HINTS & SOLUTIONS

TOPIC : QUADRATIC EQUATION

EXERCISE # 1

PART–1

A-1. $a^2 - a - 2 = 0$, $a^2 - 4 = 0$, $a^2 - 3a + 2 = 0 \Rightarrow a = 2, -1$ and $a = \pm 2$ and $a = 1, 2 \Rightarrow a = 2$
Now $(x^2 + x + 1)\, a^2 - (x^2 + 3)\, a - (2x^2 + 4x - 2) = 0$ will be an identity if $x^2 + x + 1 = 0$ & $x^2 + 3 = 0$ & $2x^2 + 4x - 2 = 0$ which is not possible.

A-2. (i) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = \left(\frac{-3}{2}\right)^2 - 2(2) = \frac{-7}{4}$ (ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = -\frac{7}{8}$

A-3. $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

(i) $\alpha + \frac{1}{\beta} + \beta + \frac{1}{\alpha} = \alpha + \beta + \frac{\alpha + \beta}{\alpha\beta} = \frac{-b}{a} + \frac{-b/a}{c/a} = -\left(\frac{b}{a} + \frac{b}{c}\right) = -b\frac{(a+c)}{ac}$

and $\left(\alpha + \frac{1}{\beta}\right)\left(\beta + \frac{1}{\alpha}\right) = \alpha\beta + \frac{1}{\alpha\beta} + 2 = \frac{c}{a} + \frac{a}{c} + 2 = \frac{(a+c)^2}{ac}$

$\therefore$ equation whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$ is

$\therefore$ $\alpha + \frac{1}{\beta}\beta + \frac{1}{\alpha} \Rightarrow acx^2 + b(a + c)x + (a + c)^2 = 0$

(ii) $\alpha^2 + 2 + \beta^2 + 2 = (\alpha + \beta)^2 - 2\alpha\beta + 4 = \frac{b^2}{a^2} - \frac{2ac}{a^2} + 4 = \frac{4a^2 + b^2 - 2ac}{a^2}$

and $(\alpha^2 + 2)(\beta^2 + 2) = \alpha^2\beta^2 + 2(\alpha^2 + \beta^2) + 4 = \frac{c^2}{a^2} + \frac{2(b^2 - 2ac)}{a^2} + 4$

$\therefore$ equation whose roots are $\alpha^2 + 2$ & $\beta^2 + 2$ is
$a^2 x^2 + (2ac - b^2 - 4a^2)x + 2b^2 + 4a^2 + c^2 - 4ac = 0$
$\Rightarrow a^2 x^2 + (2ac - b^2 - 4a^2)x + 2b^2 + (2a - c)^2 = 0$

A-4. given $\alpha^2 = 5\alpha - 3$ and $\beta^2 = 5\beta - 3$
$\Rightarrow$ α & β are the roots of $x^2 - 5x + 3 = 0$
$\therefore$ $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{25 - 6}{3} = \frac{19}{3}$ and $\frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$
$\therefore$ equation have $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}$ as its roots is $3x^2 - 19x + 3 = 0$

A-5. $x^2 + px + q = 0 \begin{smallmatrix} \nearrow \alpha \\ \searrow \beta \end{smallmatrix}$ $\Rightarrow p = -11, q = 24$
then correct equation will be $x^2 - 11x + 24 = 0$
$x^2 - 11x + 24 = 0$
$\Rightarrow$ $(x - 8)(x - 3) = 0$ $\Rightarrow x = 3, 8$

A-6. **(i)** $E = 2x^3 + 2x^2 - 7x + 72$
Given, $x = \frac{3 + 5i}{2}$
$\Rightarrow$ $2x - 3 = 5i$
$\Rightarrow$ $4x^2 + 9 - 12x = -25$
$\Rightarrow$ $4x^2 - 12x + 34 = 0$
$\Rightarrow$ $2x^2 - 6x + 17 = 0$ ....(i)
Given expression can be written as

$E = (2x^2 - 6x + 17)(x + 4) + 4 = 4$ (using (i))

(ii) $\left(x+\frac{1}{2}\right) = \frac{\sqrt{15}}{2} \Rightarrow x^2 + x + \frac{1}{4} = \frac{15}{4} \Rightarrow x^2 + x = \frac{14}{4} \Rightarrow x^2 + x = \frac{7}{2}$

$\therefore \quad 2x^3 + 2x^2 - 7x + 72 = 2x(x^2 + x) - 7x + 72 = 2x\left(\frac{7}{2}\right) - 7x + 72 = 7x - 7x + 72 = 72.$

(iii) $2^x = y \Rightarrow y^2 + 2^2 y - 32 = 0 \Rightarrow y^2 + 8y - 4y - 32 = 0$

$\Rightarrow y = 4 = 2^x \quad \because 2^x \neq -8 \Rightarrow x = 2.$

A-7. $\because \quad ax^2 + bx + c = 0 \begin{smallmatrix}\nearrow \alpha \\ \searrow \beta\end{smallmatrix} \Rightarrow \alpha + \beta = -\frac{b}{a} \Rightarrow \alpha\beta = \frac{c}{a}$

$\because$ Let $a^3 x^2 + (a\,b\,c)\,x + c^3 = 0 \begin{smallmatrix}\nearrow \gamma \\ \searrow \delta\end{smallmatrix}$

$\therefore \quad \gamma + \delta = -\frac{abc}{a^3} = \left(-\frac{b}{a}\right)\left(\frac{c}{a}\right) = (\alpha\,\beta)(\alpha + \beta) = \alpha^2\beta + \alpha\beta^2 \quad v \quad$ ....(i)

$\because \quad \gamma\,\delta = \left(\frac{c}{a}\right)^3 = (\alpha\beta)^3 = (\alpha^2\beta)(\alpha\beta^2)$ ....(ii)

From (i) and (ii) we can say that $\gamma = \alpha^2\beta$ and $\delta = \alpha\beta^2$ and $\gamma = \alpha\beta^2$ and $\delta = \alpha^2\beta$

A-8. $\alpha + \beta = p, \alpha\beta = q \Rightarrow (\alpha - 2)(\beta + 2) = r \Rightarrow \alpha\beta + 2\alpha - 2\beta - 4 = r$

$q + 2(\alpha - \beta) - 4 = r \Rightarrow 2\alpha - 2\beta = r + 4 - q \Rightarrow 2\alpha + 2\beta = 2p$

$4\alpha = r + 4 - q + 2p \Rightarrow 4\beta = 2p - (r + 4 - q) \Rightarrow 16\alpha\beta = 4p^2 - (r + 4 - q)^2$

$16q + (r + 4 - q)^2 = 4p^2.$

A-9. $\alpha\,.\,\alpha^n = \frac{c}{a} \Rightarrow \alpha = \left(\frac{c}{a}\right)^{\frac{1}{n+1}} \Rightarrow \alpha + \alpha^n = -\frac{b}{a} \Rightarrow \left(\frac{c}{a}\right)^{\frac{1}{n+1}} + \left(\frac{c}{a}\right)^{\frac{n}{n+1}} = -\frac{b}{a}$

$a^{1-\frac{1}{n+1}}\ .\ c^{\frac{1}{n+1}} + c^{\frac{n}{n+1}}.\ a^{1-\frac{n}{n+1}} + b = 0 \Rightarrow a^{\frac{n}{n+1}}\ .\ c^{\frac{1}{n+1}} + a^{\frac{1}{n+1}}.\ c^{\frac{n}{n+1}} + b = 0$

$\left(a^n\ \ c\right)^{\frac{1}{n+1}} + \left(a\ \ c^n\right)^{\frac{1}{n+1}} + b = 0$ Proved.

A-10. $S = \frac{-(2a+3)}{a+1} = -1 \Rightarrow 2a + 3 = a + 1 \Rightarrow a = -2;\ p = \frac{3a+4}{a+1} = \frac{-6+4}{-2+1} = 2$

A-11. $2x^2 + 6x + a = 0$

$\because$ Its roots are $\alpha, \beta \Rightarrow \alpha + \beta = -3$ & $\alpha\beta = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} < 2$

$\Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} < 2 \Rightarrow \frac{9-a}{a} < 1$

$\Rightarrow \frac{2a-9}{a} > 0 \Rightarrow a \in (-\infty, 0) \cup \left(\frac{9}{2}, \infty\right) \Rightarrow 2a = 11$ is least prime.

B-1. Let 3rd root be γ then $\alpha\beta\gamma = -r$ But $\alpha\beta = -1$ (given) $\Rightarrow \gamma = r$
substituting $x = \gamma = r$ in the given equation we get $r^2 + pr + q + 1 = 0$.

B-2. $x^3 + px^2 + qx + r \begin{smallmatrix}\nearrow \alpha \\ \rightarrow \beta \\ \searrow \gamma\end{smallmatrix} \Rightarrow \alpha\beta\gamma = -r \Rightarrow \left(\alpha\ \ -\frac{1}{\beta\gamma}\right)\left(\beta\ \ -\frac{1}{\gamma\alpha}\right)\left(\gamma\ \ -\frac{1}{\alpha\beta}\right)$

$= \left(\alpha + \frac{\alpha}{r}\right)\left(\beta + \frac{\beta}{r}\right)\left(\gamma + \frac{\gamma}{r}\right) = \alpha\beta\gamma\left(1 + \frac{1}{r}\right)^3 = -r\ \frac{(r+1)^3}{r^3} = -\frac{(r+1)^3}{r^2}$ **Ans.**

B-3.✍ **(i)** Let roots be α, 2α, β $\Rightarrow 3\alpha + \beta = \frac{14}{24}$, $2\alpha^2 + 3\alpha\beta = \frac{-63}{24}$, $2\alpha^2\beta = \frac{-\lambda}{24} \Rightarrow \beta = \frac{7}{12} - 3\alpha$

$$2\alpha^2 + 3\alpha\left(\frac{7}{12} - 3\alpha\right) = \frac{-21}{8} \Rightarrow 2\alpha^2 + \frac{7\alpha}{4} - 9\alpha^2 = \frac{-21}{8} \Rightarrow 0 = 7\alpha^2 - \frac{7\alpha}{4} - \frac{21}{8}$$

$$\alpha^2 - \frac{\alpha}{4} - \frac{3}{8} = 0 \Rightarrow 8\alpha^2 - 2\alpha - 3 = 0 \Rightarrow \alpha = \frac{3}{4} \text{ or } \frac{-1}{2}$$

$\alpha = \frac{3}{4} \Rightarrow$ roots are $\frac{3}{4}, \frac{3}{2}, \frac{-5}{3}$ and $\lambda = 45 \Rightarrow$ $\alpha = \frac{-1}{2}$

$\Rightarrow$ roots are $\frac{-1}{2}, -1, \frac{25}{12}$ and $\lambda = -25$

(ii) α, β, γ be roots.

