MODERN PHYSICS-I

Contents

JEE (ADVANCED) SYLLABUS

Photoelectric effect; Bohr’s theory of hydrogen-like atoms; Characteristic and continuous X-rays, Moseley’s law; de Broglie wavelength of matter waves.

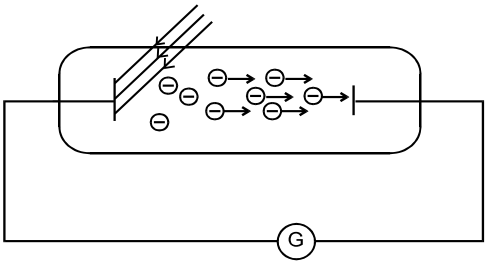
JEE (MAIN) SYLLABUS

Alpha-particle scattering experiment ; Rutherford's model of atom ; Bohr model, energy levels, hydrogen spectrum.

MODERN PHYSICS - I

1 PHOTOELECTRIC EFFECT :

When electromagnetic radiations of suitable wavelength are incident on a metallic surface then electrons are emitted, this phenomenon is called photo electric effect.



1.1 Photoelectron : The electron emitted in photoelectric effect is called photoelectron.

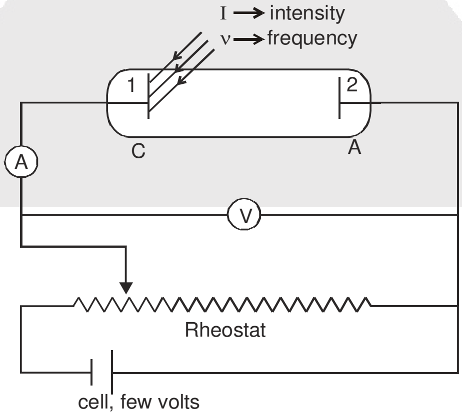
1.2 Photoelectric current : If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

1.3 Work function : The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Work functions of some photosensitive metals

Metal	Work function (ev)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

To produce photo electric effect only metal and light is necessary but for observing it, the circuit is completed. Figure shows an arrangement used to study the photoelectric effect.



Here the plate (1) is called emitter or cathode and other plate (2) is called collector or anode.

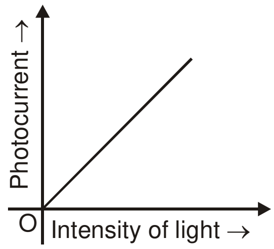
1.4 Saturation current : When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturation current, this is the maximum value of photoelectric current.

1.5 **Stopping potential :** Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.

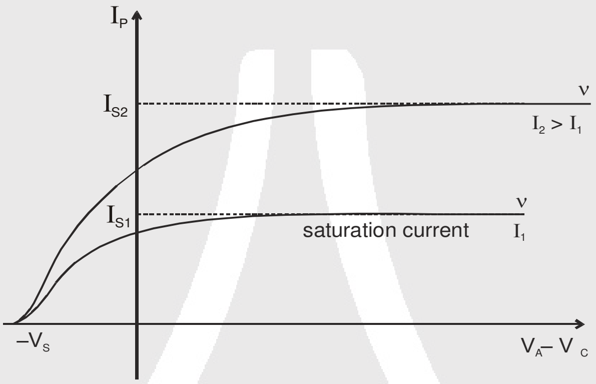
1.6 **Retarding potential :** Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

2. OBSERVATIONS : (MADE BY EINSTEIN)

2.1 A graph between intensity of light and photoelectric current is found to be a straight line as shown in figure. Photoelectric current is directly proportional to the intensity of incident radiation. In this experiment the frequency and retarding potential are kept constant.



2.2 A graph between photoelectric current and potential diffrence between cathode and anode is found as shown in figure.



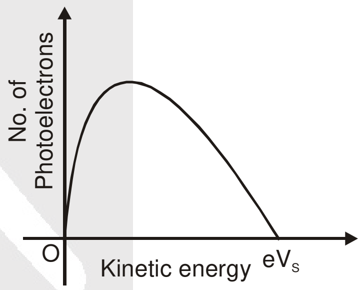
2.2 In case of saturation current,
rate of emission of photoelectrons = rate of flow of photoelectrons,
here, $v_s \rightarrow$ stopping potential and it is a positive quantity
Electrons emitted from surface of metal have different energies.
Maximum kinetic energy of photoelectron on the cathode = eV_s

$KE_{max} = eV_s$

Whenever photoelectric effect takes place, electrons are ejected out with kinetic energies ranging from

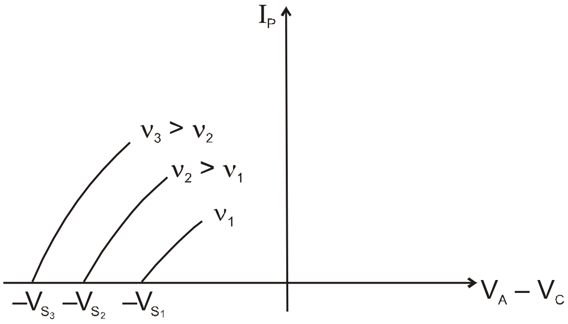
0 to $K.E_{max}$ i.e. $0 \leq KE_C \leq eV_s$

The energy distribution of photoelectron is shown in figure.



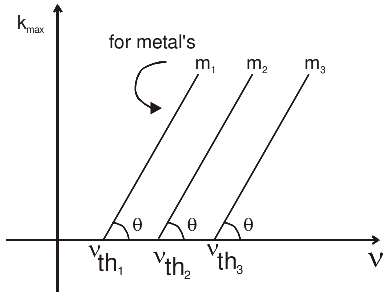
2.3 If intensity is increased (keeping the frequency constant) then saturation current is increased by same factor by which intensity increases. Stopping potential is same, so maximum value of kinetic energy is not effected.

2.4 If light of different frequencies is used then obtained plots are shown in figure.



It is clear from graph, as ν increases, stopping potential increases, it means maximum value of kinetic energy increases.

2.5 Graphs between maximum kinetic energy of electrons ejected from different metals and frequency of light used are found to be straight lines of same slope as shown in fiugre.



Graph between K_{max} and ν

m_1, m_2, m_3 : Three different metals.

It is clear from graph that there is a minimum frequency of electromagnetic radiation which can produce photoelectric effect, which is called **threshold frequency**.

ν_{th} = Threshold frequency

For photoelectric effect $\nu \geq \nu_{th}$

for no photoelectric effect $\nu < \nu_{th}$

Minimum frequency for photoelectric effect = ν_{th}

$\nu_{min} = \nu_{th}$

Threshold wavelength (λ_{th}) → The maximum wavelength of radiation which can produce photoelectric effect.

$\lambda \leq \lambda_{th}$ for photo electric effect

Maximum wavelength for photoelectric effect $\lambda_{max} = \lambda_{th}$.

Now writing equation of straight line from graph.

We have $K_{max} = A\nu + B$

When $\nu = \nu_{th}$, $K_{max} = 0$ and $B = -A\nu_{th}$

Hence **$[K_{max} = A(\nu - \nu_{th})]$**

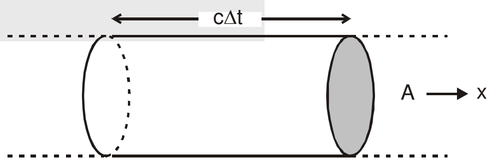
and $A = \tan\theta = 6.63 \times 10^{-34}$ J-s (from experimental data)

later on 'A' was found to be 'h'.

2.6 It is also observed that photoelectric effect is an instantaneous process. When light falls on surface electrons start ejecting without taking any time.

3. THREE MAJOR FEATURES OF THE PHOTOELECTRIC EFFECT CANNOT BE EXPLAINED IN TERMS OF THE CLASSICAL WAVE THEORY OF LIGHT.

Intensity : The energy crossing per unit area per unit time perpendicular to the direction of propagation is called the intensity of a wave. Consider a cylindrical volume with area of crosssection A and length c Δt along the X-axis. The energy contained in this cylinder crosses the area A in time Δt as the wave propagates at speed c. The energy contained.



$U = u_{av}(c.\ \Delta t)A$

The intensity is $I = \dfrac{U}{A\Delta t} = u_{av}\,c$.

In the terms of maximum electric field, $I = \dfrac{1}{2}\varepsilon_0 E_0^2 c$.

If we consider light as a wave then the intensity depends upon electric field.

If we take work function W = I . A . t, then t = $\frac{W}{IA}$

So for photoelectric effect there should be time lag because the metal has work function.
But it is observed that photoelectric effect is an instantaneous process.
Hence, light is not of wave nature.

3.1 **The intensity problem :** Wave theory requires that the oscillating electric field vector **E** of the light wave increases in amplitude as the intensity of the light beam is increased. Since the force applied to the electron is e**E**, this suggests that the kinetic energy of the photoelectrons should also increased as the light beam is made more intense. However observation shows that maximum kinetic energy is independent of the light intensity.

3.2 **The frequency problem :** According to the wave theory, the photoelectric effect should occur for any frequency of the light, provided only that the light is intense enough to supply the energy needed to eject the photoelectrons. However observations shows that there exists for each surface a characterstic cutoff frequency ν_{th}, for frequencies less than ν_{th}, the photoelectric effect does not occur, no matter how intense is light beam.

3.3 **The time delay problem :** If the energy acquired by a photoelectron is absorbed directly from the wave incident on the metal plate, the "effective target area" for an electron in the metal is limited and probably not much more than that of a circle of diameter roughly equal to that of an atom. In the classical theory, the light energy is uniformly distributed over the wavefront. Thus, if the light is feeble enough, there should be a measurable time lag, between the impinging of the light on the surface and the ejection of the photoelectron. During this interval the electron should be absorbing energy from the beam until it had accumulated enough to escape. However, no detectable time lag has ever been measured.

Now, quantum theory solves these problems in providing the correct interpretation of the photoelectric effect.

4 PLANCK'S QUANTUM THEORY :

The light energy from any source is always an integral multiple of a smaller energy value called quantum of light.hence energy Q = NE,

where E = hν and N (number of photons) = 1,2,3,....

Here energy is quantized. hν is the quantum of energy, it is a packet of energy called as **photon**.

$$E = h\nu = \frac{hc}{\lambda} \quad \text{and} \quad hc = 12400 \text{ eV Å}$$

5. EINSTEIN'S PHOTON THEORY

In 1905 Einstein made a remarkable assumption about the nature of light; namely, that, under some circumstances, it behaves as if its energy is concentrated into localized bundles, later called photons. The energy E of a single photon is given by

$$E = h\nu,$$

If we apply Einstein's photon concept to the photoelectric effect, we can write

$$h\nu = W + K_{max}, \qquad \text{(energy conservation)}$$

Equation says that a single photon carries an energy hν into the surface where it is absorbed by a single electron. Part of this energy W (called the work function of the emitting surface) is used in causing the electron to escape from the metal surface. The excess energy (hν – W) becomes the electron's kinetic energy; if the electron does not lose energy by internal collisions as it escapes from the metal, it will still have this much kinetic energy after it emerges. Thus K_{max} represents the maximum kinetic energy that the photoelectron can have outside the surface. There is complete agreement of the photon theory with experiment.

Now $IA = Nh\nu \Rightarrow N = \frac{IA}{h\nu}$ = no. of photons incident per unit time on an area 'A' when light of intensity 'I' is incident normally.

If we double the light intensity, we double the number of photons and thus double the photoelectric current; we do not change the energy of the individual photons or the nature of the individual photoelectric processes.

The second objection (the frequency problem) is met if K_{max} equals zero, we have

$h\nu_{th} = W$,

Which asserts that the photon has just enough energy to eject the photoelectrons and none extra to appear as kinetic energy. If ν is reduced below ν_{th}, $h\nu$ will be smaller than W and the individual photons, no matter how many of them there are (that is, no matter how intense the illumination), will not have enough energy to eject photoelectrons.

The third objection (the time delay problem) follows from the photon theory because the required energy is supplied in a concentrated bundle. It is not spread uniformly over the beam cross section as in the wave theory.

Hence Einstein's equation for photoelectric effect is given by

$$h\nu = h\nu_{th} + K_{max} \qquad K_{max} = \frac{hc}{\lambda} - \frac{hc}{\lambda_{th}}$$

Solved Example

Example 1. In an experiment on photo electric emission, following observations were made;

(i) Wavelength of the incident light = 1.98×10^{-7} m;

(ii) Stopping potential = 2.5 volt.

Find : (a) Kinetic energy of photoelectrons with maximum speed.

(b) Work function and

(c) Threshold frequency;

Solution : (a) Since v_s = 2.5 V, $K_{max} = eV_s$ so, K_{max} = 2.5 eV

(b) Energy of incident photon

$$E = \frac{12400}{1980} \text{eV} = 6.26 \text{ eV} \qquad W = E - K_{max} = 3.76 \text{ eV}$$

(c) $h\nu_{th} = W = 3.76 \times 1.6 \times 10^{-19}$ J $\qquad \therefore \quad \nu_{th} = \frac{3.76 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} \approx 9.1 \times 10^{14}$ Hz

Example 2. A beam of light consists of four wavelength 4000 Å, 4800 Å, 6000 Å and 7000 Å, each of intensity 1.5×10^{-3} Wm^{-2}. The beam falls normally on an area 10^{-4} m^2 of a clean metallic surface of work function 1.9 eV. Assuming no loss of light energy (i.e. each capable photon emits one electron) calculate the number of photoelectrons liberated per second.

Solution : $E_1 = \frac{12400}{4000} = 3.1$ eV, $\quad E_2 = \frac{12400}{4800} = 2.58$ eV $\qquad E_3 = \frac{12400}{6000} = 2.06$ eV

and $E_4 = \frac{12400}{7000} = 1.77$ eV

Therefore, light of wavelengths 4000 Å, 4800 Å and 6000 Å can only emit photoelectrons.

$\therefore$ Number of photoelectrons emitted per second = No. of photons incident per second)

$$= \frac{I_1A_1}{E_1} + \frac{I_2A_2}{E_2} + \frac{I_3A_3}{E_3} = IA\left(\frac{1}{E_1} + \frac{1}{E_2} + \frac{1}{E_3}\right)$$

$$= \frac{(1.5 \times 10^{-3})(10^{-4})}{1.6 \times 10^{-19}}\left(\frac{1}{3.1} + \frac{1}{2.58} + \frac{1}{2.06}\right) = 1.12 \times 10^{12}$$ **Ans.**

Example 3. A small potassium foil is placed (perpendicular to the direciton of incidence of light) a distance r (= 0.5 m) from a point light source whose output power P_0 is 1.0W. Assuming wave nature of light how long would it take for the foil to soak up enough energy (= 1.8 eV) from the beam to eject an electron? Assume that the ejected photoelectron collected its energy from a circular area of the foil whose radius equals the radius of a potassium atom (1.3×10^{-10} m).

Solution : If the source radiates uniformly in all directions, the intensity I of the light at a distance r is given by

$$I = \frac{P_0}{4\pi r^2} = \frac{1.0 \ \text{W}}{4\pi(0.5 \ \text{m})^2} = 0.32 \ \text{W/m}^2.$$

The target area A is $\pi(1.3 \times 10^{-10} \text{ m})^2$ or 5.3×10^{-20} m², so that the rate at which energy falls on the target is given by

$P = IA = (0.32 \text{ W/m}^2)(5.3 \times 10^{-20} \text{ m}^2)$

$= 1.7 \times 10^{-20}$ J/s.

If all this incoming energy is absorbed, the time required to accumulate enough energy for the electron to escape is

$$t = \left(\frac{1.8 \ \text{eV}}{1.7 \times 10^{-20} \text{J/s}}\right)\left(\frac{1.6 \ \times 10^{-19} \text{J}}{1 \ \text{eV}}\right) = 17 \text{ s.}$$

Our selection of a radius for the effective target area was some-what arbitrary, but no matter what reasonable area we choose, we should still calculate a "soak-up time" within the range of easy measurement. However, no time delay has ever been observed under any circumstances, the early experiments setting an upper limit of about 10^{-9} s for such delays.

Example 4. A metallic surface is irradiated with monochromatic light of variable wavelength. Above a wavelength of 5000 Å, no photoelectrons are emitted from the surface. With an unknown wavelength, stopping potential is 3 V. Find the unknown wavelength.

Solution : Using equation of photoelectric effect

$K_{max} = E - W$ $\quad$ ($K_{max} = eV_s$)

$$\therefore \quad 3 \text{ eV} = \frac{12400}{\lambda} - \frac{12400}{5000} = -2.48 \text{ eV} \qquad \text{or} \qquad \lambda = 2262 \text{ Å}$$

Example 5. Illuminating the surface of a certain metal alternately with light of wavelengths $\lambda_1 = 0.35$ μm and $\lambda_2 = 0.54$ μm, it was found that the corresponding maximum velocities of photo electrons have a ratio $\eta = 2$. Find the work function of that metal.

Solution : Using equation for two wavelengths

$$\frac{1}{2}mv_1^2 = \frac{hc}{\lambda_1} - W \qquad \text{....(i)}$$

$$\frac{1}{2}mv_2^2 = \frac{hc}{\lambda_2} - W \qquad \text{....(ii)}$$

Dividing Eq. (i) with Eq. (ii), with $v_1 = 2v_2$, we have $4 = \dfrac{\frac{hc}{\lambda_1} - W}{\frac{hc}{\lambda_2} - W}$

$$3W = 4\left(\frac{hc}{\lambda_2}\right) - \left(\frac{hc}{\lambda_1}\right) = \frac{4 \times 12400}{5400} - \frac{12400}{3500} = 5.64 \text{ eV}$$

$W = \dfrac{5.64}{3}$ eV = 1.88 eV $\qquad$ **Ans.**

Example 6. A photocell is operating in saturation mode with a photocurrent 4.8 μA when a monochromatic radiation of wavelength 3000 Å and power 1 mW is incident. When another monochromatic radiation of wavelength 1650 Å and power 5 mW is incident, it is observed that maximum velocity of photoelectron increases to two times. Assuming efficiency of photoelectron generation per incident to be same for both the cases, calculate,

(a) threshold wavelength for the cell (b) efficiency of photoelectron generation. [(No. of photoelectrons emitted per incident photon) × 100]

(c) saturation current in second case

Solution : (a) $K_1 = \frac{12400}{3000} - W = 4.13 - W$ .....(i)

$K_2 = \frac{12400}{1650} - W = 7.51 - W$ .....(ii)

Since $v_2 = 2v_1$ so, $K_2 = 4K_1$ .....(iiii)

Solving above equations, we get

$W = 3$ eV

$\therefore$ Threshold wavelegth $\lambda_0 = \frac{12400}{3} = 4133$ Å **Ans.**

(b) Energy of a photon in first case = $\frac{12400}{3000} = 4.13$ eV

or $E_1 = 6.6 \times 10^{-19}$ J

Rate of incident photons (number of photons per second)

$\frac{P_1}{E_1} = \frac{10^{-3}}{6.6\times10^{-19}} = = 1.5 \times 10^{15}$ per second

Number of electrons ejected = $\frac{4.8\times10^{-6}}{1.6\times10^{-19}}$ per second = 3.0×10^{13} per second

$\therefore$ Efficiency of photoelectron generation

$(\eta) = \frac{3.0\times10^{13}}{1.5\times10^{15}} \times 100 = 2\%$ **Ans.**

(c) Energy of photon in second case

$E_2 = \frac{12400}{1650} = 7.51$ eV $= 12 \times 10^{-19}$ J

Therefore, number of photons incident per second

$n_2 = \frac{P_2}{E_2} = \frac{5.0\times10^{-3}}{12\times10^{-19}} = 4.17 \times 10^{15}$ per second

Number of electrons emitted per second = $\frac{2}{100} \times 4.7 \times 10^{15} = 9.4 \times 10^{13}$ per second

$\therefore$ Saturation current in second case i = (9.4×10^{13}) (1.6×10^{-19}) amp = 15 μA **Ans.**

Example 7 Light described at a place by the equation E = (100 V/m) [sin $(5 \times 10^{15}\ s^{-1})$ t + sin $(8 \times 10^{15}\ s^{-1})$t] falls on a metal surface having work function 2.0 eV. Calculate the maximum kinetic energy of the photoelectrons.

Solution : The light contains two different frequencies. The one with larger frequency will cause photoelectrons with largest kinetic energy. This larger frequency is

$$\nu = \frac{\omega}{2\pi} = \frac{8\times10^{15}s^{-1}}{2\pi}$$

The maximum kinetic energy of the photoelectrons is

$K_{max} = h\upsilon - W$

$= (4.14 \times 10^{-15}$ eV-s) $\times \left(\frac{8\times10^{15}}{2\pi}s^{-1}\right) - 2.0$ eV

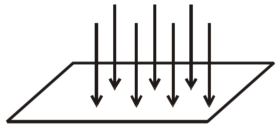
$= 5.27$ eV $- 2.0$ eV $= 3.27$ eV.

6 FORCE DUE TO RADIATION (PHOTON)

Each photon has a definite energy and a definite linear momentum. All photons of light of a particular wavelength λ have the same energy $E = hc/\lambda$ and the same magnitude of momentum $p = h/\lambda$.

When light of intensity I falls on a surface, it exerts force on that surface. Assume absorption and reflection coefficient of surface be 'a' and 'r' and assuming no transmission.

Assume light beam falls on surface of surface area 'A' perpendicularly as shown in figure.



For calculating the force exerted by beam on surface, we consider following cases.

Case : (I) a = 1, r = 0

initial momentum of the photon = $\frac{h}{\lambda}$

final momentum of photon = 0

change in momentum of photon = $\frac{h}{\lambda}$ (upward)

$\Delta P = \frac{h}{\lambda}$

energy incident per unit time = IA

no. of photons incident per unit time $= \frac{IA}{h\nu} = \frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time $= n\,\Delta P$

$= \frac{IA\lambda}{hc} \times \frac{h}{\lambda} = \frac{IA}{c}$ (upward)

force on photons = total change in momentum per unit time = $\frac{IA}{c}$ (upward)

$\therefore$ force on plate due to photons(F) = $\frac{IA}{c}$ (downward)

pressure $= \frac{F}{A} = \frac{IA}{cA} = \frac{I}{c}$

Case : (II)

when r = 1, a = 0

intial momentum of the photon $= \frac{h}{\lambda}$ (downward)

final momentum of photon $= \frac{h}{\lambda}$ (upward)

change in momentum $= \frac{h}{\lambda} + \frac{h}{\lambda} = \frac{2h}{\lambda}$

$\therefore$ energy incident per unit time = IA

no. of photons incident per unit time = $\frac{IA\lambda}{hc}$

$\therefore$ total change in momentum per unit time = n . $\Delta P = \frac{IA\lambda}{hc} \cdot \frac{2h}{\lambda} = \frac{2IA}{C}$

force = total change in momentum per unit time

$F = \frac{2IA}{c}$ (upward on photons and downward on the plate)

pressure $P = \frac{F}{A} = \frac{2IA}{cA} = \frac{2I}{c}$

Case : (III)

When $0 < r < 1 \quad a + r = 1$

change in momentum of photon when it is reflected = $\frac{2h}{\lambda}$ (upward)

change in momentum of photon when it is absorbed = $\frac{h}{\lambda}$ (upward)

no. of photons incident per unit time = $\frac{IA\lambda}{hc}$

No. of photons reflected per unit time = . $\frac{IA\lambda}{hc}$ r

No. of photon absorbed per unit time = $\frac{IA\lambda}{hc}$ (1 – r)

force due to absorbed photon (F_a) = $\frac{IA\lambda}{hc}(1-r) \cdot \frac{h}{\lambda} = \frac{IA}{c}(1-r)$ (downward)

Force due to reflected photon (F_r) = $\frac{IA\lambda}{hc} . r \frac{2h}{\lambda} = \frac{2IA\lambda}{c}$ (downward)

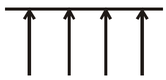
total force $= F_a + F_r$ (downward)

$$= \frac{IA}{c}(1-r) + \frac{2IAr}{c} = \frac{IA}{c}(1+r)$$

Now pressure $P = \frac{IA}{c}(1+r) \times \frac{1}{A} = \frac{I}{c}(1+r)$

Solved Example

Example 8. A plate of mass 10 gm is in equilibrium in air due to the force exerted by light beam on plate. Calculate power of beam. Assume plate is perfectly absorbing.



