\lambda + 3) + 2} = 4 \cdot (2^3)^{(5\lambda + 1)} = 4(7 + 1)^{\beta} = 4(7\mu + 1)$$

∴ शेषफल 4 है।

18. If n is an integer greater than 1, show that : $a - {}^nC_1(a-1) + {}^nC_2(a-2) - \dots + (-1)^n (a-n) = 0$.

यदि n (> 1) एक पूर्णांक है तब प्रदर्शित कीजिए : $a - {}^nC_1(a-1) + {}^nC_2(a-2) - \dots + (-1)^n (a-n) = 0$.

Sol. $S = a [{}^nC_0 - {}^nC_1 + {}^nC_2 - \dots + (-1)^n {}^nC_n] + {}^nC_1 - 2 \cdot {}^nC_2 + \dots + (-1)^{n+1} n {}^nC_n$

$$(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \dots + (-1)^n \cdot {}^nC_n x^n$$

$$n(1-x)^{n-1} = - {}^nC_1 + 2 \cdot {}^nC_2x + \dots + (-1)^n n {}^nC_n x^{n-1}$$

put $x = 1$

then $S = 0$

Hindi. $S = a [{}^nC_0 - {}^nC_1 + {}^nC_2 - \dots + (-1)^n {}^nC_n] + {}^nC_1 - 2 \cdot {}^nC_2 + \dots + (-1)^{n+1} n {}^nC_n$

$$(1-x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 - \dots + (-1)^n \cdot {}^nC_n x^n$$

$$n(1-x)^{n-1} = - {}^nC_1 + 2 \cdot {}^nC_2x + \dots + (-1)^n n {}^nC_n x^{n-1}$$

$x = 1$ रखने पर

तब $S = 0$

19. If $(1+x)^n = p_0 + p_1x + p_2x^2 + p_3x^3 + \dots$, then prove that :

यदि $(1+x)^n = p_0 + p_1x + p_2x^2 + p_3x^3 + \dots$, तब सिद्ध करो कि

$$(a) \quad p_0 - p_2 + p_4 - \dots = 2^{n/2} \cos \frac{n\pi}{4} \quad (b) \quad p_1 - p_3 + p_5 - \dots = 2^{n/2} \sin \frac{n\pi}{4}$$

Sol. $(1+x)^n = p_0 + p_1x + p_2x^2 - \dots$

$$(1-x)^n = p_0 - p_1x + p_2x^2 - \dots$$

$$(1+x)^n + (1-x)^n = 2[p_0 + p_2x^2 + p_4x^4 + \dots]$$

Put $x = i$

$$\text{then } p_0 - p_2 + p_4 - \dots = \frac{(1+i)^n + (1-i)^n}{2} = 2^{n/2} \cos \frac{n\pi}{4}$$

$$\text{and } (1+x)^n - (1-x)^n = 2[p_1x + p_3x^3 + \dots]$$

$$\text{or } p_1 - p_3 + p_5 - \dots = \frac{(1+i)^n - (1-i)^n}{2i} = 2^{n/2} \sin \frac{n\pi}{4}$$

Hindi. $(1+x)^n = p_0 + p_1x + p_2x^2 - \dots$

$$(1-x)^n = p_0 - p_1x + p_2x^2 - \dots$$

$$(1+x)^n + (1-x)^n = 2[p_0 + p_2x^2 + p_4x^4 + \dots]$$

$x = i$ रखने पर

$$\text{तब } p_0 - p_2 + p_4 \dots = \frac{(1+i)^n + (1-i)^n}{2} = 2^{n/2} \cos \frac{n\pi}{4}$$

और $(1+x)^n - (1-x)^n = 2 [p_1x + p_3x^3 + \dots]$

$$\text{या } p_1 - p_3 + p_5 \dots = \frac{(1+i)^n - (1-i)^n}{2i} = 2^{n/2} \sin \frac{n\pi}{4}$$

20. Show that if the greatest term in the expansion of $(1+x)^{2n}$ has also the greatest co-efficient, then 'x' lies between, $\frac{n}{n+1}$ & $\frac{n+1}{n}$.

