KINETIC THEORY OF GASES AND THERMODYNAMICS

KINETIC THEORY OF GASES :

Kinetic theory of gases is based on the following basic assumptions.

(a) A gas consists of very large number of molecules. These molecules are identical, perfectly elastic and hard spheres. They are so small that the volume of molecules is negligible as compared with the volume of the gas.

(b) Molecules do not have any preferred direction of motion, motion is completely random.

(c) These molecules travel in straight lines and in free motion most of the time. The time of the collision between any two molecules is very small.

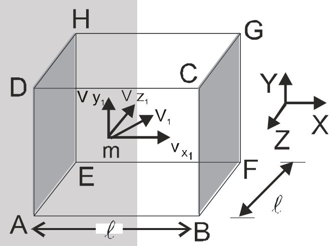
(d) The collision between molecules and the wall of the container is perfectly elastic. It means kinetic energy is conserved in each collision.

(e) The path travelled by a molecule between two collisions is called free path and the mean of this distance travelled by a molecule is called mean free path.

(f) The motion of molecules is governed by Newton's law of motion

(g) The effect of gravity on the motion of molecules is negligible.

EXPRESSION FOR THE PRESSURE OF A GAS:



Let us suppose that a gas is enclosed in a cubical box having length ℓ. Let there are ' N ' identical molecules, each having mass ' m '. Since the molecules are of same mass and perfectly elastic, so their mutual collisions result in the interchange of velocities only. Only collisions with the walls of the container contribute to the pressure by the gas molecules. Let us focus on a molecule having velocity v_1 and components of velocity $v_{x_1}, v_{y_1}, v_{z_1}$ along x, y and z-axis as shown in figure.

$v_1^2 = v^2_{x_1} + v^2_{y_1} + v^2_{z_1}$

The change in momentum of the molecule after one collision with wall BCGF

$mv_{x_1} - (-mv_{x_1}) = 2\, m\, v_{x_1}$.

The time taken between the successive impacts on the face BCGF = $\dfrac{\text{distance}}{\text{velocity}} = \dfrac{2\ell}{v_{x_1}}$

Rate of change of momentum due to collision = $\dfrac{\text{change in momentum}}{\text{time taken}} = \dfrac{2mv_{x_1}}{2\ell / v_{x_1}} = \dfrac{mv^2_{x_1}}{\ell}$

Hence the net force on the wall BCGF due to the impact of n molecules of the gas is :

$$F_x = \frac{mv^2_{x1}}{\ell} + \frac{mv^2_{x2}}{\ell} + \frac{mv^2_{x3}}{\ell} + \ldots\ldots\ldots\ldots + \frac{mv^2_{xn}}{\ell} = \frac{m}{\ell}\left(v^2_{x1} + v^2_{x2} + v^2_{x3} + \ldots\ldots\ldots\ldots\ldots + v^2_{xn}\right) = \frac{mN}{\ell} < v_x^2 >$$

where $< v_x^2 >$ = mean square velocity in x-direction. Since molecules do not favour any particular direction therefore $< v_x^2 > = < v_y^2 > = < v_z^2 >$. But $< v^2 > = < v_x^2 > + < v_y^2 > + < v_z^2 >$

$\Rightarrow \quad < v_x^2 > = \dfrac{< v^2 >}{3}$. Pressure is equal to force divided by area.

$P = \dfrac{F_x}{\ell^2} = \dfrac{M}{3\ell^3} < v^2 > = \dfrac{M}{3V} < v^2 >$. Pressure is independent of x, y, z directions.

Where ℓ^3 = volume of the container = V

M = total mass of the gas, $< v^2 >$ = mean square speed of molecules

$\Rightarrow \quad P = \dfrac{1}{3}\rho < v^2 >$

As PV = n RT , then **total translational K.E. of gas =** $\dfrac{1}{2}$ **M** $< v^2 >$ **=** $\dfrac{3}{2}$ **PV =** $\dfrac{3}{2}$ **n RT**

Translational kinetic energy of 1 molecule = $\frac{3}{2}kT$ **(it is independent of nature of gas)**

$< v^2 > = \frac{3P}{\rho}$ or **v_{rms} =** $\sqrt{\frac{3P}{\rho}} = \sqrt{\frac{3RT}{M_{mole}}} = \sqrt{\frac{3kT}{m}}$

Where v_{rms} is root mean square speed of the gas.

Pressure exerted by the gas is $P = \frac{1}{3}\rho <v^2> = \frac{2}{3} \times \frac{1}{2}\rho <v^2>$ or $P = \frac{2}{3}E$, **$E = \frac{3}{2}P$**

Thus total translational kinetic energy per unit volume (it is called energy density) of the gas is numerically equal to $\frac{3}{2}$ times the pressure exerted by the gas.

IMPORTANT POINTS :

(a) $v_{rms} \propto \sqrt{T}$ and $v_{rms} \propto \frac{1}{\sqrt{M_{mole}}}$

(b) At absolute zero, the motion of all molecules of the gas stops.

(c) At higher temperature and low pressure or at higher temperature and low density, a real gas behaves as an ideal gas.

(d) The mean free path ℓ is the average distance covered by a molecule between two successive collisions :

$$<\ell> = < v > \tau = \frac{1}{\sqrt{2}\, nd^2}$$

where n is the number density and d the diameter of the molecule.

MAXWELL'S DISTRIBUTION LAW :

Distribution Curve – A plot of $\frac{dN(v)}{dv}$ (number of molecules per unit speed interval) against v is known as Maxwell's distribution curve. The total area under the curve is given by the integral $\int_0^\infty \frac{dN(v)}{dv} dv = \int_0^\infty dN(v) = N$.

[Note : The actual formula of $\frac{dN(v)}{dv}$ is not in JEE syllabus.]

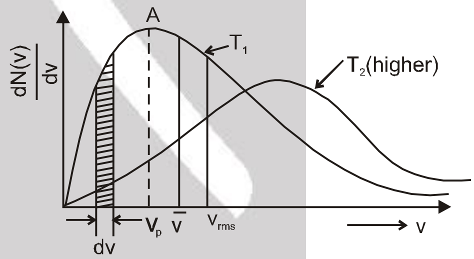


Figure shows the distribution curves for two different temperatures. At any temperature the number of molecules in a given speed interval dv is given by the area under the curve in that interval (shown shaded). This number increases, as the speed increases, upto a maximum and then decreases asymptotically towards zero. Thus, maximum number of the molecules have speed lying within a small range centered about the speed corresponding the peak (A) of the curve. This speed is called the 'most probable speed' v_p or v_{mp}.

The distribution curve is asymmetrical about its peak (the most probable speed v_p) because the lowest possible speed is zero, whereas there is no limit to the upper speed a molecule can attain. Therefore, the average speed $\bar{v}$ is slightly larger than the most probable speed v_p. The root-mean-square speed, v_{rms}, is still larger ($v_{rms} > \bar{v} > v_p$).

Average (or Mean) Speed : $\bar{v} = \sqrt{\frac{8}{\pi}\frac{kT}{m}} = 1.59\sqrt{kT/m}$. (derivation is not in the course)

RMS Speed : $v_{rms} = \sqrt{< v^2 >} = \sqrt{\frac{3kT}{m}} = 1.73\sqrt{\frac{kT}{m}}$.

Most Probable Speed : The most probable speed v_p or v_{mp} is the speed possessed by the maximum number of molecules, and corresponds to the maximum (peak) of the distribution curve. Mathematically, it is obtained by the condition.

$\frac{dN(v)}{dv} = 0$ [by substitution of formula of dN(v) (which is not in the course)]

Hence the most probable speed is $v_p = \sqrt{\frac{2kT}{m}} = 1.41\sqrt{kT/m}$.

From the above expression, we can see that $v_{rms} > \bar{v} > v_p$.

The laws which can be deduced with the help of kinetic theory of gases are below.

(a) Boyle's law
(b) Charle's law
(c) Avogadro's hypothesis
(d) Graham's law of diffusion of gases
(e) Regnault's or Gay Lussac's law
(f) Dalton's Law of Partial Pressure
(g) Ideal Gas Equation or Equation of state

DEGREE OF FREEDOM :

Total number of independent co-ordinates which must be known to completely specify the position and configuration of dynamical system is known as "degree of freedom f". Maximum possible translational degrees of freedom are three i.e. $\left(\frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2\right)$

Maximum possible rotational degrees of freedom are three i.e. $\left(\frac{1}{2}I_x\omega_x^2 + \frac{1}{2}\ I_y\omega_y^2 + \frac{1}{2}\ I_z\omega_z^2\right)$

Vibrational degrees of freedom are two i.e. (Kinetic energy of vibration and Potential energy of vibration)

Mono atomic : (all inert gases Ex. He, Ar etc.) f = 3 (translational)

Diatomic : (gases like H_2 , N_2 , O_2 etc.) f = 5 (3 translational + 2 rotational)

If temp < 70 K for diatomic molecules, then f = 3

If temp in between 250 K to 5000 K, then f = 5

If temp > 5000 K f = 7 [3 translational.+ 2 rotational + 2 vibrational]

MAXWELL'S LAW OF EQUIPARTITION OF ENERGY :

Energy associated with each degree of freedom = $\frac{1}{2}$ kT. If degree of freedom of a molecule is f, then total kinetic energy of that molecule $U = \frac{1}{2}fkT$

INTERNAL ENERGY :

The internal energy of a system is the sum of kinetic and potential energies of the molecules of the system. It is denoted by U. Internal energy (U) of the system is the function of its absolute temperature (T) and its volume (V). i.e., U = f (T, V)

In case of an ideal gas, intermolecular force is zero. Hence its potential energy is also zero. In this case, the internal energy is only due to kinetic energy, which depends on the absolute temperature of the gas. i.e. U = f (T). **For an ideal gas internal energy U = $\frac{f}{2}$ nRT.**

Solved Example

Example 1. A light container having a diatomic gas enclosed within is moving with velocity V. Mass of the gas is M and number of moles is n.

(i) What is the kinetic energy of gas w.r.t. centre of mass of the system?

(ii) What is K.E. of gas w.r.t. ground?

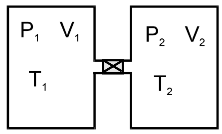
Solution : (i) K.E. = $\frac{5}{2}$ nRT

mass of gas = M
temperature T → V

(ii) Kinetic energy of gas w.r.t. ground = Kinetic energy of gas w.r.t. centre of mass + Kinetic energy of centre of mass w.r.t. ground.

K.E. = $\frac{1}{2}$ MV^2 + $\frac{5}{2}$nRT

Example 2. Two non conducting containers having volume V_1 and V_2 contain mono atomic and diatomic gases respectively. They are connected as shown in figure. Pressure and temperature in the two containers are P_1, T_1 and P_2, T_2 respectively. Initially stop cock is closed, if the stop cock is opened find the final pressure and temperature.



Solution : $n_1 = \dfrac{P_1V_1}{RT_1}$ $\quad n_2 = \dfrac{P_2V_2}{RT_2}$

$n = n_1 + n_2$ (number of moles are conserved)

Finally pressure in both parts & temperature of the both the gases will become equal.

$$\frac{P(V_1+V_2)}{RT} = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2}$$

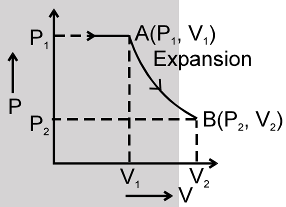
From energy conservation $\frac{3}{2}n_1RT_1 + \frac{5}{2}n_2RT_2 = \frac{3}{2}n_1RT + \frac{5}{2}n_2RT$

$$\Rightarrow \; T = \frac{(3P_1V_1+5P_2V_2)T_1T_2}{3P_1V_1T_2+5P_2V_2T_1} \;\Rightarrow\; P = \left(\frac{3P_1V_1+5P_2V_2}{3P_1V_1T_2+5P_2V_2T_1}\right)\left(\frac{P_1V_1T_2+P_2V_2T_1}{V_1+V_2}\right)$$

INDICATOR DIAGRAM :

A graph representing the variation of pressure or variation of temperature or variation of volume with each other is called indicator diagram.

Indicator diagram → (P, V)
Indicator diagram → (P, T)
Indicator diagram → (V, T)



(a) Every point of Indicator diagram represents a unique state (P, V, T) of gases.

(b) Every curve on Indicator diagram represents a unique process.

THERMODYNAMICS

Thermodynamics is mainly the study of exchange of heat energy between bodies and conversion of the same into mechanical energy and vice-versa.

THERMODYNAMIC SYSTEM

Collection of an extremely large number of atoms or molecules confined within certain boundaries such that it has a certain value of pressure (P), volume (V) and temperature (T) is called a **thermodynamic system**. Anything outside the thermodynamic system to which energy or matter is exchanged is called its surroundings. Taking into consideration the interaction between a system and its surroundings thermodynamic system is divided into three classes :

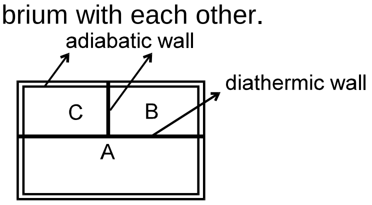
(a) **Open system :** A system is said to be an open system if it can exchange both energy and matter with its surroundings.

(b) **Closed system :** A system is said to be closed system if it can exchange only energy (not matter with its surroundings).

(c) **Isolated system :** A system is said to be isolated if it can neither exchange energy nor matter with its surroundings.

ZEROTH LAW OF THERMODYNAMICS :

If two systems (B and C) are separately in thermal equilibrium with a third one (A), then they themselves are in thermal equilibrium with each other.

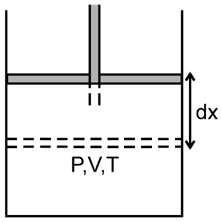


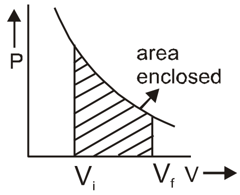
EQUATION OF STATE (FOR AN IDEAL GASES) :

The relation between the thermodynamic variables (P, V, T) of the system is called equation of state. The equation of state for an ideal gas of n moles is given by

$PV = nRT$,

WORK DONE BY A GAS :





Let P and V be the pressure and volume of the gas. If A be the area of the piston, then force exerted by gas on the piston is, $F = P \times A$.

Let the piston move through a small distance dx during the expansion of the gas. Work done for a small displacement dx is $dW = F\,dx = PA\,dx$

Since A dx = dV, increase in volume of the gas is dV

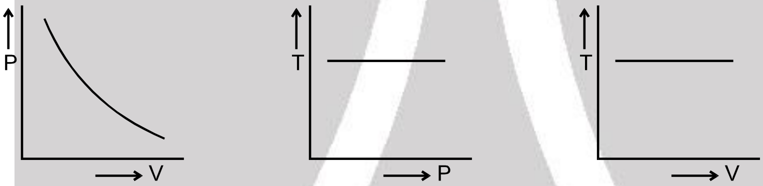
$\Rightarrow$ $dW = P\,dV$

or $W = \int dW = \int PdV$

Area enclosed under P-V curve gives work done during process.

DIFFERENT TYPES OF PROCESSES :

(a) Isothermal Process : T = constant [Boyle's law applicable] PV = constant

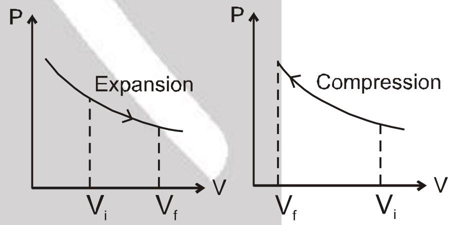


There is exchange of heat between system and surroundings. System should be compressed or expanded very slowly so that there is sufficient time for exchange of heat to keep the temperature constant.

Slope of P–V curve in isothermal process:

PV = constant = C $\Rightarrow$ $\dfrac{dP}{dV} = -\dfrac{P}{V}$

Work done in isothermal process:



$W = nRT\ \ell n\ \dfrac{V_f}{V_i}$

$$\begin{bmatrix} \text{If } V_f > V_i \text{ then } W \text{ is positive} \\ \text{If } V_f < V_i \text{ then } W \text{ is negative} \end{bmatrix}$$

$$W = \left[2.303\ nRT \log_{10} \dfrac{V_f}{V_i}\right]$$

Internal energy in isothermal process : U = f (T) $\Rightarrow$ $\Delta U = 0$

(b) Iso- choric Process (Isometric Process) :

V = constant

$\Rightarrow$ change in volume is zero

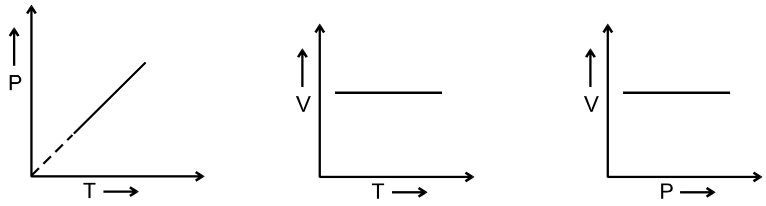
$\Rightarrow$ $\dfrac{P}{T}$ is constant

$\dfrac{P}{T} = \text{const.}$ (Gay lussac's law)

Work done in isochoric process :

Since change in volume is zero therefore $dW = P\ dV = 0$

Indicator diagram of isochoric process :



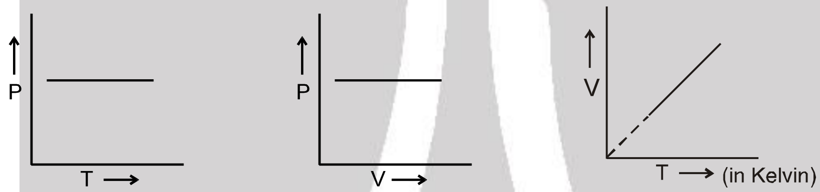
Change in internal energy in isochoric process : $\Delta U = n\,\frac{f}{2}R\,\Delta T$

Heat given in isochoric process : $\Delta Q = \Delta U = n\frac{f}{2}R\,\Delta T$

(c) Isobaric Process : Pressure remains constant in isobaric process

$\therefore$ P = constant $\Rightarrow$ $\frac{V}{T}$ = constant

Indicator diagram of isobaric process :



Work done in isobaric process : $\Delta W = P\,\Delta V = P\,(V_{final} - V_{initial}) = nR\,(T_{final} - T_{initial})$

Change in internal energy in isobaric process : $\Delta U = n\,C_V\,\Delta T$

Heat given in isobaric process : $\Delta Q = \Delta U + \Delta W$

$\Delta Q = n\,\frac{f}{2}R\,\Delta T + P\,[V_f - V_i] = n\frac{f}{2}R\,\Delta T + nR\,\Delta T$

Above expression gives an idea that to increase temperature by ΔT in isobaric process heat required is more than in isochoric process.

(d) Cyclic Process : In the cyclic process initial and final states are same therefore initial state = final state

Work done = Area enclosed under P-V diagram.

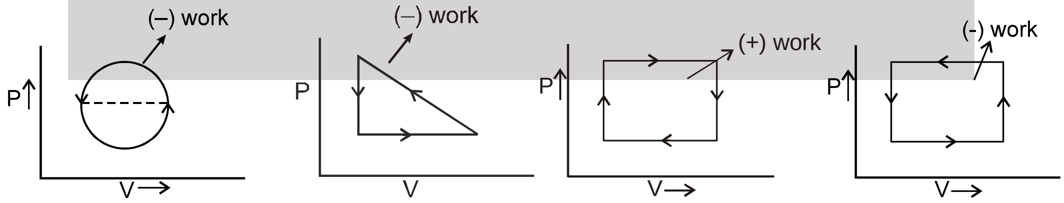
Change in internal Energy $\Delta U = 0$

$\Delta Q = \Delta U + \Delta W$

$\therefore$ $\Delta Q = \Delta W$

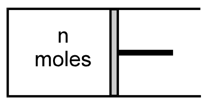
If the process on P-V curve is clockwise, then net work done is (+ve) and vice-versa.

The graphs shown below explains when work is positive and when it is negative



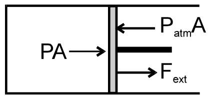
Solved Example

Example 3. The cylinder shown in the figure has conducting walls and temperature of the surrounding is T, the piston is initially in equilibrium, the cylinder contains n moles of a gas. Now the piston is displaced slowly by an external agent to make the volume double of its initial value. Find work done by external agent in terms of n, R, T



Solution : **1st Method :** Work done by external agent is positive, because F_{ext} and displacement are in the same direction. Since walls are conducting therefore temperature remains constant.

Applying equilibrium condition when pressure of the gas is P



$PA + F_{ext} = P_{atm}\,A$

$F_{ext} = P_{atm}\,A - PA$

$$W_{ext} = \int_0^d F_{ext}\,dx$$

$$= \int_0^d P_{atm}A\,dx - \int_0^d PA\,dx = P_{atm}\,A\int_0^d dx - \int_V^{2V}\frac{nRT}{V}dV$$

$= P_{atm}\,Ad - nRT\ln 2 = P_{atm}\,.\,V_0 - nRT\ln 2 = nRT\,(1 - \ln 2)$

2nd Method : Applying work energy theorem on the piston

As $W_{all} = \Delta K.E$

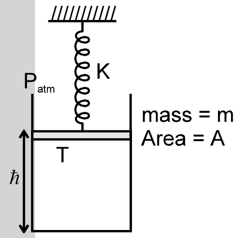
$\Delta K.E = 0$ (given)

$W_{gas} + W_{atm} + W_{ext} = 0$

$$nRT\ln\frac{V_f}{V_i} - nRT + W_{ext} = 0$$

$W_{ext} = nRT\,(1 - \ln 2)$

Example 4. A non conducting piston of mass m and area of cross section A is placed on a non conducting cylinder as shown in figure. Temperature, spring constant, height of the piston are given by T, K, h respectively. Initially spring is relaxed and piston is at rest. Find



(i) Number of moles

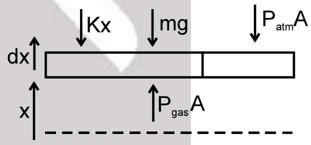
(ii) Work done by gas to displace the piston by distance d when the gas is heated slowly.

(iii) Find the final temperature

Solution : (i) $PV = nRT \Rightarrow \left(P_{atm} + \frac{mg}{A}\right)Ah = nRT$

$$\Rightarrow n = \frac{\left(P_{atm} + \frac{mg}{A}\right)Ah}{RT}$$

(ii) **1st method :** Applying Newton's law on the piston $mg + P_{atm}\,A + Kx = P_{gas}A$



$$W_{gas} = \int_0^d P_{gas}A\,dx$$

$$\int_0^d = (mg + P_{atm}\,A + Kx)\,dx.$$

$$\Rightarrow W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$

2nd method : Applying work energy theorem on the piston

$W_{all} = \Delta KE$

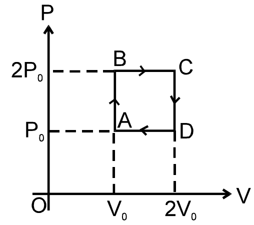
Since piston moves slowly therefore $\Delta KE = 0$

$W_{gravity} + W_{gas} + W_{atm} + W_{spring} = 0$

$$-\,mgd + W_{gas} + (-P_{atm}\,Ad) + [-(\frac{1}{2}Kd^2 - 0)] = 0$$

$$\Rightarrow W_{gas} = mgd + P_{atm}dA + \frac{1}{2}Kd^2$$

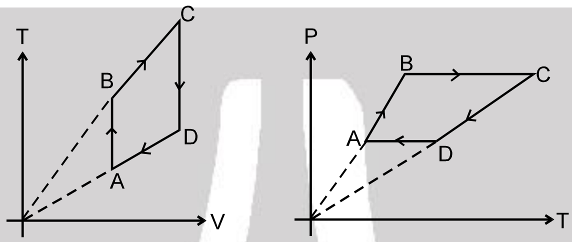
Example 5. Find out the work done in the given graph. Also draw the corresponding T-V curve and P-T curve.



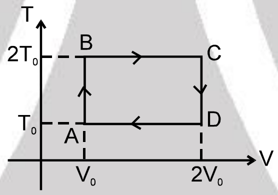
Solution : Since in P-V curves area under the cycle is equal to work done therefore work done by the gas is equal to $P_0 V_0$.

Line A B and CD are isochoric line, line BC and DA are isobaric line.

$\therefore$ the T-V curve and P-T curve are drawn as shown.



Example 6. T-V curve of cyclic process is shown below, number of moles of the gas are n find the total work done during the cycle.



Solution : Since path AB and CD are isochoric therefore work done is zero during path AB and CD. Process BC and DA are isothermal, therefore

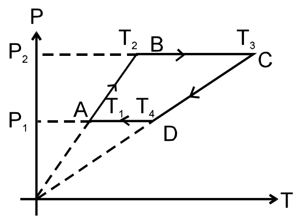
$$W_{BC} = nR2T_0 \, \ell n \, \frac{V_C}{V_B} = 2nRT_0 \, \ell n \, 2$$

$$W_{DA} = nRT_0 \, \ell n \, \frac{V_A}{V_D} = -nRT_0 \, \ell n \, 2$$

Total work done = $W_{BC} + W_{DA} = 2nRT_0 \, \ell n \, 2 - nRT_0 \, \ell n \, 2$

$= nRT_0 \, \ell n \, 2$

Example 7. P-T curve of a cyclic process is shown. Find out the work done by the gas in the given process if number of moles of the gas are n.

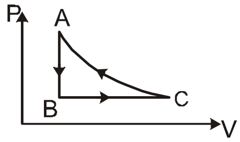


Solution : Since path AB and CD are isochoric therefore work done during AB and CD is zero. Path BC and DA are isobaric.

Hence $W_{BC} = nR\Delta T = nR(T_3 - T_2)$

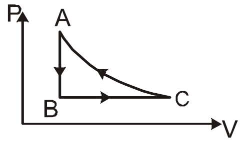
$W_{DA} = nR(T_1 - T_4)$. Total work done = $W_{BC} + W_{DA} = nR(T_1 + T_3 - T_4 - T_2)$

Example 8. In figure, a cyclic process ABCA of 3 moles of an ideal gas is given. The temperatures of the gas at B and C are 500 K and 1000 K respectively. If the work done on the gas in process CA is 2500 J then find the net heat absorbed or released by an ideal gas. Take R = 25/3 J/mol–K.



Solution : The change in internal energy during the cyclic process is zero. Hence, the heat supplied to the gas is equal to the work done by it. Hence,

$\Delta Q = W_{AB} + W_{BC} + W_{CA}$ .....(i)



The work done during the process AB is zero

$W_{BC} = P_B (V_C - V_B) = nR(T_C - T_B) = (3 \text{ mol}) (25/3 \text{ J/mol–K}) (500 \text{ K}) = 12500 \text{ J}$

As $W_{CA} = -2500$ J (given)

$\therefore \Delta Q = 0 + 12500 - 2500$ [from ......(i)]

$\Delta Q = 10$ kJ

FIRST LAW OF THERMODYNAMICS :

The first law of thermodynamics is the law of conservation of energy. It states that if a system absorbs heat dQ and as a result the internal energy of the system changes by dU and the system does a work dW, then **dQ = dU + W**. But, W = P dV

dQ = dU + P dV

which is the mathematical statement of first law of thermodynamics.

Heat gained by a system, work done by a system and increase in internal energy are taken as positive. Heat lost by a system, work done on a system and decrease in internal energy are taken as negative.

Solved Examples

Example 9. 1 gm water at 100ºC is heated to convert into steam at 100ºC at 1 atm. Find out change in internal energy of water. It is given that volume of 1 gm water at 100ºC = 1 cc, volume of 1 gm steam at 100ºC = 1671 cc. Latent heat of vaporization = 540 cal/g. (Mechanical equivalent of heat J = 4.2J/cal.)

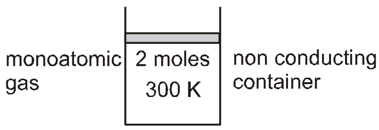
Solution : From first law of thermodynamic $\Delta Q = \Delta U + W$

$\Delta Q = mL = 1 \times 540$ cal. = 540 cal.

$$W = P\Delta V = \frac{10^5(1671-1)\times 10^{-6}}{4.2} = \frac{10^5 \times (1670) \times 10^{-6}}{4.2} = 40 \text{ cal.}$$

$\Delta U = 540 - 40 = 500$ cal.