Solution : Since plate is in air, so gravitational force will act on this

$F_{gravitational} = mg$ (downward)

$= 10 \times 10^{-3} \times 10 = 10^{-1}$ N

for equilibrium force exerted by light beam should be equal to $F_{gravitational}$

$F_{photon} = F_{gravitational}$

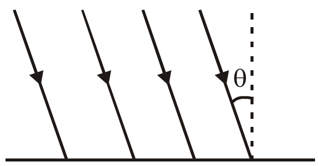
Let power of light beam be P

$\therefore \quad F_{photon} = \frac{P}{c}$

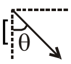
$\therefore \quad \frac{P}{c} = 10^{-1} \qquad P = 3.0 \times 10^{8} \times 10^{-1}$

$P = 3 \times 10^7$ W

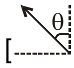
Example 9 Calculate force exerted by light beam if light is incident on surface at an angle θ as shown in figure. Consider all cases.



Solution : **Case - I** $a = 1, \quad r = 0$

initial momentum of photon(in downward direction at an angle θ with vertical) = $\frac{h}{\lambda}$ []

final momentum of photon = 0

change in momentum (in upward direction at an angle θ with vertical) = $\frac{h}{\lambda}$ []

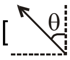
energy incident per unit time = IA cos θ

Intensity = power per unit normal area

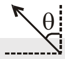
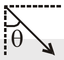
$I = \frac{P}{A\cos\theta}$ P = IA cos θ

No. of photons incident per unit time = $\frac{IA\cos\theta}{hc}.\lambda$

total change in momentum per unit time (in upward direction at an angle θ with vertical)

$= \frac{IA\cos\theta.\lambda}{hc}.\frac{h}{\lambda} = \frac{IA\cos\theta}{c}$ []

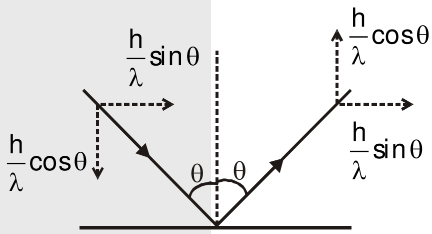
Force (F) = total change in momentum per unit time

$F = \frac{IA\cos\theta}{c}$ (direction  on photon and  on the plate)

Pressure = normal force per unit Area

Pressure = $\frac{F\cos\theta}{A}$ $P = \frac{IA\cos^2\theta}{cA} = \frac{I}{c}\cos^2\theta$

Case II : When r = 1, a = 0



∴ change in momentum of one photon

$= \frac{2h}{\lambda}\cos\theta$ (upward)

No. of photons incident per unit time

$= \frac{\text{energy incident per unit time}}{h\nu}$

$= \frac{IA\cos\theta\cdot\lambda}{hc}$

∴ total change in momentum per unit time = $\frac{IA\cos\theta\cdot\lambda}{hc} \times \frac{2h}{\lambda}\cos\theta = \frac{2IA\cos^2\theta}{c}$ (upward)

∴ force on the plate = $\frac{2IA\cos^2\theta}{c}$ (downward)

Pressure = $\frac{2IA\cos^2\theta}{cA}$ $P = \frac{2I\cos^2\theta}{c}$

Case III : 0 < r < 1, a + r = 1

change in momentum of photon when it is reflected = $\frac{2h}{\lambda}\cos\theta$ (downward)

change in momentum of photon when it is absorbed = $\frac{h}{\lambda}$ (in the opposite direction of incident beam)

energy incident per unit time = IA cos θ

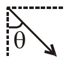
no. of photons incident per unit time = $\frac{IA\cos\theta\cdot\lambda}{hc}$

no. of reflected photon (n_r) = $\frac{IA\cos\theta.\lambda r}{hc}$

no. of absorbed photon (n_a) = $\frac{IA\cos\theta.\lambda}{hc}$ (1 – r)

force on plate due to absorbed photons $F_a = n_a . \Delta P_a$

= $\frac{IA\cos\theta.\lambda}{hc}$ (1 – r) $\frac{h}{\lambda}$

= $\frac{IA\cos\theta}{c}$ (1 – r) (at an angle θ with vertical )

force on plate due to reflected photons $F_r = n_r \Delta P_r$

= $\frac{IA\cos\theta\cdot\lambda}{hc} \times \frac{2h}{\lambda}\cos\theta$ (vertically downward)

= $\frac{IA\cos^2\theta}{c}\cdot 2r$

now resultant force is given by $F_R = \sqrt{F_r^2 + F_a^2 + 2F_aF_r\cos\theta}$

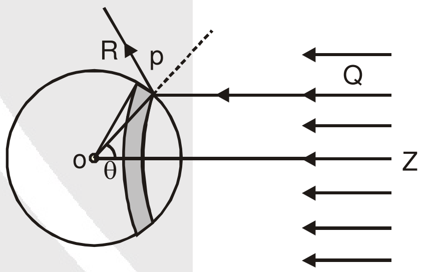
= $\frac{IA\cos\theta}{c}\sqrt{(1-r)^2 + (2r)^2\cos^2\theta + 4r(r-1)\cos^2\theta}$

and, pressure P = $\frac{F_a\cos\theta + F_r}{A} = \frac{IA\cos\theta(1-r)\cos\theta}{cA} + \frac{IA\cos^2\theta\cdot 2r}{cA}$

= $\frac{I\cos^2\theta}{c}$ (1 – r) + $\frac{I\cos^2\theta}{c}$ 2r = $\frac{I\cos^2\theta}{c}$ (1 + r)

Example 10. A perfectly reflecting solid sphere of radius r is kept in the path of a parallel beam of light of large aperture. If the beam carries an intensity I, find the force exerted by the beam on the sphere.

Solution : Let O be the centre of the sphere and OZ be the line opposite to the incident beam (figure). Consider a radius OP of the sphere making an angle θ with OZ. Rotate this radius about OZ to get a circle on the sphere. Change θ to θ + dθ and rotate the radius about OZ to get another circle on the sphere. The part of the sphere between these circles is a ring of area $2\pi r^2 \sin\theta\, d\theta$. Consider a small part ΔA of this ring at P.



Energy of the light falling on this part in time Δt is

$\Delta U = I\Delta t(\Delta A \cos\theta)$

The momentum of this light falling on ΔA is ΔU/c along QP. The light is reflected by the sphere along PR. The change in momentum is

$\Delta p = 2\frac{\Delta U}{c}\cos\theta = \frac{2}{c} I\,\Delta t\,(\Delta A\cos^2\theta)$ (direction along $\overrightarrow{OP}$)

The force on ΔA due to the light faling on it, is

$\frac{\Delta p}{\Delta t} = \frac{2}{c} I\,\Delta A\cos^2\theta.$ (direction along $\overrightarrow{PO}$)

The resultant force on the ring as well as on the sphere is along ZO by symmetry. The component of the force on ΔA along ZO

$\frac{\Delta p}{\Delta t}\cos\theta = \frac{2}{c} I\Delta A\cos^3\theta.$ (along $\overrightarrow{ZO}$)

The force acting on the ring is dF = $\frac{2}{c}I(2\pi r^2 \sin\theta \, d\theta)\cos^3\theta$.

The force on the entire sphere is F = $\int_0^{\pi/2} \frac{4\pi r^2 I}{c} \cos^3\theta \sin\theta \, d\theta$

$$= -\int_0^{\pi/2} \frac{4\pi r^2 I}{c} \cos^3\theta \, d(\cos\theta) = -\int_{\theta=0}^{\pi/2} \frac{4\pi r^2 I}{c} \left[\frac{\cos^4\theta}{4}\right]_0^{\pi/2} = \frac{\pi r^2 I}{c}$$

Note that integration is done only for the hemisphere that faces the incident beam.

7. De-BROGLIE WAVELENGTH OF MATTER WAVE

A photon of frequency ν and wavelength λ has energy.

$$E = h\nu = \frac{hc}{\lambda}$$

By Einstein's energy mass relation, E = mc^2 the equivalent mass m of the photon is given by,

$$m = \frac{E}{c^2} = \frac{h\nu}{c^2} = \frac{h}{\lambda c} \qquad .....(i)$$

or $\lambda = \frac{h}{mc}$ or $\lambda = \frac{h}{p}$ .....(ii)

Here p is the momentum of photon. By analogy de-Broglie suggested that a particle of mass m moving with speed v behaves in some ways like waves of wavelength λ (called de-Broglie wavelength and the wave is called matter wave) given by,

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad .... (iii)$$

where p is the momentum of the particle. Momentum is related to the kinetic energy by the equation,

$$p = \sqrt{2Km}$$

and a charge q when accelerated by a potential difference V gains a kinetic energy K = qV. Combining all these relations Eq. (iii), can be written as,

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{h}{\sqrt{2Km}} = \frac{h}{\sqrt{2qVm}} \text{ (de-Broglie wavelength)} \qquad ....(iv)$$

7.1 de-Broglie wavelength for an electron

If an electron (charge = e) is accelerated by a potential of V volts, it acquires a kinetic energy,

K = eV

Substituting the values of h, m and q in Eq. (iv), we get a simple formula for calculating de-Broglie wavelength of an electron.

$$\lambda(\text{in Å}) = \sqrt{\frac{150}{V(\text{in volts})}} \qquad ....(v)$$

7.2 de-Broglie wavelength of a gas molecule :

Let us consider a gas molecule at absolute temperature T. Kinetic energy of gas molecule is given by

K.E. = $\frac{3}{2}kT$; k = Boltzman constant

$$\therefore \quad \lambda_{\text{gas molecule}} = \frac{h}{\sqrt{3mkT}}$$

Solved Example

Example 11. An electron is accelerated by a potential difference of 50 volt. Find the de-Broglie wavelength associated with it.

Solution : For an electron, de-Broglie wavelength is given by, $\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{50}} = \sqrt{3}$

$= 1.73$ Å **Ans.**

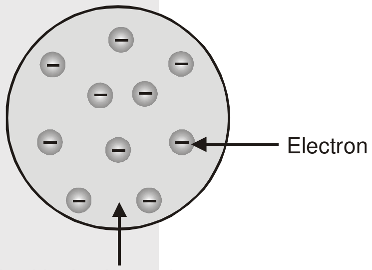
Example 12. Find the ratio of De-Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27ºC and 127ºC respectively.

Solution : de-Broglie wavelength is given by

$$\therefore \quad \frac{\lambda_{H_2}}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_{H_2}T_{H_2}}} = \sqrt{\frac{4}{2}\cdot\frac{(127+273)}{(27+273)}} = \sqrt{\frac{8}{3}}$$

8. THOMSON'S ATOMIC MODEL :

J.J. Thomson suggested that atoms are just positively charge lumps of matter with electrons embedded in them like raisins in a fruit cake. Thomson's model called the 'plum pudding' model is illustrated in figure.



Thomson played an important role in discovering the electron, through gas discharge tube by discovering cathode rays. His idea was taken seriously.

But the real atom turned out to be quite different.

9. RUTHERFORD'S NUCLEAR ATOM :

Rutherford suggested that; " All the positive charge and nearly all the mass were concentrated in a very small volume of nucleus at the centre of the atom. The electrons were supposed to move in circular orbits round the nucleus (like planets round the sun). The electronstatic attraction between the two opposite charges being the required centripetal force for such motion.

Hence $\frac{mv^2}{r} = \frac{kZe^2}{r^2}$

and total energy = potential energy + kinetic energy = $\frac{-kZe^2}{2r}$

Rutherford's model of the atom, although strongly supported by evidence for the nucleus, is inconsistent with classical physics. This model suffer's from two defects

9.1 Regarding stability of atom : An electron moving in a circular orbit round a nucleus is accelerating and according to electromagnetic theory it should therefore, emit radiation continuously and thereby lose energy. If total energy decreases then radius increases as given by above formula. If this happened the radius of the orbit would decrease and the electron would spiral into the nucleus in a fraction of second. But atoms do not collapse. In 1913 an effort was made by Neil Bohr to overcome this paradox.

9.2 Regarding explanation of line spectrum : In Rutherford's model, due to continuously changing radii of the circular orbits of electrons, the frequency of revolution of the electrons must be changing. As a result, electrons will radiate electromagnetic waves of all frequencies, i.e., the spectrum of these waves will be 'continuous' in nature. But experimentally the atomic spectra are not continuous. Instead they are line spectra.

10. THE BOHR'S ATOMIC MODEL

In 1913, Prof. Niel Bohr removed the difficulties of Rutherford's atomic model by the application of Planck's quantum theory. For this he proposed the following postulates

(1) An electron moves only in certain circular orbits, called stationary orbits. In stationary orbits electron does not emit radiation, contrary to the predictions of classical electromagnetic theory.

(2) According to Bohr, there is a definite energy associated with each stable orbit and an atom radiaties energy only when it makes a transition from one of these orbits to another. If the energy of electron in the higher orbit be E_2 and that in the lower orbit be E_1, then the frequency ν of the radiated waves is given by

$$h\nu = E_2 - E_1 \quad \text{or} \quad \nu = \frac{E_2 - E_1}{h} \qquad ...(i)$$

(3) Bohr found that the magnitude of the electron's angular momentum is quantized, and this magnitude for the electron must be integral multiple of $\frac{h}{2\pi}$. The magnitude of the angular momentum is L = mvr for a particle with mass m moving with speed v in a circle of radius r. So, according to Bohr's postulate, (n = 1, 2, 3....)

Each value of n corresponds to a permitted value of the orbit radius, which we will denote by r_n The value of n for each orbit is called **principal quantum number** for the orbit. Thus,

$$mv_nr_n = mvr = \frac{nh}{2\pi} \qquad ...(ii)$$

According to Newton's second law a radially inward centripetal force of magnitude F = $\frac{mv^2}{r_n}$ is needed by the electron which is being provided by the electrical attraction between the positive proton and the negative electron.

Thus, $$\frac{mv_n^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} \qquad ....(iii)$$

Solving Eqs. (ii) and (iii), we get

$$r_n = \frac{\varepsilon_0 n^2 h^2}{\pi m e^2} \qquad ...(iv)$$

and $$v_n = \frac{e^2}{2\varepsilon_0 nh} \qquad ...(v)$$

The smallest orbit radius corresponds to n = 1. We'll denote this minimum radius, called the **Bohr radius** as a_0. Thus,

$$a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

Substituting values of ε_0, h, p, m and e, we get

$$a_0 = 0.529 \times 10^{-10} \text{ m} = 0.529 \text{ Å} \qquad ...(vi)$$

Eq. (iv), in terms of a_0 can be written as,

$$r_n = n^2 a_0 \quad \text{or} \quad r_n \propto n^2 \qquad ...(vii)$$

Similarly, substituting values of e, ε_0 and h with n = 1 in Eq. (v), we get

$$v_1 = 2.19 \times 10^6 \text{ m/s} \qquad ...(viii)$$

This is the greatest possible speed of the electron in the hydrogen atom. Which is approximately equal to c/137 where c is the speed of light in vacuum.

Eq. (v), in terms of v_1 can be written as,

$$v_n = \frac{v_1}{n} \quad \text{or} \quad v_n \propto \frac{1}{n}$$

Energy levels : Kinetic and potential energies K_n and U_n in nth orbit are given by

$$K_n = \frac{1}{2} m v_n^2 = \frac{me^4}{8\varepsilon_0^2 n^2 h^2} \text{ and } U_n = -\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n} = -\frac{me^4}{4\varepsilon_0^2 n^2 h^2}$$

(assuming infinity as a zero potential energy level)

The total energy E_n is the sum of the kinetic and potential energies.

so, $$E_n = K_n + U_n = -\frac{me^4}{8\varepsilon_0^2 n^2 h^2}$$

Substituting values of m, e, ε_0 and h with n = 1, we get the least energy of the atom in first orbit, which is –13.6 eV. Hence,

$$E_1 = -13.6 \text{ eV} \quad \text{....(x)}$$

and $$E_n = \frac{E_1}{n^2} = -\frac{13.6}{n^2} \text{ eV} \quad \text{....(xi)}$$

Substituting n = 2, 3, 4, ..., etc., we get energies of atom in different orbits.

$$E_2 = -3.40 \text{ eV}, E_3 = -1.51 \text{ eV}, ... E_\infty = 0$$

10.1 Hydrogen Like Atoms

The Bohr model of hydrogen can be extended to hydrogen like atoms, i.e., one electron atoms, the nuclear charge is +ze, where z is the atomic number, equal to the number of protons in the nucleus. The effect in the previous analysis is to replace e^2 every where by ze^2. Thus, the equations for, r_n, v_n and E_n are altered as under:

$$r_n = \frac{\varepsilon_0 n^2 h^2}{n m z e^2} = a_0 \frac{n^2}{z} \quad \text{or} \quad r_n \propto \frac{n^2}{z} \quad \text{....(i)}$$

where $a_0 = 0.529$ Å (radius of first orbit of H)

$$v_n = \frac{ze^2}{2\varepsilon_0 nh} = \frac{z}{n} v_1 \quad \text{or} \quad v_n \propto \frac{z}{n} \quad \text{....(ii)}$$

where $v_1 = 2.19 \times 10^6$ m/s (speed of electron in first orbit of H)

$$E_n = -\frac{mz^2e^4}{8\varepsilon_0^2 n^2 h^2} = \frac{z^2}{n^2} E_1 \quad \text{or} \quad E_n \propto \frac{z^2}{n^2} \quad \text{....(iii)}$$

where $E_1 = -13.60$ eV (energy of atom in first orbit of H)

10.2 Definations valid for single electron system

(1) Ground state : Lowest energy state of any atom or ion is called ground state of the atom.

Ground state energy of H atom = –13.6 eV

Ground state energy of He^+ Ion = –54.4 eV

Ground state energy of Li^{++} Ion = –122.4 eV

(2) Excited State : State of atom other than the ground state are called its excited states.

n = 2 first excited state

n = 3 second excited state

n = 4 third excited state

$n = n_0 + 1$ n_0^{th} excited state

(3) Ionisation energy (I.E.) : Minimum energy required to move an electron from ground state to $n = \infty$ is called ionisation energy of the atom or ion

Ionisation energy of H atom = 13.6 eV

Ionisation energy of He^+ Ion = 54.4 eV

Ionisation energy of Li^{++} Ion = 122.4 eV

(4) Ionisation potential (I.P.) : Potential difference through which a free electron must be accelerated from rest such that its kinetic energy becomes equal to ionisation energy of the atom is called ionisation potential of the atom.

I.P of H atom = 13.6 V

I.P. of He^+ Ion = 54.4 V

(5) Excitation energy : Energy required to move an electron from ground state of the atom to any other exited state of the atom is called excitation energy of that state.

Energy in ground state of H atom = –13.6 eV

Energy in first excited state of H-atom = –3.4 eV

I^{st} excitation energy = 10.2 eV.

(6) Excitation Potential : Potential difference through which an electron must be accelerated from rest so that its kinetic energy becomes equal to excitation energy of any state is called excitation potential of that state.

I^{st} excitation energy = 10.2 eV.

I^{st} excitation potential = 10.2 V.

(7) Binding energy or Seperation energy : Energy required to move an electron from any state to n = ∞ is called binding energy of that state. or energy released during formation of an H-like atom/ion from n = ∞ to some particular n is called binding energy of that state.

Binding energy of ground state of H-atom = 13.6 eV

Solved Example

Example 13. First excitation potential of a hypothetical hydrogen like atom is 15 volt. Find third excitation potential of the atom.

Solution : Let energy of ground state = E_0

$E_0 = -13.6\,Z^2$ eV and $E_n = \frac{E_0}{n^2}$

n = 2, $E_2 = \frac{E_0}{4}$

given $\frac{E_0}{4} - E_0 = 15$

$-\frac{3E_0}{4} = 15$

for n = 4, $E_4 = \frac{E_0}{16}$

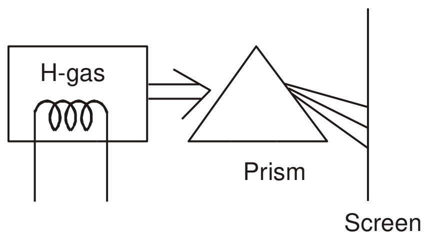
third excitation energy = $\frac{E_0}{16} - E_0 = -\frac{15}{16}E_0 = -\frac{15}{16}\cdot\left(\frac{-4\times15}{3}\right) = \frac{75}{4}$ eV

$\therefore$ third excitation potential is $\frac{75}{4}$ V

10.3 Emission spectrum of hydrogen atom :

Under normal conditions the single electron in hydrogen atom stays in ground state (n = 1). It is excited to some higher energy state when it acquires some energy from external source. But it hardly stays there for more than 10^{-8} second.

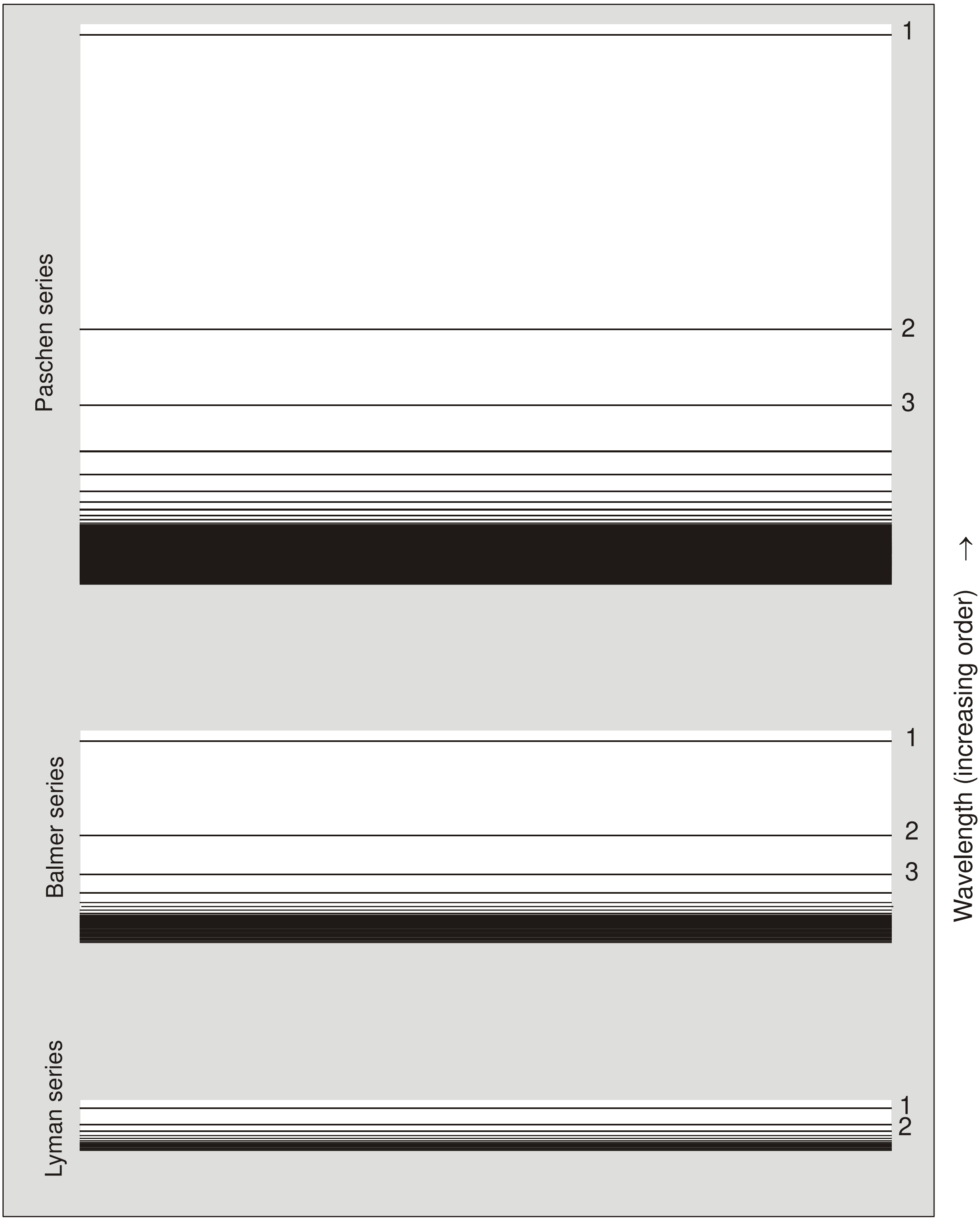
A photon corresponding to a particular spectrum line is emitted when an atom makes a transition from a state in an excited level to a state in a lower excited level or the ground level.