प्रदर्शित करो कि यदि $(1+x)^{2n}$ के प्रसार में अधिकतम पद का गुणांक भी अधिकतम है, तो 'x' का मान $\frac{n}{n+1}$ और $\frac{n+1}{n}$ के बीच में है।

- Sol.** Middle term has greatest co-efficient in this case so $r = n$
मध्य पद का गुणांक महत्तम होता है। अतः इस स्थिति में $r = n$ है

$$\begin{aligned} r &= \left[\frac{2n+1}{1+|x|} \right] \Rightarrow \frac{2n+1}{1+|x|} - 1 < n < \frac{2n+1}{1+|x|} \Rightarrow \frac{2n+1}{n+1} - 1 < |x| \text{ and तथा } |x| < 1 + \frac{1}{n} \\ &\Rightarrow \frac{n}{n+1} < x < \frac{n+1}{n} \end{aligned}$$

21. Prove that if 'p' is a prime number greater than 2, then $[(2+\sqrt{5})^p] - 2^{p+1}$ is divisible by p, where $[.]$ denotes greatest integer function.

सिद्ध करो कि यदि 'p', 2 से बड़ी एक अभाज्य संख्या है, तो $[(2+\sqrt{5})^p] - 2^{p+1}$, p से विभाजित होगा, जहाँ $[.]$ महत्तम पूर्णांक फलन है।

Sol. $[(2+\sqrt{5})^p] - 2^{p+1}$

Let $(\sqrt{5}+2)^p = I + f$ so $[(\sqrt{5}+2)^p] = I$, where I is an integer and $f \in (0, 1)$

$$(\sqrt{5}-2)^p = f' \in (0, 1)$$

$$2[pC_0 2^p + pC_2 2^{p-2} (\sqrt{5})^2 + \dots] = I + f - f'$$

$$\Rightarrow f' - f = 0 \Rightarrow f = f' \Rightarrow [(2+\sqrt{5})^p] - 2^{p+1} = 2[pC_0 2^p + pC_2 2^{p-2}.5 + \dots] - 2^{p+1}$$

$$= pC_2.2^{p-1}.5 + pC_4.2^{p-3}.5^2 + \dots$$

This is always divisible by p because for a prime number p, pC_r ($1 < r < p$) is always divisible by p.

Hindi $[(2+\sqrt{5})^p] - 2^{p+1}$

माना $(\sqrt{5}+2)^p = I + f$ इसलिए $[(\sqrt{5}+2)^p] = I$, जहाँ I एक पूर्णांक है तथा $f \in (0, 1)$

$$(\sqrt{5}-2)^p = f' \in (0, 1)$$

$$2[pC_0 2^p + pC_2 2^{p-2} (\sqrt{5})^2 + \dots] = I + f - f'$$

$$\Rightarrow f' - f = 0 \Rightarrow f = f' \Rightarrow [(2+\sqrt{5})^p] - 2^{p+1}$$

$$= 2[pC_0 2^p + pC_2 2^{p-2}.5 + \dots] - 2^{p+1}$$

$$= pC_2.2^{p-1}.5 + pC_4.2^{p-3}.5^2 + \dots$$

यह हमेशा p से भाजित है क्योंकि एक अभाज्य संख्या p हेतु pC_r ($1 < r < p$) सदैव p से भाजित है।

22. If $\sum_{r=0}^n (-1)^r \cdot {}^n C_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \dots \dots \text{to } m \text{ terms} \right] = k \left(1 - \frac{1}{2^{mn}} \right)$, then find the value of k.

यदि $\sum_{r=0}^n (-1)^r \cdot {}^n C_r \left[\frac{1}{2^r} + \frac{3^r}{2^{2r}} + \frac{7^r}{2^{3r}} + \dots \dots \text{m पदों तक} \right] = k \left(1 - \frac{1}{2^{mn}} \right)$ हो, तो k का मान ज्ञात कीजिए।