Example 10. Two moles of a monoatomic gas at 300 K are kept in a non conducting container enclosed by a piston. Gas is now compressed to increase the temperature from 300 K to 400 K. Find work done by the gas (R = $\frac{25}{3}$ J/mol–K)

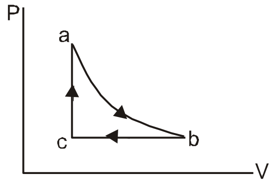


Solution : $\Delta Q = \Delta U + W$

Since container is non conducting therefore $\Delta Q = 0 = \Delta U + W$

$$\Rightarrow W = -\Delta U = -n\frac{f}{2}R\,\Delta T = -2 \times \frac{3}{2}R\,(400 - 300) = -3 \times \frac{25}{3} \times 100 \text{ J} = -2500 \text{ J}$$

Example 11. In figure, a sample of an ideal gas is taken through the cyclic process abca. 800 J of work is done by the gas during process ab. If gas absorb no heat in process ab, rejects 100 J of heat during bc and absorb 500 J of heat during process ca. Then (a) find the internal energy of the gas at b and c if it is 1000 J at a. (b) Also calculate the work done by the gas during the part bc.



Solution : (a) In process ab $\Delta Q = \Delta U + W$

$0 = U_B - 100 + 800$

$U_B = 200$ J

for Cyclic process $\Delta Q = \Delta U + W$

$400 = 0 + 800 + W_{BC}$

$W_{BC} = -400$ J

for process bc ; $\Delta Q = \Delta U + W$

$-100 = -400 + U_C - 200$

$\therefore \quad U_C = 500$ J

Example 12. Two moles of nitrogen gas is kept in a cylinder of cross-section area 10 cm^2. The cylinder is closed by a light frictionless piston. Now the gas is slowly heated such that the displacement of piston during process is 50 cm, find the rise in temperature of gas when 200 J of heat is added in it. (Atmospheric pressure = 100 kPa, R = 25/3 J/mol-K)

Solution : The change in internal energy of the gas is

$\Delta U = 5/2\ nR\ (\Delta T) = 5/2 \times 2R \times (\Delta T) = 5R \times \Delta T$

The heat given to the gas = 200 J

The work done by the gas is

$W = \Delta Q - \Delta U = 200\ J - 5R\ \Delta T$ ....(i)

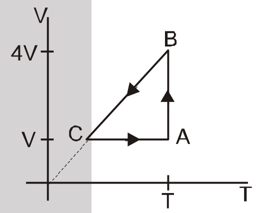
As the distance moved by the piston is 50 cm, $\therefore$ the work done is

$\Delta W = P\Delta V = PA\Delta x = 10^5 \times 10 \times 10^{-4} \times 50 \times 10^{-2}$ ....(ii)

From (i) and (ii)

$\Delta T = 18/5\ K = 3.6\ K$

Example 13. An ideal gas initially has pressure P volume V and temperature T. It is isothermally expanded to four times of its original volume, then it is compressed at constant pressure to attain its original volume V. Finally, the gas is heated at constant volume to get the original temperature T. (a) Draw V-T curve (b) Calculate the total work done by the gas in the process. (given $\ell n2 = 0.693$)



Solution : (a) V–T curve for all process is shown in figure. The initial state is represented by the point A. In the first step, it is isothermally expanded to a volume 4V. This is shown by AB. Then the pressure is kept constant and the gas is compressed to the initial volume V. From the ideal gas equation, V/T is constant at constant pressure (PV = nRT). Hence, the process is shown by a line BC which passes through the origin. At point C, the volume is V. In the final step, the gas is heated at constant volume to a temperature T. This is shown by CA. The final state is the same as the initial state.

(b) Total work done by gas, $W_{Total} = W_{AB} + W_{BC} + W_{CA}$

$W_{AB} = nRT\ \ln \dfrac{4V}{V} = 2nRT\ \ln 2 = 2PV\ \ln 2.$

Also $P_A V_A = P_B V_B$ (As AB is an isothermal process) or, $P_B = \dfrac{P_A V_A}{V_B} = \dfrac{PV}{4V} = \dfrac{P}{4}$.

In the step BC, the pressure remains constant. Hence the work done is,

$W_{BC} = \dfrac{P}{4}\ (V - 4V) = -\dfrac{3PV}{4}$.

In the step CA, the volume remains constant and so the work done is zero. The net work done by the gas in the cyclic process is

$W = W_{AB} + W_{BC} + W_{CA} = 2PV\ \ln 2 - \dfrac{3PV}{4} + 0$

Hence, the work done by the gas 0.636 PV.

Example 14. A diatomic gas is heated at constant pressure. If 105 J of heat is given to the gas, find (a) the change in internal energy of the gas (b) the work done by the gas.

Solution : Suppose the volume changes from V_1 to V_2 and the temperature changes from T_1 to T_2. The heat supplied is $\Delta Q = \Delta U + P\Delta V = \Delta U + nR\Delta T = \Delta U + \frac{2\Delta U}{f}$ $\left[\Delta U = \frac{nfR\Delta T}{2}\right]$

(a) The change in internal energy is $\Delta Q = \Delta U \left[1 + \frac{2}{f}\right]$

$105 = \Delta U\left[1 + \frac{2}{5}\right]$, $\Delta U = 75$ J

(b) The work done by the gas is $W = \Delta Q - \Delta U$
$= 105\text{ J} - 75\text{ J} = 30$ J.

Efficiency of a cycle (η) :

$$\eta = \frac{\text{total Mechanical work done by the gas in the whole process}}{\text{Heat absorbed by the gas (only + ve)}}$$

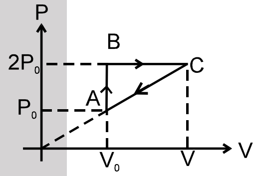
$$= \frac{\text{area under the cycle in P-V curve}}{\text{Heat injected into the system}}$$

$\eta = \left(1 - \frac{Q_2}{Q_1}\right)$ for Heat Engine,

$\eta = \left(1 - \frac{T_2}{T_1}\right)$ for Carnot cycle

Solved Examples

Example 15. n moles of a diatomic gas has undergone a cyclic process ABCA as shown in figure. Temperature at A is T_0. Find



(i) Volume at C ?
(ii) Maximum temperature ?
(iii) Total heat given to gas ?
(iv) Is heat rejected by the gas, if yes how much heat is rejected ?
(v) Find out the efficiency

Solution : (i) For process AC, $P \propto V$

$\frac{2P_0}{V_c} = \frac{P_0}{V_0} \Rightarrow V_c = 2V_0$

(ii) Since process AB is isochoric hence $\frac{P_A}{T_A} = \frac{P_B}{T_B} \Rightarrow T_B = 2T_0$

Since process BC is isobaric therefore $\frac{T_B}{V_B} = \frac{T_C}{V_C}$

$\Rightarrow T_C = 2T_B = 4T_0$ $\therefore$ $T_{max} = 4T_0$

(iii) Since process is cyclic therefore

$\Delta Q = W$ = area under the cycle = $\frac{1}{2}P_0V_0$.

(iv) Since ΔU and ΔW both are negative in process CA

$\therefore$ ΔQ is negative in process CA and heat is rejected in process CA

$\Delta Q_{CA} = W_{CA} + \Delta U_{CA} = -\frac{1}{2}[P_0 + 2P_0]V_0 - \frac{5}{2}nR(T_C - T_A)$

$= -\frac{1}{2}[P_0 + 2P_0]V_0 - \frac{5}{2}nR\left(\frac{4P_0V_0}{nR} - \frac{P_0V_0}{nR}\right) = -9P_0V_0$ (Heat rejected)

(v) η = efficiency of the cycle = $\frac{\text{work done by the gas}}{\text{heat injected}} \Rightarrow \eta = \frac{P_0V_0/2}{Q_{injected}} \times 100$

$\Delta Q_{inj} = \Delta Q_{AB} + \Delta Q_{BC}$

$= \left[\frac{5}{2}nR(2T_0 - T_0)\right] + \left[\frac{5}{2}nR(2T_0) + 2P_0(2V_0 - V_0)\right] = \frac{19}{2}P_0V_0.$

$\eta = \frac{100}{19}\%$

SPECIFIC HEAT :

The specific heat capacity of a substance is defined as the heat supplied per unit mass of the substance per unit rise in the temperature. If an amount ΔQ of heat is given to a mass m of the substance and its temperature rises by ΔT, the specific heat capacity s is given by equation

$$s = \frac{\Delta Q}{m\Delta T}$$

The molar heat capacity of a gas is defined as the heat given per mole of the gas per unit rise in the temperature. The molar heat capacity at constant volume, denoted by C_V, is :

$$C_V = \left(\frac{\Delta Q}{n\,\Delta T}\right)_{\text{constant volume}} = \frac{f}{2}R$$

and the molar heat capacity at constant pressure, denoted by C_p is,

$$C_P = \left(\frac{\Delta Q}{n\,\Delta T}\right)_{\text{constant pressure}} = \left(\frac{f}{2}+1\right)R$$

where n is the amount of the gas in number of moles and f is degree of freedom. Quite often, the term specific heat capacity or specific heat is used for molar heat capacity. It is advised that the unit be carefully noted to determine the actual meaning. The unit of specific heat capacity is J/kg-K whereas that of molar heat capacity is J/mol–K.

MOLAR HEAT CAPACITY OF IDEAL GAS IN TERMS OF R :

(i) For a monoatomic gas f = 3

$$C_V = \frac{3}{2}R,\ C_P = \frac{5}{2}R \quad \Rightarrow \quad \gamma = \frac{C_P}{C_V} = \frac{5}{3} = 1.67$$

(ii) For a diatmoc gas f = 5

$$C_V = \frac{5}{2}R,\ C_P = \frac{7}{2}R,\ \gamma = \frac{C_P}{C_V} = 1.4$$

(iii) For a Triatomic gas f = 6

$$C_V = 3R, \quad C_P = 4R$$

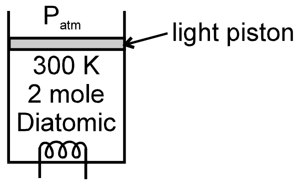
$\gamma = \frac{C_P}{C_V} = \frac{4}{3} = 1.33$ [Note for CO_2; f = 5, it is linear]

In general if f is the degree of freedom of a molecule, then,

$$C_V = \frac{f}{2}R, \quad C_P = \left(\frac{f}{2}+1\right)R, \quad \gamma = \frac{C_P}{C_V} = \left[1+\frac{2}{f}\right]$$

Solved Example

Example 16. Two moles of a diatomic gas at 300 K are enclosed in a cylinder as shown in figure. Piston is light. Find out the heat given if the gas is slowly heated to 400 K in the following three cases.



(i) Piston is free to move
(ii) If piston does not move
(iii) If piston is heavy and movable.

Solution : (i) Since pressure is constant

$$\therefore \quad \Delta Q = nC_P\,\Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300) = 700\ R$$

(ii) Since volume is constant

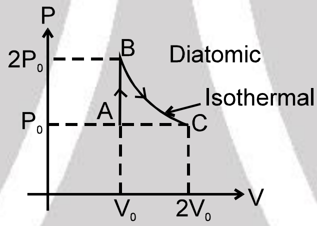
$\therefore$ W = 0 and $\Delta Q = \Delta U$ (from first law)

$$\Delta Q = \Delta U = nC_V\Delta T = 2 \times \frac{5}{2} \times R \times (400 - 300) = 500\ R$$

(iii) Since pressure is constant

$$\therefore \quad \Delta Q = nC_P\,\Delta T = 2 \times \frac{7}{2} \times R \times (400 - 300) = 700\ R$$

Example 17. P-V curve of a diatomic gas is shown in the figure. Find the total heat given to the gas in the process AB and BC



Solution : From first law of thermodynamics

$\Delta Q_{ABC} = \Delta U_{ABC} + W_{ABC}$

$$W_{ABC} = W_{AB} + W_{BC} = 0 + nR\,T_B \ln\frac{V_C}{V_B} = nR\,T_B \ln\frac{2V_0}{V_0}$$

$= nRT_B \ln 2 = 2P_0\,V_0 \ln 2$

$$\Delta U = nC_V\,\Delta T = \frac{5}{2}\,(2P_0V_0 - P_0V_0) \quad \Rightarrow \quad \Delta Q_{ABC} = \frac{5}{2}P_0V_0 + 2P_0V_0 \ln 2.$$

Example 18. From given data, calculate the value of mechanical equivalent of heat. The specific heat capacity of air at constant volume 170 cal/kg-K, $\gamma = C_p/C_v = 1.4$ and the density of air at STP is 1.29 kg/m^3. Gas constant R = 8.3 J/mol-K.

Solution : Using pV = nRT, the volume of 1 mole of air at STP is

$$V = \frac{nRT}{p} = \frac{(1\ \text{mol})\times(8.3\,\text{J/mol}-\text{K})\times(273\text{K})}{1.01\times10^5\,\text{N/m}^2} = 0.0224\text{m}^3.$$

The mass of 1 mole is, therefore, (1.29 kg/m^3) × (0.0224 m^3) = 0.029 kg.

The number of moles in 1 kg is $\frac{1}{0.029}$. The molar heat capacity at constant volume is

$$C_v = \frac{170\ \text{cal}}{(1/0.029)\ \text{mol}-\text{K}} = 4.93\ \text{cal/mol-K}.$$

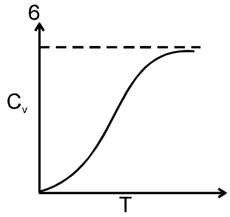
Hence, $C_p = \gamma C_v$ = 1.4 × 4.93 cal/mol-K
or, $C_p - C_v$ = 0.4 × 4.93 cal/mol-K
= 1.97 cal/mol-K.
Also, $C_p - C_v$ = R = 8.3 J/mol-K.
Thus, 8.3 J = 1.97 cal.

The mechanical equivalent of heat is $\frac{8.3\text{J}}{1.97\text{cal}}$ = 4.2 J/cal.

Average Molar Specific Heat of Metals : [Dulong and Petit law]

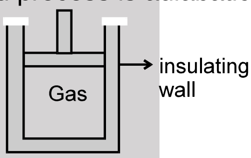
At room temperature average molar specific heat of all metals are same and is nearly equal to 3R (6 cal. mol^{-1} K^{-1}).

[**Note :** Temp. above which the metals have constant C_V is called Debye temp.]



MAYER'S EQUATION : $C_P - C_V = R$ (for ideal gases only)

Adiabatic process : When no heat is supplied or extracted from the system the process is called adiabatic. Process is sudden so that there is no time for exchange of heat. If walls of a container are thermally insulated no heat can cross the boundary of the system and process is adiabatic.



Equation of adiabatic process is given by

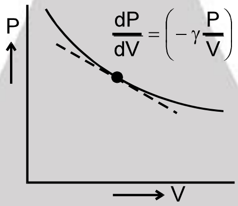
PV^{γ} = constant [Poisson Law]

$T^{\gamma} P^{1-\gamma}$ = constant

$T V^{\gamma-1}$ = constant

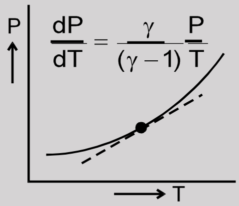
Slope of P–V–curve in adiabatic process : Since PV^{γ} is a constant

$$\therefore \quad \frac{dP}{dV} = -\gamma\left(\frac{P}{V}\right)$$

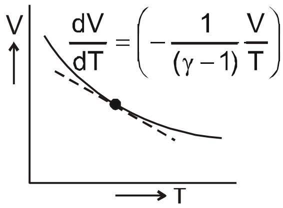


Slope of P–T–curve in adiabatic process : Since $T^{\gamma} P^{1-\gamma}$ is a constant

$$\therefore \quad \frac{dP}{dT} = -\frac{\gamma}{(1-\gamma)}\frac{P}{T} = \frac{(\gamma)}{(\gamma-1)}\frac{P}{T}$$



Slope of T–V–curve : $\dfrac{dV}{dT} = -\dfrac{1}{(\gamma-1)}\dfrac{V}{T}$



Work done in adiabatic Process :

$$\Delta W = -\Delta U = nC_V(T_i - T_f) = \frac{P_iV_i - P_fV_f}{(\gamma-1)} = \frac{nR(T_i - T_f)}{\gamma-1}$$

work done by system is (+ve) , if $T_i > T_f$ (For expansion)

work done on the system is (–ve) if $T_i < T_f$ (For compression)

Example 19. A container having slightly conducting walls contains air. The initial temperature and volume are 47ºC (equal to the temperature of the surrounding) and 400cm^3 respectively. Find the rise in the temperature if the gas is compressed to 200cm^3 (a) in a short time (b) in a long time. Take $\gamma = 1.4$. [$2^{0.4} = 1.3$]

Solution : (a) When the gas is compressed in a short time, the process is adiabatic. Thus,

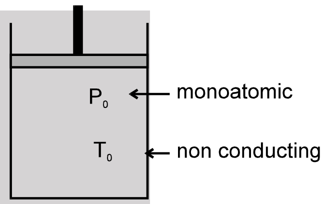
$$T_2V_2^{\gamma-1} = T_1V_1^{\gamma-1} \quad \text{or} \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = (320\text{ K}) \times \left[\frac{400}{200}\right]^{0.4} = 416\text{ K}.$$

Rise in temperature = $T_2 - T_1$ = 96 K.

(b) When the gas is compressed in a long time, the process is isothermal. Thus, the temperature remains same that is 47ºC.

$\therefore$ The rise in temperature = 0.

Example 20. An ideal monoatomic gas is enclosed in a non conducting cylinder having a piston which can move freely. Suddenly gas is compressed to 1/8 of its initial volume. Find the final pressure and temperature if initial pressure and temperature are P_0 and T_0 respectively.



Solution : Since process is adiabatic therefore

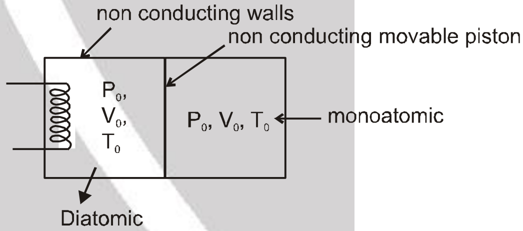
$$P_0 V^{\frac{5}{3}} = P_{final}\left(\frac{V}{8}\right)^{5/3} . \left[\gamma = \frac{C_P}{C_V} = \frac{5R}{2} / \frac{3R}{2} = \frac{5}{3}\right]$$

$P_{final} = 32\ P_0$.

Since process is adiabatic therefore

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \quad \Rightarrow \quad T_0 V_0^{2/3} = T_{final}\left(\frac{V_0}{8}\right)^{2/3} \quad \Rightarrow \quad T = 4T_0$$

Example 21. A cylindrical container having non conducting walls is partitioned in two equal parts such that the volume of each part is V_0. A movable non conducting piston is kept between the two parts. Gas on left is slowly heated so that the gas on right is compressed upto volume $\frac{V_0}{8}$.



Find pressure and temperature on both sides if initial pressure and temperature, were P_0 and T_0 respectively. Also find heat given by the heater to the gas. (Number of moles in each part is n)

Solution : Since the process on right is adiabatic therefore

PV^{γ} = constant

$\Rightarrow \quad P_0 V_0^{\gamma} = P_{final} (V_0 / 8)^{\gamma} \quad \Rightarrow \quad P_{final} = 32\ P_0$

$\quad\ T_0 V_0^{\gamma-1} = T_{final} (V_0/8)^{\gamma-1} \quad \Rightarrow \quad T_{final} = 4T_0$

Let volume of the left part is V_1

$$\Rightarrow \quad 2V_0 = V_1 + \frac{V_0}{8} \qquad \Rightarrow \quad V_1 = \frac{15V_0}{8}.$$

Since number of moles on left part remains constant therefore for the left part $\frac{PV}{T}$ = constant.

Final pressure on both sides will be same

$$\Rightarrow \quad \frac{P_0V_0}{T_0} = \frac{P_{final}V_1}{T_{final}} \quad \Rightarrow \quad T_{final} = 60\ T_0$$

$\Delta Q = \Delta U + W$

$$\Delta Q = n\frac{5R}{2}(60T_0 - T_0) + n\frac{3R}{2}(4T_0 - T_0)$$

$$\Delta Q = \frac{5nR}{2} \times 59T_0 + \frac{3nR}{2} \times 3T_0 = 152\ nRT_0$$

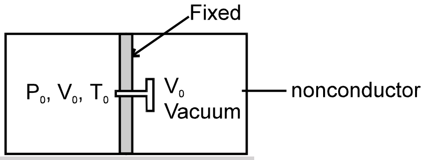
FREE EXPANSION

If a system, say a gas expands in such a way that no heat enters or leaves the system and also no work is done by or on the system, then the expansion is called the "free expansion".

$\Delta Q = 0$, $\Delta U = 0$ and $\Delta W = 0$. Temperature in the free expansion remains constant.

Solved Example

Example 22. A non conducting cylinder having volume $2V_0$ is partitioned by a fixed non conducting wall in two equal parts. Partition is attached with a valve. Right side of the partition is a vacuum and left part is filled with a gas having pressure and temperature P_0 and T_0 respectively. If valve is opened find the final pressure and temperature of the two parts.



Solution : From the first law thermodynamics $\Delta Q = \Delta U + W$

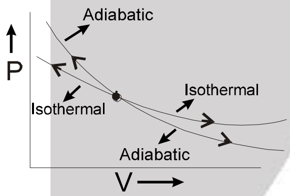
Since gas expands freely therefore W = 0, since no heat is given to gas $\Delta Q = 0$

$\Rightarrow$ $\Delta U = 0$ and temperature remains constant.

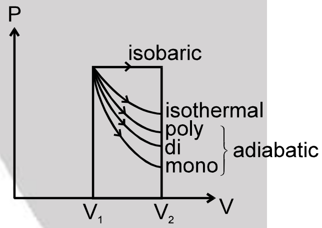
$T_{final} = T_0$

Since the process is isothermal therefore $P_0 \times V_0 = P_{final} \times 2V_0$ $\Rightarrow$ $P_{final} = P_0/2$

Comparison of slopes of an Iso-thermal and Adiabatic Curve



$$\left|\frac{dP}{dV}\right|_{adia} > \left|\frac{dP}{dV}\right|_{isothermal}$$



$W_{mono} < W_{di} < W_{poly} < W_{isothermal} < W_{isobaric}$

In compression up to same final volume: $|W_{adia}| > |W_{isothermal}|$

In Expansion up to same final volume: $W_{isothermal} > W_{adia}$

Limitations of Ist Law of Thermodynamics :

The first law of thermodynamics tells us that heat and mechanical work are interconvertible. However, this law fails to explain the following points :

(i) It does not tell us about the direction of transfer of heat.

(ii) It does not tell us about the conditions under which heat energy is converted into work.

(iii) It does not tell us whether some process is possible or not.

Mixture of non-reacting gases :

(a) Molecular weight = $\dfrac{n_1M_1 + n_2M_2}{n_1 + n_2}$, M_1 & M_2 are molar masses.

(b) Specific heat $C_V = \dfrac{n_1C_{V_1} + n_2C_{V_2}}{n_1 + n_2}$, $C_P = \dfrac{n_1C_{P_1} + n_2C_{P_2}}{n_1 + n_2}$

(c) for mixture, $\gamma = \dfrac{C_{p_{mix}}}{C_{v_{mix}}} = \dfrac{n_1C_{p_1} + n_2C_{p_2} + ...}{n_1C_{v_1} + n_2C_{v_2} + ...}$

Solved Miscellaneous Problems

Problem 1. A vessel of volume 2×10^{-2} m^3 contains a mixture of hydrogen and helium at 47º C temperature and 4.15×10^5 N/m^2 pressure. The mass of the mixture is 10^{-2} kg. Calculate the masses of hydrogen and helium in the given mixture.

Solution : Let mass of H_2 is m_1 and He is m_2

$\therefore \quad m_1 + m_2 = 10^{-2}$ kg $= 10 \times 10^{-3}$ kg .....(1)

Let P_1, P_2 are partial pressure of H_2 and He

$P_1 + P_2 = 4.15 \times 10^5$ N/m^2

for the mixture $(P_1 + P_2)\, V = \left(\frac{m_1}{M_1} + \frac{m_2}{M_2}\right) RT$

$$\Rightarrow \quad 4.15 \times 10^5 \times 2 \times 10^{-2} = \left(\frac{m_1}{2\times10^{-3}} + \frac{m_2}{4\times10^{-3}}\right) 8.31 \times 320$$

$$\Rightarrow \quad \frac{m_1}{2} + \frac{m_2}{4} = \frac{4.15\times2}{8.31\times320} = 0.00312 = 3.12 \times 10^{-3}$$

$\Rightarrow \quad 2m_1 + m_2 = 12.48 \times 10^{-3}$ kg .....(2)

Solving (1) and (2)

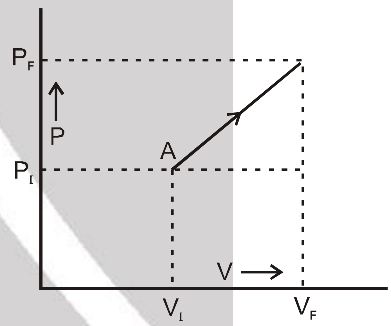
$m_1 = 2.48 \times 10^{-3}$ kg, 2.5×10^{-3} kg

and $m_2 = 7.5 \times 10^{-3}$ kg.

Problem 2. The pressure in a monoatomic gas increases linearly from 4×10^5 N m^{-2} to 8×10^5 N m^{-2} when its volume increases from 0.2 m^3 to 0.5 m^3. Calculate the following:

(a) work done by the gas. (b) increase in the internal energy.

Solution : (a) As here pressure is varying linearly with volume, work done by the gas



$W = \int PdV$ = area under P-V curve

$$W = P_I\,(V_F - V_I) + \frac{1}{2}(P_F - P_I) \times (V_F - V_I)$$

i.e., $W = 4 \times 10^5 \times 0.3 + \frac{1}{2} \times 4 \times 10^5 \times 0.3$

i.e., $W = 1.8 \times 10^5$ J

(b) The change in internal energy of a gas is given by

$$\Delta U = nC_V\,\Delta T = \frac{nR\Delta T}{(\gamma-1)} = \frac{(P_FV_F - P_IV_I)}{(\gamma-1)}$$

As the gas is monoatomic $\gamma = (5/3)$

$$\text{So, } \Delta U = \frac{10^5(8\times0.5-4\times0.2)}{[(5/3)-1]} = \frac{3}{2} \times 10^5(4 - 0.8).$$

i.e., $\Delta U = 4.8 \times 10^5$ J

Problem 3. There are two vessels. Each of them contains one mole of a monoatomic ideal gas. Initial volume of the gas in each vessel is 8.3×10^{-3} m^3 at 27º C. Equal amount of heat is supplied to each vessel. In one of the vessels, the volume of the gas is doubled without change in its internal energy, whereas the volume of the gas is held constant in the second vessel. The vessels are now connected to allow free mixing of the gas. Find the final temperature and pressure of the combined gas system.

Solution : According to Ist law of thermodynamics, $\Delta Q = \Delta U + W$

So for the vessel for which internal energy (and hence, temperature) remains constant.

$\Delta Q_1 = W = nRT \log_e (V_F/V_I)$

$\Delta Q_1 = 1 \times R \times 300 \log_e(2) = 0.693 \times 300\ R = 207.9\ R$

and for the vessel for which volume is kept constant.