Let n_i be the initial and n_f the final energy state, then depending on the final energy state following series are observed in the emission spectrum of hydrogen atom.

On Screen :

A photograph of spectral lines of the Lyman, Balmer, Paschen series of atomic hydrogen.

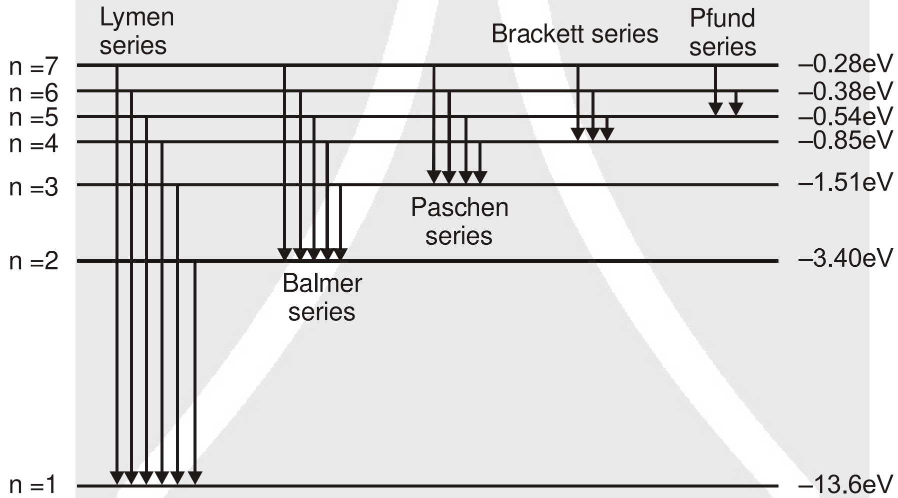


1, 2, 3..... represents the I, II & III line of Lyman, Balmer, Paschen series.

The hydrogen spectrum (some selected lines)

Name of series	Number of Line	Quantum Number			
		n_i (Lower State	n_f (Upper State)	Wavelength (nm)	Energy
Lymen	I	1	2	121.6	10.2 eV
	II	1	3	102.6	12.09 eV
	III	1	4	97	12.78 eV
	series limit	1	∞ (series limit)	91.2	13.6 eV
Balmer	I	2	3	656.3	1.89 eV
	II	2	4	486.1	2.55 eV
	III	2	5	434.1	2.86 eV
	series limit	2	∞ (series limit)	364.6	3.41 eV
Paschen	I	3	4	1875.1	0.66 eV
	II	3	5	1281.8	0.97 eV
	III	3	6	1093.8	1.13 eV
	series limit	3	∞ (series limit)	822	1.51 eV

Series limit : Line of any group having maximum energy of photon and minimum wavelength of that group is called series limit.



For the Lyman series $n_f = 1$, for Balmer series $n_f = 2$ and so on.

10.4 Wavelength of Photon Emitted in De-excitation

According to Bohr when an atom makes a transition from higher energy level to a lower level it emits a photon with energy equal to the energy difference between the initial and final levels. If E_i is the initial energy of the atom before such a transition, E_f is its final energy after the transition, and the photon's energy is $h\nu = \frac{hc}{\lambda}$, then conservation of energy gives,

$$h\nu = \frac{hc}{\lambda} = E_i - E_f \text{ (energy of emitted photon)} \quad ....(i)$$

By 1913, the spectrum of hydrogen had been studied intensively. The visible line with longest wavelength, or lowest frequency is called H_α, the next line is called H_β and so on.

In 1885, Johann Balmer, a Swiss teacher found a formula that gives the wave lengths of these lines. This is now called the Balmer series. The Balmer's formula is,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \quad ....(ii)$$

Here, n = 3, 4, 5 ...., etc.
R = Rydberg constant = 1.097×10^7 m^{-1}
and λ is the wavelength of light/photon emitted during transition,
For n = 3, we obtain the wavelength of H_α line.
Similarly, for n = 4, we obtain the wavelength of H_β line. For n = ∞, the smallest wavelength (= 3646 Å) of this series is obtained. Using the relation, $E = \frac{hc}{\lambda}$ we can find the photon energies corresponding to the wavelength of the Balmer series.

$$E = \frac{hc}{\lambda} = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = \frac{Rhc}{2^2} - \frac{Rhc}{n^2}$$

This formula suggests that,

$$E_n = -\frac{Rhc}{n^2}, \ n = 1, 2, 3..... \qquad ....(iii)$$

The wavelengths corresponding to other spectral series (Lyman, Paschen, (etc.) can be represented by formula similar to Balmer's formula.

Lyman Series : $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$, n = 2, 3, 4.....

Paschen Series : $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right)$, n = 4, 5, 6.....

Brackett Series : $\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right)$, n = 5, 6, 7.....

Pfund Series : $\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right)$, n = 6, 7, 8

The Lyman series is in the ultraviolet, and the Paschen. Brackett and Pfund series are in the infrared region.

Solved Example

Example 14. Calculate (a) the wavelength and (b) the frequency of the H_β line of the Balmer series for hydrogen.

Solution : (a) H_β line of Balmer series corresponds to the transition from n = 4 to n = 2 level. The corresponding wavelength for H_β line is,

$$\frac{1}{\lambda} = (1.097 \times 10^7)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 0.2056 \times 10^7 \quad \therefore \quad \lambda = 4.9 \times 10^{-7} \text{ m}$$ **Ans.**

(b) $\nu = \frac{c}{\lambda} = \frac{3.0 \times 10^8}{4.9 \times 10^{-7}} = 6.12 \times 10^{14}$ Hz **Ans.**

Example 15. Find the largest and shortest wavelengths in the Lyman series for hydrogen. In what region of the electromagnetic spectrum does each series lie?

Solution : The transition equation for Lyman series is given by,

$$\frac{1}{\lambda} = R\left[\frac{1}{(1)^2} - \frac{1}{n^2}\right] \qquad n = 2, 3, ...$$

for largest wavelength, n = 2

$$\frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{4}\right) = 0.823 \times 10^7$$

$\therefore$ $\lambda_{max} = 1.2154 \times 10^{-7}$ m = 1215 Å **Ans.**

The shortest wavelength corresponds to n = ∞

$$\therefore \quad \frac{1}{\lambda_{max}} = 1.097 \times 10^7\left(\frac{1}{1} - \frac{1}{\infty}\right)$$

or $\lambda_{min} = 0.911 \times 10^{-7}$ m = 911 Å **Ans.**

Both of these wavelengths lie in ultraviolet (UV) region of electromagnetic spectrum.

Example 16. How may different wavelengths may be observed in the spectrum from a hydrogen sample if the atoms are excited to states with principal quantum number n ?

Solution : From the nth state, the atom may go to (n – 1)th state, ..., 2nd state or 1st state. So there are (n – 1) possible transitions starting from the nth state. The atoms reaching (n – 1)th state may make (n – 2) different transitions. Similarly for other lower states. The total number of possible transitions is

(n – 1) + (n – 2) + (n – 3) +...........2 + 1

$= \frac{n(n-1)}{2}$ **(Remember)**

Example 17 (a) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit.

(b) How many spectral linea are observed in the emission spectrum of the above excited system?

Solution : (a) The energy in the first orbit = $E_1 = Z^2 E_0$ where $E_0 = -13.6$ eV is the energy of a hydrogen atom in ground state thus for Li^{++},

$E_1 = 9E_0 = 9 \times (-13.6 \text{ eV}) = -122.4$ eV

The energy in the third orbit is $E_3 = \frac{E_1}{n^2} = \frac{E_1}{9} = -13.6$ eV

Thus, $E_3 - E_1 = 8 \times 13.6$ eV = 108.8 eV.

Energy required to excite Li^{++} from the first orbit to the third orbit is given by

$E_3 - E_1 = 8 \times 13.6$ eV = 108.8 eV.

The wavelength of radiation required to excite Li^{++} from the first orbit to the third orbit is given by

$$\frac{hc}{\lambda} = E_3 - E_1 \quad \text{or,} \quad \lambda = \frac{hc}{E_3 - E_1} = \frac{1240\text{eV} - \text{nm}}{108.8\text{eV}} \approx 11.4\text{nm}$$

(b) The spectral lines emitted are due to the transitions n = 3 → n = 2, n = 3 → n = 1 and n = 2 → n = 1. Thus, there will be three spectral lines in the spectrum.

Example 18. Find the kinetic energy potential energy and total energy in first and second orbit of hydrogen atom if potential energy in first orbit is taken to be zero.

Solution : $E_1 = -13.60$ eV $\quad K_1 = -E_1 = 13.60$ eV $\quad U_1 = 2E_1 = -27.20$ eV

$E_2 = \frac{E_1}{(2)^2} = -3.40$ eV $\quad K_2 = 3.40$ eV and $\quad U_2 = -6.80$ eV

Now $U_1 = 0$, i.e., potential energy has been increased by 27.20 eV while kinetic energy will remain unchanged. So values of kinetic energy, potential energy and total energy in first orbit are 13.60 eV, 0, 13.60 respectively and for second orbit these values are 3.40 eV, 20.40 eV and 23.80 eV.

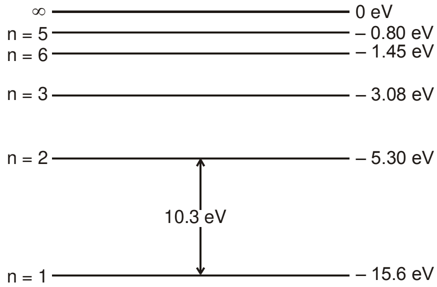
Example 19. A lithium atom has three electrons, Assume the following simple picture of the atom. Two electrons move close to the nucleus making up a spherical cloud around it and the third moves outside this cloud in a circular orbit. Bohr's model can be used for the motion of this third electron but n = 1 states are not available to it. Calculate the ionization energy of lithium in ground state using the above picture.

Solution : In this picture, the third electron moves in the field of a total charge + 3e – 2e = + e. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is :

$E_2 = \frac{E_1}{4} = \frac{-13.6 \text{ eV}}{4} = -3.4$ eV

Thus, the ionization energy of the atom in this picture is 3.4 eV.

Example 20. The energy levels of a hypothetical one electron atom are shown in the figure.



(a) Find the ionization potential of this atom.
(b) Find the short wavelength limit of the series terminating at n = 2
(c) Find the excitation potential for the state n = 3.
(d) Find wave number of the photon emitted for the transition n = 3 to n = 1.
(e) What is the minimum energy that an electron will have after interacting with this atom in the ground state if the initial kinetic energy of the electron is
(i) 6 eV (ii) 11 eV

Solution : (a) Ionization potential = 15.6 V

(b) $\lambda_{min} = \frac{12400}{5.3} = 2340$ Å

(c) $\Delta E_{31} = -3.08 - (-15.6) = 12.52$ eV
Therefore, excitation potential for state n = 3 is 12.52 volt.

(d) $\frac{1}{\lambda_{31}} = \frac{\Delta E_{31}}{12400}$ Å^{-1} $= \frac{12.52}{12400}$ Å^{-1}
$\approx 1.01 \times 10^7\ m^{-1}$

(e) (i) $E_2 - E_1 = 10.3$ eV > 6 eV.
Hence electron cannot excite the atoms. So, $K_{min} = 6$ eV.
(ii) $E_2 - E_1 = 10.3$ eV < 11 eV.
Hence electron can excite the atoms. So, $K_{min} = (11 - 10.3) = 0.7$ eV.

Example 21. A small particle of mass m moves in such a way that the potential energy $U = ar^2$ where a is a constant and r is the distance of the particle from the origin. Assuming Bohr's model of quantization of angular momentum and circular orbits, find the radius of n^{th} allowed orbit.

Solution : The force at a distance r is, $F = -\frac{dU}{dr} = -2ar$

Suppose r be the radius of nth orbit. The necessary centripetal force is provided by the above force. Thus, $\frac{mv^2}{r} = 2ar$

Further, the quantization of angular momentum gives, $mvr = \frac{nh}{2\pi}$

Solving Eqs. (i) and (ii) for r, we get $r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$ **Ans.**

Example 22. An imaginary particle has a charge equal to that of an electron and mass 100 times the mass of the electron. It moves in a circular orbit around a nucleus of charge + 4e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to the system.
(a) Derive and expression for the radius of n^{th} Bohr orbit.
(b) Find the wavelength of the radiation emitted when the particle jumps from fourth orbit to the second.

Solution : (a) We have $\frac{m_p v^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r_n^2}$ .....(i)

The quantization of angular momentum gives, $m_p v r_n = \frac{nh}{2\pi}$ ......(ii)

Solving Eqs. (i) and (ii), we get

$r = \frac{n^2 h^2 \varepsilon_0}{z\pi m_p e^2}$

Substituting m_p = 100 m

where m = mass of electron and z = 4

we get, $r_n = \frac{n^2 h^2 \varepsilon_0}{400\pi m e^2}$ **Ans.**

(b) As we know,

Energy of hydrogen atom in ground state = – 13.60 eV

and $E_n \propto \left(\frac{z^2}{n^2}\right) m$

For the given particle, $E_4 = \frac{(-13.60)\ (4)^2}{(4)^2} \times 100 = -1360$ eV

and $E_2 = \frac{(-13.60)\ (4)^2}{(2)^2} \times 100 = -5440$ eV

$\Delta E = E_4 - E_2 = 4080$ eV

$\therefore \quad \lambda$ (in Å) $= \frac{12400}{4080} = 3.0$ Å **Ans.**

Example 23. A particle known as μ-meson, has a charge equal to that of an electron and mass 208 times the mass of the electron. It moves in a circular orbit around a nucleus of charge +3e. Take the mass of the nucleus to be infinite. Assuming that the Bohr's model is applicable to this system,

(a) derive an expression for the radius of the nth Bohr orbit,

(b) find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for a hydrogen atom and

(c) find the wavelength of the radiation emitted when the μ–meson jumps from the third orbit to the first orbit.

Solution : (a) We have, $\frac{mv^2}{r} = \frac{Ze^2}{4\pi\varepsilon_0 r^2}$

or, $v^2 r = \frac{Ze^2}{4\pi\varepsilon_0 m}$ .....(i)

The quantization rule is $vr = \frac{nh}{2\pi m}$

The radius is $r = \frac{(vr)^2}{v^2 r} = \frac{4\pi\varepsilon_0 m}{Ze^2}$

$= \frac{n^2 h^2 \varepsilon_0}{Z\pi m e^2}$ ....(ii)

For the given system, Z = 3 and m = 208 m_e.

Thus $r_\mu = \frac{n^2 h^2 \varepsilon_0}{624\pi m_e e^2}$

(b) From (ii), the radius of the first Bohr orbit for the hydrogen atom is $r_h = \frac{h^2\varepsilon_0}{\pi m_e e^2}$

For $r_\mu = r_h$, $\frac{n^2h^2\varepsilon_0}{624\pi m_e e^2} = \frac{h^2\varepsilon_0}{\pi m_e e^2}$

or, $n^2 = 624$

or, $n = 25$

(c) From (i), the kinetic energy of the atom is $\frac{mv^2}{2} = \frac{Ze^2}{8\pi\varepsilon_0 r}$

and the potential energy is $-\frac{Ze^2}{4\pi\varepsilon_0 r}$

The total energy is $E_n = \frac{Ze^2}{8\pi\varepsilon_0 r}$

Using (ii), $E_n = -\frac{Z^2\pi me^4}{8\pi\varepsilon_0^2 n^2h^2} = -\frac{9\times 208 m_e{}^4}{8\varepsilon_0^2 n^2h^2} = \frac{1872}{n^2}\left(-\frac{m_e e^4}{8\varepsilon_0^2\ h^2}\right)$

But $\left(-\frac{m_e e^4}{8\varepsilon_0^2\ h^2}\right)$ is the ground state energy of hydrogen atom and hence is equal to – 13.6 eV.

From (iii), $E_n = -\frac{1872}{n^2} \times 13.6 \text{ eV} = \frac{-25459.2 \text{ eV}}{n^2}$

Thus, $E_1 = -25459.2$ eV and $E_3 = \frac{E_1}{9} = -2828.8$ eV. The energy difference is $E_3 - E_1 = 22630.4$eV.

The wavelength emitted is $\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}-\text{nm}}{22630.4 \text{ eV}}$ = 55 pm.

Solved Example

Example 24. A gas of hydrogen like atoms can absorb radiations of 68 eV. Consequently, the atoms emit radiations of only three different wavelength. All the wavelengths are equal or smaller than that of the absorbed photon.

(a) Determine the initial state of the gas atoms.
(b) Identify the gas atoms.
(c) Find the minimum wavelength of the emitted radiations.
(d) Find the ionization energy and the respective wavelength for the gas atoms.

Solution : (a) $\frac{n(n-1)}{2} = 3$

$\therefore$ n = 3

i.e., after excitation atom jumps to second excited state.

Hence $n_f = 3$. So n_i can be 1 or 2

If $n_i = 1$ then energy emitted is either equal to, greater than or less than the energy absorbed. Hence the emitted wavelength is either equal to, less than or greater than the absorbed wavelength.

Hence $n_i \neq 1$.

If $n_i = 2$, then $E_e \geq E_a$. Hence $\lambda_e \leq \lambda_0$

(b) $E_3 - E_2 = 68$ eV

$\therefore$ (13.6) $(Z^2)\left(\frac{1}{4}-\frac{1}{9}\right) = 68$

$\therefore$ Z = 6

(c) $\lambda_{min} = \frac{12400}{E_3 - E_1} = \frac{12400}{(13.6)\ (6)^2\ \left(1-\frac{1}{9}\right)} = \frac{12400}{435.2} = 28.49$ **Ans.**

(d) Ionization energy = (13.6) $(6)^2$ = 489.6 eV **Ans.**

$\lambda = \frac{12400}{489.6} = 25.33$ Å **Ans.**

Example 25. An electron is orbiting in a circular orbit of radius r under the influence of a constant magnetic field of strength B. Assuming that Bohr's postulate regarding the quantisation of angular momentum holds good for this electron, find

(a) the allowed values of the radius 'r' of the orbit.
(b) the kinetic energy of the electron in orbit
(c) The potential energy of interaction between the magnetic moment of the orbital current due to the electron moving in its orbit and the magnetic field B.
(d) The total energy of the allowed energy levels.

Solution : (a) radius of circular path

$r = \frac{mv}{Be}$ ....(i)

$mvr = \frac{nh}{2\pi}$ ....(ii)

Solving these two equations, we get

$r = \sqrt{\frac{nh}{2\pi Be}}$ and $v = \sqrt{\frac{nhBe}{2\pi m^2}}$

(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$ **Ans.**

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2}$

$= \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$

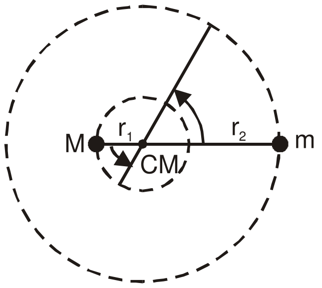
Now potential energy U = – **M . B**

$= \frac{nheB}{4\pi m}$

(d) $E = U + K = \frac{nheB}{2\pi m}$

11. EFFECT OF NUCLEUS MOTION ON ENERGY OF ATOM

Let both the nucleus of mass M, charge Ze and electron of mass m, and charge e revolve about their centre of mass (CM) with same angular velocity (ω) but different linear speeds. Let r_1 and r_2 be the distance of CM from nucleus and electron. Their angular velocity should be same then only their separation will remain unchanged in an energy level. Let r be the distance between the nucleus and the electron. Then



$Mr_1 = mr_2$

$r_1 + r_2 = r$

$\therefore \quad r_1 = \dfrac{mr}{M+m}$ and $\quad r_2 = \dfrac{Mr}{M+m}$

Centripetal force to the electron is provided by the electrostatic force. So,

$$mr_2\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}$$

or $\quad m\left(\dfrac{Mr}{M+m}\right)\omega^2 = \dfrac{1}{4\pi\varepsilon_0}\cdot\dfrac{Ze^2}{r^2}$

or $\quad \left(\dfrac{Mm}{M+m}\right) r^3\omega^2 = \dfrac{Ze^2}{4\pi\varepsilon_0}$ $\quad$ or $\quad \mu r^3\omega^2 = \dfrac{e^2}{4\pi\varepsilon_0}$

where $\quad \dfrac{Mm}{M+m} = \mu$

Moment of inertia of atom about CM, $I = Mr_1^2 + mr_2^2 = \left(\dfrac{Mm}{M+m}\right) r^2 = \mu r^2$

According to Bohr's theory, $\dfrac{nh}{2\pi} = I\,\omega$ $\quad$ or $\quad \mu r^2\omega = \dfrac{nh}{2\pi}$

Solving above equations for r, we get

$r = \dfrac{\varepsilon_0 n^2h^2}{\pi\mu e^2 Z}$ and $\quad$ **r = (0.529 Å)** $\dfrac{n^2}{Z}\cdot\dfrac{m}{\mu}$

Further electrical potential energy of the system, $U = \dfrac{-Ze^2}{4\pi\varepsilon_0 r}$ $\quad U = \dfrac{-Z^2e^4\mu}{4\varepsilon_0^2n^2h^2}$

and kinetic energy, $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}\mu r^2\omega^2$ and $K = \dfrac{1}{2}\mu v^2$

v-speed of electron with respect to nucleus. ($v = r\omega$)

here $\omega^2 = \dfrac{Ze^2}{4\pi\varepsilon_0\mu r^3}$

$\therefore \quad K = \dfrac{Ze^2}{8\pi\varepsilon_0 r} = \dfrac{Z^2e^4\mu}{8\pi\varepsilon_0^2n^2h^2}$

$\therefore$ Total energy of the system $E_n = K + U$, $E_n = -\dfrac{\mu e^4}{8\varepsilon_0^2n^2h^2}$

this expression can also be written as $\mathbf{E_n = -(13.6\ eV)}\ \dfrac{Z^2}{n^2}\cdot\left(\dfrac{\mu}{m}\right)$

The expression for E_n without considering the motion of proton is $E_n = -\dfrac{me^4}{8\varepsilon_0^2n^2h^2}$, i.e., m is replaced by μ while considering the motion of nucleus.

Solved Example

Example 26. A positronium 'atom' is a system that consists of a positron and an electron that orbit each other. Compare the wavelength of the spectral lines of positronium with those of ordinary hydrogen.

Solution : Here the two particle have the same mass m, so the reduced mass is $\mu = \dfrac{mM}{m+M} = \dfrac{m^2}{2m} = \dfrac{m}{2}$

where m is the electron mass. We know that $E_n \propto m$

$\therefore \quad \dfrac{E'_n}{E_n} = \dfrac{\mu}{m} = \dfrac{1}{2}$ energy of each level is halved.

$\therefore$ Their difference will also be halved.

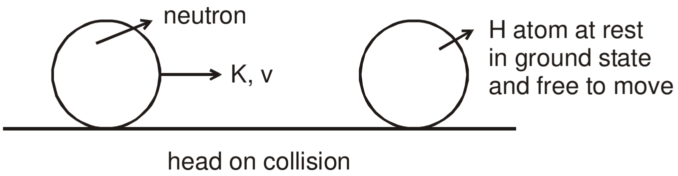
Hence $\lambda'_n = 2\lambda_n$

12. ATOMIC COLLISION

In such collisions assume that the loss in the kinetic energy of system is possible only if it can excite or ionise.

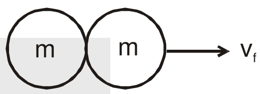
Solved Examples

Example 27



What will be the type of collision, if K = 14eV, 20.4 eV, 22 eV, 24.18 eV (elastic/inelastic/perectly inelastic)

Solution : Loss in energy (ΔE) during the collision will be used to excite the atom or electron from one level to another.



According to quantum Mechanics, for hydrogen atom.