Ans. $\frac{1}{2^n - 1}$

$$\begin{aligned} \text{Sol. } & \sum_{r=0}^n (-1)^r \cdot {}^n C_r \left[\frac{1}{2^r} + \left(\frac{3}{4}\right)^r + \left(\frac{7}{8}\right)^r + \dots \dots \text{.m terms} \right] \\ &= \sum_{r=0}^n \left[(-1)^r \cdot {}^n C_r \frac{1}{2^r} + (-1)^r \cdot {}^n C_r \left(\frac{3}{4}\right)^r + \dots \dots \text{.m terms} \right] \\ &= [{}^n C_0 - {}^n C_1 \left(\frac{1}{2}\right) + {}^n C_2 \left(\frac{1}{2}\right)^2 + \dots \dots] + \\ &[{}^n C_0 - {}^n C_1 + {}^n C_2 + \dots \dots] + \dots \dots \text{m terms} \\ &= \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \dots \dots \text{m terms} = \frac{1}{2^n} \left(\frac{1 - \frac{1}{2^{mn}}}{1 - \frac{1}{2^n}} \right) = \frac{1}{2^n - 1} \left(1 - \frac{1}{2^{mn}} \right) \end{aligned}$$

$$\begin{aligned} \text{Hindi. } & \sum_{r=0}^n (-1)^r \cdot {}^n C_r = \left[\frac{1}{2^r} + \left(\frac{3}{4}\right)^r + \left(\frac{7}{8}\right)^r + \dots \dots \text{.m terms} \right] \\ &= \left[(-1)^r \cdot {}^n C_r \frac{1}{2^r} + (-1)^r \cdot {}^n C_r \left(\frac{3}{4}\right)^r + \dots \dots \text{.m terms} \right] \\ &= [{}^n C_0 - {}^n C_1 \left(\frac{1}{2}\right) + {}^n C_2 \left(\frac{1}{2}\right)^2 + \dots \dots] + \\ &[{}^n C_0 - {}^n C_1 + {}^n C_2 + \dots \dots] + \dots \dots \text{m पद} \\ &= \left(1 - \frac{1}{2}\right)^n + \left(1 - \frac{3}{4}\right)^n + \left(1 - \frac{7}{8}\right)^n + \dots \dots \text{m पद} \\ &\frac{1}{2^n} \left(\frac{1 - \frac{1}{2^{mn}}}{1 - \frac{1}{2^n}} \right) = \frac{1}{2^n - 1} \left(1 - \frac{1}{2^{mn}} \right) \end{aligned}$$

23. Given $s_n = 1 + q + q^2 + \dots + q^n$ & $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$,

prove that ${}^{n+1}C_1 + {}^{n+1}C_2 \cdot s_1 + {}^{n+1}C_3 \cdot s_2 + \dots + {}^{n+1}C_{n+1} \cdot s_n = 2^n \cdot S_n$.

यदि $s_n = 1 + q + q^2 + \dots + q^n$ तथा $S_n = 1 + \frac{q+1}{2} + \left(\frac{q+1}{2}\right)^2 + \dots + \left(\frac{q+1}{2}\right)^n$, $q \neq 1$ हो, तो

सिद्ध करो कि ${}^{n+1}C_1 + {}^{n+1}C_2 \cdot s_1 + {}^{n+1}C_3 \cdot s_2 + \dots + {}^{n+1}C_{n+1} \cdot s_n = 2^n \cdot S_n$.

$$\text{Sol. } s_n = \frac{q^{n+1} - 1}{q - 1} \text{ and और } S_n = \frac{\left(\frac{q+1}{2}\right)^{n+1} - 1}{\frac{q+1}{2} - 1}$$

$$\begin{aligned}
 & {}^{n+1}C_1 + {}^{n+1}C_2 s_1 + \dots + {}^{n+1}C_{n+1} s_n \\
 &= {}^{n+1}C_1 + {}^{n+1}C_2 \left(\frac{q^2 - 1}{q-1} \right) + \dots + {}^{n+1}C_{n+1} \frac{q^{n+1} - 1}{q-1} \\
 &= \frac{1}{q-1} [{}^{n+1}C_1 q + {}^{n+1}C_2 q^2 + \dots + {}^{n+1}C_{n+1} q^{n+1} - {}^{n+1}C_1 - {}^{n+1}C_2 - \dots - {}^{n+1}C_{n+1}] \\
 &= \frac{1}{q-1} [(1+q)^{n+1} - 1 - 2^{n+1} + 1] = \frac{1}{q-1} [(1+q)^{n+1} - 2^{n+1}] = \left(\frac{\left(\frac{q+1}{2}\right)^{n+1} - 1}{\frac{q-1}{2}} \right) \cdot 2^n = 2^n S_n
 \end{aligned}$$

