High Level Problems (HLP)

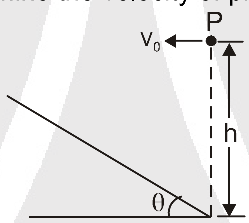
SUBJECTIVE QUESTIONS

1. A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a cm short of it when the elevation is α and goes b cm too far when the elevation is β. Show that if the velocity of projection is same in all the case, the proper elevation is $\frac{1}{2}\sin^{-1}\left[\frac{b\sin 2\alpha + a\sin 2\beta}{a+b}\right]$

2. A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If α and β be the base angles and θ the angle of projection prove that $\tan\theta = \tan\alpha + \tan\beta$.

3. A shell bursts on contact with the ground and pieces from it fly in all directions with all velocities upto 80 feet per second. Show that a man 100 feet away is in danger for $\frac{5}{\sqrt{2}}$ seconds. [Use g = 32 ft/s^2].

4. A stone is projected horizontally from a point P, so that it hits the inclined plane perpendicularly. The inclination of the plane with the horizontal is θ and the point P is at a height h above the foot of the incline, as shown in the figure. Determine the velocity of projection.



5. Two parallel straight lines are inclined to the horizontal at an angle α. A particle is projected from a point mid way between them so as to graze one of the lines and strikes the other at right angles. Show that if θ is the angle between the direction of projection and either of lines, then

$\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. The benches of a gallery in a cricket stadium are 1 m high and 1 m wide. A batsman strikes the ball at a level 1 m above the ground and hits a ball. The ball starts at 35 m/s at an angle of 53º with the horizontal. The benches are perpendicular to the plane of motion and the first bench is 110 m from the batsman. On which bench will the ball hit.

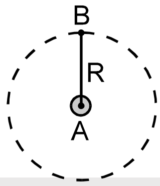
7. A ship is approaching a cliff of height 105 m above sea level. A gun fitted on the ship can fire shots with a speed of 110 ms^{-1}. Find the maximum distance from the foot of the cliff from where the gun can hit an object on the top of the cliff. [g = 10 m/s^2] **[REE 1994, 6]**

8. Shots fired simultaneously from the top and bottom of a vertical cliff with the elevation α and β respectively, strike an object simultaneously at the same point on the ground. If s is the horizontal distance of the object from the cliff, then what is the height of the cliff. ($\beta > \alpha$)

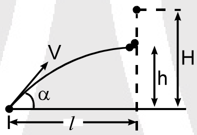
9. Some students are playing cricket on the roof of a building of height 20 m. While playing, ball falls on the ground. A person on the ground returns their ball with the minimum possible speed at angle of projection 45° with the horizontal. The speed of projection is $20\sqrt{\alpha}$ m/s. Here α is an integer. Find α

10. A cannon fires successively two shells with velocity v_0= 250 m/s; the first at the angle θ_1 = 53° and the second at the angle θ_2 = 37° to the horizontal, in the same vertical plane, neglecting the air drag, find the time interval (in sec) between firings leading to the collision of the shells. (g = 10 m/s^2).

11. A small cannon 'A' is mounted on a platform (which can be rotated so that the cannon can aim at any point) and is adjusted for maximum range R of shells. It can throw shells on any point on the shown circle (dotted) on ground. Suddenly a windstorm starts blowing in horizontal direction normal to AB with a speed $\sqrt{2}$ times the velocity of shell. At what least distance can the shell land from point B. (Assume that the velocity of the windstorm is imparted to the shell in addition to its velocity of projection. Also assume that the platform is kept stationary while projecting the shell.)



12. A body A falls freely from some altitude H (<< Re). At the moment the first body starts falling another body B is thrown from the earth's surface which collides with the first at an altitude h = H/2 . The horizontal distance of that point of collision is ℓ from the starting point of B. Find the initial velocity and the angle at which it was thrown ?