ΔE = {0, 10.2 eV, 12.09 eV, ..., 13.6 eV)

According to Newtonion mechanics

minimum loss = 0. (elastic collsion)

for maximum loss collision will be perfectly inelastic if neutron collides perfectly inelastically then,

Applying momentum conservation $m\nu_0 = 2m\nu_f$

$$v_f = \frac{v_0}{2}$$

$$\text{final K.E.} = \frac{1}{2} \times 2m \times \frac{\nu_0^2}{4} = \frac{\frac{1}{2}m\nu_0^2}{2} = \frac{K}{2} \quad \text{maximum loss} = \frac{K}{2}$$

According to classical mechanics (ΔE) = $[0, \frac{K}{2}]$

(a) If K = 14 eV,
According to quantum mechanics (ΔE) = {0, 10.2eV, 12.09 eV}
According to classical mechanics ΔE = [0, 7 eV]
loss = 0, hence it is elastic collision speed of particle changes.

(b) If K = 20.4 eV
According to classical mechanics
loss = [0, 10.2 eV]
According to quantum mechanics
loss = {0, 10.2eV, 12.09eV,.........}
loss = 0 elastic collision.
loss = 10.2eV perfectly inelastic collision

(c) If K = 22 eV
Classical mechanics ΔE =[0, 11]
Quantum mechanics ΔE = {0, 10.2eV, 12.09eV, ...}
loss = 0 elastic collision
loss = 10.2 eV inelastic collsion

(d) If K = 24.18 eV
According to classical mechanics ΔE =[0, 12.09eV]
According to quantum mechanics ΔE = {0, 10.2eV, 12.09eV, ... 13.6eV}
loss = 0 elastic collision
loss = 10.2 eV inelastic collision
loss = 12.09 eV perfectly inelastic collision

Example 28. A He^+ ion is at rest and is in ground state. A neutron with initial kinetic energy K collides head on with the He^+ ion. Find minimum value of K so that there can be an inelastic collision between these two particle.

Solution :

Here the loss during the collision can only be used to excite the atoms or electrons.
So according to quantum mechanics
loss = {0, 40.8eV, 48.3eV, ..., 54.4eV} ....(1)

$$E_n = -\frac{Z^2}{n^2}\ 13.6 \text{ eV}$$

Now according to newtonion mechanics
Minimum loss = 0
maximum loss will be for perfectly inelastic collision.
let v_0 be the initial speed of neutron and v_f be the final common speed.

so by momentum conservation $mv_0 = mv_f + 4mv_f$ $\qquad v_f = \frac{v_0}{5}$

where m = mass of Neutron
$\therefore$ mass of He^+ ion = 4m
so final kinetic energy of system

$$K.E. = \frac{1}{2} m v_f^2 + \frac{1}{2} 4m v_f^2 = \frac{1}{2}.(5m).\frac{v_0^2}{25} = \frac{1}{5}.(\frac{1}{2} m v_0^2) = \frac{K}{5}$$

maximum loss $= K - \frac{K}{5} = \frac{4K}{5}$

so loss will be $\left[0,\ \frac{4K}{5}\right]$ ....(2)

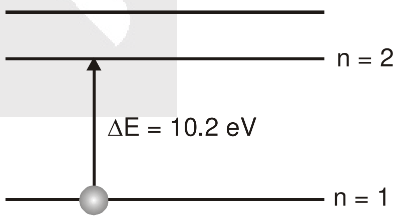
For inelastic collision there should be at least one common value other than zero in set (1) and (2)

$\therefore \quad \frac{4K}{5} > 40.8$ eV

K > 51 eV
minimum value of K = 51 eV.

Example 29 A moving hydrogen atom makes a head on collision with a stationary hydrogen atom. Before collision both atoms are in ground state and after collision they move together. What is the minimum value of the kinetic energy of the moving hydrogen atom, such that one of the atoms reaches one of the excitation state.

Solution : Let K be the kinetic energy of the moving hydrogen atom and K', the kinetic energy of combined mass after collision.



From conservation of linear momentum,

$p = p'$ or $\sqrt{2Km} = \sqrt{2K'(2m)}$

or $K = 2K'$ ....(i)

From conservation of energy, $K = K' + \Delta E$ ....(ii)

Solving Eqs. (i) and (ii), we get $\Delta E = \frac{K}{2}$

Now minimum value of ΔE for hydrogen atom is 10.2 eV.
or $\Delta E \geq 10.2$ eV

$\therefore \quad \frac{K}{2} \geq 10.2$

$\therefore \quad K \geq 20.4$ eV

Therefore, the minimum kinetic energy of moving hydrogen is 20.4 eV **Ans.**

Example 30. A neutron moving with speed v makes a head-on collision with a hydrogen atom in ground state kept at rest. Find the minimum kinetic energy of the neutron for which inelastic (completely or partially) collision may take place. The mass of neutron = mass of hydrogen = 1.67×10^{-27} kg.

Solution : Suppose the neutron and the hydrogen atom move at speed v_1 and v_2 after the collision. The collision will be inelastic if a part of the kinetic energy is used to excite the atom. Suppose an energy ΔE is used in this way. Using conservation of linear momentum and energy.

$$mv = mv_1 + mv_2 \quad \text{....(i)}$$

and $$\frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 + \Delta E \quad \text{....(ii)}$$

From (i), $v^2 = v_1^2 + v_2^2 + 2v_1v_2$,

From (ii), $v^2 = v_1^2 + v_2^2 + \frac{2\Delta E}{m}$

Thus, $2v_1v_2 = \frac{2\Delta E}{m}$

Hence, $(v_1 - v_2)^2 - 4v_1v_2 = v^2 - \frac{4\Delta E}{m}$

As $v_1 - v_2$ must be real, $v^2 - \frac{4\Delta E}{m} \geq 0$ or $\frac{1}{2}mv^2 > 2\Delta E$.

The minimum energy that can be absorbed by the hydrogen atom in ground state to go in an excited state is 10.2 eV. Thus, the minimum kinetic energy of the neutron needed for an inelastic collision is

$$\frac{1}{2}mv_{min}^2 = 2 \times 10.2 \text{ eV} = 20.4 \text{ eV}$$

Example 31. How many head-on, elastic collisions must a neutron have with deuterium nucleus to reduce its energy from 1 MeV to 0.025 eV.

Solution : Let mass of neutron = m and mass of deuterium = 2m

initial kinetic energy of neutron = K_0

Let after first collision kinetic energy of neutron and deuterium be K_1 and K_2.

Using C.O.L.M. along direction of motion $\sqrt{2mK_0} = \sqrt{2mK_1} + \sqrt{4mK_2}$

velocity of seperation = velocity of approach $\frac{\sqrt{4mK_2}}{2m} - \frac{\sqrt{2mK_1}}{m} = \frac{\sqrt{2mK_0}}{m}$

Solving equaiton (i) and (ii) we get ; $K_1 = \frac{K_0}{9}$

Loss in kinetic eneryg after first collision $\Delta K_1 = K_0 - K_1$

$$\Delta K_1 = \frac{8}{9} K_0 \quad \text{.......(1)}$$

After second collision $\Delta K_2 = \frac{8}{9} K_1 = \frac{8}{9} . \frac{K_0}{9}$

$\therefore$ Total energy loss $\Delta K = \Delta K_1 + \Delta K_2 + ... + \Delta K_n$

As, $\Delta K = \frac{8}{9}K_0 + \frac{8}{9^2}K_0 + ... + \frac{8}{9^n}K_0$

$$\Delta K = \frac{8}{9}K_0 \left(1 + \frac{1}{9} + ... + \frac{1}{9^{n-1}}\right)$$

$$\frac{\Delta K}{K_0} = \frac{8}{9}\left[\frac{1-\frac{1}{9^n}}{1-\frac{1}{9}}\right] = 1 - \frac{1}{9^n}$$

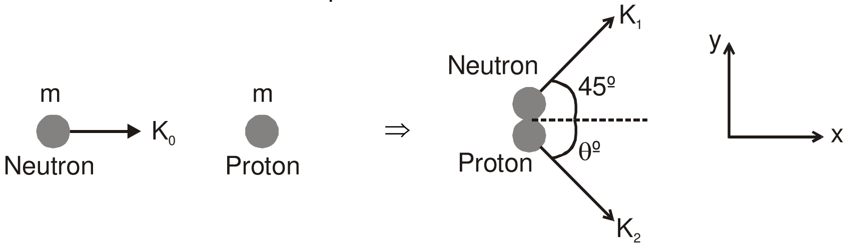
Here, $K_0 = 10^6$ eV, $\Delta K = (10^6 - 0.025)$ eV

$$\therefore \quad \frac{1}{9^n} = \frac{K_0 - \Delta K}{K_0} = \frac{0.025}{10^6} \quad \text{or} \quad 9^n = 4 \times 10^7$$

Taking log both sides and solving, we get n = 8

Example 32. A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision neutron is deflected by 45º find the number of collisions which will reduce its energy to 0.23 eV.

Solution : Mass of neutron $\approx$ mass of proton = m



From conservation of momentum in y-direction

$$\sqrt{2mK_1}\ \sin 45º = \sqrt{2mK_2}\ \sin\theta \qquad ....(i)$$

In x-direction $\sqrt{2mK_0} - \sqrt{2mK_1}\cos 45º = \sqrt{2mK_2}\cos\theta$ ....(ii)

Squaring and adding equation (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \qquad ....(iii)$$

From conservation of energy

$$K_2 = K_0 - K_1 \qquad ....(iv)$$

Solving equations (iii) and (iv), we get

$$K_1 = \frac{K_0}{2}$$

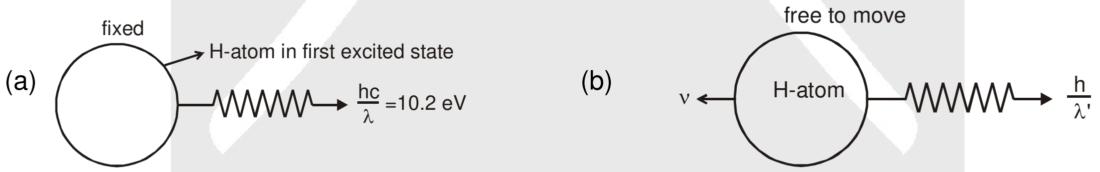
i.e., after each collision energy remains half. Therefore, after n collisions, $K_n = K_0\left(\frac{1}{2}\right)^n$

$\therefore\quad 0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n \quad\Rightarrow\quad 2^n = \frac{4.6\times10^6}{0.23}$

Taking log and solving, we get $n \approx 24$ **Ans.**

12.1 Calculation of recoil speed of atom on emission of a photon

momentum of photon = mc = $\frac{h}{\lambda}$



m - mass of atom

According to momentum conservation $m\nu = \frac{h}{\lambda'}$ ....(i)

According to energy conservation $\frac{1}{2}m\nu^2 + \frac{hc}{\lambda'}$ = 10.2 eV

Since mass of atom is very large than photon

hence $\frac{1}{2}m\nu^2$ can be neglected

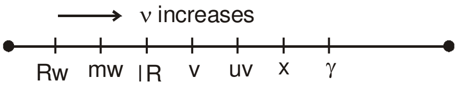
$\frac{hc}{\lambda'}$ = 10.2 eV $\qquad \frac{h}{\lambda} = \frac{10.2}{c}$ eV

$m\nu = \frac{10.2}{c}$ eV $\qquad \nu = \frac{10.2}{cm}$

recoil speed of atom = $\frac{10.2}{cm}$

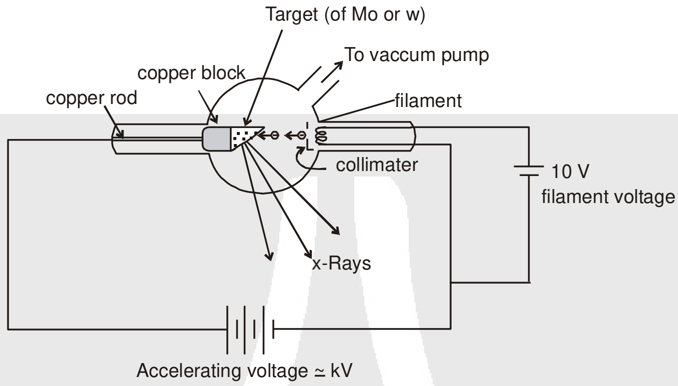
13. X-RAYS

It was discovered by **ROENTGEN**. The wavelength of x-rays is found between 0.1 Å to 10 Å. These rays are invisible to eye. They are electromagnetic waves and have speed $c = 3 \times 10^8$ m/s in vacuum. Its photons have energy around 1000 times more than the visible light.



When fast moving electrons having energy of order of several KeV strike the metallic target then x-rays are produced.

13.1 Production of x-rays by coolidge tube :



The melting point, specific heat capacity and atomic number of target should be high. When voltage is applied across the filament then filament on being heated emits electrons from it. Now for giving the beam shape of electrons, collimator is used. Now when electron strikes the target then x-rays are produced.

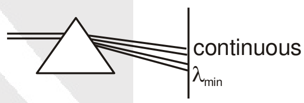
When electrons strike with the target, some part of energy is lost and converted into heat. Since, target should not melt or it can absorb heat so that the melting point, specific heat of target should be high.

Here copper rod is attached so that heat produced can go behind and it can absorb heat and target does not get heated very high.

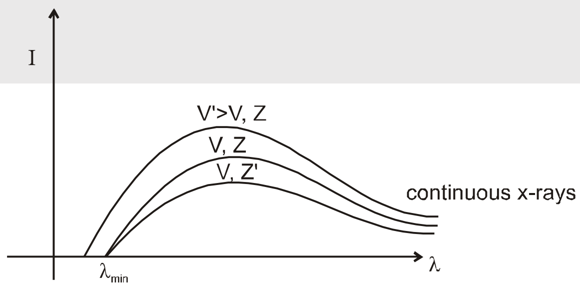
For more energetic electron, accelerating voltage is increased.

For more no. of photons voltage across filament is increased.

The x-ray were analysed by mostly taking their spectrum



13.2 Variation of Intensity of x-rays with λ is plotted as shown in figure :



13.2.1 The minimum wavelength corresponds to the maximum energy of the x-rays which in turn is equal to the maximum kinetic energy eV of the striking electrons thus

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}} \quad ; \qquad \lambda_{min} = \frac{hc}{eV} = \frac{12400}{V(\text{involts})} \text{ Å}.$$

We see that cutoff wavelength λ_{min} depends only on accelerating voltage applied between target and filament. It does not depend upon material of target, it is same for two different metals (Z and Z')

Solved Example

Example 33. An X-ray tube operates at 20 kV. A particular electron loses 5% of its kinetic energy to emit an X-ray photon at the first collision. Find the wavelength corresponding to this photon.

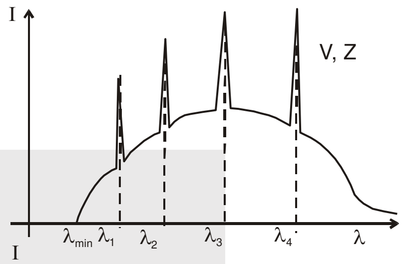
Solution : Kinetic energy acquired by the electron is K = eV = 20 × 10^3 eV.

The energy of the photon = 0.05 × 20 = 10^3 eV = 10^3 eV.

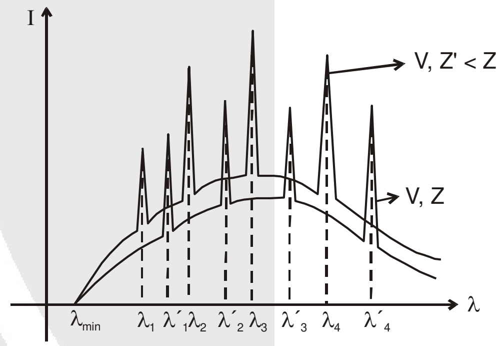
Thus, $\frac{h\nu}{\lambda} = 10^3 eV = \frac{(4.14 \times 10^{-15} eV - s) \times (3 \times 10^8 m/s)}{10^3 eV} = \frac{1242 \ \ eV - nm}{10^3 eV} = 1.24 \ \ nm$

13.2.2 Charactristic X-rays

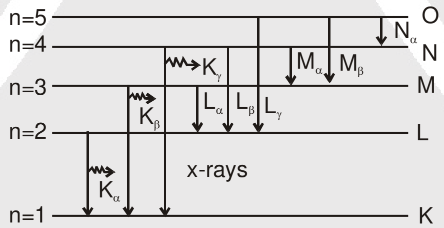
The sharp peaks obtained in graph are known as characteristic x-rays because they are characteristic of target material.



λ_1, λ_2, λ_3, λ_4, ... = charecteristic wavelength of material having atomic number Z are called **characteristic x-rays** and the spectrum obtained is called **characteristic spectrum**. If target of atomic number Z' is used then peaks are shifted.

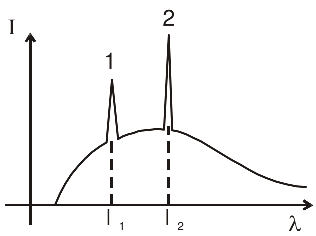


Characteristic x-ray emission occurs when an energetic electron collides with target and remove an inner shell electron from atom, the vacancy created in the shell is filled when an electron from higher level drops into it. Suppose vacancy created in innermost K-shell is filled by an electron droping from next higher level L-shell then K_α characteristic x-ray is obtained. If vaccany in K-shell is filled by an electron from M-shell, K_β line is produced and so on similarly L_α, L_β,.....M_α, M_β lines are produced.



Solved Examples

Example 34. Find which is K_α and K_β



Solution : $\Delta E = \frac{hc}{\lambda}$, $\lambda = \frac{hc}{\Delta E}$

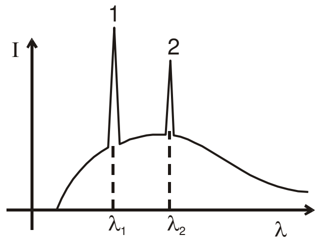
since energy difference of K_α is less than K_β

$\Delta E_{k\alpha} < \Delta E_{k\beta}$

$\lambda_{k\beta} < \lambda_{k\alpha}$

1 is K_β and 2 is K_α

Example 35



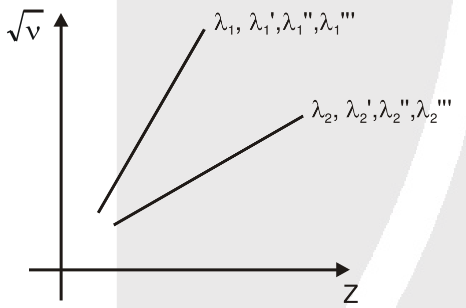
Find which is K_α and L_α

Solution : $\therefore \quad \Delta E_{K\alpha} > \Delta E_{L\alpha}$

1 is K_α and 2 is L_α

14. MOSELEY'S LAW :

Moseley measured the frequencies of characteristic x-rays for a large number of elements and plotted the sqaure root of frequency against position number in periodic table. He discovered that plot is very closed to a straight line not passing through origin.



Z_1	λ_1	λ_2
Z_2	λ_1'	λ_2'
Z_3	λ_1''	λ_2''
Z_4	λ_1'''	λ_2'''

Wavelength of charactristic wavelengths.

Moseley's observations can be mathematically expressed as $\sqrt{\nu} = a(Z - b)$

a and b are positive constants for one type of x-rays & for all elements (independent of Z).

Moseley's Law can be derived on the basis of Bohr's theory of atom, frequency of x-rays is given by

$$\sqrt{\nu} = \sqrt{CR\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \,.\, (Z - b)$$

by using the formula $\frac{1}{\lambda} = R\,z^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ with modification for multi electron system.

b → known as screening constant or shielding effect, and (Z – b) is effective nuclear charge.

for K_α line

$n_1 = 1, \quad n_2 = 2$

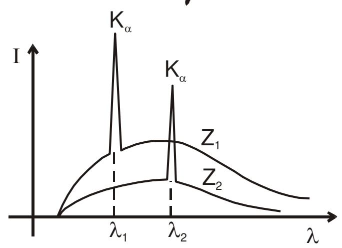
$$\therefore \quad \sqrt{\nu} = \sqrt{\frac{3RC}{4}}\ (Z - b)$$

$$\sqrt{\nu} = a(Z - b)$$

Here $a = \sqrt{\frac{3RC}{4}}$, [b = 1 for K_α lines]

Solved Example

Example 36



Find in Z_1 and Z_2 which one is greater.

Solution : $\because \sqrt{\nu} \equiv \sqrt{cR\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)}\ .\ (Z-b)$

If Z is greater then ν will be greater, λ will be less

$\therefore\ \lambda_1 < \lambda_2$

$\therefore\ Z_1 > Z_2$.

Example 37 A cobalt target is bombarded with electrons and the wavelength of its characteristic spectrum are measured. A second, fainter, characteristic spectrum is also found because of an impurity in the target. The wavelength of the K_α lines are 178.9 pm (cobalt) and 143.5 pm (impurity). What is the impurity?

Solution : Using Moseley's law and putting c/λ for ν (and assuming b = 1), we obtain

$$\sqrt{\frac{c}{\lambda_{c_0}}} = aZ_{c_0} - a$$

and $\sqrt{\frac{c}{\lambda_x}} = aZ_x - a$

Dividing yields $\sqrt{\frac{\lambda_{c_0}}{\lambda_x}} = \frac{Z_x - 1}{Z_{c_0} - 1}$

Substituting gives us $\sqrt{\frac{178.9\text{pm}}{143.5\text{pm}}} = \frac{Z_x - 1}{27 - 1}$.

Solving for the unknown, we find $Z_x = 30.0$; the impurity is zinc.

Example 38 Find the constants a and b in Moseley's equation $\sqrt{v} = a(Z-b)$ from the following data.

Element	Z	Wavelength of K_α X-ray
Mo	42	71 pm
Co	27	178.5 pm

Solution : Moseley's equation is $\sqrt{v} = a(Z-b)$

Thus, $\sqrt{\frac{c}{\lambda_1}} = a(Z_1 - b)$ ....(i)

and $\sqrt{\frac{c}{\lambda_2}} = a(Z_2 - b)$ ....(ii)

From (i) and (ii) $\sqrt{c}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right) = a(Z_1 - Z_2)$ or $a = \frac{\sqrt{c}}{(Z_1 - Z_2)}\left(\frac{1}{\sqrt{\lambda_1}} - \frac{1}{\sqrt{\lambda_2}}\right)$

$$= \frac{(3\times10^8\,\text{m/s})^{1/2}}{42-27}\left[\frac{1}{(71\times10^{-12}\text{m})^{1/2}} - \frac{1}{(178.5\times10^{-12}\text{m})^{1/2}}\right] = 5.0 \times 10^7\ (\text{Hz})^{1/2}$$

Dividing (i) by (ii),

$\sqrt{\frac{\lambda_2}{\lambda_1}} = \frac{Z_1 - b}{Z_2 - b}$ or $\sqrt{\frac{178.5}{71}} = \frac{42-b}{27-b}$ or b = 1.37

Solved Miscellaneous Problems

Problem 1. Find the momentum of a 12.0 MeV photon.

Solution : $p = \frac{E}{c}$ = 12 MeV/c.

Problem 2. Monochromatic light of wavelength 3000 Å is incident nornally on a surface of area 4 cm^2. If the intensity of the light is 15 × 10^{-2} W/m^2, determine the rate at which photons strike the surface.

Solution : Rate at which photons strike the surface

$$= \frac{IA}{hc/\lambda} = \frac{6\times10^{-5}\,J/s}{6.63\times10^{-19}\,J/photon} = 9.05 \times 10^{13} \text{ photon/s.}$$

Problem 3. The kinetic energies of photoelectrons range from zero to 4.0 × 10^{-19} J when light of wavelength 3000 Å falls on a surface. What is the stopping potential for this light ?

Solution : $K_{max} = 4.0 \times 10^{-19}\,J \times \frac{1eV}{1.6\times10^{-19}\,J} = 2.5$ eV.

Then, from $eV_s = K_{max}$, V_s = 2.5 V.

Problem 4. What is the threshold wavelength for the material in above problem ?

Solution : $2.5\text{ eV} = \frac{12.4\times10^3\,eV.Å}{3000\,Å} - \frac{12.4\times10^3\,eV.Å}{\lambda_{th}}$

Solving, λ_{th} = 7590 Å.

Problem 5. Find the de Broglie wavelength of a 0.01 kg pellet having a velocity of 10 m/s.

Solution : $\lambda = h/p = \frac{6.63\times10^{-34}\,J.s}{0.01\,kg\times10\,m/s} = 6.63 \times 10^{-23}$ Å .