- 24.** If $(1+x)^{15} = C_0 + C_1 \cdot x + C_2 \cdot x^2 + \dots + C_{15} \cdot x^{15}$, then find the value of : $C_2 + 2C_3 + 3C_4 + \dots + 14C_{15}$
 यदि $(1+x)^{15} = C_0 + C_1 \cdot x + C_2 \cdot x^2 + \dots + C_{15} \cdot x^{15}$ हो, तो $C_2 + 2C_3 + 3C_4 + \dots + 14C_{15}$ का मान ज्ञात करो।

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Sol. $(1+x)^{15} = C_0 + C_1 x + \dots + C_{15} x^{15}$

Divide by x & then differentiating both side

$$\begin{aligned}
 \frac{(1+x)^{15}}{x} &= \frac{C_0}{x} + C_1 + C_2 x + C_3 x^2 + \dots + C_{15} x^{14} \\
 \cdot \frac{1}{x} 15(1+x)^{14} - \frac{(1+x)^{15}}{x^2} &= -\frac{C_0}{x^2} + C_2 + \dots + 14 C_{15} x^{13}
 \end{aligned}$$

Put $x = 1$ then $C_2 + 2C_3 + \dots + 14C_{15} = 15 \cdot 2^{14} - 2^{15} + 1$

Hindi. $(1+x)^{15} = C_0 + C_1 x + \dots + C_{15} x^{15}$

x से भाग देकर दोनों पक्षों का अवकलन करने पर

$$\begin{aligned}
 \frac{(1+x)^{15}}{x} &= \frac{C_0}{x} + C_1 + C_2 x + C_3 x^2 + \dots + C_{15} x^{14} \\
 \cdot \frac{1}{x} 15(1+x)^{14} - \frac{(1+x)^{15}}{x^2} &= -\frac{C_0}{x^2} + C_2 + \dots + 14 C_{15} x^{13}
 \end{aligned}$$

$x = 1$ रखने पर $C_2 + 2C_3 + \dots + 14C_{15} = 15 \cdot 2^{14} - 2^{15} + 1$

- 25.** Prove that, $\frac{1}{2} {}^n C_1 - \frac{2}{3} {}^n C_2 + \frac{3}{4} {}^n C_3 - \frac{4}{5} {}^n C_4 + \dots + \frac{(-1)^{n+1} n}{n+1} \cdot {}^n C_n = \frac{1}{n+1}$

सिद्ध कीजिए कि : $\frac{1}{2} {}^n C_1 - \frac{2}{3} {}^n C_2 + \frac{3}{4} {}^n C_3 - \frac{4}{5} {}^n C_4 + \dots + \frac{(-1)^{n+1} n}{n+1} \cdot {}^n C_n = \frac{1}{n+1}$

Sol. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$

$\Rightarrow n(1+x)^{n-1} = C_1 + 2.C_2 x + 3.C_3 x^2 + \dots + n.C_n x^{n-1}$

Now multiply by x & integrate from 0 to x

$n(1+x)^{n-1} \cdot x = C_1 x + 2C_2 x^2 + \dots + nC_n x^n$

$$\Rightarrow \frac{C_1}{2} x^2 + \frac{2C_2 x^3}{3} + \frac{3C_3 x^4}{4} + \dots + \frac{n}{n+1} C_n x^{n+1} = x(1+x)^n - \frac{(1+x)^{n+1} - 1}{n+1}$$

Putting $x = -1$

$$\frac{C_1}{2} - \frac{2}{3} C_2 + \frac{3}{4} C_3 + \dots = \frac{1}{n+1}$$

Hindi. $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \dots + C_n x^n$

$\Rightarrow n(1+x)^{n-1} = C_1 + 2.C_2 x + 3.C_3 x^2 + \dots + n.C_n x^{n-1}$

x से गुणा करके 0 से x सीमाओं में समाकलन करने पर

$n(1+x)^{n-1} \cdot x = C_1 x + 2 C_2 x^2 + \dots + n C_n x^n$

$$\Rightarrow \frac{C_1}{2}x^2 + \frac{2C_2}{3}x^3 + \frac{3C_3}{4}x^4 + \dots + \frac{n}{n+1}C_n x^{n+1} = x(1+x)^n - \frac{(1+x)^{n+1} - 1}{n+1}$$