$\alpha + \gamma = 2\beta$ ....(1) ; $\alpha + \beta + \gamma = \frac{-81}{18}$ ....(2)

$\alpha\beta\gamma = \frac{-60}{18}$ ....(3)

(1), (2) $\Rightarrow \beta = \frac{-3}{2}$ Put in (1), (3)

$\Rightarrow \quad \alpha + \gamma = -3 \quad \Rightarrow \quad \alpha\gamma = \frac{20}{9}$

$\therefore \quad x^2 - (-3)x + \frac{20}{9} = 0 < \begin{matrix}\alpha \\ \gamma\end{matrix} \quad \Rightarrow \quad x = \frac{-3 \pm \sqrt{9 - 4\cdot 1\cdot \frac{20}{9}}}{2} = \frac{-5}{3}, \frac{-4}{3}$.

$\therefore$ roots are $\frac{-4}{3}, \frac{-3}{2}, \frac{-5}{3}$.

B-4. $\alpha^3 - 6\alpha^2 + 10\alpha - 3 = 0.$

Let x = 2α + 1 new root $\alpha = \frac{x-1}{2} \Rightarrow \frac{(x-1)^3}{8} - \frac{6(x-1)^2}{4} + 5(x - 1) - 3 = 0$

$(x^3 - 3x^2 + 3x - 1) - 12(x^2 - 2x + 1) + 40(x - 1) - 24 = 0 \Rightarrow x^3 - 15x^2 + 67x - 77 = 0.$

B-5.✍ $2x^3 + x^2 - 7 = 0 < \begin{matrix}\alpha \\ \beta \\ \gamma\end{matrix} \Rightarrow \alpha + \beta + \gamma = -1/2, \sum\alpha\beta = 0, \alpha\beta\gamma = 7/2$

$$\sum\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) + \frac{\beta}{\gamma} + \frac{\gamma}{\beta} + \frac{\gamma}{\alpha} + \frac{\alpha}{\gamma} = \frac{1}{\beta}(\alpha + \gamma) + \frac{1}{\alpha}(\beta + \gamma) + \frac{1}{\gamma}(\alpha + \beta)$$

$$= \frac{1}{\beta}\left(-\frac{1}{2} - \beta\right) + \frac{1}{\alpha}\left(-\frac{1}{2} - \alpha\right) + \frac{1}{\gamma}\left(-\frac{1}{2} - \gamma\right) = -\frac{1}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) - 1 - 1 - 1 = -\frac{1}{2}\left(\frac{\Sigma\alpha\beta}{\alpha\beta\gamma}\right) - 3 = -3$$

B-6. Let roots be α, α and β

$\therefore \quad \alpha + \alpha + \beta = -\frac{20}{4} \quad \Rightarrow \quad 2\alpha + \beta = -5$ ....(1)

$\therefore \quad \alpha \,.\, \alpha + \alpha\beta + \alpha\beta = -\frac{23}{4} \quad \Rightarrow \quad \alpha^2 + 2\alpha\beta = -\frac{23}{4}$ ....(2)

and $\quad \alpha^2\beta = -\frac{6}{4} = -\frac{3}{2}$ ....(3)

from equation (1) put $\beta = -5 - 2\alpha$ in (2), we get $\alpha^2 + 2\alpha(-5 - 2\alpha) = -\frac{23}{4}$

$\Rightarrow \quad 12\alpha^2 + 40\alpha - 23 = 0 \quad \therefore \quad \alpha = 1/2, -\frac{23}{6}$

(i) If $\alpha = \frac{1}{2}$ then from (1), we get $\beta = -6$

(ii) If $\alpha = -\dfrac{23}{6}$ then from (1), we get $\beta = \dfrac{8}{3}$

Note : $\because$ $\alpha = \dfrac{1}{2}$ and $\beta = -6$ also satisfy (3) but $\alpha = -\dfrac{23}{6}$ and $\beta = \dfrac{8}{3}$ does not satisfy (3)

$\therefore$ required roots are $\dfrac{1}{2}, \dfrac{1}{2}, -6$

C-1. $2 + i\sqrt{3}$ and $2 - i\sqrt{3}$ are the roots of $x^2 + px + q = 0$

$\Rightarrow$ $-p = 4 \Rightarrow p = -4$ & $q = 7$.

C-2. $x^2 - 2cx + ab = 0$ has roots real and unequal i.e. $D_1 > 0 \Rightarrow 4c^2 - 4ab > 0 \Rightarrow c^2 - ab > 0$ ....(1)

Now, $x^2 - 2(a + b)x + (a^2 + b^2 + 2c^2) = 0$

$\Rightarrow$ $D_2 = 4(a + b)^2 - 4(a^2 + b^2 + 2c^2) = -8(c^2 - ab)$

by (1) $D_2 < 0$ roots will be imaginary.

C-3. $D = 0 \Rightarrow (k + 1)^2 - 8k = 0 \Rightarrow k^2 + 1 - 6k = 0 \Rightarrow k = \dfrac{6 \pm \sqrt{36-4}}{2} \Rightarrow k = 3 \pm 2\sqrt{2}$.

C-4. $D = 0 \Rightarrow 4(b^2 - ac)^2 - 4(a^2 - bc)(c^2 - ab) = 0 \Rightarrow b(a^3 + b^3 + c^3 - 3abc) = 0$

$\Rightarrow$ Either $b = 0$ or $a^3 + b^3 + c^3 = 3abc$.

C-5. $\dfrac{1}{x-a} + \dfrac{1}{x-b} + \dfrac{1}{x-c} = 0$ ...(1)

$\Rightarrow$ $(x - b)(x - c) + (x - a)(x - c) + (x - a)(x - b) = 0$

$\Rightarrow$ $3x^2 - 2(a + b + c)x + ab + bc + ac = 0$ ...(2)

$D = 4(a + b + c)^2 - 12(ab + bc + ac) = 4[a^2 + b^2 + c^2 - ab - bc - ac] = 2[(a - b)^2 + (b - c)^2 + (c - a)^2]$

$\therefore$ $D \geq 0 \Rightarrow$ roots are always real But if $a = b = c$

Then $\dfrac{1}{x-a} + \dfrac{1}{x-b} + \dfrac{1}{x-c} = 0 \Rightarrow \dfrac{3}{x-a} = 0$

which has no real 'x'

$\Rightarrow$ this equation cannot have roots if $a = b = c$. $a = b = c$

C-6. $\dfrac{1}{(x+p)} + \dfrac{1}{(x+q)} = \dfrac{1}{r} \Rightarrow x^2 + x(p + q - 2r) + (pq - pr - qr) = 0 \begin{matrix} \nearrow \alpha \\ \searrow -\alpha \end{matrix}$

$\therefore$ $\alpha + (-\alpha) = -(p + q - 2r) = 0 \Rightarrow p + q = 2r$

&Product of roots $= pq - r(p + q) = pq - r(p + q) = pq - \dfrac{(p+q)^2}{2} = -\dfrac{1}{2}(p^2 + q^2)$

C-7. **(i)** Roots are $-2 + i\beta$, $-2 - i\beta$, γ (say) ; Sum of roots $(-2 + i\beta) + (-2 - i\beta) + \gamma = 0$; $\gamma = 4$.

Sum of products taken two at a time.

$4(-2 + i\beta) + 4(-2 - i\beta) + (4 + \beta^2) = 63$; $-16 + 4 + \beta^2 = 63$; $\beta^2 = 75$

$\beta = \pm 5\sqrt{3}$. Roots are $4, -2 \pm i5\sqrt{3}$.

(ii) Call roots as α, $\dfrac{-1}{2} + i\beta$, $\dfrac{-1}{2} - i\beta$

$\alpha - 1 = \dfrac{-b}{2}$ ....(1)

$\alpha\left(\dfrac{-1}{2} + i\beta\right) + \alpha\left(\dfrac{-1}{2} + i\beta\right) + \dfrac{1}{4} + \beta^2 = \dfrac{3}{2}$ ....(2)

$\alpha\left(\dfrac{1}{4} + \beta^2\right) = \dfrac{-1}{2}$ ....(3)

$(2) \Rightarrow \dfrac{1}{4} + \beta^2 = \dfrac{3}{2} + \alpha$

Put in (3) $\alpha\left(\dfrac{3}{2} + \alpha\right) = \dfrac{-1}{2}$; $\alpha(2\alpha + 3) = -1$. $\Rightarrow$ $\alpha = -1, \dfrac{-1}{2}$.

If $\alpha = -1$, $(3) \Rightarrow b = 4$ $\therefore$ $\alpha = \dfrac{-1}{2} \Rightarrow b = 3$

Put in (1) b = 3 or 4

C-8. Given one root is –1 + i

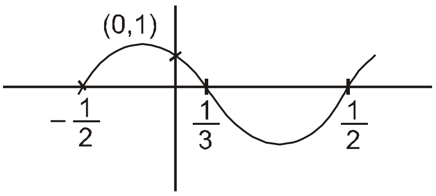
∴ 2^{nd} root will be –1 – i

∴ $x^2 + 2x + 2$ will be one factor of $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$ and $x^2 + 2x - 1$ will be another factor

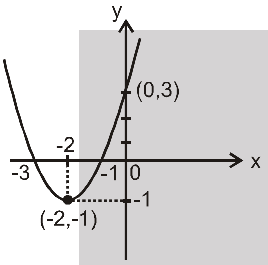
∴ The roots of given equation are $-1 \pm \sqrt{2}$ and –1 ± i.