Problem 6. Determine the accelerating potential necessary to give an electron a de Broglie wavelength of 1 Å, which is the size of the interatomic spacing of atoms in a crystal.

Solution : $V = \frac{h^2}{2m_0 e\lambda^2}$ = 151 V.

Problem 7. Determine the wavelength of the second line of the Paschen series for hydrogen.

Solution . $\frac{1}{\lambda} = (1.097 \times 10^{-3}\,Å^{-1})\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$ or λ = 12,820 Å.

Problem 8. How many different photons can be emitted by hydrogen atoms that undergo transitions to the ground state from the n = 5 state ?

Solution : No of possible transition from n = 5 are ${}^5C_2 = 10$

Answer. 10 photons.

Problem 9. An electron rotates in a circle around a nucleus with positive charge Ze. How is the electrons' velocity releated to the radius of its orbit ?

Solution : The force on the electron due to the nuclear provides the required centripetal force

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze.\ e}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow\ v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0.rm}}$$

Ans. $v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0.rm}}$.

Problem 10. (i) Calculate the first three energy levels for positronium.

(ii) Find the wavelength of the H_a line (3 → 2 transition) of positronium.

Solution : In positronium electron and positron revolve around their centre of mass

$$\frac{1}{4p\varepsilon_0}\frac{e^2}{r^2} = \frac{mv^2}{r/2} \quad ......(1)$$

$$\frac{nh}{2\pi} = 2 \times mv_{k/2} \quad ......(2)$$

From (1) & (2)

$$V = \frac{1}{2}.\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{nh} \times 2p = \frac{e^2}{4\varepsilon_0 nh}$$

$$TE = -\frac{1}{2}mv^2 \times 2 = -m.\frac{e^4}{16\varepsilon_0^2 n^2 h^2}$$

$$= -6.8\frac{1}{n^2}eV$$

(i) $E_1 = -6.8$ ev

$E_2 = -6.8 \times \frac{1}{2^2}$ eV $= -1.70$ eV

$E_3 = -6.8 \times \frac{1}{3^2}$ eV $= -0.76$ eV

(ii) ΔE (3 → 2) = $E_3 - E_2 = -0.76 - (-1.70)$ eV

= 0.94 eV

The corresponding wave length

$$\lambda = \frac{1.24\times10^4}{0.94}\ \text{Å} = 1313\ \text{Å}$$

Ans. (i) –6.8 eV, –1.7 eV, –0.76 eV ;

(ii) 1313 Å .

Problem 11. A H-atom in ground state is moving with intial kinetic energy K. It collides head on with a He^+ ion in ground state kept at rest but free to move. Find minimum value of K so that both the particles can excite to their first excited state.

Solution : Energy available for excitation = $\frac{4k}{5}$

Total energy required for excitation

= 10.2 ev + 40.8 eV

= 51.0 ev

$\therefore \quad \frac{4k}{5} = 51 \quad \Rightarrow \quad k = 63.75$ eV

Problem 12. A TV tube operates with a 20 kV accelerating potential. What are the maximum–energy X–rays from the TV set ?

Solution : The electrons in the TV tube have an energy of 20 keV, and if these electrons are brought to rest by a collision in which one X–ray photon is emitted, the photon energy is 20 keV.

Problem 13. In the Moseley relation, $\sqrt{v} = a(Z - b)$ which will have the greater value for the constant a for K_α or K_β transition ?

Solution : A is larger for the K_β transitions than for the K_α transitions.

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Photoelectric Effect

A-1. When a light of wavelength 400 nm falls on a metal of workfunction 2.5 eV, what will be the maximum magnitude of linear momentum of emitted photoelectron?

A-2. The electric field associated with a monochromatic light is given by $E = E_0 \sin (1.2 \times 10^{15} \pi t - kx)$. Find the maximum kinetic energy of the photoelectrons when this light falls on a metal surface whose work function is 2.0 eV

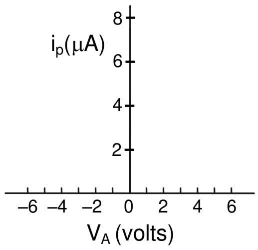
A-3. One milliwatt of light of wavelength λ = 4560 Å is incident on a cesium metal surface. Calculate the electron current liberated. Assume a quantum efficiency of η = 0.5 %. [work function for cesium = 1.89 eV] Take hc = 12400 eV-Å.

A-4. Suppose the wavelength of the incident light in photoelectric effect experiment is increased from 3000 Aº to 3040 Aº. Find the corresponding change in the stopping potential. [Take the product $hc = 12.4 \times 10^{-7}$ eV m]

A-5.✍ The magnetic field at a point associated with a light wave is $B = 2 \times 10^{-6}$ Tesla $\sin [(3.0 \times 10^{15} s^{-1})t] \sin [(6.0 \times 10^{15} s^{-1})t]$. If this light falls on a metal surface having a work function of 2.0 eV, what will be the maximum kinetic energy of the photoelectrons ?

A-6. In an experiment on photoelectric effect, light of wavelength 800 nm (less than threshold wavelength) is incident on a cesium plate at the rate of 5.0 W. The potential of the collector plate is made sufficiently positive with respect to the emitter so that the current reaches its saturation value. Assuming that on the average one of every 10^6 photons is able to eject a photoelectron, find the photo current in the circuit.

A-7.✍ In a photoelectric effect experiment, photons of energy 5 eV are incident on the photocathode of work function 3 eV. For photon intensity $I_A = 10^{15}$ m^{-2} s^{-1}, saturation current of 4.0 μ A is obtained. Sketch the variation of photocurrent i_p against the anode voltage V_a in the figure below for photon intensity I_A (curve A) and $I_B = 2 \times 10^{15}$ m^{-2} s^{-1} (curve B) (in JEE graph was to be drawn in the answer sheet itself.) **[JEE 2003, Mains 2/60]**



Section (B) : Photon emission from a source and radiation pressure

B-1. Intensity of sunlight falling normally on the earth surface is 1.4×10^3W/m^2. Assume that the light is monochromatic with average wavelength 5000Å and that no light is absorbed in between the sun and the earth's surface. The distance between the sun and the earth is 1.5×10^{11}m.

(a) Calculate the number of the photons falling per second on each square meter of earth's surface directly below the sun.

(b) How many photons are there in each cubic meter near the earth's surface at any instant ?

(c) How many photons does the sun emits per second ?

B-2. A parallel beam of monochromatic light of wavelength 663 nm is incident on a totally reflecting plane mirror. The angle of incidence is 60° and the number of photons striking the mirror per second is 5×10^{19}. Calculate the force exerted by the light beam on the mirror. ($h = 6.63 \times 10^{-34}$ J.s.)

B-3. A beam of white light is incident normally on a plane surface absorbing 70% of the light and reflecting the rest. If the incident beam carries 30 W of power, find the force exerted by it on the surface.

B-4. A sodium lamp of power 10 W is emitting photons of wavelength 590 nm. Assuming that 60% of the consumed energy is converted into light, find the number of photons emitted per second by the lamp.

Section (C) : de–Broglie wave length

C-1. Photoelectrons are liberated by ultraviolet light of wavelength 3000 Å from a metallic surface for which the photoelectric threshold wavelength is 4000 Å. Calculate the de Broglie wavelength of electrons emitted with maximum kinetic energy.

C-2. Two identical nonrelativistic particles move at right angles to each other, possessing de-Broglie wavelengths, λ_1 & λ_2. Find the de-Broglie wavelength of each particle in the frame of their centre of mass.

Section (D) : Bohr's Theory for hydrogen, hydrogen like atoms (properties)

D-1. Find the numerical value of de-Broglie wavelength of an electron in the 1st orbit of hydrogen atom assuming Bohr's atomic model. You can use standard values of the constants. Leave your answer in terms of π.

D-2. Find the radius and energy of a He^+ ion in the states (a) $n = 2$, (b) $n = 3$.

D-3. A positive hydrogen like ion having electron at its ground state ejects it, if a photon of wavelength 228 Å or less is absorbed by it. Identify the ion.

D-4. Find the temperature at which the average kinetic energy of the molecules of hydrogen equals the binding energy of its electron in ground state, assuming average kinetic energy of hydrogen gas molecule = $\frac{3}{2}kT$.

D-5. A monochromatic light source of frequency ν illuminates a metallic surface and ejects photoelectrons. The photoelectrons having maximum energy are just able to ionize the hydrogen atoms in ground state. When the whole experiment is repeated with incident radiations of frequency $\left(\frac{5}{6}\right)\nu$ the photoelectrons so emitted are able to excite the hydrogen atom which then emits a radiation of wavelength of 1215 Å. Find the frequency ν.

Section (E) : Electronic Transition in the H/H-Like atom/Species & Effect of motion of Nucleus

E-1. Find the smallest wavelength in emission spectra of (a) hydrogen, (b) He^+

E-2. Calculate the angular frequency of revolution of an electron occupying the second Bohr orbit of He^+ ion.

E-3. Find the quantum number n corresponding to the excited state of He^+ ion, if on transition to the ground state that ion emits two photons in succession with wave lengths 108.5 and 30.4 nm.

E-4. Consider a gas of hydrogen like ions in an excited state A. It emits photons having wavelength equal to the wavelength of the first line of the Lyman series together with photons of five other wavelengths. Identify the gas and find the principal quantum number of the state A.

E-5. A stationary hydrogen atom emits a photon corresponding to first line of the Lyman series. What velocity does the atom acquire ?

E-6. From the condition of the foregoing problem, find how much (in %) the energy of the emitted photon differs from the energy of the corresponding transition in a hydrogen atom.

E-7. Consider a gas consisting Li^{+2} (which is hydrogen like ion).
(a) Find the wavelength of radiation required to excite the electron in Li^{++} from n = 1 and n = 3. (Ionisation energy of the hydrogen atom equals 13.6 eV).
(b) How many spectral lines are observed in the emission spectrum of the above excited system ?

E-8. A free atom of iron emits a photon of energy 6.4 keV. Then find the recoil kinetic energy of the atom. (Take mass of iron atom = 9.3×10^{-26} kg).

Section (F) : Atomic Collisions

F-1. At what minimum kinetic energy must a hydrogen atom move for its inelastic headon collision with another stationary hydrogen atom so that one of them emits a photon? Both atoms are supposed to be in the ground state prior to the collision.

Section (G) : X-rays

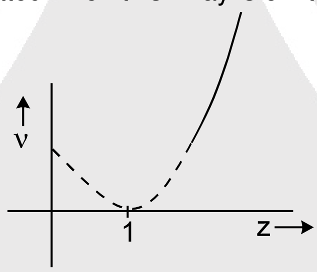
G-1. Find the cutoff wavelength for the continuous X-rays coming from an X-ray tube operating at 40 kV.

G-2. If the operating potential in an X -ray tube is increased by 0.1%, by what percentage does the cutoff wavelength decrease ?

G-3. On increasing the operating voltage in an x-ray tube to 1.5 times, the shortest wavelength decreases by 26 pm. Find the original value of operating voltage.

G-4. An X-ray tube operates at 20 kV. Suppose the electron converts 70% of its energy into a photon at each collision. Find the lowest three wavelength emitted from the tube. Neglect the energy imparted to the atom with which the electron collides.

G-5. Figure shows the variation of frequency of a characteristic x-ray and atomic number.
(i) Name the characteristic x-ray
(ii) Find the energy of photon emitted when this x-ray is emitted by a metal having z = 101.



G-6. Find the wavelength of the K_α line in copper (Z = 29), if the wave length of the K_α line in iron (Z = 26) is known to be equal to 193 pm. (Take b = 1)

G-7. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. This excited atom can make a transition to the first excited state by successively emitting two photons of energies 10.20 eV & 17.00 eV respectively. Alternatively, the atom from the same excited state can make a transition to the second excited state by sucessively emitting two photons of energies 4.25 eV and 5.95 eV respectively. Determine the values of n & Z. (Ionization energy of hydrogen atom = 13.6 eV)
[JEE 1994, 6]

G-8. Characteristic X-rays of frequency 4.2×10^{18} Hz are emitted from a metal due to transition from L- to K-shell. Find the atomic number of the metal using Moseley's law. Take Rydberg constant $R = 1.1 \times 10^7\ m^{-1}$. **[JEE '2003, Mains 2/60]**

Section (H) : for JEE Main

H-1. An electron beam of energy 10 KeV is incident on metallic foil. If the interatomic distance is 0.55Å. Find the angle of diffraction.

PART - II : ONLY ONE OPTION CORRECT TYPE

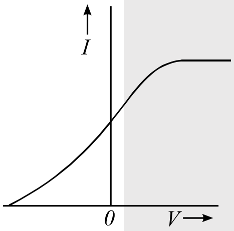
Section (A) : Photoelectric Effects

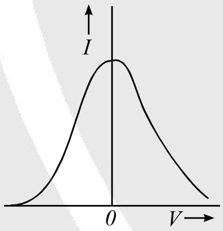
A-1. In a photoelectric experiment, if stopping potential is applied, then photocurrent becomes zero. This means that :
(A) the emission of photoelectrons is stopped
(B) the photoelectrons are emitted but are reabsorbed by the emitter metal
(C) the photoelectrons are accumulated near the collector plate
(D) the photoelectrons are dispersed from the sides of the apparatus.

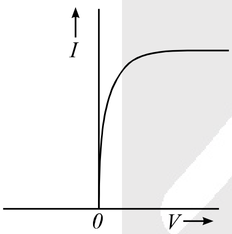
A-2. If the frequency of light in a photoelectric experiment is doubled then maximum kinetic energy of photoelectron
(A) be doubled (B) be halved
(C) become more than double (D) become less than double

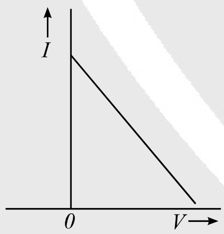
A-3. Two separate monochromatic light beams A and B of the same intensity (energy per unit area per unit time) are falling normally on a unit area of a metallic surface. Their wavelength are λ_A and λ_B respectively. Assuming that all the incident light is used in ejecting the photoelectrons, the ratio of the number of photoelectrons from beam A to that from B is
(A) $\left(\frac{\lambda_A}{\lambda_B}\right)$ (B) $\left(\frac{\lambda_B}{\lambda_A}\right)$ (C) $\left(\frac{\lambda_A}{\lambda_B}\right)^2$ (D) $\left(\frac{\lambda_B}{\lambda_A}\right)^2$

A-4. Which one of the following graphs in figure shows the variation of photoelectric current (I) with voltage (V) between the electrodes in a photoelectric cell ?

(A)


(B)


(C)


(D)


A-5. When a centimetre thick surface is illuminated with light of wavelength λ, the stopping potential is V. When the same surface is illuminated by light of wavelength 2λ, the stopping potential is V/3. The threshold wavelength for the surface is :
(A) $\frac{4\lambda}{3}$ (B) 4λ (C) 6λ (D) $\frac{8\lambda}{3}$

A-6. The anode plate in an experiment on photoelectric effect is kept vertically above the cathode plate. Light source is put on and a saturation photocurrent is recorded. An electric field is switched on which has vertically downward direction
(A) The photocurrent will increase (B) The kinetic energy of the electrons will increase
(C) The stopping potential will decrease (D) The threshold wavelength will increase

A-7. The maximum kinetic energy of photoelectrons emitted from a surface when photons of energy 6 eV fall on it is 4 eV. The stopping potential is :
(A) 2V (B) 4V (C) 6V (D) 10V

A-8. Ultraviolet light of wavelength 300 nm and intensity 1 W/m^2 falls on the surface of a photosensitive material. If one percent of the incident photons produce photoelectrons then the number of photoelectrons emitted per second from an area of 1 cm^2 of the surface is nearly **[Olympiad-2016]**
(A) 1.51×10^{13} (B) 1.51×10^{12} (C) 4.12×10^{13} (D) 2.13×10^{11}

Section (B) : Photon Emission from a source and radiation pressure

B-1. A photon of light enters a block of glass after travelling through vacuum. The energy of the photon on entering the glass block
(A) increases because its associated wavelength decreases
(B) Decreases because the speed of the radiation decreases
(C) Stays the same because the speed of the radiation and the associated wavelength do not change
(D) Stays the same because the frequency of the radiation does not change

Section (C) : de-Broglie waves

C-1. The energy of a photon of frequency ν is $E = h\nu$ and the momentum of a photon of wavelength λ is $p = h/\lambda$. From this statement one may conclude that the wave velocity of light is equal to :
(A) $3 \times 10^8\ ms^{-1}$ (B) $\frac{E}{p}$ (C) E p (D) $\left(\frac{E}{p}\right)^2$

C-2. The de Broglie wavelength of an electron moving with a velocity $1.5 \times 10^8\ ms^{-1}$ is equal to that of a photon. The ratio of the kinetic energy of the electron to that of the energy of photon is (apply non relativistic formula for electron) :
(A) 2 (B) 4 (C) $\frac{1}{2}$ (D) $\frac{1}{4}$

C-3. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non zero velocities. The ratio of the de Broglie wavelengths of the particles, λ_1/λ_2 is :
(A) $\frac{m_1}{m_2}$ (B) $\frac{m_2}{m_1}$ (C) 1 : 1 (D) $\sqrt{\frac{m_2}{m_1}}$

C-4. Let p and E denote the linear momentum and the energy of a photon. For another photon of smaller wavelength (in same medium)
(A) both p and E increase (B) p increases and E decreases
(C) p decreases and E increases (D) both p and E decreases

C-5. The de Broglie wavelength of a neutron correspoding to root mean square speed at 927ºC is λ. What will be the de Broglie wavelength of the neutron correspoding to root mean square speed at 27ºC?
(A) $\frac{\lambda}{2}$ (B) λ (C) 2λ (D) 4λ

C-6. The wavelength λ of de Broglie waves associated with an electron (mass m, charge e) accelerated through a potential difference of V is given by (h is Planck's constant) :
(A) $\lambda = h/mV$ (B) $\lambda = h/2\ meV$ (C) $\lambda = h/\sqrt{meV}$ (D) $\lambda = h/\sqrt{2meV}$

Section (D) : Bohr's atomic model of H-atom & H-Like species (Properties)

D-1. If a_0 is the Bohr radius, the radius of the n = 2 electronic orbit in triply ionized beryllium is -
(A) $4a_0$ (B) a_0 (C) $a_0/4$ (D) $a_0/16$

D-2. Consider 2 hydrogen like ions A and B. Ionization energy of A is greater than that of B. Let r, u, E and L represent the radius of the orbit, speed of the electron, energy of the atom and orbital angular momentum of the electron respectively. In ground state:
(A) $r_A > r_B$ (B) $u_A > u_B$ (C) $E_A > E_B$ (D) $L_A > L_B$

D-3. Which energy state of doubly ionized lithium (Li^{++}) has the same energy as that of the ground state of hydrogen ? Given Z for lithium = 3 :
(A) n = 1 (B) n = 2 (C) n = 3 (D) n = 4

D-4. In Bohr's model of hydrogen atom, the centripetal force is provided by the Coulomb attraction between the proton and the electron. If a_0 is the radius of the ground state orbit, m is the mass and e the charge of an electron and ε_0 is the vacuum permittivity, the speed of the electron is :

(A) zero (B) $\frac{e}{\sqrt{\varepsilon_0 a_0 m}}$ (C) $\frac{e}{\sqrt{4\pi\varepsilon_0 a_0 m}}$ (D) $\frac{\sqrt{4\pi\varepsilon_0 a_0 m}}{e}$

D-5. If an orbital electron of the hydrogen atom jumps from the ground state to a higher energy state, its orbital speed reduces to half its initial value. If the radius of the electron orbit in the ground state is r, then the radius of the new orbit would be :

(A) 2r (B) 4r (C) 8r (D) 16r

D-6. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is :

(A) – 1 (B) + 1 (C) $\frac{1}{n}$ (D) $\frac{1}{n^2}$

D-7. The innermost orbit of the hydrogen atom has a diameter of 1.06 Å. What is the diameter of the tenth orbit ?

(A) 5.3 Å (B) 10.6 Å (C) 53 Å (D) 106 Å

D-8. The orbital speed of the electron in the ground state of hydrogen is v. What will be its orbital speed when it is excited to the energy state – 3.4 eV ?

(A) 2 v (B) $\frac{v}{2}$ (C) $\frac{v}{4}$ (D) $\frac{v}{8}$

D-9. The total energy of the electron in the first excited state of hydrogen is – 3.4 eV. What is the kinetic energy of the electron in this state ?

(A) + 1.7 eV (B) + 3.4 eV (C) + 6.8 eV (D) – 13.4 eV

D-10. In above Q., the potential energy of the electron is :

(A) – 1.7 eV (B) – 3.4 eV (C) – 6.8 eV (D) – 13.4 eV

D-11. Imagine an atom made of a proton and a hypothetical particle of double the mass as that of an electron but the same charge. Apply Bohr theory to consider transitions of the hypothetical particle to the ground state. Then, the longest wavelength (in terms of Rydberge constant for hydrogen atom) is **[Olympiad 2015 (stage-1)]**

(A) $\frac{1}{2R}$ (B) $\frac{5}{3R}$ (C) $\frac{1}{3R}$ (D) $\frac{2}{3R}$

D-12. The force of attraction between the positively charged nucleus and the electron in a hydrogen atom is given by $f = k\frac{e^2}{r^2}$. Assume that the nucleus is fixed. The electron, initially moving in an orbit of radius R_1 jumps into an orbit of smaller radius R_2. The decrease in the total energy of the atom is. **[Olympiad 2016 (stage-1)]**

(A) $\frac{ke^2}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (B) $\frac{ke^2}{2}\left(\frac{R_1}{R_2^2}-\frac{R_2}{R_1^2}\right)$ (C) $\frac{ke^2}{2}\left(\frac{1}{R_2}-\frac{1}{R_1}\right)$ (D) $\frac{ke^2}{2}\left(\frac{R_2}{R_1^2}-\frac{R}{R_2^2}\right)$

D-13. It is observed that some of the spectral lines in hydrogen spectrum have wavelengths almost equal to those of the spectral lines in He^+ ion, Out of the following the transitions in He^+ that will make this possible is **[Olympiad 2016 (stage-1)]**

(A) n = 3 to n = 1 (B) n = 6 to n = 4 (C) n = 5 to n = 3 (D) n = 3 to n = 2

Section(E) : Electronic transition in the H/H-like atom/Species of effect of motion of Nucleus

E-1. Three photons coming from emission spectra of hydrogen sample are picked up. Their energies are 12.1eV, 10.2eV and 1.9eV. These photons must come from

(A) a single atom (B) two atoms
(C) three atom (D) either two atoms or three atoms

E-2. In a hypothetical atom, if transition from n = 4 to n = 3 produces visible light then the possible transition to obtain infrared radiation is :

(A) n = 5 to n = 3 (B) n = 4 to n = 2 (C) n = 3 to n = 1 (D) none of these

E-3. The ionization energy of hydrogen atom is 13.6 eV. Hydrogen atoms in the ground state are excited by electromagnetic radiation of energy 12.1 eV. How many spectral lines will be emitted by the hydrogen atoms?
(A) one (B) two (C) three (D) four

E-4. Energy levels A, B and C of a certain atom correspond to increasing values of energy, i.e. $E_A < E_B < E_C$. If λ_1, λ_2 and λ_3 are the wavelengths of radiations corresponding to transitions C to B, B to A and C to A respectively, which of the following relations is correct ?
(A) $\lambda_3 = \lambda_1 + \lambda_2$ (B) $\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 + \lambda_2}$ (C) $\lambda_1 + \lambda_2 + \lambda_3 = 0$ (D) $\lambda_3^2 = \lambda_1^2 + \lambda_2^2$

E-5. The wavelength of the first line in balmer series in the hydrogen spectrum is λ. What is the wavelength of the second line :
(A) $\frac{20\lambda}{27}$ (B) $\frac{3\lambda}{16}$ (C) $\frac{5\lambda}{36}$ (D) $\frac{3\lambda}{4}$

E-6. The frequency of the first line in Lyman series in the hydrogen spectrum is ν. What is the frequency of the corresponding line in the spectrum of doubly ionized Lithium ?
(A) ν (B) 3ν (C) 9ν (D) 27ν

E-7. A sodium atom emits a photon of wavelength 590 nm and recoils with velocity v equal to **[Olympiad 2015 (stage-1)]**
(A) 0.029 m/s (B) 0.048 m/s (C) 0.0023 m/s (D) data inadequate

Section (F) : Atomic Collisions

F-1. An electron with kinetic energy 10 eV is incident on a hydrogen atom in its ground state. The collision
(A) must be elastic (B) may be partially elastic
(C) must be completely inelastic (D) may be completely inelastic

Section (G) : X-rays

G-1. Consider a photon of continuous X-ray coming from a Coolidge tube. Energy of photon comes from
(A) the kinetic energy of the striking electron
(B) the kinetic energy of the free electrons of the target
(C) the kinetic energy of the ions of the target
(D) an atomic transition in the target

G-2. If the voltage across the filament is increased, the cutoff wavelength
(A) will increase (B) will decrease
(C) will remain unchanged (D) will change

G-3. The characteristic X-ray spectrum is emitted due to transition of
(A) valence electrons of the atom (B) inner electrons of the atom
(C) nucleus of the atom (D) both, the inner electrons and the nucleus of the atom

G-4. When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When X-rays are incident on the same cell, then :
(A) V_0 and K_{max} both increase (B) V_0 and K_{max} both decrease
(C) V_0 increases but K_{max} remains the same (D) K_{max} increases but V_0 remains the same

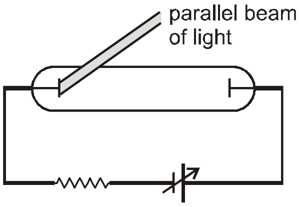
Section (H) : for JEE Main

H-1. In Davisson-Germer experiment, the filament emits **[RPET -1990]**
(A) Photons (B) Protons (C) X-rays (D) Electrons

H-2. In the Davisson and Germer experiment, the velocity of electrons emitted from the electron gun can be increased by : **[AIPMT-2011]**
(A) increasing the potential difference between the anode and filament
(B) increasing the filament current
(C) decreasing the filament current
(D) decreasing the potential difference between the anode and filament

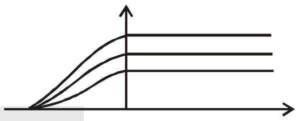
PART - III : MATCH THE COLUMN

1. In the shown experimental setup to study photoelectric effect, two conducting electrodes are enclosed in an evacuated glass-tube as shown. A parallel beam of monochromatic radiation, falls on photosensitive electrode. Assume that for each photons incident, a photoelectron is ejected if its energy is greater than work function of electrode. Match the statements in column I with corresponding graphs in column II.