$x = -1$ रखने पर

$$\frac{C_1}{2} - \frac{2}{3}C_2 + \frac{3}{4}C_3 + \dots = \frac{1}{n+1}$$

26. Prove that $\sum_{r=0}^n r^2 {}^n C_r p^r q^{n-r} = npq + n^2 p^2$, if $p + q = 1$.

सिद्ध करो कि $\sum_{r=0}^n r^2 {}^n C_r p^r q^{n-r} = npq + n^2 p^2$ होगा, जबकि $p + q = 1$ हो।

$$\begin{aligned} \text{Sol. } & \sum_{r=0}^n r^2 {}^n C_r p^r q^{n-r} = n \cdot \sum_{r=0}^n r \cdot {}^{n-1} C_{r-1} p^r q^{n-r} \\ & = n \cdot \left[\sum_{r=1}^n (r-1) \cdot {}^{n-1} C_{r-1} p^r q^{n-r} + \sum_{r=1}^n {}^{n-1} C_{r-1} \cdot p^r q^{n-r} \right] \\ & = n \left[(n-1)p^2 \sum_{r=2}^n {}^{n-2} C_{r-2} p^{r-2} q^{n-r} + p \sum_{r=1}^n {}^{n-1} C_{r-1} \cdot p^{r-1} q^{n-r} \right] \\ & = n[(n-1)p^2(p+q)^{n-2} + p(p+q)^{n-1}] \\ & = n[np^2 - p^2 + p] = n^2 p^2 - np^2 + pn = n^2 p^2 + npq \end{aligned}$$

27. Prove that : $(n-1)^2 \cdot C_1 + (n-3)^2 \cdot C_3 + (n-5)^2 \cdot C_5 + \dots = n(n+1)2^{n-3}$

सिद्ध कीजिए कि : $(n-1)^2 \cdot C_1 + (n-3)^2 \cdot C_3 + (n-5)^2 \cdot C_5 + \dots = n(n+1)2^{n-3}$

$$\begin{aligned} \text{Sol. } & (n-1)^2 {}^n C_1 + (n-3)^2 {}^n C_3 + (n-5)^2 {}^n C_5 + \dots \\ & = n^2({}^n C_1 + {}^n C_3 + {}^n C_5 + \dots) - 2n({}^n C_1 + 3{}^n C_3 + 5{}^n C_5 + \dots) + ({}^n C_1 + 9{}^n C_3 + 25{}^n C_5 + \dots) \\ & = n^2 \cdot 2^{n-1} - 2n^2({}^{n-1} C_0 + {}^{n-1} C_2 + {}^{n-1} C_4 + \dots) + n({}^{n-1} C_0 + 3{}^{n-1} C_2 + 5{}^{n-1} C_4 + \dots) \\ & = n^2 \cdot 2^{n-1} - 2n^2 \cdot (2^{n-2}) + n({}^{n-1} C_0 + {}^{n-1} C_2 + {}^{n-1} C_4 + \dots) + n(2{}^{n-1} C_2 + 4{}^{n-1} C_4 + 6{}^{n-1} C_6 + \dots) \\ & = n^2 \cdot 2^{n-1} - n^2 \cdot 2^{n-1} + n(n-1)({}^{n-2} C_1 + {}^{n-2} C_3 + {}^{n-2} C_5 + \dots) \\ & = n \cdot 2^{n-2} + n(n-1)2^{n-3} \\ & = n(n+1)2^{n-3}. \end{aligned}$$

28. Prove that ${}^n C_r + 2 {}^{n+1} C_r + 3 {}^{n+2} C_r + \dots + (n+1) {}^{2n} C_r = {}^n C_{r+2} + (n+1) {}^{2n+1} C_{r+1} - {}^{2n+1} C_{r+2}$

सिद्ध करो ${}^n C_r + 2 {}^{n+1} C_r + 3 {}^{n+2} C_r + \dots + (n+1) {}^{2n} C_r = {}^n C_{r+2} + (n+1) {}^{2n+1} C_{r+1} - {}^{2n+1} C_{r+2}$