C-9. $y = (2x - 1)(6x^2 + x - 1) = (2x - 1)(2x + 1)(3x - 1)$. Hence roots are $x = -\frac{1}{2}, \frac{1}{3}, \frac{1}{2}$

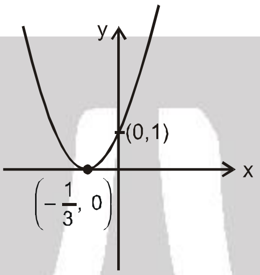
f(1) > 0



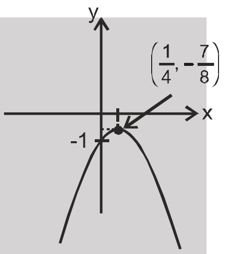
D-1. (i)



(ii)



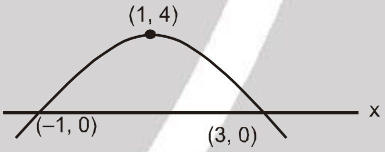
(iii)

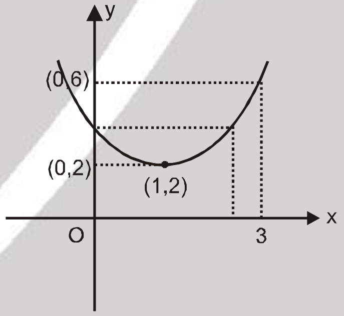


D-2. **(i)** $y = -x^2 + 2x + 3 = -(x^2 - 2x - 3) = -(x - 3)(x + 1)$

Here a < 0 and D > 0 ⇒ Range is (–∞, 4]

(ii) $f(x) = x^2 - 2x + 3 \ \forall \ x \in [0, 3]$





$y \in [2, 6] \ \forall \ x \in [0, 3]$ **Ans.**

Aliter :

$f(x) = x^2 - 2x + 3 = (x - 1)^2 + 2$

Since $0 \le x \le 3 \Rightarrow -1 \le x - 1 \le 2 \Rightarrow 0 \le (x - 1)^2 \le 4$

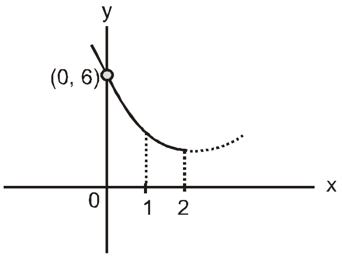
$\Rightarrow 2 \le (x - 1)^2 + 2 \le 6 \Rightarrow 2 \le f(x) \le 6$

∴ Range of f(x) is [2, 6].

(iii) $y = x^2 - 4x + 6$; $x \in (0, 1]$

Here a > 0 and D < 0

f(0) = 6 ⇒ f(1) = 3 ⇒Clearly for $x \in (0, 1] \Rightarrow y \in [3, 6)$



D-3. **(i)** $(y-1)x^2 - x + y - 1 = 0$

$\because$ $x \in R$

$\therefore$ $D \geq 0$

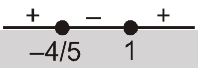
$\Rightarrow 1 - 4(y-1)^2 \geq 0 \Rightarrow (1 + 2y - 2)(1 - 2y + 2) \geq 0 \Rightarrow (2y-1)(2y-3) \leq 0 \Rightarrow \frac{1}{2} \leq y \leq \frac{3}{2}$

(ii) $y(x^2 - 2x - 9) = x^2 - 2x + 9 \quad \Rightarrow \quad (y-1)x^2 - 2(y-1)x - (y+1)9 = 0$

If $y = 1 \Rightarrow -(2)\,9 = 0$ contradiction.

$\therefore$ $y \neq 1 \quad D \geq 0 \quad \Rightarrow \quad (5y+4)(y-1) \geq 0$

$y \in \left(-\infty, \frac{-4}{5}\right] \cup (1, \infty)$



D-4. We can see for $x^2 - 8x + 17$

$D = 64 - 4(17) < 0$

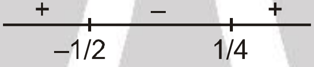
$\therefore$ $x^2 - 8x + 17$ is always +ve $\Rightarrow$ If $f(x) < 0$

$\therefore$ $kx^2 + 2(k+1)x + (9k+4) < 0 \Rightarrow k < 0$ ......(1)

& $4(k+1)^2 - 4k(9k+4) < 0 \Rightarrow k^2 + 1 + 2k - 9k^2 - 4k < 0 \Rightarrow -8k^2 - 2k + 1 < 0$

$8k^2 + 2k - 1 > 0 \Rightarrow 8k^2 + 4k - 2k - 1 > 0 \Rightarrow 4k(2k+1) - 1(2k+1) > 0$

$(2k+1)(4k-1) > 0$ ....(2)



combining (1) & (2) we get $k \in \left(-\infty, -\frac{1}{2}\right)$ (1) (2) $k \in \left(-\infty, -\frac{1}{2}\right)$

D-5. **(i)** $x^2 + (a-b)x + (1-a-b) = 0$

$\because$ $D > 0$

$\Rightarrow$ $(a-b)^2 - 4 \times 1 \times (1-a-b) > 0$

$\Rightarrow$ $a^2 + b^2 - 2ab - 4 + 4a + 4b > 0$

$\because$ $b^2 + 2b(2-a) + (a^2 + 4a - 4) > 0$

$\because$ $4(2-a)^2 - 4 \times 1 \times (a^2 + 4a - 4) < 0$

$4 + a^2 - 4a - a^2 - 4a + 4 < 0$

$\Rightarrow$ $8a - 8 > 0 \quad \Rightarrow \quad a > 1$

(ii) $(a-b)^2 - 4 \cdot 1 \cdot (1-a-b) \leq 0$

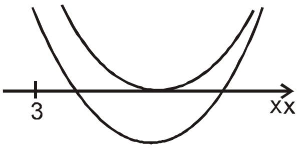
$\Rightarrow$ $b^2 + (4-2a)b + (a^2 + 4a - 4) \leq 0, \forall b \in R$

as cofficient of $b^2 = 1$, positive it in not possible.

$\therefore$ $a \in \phi$.

E-1. For both roots to exceed 3

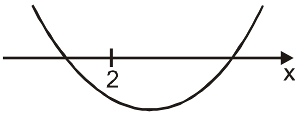
(i) $D \geq 0 \Rightarrow 36a^2 - 8 + 8a - 36a^2 \geq 0 \Rightarrow a \geq 1$



(ii) $f(3) > 0 \Rightarrow 9 - 18a + 2 - 2a + 9a^2 > 0 \Rightarrow 9a^2 - 20a + 11 > 0 \Rightarrow a \in (-\infty, 1) \cup \left(\frac{11}{9}, \infty\right)$

(iii) $\frac{-b}{2a} > 3 \Rightarrow 3a > 3 \Rightarrow a > 1 \qquad \therefore \qquad$ (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow a > \frac{11}{9}$.

E-2. Here for one root to exceed 2 and other to be smaller than 2, $f(2) < 0$

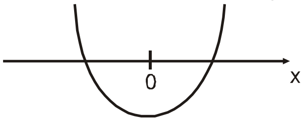


$\Rightarrow$ $4 - 2k - 2 + k^2 + k - 8 < 0$

$\Rightarrow$ $k^2 - k - 6 < 0$

$\Rightarrow$ $-2 < k < 3$.

E-3. Here coefficient of x^2 is always positive



$\therefore$ $f(0) < 0$

$\Rightarrow$ $(a^2 + 4)(a - 2)(a + 2) < 0$

$\Rightarrow$ $a \in (-2, 2)$

E-4.✍ (i) $D > 0$

$4a^2 - 4(a^2 - 1) > 0$

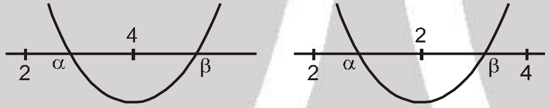
$4 > 0 \quad \forall\, x \in R$

(ii) $f(2)\, f(4) < 0$

$(4 - 4a + a^2 - 1)(16 - 8a + a^2 - 1) < 0$

$(a - 3)^2 (a - 1)(a - 5) < 0$

$a \in (1, 5) - \{3\}$



E-5.✍ $x^2 + 2(k - 3)x + 9 = 0$ ......(i)

Roots α, β of equation (i) are distinct & lies between –6 and 1

$D > 0 \Rightarrow 4(K - 3)^2 - 36 > 0 \Rightarrow k(k - 6) > 0$

$\Rightarrow$ $k \in (-\infty, 0) \cup (6, \infty)$ ......(ii)

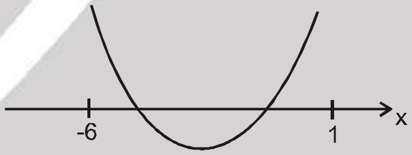
$f(1) > 0 \Rightarrow 1 + 2(k - 3) + 9 > 0$

$\Rightarrow$ $2k + 4 > 0$

$\Rightarrow$ $k \in (-2, \infty)$ ......(iii)

$f(-6) > 0 \Rightarrow 36 - 12(k - 3) + 9 > 0$

$\Rightarrow$ $4k - 27 < 0 \Rightarrow k \in \left(-\infty, \dfrac{27}{4}\right)$ ......(iv)



$-6 < -\dfrac{b}{2a} < 1 \Rightarrow -6 < \dfrac{-2(K - 3)}{2} < 1$

$\Rightarrow -1 < k - 3 < 6 \quad \Rightarrow 2 < k < 9$ ......(v)

(ii) $\cap$ (iii) $\cap$ (iv) $\cap$ (v) $\Rightarrow k \in \left(6, \dfrac{27}{4}\right)$.