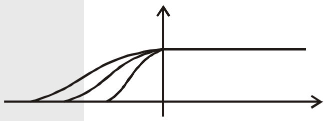


Column-I / **Column-II**

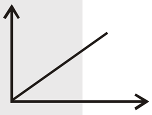
(A) Saturation photocurrent (for same metal) versus intensity of radiation is represented by

(p) 

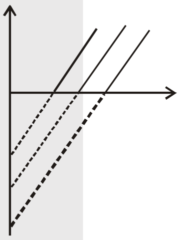
(B) Maximum kinetic energy of ejected photoelectrons versus frequency for electrodes of different work function is represented by

(q) 

(C) Photo current versus applied voltage for different intensity of radiation (for same metal) is represented by

(r) 

(D) Photo current versus applied voltage at constant intensity of radiation for electrodes of different work function.

(s) 

2. The energy, the magnitude of linear momentum, magnitude of angular momentum and orbital radius of an electron in a hydrogen atom corresponding to the quantum number n are E, p, L and r respectively. Then according to Bohr's theory of hydrogen atom, match the expressions in column-I with statement in column-II.

Column-I	**Column-II**
(A) Epr	(p) is independent of n.
(B) $\frac{p}{E}$	(q) is directly proportional to n
(C) Er	(r) is inversely proportional to n.
(D) pr	(s) is directly proportional to L.

3. In each situation of column I a physical quantity related to orbiting electron in a hydrogen like atom is given.The terms 'Z' and 'n' given in column-II have usual meaning in Bohr's theory. Match the quantities in column-I with the terms which depend on quantity given in column-II.

Column I	**Column II**
(A) Frequency of orbiting electron	(p) is directly proportional to Z^2
(B) Angular momentum of orbiting electron	(q) is directly proportional to n.
(C) Magnetic moment of orbiting electron	(r) is inversely proportional to n^3
(D) The average current due to orbiting of electron	(s) is independent of Z

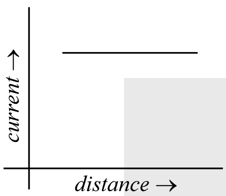
Exercise-2

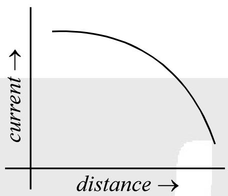
✍ **Marked Questions can be used as Revision Questions.**

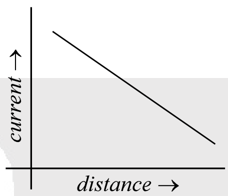
PART - I : ONLY ONE OPTION CORRECT TYPE

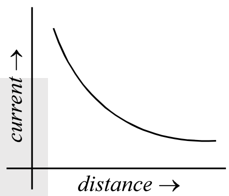
1. The photoelectrons emitted from a metal surface :
(A) Are all at rest
(B) Have the same kinetic energy
(C) Have the same momentum
(D) Have speeds varying from zero up to a certain maximum value

2. A point source causes photoelectric effect from a small metal plate. Which of the following curves may represent the saturation photocurrent as a function of the distance between the source and the metal?

(A) 

(B) 

(C) 

(D) 

3.✍ In a photoelectric experiment, with light of wavelength λ, the fastest electron has speed v. If the exciting wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will become

(A) $v\sqrt{\frac{3}{4}}$ (B) $v\sqrt{\frac{4}{3}}$ (C) less than $v\sqrt{\frac{3}{4}}$ (D) greater than $v\sqrt{\frac{4}{3}}$

4. In a photoelectric experiment, the frequency and intensity of a light source are both doubled. Then consider the following statements.
(i) The saturation photocurrent remains almost the same.
(ii) The maximum kinetic energy of the photoelectrons is doubled.
(A) Both (i) and (ii) are true (B) (i) is true but (ii) is false
(C) (i) is false but (ii) is true (D) both (i) and (ii) are false

5. When a monochromatic point source of light is at a distance of 0.2 m from a photoelectric cell, the cut-off voltage and the saturation current are respectively 0.6 V and 18 mA. If the same source is placed 0.6 m away from the cell, then :
(A) the stopping potential will be 0.2 V (B) the stopping potential will be 1.8 V
(C) the saturation current will be 6.0 mA (D) the saturation current will be 2.0 mA

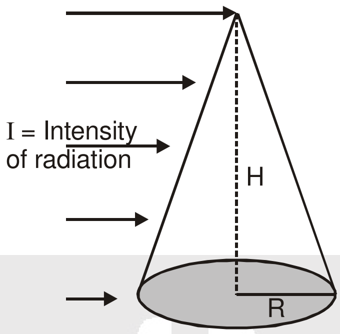
6. An image of the sun is formed by a lens of focal length 30 cm on the metal surface of a photo-electric cell and it produces a current I. The lens forming the image is then replaced by another lens of the same diameter but of focal length 15 cm. The photoelectric current in this case will be : (In both cases the plate is kept at focal plane and normal to the axis lens). (Assume saturation current only).
(A) I/2 (B) 2 I (C) I (D) 4 I

7.✍ The work function of a certain metal is $\frac{hC}{\lambda_0}$. When a monochromatic light of wavelength $\lambda < \lambda_0$ is incident such that the plate gains a total power P. If the efficiency of photoelectric emission is η% and all the emitted photoelectrons are captured by a hollow conducting sphere of radius R already charged to potential V, then neglecting any interaction between plate and the sphere, expression of potential of the sphere at time t is (e= 1.6×10^{-19} C) :

(A) $V + \frac{100\eta\lambda Pet}{4\pi\varepsilon_0 RhC}$ (B) $V - \frac{\eta\lambda Pet}{400\pi\varepsilon_0 RhC}$ (C) V (D) $\frac{\lambda Pe\,t}{4\pi\varepsilon_0 RhC}$

8. Radiation pressure on any surface (for a given intensity):
(A) is dependent on wavelength of the light used
(B) is dependent on nature of surface
(C) is dependent on frequency and nature of surface
(D) depends on the nature of source from which light is coming and on nature of surface on which it is falling.

9. The radiation force experienced by body exposed to radiation of intensity I, assuming surface of body to be perfectly absorbing is :



(A) $\frac{\pi R^2 I}{c}$ (B) $\frac{\pi RHI}{c}$ (C) $\frac{IRH}{2c}$ (D) $\frac{IRH}{c}$

10. Which one of the following statements is NOT true for de Broglie waves ?
(A) All atomic particles in motion have matter waves of some de-Broglie wavelengths associated with them
(B) The higher the momentum, the longer is the wavelength
(C) The faster the particle, the shorter is the wavelength
(D) For the same velocity, a heavier particle has a shorter wavelength

11. An α-particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of :
(A) 1 Å (B) 10^{-10} cm (C) 10^{-12} cm (D) 10^{-15} cm

12. An energy of 24.6 eV is required to remove one of the electrons from a neutral helium atom. The energy (In eV) required to remove both the electrons from a neutral helium atom is : **[JEE 1995, 1]**
(A) 38.2 (B) 49.2 (C) 51.8 (D) 79.0

13. An atom consists of three energy levels given by a ground state with energy $E_0 = 0$, the first excited state with energy $E_1 = K$ and the second excited state with energy $E_2 = 2K$ where $K > 0$. The atom is initially in the ground state. Light from a laser which emits photons with energy 1.5K is shined on the atom. Which of the following is/are correct ?
(A) The photons are absorbed, putting one atom in a state E_1 and one atom in a state E_2.
(B) A photon will always be absorbed, but half the time the atom will go into the state with energy K and the other half into the state with energy 2K. In this way, energy will be conserved on the average.
(C) The atom absorbs a photon, goes into the first excited state with energy K and emits a photon with energy 0.5 K to conserve energy.
(D) The atom does not absorb any photon and stays in the ground state.

14. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n – 1). If n >> 1, the frequency of radiation emitted is proportional to : **[Olympiad 2011]**
(A) $\frac{1}{n^2}$ (B) $\frac{1}{n^3}$ (C) n^2 (D) $\frac{1}{n^4}$

15. The relation between λ_1: wavelength of series limit of Lyman series, λ_2: the wavelength of the series limit of Balmer series & λ_3: the wavelength of first line of Lyman series is :

(A) $\lambda_1 = \lambda_2 + \lambda_3$ (B) $\lambda_3 = \lambda_1 + \lambda_2$ (C) $\lambda_2 = \lambda_3 - \lambda_1$ (D) $\frac{1}{\lambda_1} - \frac{1}{\lambda_2} = \frac{1}{\lambda_3}$

16. Ultraviolet light of wavelengths λ_1 and λ_2 when allowed to fall on hydrogen atoms in their ground state is found to liberate electrons with kinetic energy 1.8 eV and 4.0 eV respectively. Find the value of $\frac{\lambda_1}{\lambda_2}$.

(A) $\frac{7}{8}$ (B) $\frac{8}{7}$ (C) $\frac{9}{20}$ (D) $\frac{20}{9}$

17. In a discharge tube when 200 volt potential difference is applied 6.25×10^{18} electrons move from cathode to anode and 3.125×10^{18} singly charged positive ions move from anode to cathode in one second. Then the power of tube is:

(A) 100 watt (B) 200 watt (C) 300 watt (D) 400 watt

18. An X-ray photon of wavelength λ and frequency ν collides with an intially stationary electron (but free to move) and bounces off. If λ' and ν' are respectively the wavelength and frequency of the scattered photon, then :

(A) $\lambda' = \lambda$; $\nu' = \nu$ (B) $\lambda' < \lambda$; $\nu' > \nu$ (C) $\lambda' > \lambda$; $\nu' > \nu$ (D) $\lambda' > \lambda$; $\nu' < \nu$

19. The wavelengths of K_α x-rays of two metals 'A' and 'B' are $\frac{4}{1875R}$ and $\frac{1}{675R}$ respectively, where 'R' is rydberg constant. The number of elements lying between 'A' and 'B' according to their atomic numbers is

(A) 3 (B) 6 (C) 5 (D) 4

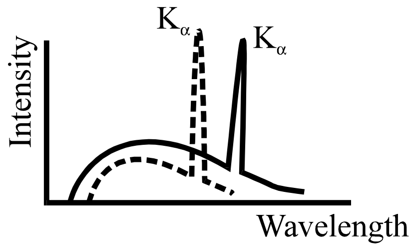
20. An X-ray tube is operated at 66 kV. Then, in the continuous spectrum of the emitted X-rays :

(A) wavelengths 0.01 nm and 0.02 nm will both be present
(B) wavelengths 0.01 nm and 0.02 nm will both be absent
(C) wavelengths 0.01 nm will be present but wavelength 0.02 nm will be absent
(D) wavelength 0.01 nm will be absent but wavelength 0.02 nm will be present

21. For the structural analysis of crystals, X-rays are used because :

(A) X-rays have wavelength of the order of the inter-atomic spacing
(B) X-rays are highly penetrating radiations
(C) Wavelength of X-rays is of the order of nuclear size
(D) X-rays are coherent radiations

22. Given curve shows the intensity-wavelength relations of X-rays coming from two different Coolidge tubes A and B. The dark curve represents the relation for the tube A in which the potential difference between the target and the filament is V_A and the atomic number of the target material is Z_A. Similarly dotted curve is for tube B. Respective quantities are V_B and Z_B for the tube B. Then,



(A) $V_A > V_B$, $Z_A > Z_B$ (B) $V_A > V_B$, $Z_A < Z_B$ (C) $V_A < V_B$, $Z_A > Z_B$ (D) $V_A < V_B$, $Z_A < Z_B$

23.✍ If λ_{min} is minimum wavelength produced in X-ray tube and $\lambda_{k\alpha}$ is the wavelength of k_α line. As the operating tube voltage is increased.

(A) $(\lambda_k - \lambda_{min})$ increases
(B) $(\lambda_k - \lambda_{min})$ decreases
(C) $\lambda_{k\alpha}$ increases
(D) $\lambda_{k\alpha}$ decreases

24. According to Moseley's law the ratio of the slopes of graph between $\sqrt{\nu}$ and Z for K_β and K_α is :

(A) $\sqrt{\frac{32}{27}}$ (B) $\sqrt{\frac{27}{32}}$ (C) $\sqrt{\frac{33}{22}}$ (D) $\sqrt{\frac{22}{33}}$

25. If the frequency of K_α X-ray emitted from element with atomic number 31 is f, then the frequency of K_α X-ray emitted from the element with atomic number 51 would be (assume that screening constant for K_α is 1) :

(A) $\frac{5}{3}$ f (B) $\frac{51}{31}$ f (C) $\frac{9}{25}$ f (D) $\frac{25}{9}$ f

26. An α particle with a kinetic energy of 2.1 eV makes a head on collision with a hydrogen in ground state atom moving towards it with a kinetic energy of 8.4 eV. The collision.

(A) must be perfectly elastic
(B) may be perfectly inelastic
(C) may be inelastic
(D) must be perfectly inelastic

27._ The photoelectric threshold wavelength of tungsten is 230 nm. The energy of electrons ejected from its surface by ultraviolet light of wavelength 180 nm is **[Olympiad (State-1) 2017]**

(A) 0.15 eV (B) 1.5 eV (C) 15 eV (D) 1.5 keV

28._ In an X ray tube the electrons are expected to strike the target with a velocity that is 10% of the velocity of light. The applied voltage should be **[Olympiad (State-1) 2017]**

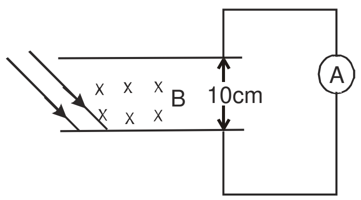
(A) 517.6 V (B) 1052 V (C) 2.559 kV (D) 5.680 kV

29._ In and atom an electron excites to the fourth orbit. When it jumps back to the energy levels a spectrum is formed. Total number of spectral lines in this spectrum would be **[Olympiad (State-1) 2017]**

(A) 3 (B) 4 (C) 5 (D) 6

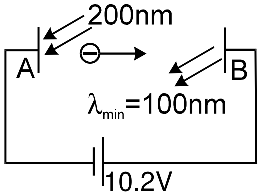
PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1.✍ In an experiment on photoelectric effect, the separation between emitter and the collector plates is 10 cm. Plates are connected through an ammeter without any cell. A Magnetic field B exists parallel to the plates. The work function of the emitter is 2.39eV and the light of wavelengths between 400 nm and 600 nm is incident on it. If minimum value of B for which the current registered by the ammeter is zero is n × 10^{-6} T. Then find out value of n (Neglect any effect of space charge). (Assume emission of photo electron to be randomly every possible direction)



2. A light beam of wavelength 400 nm is incident on a metal plate of work function 2.2 eV. A particular electron absorbs a photon and makes some collisions before coming out of the metal. Assuming that 10% of the instantaneous energy is lost to the metal in each collision. Find the minimum number of collisions the electron can suffer before it becomes unable to come out of metal. (Use hc = 12400 eV Å)

3. In the figure shown electromagnetic radiations of wavelength 200nm are incident on the metallic plate A. The photo electrons are accelerated by a potential difference 10.2 eV. These electrons strike another metal plate B from which electromagnetic radiations are emitted. The minimum wavelength of the emitted photons is 100nm. Find the work function of the metal 'A' (in eV). Use hc = 12400 eVÅ,



4. Consider Bohr's theory for hydrogen atom. The magnitude of angular momentum, orbit radius and frequency of the electron in n^{th} energy state in a hydrogen atom are, r & f respectively. Find out the value of ' x ', if $(fr\ell)$ is directly proportional to n^x.

5. The first excitation potential of He^+ ion is n, and the ionization potential of Li^{++} ion is m then find out value of $\frac{m}{n}$

6. A neutron moving with a speed v strikes a hydrogen atom in ground state moving towards it with the same speed. If the minimum speed of the neutron for which elastic collision dose not take place is 3.13×10^n m/s, then find out the value of n. (The mass of neutron = mass of hydrogen = 1.67×10^{-27}kg)

7. Electrons in hydrogen-like atoms (Z = 3) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 V. The work function of the metal = x ev. Then find x (Rydberg constant = 1.094×10^7 m^{-1}) **[JEE 1990; 7m]**

8. An electron of energy 20 eV collides with a hydrogen atom in the ground state. As a result of the colision, the atom is excited to a higher energy state and the electron is scattered with reduced velocity. The atom subsequently returns to its ground state with emission of radiation of wavelength 1.216×10^{-7} m. If the velocity of the scattered electron is 1.86×10^n m/s then find n.

9. Calculate the value of X if magnetic field strength at the centre of a hydrogen atom caused by an electron moving along the first Bohr orbit is $\frac{x}{2}$T :

10. Radiation from a hydrogen discharge tube (energy of photons ≤ 13.6 eV) goes through a filter which transmits only waves of wavelength greater than 4400 Å and is incident on a metal of work function 2.0 eV. If stopping potential is $n \times 10^{-2}$ volts. Find the value of 'n'

11. The ionization energy of a hydrogen like Bohr atom is 4 Rydberg. If the wavelength of radiation emitted when the electron jumps from the first excited state to the ground state is N-m and if the radius of the first orbit of this atom is r-m then the value of $\frac{N}{r}$ = P × 10^2 then, value of P. (Bohr radius of hydrogen = 5×10^{-11} m; 1 Rydberg = 2.2×10^{-18} J)

PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. Photoelectric effect supports particle nature of light because
(A) there is a minimum frequency below which no photoelectrons are emitted
(B) the maximum kinetic energy of photoelectrons depends only on the frequency of light and is independent of intensity.
(C) even when the metal surface is illuminated with very small intensity the photoelectrons (if $\nu \geq \nu_{th}$) leave the surface immediately
(D) electric charge of the photoelectrons is quantized

2. Select the correct alternative(s):
When photons of energy 4.25 eV strike the surface of a metal A, the ejected photo electrons have maximum kinetic energy T_A eV and de Broglie wave length λ_A. The maximum kinetic energy of photo electrons liberated from another metal B by photons of energy 4.70 eV is $T_B = (T_A - 1.50)$ eV. If the de-Brogleie wave length of these photo electrons is $\lambda_B = 2\lambda_A$, then: **[JEE 1994, 2] [Olympiad 2015 (stage-1)]**
(A) the work function of A is 2.25 eV
(B) the work function of B is 4.20 eV
(C) $T_A = 2.00$ eV
(D) $T_B = 2.75$ eV

3. Consider a hypothetical hydrogen like atom. The wavelength in Å for the spectral lines for transition from n = p to n = 1 are given by -

$$\lambda = \frac{1500\ \ p^2}{p^2 - 1}$$

where p = 2, 3, 4, ... (given hc = 12400 eV/Å)
(A) The wavelength of the least energetic and the most energetic photons in this series is 2000 Å, 1500 Å.
(B) Difference between energies of fourth and third orbit is 0.40 eV.
(C) Energy of second orbit is 6.2 eV
(D) The ionisation potential of this element is 8.27 V.

4. A sample of hydrogen atom gas contains 100 atoms. All the atoms are excited to the same n^{th} excited state. The total energy released by all the atoms is $\frac{4800}{49}$ Rch (where Rch = 13.6 eV), as they come to the ground state through various types of transitions. Find
(A) maximum energy of the emitted photon will be less than $\frac{48}{49}$ Rch.
(B) maximum energy of the emitted photon may be grater than $\frac{48}{49}$ Rch
(C) the value of n = 6
(D) total number of photons that can be emitted by this sample may be less than 600.

5. One hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975 Å. You may assume the ionization energy for hydrogen atom is 13.6eV **[JEE 1982; 5M]**
(A) Total number of lines in emission spectrum would be 6.
(B) Energy difference between 3rd and 4th orbit is 0.66 eV.
(C) logest wavelength in emission spectrum would be 1.875 μm.
(D) smallest wavelenth in emission spectrum would be 975 Å.

6. Consider an electron orbiting the nucleus with speed v in an orbit of radius r. The ratio of the magnetic moment to the orbital angular momentum of the electron is independent of : **[Olympiad 2011]**
(A) radius r
(B) speed v
(C) charge of electron e
(D) mass of electron m_e

7. Consider a metal used to produced some charateristic X-rays. Energy of X-rays are given by E and wavelength as represented by λ. Then which of the following is true :
(A) $E(K_\alpha) > E(K_\beta) > E(K_\gamma)$
(B) $E(M_\alpha) > E(L_\alpha) > E(K_\alpha)$
(C) $\lambda(K_\alpha) > \lambda(K_\beta) > \lambda(K_\gamma)$
(D) $\lambda(M_\alpha) > \lambda(L_\alpha) > \lambda(K_\alpha)$

8. The potential difference applied to an X-ray tube is increased. As a result, in the emitted radiation,
(A) the intensity increases
(B) the minimum wavelength increases
(C) the intensity remains unchanged
(D) the minimum wavelength decreases

9. X-ray falling on a material
(A) exerts a force on it
(B) transfers energy to it
(C) transfers momentum to it
(D) transfers impulse to it

10. In an x–ray tube the voltage applied is 20 kV. The energy required to remove an electron from K shell is 19.9 KeV. In the x–rays emitted by the tube (hc = 12420 evÅ)
(A) minimum wavelength will be 62.1 pm
(B) energy of the characterstic x–rays will be equal to or less than 19.9 KeV
(C) L_α x–ray may be emitted
(D) L_α x–ray will have energy 19.9 KeV

11.✍ In an X-ray tube the accelerating voltage is 20 kV. Two targets A and B are used one by one. For 'A' the wavelength of the K_α line is 62 pm. For 'B' the wavelength of the L_α line is 124 pm. The energy of the 'B' ion with vacancy in 'M' shell is 5.5 keV higher than the atom of B. [Take hc = 12400 eVÅ]
(A) Value of λ_{min} is 0.62 Å.
(B) A will emitte K_α– photon.
(C) B will emitte L – photons.
(D) minimum wavelength (in Å) of the characteristic X-ray that will be emitted by 'B' is 0.8 Å.

12. When Z is doubled in a hydrogen like atom, which of the following statements are consistent with Bohr's theory?
(A) Energy of a state is double (B) Radius of an orbit is doubled.
(C) Velocity of electrons in an orbit is doubled. (D) Radius of an orbit is halved.