Sol. Let ${}^n C_r + 2 {}^{n+1} C_r + 3 {}^{n+2} C_r + \dots = S$

$S = \text{co-efficient of } x^r \text{ in } (1+x)^n + 2(1+x)^{n+1} + 3(1+x)^{n+2} + \dots$

$$\text{Let } S' = (1+x)^n + 2(1+x)^{n+1} + 3(1+x)^{n+2} + \dots + (n+1)(1+x)^{2n} \quad \dots \quad (1)$$

$$(1+x)S' = (1+x)^{n+1} + 2(1+x)^{n+2} + \dots + (n+1)(1+x)^{2n+1} \quad \dots \quad (2)$$

(1) - (2) :

$$\rightarrow S' = (1+x)^n + (1+x)^{n+1} + \dots + (1+x)^{2n} - (n+1)(1+x)^{2n+1}$$

$$\Rightarrow \rightarrow S' = (1+x)^n \left[\frac{(1+x)^{n+1} - 1}{x} \right] - (n+1)(1+x)^{2n+1} \Rightarrow S' = \frac{(1+x)^{2n+1} - (1+x)^n}{-x^2} + \frac{(n+1)(1+x)^{2n+1}}{x}$$

Now $S = \text{co-efficient of } x^r \text{ in } S'$

$$= - {}^{2n+1} C_{r+2} + {}^n C_{r+2} + (n+1) {}^{2n+1} C_{r+1}$$

Hindi. माना ${}^n C_r + 2 {}^{n+1} C_r + 3 {}^{n+2} C_r + \dots = S$

$S = (1+x)^n + 2(1+x)^{n+1} + 3(1+x)^{n+2} + \dots$ में x^r का गुणांक

$$\text{माना } S' = (1+x)^n + 2(1+x)^{n+1} + 3(1+x)^{n+2} + \dots + (n+1)(1+x)^{2n} \quad \dots \quad (1)$$



31. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_n x^n$, then show that the sum of the products of the C_i 's taken two at a time, represented by $\sum_{0 \leq i < j \leq n} C_i C_j$ is equal to $2^{2n-1} - \frac{2n!}{2(n!)^2}$.

यदि $(1+x)^n = C_0 + C_1x + C_2x^2 + \dots + C_n x^n$, तब दर्शाओं कि दो C_i 's को एक साथ लेने पर उनके गुणनफलनों का योग जोकि $\sum_{0 \leq i < j \leq n} C_i C_j$ द्वारा प्रदर्शित होता है $2^{2n-1} - \frac{2n!}{2(n!)^2}$ के बराबर है।

Sol. $(1+x)^n = C_0 + C_1x + \dots + C_n x^n$

$$S = \sum \sum C_i C_j = \frac{1}{2} [(C_0 + C_1 + C_2 + \dots + C_n)^2 - (C_0^2 + C_1^2 + C_2^2 + \dots + C_n^2)]$$

$$\Rightarrow 2S = 2^{2n} - 2^n C_n \quad \Rightarrow S = 2^{2n-1} - \frac{2^n C_n}{2}$$

32. If a_0, a_1, a_2, \dots be the coefficients in the expansion of $(1 + x + x^2)^n$ in ascending powers of x, then prove that :

(i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - \dots = 0$

(ii) $a_0 a_2 - a_1 a_3 + a_2 a_4 - \dots + a_{2n-2} a_{2n} = a_{n+1}$

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$; where $E_1 = a_0 + a_3 + a_6 + \dots$; $E_2 = a_1 + a_4 + a_7 + \dots$ & $E_3 = a_2 + a_5 + a_8 + \dots$

यदि $a_0, a_1, a_2, \dots, (1 + x + x^2)^n$ के प्रसार में x की बढ़ती हुई घातों के गुणांक हैं, तो सिद्ध करो कि :

(i) $a_0 a_1 - a_1 a_2 + a_2 a_3 - \dots = 0$

(ii) $a_0 a_2 - a_1 a_3 + a_2 a_4 - \dots + a_{2n-2} a_{2n} = a_{n+1}$

(iii) $E_1 = E_2 = E_3 = 3^{n-1}$; जहाँ $E_1 = a_0 + a_3 + a_6 + \dots$; $E_2 = a_1 + a_4 + a_7 + \dots$ एवं $E_3 = a_2 + a_5 + a_8 + \dots$

Sol. $(1 + x + x^2)^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots + a_{2n} x^{2n}$

taking $-\frac{1}{x}$ in place of x.