F-1.✍ If α is one of the root of $a_1x^2 + b_1x + c_1 = 0$. Then $\dfrac{1}{\alpha}$ will be a root of $ax^2 + bx + c = 0$

$\Rightarrow c\alpha^2 + b\alpha + a = 0$ & $a_1\alpha^2 + b_1\alpha + c_1 = 0$ have one common root.

$\therefore$ applying the condition for one common root we get $(aa_1 - cc_1)^2 = (bc_1 - ab_1)(b_1c - a_1b)$

F-2. Given equation are

$x^2 - 11x + a = 0$ ....(i)

$x^2 - 14x + 2a = 0$ ....(ii)

Multiplying equation (i) by 2 and then subtracting, we get $x^2 - 8x = 0 \Rightarrow x = 0, 8$

If $x = 0, \quad a = 0$

If $x = 8, \quad a = 24$

F-3. $ax^2 + bx + c = 0 \Rightarrow bx^2 + cx + a = 0$ have a common root, say α

$$\therefore \quad a\alpha^2 + b\alpha + c = 0 \Rightarrow \qquad b\alpha^2 + c\alpha + a = 0 \Rightarrow \frac{\alpha^2}{ab - c} = \frac{\alpha}{bc - a^2} = \frac{1}{ac - b^2}$$

$$\alpha^2 = \frac{ab - c^2}{ac - b^2}, \quad \alpha = \frac{bc - a^2}{ac - b^2} \Rightarrow \left(\frac{ab - c^2}{ac - b^2}\right) = \left(\frac{bc - a^2}{ac - b^2}\right)^2 \Rightarrow \quad (ab - c^2)(ac - b^2) = (bc - a^2)^2$$

$$\Rightarrow a^3 + b^3 + c^3 = 3abc \qquad [\because a \neq 0] \qquad \Rightarrow \frac{a^3 + b^3 + c^3}{abc} = 3 \quad \text{Ans.}$$

<u>Aliter :</u>

By observation, x = 1 is the common root

$\therefore$ $a + b + c = 0$ $\therefore$ $a^3 + b^2 + c^3 = 3abc$ or $= 3$.

F-4. Let α is the common root hence $\alpha^2 + p\alpha + q = 0 \Rightarrow \alpha^2 + q\alpha + p = 0$

$$\frac{\alpha^2}{p^2 - q^2} = \frac{\alpha}{q - p} = \frac{1}{q - p} \Rightarrow \alpha^2 = -(p + q), \quad \alpha = 1 \quad \Rightarrow -(p + q) = 1 \Rightarrow p + q + 1 = 0$$

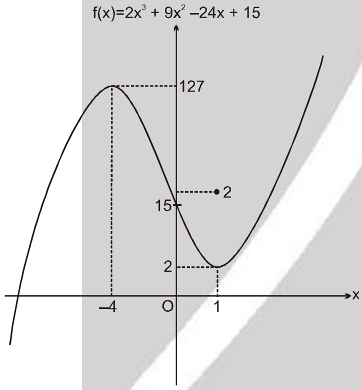
Let other roots be β and δ then $\alpha + \beta = -p$, $\alpha\beta = q \Rightarrow \alpha + \delta = -q$, $\alpha\delta = p$

$$\beta - \delta = q - p, \quad \frac{\beta}{\delta} = \frac{q}{p} \Rightarrow \frac{\beta - \delta}{\delta} = \frac{q - p}{p} \Rightarrow \frac{q - p}{\delta} = \frac{q - p}{p} \Rightarrow \quad \delta = p \Rightarrow \beta = q$$

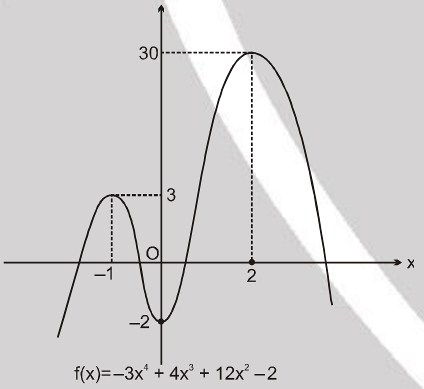
Equation having β, δ as roots

$x^2 - (\beta + \delta)x + \beta\delta = 0 \quad \Rightarrow x^2 - (p + q)x + pq = 0 \qquad \Rightarrow \qquad x^2 + x + pq = 0$ $[\because p + q = -1]$

F-5.✍ (i)



(ii)



F-6.✍ $f(x) = x^3 - 3x^2 + 2$

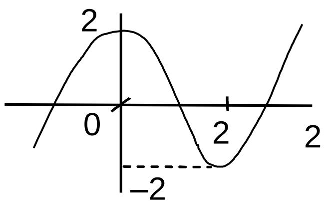
$f'(x) = 3x^2 - 6x = 3x(x - 2) = 0$

$f(0) = 2$

$f(2) = 8 - 12 + 2 = -2$

(i) $k \in [-2, 2]$

(ii) $k \in (-\infty, -2)$ $(2, \infty)$



PART - II

A-1. $x = 1$ is root. Let other root $= \alpha$

$\therefore$ Product of the roots $= (1)(\alpha) = \dfrac{a-b}{b-c} \quad \Rightarrow \quad$ roots are $1, \dfrac{a-b}{b-c}$

A-2. $\alpha + \beta = -p \quad \Rightarrow \quad \alpha\beta = q \quad \Rightarrow \quad \gamma + \delta = -p \quad \Rightarrow \quad \gamma\delta = -r$

$(\alpha - \gamma)(\alpha - \delta) = \alpha^2 - \alpha(\gamma + \delta) + \gamma\delta = \alpha^2 + p\alpha - r = \alpha(\alpha + p) - r = -\alpha\beta - r = -q - r = -(q + r)$

A-3. $(\alpha - \beta) = 4 \quad \Rightarrow \quad (\alpha - \beta)^2 = 16 \quad \Rightarrow \quad (\alpha + \beta)^2 - 4\alpha\beta = 16$

$\Rightarrow \quad 9 - 4\alpha\beta = 16 \quad \Rightarrow \quad \alpha\beta = -\dfrac{7}{4} \quad \Rightarrow \quad$ equation is $x^2 - 3x - \dfrac{7}{4} = 0$

A-4. $3x^2 + px + 3 = 0 \begin{matrix} \alpha^2 \\ \alpha \end{matrix} \quad \because \quad \alpha + \alpha^2 = -\dfrac{p}{3}$ .... (i)

$\alpha^3 = 1, \quad \Rightarrow \quad \alpha = 1, \omega, \omega^2 \quad \because \quad \alpha \neq 1$

$\therefore \quad \alpha = \omega$ or $\alpha = \omega^2$ put is (i) $\quad \therefore \quad p = 3$

A-5. S_1 : $x^2 - bx + c = 0 \begin{matrix} \alpha \\ \beta \end{matrix}$

$\because \quad |\alpha - \beta| = 1 \quad \Rightarrow \quad (\alpha - \beta)^2 = 1 \quad \Rightarrow \quad b^2 - 4c = 1.$

S_2 : $\because \quad \alpha + \beta = 1$ and $\alpha\beta = 3$

$\because \quad \alpha^4 + \beta^4 = (\alpha^2 + \beta^2)^2 - 2\alpha^2\beta^2 = [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2(\alpha\beta)^2 = (1 - 6)^2 - 2(9) = 25 - 18 = 7$

S_3 : $\because \quad \Sigma\alpha = 7 \Rightarrow \Sigma\alpha\beta = 16 \Rightarrow \alpha\beta\gamma = 12$

$\therefore \quad \Sigma\alpha^2 = (\Sigma\alpha)^2 - 2(\Sigma\alpha\beta) = 49 - 32$

$\therefore \quad \alpha^2 + \beta^2 + \gamma^2 = 17$

B-1. Let the roots be $\alpha, \beta, -\beta$ then $\quad \alpha + \beta - \beta = p$

$\Rightarrow \quad \alpha = p$ ...(1) and $\quad \alpha\beta - \alpha\beta - \beta^2 = q \quad \Rightarrow \quad \beta^2 = -q$ ...(2)

also $-\alpha\beta^2 = r \Rightarrow pq = r$ [using (1)].

B-2.✍ $x^3 - x - 1 = 0 \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix}$ then $\alpha^3 - \alpha - 1 = 0$ ....(1)

Let $\dfrac{1+\alpha}{1-\alpha} = y \Rightarrow \alpha = \dfrac{y-1}{y+1}$ from equation (1) $\left(\dfrac{y-1}{y+1}\right)^3 - \left(\dfrac{y-1}{y+1}\right) - 1 = 0 \quad y^3 + 7y^2 - y + 1 = 0 \begin{matrix} \frac{1+\alpha}{1-\alpha} \\ \frac{1+\beta}{1-\beta} \\ \frac{1+\gamma}{1-\gamma} \end{matrix}$

then $\dfrac{1+\alpha}{1-\alpha} + \dfrac{1+\beta}{1-\beta} + \dfrac{1+\gamma}{1-\gamma} = -7$ **Ans.**

B-3.✍ Clearly $(x - a)(x - b)(x - c) = -(x - \alpha)(x - \beta)(x - \gamma)$

$\therefore$ if α, β, γ are the roots of given equation

then $(x - \alpha)(x - \beta)(x - \gamma) + d = 0$ will have roots a, b, c.