$\Delta Q_2 = \Delta U = nC_V\ \Delta T$ [as W = 0]

i.e., $\Delta Q_1 = 1(3/2)R\ \Delta T$

According to given problem $\Delta Q_1 = \Delta Q_2$ i.e.,

$207.9R = (3/2)R\Delta T$, i.e. $\Delta T = 138.6$

i.e., $T_F - T_I = 138.6$ with $T_I = 300$ K

So, $T_F = 300 + 138.6 =$ **438.6 K**

Now when the free mixing of gases is allowed

$U_1 + U_2 = U$

$n_1(C_V)_1T_1 + n_2 (C_V)_2T_2 = nC_VT$

with $n = n_1 + n_2$

Here $n_1 = n_2 = 1$

and $(C_V)_1 = (C_V)_2 = C_V$

So $1 \times 300 + 1 \times 438.6 = 2T$,

i.e., T = 369.3 K

Further for the mixture from PV = nRT with V = V + 2V = 3V and $n = n_1 + n_2 = 2$, we have

$P = \frac{nRT}{3V} = \frac{2 \times 8.3 \times 369.3}{3 \times 8.3 \times 10^{-3}} = \mathbf{2.462 \times 10^5\ N/m^2}$

Problem 4. A gaseous mixture enclosed in a vessel of volume V consists of one gram mole of a gas A with $\gamma = \frac{C_p}{C_v} = \frac{5}{3}$ and another gas B with $\gamma = \frac{7}{5}$ at a certain temperature T. The gram molecular weights of the gases A and B are 4 and 32 respectively. The gases A and B do not react with each other and are assumed to be ideal. The gaseous mixture follows the equation; $PV^{19/13}$ = constant in adiabatic processes.

(a) Find the number of gram moles of the gas B in the gaseous mixture.

(b) Compute the speed of sound in the gaseous mixture at T = 300 K.

(c) If T is raised by 1 K from 300 K, find the percentage change in the speed of sound in the gaseous mixture.

Solution : (a) As for ideal gas $C_P - C_V = R$ and $\gamma = (C_P/C_V)$,

So $\gamma - 1 = \frac{R}{C_V}$ or $C_V = \frac{R}{(\gamma - 1)}$

$\therefore\ (C_V)_1 = \frac{R}{(5/3) - 1} = \frac{3}{2}R$; $(C_V)_2 = \frac{R}{(7/5) - 1} = \frac{5}{2}R$

and $(C_V)_{mix} = \frac{R}{(19/13) - 1} = \frac{13}{6}R$

Now from conservation of energy, i.e., $\Delta U = \Delta U_1 + \Delta U_2$,

$(n_1 + n_2)\ (C_V)_{mix}\ \Delta T = [n_1(C_V)_1 + n_2(C_V)_2]\Delta T$

i.e., $(C_V)_{mix} = \frac{n_1(C_V)_1 + n_2(C_V)_2}{n_1 + n_2}$

We have $\frac{13}{6}R = \frac{1 \times \frac{3}{2}R + n \times \frac{5}{2}R}{1 + n} = \frac{(3 + 5n)}{2(1 + n)}$

or, $13 + 13n = 9 + 15n$, n = 2 mole.

(b) Molecular weight of the mixture will be given by

$$M = \frac{n_A M_A + n_B M_B}{n_A + n_B} = \frac{(1)(4) + 2(32)}{1+2}$$

M = 22.67

Speed of sound in a gas is given by

$$v = \sqrt{\frac{\gamma RT}{M}}$$

Therefore, in the mixture of the gas

$$v = \sqrt{\frac{(19/13)(8.31)(300)}{22.67 \times 10^{-3}}} \text{ m/s}$$

$v \approx 401$ m/s

(c) $v \propto \sqrt{T}$

or $v = KT^{1/2}$ ......(2)

$$\Rightarrow \frac{dv}{dT} = \frac{1}{2} KT^{-1/2} \quad \Rightarrow dv = K\left(\frac{dT}{2\sqrt{T}}\right)$$

$$\Rightarrow \frac{dv}{v} = \frac{K}{v}\left(\frac{dT}{2\sqrt{T}}\right) \quad \Rightarrow \frac{dv}{v} = \frac{1}{\sqrt{T}}\left(\frac{dT}{2\sqrt{T}}\right) = \frac{1}{2}\left(\frac{dT}{T}\right)$$

$$\Rightarrow \frac{dv}{v} \times 100 = \frac{1}{2}\left(\frac{dT}{T}\right) \times 100 = \frac{1}{2}\left(\frac{1}{300}\right) \times 100 = 0.167 = \frac{1}{6}$$

Therefore, percentage change in speed is 0.167%.

HANDOUT KTG & THERMODYNAMICS

Reversible and Irreversible Process

A thermodynamical process taking a system from initial state i to final state f is reversible, if the process can be turned back such that both, the system and the surroundings return to their original states, with no other change anywhere else in the universe.

For a process to be reversible, the following conditions must be satisfied :

1. The process should proceed at an extremely slow rate, i.e., process is quasi-static so that the system is in equilibrium with surroundings at every stage, i.e.

(i) The system remains in mechanical equilibrium, i.e., there is no unbalanced force,

(ii) The system remains in thermal equilibrium, i.e., all parts of the system and the surroundings remain at the same temperature.

(iii) The system remains in chemical equilibrium, i.e., the internal structure of the system does not change.

2. The system should be free from dissipative forces like friction, inelasticity, viscosity, etc. This is because energy spent against such forces cannot be recovered.

As all the conditions mentioned above are of an idealized nature, no process in nature is truly reversible.

Infact, reversibility is an idealized concept which can never be attained. It can at best be approximated.

Some of the examples of approximately reversible processes are :

(i) An ideal gas allowed to expand slowly and then compressed slowly in a cylinder fitted with frictionless movable piston.

(ii) Electrolysis can be taken as a reversible process provided resistance offered by electrolyte is zero.

(iii) Slow compression and expansion of a spring can also be treated as a reversible process.

Now answer the following questions :

Q.1 What are irreversible process

Ans. A process, which does not satisfy any of the conditions for reversible process is called an irreversible process.

Infact, all spontaneous processes of nature are irreversible processes. For example, transfer of heat from a hot body to a cold body, ordinary expansion of a gas, diffusion of gases, stopping of moving body through friction etc. are all irreversible processes.

Q.2 What are fundamental cause of irreversibility ?

Ans. Irreversibility arises mainly from two causes :

(i) Many processes like free expansion or an explosive chemical reaction take the system to non equilibrium states.

(ii) Most processes involve friction, viscosity and other dissipative effects.

As the dissipative effects are present everywhere, and they can be minimised only and cannot be fully eliminated, therefore, most processes we deal with are irreversible processes.

Q.3 Give some example of irreversible process.

Ans. Examples of irreversible processes are :

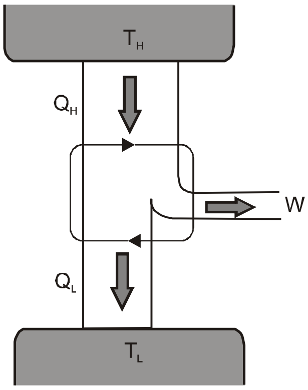
(i) Most of the chemical reactions are irreversible, because they involve changes in the internal structure of the constituents.

(ii) The decay of organic matter is an irreversible process.

(iii) Rusting of iron is an irreversible process.

(iv) Adiabatic (sudden) compression and expansion of a gas are irreversible processes.

HEAT ENGINES



We have seen that when mechanical work is done on a system, its internal energy increases (remember, we assume that the system does not have any systematic motion). The reverse process in which mechanical work is obtained at the expense of internal energy is also possible. Heat engines are devices to perform this task. The basic activity of a heat engine is shown in figure. It takes some heat from bodies at higher temperature, converts a part of it into the mechanical work and delivers the rest to bodies at lower temperature.

The substance inside the engine comes back to the original state. A process in which the final state of a system is the same as its initial state, is called a cyclic process. An engine works in cyclic process.

Efficiency

Suppose an engine takes an amount Q_H of heat from high-temperature bodies, converts a part W of it into work and rejects an amount Q_L of heat to low-temperature bodies. If the final state of the substance inside the engine is the same as the initial state, there is no change in its internal energy. By first law of thermodynamics, $W = Q_H - Q_L$.

The efficiency of the engine is defined as $\eta = \dfrac{\text{work done by the engine}}{\text{heat supplied to it}}$

$$= \frac{W}{Q_H} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$$

We now describe carnot engine

The significance of the concept of reversibility :

The main concern of thermodynamics is the efficiency with which heat can be converted into mechanical work. It turned out that a heat engine based on idealised reversible processes achieves the highest possible efficiency. All other engines involving irreversibility of some kind have much lower efficiency because there is energy loss in friction viscous forces etc. Therefore, reversibility is an important concept in thermodynamics.

Comprehension :

A steam engine delivers 5.4×10^8 J of work per minute and take 3.6×10^9 J of heat per minute from its boiler.

Now answer the following questions :

1. Which engine have more efficiency ?
(A) Reversible engine (B) Irreversible engine
(C) Both have same efficiency (D) Can't say

Ans. **(A)**

2. What is the efficiency of the engine ?
(A) 10% (B) 15% (C) 20% (D) 25%

Ans. **(B)**

3. How much heat is wasted per minute ?
(A) 3.06×10^9 (B) 3.06×10^8 (C) 3.6×10^9 (D) 3.6×10^8

Ans. **(A)**

4. If this heat engine is used as water pump, then how much water per minute can be transferred at a building of height 30 meter ?
(A) 1.8×10^6 Kg (B) 1.8×10^5 Kg (C) 5.4×10^4 Kg (D) 5.4×10^3 Kg

Ans. **(A)**

Sol. Work done per minute output = 5.4×10^8 J

Heat absorbed per minute, input = 3.6×10^9 J

Efficiency, $\eta = \frac{5.4 \times 10^8}{3.6 \times 10^9} = 0.15$

% η = 0.15 × 100 = 15

Heat energy wasted/minute

= Heat energy absorbed/minute – Useful work done/minute

= $3.6 \times 10^9 - 5.4 \times 10^8 = (3.6 \times 0.54) \times 10^9 = 3.06 \times 10^9$ J.

mgh = W

$m \times 10 \times 30 = 5.4 \times 10^8$

$\Rightarrow$ $m = 1.8 \times 10^6$ kg

Comprehension

The first law of thermodynamics establishes the essential equivalence between the heat energy and mechanical work and says that the two can be converted into each other. Further, 4.18 joule of mechanical work are required to produce one calorie of heat and vice-versa. However, this law has the following limitations :

1. The first law does not indicate the direction in which the change can occur.

For example (i) when two bodies at different temperatures are put in thermal contact with each other, heat flows from the body at higher temperature to the body at lower temperature. We now know that heat cannot flow from the body at lower temperature to the body at higher temperature, although first law of thermodynamics is not violated.

(ii) When a moving car is stopped by applying brakes, work done against friction is converted into heat. When the car cools down, it does not start moving with the conversion of all its heat energy into mechanical work.

(iii) When a bullet strikes a target, kinetic energy of the bullet is converted into heat energy. But heat energy developed in the target cannot be converted back into mechanical energy of the bullet enabling it to fly back.

2. The first law gives no ideal about the extent of change

Our observations and experience tell that there appears to be no restriction on conversion of mechanical work into heat. But there are severe restrictions on the reverse process, i.e, conversion of heat energy into mechanical energy.

We know that heat is not converted into mechanical energy all by itself. An external agency called heat engine is required for the purpose.

No heat engine can convert all the heat energy received from the source into mechanical energy. The first law of thermodynamics is silent about all this.

3. The first law of thermodynamics gives no information about the source of heat, i.e., whether it is a hot or a cold body.

These limitations lead to the formulation of another law called the "**second law of thermodynamics**".

SECOND LAW OF THERMODYNAMICS

This law specifies the conditions for the conversion of heat into work. There are several statements of this law but the following two are the most significant :

(i) Kelvin-Planck Statement

" It is impossible to construct an engine, operating in a cycle, which will produce no effect other than extracting heat from a reservoir and performing an equivalent amount of work ." In simple words, it is not possible to get a continuous supply of work from a body by cooling it to temperature lower than that of the surroundings.

This form of the law is applicable to heat engines. The working substance of a heat engine, operating in a cycle, cannot convert all the extracted heat into work. It must reject a part of the heat to the sink at a lower temperature. So, in order to convert heat into wrok, it is necessary to have both source and sink. Since all the heat extracted from the source can never be converted into work therefore the efficiency of the engine is never one.

(ii) Rudlope Classius Statement

"It is impossible to make heat flow from a body at a lower temperature to a body at a higher temperature without doing external work on the working substance". In simple words, heat cannot by itself, flow from a body at a lower temperature to a body at a higher temperature.

This form of the law is applicable to ice plants and refrigerators. Ths refrigerant absorbs heat from inside the refrigerator and rejects a greater quantity of heat to the surroundings (at higher temperature) with the help of an external agency say an electric motor. In ammonia ice plant heat is absorbed from the brine solution at a lower temperature and rejected into water at a higher temperature. This is achieved with the help of an external agency like a pump.

Now answer the following questions :

1. What forbids the complete conversion of work into heat?

Ans. Second law of thermodynamics.

2. Can mechanical work be completely converted into heat. Is reverse also possible?

Ans. The mechanical work can be completely converted into heat but heat extracted from some body cannot be completely converted into useful work.

3. **Statement-1 :** It is not possible for a system unaided by any external agency to transfer heat from a body at lower temperature to another body at higher temperature.

Statement-2 : It is not possible to violate the second law of thermodynamics.

Then which of the combination is true.

(A) T, T (B) T, F (C) F, T (D) F, F

Ans. **(A)**

4. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of :

(A) second law of thermodynamics (B) conservation of momentum

(C) conservation of mass (D) first law of thermodynamics

Ans. **(A)**

Sol. Heat cannot flow itself from a lower temperature to a body of higher temperature. This corresponds to second law of thermodynamics.

Comprehension :

Changes in energy within a closed system do not set the direction of irreversible processes. Rather that direction is set by another that we shall discuss hear that is change in entropy ΔS of the system.

ENTROPY

Like pressure, volume temperature internal energy etc. we have another thermodynamic variable of a system named entropy. In a given equilibrium state, the system has a definite value of entropy. If the system has a temperature T (in absolute scale) and a small smount of heat ΔQ is given to it, we define the change in the entropy of the system as

$$\Delta S = \frac{\Delta Q}{T} \qquad ....(i)$$

In general, the temperature of the system may change during a process. If the process is reversible, the change in entropy is defined as

$$S_f - S_i = \int_i^f \frac{\Delta Q}{T} \qquad ....(ii)$$

In an adiabatic reversible process, no heat is given to the system, The entropy of the system remains constant in such a process.

Entropy is related to the disorder in the system. Thus, if all the molecules in a given sample of a gas are made to move in the same direction with the same velocity, the entropy will be smaller then that in the actual situation in which the molecules move randomly in all directions.

An interesting fact about entropy is that it is not a conserved quantity. More interesting is the fact that entropy can be created but cannot be destroyed. Once some entropy is created in a process, the universe has to carry the burden of that entropy for ever. The second law of thermodynamics may be stated in terms of entropy as follows.

It is not possible to have a process in which the entropy of an isolated system is decreased.

Now answer the following questions :

1. Which has more entropy a crowd or a militry force ?

Ans. Crowd has more entropy due to randomeness.

2. What is the change in entropy for an adiabatic process ?

Ans. Zero.

3. When you make ice cubes from water, the entropy of water

(A) does not change

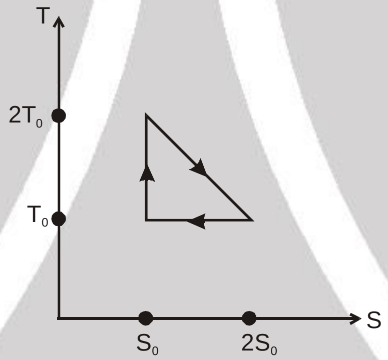
(B) increases

(C) decreases

(D) may either increase or decrease depending on the process used.

Ans. (C)

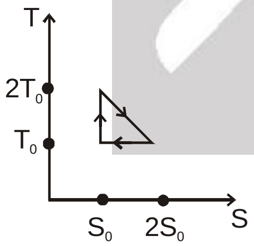
4. The temperature-entropy diagram of a reversible engine cycle is given in the figure. Its efficiency is :



(A) $\frac{1}{2}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$

Ans. **(C)**

Sol. According to the figure



$Q_1 = T_0S_0 + \frac{1}{2}\ T_0S_0 = \frac{3}{2}T_0S_0$

$Q_2 = T_0\ (2S_0 - S_0) = T_0S_0$

$Q_3 = 0$

$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

$= 1 - \frac{Q_2}{Q_1} = 1 - \frac{2}{3} = \frac{1}{3}$

Comprehension

We cannot move a ship in an ocean by utilising the energy of the ocean. Why? Explanation is here.

CARNOT'S IDEAL HEAT ENGINE

It is an ideal heat engine which is free from all the imperfections of an actual engine. So, it cannot be realised in actual practice. It was conceived by Niolas Le'onard Sadi Carnot, a French Engineer. This engine serves us a standard by which the performance of actual engines can be judged. It consists essentially of the following parts.

(i) Source. It serves as source of heat. It is maintained at a constant high temperature T_1K. It has infinite thermal capacity i.e., any amount of heat may be extracted from it at a constant temperature T_1.

(ii) Sink. It is a cold body maintained at constant low temperature T_2K. It also has infinite thermal capacity, i.e., any amount of heat rejected to it will not affect its temperature.

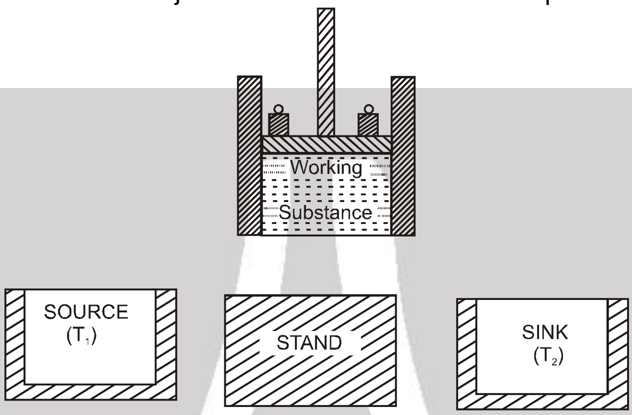


Fig. Carnot's ideal heat engine

(iii) Insulating stand. It is a perfectly non-conducting pad.

(iv) A cylinder : With perfectly non-conducting walls but with a perfectly conducting bottom. It is fitted with a perfectly non-conducting and frictionaless piston over which some weights are placed. One mole of an ideal gas is enclosed in the cylinder. The ideal gas acts as the working substance.

The working substance is subjected to the following four successive reversible operations so as to complete a reversible cycle. This cycle is called Carnot's cycle. The reversibility of operations is a very important assumption because our aim is to find out the maximum efficiency attainable by engine where all sources of irreversibilities are absent.

To begin with let the pressure, volume and temperature be P_1, V_1 and T_1 respectively. The state of the working substance is represented by the point a in the P-V diagram.

(1) Operation I (Isothermal Expansion). The cylinder is placed on the source. The piston is allowed to move out infinitely slowly by reducing very gradually the weights on the piston. The gas expands extremely slowly. As the gas expands, its temperature tends to fall. But since it is in thermal contact with the heat source therefore it will extract a certain amount of heat Q_1 from the source. In this way the temperature of the gas will remain T_1 throughout the process of expansion. In other words, the gas expands isothermally at temperature T_1K. This isothermal expansion is represented by the curve AB on the indicator diagram. Let W_1 be the work done by the gas in expanding from volume V_1 to volume V_2. The pressure decreases from P_1 to P_2.

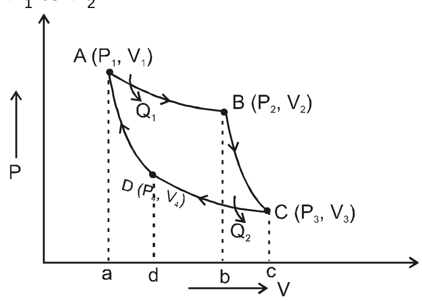


Fig. P-V Diagram of Carnot's cycle

Applying first law of thermodynamics,

$$Q_1 = W_1 = \int_{V_1}^{V_2} PdV = RT_1 \log_e \frac{V_2}{V_1} = \text{Area AB ba A}$$

(2) Operation II (Adiabatic Expansion). The cylinder is placed on the insulated stand and the piston is allowed to move out. The gas expands adiabatically from volume V_2 to volume V_3 till its temperature falls to T_2K.

The pressure falls from P_2 to P_3.

The adiabatic expansion is represented by the curve BC in the indicator diagram.

The work done by the gas is given by

$$W_2 = \int_{V_2}^{V_3} PdV = \frac{R}{\gamma - 1}(T_1 - T_2) = \text{Area BC cb B}$$

(3) Operation III (Isothermal Compression) The cylinder is placed on the sink and the gas is isothermally compressed until the pressure and volume become P_4 and V_4 respectively. The operation is represented by the isothermal curve CD. The heat Q_2 developed in compression is absorbed by the sink. Let W_3 be the work done on the gas.

Applying first law of thermodynamics.

$$Q_2 = W_3 = -\int_{V_3}^{V_4} PdV = -RT_2 \log_e \frac{V_4}{V_3} = RT_2 \log_e \frac{V_3}{V_4} = \text{Area C cd DC}$$

(4) Operation IV (Adiabatic Compression) The cylinder is placed on the insulating stand and the gas is compressed adiabatically till it attains its initial pressure P_1 volume V_1 and temperature T_1. The adiabatic compression is represented by the curve DA in the indicator diagram. Let W_4 be the work done on the gas.

Then $W_4 = -\int_{V_4}^{V_1} PdV = \frac{R}{\gamma - 1}(T_1 - T_2) = \text{Area AD da A}$

Let W be the net external work done by the working substance during one cycle.

Then, W = Work done by the gas – Work done on the gas

$= W_1 + W_2 - W_3 - W_4 = W_1 - W_3 \qquad [\therefore W_2 = W_4]$

= area AB ba A – area C cd DC = area ABCDA

The working substance can be taken through the cycle again and again. In this way, more and more work can be done by the engine.

Thermal Efficiency of a Carnot engine is defined as the ratio of the external work done in one cycle to corresponding amount of heat extracted from the source.

Since the working substance is restored to its initial state therefore there is no change in its internal energy.

Applying first law of thermodynamics,

$W = Q_1 - Q_2$

W, Q_1 and Q_2 are all measured in the same units, i.e, either in units of heat or in units of work.

Thermal efficiency, $\eta = \frac{\text{external work done}}{\text{heat extracted}}$

or $\qquad \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

Here, $Q_1 = W_1 = RT_1 \log_e \frac{V_2}{V_1}$ and $Q_2 = W_3 = RT_2 \log_e \frac{V_3}{V_4}$

Now, $\frac{Q_2}{Q_1} = \frac{RT_2 \log_e\left(\frac{V_3}{V_4}\right)}{RT_1 \log_e\left(\frac{V_2}{V_1}\right)}$ or $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \frac{\log_e\left(\frac{V_3}{V_4}\right)}{\log_e\left(\frac{V_2}{V_1}\right)}$ ....(i)

The points B and C lie on the same adiabatic.

$\therefore \qquad T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$ ....(ii)

The points A and D lie on the same adiabatic.

$\therefore \qquad T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1}$ ....(iii)

Dividing (2) by (3) be get

$$\left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad \text{Put in eq. (i)}$$

Then from equation (1), $\frac{Q_2}{Q_1} = \frac{T_2}{T_1} \Rightarrow \eta = 1 - \frac{T_2}{T_1}$

Results (i) The efficiency of the Carnot's ideal engine is independent of the nature of the working substance. It depends only upon the temperatures of the source and sink. The greater the difference between the two temperatures, higher is the efficiency of the Carnot engine.

(ii) Efficiency is the same for all reversible engines working between temperatures T_1 and T_2.

(iii) η is always less than one. The value of η can be one only if $T_2 = 0$ i.e., if the sink is at absolute zero of temperature. Since the absolute zero of temperature cannot be attained therefore η cannot be equal to one.

(iv) When $T_2 = T_1$, then $\eta = 0$.

So, heat cannot be converted into work without a temperature difference. In other words, heat can be converted into work only if a sink at a lower temperature is available. This explains as to why the large amount of heat energy of sea water cannot be used for deriving mechanical work.

SOLVED EXAMPLE

Example 1. How is the efficiency of a Carnot engine affected by the nature of the working substance?

Solution : The efficiency is independent of the nature of the working substance.

Example 2. A Carnot engine operates between 227ºC and 127ºC. If it absorbs 60 × 10^4 calorie at higher temperature, how much work per cycle can the engine perform.

Solution : $T_1 = (227 + 273)$ K = 500 K

$T_2 = (127 + 273)$ K = 400 K

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{400}{500} = \frac{100}{500} = \frac{1}{5}$$

But $\eta = \frac{W}{Q_1}$ or $W = \eta Q_1$

or $W = (1/5) \times 60 \times 10^4$ cal [$\because Q_1 = 60 \times 10^4$ cal]

$= 12 \times 10^4$ cal $= 12 \times 10^4 \times 4.2$ J [$\because$ 4.2 J = 1 cal] $= 5.04 \times 10^5$ J

Example 3. A carnot cycle is performed by air initially at 927ºC. Each stage represents a compression or expansion in the ratio 1 : 32. Calculate (i) the lowest temperature (ii) efficiency of the cycle. Given : $\gamma = 1.4$.

Solution : $T_1 = (927 + 273)$ K = 1200 K

$$\frac{V_1}{V_2} = \frac{1}{6}, \gamma = 1.4$$

(i) $T_2 V_2^{\gamma-1} = T_1 V_1^{\gamma-1}$ or $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

or $T_2 = 1200\left(\frac{1}{32}\right)^{1.4-1} = 300$ K

(ii) Efficiency, $\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{1200} = 0.75$

% age $\eta = 0.75 \times 100 = 75\%$

EXERCISE

1. Can the carnot engine be realised in actual practice

2. A carnot's reversible heat engine works between 300K and 600K. In each cycle of operations, the engine draws 1000 J of energy from the source at 600 K. Calculate
(i) the energy rejected to the sink at 300 K.
(ii) the external work done by the engine.
(iii) the efficiency of the engine.

3. A power station uses superheated steam (at high pressure) at approximately 473ºC. The cold sink corresponds to the temperature at which steam condenses at atmospheric pressure, that is 100ºC. What is the maximum theoretical efficiency for the power station ?
temperature at which steam condenses at atmospheric pressure, that is 100ºC. What is the maximum theoretical efficiency for the power station ?

4. A carnot engine works between ice point and steam point. It is desired to increase the efficiency by 20% (a) making temperature of the source constant (b) making temperature of the sink constant. Calculate the change in temperature in two cases. Which one of these will you prefer and why ?

5. Two carnot engies A and B are operated in series. The first one A receives heat at 800 K and rejects to a reservoir at temperature T K. The second engine B receives the heat rejected by the first engine and in turn temperature T K for the following situations.
(i) The outputs of the two engines are equal.
(ii) The efficiencies of the two engines are equal.

6. Draw temperature entropy diagram for carnot cycle and calculate efficiency & prove it only depends on temperature of hot and cold bodies.

7. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is :
(A) 0.5 (B) 0.75 (C) 0.99 (D) 0.25

8. A Carnot engine operating between temperatures T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 K, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are, respectively :
(A) 372 K and 310 K (B) 372 K and 330 K (C) 330 K and 268 K (D) 310 K and 248 K

9. Which statement is incorrect ?
(A) All reversible cycles have same efficiency
(B) Reversible cycle has more efficiency than an irreversible one
(C) Carnot cycle is a reversible one
(D) Carnot cycle has the maximum efficiency in all cycles

10. Even Carnot engine cannot give 100% efficiency because we cannot :
(A) prevent radiation (B) find ideal sources
(C) reach absolute zero temperature (D) eliminate friction

11. A Carnot engine takes 3×10^6 cal of heat from a reservoir at 627ºC and gives it to a sink at 27ºC. The work done by the engine is :
(A) 4.2×10^6 J (B) 8.4×10^6 J (C) 16.8×10^6 J (D) zero

12. A Carnot engine, whose efficiency is 40%, takes in heat from a source maintained at a temperature of 500K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be :
(A) efficiency of carnot engine cannot be made larger than 50%
(B) 1200 K
(C) 750 K
(D) 600 K

Assertion and Reason Problems

In each problem a statement of assertion (A) is given and a corresponding statement of reason (R) is given just below it. Of the statements mark the correct answer as :

(A) If both A and R are true and R is the correct explanation of A.