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + \dots$$

(i) $\therefore a_0 a_1 - a_1 a_2 + a_2 a_3 - \dots = \text{coefficient of } x \text{ in } (1 + x + x^2)^n \left(\frac{x^2 - x + 1}{x^2}\right)^n$
 $= \text{coefficient of } x \text{ in } \frac{1}{x^{2n}} (x^4 + x^2 + 1)^n$
 $= 0$

(ii) $a_0 a_2 - a_1 a_3 + a_2 a_4 - \dots = \text{coeff. of } x^2 \text{ in } \frac{1}{x^{2n}} (x^4 + x^2 + 1)^n$
 $= a_{n+1}$

(iii) putting $x = 1, \omega$ & ω^2 respectively we get

$$3^n = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + \dots \quad (1)$$

$$0 = a_0 + a_1 \omega + a_2 \omega^2 + a_3 \omega^3 + a_4 \omega^4 + a_5 \omega^5 + a_6 \omega^6 + \dots \quad (2)$$

$$0 = a_0 + a_1 \omega^2 + a_2 \omega^4 + a_3 \omega^6 + a_4 \omega^8 + a_5 \omega^{10} + a_6 \omega^{12} + \dots \quad (3)$$

on adding

$$3^n = 3(a_0 + a_3 + a_6 + \dots)$$

$$\Rightarrow E_1 = 3^{n-1}$$

(1) + ω^2 (2) + ω (3) gives

$$3^n = 3(a_1 + a_4 + a_7 + \dots)$$

$$\Rightarrow E_2 = 3^{n-1}$$

Similarly

(1) + $\omega(2) + \omega^2(3)$ gives

$$E_3 = 3^{n-1}.$$

Hindi. $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots + a_{2n}x^{2n}$

x के स्थान पर $-\frac{1}{x}$ रखने पर

$$\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)^n = a_0 - \frac{a_1}{x} + \frac{a_2}{x^2} - \frac{a_3}{x^3} + \dots$$

$$(i) \quad \therefore a_0a_1 - a_1a_2 + a_2a_3 \dots = (1 + x + x^2)^n \left(\frac{x^2 - x + 1}{x^2}\right)^n \text{ में } x \text{ का गुणांक}$$

$$= \frac{1}{x^{2n}} (x^4 + x^2 + 1)^n \text{ में } x \text{ का गुणांक}$$

$$= 0$$

$$(ii) \quad a_0a_2 - a_1a_3 + a_2a_4 \dots = \frac{1}{x^{2n}} (x^4 + x^2 + 1)^n \text{ में } x^2 \text{ का गुणांक}$$

$$= a_{n+1}$$

(iii) $x = 1, \omega, \text{ और } \omega^2$ क्रमशः रखने पर

$$3^n = a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + \dots \quad (1)$$

$$0 = a_0 + a_1\omega + a_2\omega^2 + a_3\omega^3 + a_4\omega^4 + a_5\omega^5 + a_6\omega^6 + \dots \quad (2)$$

$$0 = a_0 + a_1\omega^2 + a_2\omega^4 + a_3\omega^6 + a_4\omega^8 + a_5\omega^{10} + a_6\omega^{12} + \dots \quad (3)$$

(1), (2) व (3) को जोड़ने पर

$$3^n = 3(a_0 + a_3 + a_6 + \dots)$$

$$\Rightarrow E_1 = 3^{n-1}$$

(1) + $\omega^2(2) + \omega(3)$ से प्राप्त होता है—

$$3^n = 3(a_1 + a_4 + a_7 + \dots)$$

$$\Rightarrow E_2 = 3^{n-1}$$

इसी प्रकार

(1) + $\omega(2) + \omega^2(3)$ से प्राप्त होता है—

$$E_3 = 3^{n-1}.$$