13. Two guns situated on the top of a hill of height 10m fire one shot each with the same speed $5\sqrt{3}$ m/s at some interval of time. One gun fires horizontaly and other fires upwards at an angle of 60° with the horizontal. The shots collide in air at point P. Find : **[JEE 1996, 5]**
(a) The time interval between the firings and
(b) the coordinates of the point P. Take origin of the coordinate system at the foot of the hill right below the muzzle and trajectories in x-y plane

14. A small ball rolls of the top of a stairway horizontally with a velocity of 4.5 m s^{-1}. Each step is 0.2 m high and 0.3 m wide. If g is 10 ms^{-2}, then the ball will strike the nth step where n is equal to (assume ball strike at the edge of the step).

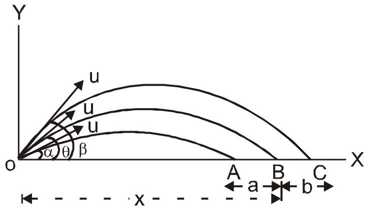
15. A particle moves along the parabolic path x = y^2 + 2y + 2 in such a way that the y-component of velocity vector remains 5m/s during the motion. The magnitude of the acceleration of the particle (in m/s^2) is :

16. A building 4.8 m high 2b meters wide has a flat roof. A ball is projected from a point on the horizontal ground 14.4 m away from the building along its width. If projected with velocity 16 m/s at an angle of 45° with the ground, the ball hits the roof in the middle, find the width 2b. Also find the angle of projection so that the ball just crosses the roof if projected with velocity $10\sqrt{3}$ m/s.(g=10m/s^2)

17. A vertical pole has a red mark at some height. A stone is projected from a fixed point on the ground. When projected at an angle of 45° it hits the pole orthogonally 1 m above the mark. When projected with a different speed at an angle of $\tan^{-1}(3/4)$, it hits the pole orthogonally 1.5 m below the mark. Find the speed and angle of projection so that it hits the mark orthogonally to the pole. [g = 10 m/sec^2] **[REE 1996, 6]**

Answers

1.



$$OA = x - a = \frac{u^2 \sin 2\alpha}{g} \quad ...(1)$$

$$OC = x + b = \frac{u^2 \sin 2\beta}{g} \quad ...(2)$$

$$OB = x = \frac{u^2 \sin 2\theta}{g} \quad ...(3)$$

From eqs. (1) and (2)

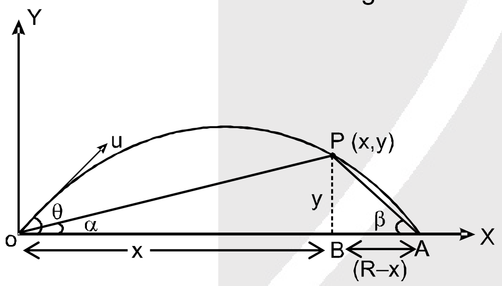
$$x\,(b + a) = \left(\frac{b \sin 2\alpha + a \sin 2\beta}{g}\right) u^2$$

Substituting the value of x from eq. (3), we get

$$\frac{u^2 \sin 2\theta}{g}\,(b + a) = \left(\frac{b \sin 2\alpha + a \sin 2\beta}{g}\right) u^2$$

Solving this equation, we will get θ.

2. The situation is shown in the fig.



From fig $\quad \tan\alpha + \tan\beta = \dfrac{y}{x} + \dfrac{y}{(R - x)}$

where R is the range.

$$\therefore \quad \tan\alpha + \tan\beta = \frac{y(R - x) + xy}{x(R - x)}$$

$$\text{or } \tan\alpha + \tan\beta = \frac{y}{x} \times \frac{R}{(R - x)} \quad ....(1)$$

$$\text{but } \quad y = x \tan\theta \left(1 - \frac{x}{R}\right)$$

$$\text{or } \tan\theta = \frac{y}{x} \times \frac{R}{(R - x)} \quad ....(2)$$

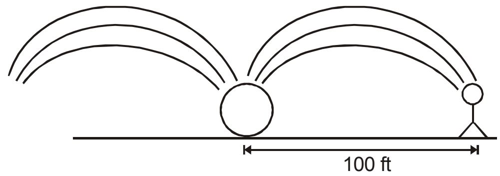
From equations (1) and (2), we have

$\tan\theta = \tan\alpha + \tan\beta$.