(B) If both A and R are true but R is not the correct explanation of A.

(C) If A is true but R is false.

(D) If both A and R are false.

(E) If A is false but R is true.

13. **Assertion (A) :** The efficiency of a carnot engine is determined mainly by the temperature of the source and not by the temperature of the sink

Reason (R) : The efficiency of a carnot engine can be 100% if the temperature of the source is infinite or the temperature of the sink is 0 K.

14. **Assertion (A) :** No heat engine working between two given temperature of source and sink can be more efficient than a perfectly reversible engine working between the same two temperatures.

Reason (R) : Irreversible engines are highly efficient engines.

Comprehension

REFRIGERATOR (HEAT PUMP)

An ideal refrigerator may be regarged as a carnot's ideal heat engine working in the reverse order.

In an actual refrigerator, the vapours of some low boiling point liquid (ammonia or freon – 12) act as the working substance. The working substance absorbs a certain quantity of heat Q_2 from the cold body or sink at lower temperature T_2. In a household refrigerator, the ice cubes in the freezer compartment and food constitute the cold body. A certain amount of work W is performed by the compressor of the refrigerator on the working substance. The compressor is operated by an electric motor. The quantity of heat Q_1 is rejected to the hot body (atmospher) at temperature T_1 K by the radiator (fixed at the back of the refrigerator).

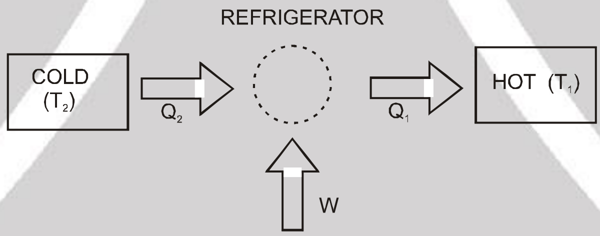


Fig. Refrigerator

Coefficient of performance. It measures the efficiency of a refrigerator.

It is defined as the ratio of the quantity of heat extracted per cycle from the contents of the refrigerator to the mechanical work W done by the external agency to do so.

It is denoted by β or K or ω.

$$\therefore \qquad \beta = \frac{Q_2}{W}$$

Smaller the amount of mechanical work done in removing heat Q_2, greater will be the coefficient of performance.

$$W = Q_1 - Q_2 \qquad \therefore \qquad \beta = \frac{Q_2}{Q_1 - Q_2} \qquad \text{...........(i)}$$

This expression may be put in another form also.

$$\beta = \frac{1}{\frac{Q_1}{Q_2} - 1}$$

[Dividing the numerator and denominator of equation (1) by Q_2.]

But $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$ $\therefore$ $\beta = \frac{1}{\frac{T_1}{T_2} - 1}$ ....(ii)

or $\beta = \frac{T_2}{T_1 - T_2}$

Discussion (i) In actual practice, β varies from 2 to 6. For an actual refrigerator, the value of β is less than that calculated from equations (1) or (2).

(ii) Lesser the difference in the temperatures of the cooling chamber and the atmosphere, higher is the coefficient of performance of the refrigerator.

(iii) In a heat engine the efficiency can never exceed 100%. But in the case of a refrigerator, the coefficient of performance may be much higher than 100%

(iv) As the refrigerator works, T_2 goes on decreasing due to formation of too much ice. There is practically no change in T_1. This decreases the value of β. However, if the refrigerator is defrosted, T_2 shall increase and consequenctly the value of β. So, it is necessary to defrost the refrigerator.

Solved Example :

1. Refrigerator transfers heat from a cold body to a hot body. Does this not violate the second law of thermodynamics

Ans. No. This is because external work is being performed.

2. Is coefficient of performance of a refrigerator constant?

Ans. No, the coefficient of performance of refrigerator decreases with decrease in its inside temperature.

Now answer the following Questions :

1. Can we increase the coefficient of performance of a refrigerator by increasing the amount of working substance

(A) No (B) Yes

(C) some time yes, some time no (D) can't say

Ans. **(A)**

2. The door of an operating refrigerator is kept open in a closed room. Then temperature of room will be :

(A) increases slightly (B) decreases slightly

(C) may increase or decrease (D) can't say

Ans. **(A)**

Ans. The room will be slightly warmed.

3. A Carnot engine, having an efficiency of η = 1/10 as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is

(1) 99 J (2) 90 J (3) 1 J (4) 100 J

Ans. **(2)**

Sol. For Carnot engine using as refrigerator

$$W = Q_2\left(\frac{T_1}{T_2} - 1\right)$$

It is given $\eta = \frac{1}{10}$ $\Rightarrow$ $\eta = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{9}{10}$

So, Q_2 = 90 J (as W = 10 J)

EXERCISE

15. Calculate the coefficient of performance of a refrigerator working between –3ºC and 27ºC.

16. A refrigerator is to maintain eatables kept inside at 9ºC. If room temperature is 36ºC, calculate the coefficient of performance.

17. A carnot refrigerator works between temperature limits of 0ºC and 27ºC. If 10 kg of water at 0ºC is converted into ice at 0ºC, calculate (a) the heat rejected (b) the energy supplied to the refrigerator. Given : latent heat of ice = 80 kcal kg^{-1}.

1. It does not indicate the direction in which the change can proceed.

Illustrations.

(i) When a hot body is brought in thermal contact with a cold body, heat always flows from the hot body to the cold body. Why heat does not flow from the cold body to the hot body? The first law of thermodynamics is silent about it. So, this law does not indicate the direction of heat transfer.

(ii) It is not possible for a ship to use the huge amounts of heat of the sea waters to operate its engine. What prevents the conversion of heat into work ? The first law is again silent about this. So, the first law of thermodynamics does not specify the conditions under which heat is converted into work.

2. The first law of thermodynamics gives no idea about the extent to which the change takes place.

It has been observed that no heat engine can convert all the heat extracted from the source into mechanical energy. Why the whole of the heat cannot be converted into mechanical energy. The first law is silent about this question.

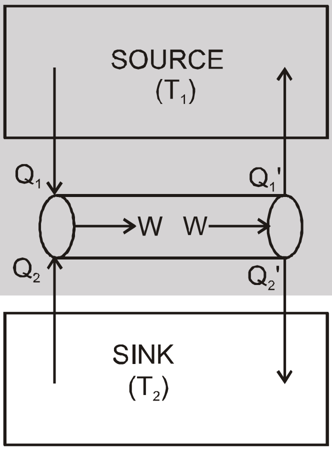
CARNOT'S THEOREM

Carnot showed that no engine can be more efficient than the perfectly reversible engine. This is known as Carnot's theorem and is stated as follows :

No heat engine working between given temperatures can have efficiency greater than that of a reversible engine working between the same temperatures.

Consider two engines – an irreversible engine A and a reversible engine B. Let the two engines be coupled such that A drives B backwards. So B is acting as a refrigerator.

A absorbs heat Q_1 from the source performs work W and rejects heat Q_2 to the sink.



Efficiency of A, $\eta_A = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1}$

B absorbs heat Q_2, from the sink gets work W performed on it and rejects heat Q_1 to the source.

Efficiency of B, $\eta_B = \frac{W}{Q_1'} = \frac{Q_1' - Q_2'}{Q_1'}$

Suppose A is more efficient than B.

Then, $\eta_A > \eta_B$

$\therefore \quad \frac{W}{Q_1} > \frac{W}{Q_1'} \quad$ or $\quad Q_1 < Q_1'$

$\therefore$ $Q_1 - Q_1$ is positive. So the net quantity of heat given to the source at temperature T_1 by the compound engine AB is $(Q_1' - Q_1)$.

Again, $W = Q_1 - Q_2 = Q_1' - Q_2'$ or $\quad Q_2' - Q_2 = Q_1' - Q_2$

$\therefore \quad Q_2' - Q_2$ is positive. $\quad [\therefore Q_1' - Q_1$ is + ve]

So, the net quantity of heat taken from the sink at temperature T_2 by the compound engine AB is $(Q_2' - Q_2)$.

Thus, the compound engine AB is a self-acting device which is transferring heat from lower temperature to higher temperature without any work being done by the external agency. This is forbidden by second law of thermodynamics. So, our assumption that engine A is more efficient than B is wrong.

This proves Carnot's theorem.

Example 1. Three ideal engines operate between reservoir temperatures of (a) 400 K and 500 K; (b) 600 K and 800 K, and (c) 400 K and 600 K. Rank the engines according to their thermal efficiencies, greatest first.

Answer : c, b, a.

Solution : (c) $\eta = 1 - \frac{400}{600} = 1 - \frac{2}{3} = \frac{1}{3}$ (b) $\eta = 1 - \frac{600}{800} = 1 - \frac{3}{4} = \frac{1}{4}$ (a) $\eta = 1 - \frac{400}{500} = 1 - \frac{4}{5} = \frac{1}{5}$

Example 2. Five moles of an ideal gas are taken in a Carnot engine working between 100ºC and 30ºC. The useful work done in one cycle is 420 joule. Calculate the ratio of the volume of the gas at the end and beginning of the isothermal expansion. Give : R = 8.4 J mol^{-1} K^{-1}.

Solution : $T_1 = (100 + 273)$ K = 373 K

$T_2 = (30 + 273)$ K = 303 K

Useful work, $W = Q_1 - Q_2 = 420$ J ....(i)

Let W_1 and W_2 be the works done during isothermal expansion at T_1 and isothermal compression at T_2.

Now $\frac{Q_1}{Q_2} = \frac{W_1}{W_2} = \frac{T_1}{T_2} = \frac{373}{303}$

$\therefore \quad Q_1 = \frac{373}{303} Q_2$

From equation $W = \frac{373}{303} Q_2 - Q_2$

or $\quad 420 = \left(\frac{373}{303} - 1\right) Q_2 \quad$ or $\quad Q_2 = 1818$ J

Again : $Q_1 = \frac{373}{303} Q_2 = \frac{373}{303} \times 1818\,J = 2238\,J$,

Heat used in isothermal expansion is given by

$$Q_1 = \mu RT \log_e \frac{V_2}{V_1} = 2.3026 \times \mu RT \log_{10} \frac{V_2}{V_1}$$

or $\quad 2238 = 2.3026 \times 5 \times 8.4 \times 373 \log_{10} \frac{V_2}{V_1}$

On simplification, $\frac{V_2}{V_1} = 1.15$

EXERCISE

18. What type of process is carnot's cycle

19. One mole of an ideal gas is taken in a carnot engine working between 27ºC and 227ºC. The useful work done in one cycle is 600J. Calculate the ratio of volume of gas at the end and beginning of the isothermal expansion. Given : R = 8.3 J $mole^{-1}$ K^{-1}.

20. A carnot engine takes in 1000 k cal of heat from a reservoir at 627ºC and exhausts heat to sink at 27ºC. What is its efficiency? When will its efficiency be 100% ?

Example 1. You wish to increase the coefficient of performance of an ideal refrigerator. You can be do so by (a) running the cold chamber at a slightly higher temperature, (b) running it at a slightly lower temperature, (c) moving the unit to a slightly warmer room, or (d) moving it to a slightly cooler room. The temperature changes are to be the same in all four cases. List the changes according to the resulting coefficients of performance, greatest first.

Answer : (a), (b), (c), (d)

Solution : Closer the temperatures of the two reservoirs to each other higher is the value of coefficient of performance.

Example 2. Assuming a domestic refrigerator as reversible engine working between melting point of ice and the room temperature of 27ºC, calculate the energy in joule that must be supplied to freeze one kg of water. Given : temperature of water – 0ºC, L = 80 cal g^{-1}.

Solution : T_1 = (27 + 273) K = 300 K

T_2 = (0 + 273) K = 273 K

Heat to be removed, $Q_2 = mL$

where m is the mass of water and L is the latent heat.

$\therefore \quad Q_2 = 1000 \times 80 \text{ cal} = 8 \times 10^4 \text{ cal}$

$[\therefore\ m = 1 \text{ kg} = 1000 \text{ g}]$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} \quad \text{or} \quad Q_1 = \frac{T_1}{T_2} \times Q_2 \quad \text{or} \quad Q_1 = \frac{300}{273} \times 8 \times 10^4 \text{ cal} = 87912.1 \text{ cal}$$

Energy required to be supplied, $W = Q_1 - Q_2$

or $\quad W = (87912.1 - 80{,}000) \text{ cal} = 9712.1 \text{ cal}$

$= 7912.1 \times 4.2 \text{ J} = 33230.8 \text{ J}$

EXERCISE

21. How much energy in watt-hour may be required to convert 2 kg of water into ice at 0ºC assuming that the refrigerator is ideal? Given : temperature of freezer = – 15ºC, room temperature = 25ºC and initial temperature of water = 25ºC.

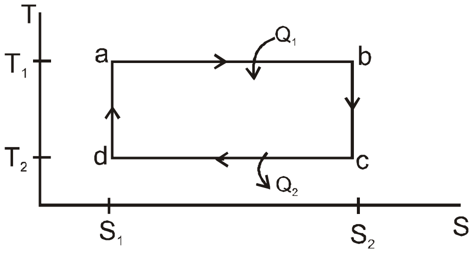
ANSWER KEY OF EXERCISE

1. No. It is an ideal heat engine **2.** (i) 500 J (ii) 500 J (iii) 50% **3.** 50%

4. (a) 20ºC (b) 129.5ºC; Process (a) In which temperature of sink has to be redused by 20ºC will be prefereable

5. (i) 550 K (ii) 489.9 K

6. The basic process of a Carnot engine, described above, is again shown in figurein a T-S (temperature-entropy) diagram.



The points a, b, c and d represent the same states as in figure. Let the entropy in state a be S_1. An amount Q_1 of heat is supplied to the system in the isothermal process ab at the temperature T_1. The entropy increases in this part as heat is supplied to the system. Also, by definition,

$$S_2 - S_1 = \frac{Q_1}{T_1} \qquad \text{........(i)}$$

The entropy remains constant in the part bc as it describes an adiabatic process. So the entropy in state c is S_2. In the part cd, the system gives a heat Q_2 at the lower temperature T_2 and its entropy is decreased. The part da represents an adiabatic process and the entropy remains constant. As the entropy in state a is S_1, the entropy in state d is also S_1, Using the definition of change in entropy for the process cd,

$$S_1 - S_2 = \frac{Q_2}{T_2} \qquad \text{........(ii)}$$

From (i) and (ii),

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \qquad \text{or} \qquad \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

The efficiency of the engine is

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \qquad \text{........(iii)}$$

Thus, the efficiency of the engine depends only on the temperatures of the hot and cold bodies between which the engine works.

7. (B) **8.** **(A)** **9.** (A) **10.** (C) **11.** (B) **12.** (C) **13.** (E)

14. (C) **15.** 9 **16.** $T_1 = 36^oC = (36 + 273)\ K = 309\ K$, $T_2 = 9^oC = (9 + 273)\ K = 282\ K$

Coefficient of performance $= \dfrac{T_2}{T_1 - T_2} = \dfrac{282}{309 - 282} = \dfrac{282}{27} = 10.4$

17. 879.1 kcal, 3.32×10^5 J

(i) Production of heat on passing an electric current through a wire. The same amount of heat is produced in a given time, even when the direction of current is reversed.

(ii) Mixing of two different substances like water and alcohol.

(iii) The formation of solution of a solid in water etc.

18. Cyclie process.

19. 1.43

20. 66.67

The efficiency will be 100 % if $T_2 = 0$, i.e., if the sink is maintained at absolute zero of temperature.

21. 37.98 watt hour

Exercise-1

Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Kinetic Theory of Gases

A-1. Find the average momentum of molecules of hydrogen gas in a container at temperature 300 K.

A-2. A cubical container having each side as ℓ is filled with a gas having N molecules in the container. Mass of each molecule is m. If we assume that at every instant half of the molecules are moving towards the positive x-axis and half of the molecules are moving towards the negative x-axis. Two walls of the container are perpendicular to the x-axis. Find the net force acting on the two walls given? Assume that all the molecules are moving with speed v_0.

Section (B) : Root mean square speed, Kinetic Energy and equation of state

B-1. The speeds of three molecules are 3V, 4V and 5V respectively. Find their rms speed.

B-2. At room temperature (300 K), the rms speed of the molecules of a certain diatomic gas is found to be 1930 m/s. Can you guess name of the gas ? Find the temperature at which the rms speed is double of the speed in part one (R = 25/3 J/mol – k)

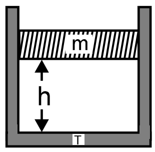
B-3. A gas is filled in a rigid container at pressure P_0. If the mass of each molecule is halved keeping the total number of molecules same and their r.m.s. speed is doubled then find the new pressure.

B-4. Butane gas burns in air according to the following reaction, $2C_4H_{10} + 13\ O_2 \longrightarrow 10\ H_2O + 8\ CO_2$. Suppose the initial and final temperatures are equal and high enough so that all reactants and products act as perfect gases. Two moles of butane are mixed with 13 moles of oxygen and then completely reacted. Find the final pressure (if the volume remains unchanged and the pressure before reaction is P_0) ?

B-5. At a pressure of 3 atm air (treated as an ideal gas) is pumped into the tubes of a cycle rickshaw. The volume of each tube at given pressure is 0.004 m^3. One of the tubes gets punctured and the volume of the tube reduces to 0.0008 m^3. Find the number of moles of air that have leaked out? Assume that the temperature remains constant at 300 K. (R = 25/3 J mol^{-1} K^{-1})

B–6. (i) A conducting cylinder whose inside diameter is 4.00 cm contains air compressed by a piston of mass m = 13.0 kg, which can slide freely in the cylinder shown in the figure.

The entire arrangement is immersed in a water bath whose temperature can be controlled. The system is initially in equilibrium at temperature t_i = 20°C. The initial height of the piston above the bottom of the cylinder is h_i = 4.00 cm. P_{atm} = 1 × 10^5 N/m^2 and g = 10 m/s^2. If the temperature of the water bath is gradually increased to a final temperature t_f = 100°C. Find the height h_f of the piston (in cm) at that instant?



(ii) In the above question, if we again start from the initial conditions and the temperature is again gradually raised, and weights are added to the piston to keep its height fixed at h_i. Find the value of the added mass when the final temperature becomes t_f = 100°C?

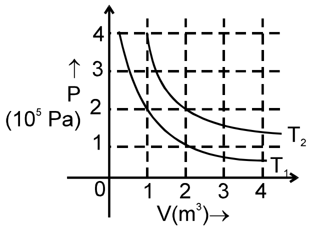
Section (C) : Maxwell's distribution of speed

C-1. Find the temperature at which average speed of oxygen molecule be sufficient so as to escape from the earth? (Escape speed from the earth is 11.0 km/sec, R = 25/3 J-$mol^{-1}K^{-1}$).

C-2. Find the average of magnitude of linear momentum of helium molecules in a sample of helium gas at temperature of 150 π K. Mass of a helium molecules = (166/3) × 10^{-27} kg and R = 25/3 J-mol^{-1} K^{-1}

C-3. The mean speed of the molecules of a hydrogen sample equals the mean speed of the molecules of helium sample. Calculate the ratio of the temperature of the hydrogen sample to the temperature of the helium sample.

C-4. The following graph shows two isotherms for a fixed mass of an ideal gas. Find the ratio of r.m.s. speed of the molecules at temperatures T_1 amd T_2 ?



Section (D) : Law of equipartition and internal energy

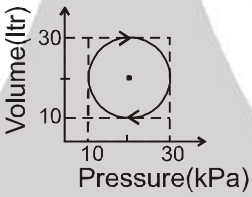
D-1. 16 g of oxygen at 37ºC is mixed with 14 g of nitrogen at 27ºC. Find the temperature of the mixture?

D-2. 0.040 g of He is kept in a closed container initially at 100.0ºC. The container is now heated. Neglecting the expansion of the container, calculate the temperature at which the internal energy is increased by 12J. $\left[R = \frac{25}{3} J - mol^{-1} - k^{-1}\right]$

D-3. Show that the internal energy of the air (treated as an ideal gas) contained in a room remains constant as the temperature changes between day and night. Assume that the atmospheric pressure around remains constant and the air in the room maintains this pressure by communicating with the surrounding through the windows etc.

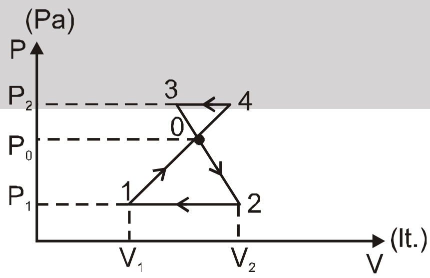
Section (E) : Calculation of work

E-1. Find the work done by gas going through a cyclic process shown in figure?



E-2. An ideal gas is compressed at constant pressure of 10^5 Pa until its volume is halved. If the initial volume of the gas as 3.0×10^{-2} m^3, find the work done on the gas?

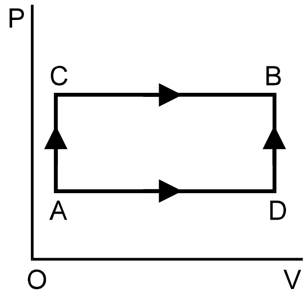
E-3. Find the work done by an ideal gas during a closed cycle $1 \rightarrow 4 \rightarrow 3 \rightarrow 2 \rightarrow 1$ shown in figure if $P_1 = 10^5$ Pa, $P_0 = 3 \times 10^5$ Pa, $P_2 = 4 \times 10^5$ Pa, $V_2 - V_1 = 10$ litre, and segments 4-3 and 2-1 of the cycle are parallel to the V-axis ?



E-4. Find the expression for the work done by a system undergoing isothermal compression (or expansion) from volume V_1 to V_2 at temperature T_0 for a gas which obeys the van der waals equation of state, $(p + an^2 / V^2)(V - bn) = nRT$?

Section (F) : First Law of thermodynamics

F-1. In given figure, when a thermodynamic system is taken from state A to state B via path ACB, 100 cal of heat given to the system and 60 cal work is done by the gas. Along the path ADB, the work done by the gas is 20 cal. Find the heat flowing into the system in this case?

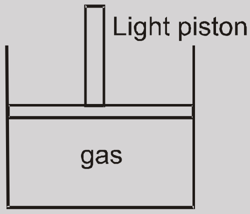


F-2. A cylinder fitted with a piston contains an ideal monoatomic gas at a temperature of 400 K. The piston is held fixed while heat ΔQ is given to the gas, It is found the temperature of the gas has increased by 20 K. In an isobaric process the same ΔQ heat is supplied slowly to it. Find the change in temperature in the second process?

F-3. When 1 g of water at 0ºC and 1×10^5 N m^{-2} pressure is converted into ice of volume 1.091 cm^3, find the work done by water ? (ρ_w = 1 gm/cm^3)

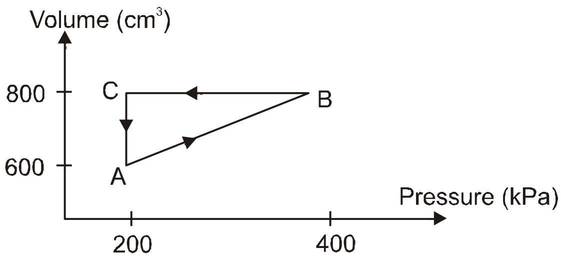
F-4. An ideal gas is taken through a cyclic thermodynamic process through four steps. The amounts of heat involved in these steps are Q_1 = 5960 J, Q_2 = – 5585 J, Q_3 = – 2980 J and Q_4 = 3645 J respectively. The corresponding works involved are W_1 = 2200 J, W_2 = – 825 J, W_3 = – 1100 J and W_4 respectively.
(i) Find the value of W_4.
(ii) What is the efficiency of the cycle ?

F-5. In given figure, gas is slowly heated for sometime. During the process, the increase in internal energy of the gas is 10 J and the piston is found to move out by 25 cm, then find the amount of heat supplied. The area of cross-section of cylinder = 40 cm^2 and atmospheric pressure = 100 kPa

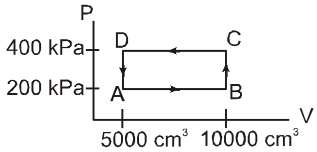


F-6. Find the change in the internal energy of 2kg of water as it is heated from 0ºC to 4ºC. The specific heat capacity of water is 4200 J/kg-K and its densities at 0ºC and 4ºC are 999.9kg/m^3 and 1000 kg/m^3 respectively. Atmospheric pressure = 10^5 Pa.

F-7. In given figure, An ideal gas a gas is taken through a cyclic process ABCA, calculate the value of mechanical equivalent of heat (J) when 4.8 cal of heat is given in the process ?



F-8. In given figure, one mole of an ideal gas (γ = 7/5) is taken through the cyclic process ABCDA. Take R = $\frac{25}{3}$ J/mol–K



(a) Find the temperature of the gas in states A, B, C and D.
(b) Find the amount of heat supplied/released in processes AB, BC, CD and DA.
(c) Find work done by gas during cyclic process.

Section (G) : Specific heat capacities of gases

G-1. If γ be the ratio of specific heats (C_p & C_v) for a perfect gas, Find the number of degrees of freedom of a molecule of the gas? :

G-2. Internal energy of two moles of an ideal gas at a temperature of 127ºC is 1200 R. Then find the molar specific heat of the gas at constant pressure?

G-3. Ideal monoatomic gas is taken through a process dQ = 2dU. Find the molar heat capacity (in terms of R) for the process ? (where dQ is heat supplied and dU is change in internal energy)

G-4. Calculate the value of mechanical equivalent of heat from the following data. Specific heat capacity of air at constant volume and at constant pressure are 4.93 cal/mol–K and 6.90 cal/mol–K respectively. Gas constant R = 8.3 J/mol–K.

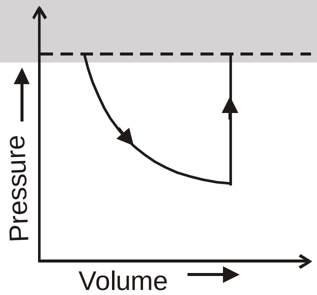
G-5. When 100 J of heat is given to an ideal gas it expands from 200 cm^3 to 400 cm^3 at a constant pressure of 3×10^5 Pa. Calculate (a) the change in internal energy of the gas, (b) the number of moles in the gas if the initial temperature is 400 K, (c) the molar heat capacity C_p at constant pressure and (d) the molar heat capacity C_v at constant volume. $\left[R = \frac{25}{3} J/mol - k\right]$

G-6. The temperature of 5 mol of a gas which was held at constant volume was changed from 100ºC to 120ºC. The changes in internal energy was found to be 80 J. Find the molar heat capacity of the gas at constant volume?

G-7. For a gas, γ = 9/7. What is the number of degrees of freedom of the molecules of this gas ?

Section (H) : Adiabatic process and free expansion

H-1. In given figure, a sample of an ideal gas initially having internal energy U_1 is allowed to expand adiabatically performing work W. Heat Q is then supplied to it, keeping the volume constant at its new value, until the pressure rised to its original value. The internal energy is then U_2.



Find the increase in internal energy ($U_2 - U_1$) ?

H-2. One mole of an ideal monoatomic gas $\left(\gamma = \frac{5}{3}\right)$ is mixed with one mole of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$.

(γ denotes the ratio of specific heat at constant pressure, to that at constant volume) find γ for the mixture ?

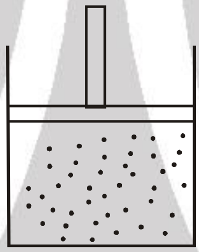
H-3. The pressure and density of a diatomic gas $\left(\gamma = \frac{7}{5}\right)$ change adiabatically from (P, d) to (P′, d′). If $\frac{d'}{d}$ = 32, then find the value of $\frac{P'}{P}$?

H-4. An ideal gas ($\gamma = \frac{5}{3}$) is adiabatically compressed from 640 cm^3 to 80 cm^3. If the initial pressure is P then find the final pressure?