B-4_. $\alpha + \beta + \gamma = 0 \quad \Rightarrow \quad \dfrac{\alpha^3 + \beta^3 + \gamma^3}{\alpha^2 + \beta^2 + \gamma^2} = \dfrac{3\alpha\beta\gamma}{-2(\alpha\beta + \beta\gamma + \gamma\alpha)} = \dfrac{3b}{2a}$

B-5_.✍ Let roots are $\alpha, -\alpha, \beta, \gamma$ then $\beta + \gamma = 2$ and $-\alpha^2(\beta + \gamma) = -8$

$\Rightarrow \quad \alpha^2 = 4 \quad \Rightarrow \quad \alpha \pm 2$

$\Rightarrow \quad 2^4 - 2(2^3) + a(2)^2 + 8(2) + b = 0$

$\Rightarrow \quad 4a + b = -16$

C-1. $\alpha + \beta = \sqrt{3}$

$\sqrt{3} + 2 + \beta = \sqrt{3} \quad \Rightarrow \beta = -2$

C-2. $D_1 = b^2 - 4.2.c > 0 \Rightarrow b^2 - 8c > 0$

$D_2 = (b - 4c)^2\ 4.2c.\ (2c - b + 1) = b^2 + 16c^2 - 8bc - 16c^2 + 8bc - 8c = b^2 - 8c > 0$

C-3. $\alpha + \alpha^2 = -\ell, \alpha^3 = m$

$\alpha^6 + \alpha^3 + 3\alpha^2(\alpha + \alpha^2) = -\ell^3$

$\Rightarrow \quad m^2 + m + 3m(-\ell) + \ell^3 = 0 \quad \Rightarrow \quad m^2 + m(1 - 3\ell) + \ell^3 = 0$

$\Rightarrow \quad (1 - 3\ell)^2 - 4\ell^3 \geq 0$ {because $m \in R$}

$\Rightarrow \quad 4\ell^3 - 9\ell^2 + 6\ell - 1 \leq 0 \quad \Rightarrow \quad (\ell - 1)^2(4\ell - 1) \leq 0 \quad \Rightarrow \quad \ell \in (-\infty, \frac{1}{4}] \cup \{1\}$

C-4. $D = b^2 - 4ac = 20d^2 \quad \Rightarrow \quad \sqrt{D} = 2\sqrt{5}d$ So roots are irrational.

C-5. $D = b^2 - 4ac = b^2 - 4a(-4a - 2b) = b^2 + 16a^2 + 8ab$

Since $ab > 0 \Rightarrow D > 0$. So equation has real roots.

C-6. For integral roots, D of equation should be perfect sq.

∵ $D = 4(1+n)$

By observation, for $n \in N$, D should be perfect sq. of even integer.

So $D = 4(1+n) = 6^2, 8^2, 10^2, 12^2, 14^2, 16^2, 18^2, 20^2$. No. of values of n = 8.

D-1. $x^2 + bx + c = 0 \begin{matrix} \nearrow \alpha \\ \searrow \beta \end{matrix}$

∵ $\alpha + \beta = -b$

$\Rightarrow \quad \alpha\beta = c$

∵ Sum is +ve and product is – ve.

∴ $\alpha < 0 < \beta < |\alpha|$

D-2. $a > 0$ & $c < 0$ is satisfied by (B) only [∵ $f(0) = 0$ & $a > 0$] Further in (B)

$-\frac{b}{2a} > 0 \quad \Rightarrow \quad b < 0 \quad$ [∵ $a > 0$].

D-3. For $y = ax^2 + bx + c$ to have the sign always same of 'a' $b^2 - 4ac < 0 \Rightarrow 4ac > b^2$

D-4. Here for $D < 0$, entire graph will be above x-axis (∵ $a > 0$)

$\Rightarrow \quad (k - 1)^2 - 36 < 0 \Rightarrow \quad (k - 7)(k + 5) < 0 \quad \Rightarrow \quad -5 < k < 7$

D-5. Let $f(x) = ax^2 - bx + 1$. Given $D < 0$ & $f(0) = 1 > 0$

∴ possible graph is as shown

i.e. $f(x) > 0 \ \ x \in R$ or $f(-1) > 0 \Rightarrow f(-1) = a + b + 1 > 0$

D-6. $x^2 + ax + b = 0 \quad \Rightarrow \quad a + b = -a \quad \Rightarrow \quad 2a + b = 0$ and $ab = b$

$ab - b = 0 \quad \Rightarrow \quad b(a - 1) = 0 \quad \Rightarrow \quad$ Either $b = 0$ or $a = 1$

But $b \neq 0$ (given)

∴ $a = 1$

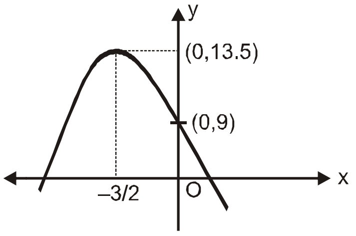
∴ $b = -2$

∴ $f(x) = x^2 + x - 2$

Least value occurs at $x = -\frac{1}{2}$

Least value = $\frac{1}{4} - \frac{1}{2} - 2 = -\frac{9}{4}$

D-7.



$y = -2x^2 - 6x + 9$

$\therefore \quad \frac{-b}{2a} = \frac{6}{2(-2)} = -\frac{3}{2} = -1.5$ & D = 36 – 4(–2) (9) = 36 + 72 = 108

$\therefore \quad -\frac{D}{4a} = -\frac{108}{4(-2)} = +\frac{108}{8} = 13.5$

$\Rightarrow y \in (-\infty, 13.5]$

D-8. min. f(x) > max. g(x)

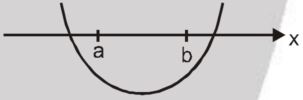
$\Rightarrow \quad -b^2 + 2c^2 > b^2 + c^2$

$\Rightarrow \quad c^2 > 2b^2$

$\Rightarrow \quad |c| > |b|\sqrt{2}$

E-1. (x – a) (x – b) – 1 = 0. Let f(x) = (x – a) (x – b) – 1 ⇒ f(a) = –1 ⇒ f(b) = – 1

∵ the graph will be mouth opening upwards.



∴ (D) will be correct

E-2. $x^2 - 2px + (8p - 15) = 0$

f(1) < 0 and f(2) < 0

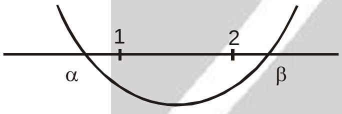
⇒ f(1) = 1 – 2p + 8p – 15 < 0

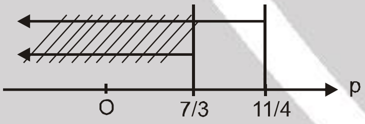
⇒ p < 7/3

and f(2) = 4 – 4p + 8p – 15 < 0

$\Rightarrow 4p - 11 < 0 \quad \Rightarrow p < \frac{11}{4}$

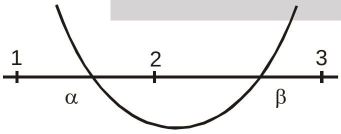
Hence $p \in (-\infty, \ 7/3)$ Ans.

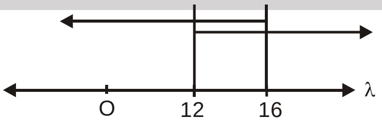




E-3. $4x^2 - 16x + \lambda = 0$

f(1) > 0 and f(2) < 0 and f(3) > 0





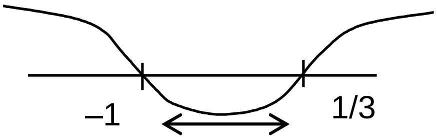
f(1) = 4 – 16 + λ > 0	⇒	λ > 12	...(i)
f(2) = 16 – 32 + λ < 0	⇒	λ < 16	...(ii)
f(3) = 36 – 48 + λ > 0	⇒	λ > 12	...(iii)

by (i) ∩ (ii) ∩ (iii)

12 < λ < 16. So λ = 13, 14, 15 has 3 integral solutions.

E-4. $D \geq 0$

$(k-1)^2 - 4k^2 \geq 0 \Rightarrow (k+1)(3k-1) \leq 0$



Case- I Exactly one in (1, 2)

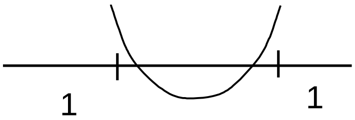
$f(1)\,f(2) < 0 \Rightarrow (1-k+1+1)(4-2k+2+k^2)<0$

$\Rightarrow (3-k)(k^2-2k+6) < 0$

$\Rightarrow 3-k < 0 \Rightarrow k > 3$

if one roots is – 1 then k = 3

$-1 \times k = 9 \Rightarrow k = -9 \Rightarrow k \neq 3$



if one root is 2 then $k^2 - 2k + 6 = 0$ not possible

$\Rightarrow k \in \phi$

Case-II If both roots lie in (1, 2)

$f(1) > 0$ & $f(2) > 0$

$3-k > 0 \Rightarrow k < 3$ & $k^2 - 2k + 6 > 0 \Rightarrow k \in \phi$

F-1. $x^2(6k+2) + rx + (3k-1) = 0 \quad \Rightarrow \quad x^2(12k+4) + px + 6k - 2 = 0$

For both roots common, $\frac{6k+2}{12k+4} = \frac{r}{p} = \frac{3k-1}{2(3k-1)}$

$\Rightarrow \frac{r}{p} = \frac{1}{2}$

$\Rightarrow 2r - p = 0$ Ans.

F-2. $3x^2 - 17x + 10 = 0 \quad \Rightarrow \quad x = \frac{2}{3}$ or 5

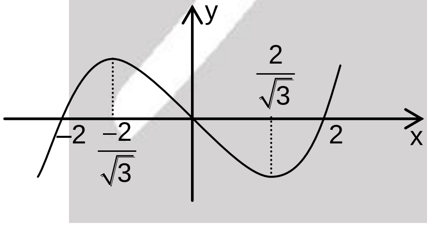
If x = 5 is comman $\Rightarrow \lambda = 0$

If $x = \frac{2}{3}$ is comman $\Rightarrow \lambda = \frac{26}{9}$; Sum = $\frac{26}{9}$

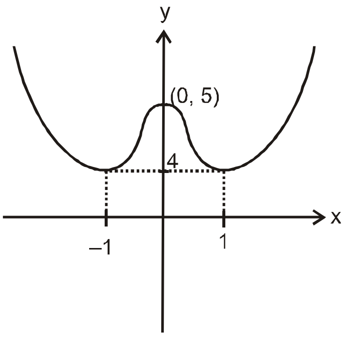
F-3. $D_1 = 4a^2b^2 - 8a^2b^2 = -4a^2b^2 < 0$ img. root ; $D_2 = 4p^2q^2 - 4p^2q^2 = 0$ equal, real roots

So no common roots.