3. According to given problem u = 80 f / s

$$\text{Range} = \frac{u^2 \sin 2\theta}{g}$$

$$\sin 2\theta = \frac{100 \times 32}{(80)^2} = 1/2$$



$\theta = 15°$ For same Range $\theta = 15°, 75°$

Thus there will be two time of flight

$$T_1 = \frac{2u \sin 15°}{g} = \frac{2 \times 80 \times \sin 15°}{32} \text{ (minimum time)}$$

$$\sin 15° = \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

$$T_2 = \frac{2u \sin 75°}{g} = \frac{2 \times 8 \times \sin 75°}{32} \text{ (maximum time)}$$

$$\sin 75° = \frac{\sqrt{3} + 1}{2\sqrt{2}}$$

Danger time = Maximum time – Minimum time = $(T_2 - T_1)$

$$= \frac{2 \times 80}{32}\,[\sin 75° - \sin 15°]$$

$$= \frac{2 \times 80}{32}\left[\frac{\sqrt{3} + 1}{2\sqrt{2}} - \frac{\sqrt{3} - 1}{2\sqrt{2}}\right] = \frac{5}{\sqrt{2}} \text{ sec.}$$

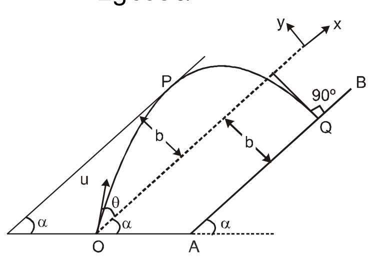
4. $v_0 = \sqrt{\dfrac{2gh}{2 + \cot^2\theta}}$

5. Consider the motion of the particle from O to P. The velocity v_y at P is zero.

$$v_y^2 = u_y^2 + 2a_y s_y$$

$$\therefore \quad 0 = (u \sin\theta)^2 - 2\,(g \cos\alpha)\, b$$

$$\text{or } \quad b = \frac{u^2 \sin^2\theta}{2g \cos\alpha} \quad ....(i)$$



Now, consider the motion of the particle from O to Q.

The particle strikes the point Q at 90º to AB, i.e., its velocity along x-direction is zero.

Using $v_x = u_x + a_x t$, we have

$0 = u \cos\theta - (g \sin\alpha)t$

or $t = \dfrac{u\cos\theta}{g\sin\alpha}$ ...(ii)

For motion in y-direction, $s_y = u_y t + \dfrac{1}{2} a_y t^2$

or $-b = u \sin\theta$

$$\left(\frac{u\cos\theta}{g\sin\alpha}\right) + \frac{1}{2}(-g\cos\alpha)\left(\frac{u\cos\theta}{g\sin\alpha}\right)^2 \text{ ...(iii)}$$

From Eqs. (i) and (iii)

or $-\dfrac{u^2\sin^2\theta}{2g\cos\alpha} = \dfrac{u^2\sin\theta\cos\theta}{g\sin\alpha} - \dfrac{gu^2\cos\alpha\cos^2\theta}{2g^2\sin^2\alpha}$

or $-\dfrac{\sin^2\theta}{2\cos\alpha} = \dfrac{\sin\theta\cos\theta}{\sin\alpha} - \dfrac{\cos\alpha\cos^2\theta}{2\sin^2\alpha}$

Solving, we get $\tan\theta = \left(\sqrt{2}-1\right)\cot\alpha$

6. 6th step

7. 1100 m

8. $h = s(\tan\beta - \tan\alpha)$.