H-5. In an adiabatic process, the pressure is increased by $\frac{2}{3}\%$. If $\gamma = \frac{3}{2}$, then find the decreases in volume (approximately)?

H-6. An ideal gas at pressure 4×10^5 Pa and temperature 400 K occupies 100 cc. It is adiabatically expanded to double of its original volume. Calculate (a) the final pressure, (b) final temperature and (c) work done by the gas in the process (γ = 1.5) :

H-7. In fig, the walls of the container and the piston are weakly conducting. The initial pressure, volume and temperture of the gas are 200 K Pa, 800 cm^3 and 100 K resp. Find the pressure and the temperature of the gas if it is (a) slowly compressed (b) suddenly compressed to 200 cm^3 (γ = 1.5).



H-8. When the state of a system changes form A to B adiabatically the work done on the system is 322 Joule. If the state of the same system is changed from A to B by another process, and heat required is 50 calories of heat is required then find work done on the system in this process? (J = 4.2 J/cal)

Section (I) : Polytropic Process

I-1. Find the molar heat capacity (in terms of R) of a monoatomic ideal gas undergoing the process : $PV^{1/2}$ = constant ?

I-2. If Q amount of heat is given to a diatomic ideal gas in a process in which the gas perform a work $\frac{2Q}{3}$ on its surrounding. Find the molar heat capacity (in terms of R) for the process.

I-3. One mole of a gas expands with temperature T such that its volume, $V = kT^2$, where k is a constant. If the temperature of the gas changes by 60º C then find the work done by the gas? (R = 25/3 J/mol-K).

Section (J) : For jee-main

J-1 A Carnot engine takes 10^3 kilocalories of heat from a reservoir at 627ºC and exhausts it to a sink at 27ºC. What will be the efficiency of the engine ?

J-2 In the above problem, what will be the work performed by the engine ?

J-3 The efficiency of Carnot's engine is 50%. The temperature of its sink is 7ºC. To increase its efficiency to 70%. What is the increase in temperature of the source ?

J-4 A Carnot engine work as refrigerator in between 0ºC and 27ºC. How much energy is needed to freeze 10 kg ice at 0ºC.

J-5 What is the work efficiency coefficient in above question ?

J-6 A Carnot engine works as a refrigerator in between 250K and 300K. If it acquires 750 calories from heat source at low temperature, then what is the heat generated at higher temperature. (in calories)?

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Kinetic Theory of gases

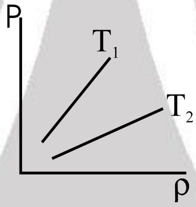
A-1. When an ideal gas is compressed isothermally then its pressure increases because :
(A) its potential energy decreases
(B) its kinetic energy increases and molecules move apart
(C) its number of collisions per unit area with walls of container increases
(D) molecular energy increases

A-2. Which of the following is correct for the molecules of a gas in thermal equilibrium ?
(A) All have the same speed
(B) All have different speeds which remain constant
(C) They have a certain constant average speed
(D) They do not collide with one another.

Section (B) : Root mean square speed, Kinetic Energy and Equation of state

B-1. The temperature at which the r.m.s velocity of oxygen molecules equal that of nitrogen molecules at 100ºC is nearly :
(A) 426.3 K (B) 456.3 K (C) 436.3 K (D) 446.3 K

B-2. Figure shows graphs of pressure vs density for an ideal gas at two temperatures T_1 and T_2.



(A) $T_1 > T_2$ (B) $T_1 = T_2$ (C) $T_1 < T_2$ (D) any of the three is possible

B-3. Suppose a container is evacuated to leave just one molecule of a gas in it. Let v_{mp} and v_{av} represent the most probable speed and the average speed of the gas, then
(A) $v_{mp} > v_{av}$ (B) $v_{mp} < v_{av}$ (C) $v_{mp} = v_{av}$ (D) none of these

B-4. The average speed of nitrogen molecules in a gas is v. If the temperature is doubled and the N_2 molecule dissociate into nitrogen atoms, then the average speed will be
(A) v (B) $v\sqrt{2}$ (C) 2 v (D) 4v

B-5. Four containers are filled with monoatomic ideal gases. For each container, the number of moles, the mass of an individual atom and the rms speed of the atoms are expressed in terms of n, m and v_{rms} respectively. If T_A, T_B, T_C and T_D are their temperatures respectively then which one of the options correctly represents the order ?

	A	B	C	D
Number of moles	n	3n	2n	n
Mass	4m	m	3m	2m
Rms speed	V_{rms}	$2V_{rms}$	V_{rms}	$2V_{rms}$
Temperature	T_A	T_B	T_C	T_D

(A) $T_B = T_C > T_A > T_D$ (B) $T_D > T_A > T_C > T_B$ (C) $T_D > T_A = T_B > T_C$ (D) $T_B > T_C > T_A > T_D$

B-6. For a gas sample with N_0 number of molecules, function N(V) is given by : $N(V) = \frac{dN}{dV} = \left(\frac{3\ N_0}{V_0^3}\right)V^2$ for $0 < V < V_0$ and N(V) = 0 for $V > V_0$. Where dN is number of molecules in speed range V to V+ dV. The rms speed of the molecules is :

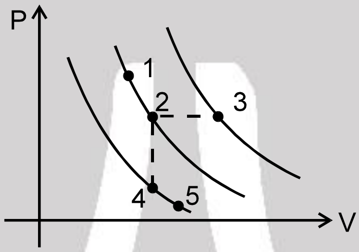
(A) $\sqrt{\frac{2}{5}}V_0$ (B) $\sqrt{\frac{3}{5}}V_0$ (C) $\sqrt{2}V_0$ (D) $\sqrt{3}V_0$

Section (C) : Maxwell's distribution of speed

C-1. Three closed vessels A, B, and C are at the same temperature T and contain gases which obey the Maxwell distribution of speed. Vessel A contains only O_2, B only N_2 and C a mixture of equal quantities of O_2 and N_2. If the average speed of O_2 molecules in vessel A is V_1, that of the N_2 molecules in vessel B is V_2, the average speed of the O_2 molecules in vessel C will be :

(A) $(V_1 + V_2)/2$ (B) V_1 (C) $(V_1V_2)^{1/2}$ (D) $\frac{V_1}{2}$

C-2. A certain gas is taken to the five states represented by dots in the graph. The plotted lines are isotherms. Order of the most probable speed v_p of the molecules at these five states is :



(A) $V_{P\text{ at }3} > V_{P\text{ at }1} = V_{P\text{ at }2} > V_{P\text{ at }4} = V_{P\text{ at }5}$
(B) $V_{P\text{ at }1} > V_{P\text{ at }2} = V_{P\text{ at }3} > V_{P\text{ at }4} > V_{P\text{ at }5}$
(C) $V_{P\text{ at }3} > V_{P\text{ at }2} = V_{P\text{ at }4} > V_{P\text{ at }1} > V_{P\text{ at }5}$
(D) Insufficient information to predict the result.

Section (D) : Law of equipartition and internal energy

D-1. The pressure of an ideal gas is written as $E = \frac{3PV}{2}$. Here E stands for

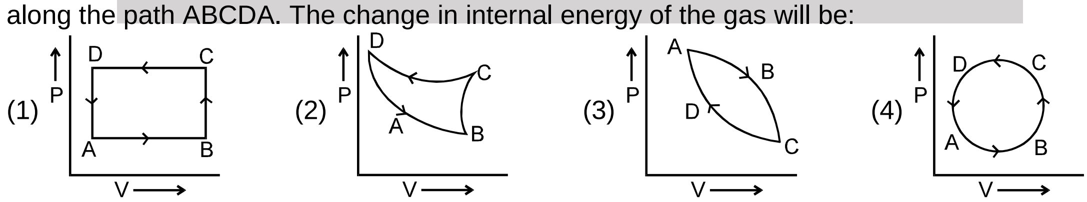
(A) average translational kinetic energy (B) rotational kinetic energy
(C) total kinetic energy. (D) None of these

D-2. The quantities which remain same for all ideal gases at the same temperature is/are ?
(A) the kinetic energy of equal moles of gas (B) the kinetic energy of equal mass of gas
(C) the number of molecules of equal moles of gas (D) the number of molecules of equal mass of gas

D-3. The quantity $\frac{2U}{fkT}$ represents (where U = internal energy of gas)
(A) mass of the gas (B) kinetic energy of the gas
(C) number of moles of the gas (D) number of molecules in the gas

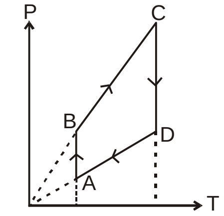
Section (E) : Calculation of work

E-1. In the following figures (1) to (4), variation of volume by change of pressure is shown. A gas is taken along the path ABCDA. The change in internal energy of the gas will be:

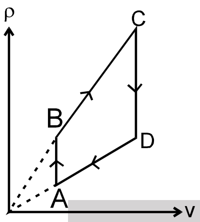


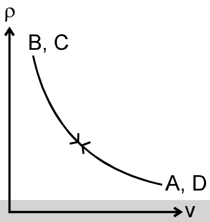
(A) positive in all cases from (1) to (4)
(B) positive in cases (1), (2) and (3) but zero in case (4)
(C) negative in cases (1), (2) and (3) but zero in case (4)
(D) zero in all the four cases.

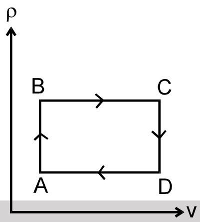
E-2. Pressure versus temperature graph of an ideal gas is as shown in figure.

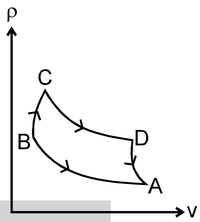


Corresponding density (ρ) versus volume (v) graph will be :

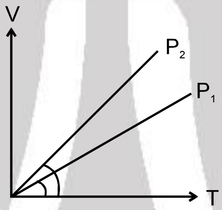
(A) 

(B) 

(C) 

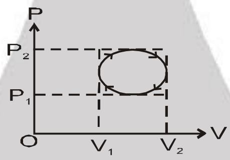
(D) 

E-3. In the following V-T diagram what is the relation between P_1 and P_2 :



(A) $P_2 = P_1$ (B) $P_2 > P_1$ (C) $P_2 < P_1$ (D) cannot be predicted

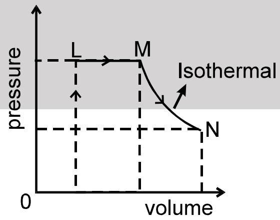
E-4. In a cyclic process shown on the P – V diagram the magnitude of the work done is :



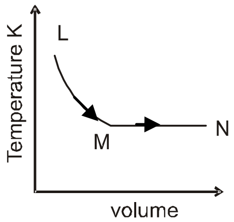
(A) $\pi\left(\frac{P_2 - P_1}{2}\right)^2$ (B) $\pi\left(\frac{V_2 - V_1}{2}\right)^2$

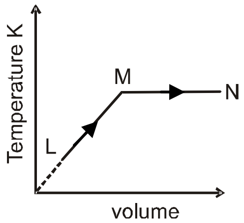
(C) $\frac{\pi}{4}\,(P_2 - P_1)\,(V_2 - V_1)$ (D) $\pi\,(P_2V_2 - P_1V_1)$

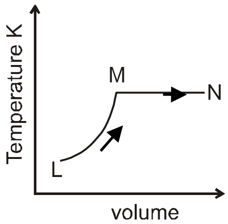
E-5. A fixed mass of an ideal gas undergoes changes of pressure and volume starting at L, as shown in Figure.

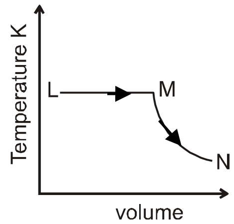


Which of the following is correct :

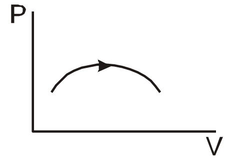
(A) 

(B) 

(C) 

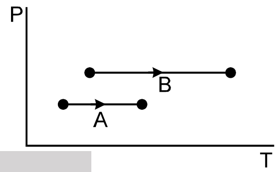
(D) 

E-6. In figure, P-V curve of an ideal gas is given. During the process, the cumulative work done by the gas



(A) continuously increases
(B) continuously decreases
(C) first increases then decreases
(D) first decreases then increases

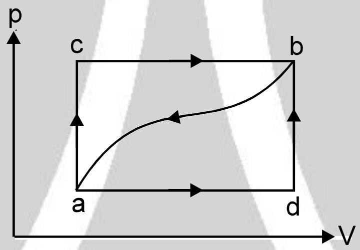
E-7. In given figure, let ΔW_1 and ΔW_2 be the work done by the gas in process A and B respectively then (given change in volume is same in both process)



(A) $\Delta W_1 > \Delta W_2$
(B) $\Delta W_1 = \Delta W_2$
(C) $\Delta W_1 < \Delta W_2$
(D) Nothing can be said about the relation between ΔW_1 and ΔW_2

Section (F) : First law of thermodynamics

F-1. When a system is taken from state 'a' to state 'b' along the path 'acb', it is found that a quantity of heat Q = 200 J is absorbed by the system and a work W = 80J is done by it. Along the path 'adb', Q = 144J. The work done along the path 'adb' is



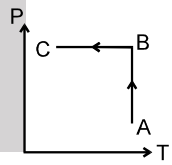
(A) 6J (B) 12 J (C) 18 J (D) 24 J

F-2. In the above question, if the work done on the system along the curved path 'ba' is 52J, heat abosrbed is
(A) – 140 J (B) – 172 J (C) 140 J (D) 172 J

F-3. In above question, if U_a = 40J, value of $U_b$will be
(A) – 50 J (B) 100 J (C) – 120 J (D) 160 J

F-4. In above question, if U_d = 88 J, heat absorbed for the path 'db' is
(A) – 72 J (B) 72 J (C) 144 J (D) – 144 J

F-5. Ideal gas is taken through process shown in figure:



(A) In process AB, work done by system is positive
(B) In process AB, heat is rejected out of the system.
(C) In process AB, internal energy increases
(D) In process AB internal energy decreases and in process BC internal energy increases.

Section (G) : Specific heat capacities of gases

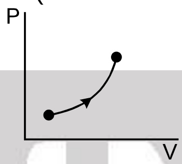
G-1. The value of the ratio C_p/C_v for hydrogen is 1.67 at 30 K but decreases to 1.4 at 300 K as more degrees of freedom become active. During this rise in temperature (assume H_2 as ideal gas),
(A) C_p remains constant but C_v increases
(B) C_p decreases but C_v increases
(C) Both C_p and C_v decrease by the same amount
(D) Both C_p and C_v increase by the same amount

G-2. Boiling water is changing into steam. Under this condition, the specific heat of water is
(A) zero (B) one (C) Infinite (D) less than one

G-3. For an ideal gas, the heat capacity at constant pressure is larger than than that at constant volume because
(A) positive work is done during expansion of the gas by the external pressure
(B) positive work is done during expasion by the gas against external pressure
(C) positive work is done during expansion by the gas against intermolecular forces of attraction
(D) more collisions occur per unit time when volume is kept constant.

G-4. A gas has :
(A) one specific heat only
(B) two specific heats only
(C) infinite number of specific heats
(D) no specific heat

G-5. If molar heat capacity of the given process (as shown in figure) is C, then



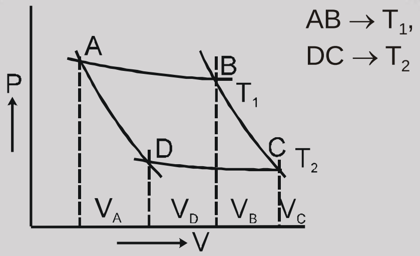
(A) $C < C_V$
(B) $C = 0$
(C) $C > C_v$
(D) $C = C_v$

G-6. For small positive coefficient of expansion in case of solid,
(A) $C_p - C_v = R$
(B) $C_p - C_v = 2R$
(C) C_p is slightly greater than C_v
(D) C_p is slightly less than C_v

Section (H) : Adiabatic process and free expansion

H-1. A gas is contained in a metallic cylinder fitted with a piston. The gas is suddenly compressed by pushing piston downward and is maintained at this position. After this process, as time passes the pressure of the gas in the cylinder
(A) increases
(B) decreases
(C) remains constant
(D) increases or decreases depending on the nature of the gas.

H-2. In the following P–V diagram of an ideal gas, AB and CD are isothermal where as BC and DA are adiabatic process. The value of V_B/V_C is



(A) $= V_A/V_D$
(B) $< V_A/V_D$
(C) $> V_A/V_D$
(D) cannot say

H-3. Two samples 1 and 2 are initially kept in the same state. The sample 1 is expanded through an isothermal process where as sample 2 through an adiabatic process upto the same final volume. The final temperature in process 1 and 2 are T_1 and T_2 respectively, then
(A) $T_1 > T_2$
(B) $T_1 = T_2$
(C) $T_1 < T_2$
(D) The relation between T_1 and T_2 cannot be deduced.

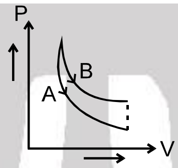
H-4. Let P_1 and P_2 be the final pressure of the samples 1 and 2 respectively in the previous question then :
(A) $P_1 < P_2$
(B) $P_1 = P_2$
(C) $P_1 > P_2$
(D) The relation between P_1 and P_2 cannot be deduced.

H-5. Let ΔW_1 and ΔW_2 be the work done by the systems 1 and 2 respectively in the previous question then :
(A) $\Delta W_1 > \Delta W_2$
(B) $\Delta W_1 = \Delta W_2$
(C) $\Delta W_1 < \Delta W_2$
(D) The relation between W_1 and W_2 cannot be deduced.

H-6. When an ideal gas undergoes an adiabatic change causing a temparture change ΔT
(i) there is no heat gained or lost by the gas
(ii) the work done by the gas is equal to change in internal energy
(iii) the change in internal energy per mole of the gas is $C_v \Delta T$, where C_v is the molar heat capacity at constnat volume.
(A) (i), (ii), (iii) correct (B) (i), (ii) correct (C) (i), (iii) correct (D) (i) correct

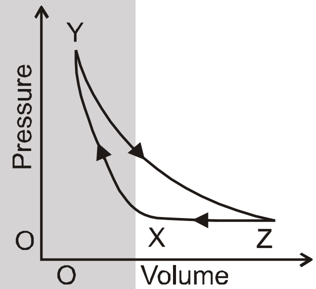
H-7. A given quantity of a gas is at pressure P and absolute temperature T. The isothermal bulk modulus of the gas is:
(A) $\frac{2}{3}P$ (B) P (C) $\frac{3}{2}P$ (D) 2P

H-8. In figure, A and B are two adiabatic curves for two different gases. Then A and B corresponds to :



(A) Ar and He respectively (B) He and H_2 respectively
(C) O_2 and H_2 respectively (D) H_2 and He respectively

H-9. In given figure, a fixed mass of an ideal gas undergoes the change represented by XYZX below. Which one of the following sets could describe these of changes ?



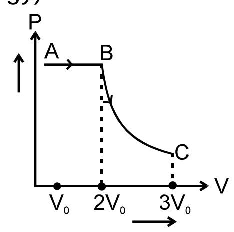
	X Y	Y Z	Z X
(A)	isothermal expansion	adiabatic compression	compression at constant pressure
(B)	adiabatic expansion	isothermal compression	pressure reduction constant volume
(C)	isothermal compression	adiabatic expansion	compression at constant pressure
(D)	adiabatic compression	isothermal expansion	compression at constant pressure

H-10. During an adiabatic process, the pressure of a gas is found to be proportional to the cube of its absolute temperature. The ratio C_p/C_v for the gas is : **[AIEEE - 2003, 4/300]**
(A) 4/3 (B) 2 (C) 5/3 (D) 3/2

Section (I) : Polytrotic Process

I-1. A gas undergoes a process in which its pressure P and volume V are related as VP^n = constant. The bulk modulus of the gas in the process is :
(A) nP (B) $P^{1/n}$ (C) P/n (D) P^n

I-2. One mole of a gas is subjected to two process AB and BC, one after the other as shown in the figure. BC is represented by PV^n = constant. We can conclude that (where T = temperature, W = work done by gas, V = volume and U = internal energy).



(A) $T_A = T_B = T_C$ (B) $V_A < V_B$, $P_B < P_C$ (C) $W_{AB} < W_{BC}$ (D) $U_A < U_B$

I-3. The molar heat capacity C for an ideal gas going through a process is given by C = $\frac{a}{T}$, where 'a' is a constant. If $\gamma = \frac{C_p}{C_v}$, the work done by one mole of gas during heating from T_0 to ηT_0 will be :

(A) $a \ell n \eta$ (B) $\frac{1}{a\ell n\eta}$ (C) $a\ell n\eta - \left(\frac{\eta-1}{\gamma-1}\right)RT_0$ (D) $a\ell n\eta - (\gamma - 1)RT_0$

I-4. One mole of an ideal gas undergoes a process in which $T = T_0 + aV^3$, where T_0 and 'a' are positive constants and V is volume. The volume for which pressure will be minimum is

(A) $\left(\frac{T_0}{2a}\right)^{1/3}$ (B) $\left(\frac{T_0}{3a}\right)^{1/3}$ (C) $\left(\frac{a}{2T_0}\right)^{2/3}$ (D) $\left(\frac{a}{3T_0}\right)^{2/3}$

I-5. In the above question, minimum pressure attainable is

(A) $\frac{3}{4}\left(a^{5/3}R^{2/3}T_0^{2/3}\right)2^{1/3}$ (B) $\frac{3}{2}\left(a^{2/3}RT_0^{2/3}\right)3^{1/2}$ (C) $\frac{3}{2}\left(a^{1/2}R^{2/3}T_0^{3/4}\right)4^{1/3}$ (D) $\frac{3}{2}\left(a^{1/3}RT_0^{2/3}\right)2^{1/3}$

I-6. In a certain gas, the ratio of the speed of sound and root mean square speed is $\sqrt{\frac{5}{9}}$. The molar heat capacity of the gas in a process given by PT = constant is (Take R = 2 cal/mole K). Treat the gas as ideal.

(A) $\frac{R}{2}$ (B) $\frac{3R}{2}$ (C) $\frac{5R}{2}$ (D) $\frac{7R}{2}$

I-7. A polytropic process for an ideal gas is represented by equation PV^n = constant. If γ is ratio of specific heats $\left(\frac{C_p}{C_v}\right)$, then value of n for which molar heat capacity of the process is negative, is given as :

(A) $\gamma > n$ (B) $\gamma > n > 1$ (C) $n > \gamma$ (D) none, as it is not possible

Section (J) : For JEE Main

J-1. A Carnot working between 300K and 600K has work output of 800 J per cycle. What is amount of heat energy supplied to the engine form source per cycle

(A) 1800 J/cycle (B) 1000 J/cycle (C) 2000 J/cycle (D) 1600 J/cycle

J-2. The coefficient of performance of a carnot refrigertor working between 30° C and 0° C is

(A) 10 (B) 1 (C) 9 (D) 0

J-3. If the door of a refrigerator is kept open then which of the following is ture

(A) Room is cooled (B) Room is heated
(C) Room is either cooled or heated (D) Room is neither cooled nor heated

J-4. A scientist says that the efficiency of his heat engine which operates at source temperature 127°C and sink temperature 27° C is 26% then

(A) It is impossible (B) It is possible but less probable
(C) It is quite probable (D) Data are incomplete

J-5. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of : **[AIEEE - 2003, 4/300]**

(A) second law of thermodynamics (B) conservation of momentum
(C) conservation of mass (D) first law of thermodynamics

PART - III : MATCH THE COLUMN

1. An ideal monoatomic gas undergoes different types of processes which are described in column-I. Match the corresponding effects in column-II. The letters have usual meaning.

Column-I	**Column-II**
(A) $P = 2V^2$	(p) If volume increases then temperature will also increases.
(B) PV^2 = constant	(q) If volume increases then temperature will decreases.
(C) $C = C_V + 2R$	(r) For expansion, heat will have to be supplied to the gas.
(D) $C = C_V - 2R$	(s) If temperature increases then work done by gas is positive.

2.✍ The figures given below show different processes (relating pressure P and volume V) for a given amount for an ideal gas. W is work done by the gas and ΔQ is heat absorbed by the gas.

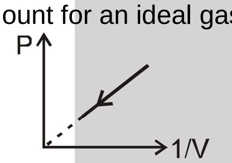


fig (i)

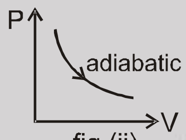


fig (ii)

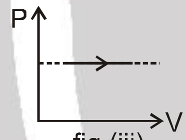


fig (iii)

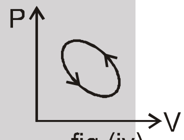


fig (iv)

Column-I	**Column-II**
(A) In Figure (i)	(p) $\Delta Q > 0$.
(B) In Figure (ii)	(q) $W < 0$.
(C) In Figure (iii)	(r) $\Delta Q < 0$.
(D) In Figure (iv) (for complete cycle)	(s) $W > 0$.

Exercise-2

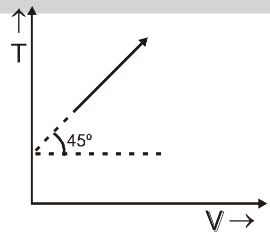
✍ Marked Questions can be used as Revision Questions.

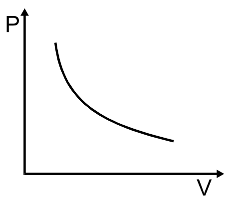
PART - I : ONLY ONE OPTION CORRECT TYPE

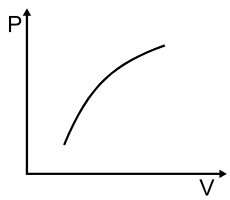
1. The molar heat capacity at constant presure of nitrogen gas at STP is nearly 3.5 R. Now when the temperature is increased, it gradually increases and approaches 4.5 R. The most appropriate reason for this behaviour is that at high temperatures

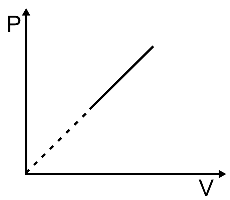
(A) nitrogen does not behave as an ideal gas
(B) nitrogen molecules dissociate in atoms
(C) the molecules collides more frequently
(D) molecular vibration gradually become effective

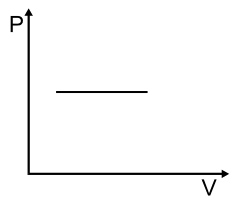
2. The given curve represents the variation of temperature as a function of volume for one mole of an ideal gas. Which of the following curves best represents the variation of pressure as a function of volume ?



(A)


(B)


(C)


(D)


3. Consider a hypothetical gas with molecules that can move along only a single axis. The following table gives four situations, the velocities in meter per second of such a gas having four molecules. The plus and minus sign refer to the direction of the velocity along the axis.