F-4. (C)



F-5. (D)



PART - III

1. (A) $x^2 - 8x + k = 0 \left\langle \begin{matrix}\alpha \\ \alpha + 4 = \beta\end{matrix}\right.$ $\quad\because\quad (\beta - \alpha)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$\Rightarrow \quad 16 = 64 - 4k \Rightarrow 4k = 48 \Rightarrow \quad k = 12$

(B) $x^2 + 2x - 4 = 0 \quad \Rightarrow \frac{1}{x^2} + \frac{2}{x} - 4 = 0$ has roots $\frac{1}{\alpha}, \frac{1}{\beta} \quad \Rightarrow \quad -4x^2 + 2x + 1 = 0$

$4x^2 - 2x - 1 = 0 \Rightarrow x^2 - \frac{1}{2}x - \frac{1}{4} = 0 \Rightarrow \quad q + r = \frac{-1}{2} - \frac{1}{4} = \frac{-3}{4} \Rightarrow \; 4 = \frac{-3}{q+r}$

(C) $\alpha + \beta = 0, \;\; \alpha\beta = \frac{c}{a} \;\Rightarrow\; \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 - \alpha\beta + \beta^2) = 0.$

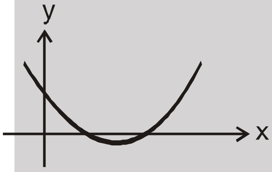
(D) $x^2 - kx + 36 = 0 \left\langle \begin{matrix}\alpha \\ \beta\end{matrix}\right. \Rightarrow \alpha + \beta = k, \alpha\beta = 36 \Rightarrow 36 = (1)(36) = (2)(18) = (3)(12) = (4)(9) = (6)(6)$

or $36 = (-1)(-36) = (-2)(-18) = (-3)(-12) = (-4)(-9) = (-6)(-6)$ i.e. 10 values of k are possible.

2.✍ $f(x) = ax^2 + bx + c \; (a \neq 0)$ $\qquad a + b = -b/a$

$D = b^2 - 4ac$ $\qquad \alpha b = c/a$

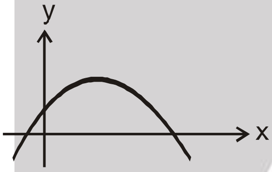
(P)



from graph

$D > 0 \; \frac{-b}{a} > 0 \quad \Rightarrow \quad b < 0 \quad ; \quad a > 0 \; \frac{c}{a} > 0 \quad \Rightarrow \quad c > 0$

(q)

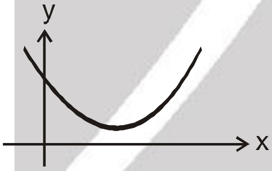


$D > 0$

$a < 0 \qquad abc < 0$

$f(0) > 0 \quad \Rightarrow \quad c > 0, \frac{abc}{D} < 0 \quad \Rightarrow \frac{-b}{2a} > 0 \quad \Rightarrow b > 0$

(r)

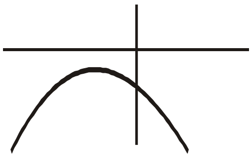


$D < 0$

$f(0) > 0 \quad \Rightarrow \quad C > 0 \quad abc < 0$

$\Rightarrow \quad a > 0 \quad \frac{abc}{D} > 0 \quad \Rightarrow \frac{-b}{2a} > 0 \quad \Rightarrow b < 0$ (A, D)

(s)



$D < 0$

$a < 0$

$f(0) < 0 \Rightarrow \quad C < 0 \quad \Rightarrow \frac{-b}{2a} < 0 \quad \Rightarrow \quad b < 0$

$abc < 0 \Rightarrow \quad \frac{abc}{D} > 0$ (A, D)

3.✍ **(A) q, s, t (B) p, t (C) r (D) q, s.**

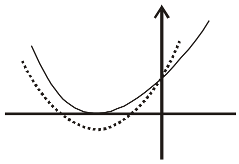
Exercise # 2

PART - I

1. $a > 0, b > 0$ and $c > 0 \quad \Rightarrow \quad ax^2 + bx + c = 0 \begin{matrix} \diagup\ \alpha \\ \diagdown\ \beta \end{matrix}$

$\alpha + \beta = - b/a = -$ ve, $\alpha\beta = \dfrac{c}{a} = +$ ve

–ve real part



2. $x^2 + 2ax + b = 0 \begin{matrix} \diagup\ \alpha \\ \diagdown\ \beta \end{matrix} \quad \Rightarrow \quad 0 < |\alpha - \beta| \le 2m \quad \Rightarrow \quad 0 < \sqrt{(\alpha+\beta)^2 - 4\ \ \alpha\beta} \le 2m$

$0 < 4a^2 - 4b \le 4m^2 \quad \Rightarrow \quad a^2 - m^2 \le b < a^2 \quad \Rightarrow \quad b \in [a^2 - m^2, a^2)$

3. Sum of roots < 1

$\Rightarrow \quad \lambda^2 - 5\lambda + 5 < 1 \quad \Rightarrow \quad (\lambda - 1)(\lambda - 4) < 0 \quad \Rightarrow \quad 1 < \lambda < 4 \qquad ...(1)$

$\Rightarrow$ Product of roots < 1

$\Rightarrow \quad 2\lambda^2 - 3\lambda - 5 < 0 \quad \Rightarrow \quad (2\lambda - 5)(\lambda + 1) < 0 \quad \Rightarrow \quad -1 < \lambda < \dfrac{5}{2} \qquad ...(2)$

(1) & (2) $\quad \Rightarrow \quad 1 < \lambda < \dfrac{5}{2}$.

4. Dis. of $x^2 + px + 3q$ is $p^2 - 12q \equiv D_1$

Dis. of $-x^2 + rx + q$ is $r^2 + 4q \equiv D_2$

Dis. of $-x^2 + sx - 2q$ is $s^2 - 8q \equiv D_3$

Case 1 : If $q < 0$, then $D_1 > 0$, $D_3 > 0$ and D_2 may or may not be positive

Case 2 : If $q > 0$, then $D_2 > 0$ and D_1, D_3 may or may not be positive

Case 3 : If $q = 0$, then $D_1 \ge 0$, $D_2 \ge 0$ and $D_3 \ge 0$

from **Case 1, Case 2** and **Case 3** we can say that the given equation has atleast two real roots.

5.✍ We, know that $a + b > c$, $b + c > a$ and $c + a > b \quad \Rightarrow \quad c - a < b,\ a - b < c,\ b - c < a$

squaring on both sides and adding $(c - a)^2 + (a - b)^2 + (b - c)^2 < a^2 + b^2 + c^2$

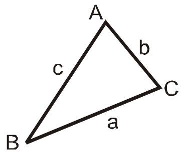
$a^2 + b^2 + c^2 - 2(ab + bc + ca) < 0 \quad \Rightarrow \quad (a + b + c)^2 - 4(ab + bc + ca) < 0$

$\Rightarrow \quad \dfrac{(a+b+c)^2}{ab+bc+ca} < 4 \qquad ....(i)$

Now roots of equation $x^2 + 2(a + b + c)x + 3\lambda(ab + bc + ca) = 0$ are real, then $D \ge 0$

$\Rightarrow \quad 4(a + b + c)^2 - 4.\ 3\lambda(ab + bc + ca) \ge 0 \quad \Rightarrow \quad \dfrac{(a+b+c)^2}{ab+bc+ca} \ge 3\lambda$

So $\quad 3\lambda \le \dfrac{(a+b+c)^2}{ab+bc+ca} < 4 \quad \Rightarrow \quad \lambda < \dfrac{4}{3}$



6.✍ Let biqhadratic is $ax^4 + bx^3 + cx^2 + dx + e = 0$

$\Rightarrow$ $a + b + c + d + e = 0$ as a, b, c, d, e $\in \{-9, -5, 3, 4, 7\}$

Hence x = 1 is a root. So real root will be atleast two.

$ax^4 + bx^3 + cx^2 + dx + e = 0 \Rightarrow a + b + c + d + e = 0$ a, b, c, d, e $\in \{-9, -5, 3, 4, 7\}$

7.✍ $x^2 + px + q = 0 \Rightarrow D_1 = p^2 - 4q$ .....(1)

$x^2 + rx + s = 0 \Rightarrow D_2 = r^2 - 4s$ .....(2)

$D_1 + D_2 = p^2 + r^2 - 4(q + s)$ $[\because pr = 2(q + s)] = (p - r)^2 > 0$

Since $D_1 + D_2$ is +ve, so atleast

one of the equation has real roots.

8. $\pi^x = -2x^2 + 6x - 9 \Rightarrow D = 36 - 4(-2)(-9) = 36 - 72 < 0$ & $a < 0$

So quadratic expression $-2x^2 + 6x - 9$ is always negative whereas π^x is always +ve

$\therefore$ Equation will not hold for any x.

$\therefore$ $x \in \phi$ So $\pi^x = -2x^2 + 6x - 9$ has no solution.