13. Let A_n be the area enclosed by the n^{th} orbit in a hydrogen atom. The graph of ℓn (A_n / A_1) against In(n)
(A) will pass through the origin
(B) will have certain points lying on a straight line with slope 4
(C) will be a monotonically increasing nonlinear curve
(D) will be a circle

PART - IV : COMPREHENSION

Comprehension-1

A physicist wishes to eject electrons by shining light on a metal surface. The light source emits light of wavelength of 450 nm. The table lists the only available metals and their work functions.

Metal		W_0(eV)
Barium	–	2.5
Lithium	–	2.3
Tantalum	–	4.2
Tungsten	–	4.5

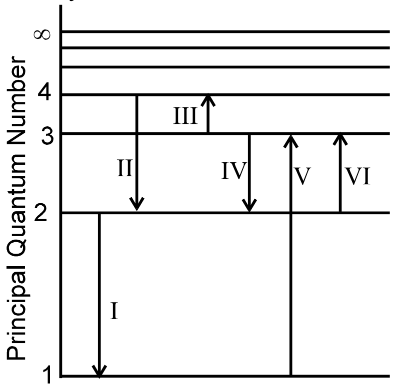
1. Which metal(s) can be used to produce electrons by the photoelectric effect from given source of light ?
(A) Barium only (B) Barium or lithium
(C) Lithium, tantalum or tungsten (D) Tungsten or tantalum

2. Which option correctly identifies the metal that will produce the most energetic electrons and their energies ?
(A) Lithium, 0.45 eV (B) Tungusten, 1.75 eV (C) Lithium, 2.30 eV (D) Tungusten, 2.75 eV

3. Suppose photoelectric experiment is done separately with these metals with light of wavelength 450 nm. The maximum magnitude of stopping potential amongst all the metals is
(A) 2.75 volt (B) 4.5 volt (C) 0.45 volt (D) 0.25 volt

Comprehension-2

The figure shows an energy level diagram for the hydrogen atom. Several transitions are marked as I, II, III, _______. The diagram is only indicative and not to scale.



4. In which transition is a Balmer series photon absorbed ?
(A) II (B) III (C) IV (D) VI

5. The wavelength of the radiation involved in transition II is
(A) 291 nm (B) 364 nm (C) 487 nm (D) 652 nm

6. Which transition will occur when a hydrogen atom is irradiated with radiation of wavelength 103nm?
(A) I (B) II (C) IV (D) V

Comprehension-3

Assume that the de Broglie wave associated with an electron can form a standing wave between the atoms arranged in a one dimensional array with nodes at each of the atomic sites. It is found that one such standing wave is formed if the distance ' d ' between the atoms of the array is 2 Å. A similar standing wave is again formed if ' d ' is increased to 2.5 Å but not for any intermediate value of d.

7. Find the energy of the electrons in eV
(A) 302 eV (B) 151 eV (C) 75.5 eV (D) 75.5×10^6 eV

8. The least value of d for which the standing wave of the type described above can form.
(A) 0.4 Aº (B) 0.5 Aº (C) 2 Aº (D) 1 Aº

Comprehension-4 ✍

A uniform magnetic field B exists in a region. An electron is given velocity perpendicular to the magnetic field.Assuming Bohr's quantization rule for angular momentum.

9.✍ Calculate the radius of the nth orbit
(A) $\sqrt{\frac{nh}{2\pi eB}}$ (B) $\sqrt{\frac{nheB}{2\pi}}$ (C) $\sqrt{\frac{nhe}{2\pi B}}$ (D) $\sqrt{\frac{nhB}{2\pi e}}$

10.✍ Calculate the minimum possible speed of the electron.
(A) $\sqrt{\frac{heB}{nm^2}}$ (B) $\sqrt{\frac{he}{2\pi Bm^2}}$ (C) $\sqrt{\frac{h \cdot eB}{2\pi m^2}}$ (D) $\sqrt{\frac{hem^2}{2\pi B}}$

Comprehension-5.

A neutron beam, in which each neutron has same kinetic energy, is passed through a sample of hydrogen like gas (but not hydrogen) in ground state. Due to collision of neutrons with the ions of the gas, ions are excited and then they emit photons. Six spectral lines are obtained in which one of the lines is of wavelength (6200/51) nm.

11. Which gas is this ?
(A) H (B) D (C) He^+ (D) Li^{+2}

12. What is the minimum possible value of kinetic energy of the neutrons for this to be possible. The mass of neutron and proton can be assumed to be nearly same. Use hc = 12400 eVÅ.
(A) 51 eV (B) 54.4 eV (C) 63.75 eV (D) 69 eV.

Exercise-3

✍ **Marked Questions can be used as Revision Questions.**
* **Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **[JEE 2007, 3/81]**
(A) 802 nm (B) 823 nm (C) 1882 nm (D) 1648 nm

2. **STATEMENT-1 :** If the accelerating potential in an X-ray tube is increased, the wavelengths of the characteristic X-rays do not change. **[JEE 2007, 3/81]**
because
STATEMENT-2 : When an electron beam strikes the target in an X-ray tube, part of the kinetic energy is converted into X-ray energy.
(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True.

3. Electrons with de-Broglie wavelength λ fall on the target in an X-ray tube. The cut-off wavelength of the emitted X-rays is **[JEE 2007, 3/81]**
(A) $\lambda_0 = \frac{2mc\lambda^2}{h}$ (B) $\lambda_0 = \frac{2h}{mc}$ (C) $\lambda_0 = \frac{2m^2c^2\lambda^3}{h^2}$ (D) $\lambda_0 = \lambda$

4. Which one of the following statements is WRONG in the context of X-rays generated from a X-ray tube? **[JEE 2008, 4/163]**
(A) Wavelength of characteristic X-rays decreases when the atomic number of the target increases
(B) Cut-off wavelength of the continuous X-rays depends on the atomic number of the target
(C) Intensity of the characteristic X-rays depends on the electrical power given to the X-ray tube
(D) Cut-off wavelength of the continuous X-rays depends on the energy of the electrons in the X-ray tube

Paragraph for Question Nos. 5 to 7

In a mixture of H – He^+ gas (He^+ is singly ionized He atom), H atoms and He^+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He^+ ions (by collisions). Assume that the Bohr model of atom is exactly valid. **[IIT-JEE 2008, 4×3/163]**

5.✍ The quantum number n of the state finally populated in He^+ ions is :
(A) 2 (B) 3 (C) 4 (D) 5

6.✍ The wavelength of light emitted in the visible region by He^+ ions after collisions with H atoms is
(A) 6.5×10^{-7} m (B) 5.6×10^{-7} m (C) 4.8×10^{-7} m (D) 4.0×10^{-7} m

7.✍ The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He^+ ion is :
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2

Paragraph for Question Nos. 8 to 10

When a particle is restricted to move along x-axis between x = 0 and x = a, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends x = 0 and x = a. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de-Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E = \frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1,2,3,......., (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line x = 0 to x = a. Take $h = 6.6 \times 10^{-34}$ J s and $e = 1.6 \times 10^{-19}$ C. **[IIT-JEE 2009, 3×4/160, –1]**

8. The allowed energy for the particle for a particular value of n is proportional to :

(A) a^{-2} (B) $a^{-3/2}$ (C) a^{-1} (D) a^2

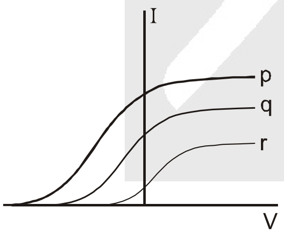
9. If the mass of the particle is $m = 1.0 \times 10^{-30}$ kg and a = 6.6 nm, the energy of the particle in its ground state is closest to :

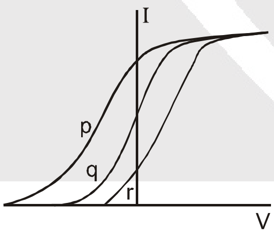
(A) 0.8 meV (B) 8 meV (C) 80 meV (D) 800 meV

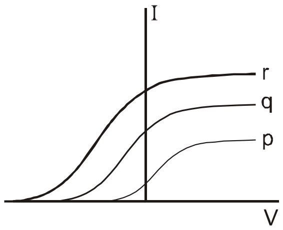
10. The speed of the particle, that can take discrete values, is proportional to :

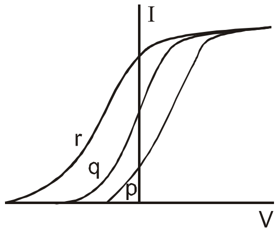
(A) $n^{-3/2}$ (B) n^{-1} (C) $n^{1/2}$ (D) n

11. Photoelectric effect experiments are performed using three different metal plates p, q and r having work functions $\phi_p = 2.0$ eV, $\phi_q = 2.5$ eV and $\phi_r = 3.0$ eV respectively. A light beam containing wavelengths of 550 nm, 450 nm and 350 nm with equal intensities illuminates each of the plates. The correct I-V graph for the experiment is [Take hc = 1240 eV nm] **[JEE 2009, 3/160, –1]**

(A)


(B)


(C)


(D)


12. An α-particle and a proton are accelerated from rest by a potential difference of 100V. After this, their de-Broglie wavelength are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$, to the nearest integer, is: **[JEE 2010, 3/163]**

Paragraph for questions 13 to 15

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **[JEE 2010, 9/163, –1]**

13. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is : **[JEE 2010, 3/163, –1]**

(A) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (B) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$ (C) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (D) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

14. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi}\times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its centre of mass is close to (Take h = $2\pi \times 10^{-34}$ J s) **[JEE 2010, 3/163, –1]**

(A) 2.76×10^{-46} kg m^2 (B) 1.87×10^{-46} kg m^2 (C) 4.67×10^{-47} kg m^2 (D) 1.17×10^{-47} kg m^2

15. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. = $\frac{5}{3}\times 10^{-27}$ kg, is close to : **[JEE 2010, 3/163, –1]**

(A) 2.4×10^{-10} m (B) 1.9×10^{-10} m (C) 1.3×10^{-10} m (D) 4.4×10^{-11} m

16. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 Å. The wavelength of the second spectral line in the Balmer series of singly ionized helium atom is : **[JEE 2010, 3/160, –1]**

(A) 1215 Å (B) 1640 Å (C) 2430 Å (D) 4687 Å

Paragraph for Question 17 to 18

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begin to oscillate about the positive ions with a natural angular frequency 'ω_p', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_p all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[JEE 2011, 3 × 2/160, –1]**

17. Taking the electronic charge as ***'e'*** and the permitlivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_p.

(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$ (C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

18. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons N = 4 x 10^{27} m^{-3}. Take $\varepsilon_0 = 10^{-11}$ and m = 10^{-30}, where these quantities are in proper SI units.

(A) 800 nm (B) 600 nm (C) 300 nm (D) 200 nm

19. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulating thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is $A \times 10^Z$ (where $1 < A < 10$). The value of 'Z' is **[JEE 2011, 4/160]**

20. A pulse of light of duration 100 ns is absorbed completely by a small object initially at rest. Power of the pulse is 30mW and the speed of light is 3×10^8 ms^{-1}. The final momentum of the object is : **[JEE (Advanced) 2013; 2/60, –1]**
(A) 0.3×10^{-17} kg ms^{-1} (B) 1.0×10^{-17} kg ms^{-1} (C) 3.0×10^{-17} kg ms^{-1} (D) 9.0×10^{-17} kg ms^{-1}

21. The work functions of Silver and Sodium are 4.6 and 2.3 eV, respectively. The ratio of the slope of the stopping potential versus frequency plot for Silver to that of Sodium is : **[JEE (Advanced) 2013; 4/60, –1]**

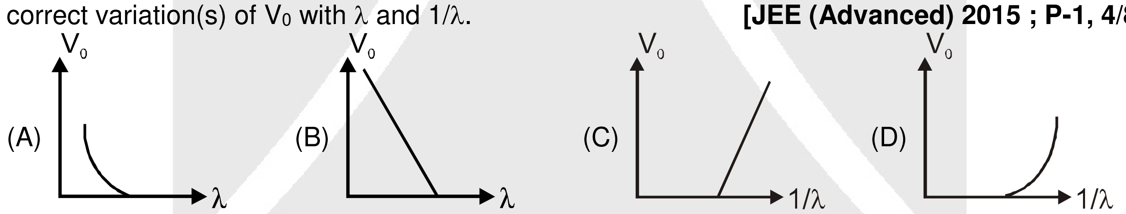
22.* The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s) when the atom de-excites is (are) : **[JEE (Advanced) 2013; 3/60, –1]**
(A) $\frac{9}{32R}$ (B) $\frac{9}{16R}$ (C) $\frac{9}{5R}$ (D) $\frac{4}{3R}$

23. If λ_{Cu} is the wavelength of K_α X-ray line of copper (atomic number 29) and λ_{Mo} is the wavelength of the K_α X-ray line of molybdenum (atomic number 42), then the ratio $\lambda_{Cu}/\lambda_{Mo}$ is close to **[JEE (Advanced) 2014; 3/60, –1]**
(A) 1.99 (B) 2.14 (C) 0.50 (D) 0.48

24. A metal surface is illuminated by light of two different wavelengths 248 nm and 310 nm. The maximum speeds of the photoelectrons corresponding to these wavelengths are u_1 and u_2, respectively. If the ratio $u_1 : u_2 = 2 : 1$ and hc = 1240 eV nm, the work function of the metal is nearly **[JEE (Advanced) 2014; 3/60, –1]**
(A) 3.7 eV (B) 3.2 eV (C) 2.8 eV (D) 2.5 eV

25. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is (hc = 1242 eV nm) **[JEE (Advanced) 2015 ; P-1,4/88]**

26.* For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and 1/λ. **[JEE (Advanced) 2015 ; P-1, 4/88, –2]**



27. An electron in an excited state of Li^{2+} ions has angular momentum $3h/2\pi$. The de-Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **[JEE(Advanced) 2015 ; P-2,4/88]**

28. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_0) are given below : **[JEE (Advanced) 2016; P-1, 3/62, –1]**

λ (μm)	V_0 (Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ ms^{-1} and $e = 1.6 \times 10^{-19}$ C, Planck's constant (in units of J s) found from such an experiment is)
(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

29.* Highly excited states for hydrogen-like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n >> 1. Which of the following statement (s) is (are) true? **[JEE (Advanced) 2016; P-1, 4/62, –2]**

(A) Relative change in the radii of two consecutive orbitals does not depend on Z.

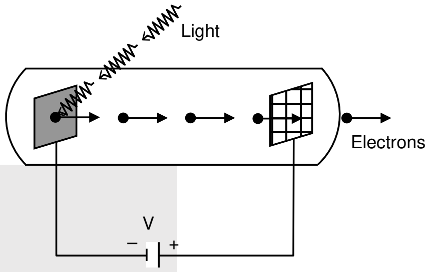
(B) Relative change in the radii of two consecutive orbitals varies as 1/n

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

30. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking hc/e = 1.237×10^{-6} eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is : **[JEE (Advanced) 2016; P-1, 3/62]**

31. Light of wavelength λ_{ph} falls on a cathode plate inside a vacuum tube as shown in the figure. The work function of the cathode surface is ϕ and the anode is a wire mesh of conducting material kept at a distance d from the cathode. A potential difference V is maintained between the electrodes. If the minimum de Broglie wavelength of the electrons passing through the anode is λ_e, which of the following statement(s) is (are) true ? **[JEE (Advanced) 2016; P-2, 3/62, –1]**



(A) λ_e increases at the same rate as λ_{ph} for $\lambda_{ph} < hc/\phi$.

(B) For large potential difference ($V >> \phi/e$), λ_e is approximately halved if V is made four times.

(C) λ_e is approximately halved, if d is doubled

(D) λ_e decreases with increase in ϕ and λ_{ph}.

32. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $V_i/V_f = 6.25$, then the smallest possible n_f is : **[JEE (Advanced) 2017; P-1, 3/61]**

33. A photoelectric material having work-function ϕ_0 is illuminated with light of wavelength λ $\left(\lambda < \frac{hc}{\phi_0}\right)$. The fastest photoelectron has a de Broglie wavelength λ_d. A change in wavelength of the incident light by $\Delta\lambda$ results in a change $\Delta\lambda_d$ in λ_d. Then the ratio $\frac{\Delta\lambda_d}{\Delta\lambda}$ is proportional to :**[JEE (Advanced) 2017; P-2, 3/61, –1]**

(A) $\frac{\lambda_d^3}{\lambda^2}$ (B) $\frac{\lambda_d^3}{\lambda}$ (C) $\frac{\lambda_d^2}{\lambda^2}$ (D) $\frac{\lambda_d}{\lambda}$

34. In a photoelectric experiment a parallel beam of monochromatic light with power of 200W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficiency is 100%. A potential difference of 500V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of is __________. Mass of the electron $m_e = 9 \times 10^{-31}$kg and 1.0 eV = 1.6×10^{-19}J. **[JEE (Advanced) 2018; P-2, 3/60]**

35. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is __________. **[JEE (Advanced) 2018; P-2, 3/60]**

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The time taken by a photoelectron to come out after the photon strikes is approximately **[AIEEE 2006 3/180]**

(1) 10^{-1}s (2) 10^{-4} s (3) 10^{-10} s (4) 10^{-16} s

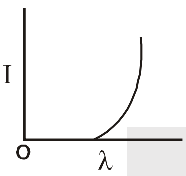
2. An alpha nucleus of energy $\frac{1}{2}$ mv^2 bombards a heavy nuclear target of charge Ze. Then the distance of closest approach for the alpha nucleus will be proportional to : **[AIEEE 2006 ; 3/180]**

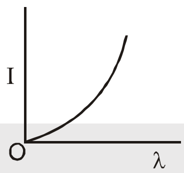
(1) $\frac{1}{Ze}$ (2) v^2 (3) $\frac{1}{m}$ (4) $\frac{1}{v^4}$

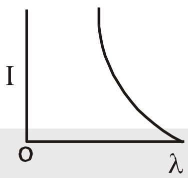
3. The threshold frequency for a metallic surface corresponds to an energy of 6.2 eV, and the stopping potential for a radiation incident on this surface is 5V. The incident radiation lies in **[AIEEE 2006 ; 3/180]**

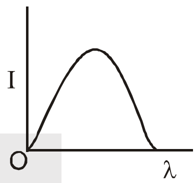
(1) X-ray region (2) ultra-violet region (3) infra-red region (4) visible region

4. The anode voltage of a photocell is kept fixed. The wavelength of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[AIEEE 2006 ; 3/180]**

(1) 

(2) 

(3) 

(4) 

5. Photon of frequency ν has a momentum associated with it. If c is the velocity of light, the momentum is: **[AIEEE 2007 ; 3/120, –1]**

(1) ν/c (2) $h\nu c$ (3) $h\nu/c^2$ (4) $h\,\nu/c$

6. Which of the following transitions in hydrogen atoms emit photons of highest frequency ? **[AIEEE 2007 ; 3/120, –1]**

(1) n = 2 to n = 6 (2) n = 6 to n = 2 (3) n = 2 to n = 1 (4) n = 1 to n = 2

7.✍ Suppose an electron is attracted towards the origin by a force k/r where 'k' is a constant and 'r' is the distance of the electron from the origin. By applying Bohr model to this system, the radius of the nth orbital of the electron is found to be 'r_n' and the kinetic energy of the electron to be 'T_n'. Then which of the following is true? **[AIEEE 2008 ; 3/105, –1]**

(1) T_n independent of n, $r_n \propto n$ (2) $T_n \propto \frac{1}{n}$, $r_n \propto n$

(3) $T_n \propto \frac{1}{n}$ n_1, $r_n \propto n^2$ (4) $T_n \propto \frac{1}{n^2}$, $r_n \propto n^2$

8. The transition from the state n =4 to n =3 in a hydrogen like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition from: **[AIEEE 2009 ; 4/144, –1]**

(1) $3 \rightarrow 2$ (2) $4 \rightarrow 2$ (3) $5 \rightarrow 4$ (4) $2 \rightarrow 1$

9. The surface of a metal is illuminated with the light of 400 nm. The kinetic energy of the ejected photoelectrons was found to be 1.68 eV. The work fuction of the metal is : (hc = 1240 eV.nm) **[AIEEE 2009 ; 4/144, –1]**

(1) 1.41 eV (2) 1.51 eV (3) 1.68 eV (4) 3.09 eV

10. **Statement-1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase. **[AIEEE 2010 ; 4/144, –1]**

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(2) Statement-1 is true, Statement-2 is true; Statement-2 is not the correct explanation of Statement-1
(3) Statement-1 is false, Statement-2 is true.
(4) Statement-1 is true, Statement-2 is false.

11. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called : **[AIEEE 2010 ; 4/144, –1]**

(1) X-rays (2) ultraviolet rays (3) microwaves (4) γ-rays

12. Energy required for the electron excitation in Li^{++} from the first to the thrid Bohr orbit is : **[AIEEE 2011 ; 4/120, –1]**

(1) 12.1 eV (2) 36.3 eV (3) 108.8 eV (4) 122.4 eV

13. This question has statement-1 and statement-2. Of the four choices given after the statements, choose the one that best describes the two statements : **[AIEEE 2011 ; 4/120, –1]**

Statement-1: A metallic surface is irradiated by a monochromatic light of frequency $\nu > \nu_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement-1 is true, statement-2 is false.
(2) Statement-1 is true, Statement-2 is true, Statement-2 is the correct explanation of Statement-1
(3) Statement-1 is true, Statement-2 is true, Statement-2 is not the correct explanation of Statement-1
(4) Statement-1 is false, Statement –2 is true

14. After absorbring a slowly moving neutron of Mass m_N (momentum ≈ 0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1$ ($6m_1 = M + m_N$) respectively. If the de Broglic wavelength of the nucleus with mass m_1 is λ, the de Broglie wevelength of the nucleus will be: **[AIEEE 2011 ; 11 May; 4/120, –1]**

(1) 5λ (2) $\lambda/5$ (3) λ (4) 25λ

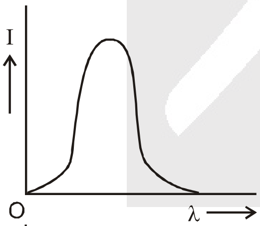
15. Hydrogen atom is excited from ground state to another state with principal quantum number equal to 4. Then the number of spectral lines in the emission spectra will be : **[AIEEE 2012 ; 4/120, –1]**

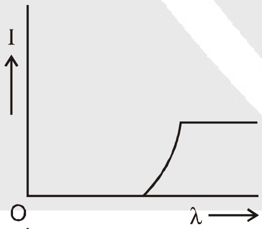
(1) 2 (2) 3 (3) 5 (4) 6

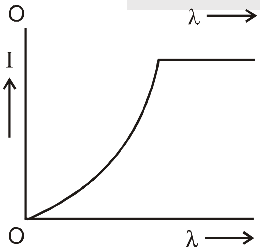
16. A diatomic molecule is made of two masses m_1 and m_2 which are separated by a distance r. If we calculate its rotational energy by applying Bohr's rule of angular momentum quantization, its energy will be given by : (n is an integer) ($\hbar = \frac{h}{2\pi}$) **[AIEEE 2012 ; 4/120, –1]**

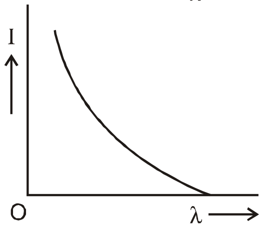
(1) $\frac{(m_1+m_2)^2 n^2 \hbar^2}{2m_1^2 m_2^2 r^2}$ (2) $\frac{n^2\hbar^2}{2(m_1+m_2)r^2}$ (3) $\frac{2n^2\hbar^2}{2(m_1+m_2)r^2}$ (4) $\frac{(m_1+m_2)n^2\hbar^2}{2m_1 m_2 r^2}$

17. The anode voltage of a photocell is kept fixed. The wavelength λ of the light falling on the cathode is gradually changed. The plate current I of the photocell varies as follows : **[JEE (Main) 2013 ; 4/120]**

(1)


(2)


(3)


(4)


18. In a hydrogen like atom electron makes transition from an energy level with quantum number n to another with quantum number (n–1). If n>>1, the frequency of radiation emitted is proportional to : **[JEE (Main) 2013 ; 4/120]**

(1) $\frac{1}{n}$ (2) $\frac{1}{n^2}$ (3) $\frac{1}{n^{3/2}}$ (4) $\frac{1}{n^3}$

19. The radiation corresponding to 3 → 2 transition of hydrogen atom falls on a metal surface to produce photoelectrons. These electrons are made to enter a magnetic field of 3×10^{-4} T. If the radius of the largest circular path followed by these electrons is 10.0 mm, the work function of the metal is close to : **[JEE (Main) 2014 ; 4/120, –1]**

(1) 1.8 eV (2) 1.1 eV (3) 0.8 eV (4) 1.6 eV

20. Hydrogen ($_1H^1$), Deuterium ($_1H^2$), singly ionised Helium $(_2He^4)^+$ and doubly ionised lithium $(_3Li^6)^{++}$ all have one electron around the nucleus. Consider an elelctron transition from n = 2 to n = 1. If the wave lengths of emitted radiation are λ_1, λ_2, λ_3, and λ_4 respectively then approximately which one of the following is correct ? **[JEE (Main) 2014 ; 4/120, –1]**

(1) $4\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (2) $\lambda_1 = 2\lambda_2 = 2\lambda_3 = \lambda_4$ (3) $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$ (4) $\lambda_1 = 2\lambda_2 = 3\lambda_3 = 4\lambda_4$

21. As an electron makes a transition from an excited state to the ground state of a hydrogen-like atom/ion **[JEE (Main) 2015 ; 4/120, –1]**

(1) its kinetic energy increases but potential energy and total energy decrease
(2) kinetic energy, potential energy and total energy decrease
(3) kinetic energy decreases, potential energy increases but total energy remains same
(4) kinetic energy and total energy decrease but potential energy increases

22. Match List-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list : **[JEE (Main) 2015; 4/120, –1]**

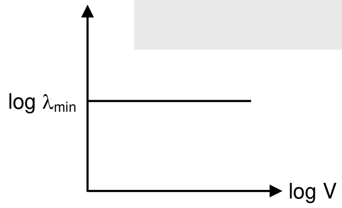
	List - I		List - II
(A)	Franck-Hertz experiment	(i)	Particle nature of light
(B)	Photo-electric experiment	(ii)	Discrete energy levels of atom
(C)	Davisson-Germer experiment	(iii)	Wave nature of electron
		(iv)	Structure of atom

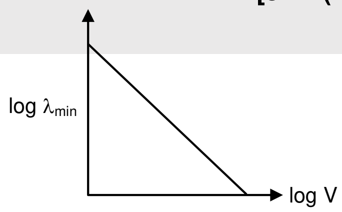
(1) (A) - (i) (B) - (iv) (C) - (iii) (2) (A) - (ii) (B)-(iv) (C) - (iii)
(3) (A) - (ii) (B) (i) (C) - (iii) (4) (A) - (iv) (B) - (iii) (C) - (ii)

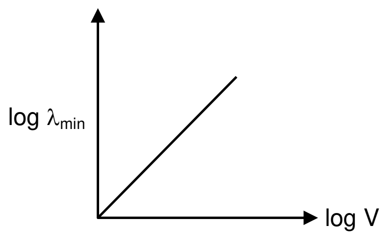
23. Radiation of wavelength λ, is incident on a photocell. The fastest emitted electron has speed 'v'. If the wavelength is changed to $\frac{3\lambda}{4}$, the speed of the fastest emitted electron will be :**[JEE(Main) 2016 ; 4/120, –1]**

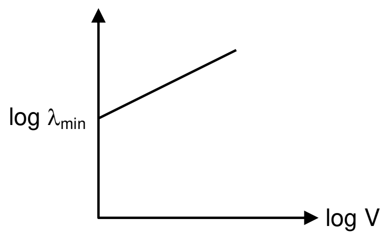
(1) $< v\left(\frac{4}{3}\right)^{1/2}$ (2) $= v\left(\frac{4}{3}\right)^{1/2}$ (3) $= v\left(\frac{3}{4}\right)^{1/2}$ (4) $> v\left(\frac{4}{3}\right)^{1/2}$

24. An electron bean is acceleration by a potential difference V to hit a metallic target to produce X-rays. It produces continuous as well as characteristic X-rays. If λ_{min} is the smallest possible wavelength of X-ray in the spectrum, the variation of log λ_{min} with log V is correctly represented in : **[JEE (Main) 2017 ; 4/120, –1]**

(1) 

(2) 

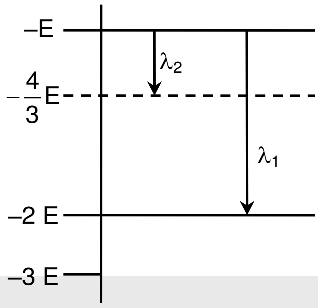
(3) 

(4) 

25. A particle A of mass m and initial velocity υ collides with a particle B of mass m/2 which is at rest. The collision is head on, and elastic. The ratio of the de-Broglie wavelengths λ_A to λ_B after the collision is : **[JEE (Main) 2017 ; 4/120, –1]**

(1) $\frac{\lambda_A}{\lambda_B} = \frac{1}{2}$ (2) $\frac{\lambda_A}{\lambda_B} = \frac{1}{3}$ (3) $\frac{\lambda_A}{\lambda_B} = 2$ (4) $\frac{\lambda_A}{\lambda_B} = \frac{2}{3}$

26. Some energy levels of a molecule are shown in the figure. The ratio of the wavelengths r = λ_1/λ_2, is given by : **[JEE (Main) 2017; 4/120, –1]**



(1) $r = \frac{1}{3}$ (2) $r = \frac{4}{3}$ (3) $r = \frac{2}{3}$ (4) $r = \frac{3}{4}$

27. If the series limit frequency of the Lyman series is ν_L, then the series limit frequency of the Pfund series is **[JEE (Main) 2018; 4/120, –1]**

(1) $\nu_L/16$ (2) $\nu_L/25$ (3) $25\nu_L$ (4) $16\nu_L$

28. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in transition from the n^{th} state to the ground state. For large n, (A, B are constants) **[JEE (Main) 2018; 4/120, –1]**

(1) $\Lambda_n^2 \approx A + B\lambda_n^2$ (2) $\Lambda_n^2 \approx \lambda$ (3) $\Lambda_n \approx A + \frac{B}{\lambda_n^2}$ (4) $\Lambda_n \approx A + B\lambda_n$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. $\frac{P^2}{2m} = \left(\frac{1.24\times10^4}{4000} - 2.5\right)eV = 0.6eV$;

$P = \sqrt{2\times9.1\times10^{-31}\times0.6\times1.6\times10^{-19}}$

$= 4.2 \times 10^{-25}$ kg.m/s

A-2. $(0.6\times10^{15}h - 2e)$ J = 0.48 eV

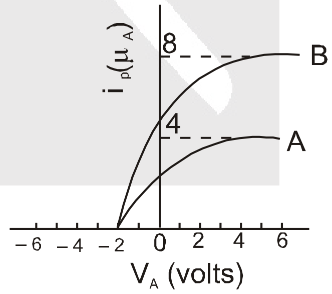
A-3. $I = \eta \cdot \frac{P}{E_\lambda}\times\frac{e}{100} = 1.84 \times 10^{-6}$ amp

A-4. $dV_s = \frac{hc}{e}\cdot\frac{d\lambda}{\lambda^2} = -\frac{hc}{228e} \times 10^7 = -5.5\times10^{-2}$ volt

A-5. $\left(\frac{9\times10^{15}}{2\pi e}h - 2\right)$ eV = 3.93 eV

A-6. $\frac{P\lambda}{hc\times10^6}e$ A = 3.2 μ A

A-7.



Section (B) :

B-1. (a) $N = \frac{7\times10^{-4}}{hc} = 3.5 \times 10^{21}$,

(b) $\frac{7\times10^{-4}}{hc^2} = 1.2 \times 10^{13}$,

(c) $\frac{7\times10^{-4}\times4\pi(1.5\times10^{11})^2}{hc} = 9.9 \times 10^{44}$

B-2. 5×10^{-8} N

B-3. $\dfrac{1.3\times30}{3\times10^{8}} = 1.3\times10^{-7}$ N

B-4. $\dfrac{354\times10^{-8}}{hc} = 1.77 \times 10^{19}$

Section (C) :

C-1. $\lambda_d = \sqrt{\dfrac{h.\lambda.\lambda_{th}}{2m.c.(\lambda_{th}-\lambda)}} = \sqrt{\dfrac{6\times10^{-7}h}{m_e c}}$ m = 12.08 Å

C-2. $\lambda = \dfrac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

Section (D) :

D-1. $\lambda = 2\pi r = 2\pi \times 0.529$ Å $= 1.058\ \pi$ Å

D-2. (a) $r = 0.529 \times \dfrac{2^2}{2} = 1.058$ Å ;

$E = -13.6 \times \dfrac{2^2}{2^2} = -13.6$ eV

(b) $r = 0.529 \times \dfrac{3^2}{2} = 2.38$ Å ;

$E = -13.6 \times \dfrac{2^2}{3^2} = -6.04$ eV.

D-3. He^{+1}

D-4. $K = T = \dfrac{2E_0}{3K} = 1.05 \times 10^5$ K

D-5. $\upsilon = \dfrac{6}{h}\left[13.6e - \dfrac{hc\times10^{10}}{1215}\right] = 5\times10^{15}$ Hz,

Section (E) :

E-1. (a) 91 nm (b) 23 nm

E-2. $\dfrac{v_0}{r_0}.\dfrac{z^2}{n^3} = \dfrac{2.19\times10^6}{0.529\times10^{-10}}\times\dfrac{Z^2}{n^3} = 2.07 \times 10^{16}\ s^{-1}$

E-3. n = 5 **E-4.** He^+ 4,

E-5. $\dfrac{E}{c}.\dfrac{1}{m} = \dfrac{13.6\times3e}{4cm_p} = 3.25$ m/s

E-6. $\dfrac{(E - E')}{E} \times 100 = 0.55 \times 10^{-6}$ %

E-7. (a) $\dfrac{hc}{13.6\times8e} = 113.7$ Å (b) 3

E-8. $K_{recoil} = \left(\dfrac{6.4\times10^3 e}{c}\right)^2 \times \dfrac{1}{2\times(9.3\times10^{26})}$ J

$= 3.9 \times 10^{-4}$ eV

Section (F) :

F-1. $T_{min} = \dfrac{T}{2} = 10.2 = 20.4$ eV

Section (G) :

G-1. $\lambda = \dfrac{hc}{40\times10^3 e}$ m = 31.05 pm

G-2. approximately 0.1%

G-3. $v_1 = \dfrac{hc}{1.5e(\lambda_1+26\times10^{-12})}$ V = 15.9 kV

G-4. $\lambda_1 = \dfrac{hc}{20\times10^3\times(0.7)e} = 88.6$ pm,

$\lambda_2 = \dfrac{hc}{20\times10^3\times(0.7\times0.3)e} = 295.6$ pm,

$\lambda_3 = \dfrac{hc}{20\times10^3\times0.7\times(0.3)^2 e} = 985.6$ pm,

G-5. (i) k_α (ii) 102 keV.

G-6. $\lambda_1 = \left(\dfrac{26-1}{29-1}\right)^2$ 193 pm = 154 pm

G-7. n = 6, Z = 3 **G-8.** 42

Section (H) :

H-1. $\phi = \sin^{-1}(0.2231) \approx 12.89º$

PART - II

Section (A) :

A-1.	(B)	**A-2.**	(C)	**A-3.**	(A)
A-4.	(A)	**A-5.**	(B)	**A-6.**	(B)
A-7.	(B)	**A-8.**	(B)		

Section (B) :

B-1. (D)

Section (C) :

C-1.	(B)	**C-2.**	(D)	**C-3.**	(C)
C-4.	(A)	**C-5.**	(C)	**C-6.**	(D)

Section (D) :

D-1.	(B)	**D-2.**	(B)	**D-3.**	(C)
D-4.	(C)	**D-5.**	(B)	**D-6.**	(A)
D-7.	(D)	**D-8.**	(B)	**D-9.**	(B)
D-10.	(C)	**D-11.**	(D)	**D-12.**	(C)
D-13.	(B)				

Section (E) :

E-1.	(D)	**E-2.**	(D)	**E-3.**	(C)
E-4.	(B)	**E-5.**	(A)	**E-6.**	(C)
E-7.	(A)				

Section (F) :

F-1. (A)

Section (G) :

G-1.	(A)	**G-2.**	(C)	**G-3.**	(B)
G-4.	(A)				

Section (H) :

H-1. (D) **H-2.** (A)

PART - III

1. (A) $\rightarrow$ r, (B) $\rightarrow$ s, (C) $\rightarrow$ p, (D) $\rightarrow$ q
2. (A) $\rightarrow$ r ; (B) $\rightarrow$ q,s ; (C) $\rightarrow$ p ; (D) $\rightarrow$ q,s
3. (A) $\rightarrow$p,r ; (b) $\rightarrow$ q,s ; (C) $\rightarrow$ q,s ; (D) $\rightarrow$ p,r

EXERCISE-2

PART - I

1.	(D)	**2.**	(D)	**3.**	(D)
4.	(B)	**5.**	(D)	**6.**	(C)
7.	(B)	**8.**	(B)	**9.**	(D)
10.	(B)	**11.**	(C)	**12.**	(D)
13.	(D)	**14.**	(B)	**15.**	(D)
16.	(B)	**17.**	(C)	**18.**	(D)
19.	(D)	**20.**	(D)	**21.**	(A)
22.	(B)	**23.**	(A)	**24.**	(A)
25.	(D)	**26.**	(C)	**27.**	(B)
28.	(C)	**29.**	(A)		

PART - II

1.	57	**2.**	4	**3.**	4
4.	0	**5.**	3	**6.**	4
7.	2	**8.**	6	**9.**	25
10.	55	**11.**	12		

PART - III

1.	(ABC)	**2.**	(ABC)	**3.**	(ABD)
4.	(CD)	**5.**	(BCD)	**6.**	(AB)
7.	(CD)	**8.**	(AD)	**9.**	(ABCD)
10.	(ABC)	**11.**	(ACD)	**12.**	(CD)
13.	(AB)				

PART - IV

1.	(B)	**2.**	(A)	**3.**	(C)
4.	(D)	**5.**	(C)	**6.**	(D)
7.	(B)	**8.**	(B)	**9.**	(A)
10.	(C)	**11.**	(C)	**12.**	(C)

EXERCISE-3

PART - I

1.	(B)	**2.**	(B)	**3.**	(A)
4.	(B)	**5.**	(C)	**6.**	(C)
7.	(A)	**8.**	(A)	**9.**	(B)
10.	(D)	**11.**	(A)	**12.**	3
13.	(D)	**14.**	(B)	**15.**	(C)
16.	(A)	**17.**	(C)	**18.**	(B)
19.	7	**20.**	(B)	**21.**	1
22.	(AC)	**23.**	(B)	**24.**	(A)
25.	2	**26.**	(AC)	**27.**	2
28.	(B)	**29.**	(ABD)	**30.**	6
31.	(B)	**32.**	(5)	**33.**	(A)
34	24.00	**35.**	3		

PART – II

1.	(3)	**2.**	(3)	**3.**	(2)
4.	(3)	**5.**	(4)	**6.**	(3)
7.	(1)	**8.**	(3)	**9.**	(1)
10.	(4)	**11.**	(2)	**12.**	(3)
13.	(4)	**14.**	(3)	**15.**	(4)
16.	(4)	**17.**	(4)	**18.**	(4)
19.	(2)	**20.**	(3)	**21.**	(1)
22.	(3)	**23.**	(4)	**24.**	(2)
25.	(3)	**26.**	(1)	**27.**	(2)
28.	(3)				

High Level Problems (HLP)

SUBJECTIVE QUESTIONS

1. A small particle of mass m moves in such a way that the potential energy $U = \frac{1}{2}mb^2r^2$, where b is a constant and r is the distance of the particle from the origin (Nucleus). Assuming Bohr model of quantization of angular momentum and circular orbits, show that radius of the nth allowed orbit is proportional to $\sqrt{n}$.

2. Suppose the potential energy between electron & proton at a distance r is given by $\frac{-ke^2}{3r^3}$. Use Bohr's theory to obtain energy levels of such a hypothetical hydrogen atom.

3. In a transition to a state of excitation energy 10.19 eV a hydrogen atom emits a 4890 Aº photon. Determine the binding energy of the initial state. Also find the nature of transition?

4. Suppose in certain conditions only those transitions are allowed to hydrogen atoms in which the principal quantum number n change by 2 (i) Find the smallest wavelength emitted by hydrogen (ii) List the wavelengths emitted by hydrogen in the visible range (380 nm to 780 nm)

5. Find the velocity of photoelectrons liberated by electromagnetic radiation of wavelength λ = 18.0 nm from stationary He^+ ions in the ground state.

6. (I) Find the maximum wavelength λ of light which can ionize a H-atom in ground state.
(II) Light of wavelength λ is incident on a H-atom which is in its first excited state. Find the kinetic energy of the electron coming out.

7. A beam of monochromatic light of wavelength λ ejects photoelectrons from a cesium surface (Φ = 1.9 eV) These photoelectrons are made to collide with hydrogen atoms in ground state .Find the maximum value of λ for which (a) hydrogen atoms may be ionised (b) hydrogen atoms may get excited from the ground state to the first excited state and (c) the excited hydrogen atoms may emit visible light.

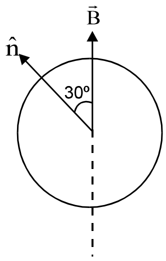
8. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wave length 975Aº. How many different lines are possible in the resulting spectrum? Calculate the longest wavelength among them. You may assume the ionization energy of hydrogen atom as 13.6 eV.

9. Average life time of a hydrogen atom excited to n = 2 state is 10^{-8} s. Find the number of revolutions made by the electrons on the average before it jumps to ground state.

10. In a hydrogen like ionized atom a single electron is orbiting around a stationary positive charge. If a spectral line of λ equal to 4861Å is observed due to transition from n = 12 to n = 6. What is the wavelength of a spectral line due to transition from n = 9 to n = 6 and also identify the element.

11. For atoms of light and heavy hydrogen (H and D) find the difference;
(a) between the binding energies of their electrons in the ground state.
(b) between the wavelengths of first lines of the Lyman series.

12. An electron in the ground state of hydrogen atoms is revolving in anti clock wise direction in a circular orbit of radius R. **[JEE 1996, 5]**



(i) Obtain an expression for the orbital magnetic dipole moment of the electron.
(ii) The atom is placed in a uniform magnetic induction, such that the plane normal to the electron orbit make an angle of 30º with the magnetic induction. Find the torque experienced by the orbiting electron.

13. A proton and an electron, both at rest initially, combine to form a hydrogen atom in ground state. A single photon is emitted in this process. What is the wavelength ?

14. A neutron of kinetic energy 65 eV collides inelastically with a singly ionized helium atom at rest. It is scattered at an angle of 90º with respect of its original direction. **[JEE 1993; 9 + 1M]**
(a) Find the allowed values of the energy of the neutron and that of the atom after the collision.
(b) If the atom gets de-excited subsequently by emitting radiation, find the frequencies of the emitted radiation. [Given : Mass of He atom = 4 × (mass of neutrons Ionization energy of H atom = 13.6 eV]

15. Suppose the potential energy between electron and proton at a distance r is given by $U = ke\ \ell n \frac{r}{a}$, where r < a and k, e, a are positive constants. Use Bohr's theory to obtain the energy of nth energy level for such an atom.

16. A positronium consists of an electron and a positron revolving about their common centre of mass. Derive and calculate
(i) Separation between the electron and positron in their first excited state.
(ii) Kinetic energy of the electron in ground state.

17. In a photo electric effect set – up, a point source of light of power 3.2×10^{-3} W emits mono energetic photons of energy 5.0 eV. The source is located at a distance of 0.8 m from the centre of a stationary metallic sphere of work function 3.0 eV & of radius 8.0×10^{-3}m. The efficiency of photo electrons emission is one for every 10^6 incident photons. Assume that the sphere is isolated and initially neutral, and that photo electrons are instantly swept away after emission. **[JEE 1995, 10]**
(a) Calculate the number of photo electrons emitted per second.
(b) Find the ratio of the wavelength of incident light to the De – Broglie wave length of the fastest photo electrons emitted.
(c) It is observed that the photo electron emission stops at a certain time t after the light source is switched on. Why?
(d) Evaluate the time t.

18. The K_β x-ray of argon has a wavelength of 0.36 nm . The minimum energy required to take out the outermost electron from argon atom is 16.53 eV. Find the energy (in KeV) needed to knock out an electrons from the K shell of an argon.

19. A schwarzschild black hole is characterized by its mass M and a mathematical spherical surface of radius $R_s = \frac{2GM}{C^2}$ called the event horizon. If the radial distance of an object r from the black hole is such that r < R_s, then the object is "swallowed" by the black hole and r rapidly decreased to the singular point r = 0 **[Olympiad_2015]**
(a) Suppose a black hole of mass M "captures" a proton to form a "black hole proton atom (BHP)" in circular orbit. Find the smallest radius r_B of this atom.
(b) Obtain a numerical upper bound on M such that a stable BHP may exist.
(c) Find the minimum energy F_{min}, in Mev, reuired to dissociate this BHP atom from the ground state.
(d) In 1974, Stephen Hawking showed that quantum effects cause black to radiate like a black body with temperature $T_{BH} = \frac{10^{23}K}{M}$. Discuss then the possibility of the existence of a stable BHP atom.

HLP Answers

2. $E = \left(\frac{nh}{2\pi}\right)^6 \frac{1}{6\,(Ke^2)^2\,m^3}$

3. $\frac{13.6}{(4)^2} = 0.85\text{eV}(n = 4 \text{ to } n = 2)$

4. (a) $\frac{9}{8R} = 103$ nm (b) $\frac{16}{3R} = 487$ nm

5. $\sqrt{\frac{2}{m_e}\cdot\left[\frac{10^9 hc}{18} - 54.4\,e\right]} = 2.2 \times 10^6$ m/s

6. (I) 913Å, (II) 10.2 eV

7. (a)$\lambda = \frac{hc}{(13.6+1.9)\text{ eV}} = 80$ nm;(b) $\lambda = \frac{hc}{(10.2+1.9)\text{ eV}} = 102$ nm ; (c) $\frac{hc}{(12.08+1.9)\text{ eV}} = 89$ nm

8. 6, $\lambda_{min} = \frac{16\times 9}{7R} = 18800$ Aº,

9. $10^{-8} \times \frac{2.19\times 10^6}{2\pi(0.529\times 10^{-10})} \times \frac{(1)^2}{(2)^3} = 8.2 \times 10^6$

10. 6563 Å, Z = 3

11. $E_D - E_H = 3.7$ meV, $\lambda_H - \lambda_D = 33$ pm

12. (i) $\frac{he}{4\pi m}$ (ii) $\frac{h\,e\,B}{8\,\pi\,m}$

13. 912 Å

14. (a) 6.36 eV, 0.312 eV (of neutron), 17.84 eV, 16.328 eV (of atom)
(b) 1.82×10^{15} Hz, 11.67×10^{15} Hz, 9.84×10^{15} Hz.

15. $\frac{1}{2}ke\left(1 + 2\ln\left(\frac{nh}{2\pi\sqrt{kema^2}}\right)\right)$

16. (i) $r_0 = \frac{2h^2}{\pi^2 m} \times \frac{4\pi\varepsilon_0}{e^2} = 4.23$ Å (ii) $\frac{1}{2}m\left(\frac{e^2\pi}{4\pi\varepsilon_0 h}\right)^2$ J = 3.4 eV

17. (a) $10^5\ s^{-1}$ (b) 286.18 (d) $\frac{1000}{9}$ sec = 111s

18. $\left[\frac{hc}{0.36\times 10^{-9}e} + 16.53\right]$eV = 4 KeV

19. (a) $r_B = \frac{h^2}{GMm^2}$ (b) $M < 2 \times 10^{11}$ Kg. (c) $E_{min.} = 55$MeV

(d) For M = 10^{11} Kg. $T_{BH} = \frac{10^{23}K}{10^{11}} = 10^{12}$K At this temperature thermal energies kT_{BH} = 82 MeV. The dissociation energy required is 55 MeV. Thus the BHP is thermally unstable.