Situation	Velocities			
a	–2	+3	–4	+5
b	+1	–3	+4	–6
c	+2	+3	+4	+5
d	+3	+3	–4	–5

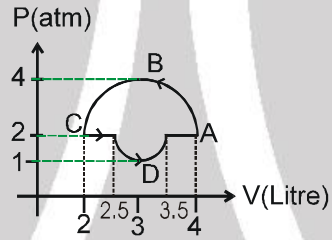
In which situation root-mean-square speed of the molecules is greatest

(A) a (B) b (C) c (D) d

4. The value of $C_P - C_V$ is 1.09 R for a gas sample in state A and is 1.00 R in state B. Let T_A, T_B denote the temperature and P_A and P_B denote the pressure of the states A and B respectively. Then

(A) $P_A < P_B$ and $T_A > T_B$ (B) $P_A > P_B$ and $T_A > T_B$ (C) $P_A = P_B$ and $T_A < T_B$ (D) $P_A > P_B$ and $T_A < T_B$

5. Find work done by the gas in the process shown in figure :

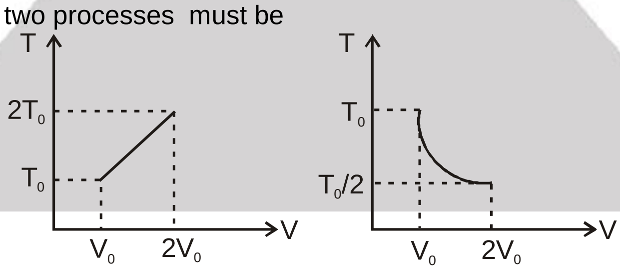


(A) $\frac{5}{2}\pi$ atm L (B) $\frac{5}{2}$ atm L (C) $-\frac{3}{2}\pi$ atm L (D) $-\frac{5}{4}\pi$ atm L

6. An ideal monoatomic gas is initially in state 1 with pressure P_1 = 20 atm and volume V_1 = 1500 cm^3. It is then taken to state 2 with pressure $P_2 = 1.5\ P_1$ and volume $V_2 = 2V_1$. The change in internal energy from state 1 to state 2 is equal to

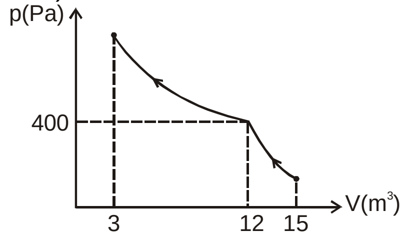
(A) 2000 J (B) 3000 J (C) 6000 J (D) 9000 J

7. For two thermodynamic process temperature and volume diagram are given. In first process, it is a straight line having initial and final coordinates as (V_0, T_0) and ($2V_0$, $2T_0$), where as in second process it is a rectangular hyperbola having initial and final coordinates (V_0, T_0) and ($2V_0$, $T_0/2$). Then ratio of work done ($W_1 : W_2$) in the two processes must be



(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) None of these

8. Curve in the figure shows an adiabatic compression of an ideal gas from 15 m^3 to 12 m^3, followed by an isothermal compression to a final volume of 3.0 m^3. There are 2.0 moles of the gas. Total heat supplied to the gas is equal to : (ℓn2 = 0.693)

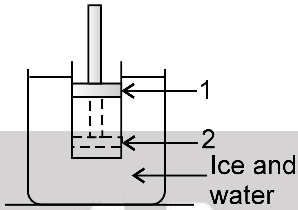


(A) 4521 J (B) –4521 J (C) –6653 J (D) –8476 J

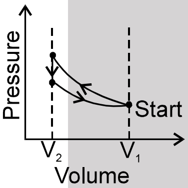
9. P_i, V_i are initial pressure and volumes and V_f is final volume of a gas in a thermodynamic process respectively. If PV^n = constant, then the amount of work done by gas is : ($\gamma = C_p/C_v$). Assume same, initial state & same final volume in all processes.

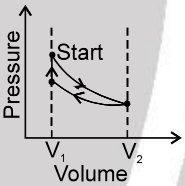
(A) minimum for $n = \gamma$ (B) minimum for n = 1 (C) minimum for n = 0 (D) minimum for $n = \frac{1}{\gamma}$

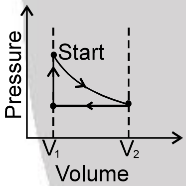
10. Figure shows a conducting cylinder containing gas and closed by a movable piston. The cylinder is submerged in an ice-water mixture. The piston is quickly pushed down from position (1) to position (2). The piston is held at position (2) until the gas is again at 0°C and then is slowly raised back to position (1).

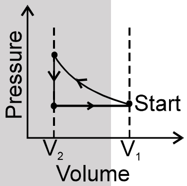


P-V diagram for the above process will be

(A) 

(B) 

(C) 

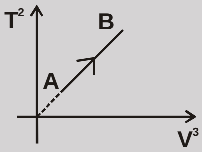
(D) 

11. Two different ideal diatomic gases A and B are initially in the same state. A and B are then expanded to same final volume through adiabatic and isothermal process respectively. If P_A, P_B and T_A, T_B represents the final pressure and temperatures of A and B respectively then:

(A) $P_A < P_B$ and $T_A < T_B$ (B) $P_A > P_B$ and $T_A > T_B$

(C) $P_A > P_B$ and $T_A < T_B$ (D) $P_A < P_B$ and $T_A > T_B$

12. If ideal diatomic gas follows the process, as shown in graph, where T is temperature in kelvin and V is volume (m^3), then molar heat capacity for this process will be [in terms of gas constant R] :



(A) $\frac{7R}{2}$ (B) 5R (C) $\frac{19R}{6}$ D) $\frac{11R}{2}$

13. A mono–atomic ideal gas is compressed from volume V to V/2 through various process. For which of the following processes final pressure will be maximum :

(A) isobaric (B) isothermal (C) adiabatic (D) PV^2 = constant

14. 4 moles of H_2 at 500 K is mixed with 2 moles of He at 400K .The mixture attains a temperature T and volume V. Now the mixture is compressed adiabatically to a volume V' and temperature T'. If $\frac{T'}{T} = \left(\frac{V}{V'}\right)^n$, find the value of 13n.

(A) 4 (B) 6 (C) 5 (D) 13

15. A coal based thermal power plant producing electricity operates between the temperatures 27ºC and 227ºC. The plant works at 80% of its maximum theoretical efficiency. Complete burring of 1 kg of coal yields 3600 KJ of heat. A house needs 10 units of electricity each day. Coal used for supplying the amount of energy for the house in one year is **[Olympiad 2014 (stage –1)]**

(A) 1141 kg (B) 580 kg (C) 605 kg (D) 765 kg

16. Two identical rooms in a house are connected by an open doorway. The temperatures in the two rooms are maintained at two different values. Therefore
(A) The room with higher temperature contains more amount of air.
(B) The room with lower temperature contains more amount of air.
(C) Both the rooms contain the same amount of air.
(D) The room with higher pressure contains more amount of air.

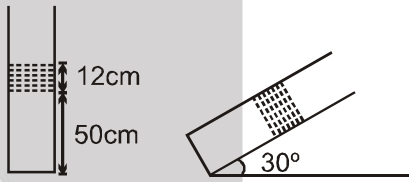
17. A gas is made to undergo a change of state from an initial state to a final state along different paths by adiabatic process only. Therefore.
(A) The work done is different for different paths
(B) The work done is the same for all paths
(C) There is no work done as there is no transfer of energy
(D) The total internal energy of the system will not change

18._ Two moles of hydrogen are mixed with n moles of helium. The root mean square speed of gas molecules in the mixture is $\sqrt{2}$ times the speed of sound in the mixture. Then n is.
[Olympiad (Stage-1) 2017]
(A) 3 (B) 2 (C) 1.5 (D) 2.5

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

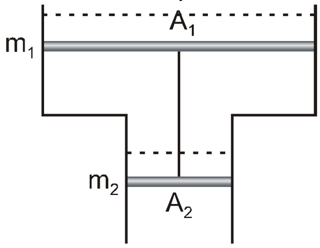
1. A vessel of volume V = 5 litre contains 1.4 g nitrogen and 0.4 g of He at 1500 K. If 30% of the nitrogen molecules are disassociated into atoms then the gas pressure bcomes $\frac{N}{8} \times 10^5$ N/m^2. Find N (Assume T constant). $\left[R = \frac{25}{3} J/mol\ K\right]$

2. In given figure, an ideal gas is trapped between a mercury column and the closed end of a uniform vertical tube. The upper end of the tube is open to the atmosphere. Initialy the lengths of the mercury column and the trapped air column are 12 cm and 50 cm respectively. When the tube is tilted slowly in a vertical plane through an angle of 30º with horizontal then the new length of air column is $\frac{x}{41}$ m. Find x. Assuming the temperature to remain constant. (P_{atm} =76 cm of Hg)



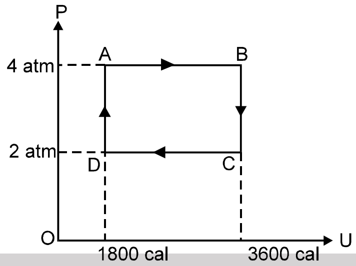
3. Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 litres of gas at 300 K and 2 × 10^5 N m^{-2} pressure while vessel B has 4 litres of gas at 350 K and 4 × 10^5 Nm^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. The new pressure will be $\frac{310}{93} \times 10^n$ (N/m^2). Find n.

4. Consider a vertical tube open at both ends. The tube consists of two parts, each of different cross-sections and each part having a piston which can move smoothly in respective tubes. The two pistons are joined together by an inextensible wire. The combined mass of the two piston is 5 kg and area of cross-section of the upper piston is 10 cm^2 greater than that of the lower piston. Amount of gas enclosed by the pistons is one mole. When the gas is heated slowly, pistons move by 50 cm as shown in figure. The rise in the temperature of the gas, in the form $\frac{X}{R}$ K where R is universal gas constant. Use g = 10 m/s^2 and outside pressure = 10^5 N/m^2). Fill value of X.

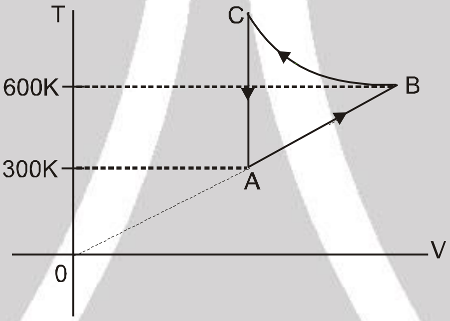


5. When 2g of gas A is introduced into an evacuated flask kept at 25^0C the pressure is found to be 1atm. If 3 g of another gas B is then added to the same flask the total pressure becomes 1.5atm. The ratio of molecular weight of A and B is 1 : n. Find n.

6. Two moles of an ideal monoatomic gas undergo a cyclic process which is indicated on a P-U diagram, where U is the internal energy of the gas. The work done by the gas in the cycle is $k \times 10^2$ ln 2. Find k.



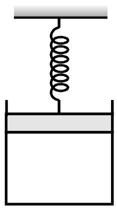
7. In figure, a sample of 3 moles of an ideal gas is undergoing through a cyclic process ABCA. A total of 1500 J of heat is withdrawn from the sample in the process. The work done by the gas during the part BC is –P kJ. FIND P. ($R = \frac{25}{3}$ J/mole K)



8. During the expansion process the volume of the gas changes from $4m^3$ to $6m^3$ while the pressure changes according to P = 30V + 100 where pressure is in Pa and volume is in m^3. The work done by gas is N $\times 10^2$ J. Find N.

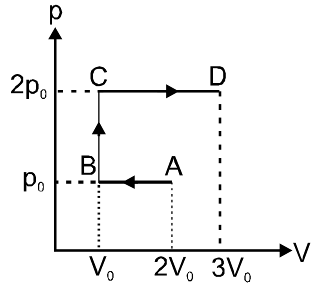
9. A balloon containing an ideal gas has a volume of 10 liter and temperature of 17ºC. If it is heated slowly to 75ºC, the work done by the gas inside the balloon is 2×10^x J. Find x. (neglect elasticity of the balloon and take atmospheric pressure as 10^5 Pa).

10. One mole of an ideal gas is kept enclosed under a light piston (area=10^{-2} m^2) connected by a compressed spring (spring constant 100 N/m). The volume of gas is 0.83 m^3 and its temperature is 100K. The gas is heated so that it compresses the spring further by 0.1 m. The work done by the gas in the process is $N \times 10^{-1}$ J. Find N. (Take R = 8.3 J/K-mole and suppose there is no atmosphere).



11. An adiabatic cylindrical tube is fitted with an adiabatic separator as shown in figure. Initially separator is in equilibrium and divides a tube in two equal parts. The separator can be slide into the tube by an external mechanism. An ideal gas (γ = 1.5) is injected in the two sides at equal pressures and temperatures. Now separator is slid to a position where it divides the tube in the ratio 7 : 3. The ratio of the temperatures in the two parts of the vessel is $\sqrt{n} : \sqrt{7}$ find n.

12. P–V indicator diagram for a given sample of monoatomic ideal gas is shown in figure. If the average molar specific heat capacity of the system for the process ABCD is $\frac{xR}{4}$ than find value of x : (R is a universal gas constant)



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

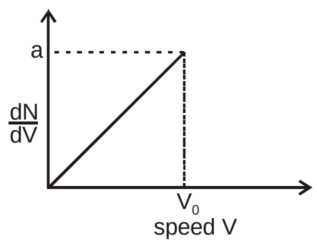
1. In a mixture of nitrogen and helium kept at room tempertaure. As compared to a helium molecule nitrogen molecule hits the wall
(A) With greater average speed
(B) with smaller average speed
(C) with greater average kinetic energy
(D) with smaller average kinetic energy.

2. Consider a collision between an argon molecule and a nitrogen molecule in a mixture of argon and nitrogen kept at room temperature. Which of the following are possible ?
(A) The kinetic energies of both the molecules decrease.
(B) The kinetic energies of both the molecules increase
(C) The kinetic energy of the argon molecule increases and that of the nitrogen molecules decrease.
(D) The kinetic energy of the nitrogen molecules increases and that of the argon molecule decrease.

3. An ideal gas of one mole is kept in a rigid container of negligible heat capacity. If 25 J of heat is supplied the gas temperature raises by 2°C. Then the gas may be
(A) helium
(B) argon
(C) oxygen
(D) carbon dioxide

4. Pick the correct statement (s) :
(A) The rms translational speed for all ideal-gas molecules at the same temperature is not the same but it depends on the mass.
(B) Each particle in a gas has average translational kinetic energy and the equation $\frac{1}{2}mv^2_{rms} = \frac{3}{2}kT$ establishes the relationship between the average translational kinetic energy per particle and temperature of an ideal gas. It can be concluded that single particle has a temperature.
(C) Temperature of an ideal gas is doubled from 100°C to 200°C. The average kinetic energy of each particle is also doubled.
(D) It is possible for both the pressure and volume of a monoatomic ideal gas to change simultaneously without causing the internal energy of the gas to change.

5. Graph shows a hypothetical speed distribution for a sample of N gas particle (for $V > V_0$; $\frac{dN}{dV} = 0$)



(A) The value of aV_0 is 2N.
(B) The ratio V_{avg}/V_0 is equal to 2/3.
(C) The ratio V_{rms}/V_0 is equal to $1/\sqrt{2}$.
(D) Three fourth of the total particle has a speed between 0.5 V_0 and V_0.

6. A system undergoes a cyclic process in which it absorbs Q_1 heat and gives out Q_2 heat. The efficiency of the process is η and work done is W. Select correct statement:

(A) $W = Q_1 - Q_2$ (B) $\eta = \frac{W}{Q_1}$ (C) $\eta = \frac{Q_2}{Q_1}$ (D) $\eta = 1 - \frac{Q_2}{Q_1}$

7. The pressure P and volume V of an ideal gas both decreases in a process.
(A) The work done by the gas is negative
(B) The work done by the gas is positive
(C) The temperature of the gas must decrease
(D) Heat supplied to the gas is equal to the change in internal energy.

8. An ideal gas can be taken from initial state 1 to final state 2 by two different process. Let ΔQ and W represent the heat given and work done by the system. Then which quantities is/are same in both process (where ΔU = internal energy of gas)
(A) ΔQ (B) W (C) ΔU (D) ΔQ - W

9. The following sets of values for C_v and C_p of an ideal gas have been reported by different students. The units are cal mole^{-1} K^{-1}. Which of these sets is most reliable ?
(A) $C_v = 3, C_p = 5$ (B) $C_v = 4, C_p = 6$ (C) $C_v = 3, C_p = 2$ (D) $C_v = 3, C_p = 4.2$

10. For an ideal gas :
(A) the change in internal energy in a constant pressure process from temperature T_1 to T_2 is equal to $nC_v(T_2 - T_1)$, where C_v is the molar specific heat at constant volume and n the number of moles of the gas.
(B) the change in internal energy of the gas and the work done by the gas are equal in magnitude in an adiabatic process.
(C) the internal energy does not change in an isothermal process.
(D) no heat is added or removed in an adiabatic process.

11. A gaseous mixture consists of equal number of moles of two ideal gases having adiabatic exponents γ_1 and γ_2 and molar specific heats at constant volume C_{v_1} and C_{v_2} respectively. Which of the following statements is/are correct ?
(A) Adiabatic exponent for gaseous mixture is equal to $\frac{\gamma_1 + \gamma_2}{2}$
(B) Molar specific heat at constant volume for gaseous mixture is equal to $\frac{C_{v_1} + C_{v_2}}{2}$
(C) Molar specific heat at constant pressure for gaseous mixture is equal to $\frac{C_{v_1} + C_{v_2} + R}{2}$
(D) Adiabatic exponent for gaseous mixture is $1 + \frac{2R}{C_{v_1} + C_{v_2}}$

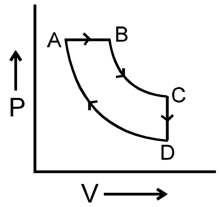
12. Let n_1 and n_2 moles of two different ideal gases be mixed. If adiabatic coefficient of the two gases are γ_1 and γ_2 respectively, then adiabatic coefficient γ of the mixture is given through the relation

(A) $(n_1 + n_2)\,\gamma = n_1\,\gamma_1 + n_1\,\gamma_2$ (B) $\frac{(n_1+n_2)}{\gamma-1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}$ **[Olympiad 2011-12]**

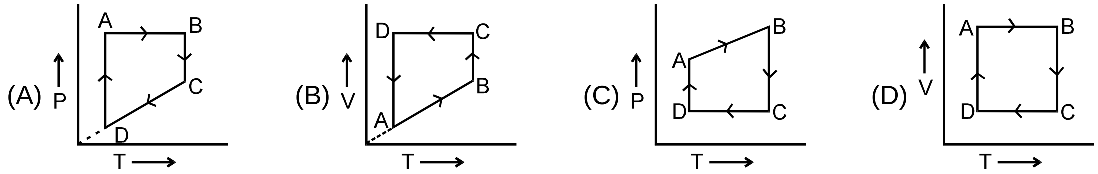
(C) $(n_1+n_2)\,\frac{\gamma}{\gamma-1} = n_1\frac{\gamma_1}{\gamma_1-1} + n_2\frac{\gamma_2}{\gamma_2-1}$ (D) $(n_1 + n_2)(\gamma - 1) = n_1(\gamma_1 - 1) + n_2(\gamma_2 - 1)$

13. An ideal gas can be expanded from an initial state to a certain volume through two different processes (i) PV^2 = constant and (ii) $P = KV^2$ where K is a positive constant. Then
(A) Final temperature in (i) will be greater then in (ii)
(B) Final temperature in (ii) will be greater then in (i)
(C) Total heat given to the gas in (i) case is greater than in (ii)
(D) Total heat given to the gas in (ii) case is greater than in (i)

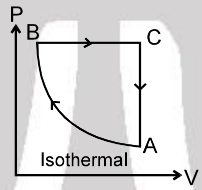
14. A cyclic process ABCD is shown in the P–V diagram. (BC and DA are isothermal)



Which of the following curves represents the same process?



15. A cyclic process of an ideal monoatomic gas is shown in figure. The correct statement is (are) :



(A) Work done by gas in process AB is more than that of the process BC.
(B) net heat energy has been supplied to the system.
(C) temperature of the gas is maximum in state B.
(D) in process CA, heat energy is rejected out by system.

16. A gas kept in a container, if the container is of finite conductivity, then the process
(A) must be very nearly adiabatic
(B) must be very nearly isothermal
(C) may be very nearly adiabatic
(D) may be very nearly isothermal

17. Oxygen, nitrogen and helium gas are kept in three identical adiabatic containers P, Q and R respectively at equal pressure. When the gases are pushed to half their original volumes. (Initial temperature is same)
(A) The final temperature in the three containers will be the same.
(B) The final pressures in the three containers will be the same.
(C) The pressure of oxygen and nitrogen will be the same but that of helium will be different.
(D) The temperature of oxygen and nitrogen will be the same but that of helium will be different

18. During an experiment, an ideal gas is found to obey a condition $\frac{P^2}{\rho}$ = constant [ρ = density of the gas]. The gas is initially at temperature T, pressure P and density ρ. The gas expands such that density changes to $\frac{\rho}{2}$
(A) The pressure of the gas changes to $\sqrt{2}$ P.
(B) The temperature of the gas changes to $\sqrt{2}$ T.
(C) The graph of the above process on the P-T diagram is parabola.
(D) The graph of the above process on the P-T diagram is hyperbola.

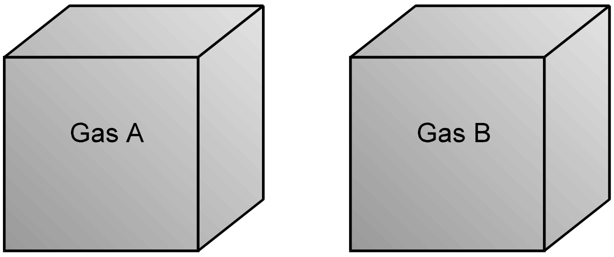
19. Which of the following statement/s in case of a thermodynamic process is /are correct ? **[Olympiad 2015 (stage-1)]**
(A) $\Delta E_{int} = W$ indicates an adiabatic process
(B) $\Delta E_{int} = Q$ suggests an isochoric process
(C) $\Delta E_{int} = 0$ is true for a cyclic process
(D) $\Delta E_{int} = -W$ indicates an adiabatic

20. If a system is made to undergo a change from an initial state to a final state by adiabatic process only, then **[Olympiad (Stage-1) 2017]**
(A) the work done is different for different paths connecting the two states
(B) there is no work done since there is no transfer of heat
(C) the internal energy of the system will change
(D) the work done is the same for all adiabatic paths.

PART - IV : COMPREHENSION

Comprehension # 1

Two closed identical conducting containers are found in the laboratory of an old scientist. For the verification of the gas some experiments are performed on the two boxes and the results are noted.



Experiment -1:

When the two containers are weighed W_A = 225 g, W_B = 160 g and mass of evacuated container W_C = 100 g.

Experiment -2:

When the two containers are given same amount of heat same temperature rise is recorded. The pressure changes found are ΔP_A = 2.5 atm. ΔP_B = 1.5 atm.

Required data for unknown gas :

Mono (molar mass)	He 4g	Ne 20g	Ar 40 g	Kr 84 g	Xe 131 g	Rd 222 g
Dia (molar mass)	H_2 2g	F_2 19 g	N_2 28g	O_2 32g	Cl_2 71 g	

1. Identify the type of gas filled in container A and B respectively.
(A) Mono, Mono (B) Dia, Dia (C) Mono, Dia (D) Dia, Mono.

2. Identify the gas filled in the container A and B.
(A) N_2, Ne (B) He, H_2 (C) O_2, Ar (D) Ar, O_2

3. Total number of molecules in 'A' (here N_A = Avagadro number)
(A) $\frac{125}{64}N_A$ (B) 3.125 N_A (C) $\frac{125}{28}N_A$ (D) 31.25 N_A

4. The initial internal energy of the gas in container 'A', If the container were at room temperature 300K initially
(A) 1406.25 cal (B) 1000 cal (C) 2812.5 cal (D) none of these

Comprehension # 2

A mono atomic ideal gas is filled in a non conducting container. The gas can be compressed by a movable non conducting piston. The gas is compressed slowly to 12.5% of its initial volume.

5. The percentage increase in the temperature of the gas is
(A) 400% (B) 300% (C) – 87.5% (D) 0%

6. The ratio of initial adiabatic bulk modulus of the gas to the final value of adiabatic bulk modulus of the gas is
(A) 32 (B) 1 (C) 1/32 (D) 4

7. The ratio of work done by the gas to the change in internal energy of the gas is
(A) 1 (B) –1 (C) ∞ (D) 0

Comprehension # 3

An ideal gas initially at pressure p_0 undergoes a free expansion (expansion against vaccum under adiabatic conditions) until its volume is 3 times its initial volume. The gas is next adiabatically compressed back to its original volume. The pressure after compression is $3^{2/3}$ p_0.

8. The pressure of the gas after the free expansion is :

(A) $\frac{p_0}{3}$ (B) $p_0^{1/3}$ (C) p_0 (D) $3p_0$

9. The gas

(A) is monoatomic.

(B) is diatomic.

(C) is polyatomic.

(D) type is not possible to decide from the given information.

10. What is the ratio of the average kinetic energy per molecule in the final state to that in the initial state ?

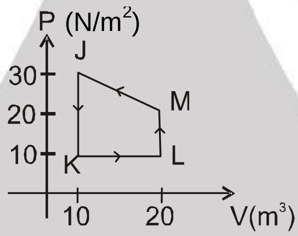
(A) 1 (B) $3^{2/3}$ (C) $3^{1/3}$ (D) $3^{1/6}$

Exercise-3

✍ Marked Questions can be used as Revision Questions.
**** Marked Questions may have more than one correct option.***

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. Match the following for the given process : **[JEE 2006, 6/184]**



(A) Process J → K (p) W > 0

(B) Process K → L (q) W < 0

(C) Process L → M (r) Q > 0

(D) Process M → J (s) Q < 0

2. **Statement-1 :** The total translational kinetic energy of all the molecules of a given mass of an ideal gas is 1.5 times the product of its pressure and its volume. Because **[JEE 2007; 3/162)]**

Statement-2 : The molecules of a gas collide with each other and the velocities of the molecules change due to the collision.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1

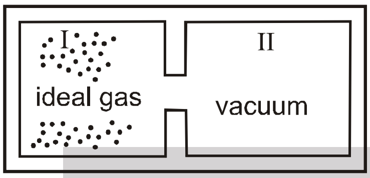
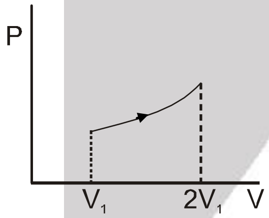
(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

3. An ideal gas is expanding such that PT^2 = constant. The coefficient of volume expansion of the gas is

(A) $\frac{1}{T}$ (B) $\frac{2}{T}$ (C) $\frac{3}{T}$ (D) $\frac{4}{T}$ **[JEE 2008' 3/163]**

4. **Column I** contains a list of processes involving expansion of an ideal gas. Match this with **Column II** describing the thermodynamic change during this process. Indicate your answer by darkening the appropriate bubbles of the 4 × 4 matrix given in the ORS. **[JEE 2008' 6/163]**

Column I	**Column II**
(A) An insulated container has two chambers separated by a valve. Chamber I contains an ideal gas and the Chamber II has vacuum. The valve is opened. 	(p) The temperature of the gas decreases
(B) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^2}$, where V is the volume of the gas.	(q) The temperature of the gas increases or remains constant
(C) An ideal monoatomic gas expands to twice its original volume such that its pressure $P \propto \frac{1}{V^{4/3}}$, where V is its volume	(r) The gas loses heat
(D) An ideal monoatomic gas expands such that its pressure P and volume V follows the behaviour shown in the graph 	(s) The gas gains heat

5.* C_v and C_p denote the molar specific heat capacities of a gas at constant volume and constant pressure, respectively. Then **[JEE, 2009, 4/160, –1]**

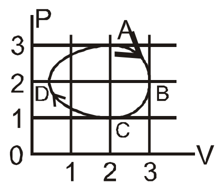
(A) $C_p - C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(B) $C_p + C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

(C) C_p/C_v is larger for a diatomic ideal gas than for a monoatomic ideal gas

(D) $C_p.C_v$ is larger for a diatomic ideal gas than for a monoatomic ideal gas

6.* The figure shows the P-V plot of an ideal gas taken through a cycle ABCDA. The part ABC is a semi-circle and CDA is half of an ellipse. Then, **[JEE, 2009, 4/160, –1]**



(A) the process during the path A → B is isothermal

(B) heat flows out of the gas during the path B → C → D

(C) work done during the path A → B → C is zero

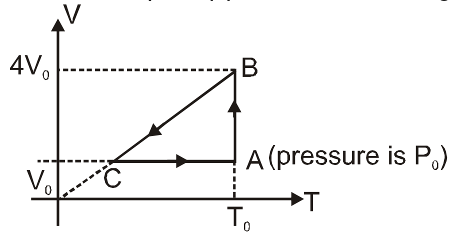
(D) positive work is done by the gas in the cycle ABCDA

7. A real gas behaves like an ideal gas if its **[JEE, 2010, 3/163, –1]**

(A) pressure and temperature are both high (B) pressure and temperature are both low

(C) pressure is high and temperature is low (D) pressure is low and temperature is high

8.* One mole of an ideal gas in initial state A undergoes a cyclic process ABCA, as shown in the figure. Its pressure at A is P_0. Choose the correct option(s) from the following : **[JEE, 2010, 3/163]**



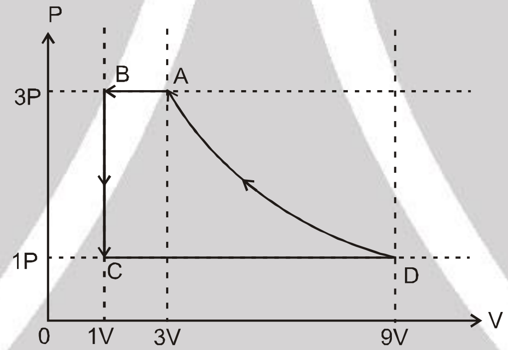
(A) Internal energies at A and B are the same
(B) Work done by the gas in process AB is $P_0V_0 \ln 4$
(C) Pressure at C is $\frac{P_0}{4}$
(D) Temperature at C is $\frac{T_0}{4}$

9. A diatomic ideal gas is compressed adiabatically to $\frac{1}{32}$ of its initial volume. If the initial temperature of the gas is T_i (in Kelvin) and the final temperature is aT_i, the value of a is : **[JEE, 2010, 3/163]**

10. 5.6 liter of helium gas at STP is adiabatically compressed to 0.7 liter. Taking the initial temperature to be T_1, the work done in the process is : **[JEE, 2011, 3/160, –1]**

(A) $\frac{9}{8}RT_1$ (B) $\frac{3}{2}RT_1$ (C) $\frac{15}{8}RT_1$ (D) $\frac{9}{2}RT_1$

11. One mole of a monatomic ideal gas is taken through a cycle ABCDA as shown in the P-V diagram. Column II gives the characteristics involved in the cycle. Match them with each of the processes given in Column I. **[JEE, 2011, 8/160]**



Column I	Column II
(A) Process A → B	(p) Internal energy decreases
(B) Process B → C	(q) Internal energy increases
(C) Process C → D	(r) Heat is lost
(D) Process D → A	(s) Heat is gained
	(t) Work is done on the gas.