9.✍ $(\lambda + 2)(\lambda - 1)x^2 + (\lambda + 2)x - 1 < 0 \ \forall x \in R \Rightarrow (\lambda + 2)(\lambda - 1) < 0$

$\Rightarrow -2 < \lambda < 1$ ...(1) $(a < 0)$

and $(\lambda + 2)^2 + 4(\lambda + 2)(\lambda - 1) < 0$ $(D < 0)$

$\Rightarrow (\lambda + 2)(\lambda + 2 + 4\lambda - 4) < 0 \Rightarrow (\lambda + 2)(5\lambda - 2) < 0$

$\Rightarrow -2 < \lambda < \frac{2}{5}$ ...(2)

(1) & (2) $\Rightarrow \lambda \in \left(-2, \frac{2}{5}\right)$ Also $\lambda = -2 \Rightarrow 0 < 1$ which is true

$\therefore$ Required interval is $\lambda \in \left[-2, \frac{2}{5}\right)$

10. $C_1 : b^2 - 4ac \geq 0$; C_2 : a, –b, c are of same sign $ax^2 + bx + c = 0$ has real roots then $D \geq 0$ i.e. C_1 must be satisfied

(i) Let a, –b, c > 0 then $-\frac{b}{2a} > 0$

(ii) Let a, – b, c < 0 then $-\frac{b}{2a} > 0$

Hence, for roots to be + ve, C_2 must be satisfied. Thus both C_1, C_2 are satisfied

11.✍ Let $y = \frac{x^2 - x + c}{x^2 + x + 2c}$; $x \in R$ and $y \in R$ $\Rightarrow (y - 1)x^2 + (y + 1)x + 2yc - c = 0$

$\because$ $x \in R \Rightarrow D \geq 0 \Rightarrow (y + 1)^2 - 4c(y - 1)(2y - 1) \geq 0$

$\Rightarrow y^2 + 1 + 2y - 4c[2y^2 - 3y + 1] \geq 0 \Rightarrow (1 - 8c)y^2 + (2 + 12c)y + 1 - 4c \geq 0$ .... (1)

Now for all $y \in R$ (1) will be true if $1 - 8c > 0 \Rightarrow c < \frac{1}{8}$ and $D \leq 0$

$\Rightarrow 4(1 + 6c)^2 - 4(1 - 8c)(1 - 4c) \leq 0 \Rightarrow 1 + 36c^2 + 12c - 1 - 32c^2 + 12c \leq 0$

$\Rightarrow 4c^2 + 24c \leq 0 \Rightarrow -6 \leq c \leq 0$

But c = –6 and c = 0 will not satisfy given condition

$\therefore$ $c \in (-6, 0)$

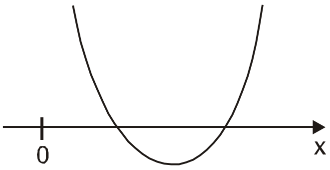
12. $(2 - x)(x + 1) = p \Rightarrow x^2 - x + (p - 2) = 0$ ...(1)

(1) has both roots distinct & positive

$\therefore$ (i) $D > 0$ (ii) $f(0) > 0$ (iii) $\frac{-b}{2a} > 0$

(i) $D > 0 \Rightarrow p < \frac{9}{4}$ (ii) $f(0) > 0 \Rightarrow p > 2$ (iii) $\frac{-b}{2a} = \frac{1}{2} > 0$ (always true)

$\therefore$ (i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow p \in \left(2,\ \frac{9}{4}\right)$.



13.✍ $(a - 1)(x^2 + x + 1)^2 - (a + 1)(x^4 + x^2 + 1) = 0$ ....(1)

$\because$ $x^4 + x^2 + 1 = (x^2 + x + 1)\ (x^2 - x + 1)$

$\therefore$ (1) becomes

$\Rightarrow$ $(x^2 + x + 1)\ [(x^2 + x + 1)(a - 1) - (a + 1)(x^2 - x + 1)] = 0$

$\Rightarrow$ $(x^2 + x + 1)(x^2 - ax + 1) = 0$

Here two roots are imaginary and for other two roots to be real D > 0

$\Rightarrow$ $a^2 - 4 > 0$ $\Rightarrow$ $a \in (-\infty, -2) \cup (2, \infty)$

14.✍ $x^3 + 5x^2 + px + q = 0 \begin{matrix}\alpha\\ \beta\\ x_1\end{matrix}$ $\Rightarrow$ $\alpha + \beta + x_1 = -5, \alpha\ \beta + \beta\ x_1 + \alpha\ x_1 = p$ ...(1)

$x^3 + 7x^2 + px + r = 0 \begin{matrix}\alpha\\ \beta\\ x_2\end{matrix}$ $\Rightarrow$ $\alpha + \beta + x_2 = -7, \alpha\beta + \beta x_2 + \alpha x_2 = p$ ...(2)

Subtracting (2) from (1)

$\alpha\ \beta + \beta\ x_1 + \alpha\ x_1 = p$ $\Rightarrow$ $\frac{\alpha\ \ \beta + \beta x_2 + \alpha x_2 = p}{\alpha\ \ (x_1 - x_2) + \beta\ \ (x_1 - x_2) = 0}$ $\Rightarrow$ $(x_1 - x_2)(\alpha - \beta) = 0\ [x_1 \neq x_2]$

$\therefore$ $\alpha + \beta = 0$ $\Rightarrow$ $x_1 = -5 \Rightarrow$ $x_2 = -7$

15.✍ $\because$ $a^2 + b^2 + c^2 = 1$ $\Rightarrow$ $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) \geq 0$

$\Rightarrow$ $1 + 2(ab + bc + ca) \geq 0$ $\Rightarrow$ $(ab + bc + ca) \geq -\frac{1}{2}$ ....(1)

$\because$ $a^2 + b^2 + c^2 - (ab + bc + ca) \geq 0$ $\Rightarrow$ $(ab + bc + ca) \leq 1$ ....(2)

$\therefore$ From (1) and (2) we can say that $(ab + bc + ca) \in \left[-\frac{1}{2}, 1\right]$

PART - II

1.✍ $\left(x^2 + 3x + 2\right)\left(x^2 + 3x\right) = 120$

Let $x^2 + 3x = y \Rightarrow y^2 + 2y - 120 = 0$ $\Rightarrow$ $(y + 12)(y - 10) = 0$

$\Rightarrow$ $y = -12 \Rightarrow x^2 + 3x + 12 = 0$ $\Rightarrow$ $x \in \phi$

$y = 10 \Rightarrow x^2 + 3x - 10 = 0$ $\Rightarrow$ $(x + 5)(x - 2) = 0$

$\Rightarrow$ $x = \{-5, 2\}$

x = 2, – 5 are only two integer roots.

2.✍ $\left(5 + 2\sqrt{6}\right)^{x^2-3} + \frac{1}{\left(5 + 2\sqrt{6}\right)^{x^2-3}} = 10$

$\Rightarrow$ $t + \frac{1}{t} = 10$

$\Rightarrow$ $t^2 - 10t + 1 = 0$

$t = \frac{10 \pm \sqrt{96}}{2} = 5 \pm 2\sqrt{6}$

$\Rightarrow$ $(5 + 2\sqrt{6})^{x^2-3} = (5 + 2\sqrt{6})$ or $\frac{1}{5 + 2\sqrt{6}}$

$\Rightarrow$ $x^2 - 3 = 1$ or $x^2 - 3 = -1$

$\Rightarrow$ x = 2 or –2 or $-\sqrt{2}$ or $\sqrt{2}$ Product 8

3.✍ $x^2 + px + 1 = 0 \begin{matrix} \nearrow a \\ \searrow b \end{matrix}$ $a + b = -p,\ ab = 1$; $x^2 + qx + 1 = 0 \begin{matrix} \nearrow c \\ \searrow d \end{matrix}$ $c + d = -q,\ cd = 1$

$a + b = -p,\ ab = 1 \Rightarrow c + d = -q,\ cd = 1$

RHS $= (a - c)(b - c)(a + d)(b + d) = (ab - ac - bc + c^2)(ab + ad + bd + d^2)$

$= (1 - ac - bc + c^2)(1 + ad + bd + d^2)$

$= 1 + ad + bd + d^2 - ac - a^2cd - abcd - acd^2 - bc - abcd - b^2cd - bcd^2 + c^2 + adc^2 + bdc^2 + c^2d^2$

$= 1 + ad + bd + d^2 - ac - a^2 - 1 - ad - bc - 1 - b^2 - bd + c^2 + ac + bc + 1$ $[\because ab = cd = 1]$

$= c^2 + d^2 - a^2 - b^2 = (c + d)^2 - 2cd - (a + b)^2 + 2ab = q^2 - 2 - p^2 + 2 = q^2 - p^2 =$ LHS. **Proved.**

Aliter:

RHS $= (ab - c(a + b) + c^2)(ab + d(ab + d(a + b) + d^2) = (c^2 + pc + 1)(1 - pd + d^2)$ ...(1)

Since c & d are the roots of the equation $x^2 + qx + 1 = 0$

$\therefore\ c^2 + qc + 1 = 0 \Rightarrow c^2 + 1 = -qc$ & $d^2 + qd + 1 = 0 \Rightarrow d^2 + 1 = -qd$.

$\therefore$ (i) Becomes $= (pc - qc)(-pd - qd) = c(p - q)(-d)(p + q) = -cd\,(p^2 - q^2)$

$= cd\,(q^2 - p^2) = q^2 - p^2 =$ LHS. **Proved.**

4. $\because$ α, β are roots of $\lambda x^2 - (\lambda - 1)x + 5 = 0$

$\therefore$ $\alpha + \beta = \dfrac{\lambda - 1}{\lambda}$ and $\alpha\beta = \dfrac{5}{\lambda}$

$\because$ $\dfrac{\alpha}{\beta} + \dfrac{\beta}{\alpha} = 4 \Rightarrow \dfrac{\alpha^2 + \beta^2}{\alpha\beta} = 4 \Rightarrow (\alpha + \beta)^2 = 6\,\alpha\,\beta$

$\Rightarrow \dfrac{(\lambda - 1)^2}{\lambda^2} = \dfrac{30}{\lambda} \Rightarrow \lambda^2 - 32\lambda + 1 = 0$ ....(1)

$\because$ λ_1, λ_2 are roots of (1) $\therefore$ $\lambda_1 + \lambda_2 = 32$ and $\lambda_1\lambda_2 = 1$

$$\therefore \quad \frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{(\lambda_1 + \lambda_2)^2 - 2\lambda_1\lambda_2}{\lambda_1\lambda_2} = \frac{(32)^2 - 2}{1} = 1022 \Rightarrow \left(\frac{\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1}}{14}\right) = 73$$

5.✍ $\alpha + 2\alpha = -\dfrac{\ell}{\ell - m} \Rightarrow \alpha = -\dfrac{\ell}{3(\ell - m)}$ Also $2\alpha^2 = \dfrac{1}{\ell - m} \Rightarrow \dfrac{2\ell^2}{9(\ell - m)^2} = \dfrac{1}{\ell - m}$

$\Rightarrow 2\ell^2 - 9\ell + 9m = 0 \Rightarrow \ell \in R \Rightarrow D \ge 0 \Rightarrow 81 - 72m \ge 0 \Rightarrow m \le \dfrac{9}{8}$.