9. 2

10. 10sec

11. Let the speed of shell be u and the speed of wind be v.

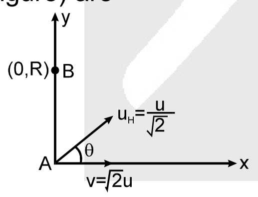
The time of flight T remains unchanged due to windstorm

$T = \dfrac{\sqrt{2}u}{g}$ ....(1)

Horizontal component of velocity of shell in absence of air

$u_H = \dfrac{u}{\sqrt{2}}$ ....(2)

Hence the net x and y component of velocity of shell (see figure) are



$u_x = \sqrt{2}\,u + \dfrac{u}{\sqrt{2}} \cos\theta$ ....(3a)

$u_y = \dfrac{u}{\sqrt{2}} \sin\theta$ ....(3b)

$\therefore$ The x and y coordinate of point P where shell lands is

$x = u_xT = (\sqrt{2}\,u + \dfrac{u}{\sqrt{2}} \cos\theta)\dfrac{\sqrt{2}u}{g} = 2R + R\cos\theta$

..........(4a)

$y = u_yT = \left(\dfrac{u}{\sqrt{2}}\sin\theta\right) \dfrac{\sqrt{2}u}{g} = R\sin\theta$ ....(4b)

$\therefore$ The distance S between B and P is given by

$S^2 = (x-0)^2 + (y-R)^2 = (2R + R\cos\theta)^2 + (R\sin\theta - R)^2$

$= R^2 [6 + 4\cos\theta - 2\sin\theta]$

$= R^2 \left[6 + \sqrt{20}\left(\dfrac{4\cos\theta}{\sqrt{20}} - \dfrac{2\sin\theta}{\sqrt{20}}\right)\right]$

$\therefore$ $S_{minimum} = R\sqrt{6-\sqrt{20}}$

$= R\sqrt{6-2\sqrt{5}}$ or $R(\sqrt{5}-1)$ **Ans.**

Alternate : Circle in fig. (1) Represents locus of all points where shell lands on the ground in absence of windstorm.

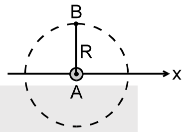


fig.1

Let the speed of shell be 'u' and the speed of wind be $v = \sqrt{2}\,u$. Let T be the time of flight, which remains unaltered even when the windstorm blows. Since R is the maximum range angle of projection is 45º with the horizontal.

Then $R = \dfrac{u}{\sqrt{2}}\,T$ .... (1)

As a result of flow of wind along x-axis, there is an additional shift (Δx) of the shell along x-axis in time of flight.

$\Delta x = vT = \sqrt{2}\,uT = 2R.$

Hence locus of all points where shell lands on ground shifts along x-axis by 2R as shown in fig. (2).

From the fig (2).

$BC = \sqrt{R^2 + (2R)^2}$

$= \sqrt{5R^2} = \sqrt{5}\,R$

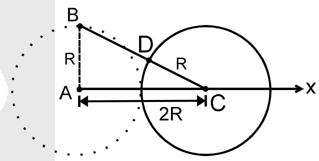


fig.2

Hence the minimum required distance is

$BD = BC - DC = \sqrt{5}\,R - R =$ **$(\sqrt{5} - 1)$ R Ans.**

12. $v_0 = \sqrt{gH\left(1+\dfrac{\ell^2}{H^2}\right)}$, $\tan\alpha = \dfrac{H}{\ell}$

13. (a) 1s (b) $5\sqrt{3}$ m. 5 m

14. 9

15. 50

16. width of the roof is 9.6 m

$\theta = \tan^{-1} 3/2$ or $\theta = 45°$

17. $\dfrac{\sqrt{3620}}{3}$ m/s, $\tan^{-1}\left(\dfrac{9}{10}\right)$