12. A mixture of 2 moles of helium gas (atomic mass = 4 amu), and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(\text{helium})}{v_{rms}(\text{argon})}\right)$ is :
[IIT-JEE 2012, P-1 : 3/70, –1]

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

13. Two moles of ideal helium gas are in a rubber balloon at 30° C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C.The amount of heat required in raising the temperature is nearly
(take R = 8.31 J/mol.K) **[IIT-JEE 2012, Paper-2 : 3/66, –1]**

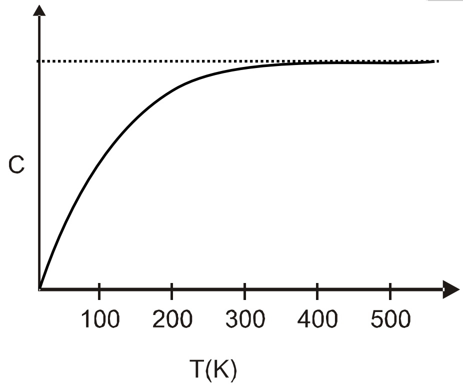
(A) 62J (B) 104 J (C) 124 J (D) 208 J

14. Two non-reactive monoatomic ideal gases have their atomic masses in the ratio 2 : 3. The ratio of their partial pressures, when enclosed in a vessel kept at a constant temperature, is 4 : 3. The ratio of their densities is: **[JEE (Advanced) 2013, 3/60,–1]**

(A) 1 : 4 (B) 1 : 2 (C) 6 : 9 (D) 8 : 9

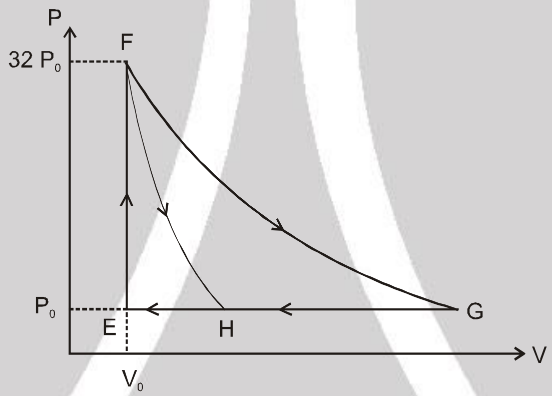
15.* The figure below shows the variation of specific heat capacity (C) of a solid as a function of temperature (T). The temperature is increased continuously from 0 to 500 K at a constant rate. Ignoring any volume change, the following statement(s) is (are) correct to a reasonable approximation.

[JEE (Advanced) 2013, 2/60, –1]



(A) the rate at which heat is absorbed in the range 0–100 K varies linearly with temperature T.
(B) heat absorbed in increasing the temperature from 0–100 K is less than the heat required for increasing the temperature from 400–500 K.
(C) there is no change in the rate of heat absorbtion in the range 400–500 K.
(D) the rate of heat absorption increases in the range 200–300 K.

16. One mole of a monatomic ideal gas is taken along two cyclic processes $E \rightarrow F \rightarrow G \rightarrow E$ and $E \rightarrow F \rightarrow H \rightarrow E$ as shown in the PV diagram. The processes involved are purely isochoric, isobaric, isothermal or adiabatic.



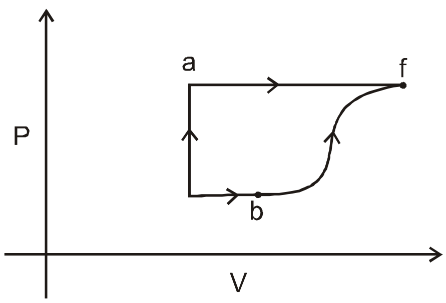
Match the paths in List I with the magnitudes of the work done in List II and select the correct answer using the codes given below the lists. **[JEE (Advanced) 2013 ; 9/60]**

List I		**List II**
P. $G \rightarrow E$	1.	$160\ P_0V_0 \ln 2$
Q. $G \rightarrow H$	2.	$36\ P_0V_0$
R. $F \rightarrow H$	3.	$24\ P_0V_0$
S. $F \rightarrow G$	4.	$31\ P_0V_0$

Codes :

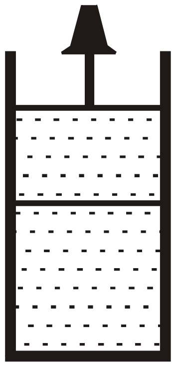
	P	Q	R	S
(A)	4	3	2	1
(B)	4	3	1	2
(C)	3	1	2	4
(D)	1	3	2	4

17. A thermodynamic system is taken form an initial state i with internal energy U_i = 100 J to the final state f along two different paths iaf and ibf, as schematically shown in the figure. The work done by the system along the paths af, ib and bf are W_{af} = 200 J, W_{ib} = 50 J and W_{bf} = 100 J respectively. The heat supplied to the system along the path iaf, ib and bf are Q_{iaf},Q_{bf} and Q_{ib} respectively.If the internal energy of the sytem in the state b is U_b = 200 J and Q_{iaf} = 500 J, the ratio Q_{bf}/Q_{ib} is: **[JEE (Advanced) 2014,P-1, 3/60]**



Paragraph For Questions 18 to 19

In the figure a container is shown to have a movable (without friction) piston on top. The container and the piston are all made of perfectly insulating material allowing no heat transfer between outside and inside the container. The container is divided into two compartments by a rigid partition made of a thermally conducting material that allows slow transfer of heat.



The lower compartment of the container is filled with 2 moles of an ideal monatomic gas at 700 K and the upper compartment is filled with 2 moles of an ideal diatomic gas at 400 K. The heat capacities per mole of an ideal monatomic gas are $C_V = \frac{3}{2}R$, $C_P = \frac{5}{2}R$, and those for an ideal diatomic gas are $C_V = \frac{5}{2}R$, $C_P = \frac{7}{2}R$.

18. Consider the partition to be rigidly fixed so that it does not move. When equilibrium is achieved, the final temperature of the gases will be : **[JEE (Advanced) 2014, 3/60, –1]**

(A) 550 K (B) 525 K (C) 513 K (D) 490 K

19. Now consider the partition to be free to move without friction so that the pressure of gases in both compartments is the same. Then total work done by the gases till the time they achieve equilibrium will be : **[JEE (Advanced) 2014, 3/60, –1]**

(A) 250 R (B) 200 R (C) 100 R (D) –100 R

20.* A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gases are ideal, the correct statement(s) is (are) **[JEE (Advanced) 2015 ; 4/88, –2]**

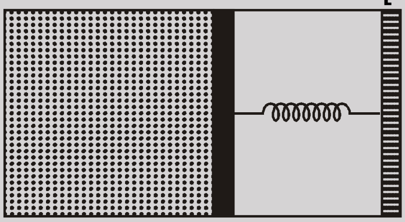
(A) The average energy per mole of the gas mixture is 2RT.

(B) The ratio speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

21.* An ideal monoatomic gas is confined in a horizontal cylinder by a spring loaded piston (as shown in the figure). Initially the gas is at temperature T_1, pressure P_1 and volume V_1 and the spring is in its relaxed state. the gas is then heated very slowly to temperature T_2, pressure P_2 and volume V_2. During this process the piston moves out by a distance x. Ignoring the friction between the piston and the cylinder, the correct statement(s) is(are) **[JEE (Advanced) 2015 ; P-2,4/88, –2]**



(A) If $V_2 = 2V_1$ and $T_2 = 3T$, then the energy stored in the spring is $\frac{1}{4}P_1V_1$

(B) If $V_2 = 2V_1$ and $T_2 = 3T_1$, then the change in internal energy is $3P_1V_1$

(C) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the work done by the gas is $\frac{7}{3}P_1V_1$

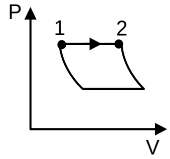
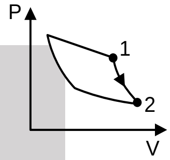
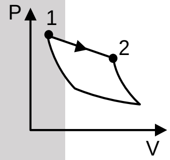
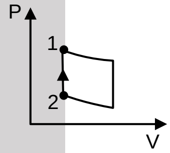
(D) If $V_2 = 3V_1$ and $T_2 = 4T_1$, then the heat supplied to the gas is $\frac{17}{6}P_1V_1$

22. A gas is enclosed in a cylinder with a movable frictionless piston. Its initial thermodynamic state at pressure $P_i = 10^5$ Pa and volume $V_i = 10^{-3}$ m^3 changes to a final state at $P_f = (1/32) \times 10^5$ Pa and $V_f = 8 \times 10^{-3}$ m^3 in an adiabatic quasi-static process, such that P^3V^5 = constant. Consider another thermodynamic process that brings the system from the same initial state to the same final state in two steps : an isobaric expansion at P_i followed by an isochoric (isovolumetric) process at volume V_f. The amount of heat supplied to the system in the two-step process is approximately. **[JEE (Advanced) 2016; P-2, 3/62, –1]**

(A) 112 J (B) 294 J (C) 588 J (D) 813 J

Answer Q.23, Q.24 and Q.25 by appropriately matching the information given in the three columns of the following table.

An ideal gas is undergoing a cyclic thermodynamic process in different ways as shown in the corresponding P – V diagrams in column 3 of the table. Consider only the path from state 1 to state 2. W denotes the corresponding work done on the system. The equations and plots in the table have standard notations as used in thermodynamic process. Here γ is the ratio of heat capacities at constant pressure and constant volume. The number of moles in the gas is n.

	Column-1		Column-2		Column-3
(I)	$W_{1\to2} = \frac{1}{\gamma-1}(P_2V_2 - P_1V_1)$	(i)	Isothermal	(P)	
(II)	$W_{1\to2} = -PV_2 + PV_1$	(ii)	Isochoric	(Q)	
(III)	$W_{1\to2} = 0$	(iii)	Isobaric	(R)	
(IV)	$W_{1\to2} = -nRT\ln\left(\frac{V_2}{V_1}\right)$	(iv)	Adiabatic	(S)	

23. Which of the following options is the only correct representation of a process in which $\Delta U = \Delta Q - P\Delta V$?
[JEE (Advanced) 2017; P-1, 3/61, –1]

(A) (II) (iii) (P) (B) (II) (iii) (S) (C) (III) (iii) (P) (D) (II) (iv) (R)

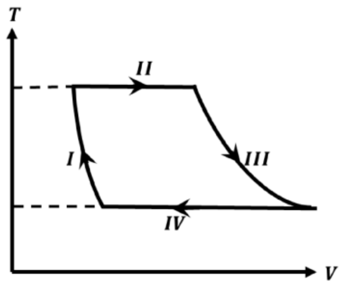
24. Which one of the following options is the correct combination ? **[JEE (Advanced) 2017;P-1, 3/61, –1]**

(A) (II) (iv) (P) (B) (IV) (ii) (S) (C) (II) (iv) (R) (D) (III) (ii) (S)

25. Which one of the following options correctly represents a thermodynamics process that is used as a correction in the determination of the speed of sound in an ideal gas ?
[JEE (Advanced) 2017; P-1, 3/61, –1]

(A) (III) (iv) (R) (B) (I) (ii) (Q) (C) (IV) (ii) (R) (D) (I) (iv) (Q)

26*. One mole of a monatomic ideal gas undergoes a cyclic process as shown in the figure (where V is the volume and T is the temperature). Which of the statements below is (are) true?
[JEE Advanced 2018; P-1, 4/60, –2]

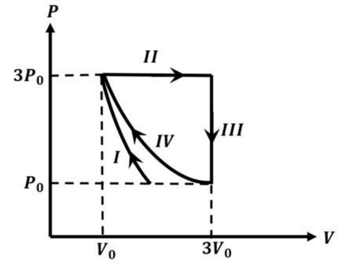


(A) Process I is an isochoric process
(B) In process II, gas absorbs heat
(C) In process IV, gas releases heat
(D) Processes I and III are **not** isobaric

27. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $8.0 J mol^{-1}K^{-1}$, the decrease in its internal energy, in Joule, is____. **[JEE Advanced 2018; P-2, 3/60]**

28. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-1 with the corresponding statements in List-II. **[JEE Advanced 2018; P-2, 3/60, –1]**

	List-I		**List-II**
P.	In process I	1.	Work done by the gas is zero
Q.	In process II	2.	Temperature of the gas remains unchanged
R.	In process III	3.	**No** heat is exchanged between the gas and its surroundings
S.	In process IV	4.	Work done by the gas is $6P_0V_0$



(A) P → 4; Q → 3; R → 1; S → 2 (B) P → 1; Q → 3; R → 2; S → 4
(C) P → 3; Q → 4; R → 1; S → 2 (D) P → 3; Q → 4; R → 2; S → 1

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. Two rigid boxes containing different ideal gases are placed on a table. Box A contains one mole of nitrogen at temperature T_0, while box B contains one mole of helium at temperature $(7/3)T_0$. The boxes are then put into thermal contact with each other, and heat flows between them until the gases reach a common final temperature. (Ignore the heat capacity of boxes). Then, the final temperature of the gases, T_f in terms of T_0 is : **[AIEEE - 2006, 4½/180]**

(1) $T_f = \frac{3}{7}T_0$ (2) $T_f = \frac{7}{3}T_0$ (3) $T_f = \frac{3}{2}T_0$ (4) $T_f = \frac{5}{2}T_0$

2. The work of 146 kJ is performed in order to compress one kilo mole of a gas adiabatically and in this process the temperature of the gas increases by 7 °C. The gas is ($R = 8.3$ J mol^{-1} K^{-1}) **[AIEEE - 2006, 3/180]**

(1) diatomic (2) triatomic
(3) mixture of monoatomic and diatomic (4) monoatomic

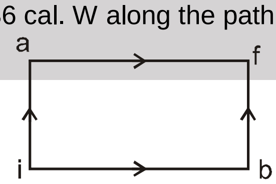
3. A Carnot engine, having an efficiency of $\eta = 1/10$ as heat engine, is used as a refrigerator. If the work done on the system is 10 J, the amount of energy absorbed from the reservoir at lower temperature is **[AIEEE - 2007, 3/120]**

(1) 99 J (2) 90 J (3) 1 J (4) 100 J

4. If C_p and C_v denote the specific heats of nitrogen per unit mass at constant pressure and constant volume respectively, then **[AIEEE - 2007, 3/120]**

(1) $C_p - C_v = R/28$ (2) $C_p - C_v = R/14$ (3) $C_p - C_v = R$ (4) $C_p - C_v = 28R$

5. When a system is taken from state i to state f along the path iaf, it is found that Q = 50 cal and W = 20 cal. Along the path ibf Q = 36 cal. W along the path ibf is : **[AIEEE - 2007, 3/120]**



(1) 6 cal (2) 16 cal (3) 66 cal (4) 14 cal

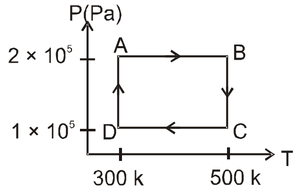
6. An insulated container of gas has two chambers separated by an insulating partition. One of the chambers has volume V_1 and contains ideal gas at pressure p_1 and temperature T_1. The other chamber has volume V_2 and contains ideal gas at pressure p_2 and temperature T_2. If the partition is removed without doing any work on the gas, the final equilibrium temperature of the gas in the container will be – **[AIEEE - 2008, 3/105]**

(1) $\frac{T_1T_2(p_1V_1+p_2V_2)}{p_1V_1T_2+p_2V_2T_1}$ (2) $\frac{p_1V_1T_1+p_2V_2T_2}{p_1V_1+p_2V_2}$ (3) $\frac{p_1V_1T_2+p_2V_2T_1}{p_1V_1+p_2V_2}$ (4) $\frac{T_1T_2(p_1V_1+p_2V_2)}{p_1V_1T_1+p_2V_2T_2}$

Directions : Question number 7, 8 and 9 are based on the following paragraph.

Two moles of helium gas are taken over the cycle ABCDA, as shown in the P-T diagram.

[AIEEE - 2009, 4×3/144]



7. Assume the gas to be ideal the magnitude of work done on the gas in taking it from A to B is :
(1) 200 R (2) 300 R (3) 400 R (4) 500 R

8. The work done on the gas in taking it from D to A is
(1) –414 R (2) + 414 R (3) – 690 R (4) + 690 R

9. The magnitude of net work done on the gas in the cycle ABCDA is:
(1) Zero (2) 276 R (3) 1076 R (4) 1904 R

10. One kg of a diatomic gas is at a pressure of 8×10^4 N/m^2. The density of the gas is 4 kg/m^3. What is the energy of the gas due to its thermal rnotion? **[AIEEE - 2009, 4/144]**
(1) 5×10^4J (2) 6×10^4J (3) 7×10^4J (4) 3×10^4J

11. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is : **[AIEEE - 2010, 4/144, –1]**
(1) 0.5 (2) 0.75 (3) 0.99 (4) 0.25

12. 100g of water is heated from 30ºC to 50ºC ignoring the slight expansion of the water, the change in its internal energy is (specific heat of water is 4184 J/Kg/K) : **[AIEEE - 2011, 4/120, –1]**
(1) 4.2 kJ (2) 8.4 kJ (3) 84 kJ (4) 2.1 kJ

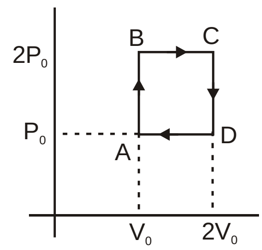
13. A Carnot engine operating between temperatures T_1 and T_2 has effeiciency 1/6. When T_2 is lowered by 62 K, its efficiency increases to 1/3. Then T_1 and T_2 are, respectively : **[AIEEE - 2011, 4/120, –1]**
(1) 372 K and 310 K (2) 372 K and 330 K (3) 330 K and 268 K (4) 310 K and 248 K

14. Three perfect gases at absolute temperature T_1, T_2 and T_3 are mixed. The masses of molecules are m_1,m_2 and m_3 and the number of molecules are n_1,n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is : **[AIEEE - 2011, 4/120, –1]**
(1) $\frac{(T_1+T_2+T_3)}{3}$ (2) $\frac{n_1T_1+n_2T_2+n_3T_3}{n_1+n_2+n_3}$ (3) $\frac{n_1T_1^2+n_2T_2^2+n_3T_3^2}{n_1T_1+n_2T_2+n_3T_3}$ (4) $\frac{n_1^2T_1^2+n_2^2T_2^2+n_3^2T_3^2}{n_1T_1+n_2T_2+n_3T_3}$

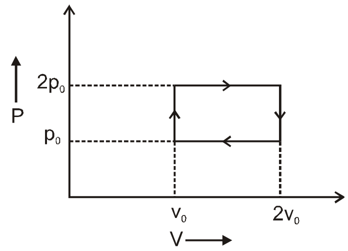
15. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by : **[AIEEE - 2011, 4/120, –1]**
(1) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2K$ (2) $\frac{(\gamma-1)}{2\gamma R}Mv^2K$ (3) $\frac{\gamma Mv^2}{2R}K$ (4) $\frac{(\gamma-1)}{2R}Mv^2K$

16. A container with insulating walls is divided into equal parts by a partition fitted with a valve. One part is filled with an ideal gas at a pressure P and temperature T, whereas the other part is completely evacuated. If the valve is suddenly opened, the pressure and temperature of the gas will be :
[AIEEE 2011, 11 May; 4/120, –1]
(1) $\frac{P}{2},\frac{T}{2}$ (2) P, T (3) P, $\frac{T}{2}$ (4) $\frac{P}{2}$, T

17. Helium gas goes through a cycle ABCDA (consisting of two isochoric and isobaric lines) as shown in figure. Efficiency of this cycle is nearly : (Assume the gas to be close to ideal gas) **[AIEEE 2012 ; 4/120, –1]**
(1) 15.4% (2) 9.1%
(3) 10.5% (4) 12.5%



18. The above p-v diagram represents the thermodynamic cycle of an engine, operating with an ideal monoatomic gas. The amount of heat, extracted from the source in a single cycle is : **[JEE (Main) 2013, 4/120, –1]**



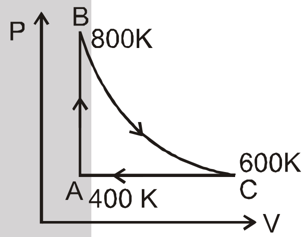
(1) p_0v_0 (2) $\left(\frac{13}{2}\right)p_0v_0$

(3) $\left(\frac{11}{2}\right)p_0v_0$ (4) $4p_0v_0$

19. An ideal gas enclosed in a vertical cylindrical container supports a freely moving piston of mass M. The piston and the cylinder have equal cross sectional area A. When the piston is in equilibrium, the volume of the gas is V_0 and its pressure is P_0. The piston is slightly displaced from the equilibrium position and released. Assuming that the system is completely isolated from its surrounding, the piston executes a simple harmonic motion with frequency : **[JEE (Main) 2013, 4/120]**

(1) $\frac{1}{2\pi}\frac{A\gamma P_0}{V_0M}$ (2) $\frac{1}{2\pi}\frac{V_0MP_0}{A^2\gamma}$ (3) $\frac{1}{2\pi}\sqrt{\frac{A^2\gamma P_0}{MV_0}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{MV_0}{A\gamma P_0}}$

20. One mole of diatomic ideal gas undergoes a cyclic process ABC as shown in figure. The process BC is adiabatic. The temperatures at A, B and C are 400K, 800K and 600 K respectively. Choose the correct statement : **[JEE (Main) 2014, 4/120,–1]**



(1) The change in internal energy in whole cyclic process is 250 R.
(2) The change in internal energy in the process CA is 700 R
(3) The change in internal energy in the process AB is – 350 R
(4) The change in internal energy in the process BC is – 500 R

21. An open glass tube is immersed in mercury in such a way that a length of 8 cm extends above the mercury level. The open end of the tube is then closed and sealed and the tube is raised vertically up by additional 46 cm. What will be length of the air column above mercury in the tube now ?
(Atmospheric pressure = 76 cm of Hg) **[JEE (Main) 2014, 4/120,–1]**

(1) 16 cm (2) 22 cm (3) 38 cm (4) 6 cm

22. Consider a spherical shell of radius R at temperature T. The black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $P = \frac{1}{3}\left(\frac{U}{V}\right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is **[JEE (Main) 2015; 4/120, –1]**

(1) $T \propto e^{-R}$ (2) $T \propto e^{-3R}$ (3) $T \propto 1/R$ (4) $T \propto 1/R^3$

23. A solid body of constant heat capacity 1 J/°C is being heated by keeping it in contact with reservoirs in two ways:
(i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
(ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat.
In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy changes of the body in the two cases respectively is **[JEE (Main) 2015 ; 4/120, –1]**

(1) $\ell n\ 2, 4\ell n 2$ (2) $\ell n\ 2, \ell n\ 2$ (3) $\ell n\ 2, 2\ \ell n\ 2$ (4) $2\ \ell n\ 2, 8\ \ell n\ 2$

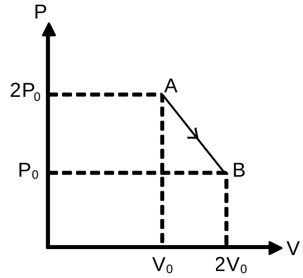
24. Consider an ideal gas confined in an isolated closed chamber. As the gas undegoes an adiabatic expansion, the average time of collision between molecules increases as V^q, where V is the volume of the gas. The value of q is : $\left(\gamma = \frac{C_P}{C_V}\right)$ **[JEE (Main) 2015 ; 4/120, –1]**

(1) $\frac{3\gamma+5}{6}$ (2) $\frac{3\gamma-5}{6}$ (3) $\frac{\gamma+1}{2}$ (4) $\frac{\gamma-1}{2}$

25. An ideal gas undergoes a quasi static, reversible process in which its molar heat capacity C remains constant. If during this process the relation of pressure P and volume V is given by PV^n = constant, then n is given by (Here Cp and C_V are molar specific heat at constant pressure and constant volume, respectively) : **[JEE (Main) 2016, 4/120, –1]**

(1) $n = \frac{C-C_p}{C-C_V}$ (2) $n = \frac{C_p-C}{C-C_V}$ (3) $n = \frac{C-C_V}{C-C_p}$ (4) $n = \frac{C_p}{C_V}$

26. 'n' moles of an ideal gas undergoes a process A → B as shown in the figure. The maximum temperature of the gas during the process will be : **[JEE (Main) 2016, 4/120, –1]**



(1) $\frac{3P_0V_0}{2nR}$ (2) $\frac{9P_0V_0}{2nR}$

(3) $\frac{9P_0V_0}{nR}$ (4) $\frac{9P_0V_0}{4nR}$

27. C_p and C_v are specific heats at constant pressure and constant volume respectively. It is observed that
$C_p - C_v = a$ for hydrogen gas
$C_p - C_v = b$ for nitrogen gas
The correct relation between a and b is : **[JEE (Main) 2017, 4/120, –1]**

(1) a = 28 b (2) $a = \frac{1}{14}b$ (3) a = b (4) a = 14 b

28. The temperature of an open room of volume 30 m³ increased from 17ºC to 27ºC due to the sunshine. The atmospheric pressure in the room remains 1×10^5 Pa. If n_i and n_f are the number of molecules in the room before and after heating, then $n_f - n_i$ will be : **[JEE (Main) 2017, 4/120, –1]**

(1) -2.5×10^{25} (2) -1.61×10^{23} (3) 1.38×10^{23} (4) 2.5×10^{25}

29. Two moles of an ideal monoatomic gas occupies a volume V at 27°C. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy. **[JEE (Main) 2018; 4/120, –1]**

(1) (a) 189 K (b) –2.7 kJ (2) (a) 195 K (b) 2.7 kJ
(3) (a) 189 K (b) 2.7 kJ (4) (a) 195 K (b) –2.7 kj

Answers

EXERCISE-1

PART - I

Section (A) :

A-1 zero **A-2.** $\left(\frac{mv_0^2}{\ell}\right)$N

Section (B) :

B-1. $\sqrt{\frac{50}{3}}$ V **B-2.** H_2, 1200 K

B-3. $2P_0$ **B-4.** $\frac{6P_0}{5}$ **B-5.** $\frac{112}{250}$ mole

B-6. (i) $\frac{1492}{293}$ cm (ii) $\frac{320\pi}{293}\left(1+\frac{13}{4\pi}\right)$ kg

Section (C) :

C-1. $\frac{1452\pi}{25}\times10^3$ K **C-2.** $\frac{83}{3\sqrt{10}}\times10^{-23}$ kg-m/s

C-3. 1 : 2 **C-4.** 1 : $\sqrt{2}$

Section (D) :

D-1. 32ºC **D-2.** 196ºC

D-3. $U=\frac{fnRT}{2}=\frac{f}{2}PV=\frac{f}{2}P_{atm}\,.\,V_{room}$ = constant.

Section (E) :

E-1. $-100\,\pi$J **E-2.** 1500J

E-3. 750J

E-4. $nRT_0\,\ell n\left(\frac{V_2-nb}{V_1-nb}\right)+an^2\left(\frac{V_1-V_2}{V_1V_2}\right)$

Section (F) :

F-1. 60 cal **F-2.** 12 K **F-3.** 0.0091 J

F-4. (i) 765 J; (ii) $\frac{208}{1921}$

F-5. 110 J **F-6.** (33600 + 0.02) J

F-7. 25/6 J/cal

F-8. (a) 120 K, 240 K, 480 K, 240 K,
(b) 3500 J, 5000 J, 7000 J, 2500 J
(c) –1000 J

Section (G) :

G-1. $\frac{2}{\gamma-1}$ **G-2.** 2.5 R **G-3.** 3 R

G-4. 4.2J/cal

G-5. (a) 40 J (b) $\frac{9}{500}$ moles
(c) $\frac{125}{9}$ J/mol–K (d) $\frac{50}{9}$ J/mol–K

G-6. $0.8\ JK^{-1}$ **G-7.** 7

Section (H) :

H-1. Q – W **H-2.** $\frac{3}{2}$ **H-3.** 128

H-4. 32P **H-5.** 4/9 %

H-6. (a) $\sqrt{2}\times10^5$ Pa (b) $200\sqrt{2}$ K
(c) $40(2-\sqrt{2})$ J

H-7. (a) 800 kPa, 100 K (b) 1600 kPa, 200 K

H-8. 112 joule

Section (I) :

I-1.	$\frac{7}{2}R$	**I-2.**	7.5 R	**I-3.**	1000 J

Section (J) :

J-1	66.6 %	**J-2**	2.8×10^6 Joule
J-3	373.3 K	**J-4**	879 kcal
J-5	10.13	**J-6**	900 Calories

PART - II

Section (A) :

A-1.	(C)	**A-2.**	(C)

Section (B) :

B-1.	(A)	**B-2.**	(A)	**B-3.**	(C)
B-4.	(C)	**B-5.**	(C)	**B-6.**	(B)

Section (C) :

C-1.	(B)	**C-2.**	(A)

Section (D) :

D-1.	(A)	**D-2.**	(C)	**D-3.**	(D)

Section (E) :

E-1.	(D)	**E-2.**	(B)	**E-3.**	(C)
E-4.	(C)	**E-5.**	(B)	**E-6.**	(A)
E-7.	(C)				

Section (F) :

F-1.	(D)	**F-2.**	(B)	**F-3.**	(D)
F-4.	(B)	**F-5.**	(B)		

Section (G) :

G-1.	(D)	**G-2.**	(C)	**G-3.**	(B)
G-4.	(C)	**G-5.**	(C)	**G-6.**	(C)

Section (H) :

H-1.	(B)	**H-2.**	(A)	**H-3.**	(A)
H-4.	(C)	**H-5.**	(A)	**H-6.**	(C)
H-7.	(B)	**H-8.**	(B)	**H-9.**	(D)
H-10.	(D)				

Section (I) :

I-1.	(C)	**I-2.**	(D)	**I-3.**	(C)
I-4.	(A)	**I-5.**	(D)	**I-6.**	(D)
I-7.	(B)				

Section (J) :

J-1.	(D)	**J-2.**	(C)	**J-3.**	(B)
J-4.	(A)	**J-5.**	(A)		

PART - III

1. (A) → p, r, s ; (B) → q ; (C) → p, r, s ; (D) → q, r

2. (A) → p, s ; (B) → s ; (C) → p, s ; (D) → q, r

EXERCISE-2

PART - I

1.	(D)	**2.**	(A)	**3.**	(B)
4.	(D)	**5.**	(D)	**6.**	(D)
7.	(B)	**8.**	(C)	**9.**	(A)
10.	(A)	**11.**	(A)	**12.**	(C)
13.	(D)	**14.**	(B)	**15.**	(A)
16.	(B)	**17.**	(B)	**18.**	(B)

PART - II

1.	33	**2.**	22	**3.**	5
4.	75	**5.**	3	**6.**	12
7.	9	**8.**	5	**9.**	2
10.	15	**11.**	3	**12.**	9

PART - III

1.	(BC)	**2.**	(CD)	**3.**	(AB)
4.	(AD)	**5.**	(ABCD)	**6.**	(ABD)
7.	(AC)	**8.**	(CD)	**9.**	(AB)
10.	(ABCD)	**11.**	(BD)	**12.**	(BC)
13.	(BD)	**14.**	(AB)	**15.**	(BD)
16.	(CD)	**17.**	(CD)	**18.**	(BD)
19.	(BCD)	**20.**	(CD)		

PART - IV

1.	(C)	**2.**	(D)	**3.**	(B)
4.	(C)	**5.**	(B)	**6.**	(C)
7.	(B)	**8.**	(A)	**9.**	(A)
10.	(B)				

EXERCISE-3

PART - I

1. (A) → s; (B) → p, r; (C) → r; (D) → q, s

2. (B) **3.** (C)

4. (A) → (q); (B) → (p, r); (C) → p,s; (D) → (q, s)

5. (BD) **6.** (BD) **7.** (D)

8. (ABCD) **9.** 4 **10.** (A)

11. (A) → p,r,t; (B) → p,r; (C) → q,s; (D) → r, t

12.	(D)	**13.**	(D)	**14**	(D)
15.	(BCD)	**16.**	(A)	**17.**	2
18.	(D)	**19.**	(D)	**20.**	(ABD)
21.	(ABC)	**22.**	(C)	**23.**	(A)
24.	(D)	**25.**	(D)	**26.**	(BCD)
27.	900 J	**28.**	(C)		

PART - II

1.	(3)	**2.**	(1)	**3.**	(2)
4.	(1)	**5.**	(1)	**6.**	(1)
7.	(3)	**8.**	(2)	**9.**	(2)
10.	(1)	**11.**	(2)	**12.**	(2)
13.	(1)	**14.**	(2)	**15.**	(4)
16.	(4)	**17.**	(1)	**18.**	(2)
19.	(3)	**20.**	(4)	**21.**	(1)
22.	(3)	**23.**	(2)	**24.**	(3)
25.	(1)	**26.**	(4)	**27.**	(4)
28.	(1)	**29.**	(1)		

High Level Problems (HLP)

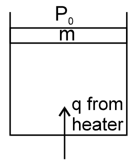
SUBJECTIVE QUESTIONS

1. A vessel of volume V is evacuated by means of a piston air pump. In one stroke the piston is pulled out to make the volume of gas V + ΔV then ΔV volume from this is taken out leaving volume V in the cylinder. How many strokes are needed to reduce the pressure in the vessel to $1/\eta$ times the initial pressure? The process is assumed to be isothermal, and the gas is an ideal.

2. Find the pressure of air in a vessel being evacuated as a function of evacuation time t. The vessel volume is V, the initial pressure is p_0. The process is assumed to be isothermal, and the evacuation rate equal to C and independent of pressure.
Note: The evacuation rate is the gas volume being evacuated per unit time, with that volume being measured under the gas pressure attained by that moment.

3. Find the maximum attainable temperature of an ideal gas in the following process : where (a) $p = p_0 - \alpha V^2$; (b) $p = p_0 e^{-\beta V}$, where p_0, α and β are positive constants and V is the volume of one mole of gas.

4. Two moles of an ideal monoatomic gas are contained in a vertical cylinder of cross sectional area A as shown in the figure. The piston is frictionless and has a mass m. At a certain instant a heater starts supplying heat to the gas at a constant rate q J/s. Find the steady velocity of the piston under isobaric condition. All the boundaries are thermally insulated.

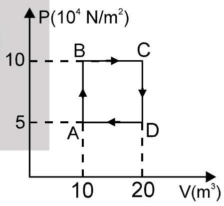


5. A piston can freely move inside a horizontal cylinder closed from both ends. Initially, the piston separates the inside space of the cylinder into two equal parts each of volume V_0, in which an ideal gas is contained under the same pressure p_0 and at the same temperature. What work has to be performed in order to increase isothermally the volume of one part of gas η times compared to that of the other by slowly moving the piston?

6. At 27º C two moles of an ideal monoatomic gas occupy a volume V. The gas expands adiabatically to a volume 2V. Calculate : **[JEE 1996, 5/100]**
(i) the final temperature of the gas, (ii) change in its internal energy and (iii) the work done by the gas during the process. (Take R = $\frac{25}{3}$ J/mol-K)

7. A vertical hollow cylinder contains an ideal gas. The gas is enclosed by a 5kg movable piston with an area of cross-section 5×10^{-3} m^2. Now, the gas is heated slowly from 300 K to 350 K and the piston rises by 0.1 m. The piston is now clamped at this position and the gas is cooled back to 300 K. Find the difference between the heat energy added during heating process and energy lost during the cooling process. [1 atm pressure = 10^5 N m^{-2}] **[REE 1996, 5]**

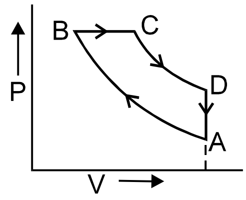
8. A sample of 2 kg of monoatomic Helium (assumed ideal) is taken through the process ABC and another sample of 2 kg of the same gas is taken through the process ADC as in figure. Given, molecular mass of Helium = 4.
(i) What is the temperature of Helium in each of the states A, B, C and D ?
(ii) Is there any way of telling afterwards which sample of Helium went through
(iii) How much is the heat involved in each of the processes ABC and ADC.
[JEE 1997, 5/100]



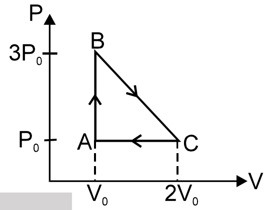
9. Two moles of an ideal monoatomic gas are confined within a cylinder by a massless and frictionless spring loaded piston of cross-sectional area 4×10^{-3} m^2. The spring is, initially in its relaxed state. Now the gas is heated by an electric heater, placed inside the cylinder, for some time. During this time, the gas expands and does 50 J of work in moving the piston through a distance 0.10 m. The temperature of the gas increases by 50 K. Calculate the spring constant and the heat supplied by the heater. $P_{atm} = 1 \times 10^5$ N/m^2.R = 8.314 J/mol-K **[REE 1997, 5]**

10. Two vessels A and B, thermally insulated, contain an ideal monoatomic gas. A small tube fitted with a valve connects these vessels. Initially the vessel A has 2 liters of gas at 300 K and 2×10^5 N m^{-2} pressure while vessel B has 4 liters of gas at 350 K and 4×10^5 Nm^{-2} pressure. The valve is now opened and the system reaches equilibrium in pressure and temperature. Calculate the new pressure and temperature. (R = $\frac{25}{3}$ J/mol-K) **[REE 1997, 5]**

11. One mole of a diatomic ideal gas (γ = 1.4) is taken through a cyclic process starting from point A. The process A $\rightarrow$ B is an adiabatic compression. B$\rightarrow$C is isobaric expansion. C $\rightarrow$ D an adiabatic expansion and D $\rightarrow$ A is isochoric as shown in P-V diagram. The volume ratios are $\frac{V_A}{V_B} = 16$ & $\frac{V_C}{V_B} = 2$ and the temperature at A is T_A = 300 K. Calculate the temperature of the gas at the points B and D and find the efficiency of the cycle. **[JEE 1997, 5/100]**



12. One mole of an ideal monoatomic gas is taken around the cyclic process ABCA as shown in the figure. Calculate, **[JEE1998, 8/200]**
(a) the work done by the gas ;
(b) the heat rejected by the gas in the path CA and the heat absorbed by the gas in the path AB ;
(c) the net heat absorbed by the gas in the path BC ;
(d) the maximum temperature attained by the gas during the cycle.



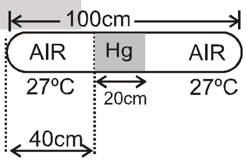
13. Two moles of a monatomic gas, initially at pressure P_1 and volume V_1, undergo an adiabatic compression until its volume becomes V_2. Then the gas is given heat Q at constant volume V_2.
(a) Sketch the complete process on a P-V diagram.
(b) Find the total work done by the gas, the total change in its internal energy and the final temperature of the gas.[Give your answers in terms of P_1, V_1, V_2 and Q and R.] **[JEE 1999, 2 + 8 / 200]**

14. A weightless piston divides a thermally insulated cylinder into two parts of volumes V and 3V. 2 moles of an ideal gas at pressure P = 2 atmosphere are confined to the part with volume V = 1 liter. The remainder of the cylinder is evacuated. The piston is now released and the gas expands to fill the entire space of the cylinder. The piston is then pressed back to the initial position. Find the increase of internal energy in the process and final temperature of the gas. The ratio of the specific heats of the gas, γ = 1.5. **[REE -1999]**

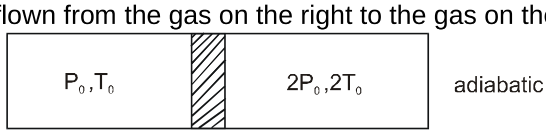
15. Two containers A and B of equal volume $V_0/2$ each are connected by a narrow tube which can be closed by a valve. The containers are fitted with pistons which can be moved to change the volumes. Initially, the valve is open and the containers contain an ideal gas ($C_p/C_v = \gamma$) at atmospheric pressure P_0 and atmospheric temperature $2T_0$. The walls of the containers A are highly conducting and of B are non-conducting. The valve is now closed and the pistons are slowly pulled out to increase the volumes of the containers to double the original value. (a) Calculate the temperatures and pressures in the two containers. (b) The valve is now opened for sufficient time so that the gases acquire a common temperature and pressure. Find the new values of the temperature and the pressure.

16. In given figure, an adiabatic cylindrical tube of volume $2V_0$ is divided in two equal parts by a frictionless adiabatic separator. An ideal gas in left side of a tube having pressure P_1 and temperature T_1 where as in the right side having pressure P_2 and temperature T_2. $C_p/C_v = \gamma$ is the same for both the gases. The separator is slid slowly and is released at a position where it can stay in equilibrium. Find (a) the final volumes of the two parts, (b) the heat given to the gas in the left part and (c) the final common pressure of the gases.

17. In the given figure a glass tube lies horizontally with the middle 20 cm containing mercury. The two ends of the tube contains air at 27ºC and at a pressure 76 cm of mercury. Now the air column on one side is maintained at 0ºC and the other side is maintained at 127ºC. Find the new length of the air column on the cooler side. Neglect the changes in the volume of mercury and of the glass.



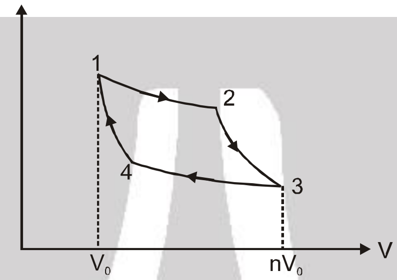
18. A cylindrical tube with adiabatic walls having volume $2V_0$ contains an ideal monoatomic gas as shown in figure. The tube is divided into two equal parts by a fixed super conducting wall. Initially, the pressure and the temperature are P_0, T_0 on the left and $2P_0$, $2T_0$ on the right. When system is left for sufficient amount of time the temperature on both sides becomes equal (a) Find work done by the gas on the right part? (b) Find the final pressures on the two sides. (c) Find the final equilibrium temperature. (d) How much heat has flown from the gas on the right to the gas on the left?



19. An ideal gas ($C_p/C_v = \gamma$) having initial pressure P_0 and volume V_0. (a) The gas is taken isothermally to a pressure $2P_0$ and then adiabatically to a pressure $4P_0$. Find the final volume. (b) The gas is brought back to its initial state. It is adiabatically taken to a pressure $2P_0$ and then isothermally to a pressure $4P_0$. Find the final volume.

20. Two samples A and B of the same gas have equal volumes and pressures. The gas in sample A is expanded isothermally to four times of its initial volume and the gas in B is expanded adiabatically to double its volume. It work done in isothermal process is twice that of adiabatic process, then show that γ satisfies the equation $1 - 2^{1-\gamma} = (\gamma - 1)$ In 2.

21. A Carnot engine cycle is shown in the Fig. (2). The cycle runs between temperatures $T_H = \alpha T_0$ and $T_L = T_0$ ($\alpha > 1$). Minimum and maximum volume at state 1 and state 3 are V_0 and nV_0 respectively. The cycle uses one mole of an ideal gas with $C_P / C_V = \gamma$. Here C_P and C_V are the specific heats at constant pressure and volume respectively. You must express all answers in terms of the given parameters $\{\alpha, n, T_0, V_0, ?\}$ and universal gas constant R. **[Olympiad 2011]**



(a) Find P, V, T for all the states
(b) Calculate the work done by the engine in each process: W_{12}, W_{23}, W_{34}, W_{41}.
(c) Calculate Q, the heat absorbed in the cycle.

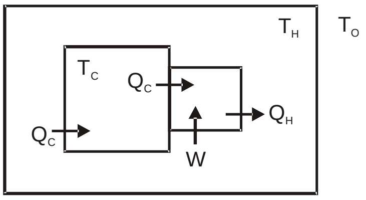
22. A thermally insulated piston divides a nonconducting container in two compartments, right compartment of 2V, T and 2P, while in the left compartment the respective values are V, T and P. Total moles in total system of both compartments is 5 moles (same molar mass). If the piston can slide freely, and in the final equilibrium position, volume of right compartment is $\frac{xV}{5}$ then find the value of x.

23. **Cloud formation condition**
Consider a simplified model of cloud formation. Hot air in contact with the earth's surface contains water vapor. This air rises convectively till the water vapor content reaches its saturation pressure. When this happens, the water vapor starts condensing and droplets are formed. We shall estimate the height at which this happens. We assume that the atmosphere consists of the diatomic gases oxygen and nitrogen in the mass proportion 21:79 respectively. We further assume that the atmosphere is an ideal gas, g the acceleration due to gravity is constant and air processes are adiabatic. Under these assumptions one can show that the pressure is given by **[OLYMPIAD 2012]**

$$p = p_0\left(\frac{T_0 - \Gamma z}{T_0}\right)^{\alpha}$$

Here p_0 and T_0 is the pressure and temperature respectively at sea level ($z = 0$), Γ is the lapse rate (magnitude of the change in temperature T with height z above the earth's surface, i.e. $\Gamma > 0$).
(a) Obtain an expression for the lapse rate Γ in terms of γ, R, g and m_a. Here γ is the ratio of specific heat at constant pressure to specific heat at constant volume; R, the gas constant; and m_a, the relevant molar mass.
(b) Estimate the change in temperature when we ascend a height of one kilometer ?
(c) Show that pressure will depend on height as given by Eq. (1). Find an explicit expression for exponent α in terms of γ.
(d) According to this model what is the height to which the atmosphere extends? Take $T_0 = 300$ K and $p_0 = 1$ atm.

24. It is well known that the temperature of a closed room goes up if the refrigerator is switched on inside it. A refrigerator compartment set to temperature T_c is turned on inside a hut in Leh (Ladakh). The atmosphere (outside the hut) can be considered to be a vast reservoir at constant temperature T_o. Walls of hut and refrigerator compartment are conducting. The temperature of the refrigerator compartment is maintained at T_c with the help of a compressor engine. We explain the working of the refrigerator engine and the heat flow with the help of the associated figure.



The larger square is the refrigerator compartment with heat leak per unit time Q_c into it from the room. The same heat per unit time Q_c is pumped out of it by the engine (also called compressor and indicated by the smaller square in thick). The compressor does work W and rejects heat per unit time Q_H into the hut. The thermal conductance (in units of watt per kelvin) of the walls of the compartment and hut respectively are K_c and K_H. After a long time it is found that temperature of the hut is T_H. The compressor works as a reverse Carnot engine and it does not participate in heat conduction process.

[Olympiad 2014]

(a) State the law of heat conduction for the walls of the hut and the refrigerator compartment.
(b) We define the dimensionless quantities $k = K_H/K_c$, $h = T_H/T_o$ and $c = T_c/T_o$. Express h in terms of c and k.
(c) Calculate stable temperature T_H given $T_o = 280.0$ K, $T_c = 252.0$ K and $k = 0.90$.
(d) Now another identical refrigerator is put inside the hut. T_c and T_o do not change but T_H, the hut temperature will change to T'_H. State laws of heat conduction for hut and one of the two identical refrigerator compartments.
(e) Assume that dimensionless quantities k and c do not change. Let $h' = T'_H / T_o$. Obtain an expression for h′.

HLP Answers

1. $n = \dfrac{\ln\eta}{\ln(1+\Delta V/V)}$ **2.** $p = p_0 e^{-Ct/V}$ **3.** (a) $T_{max} = \dfrac{2}{3}(p_0/R)\sqrt{p_0/3\alpha}$](b) $T_{max} = p_0/e\beta R$.

4. $\dfrac{2q}{5(mg + P_0A)}$ **5.** $W = p_0V_0 \ln[(\eta+1)^2/4\eta]$

6. (i) $300\left(\dfrac{1}{2}\right)^{2/3}$ K (ii) $7500\left(2^{-2/3}-1\right)$ J (iii) $-7500\left(2^{-2/3}-1\right)$ J **7.** 55 J

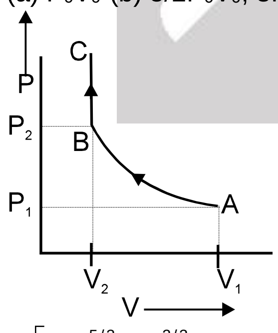
8. (i) $T_A = 120$ K, $T_B = 241$ K, $T_C = 481$ K, $T_D = 241$ K, (ii) No, (iii) $\Delta Q_{ABC} = \dfrac{13}{4}\times10^6$ J ; $\Delta Q_{ADC} = \dfrac{11}{4}\times10^6$ J

9. K = 2000 N/m, Q = 923 Joules approx. **10.** $P = \dfrac{10}{3}\times10^5$ N/m^2, $T = \dfrac{10500}{31}$ K $\simeq 338.71$ K

11. $T_B = 600 \times 2^{3/5}$ K, $T_D = 1200 \times 2^{-3/5}$ K, $\eta = 61.37\%$

12. (a) P_0V_0 (b) $5/2P_0V_0$, $3P_0V_0$ (c) $1/2P_0V_0$ (d) $25P_0V_0/8R$

13.



, $W = \dfrac{3}{2}P_1V_1\left[1-\left(\dfrac{V_1}{V_2}\right)^{2/3}\right]$; $\Delta U = Q - \dfrac{3}{2}P_1V_1\left[1-\left(\dfrac{V_1}{V_2}\right)^{2/3}\right]$,$T_{final} = \dfrac{Q}{3R} + \dfrac{P_1V_1}{2R}\left(\dfrac{V_1}{V_2}\right)^{2/3}$

$= \left[\dfrac{P_1V_1^{5/3}\cdot V_2^{-2/3}}{2\ R} + \dfrac{Q}{3\ R}\right]$

14. 400 J, 24 K **15.** (a) $2T_0$, $\dfrac{p_0}{2}$ in the vessel A and $\dfrac{T_0}{2^{\gamma-2}}$, $p_0/2^\gamma$ in vessel B, (b) $2T_0$, $P_0/2$

16. (a) $\dfrac{2p_1^{1/\gamma}V_0}{A}$, $\dfrac{2p_2^{1/\gamma}V_0}{A}$, (b) zero, (c) $(A/2)^\gamma$ where $A = p_1^{1/\gamma} + p_2^{1/\gamma}$ **17.** $\ell = \dfrac{21840}{673}$ cm

18. (a) zero (b) $\frac{3P_0}{2}$ on the left and $\frac{3P_0}{2}$ on the right (c) $\frac{3T_0}{2}$ (d) $\frac{3P_0V_0}{4}$

19. $\frac{V_0}{2^{\frac{\gamma+1}{\gamma}}}$ in each cases

20. Work done by gas A in isothermal process $W_A = P_0 V_0 \ell n\left(\frac{4V_0}{V_0}\right) = 2P_0 V_0 \ell n\, 2$

Work done by gas B in adiabatic process $W_B = \frac{P_iV_i - P_fV_f}{\gamma-1} = \frac{P_0V_0 - P_0(2)^{-\gamma}2V_0}{\gamma-1} = \frac{P_0V_0(1-2^{1-\gamma})}{\gamma-1}$

According to question. $W_A = 2W_B$

$2P_0 V_0 \ell n\, 2 = \frac{2P_0V_0(1-2^{1-\gamma})}{\gamma-1} \Rightarrow 1 - 2^{1-\gamma} = (\gamma - 1)\, \ell n\, 2$

21. (a) $P_1 = \frac{R\alpha T_0}{V_0}$, $P_2 = \frac{\alpha^{\frac{\gamma}{\gamma-1}}RT_0}{nV_0}$, $P_3 = \frac{RT_0}{nV_0}$, $P_4 = \frac{RT_0}{\alpha^{\frac{1}{\gamma-1}}V_0}$

$V_1 = V_0$, $V_2 = \frac{nV_0}{\alpha^{\frac{1}{\gamma-1}}}$, $V_3 = nV_0$, $V_4 = \alpha^{\frac{1}{\gamma-1}}.V_0$

$T_1 = \alpha T_0$, $T_2 = \alpha T_0$, $T_3 = T_0$, $T_4 = T_0$

(b) $W_{12} = R\,\alpha\,T_0 \ln\left(\frac{V_2}{V_1}\right) = \alpha R T_0 \ln\left(\frac{n}{\alpha^{\frac{1}{\gamma-1}}}\right)$; $W_{23} = -\frac{R}{\gamma-1}(T_0 - \alpha\ T_0)$

$W_{34} = RT_0\left(\frac{\alpha^{\frac{1}{\gamma-1}}}{n}\right)$; $W_{41} = -\frac{R}{\gamma-1}(\alpha T_0 - T_0)$; (c) $Q = RT_0(\alpha - 1)\ \ell n\left(\frac{n}{\alpha^{\left(\frac{1}{\gamma-1}\right)}}\right)$.

22. 12

23. (a) $\Gamma = \frac{dT}{dz} = \left(\frac{\rho T}{P}\right)g\frac{\gamma-1}{\gamma} = \frac{m_a}{R}g\frac{(\gamma-1)}{\gamma}$ (b) Change in temperature = 9.9 Kelvin

(c) Comparing $\alpha = \frac{\gamma}{\gamma-1}$ (d) $z = \frac{T_0}{\Gamma} = \frac{300}{9.9} = 30.3$ km

24. (a) For hut : $Q_H - Q_C = K_H (T_H - T_o)$. For refrigerator compartment : $Q_c = K_c (T_H - T_c)$

(b) $h^2 - h(2c + kc) + c^2 + kc = 0$; $h = \frac{(2c+kc) \pm \sqrt{(2c+kc)^2 - 4(c^2+kc)}}{2}$

(c) h = 1.02 (choosing – sign) $\Rightarrow$ $T_H = 284.7$K

(d) For hut : $2(Q'_H - Q'_H) = K_H (T'_H - T_o)$
For refrigerator compartment : $Q'_c = K_c(T'_H - T_c)$

(e) $h'^2 - h'\left(2c + \frac{k}{2}c\right) + c^2 + \frac{k}{2}c = 0$

$$h' = \frac{\left(2c+\frac{k}{2}c\right) \pm \sqrt{\left(2c+\frac{k}{2}c\right)^2 - 4\left(c^2+\frac{k}{2}c\right)}}{2}$$