6. $\alpha.\beta = b\ ;\ \gamma\delta = b - 2 \Rightarrow \alpha\beta\gamma\delta = b(b - 2) = 24$

$\therefore$ $bx^2 + ax + 1 = 0$ has roots $\dfrac{1}{\alpha}, \dfrac{1}{\beta} \Rightarrow \dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{-a}{b}$

$(b - 2)x^2 - ax + 1 = 0$ has root $\dfrac{1}{\gamma}, \dfrac{1}{\delta} \Rightarrow \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{a}{b - 2}$

$\dfrac{1}{\gamma} + \dfrac{1}{\beta} + \dfrac{1}{\gamma} + \dfrac{1}{\delta} = \dfrac{-a}{b} + \dfrac{a}{b - 2} = \dfrac{5}{6}\ ;\ \dfrac{+2a}{b(b - 2)} = \dfrac{5}{6}\ ;\ \dfrac{+2a}{24} = \dfrac{5}{6}\ ;$ a = 10.

7.✍ $a^3 + b^3 + (-9)^3 = 3 \cdot a \cdot b\,(-9) \Rightarrow a + b - 9 = 0$ or $a = b = -9$. Which is rejected.

As $a > b > -9 \Rightarrow a + b - 9 = 0 \Rightarrow x = 1$ is a root

other root $= \dfrac{-9}{a}$. $\therefore$ $\alpha = \dfrac{-9}{a}$, $\beta = 1 \Rightarrow 4\beta - a\alpha = 4 - a\left(\dfrac{-9}{a}\right) = 4 + 9 = 13$.

8.✍ Let $t^2 - 2t + 2 = k \Rightarrow \alpha^2 - 6k\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\,k\alpha$

$a_{100} - 2a_{98} = \alpha^{100} - 2.\alpha^{98} - \beta^{100} + 2.\beta^{98} = \alpha^{98}(\alpha^2 - 2) - \beta^{98}(\beta^2 - 2) = 6k\,(\alpha^{99} - \beta^{99})$ $a_{100} - 2a_{98} = 6k.a_{99}$

$\dfrac{a_{100} - 2a_{98}}{a_{99}} = 6k = 6(t^2 - 2t + 2) = 6[(t - 1)^2 + 1]$ $\therefore$ min. value of $\dfrac{a_{100} - 2a_{98}}{a_{99}}$

9. $x^4 - Kx^3 + Kx^2 + Lx + M = 0 \begin{matrix}\nearrow \alpha \\ \nearrow \beta \\ \searrow \gamma \\ \searrow \delta\end{matrix} \Rightarrow \quad \sum \alpha = K, \; \sum \alpha\beta = K, \; \sum \alpha\beta\gamma = -L$

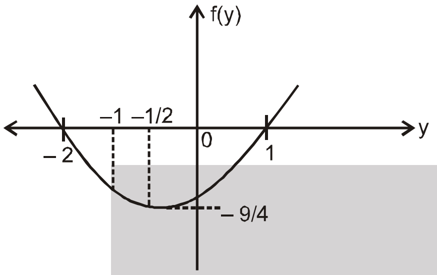
$\alpha\beta\gamma\delta = M \quad \Rightarrow \quad \alpha^2 + \beta^2 + \gamma^2 + \delta^2 = (\alpha + \beta + \gamma + \delta)^2 - 2\sum \alpha\beta$

$K^2 - 2K = (K - 1)^2 - 1 \quad \Rightarrow \quad (\alpha^2 + \beta^2 + \gamma^2 + \delta^2)_{\min} = -1$

10. $y = \frac{2x}{1 + x^2} \quad \Rightarrow \quad x^2y - 2x + y = 0 \quad \forall\, x \in R$

$D \geq 0 \quad 4 - 4y^2 \geq 0 \quad \Rightarrow \quad y \in [-1, 1]$ Now $f(y) = y^2 + y - 2$

$\Rightarrow f(y) \in \left[-\frac{9}{4}, 0\right] \quad \Rightarrow \quad a = \frac{-9}{4}, b = 0 \Rightarrow b - 4a = 0 - 4\left(\frac{-9}{4}\right) = 9$. **Ans.**



11. Let α, β, γ be the roots of $x^3 - Ax^2 + Bx - C = 0$ ...(1)
the roots of $x^3 + Px^2 + Qx - 19 = 0$ will be $(\alpha + 1), (\beta + 1), (\gamma + 1)$

$\therefore \quad (\alpha + 1)(\beta + 1)(\gamma + 1) = 19 \quad \Rightarrow \quad (\alpha\beta + \alpha + \beta + 1)(\gamma + 1) = 19$

$\Rightarrow \quad \alpha\beta\gamma + \alpha\gamma + \beta\gamma + \alpha\beta + \alpha + \beta + \gamma + 1 = 19 \quad \Rightarrow \quad C + B + A = 18$ [using (1)].

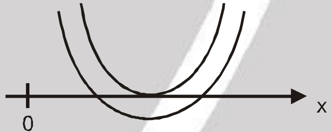
12. $\alpha + 2\alpha = 12x \quad \Rightarrow \quad \alpha = 4x \quad \Rightarrow \quad (\alpha)(2\alpha) = -f(x) - 64x$

$\Rightarrow \quad f(x) = -(32x^2 + 64x) \quad \Rightarrow \quad f(x) = -32(x^2 + 2x) \quad \Rightarrow \quad f(x) = -32((x + 1)^2 - 1)$

$\Rightarrow \quad f(x) \leq 32. \quad \Rightarrow$ Maximum value of f(x) is 32 f(x)

13. **Case-I :** Both the roots are positive $x^2 + 2(K - 1)x + (K + 5) = 0$

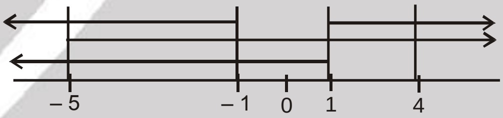
(i) $D \geq 0 \quad \Rightarrow \quad 4(K - 1)^2 - 4(K + 5) \geq 0 \Rightarrow \quad (K + 1)(K - 4) \geq 0$



(case-I)

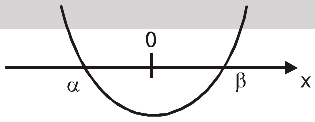
(ii) $f(0) > 0 \quad \Rightarrow \quad K + 5 > 0 \quad \Rightarrow \quad K > -5$

(iii) $-\frac{b}{2a} > 0 \quad \Rightarrow \quad \frac{2(1-k)}{2} > 0 \quad \Rightarrow \quad K < 1$



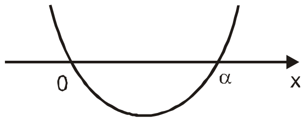
$K \in (-\infty, -1]$ ... (i)

Case-II : One root is +ve and other root is –ve $f(0) < 0 \Rightarrow k + 5 < 0 \Rightarrow \quad K < -5$ ... (ii)



(case-II)

Case-III : One root is zero and other is +ve $f(0) = 0$ & $\frac{-b}{2a} > 0 \quad \Rightarrow K = -5$ ... (iii)



(case-III)

Union of all the three cases give $K \in (-\infty, -1] = (-\infty, -b] \Rightarrow b = 1$. **Ans.**

14. **case- I :** Both roots are greater than 2.

or one root is 2 & other is greater than 2

$D \geq 0 \Rightarrow (a-3)^2 - 4a \geq 0 \Rightarrow a^2 - 10\,a + 9 \geq 0 \quad (a-1)(a-9) \geq 0$

$a \in (-\infty, 1] \cup [9, \infty)$ ... (i)

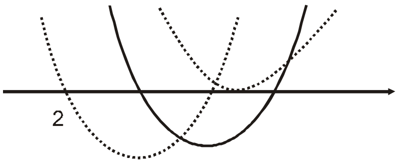
$\frac{-b}{2a} > 2 \Rightarrow \frac{a-b}{2} > 2 \Rightarrow a > 7$ .. (ii)

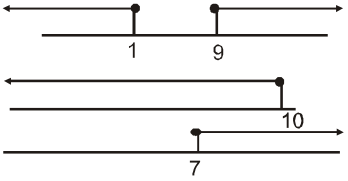
$f(2) \geq 0 \Rightarrow 4 - 2(a-3) + a \geq 0$

$-a + 10 \geq 0 \Rightarrow a \leq 10$ ... (iii)

(i) ∩ (ii) ∩ (iii) gives

$a \in [9, 10]$ ... (iv)





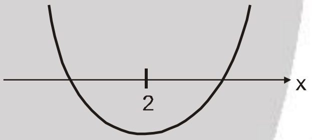
Case-II : One root is greater than 2

$f(2) < 0 \Rightarrow -a + 10 < 0$

$\Rightarrow a > 10 \Rightarrow a \in (10, \infty)$ ....(v)

(iv) ∪ (v) gives final answer as $a \in [9, \infty)$

⇒ Least value of 7a is 63.



15. $\begin{vmatrix} 3 & a \\ 2 & b \end{vmatrix} \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix}^2 \Rightarrow (3b - 2a)(a - b) = (3 - 2)^2$

$\Rightarrow 5ab - 3b^2 - 2a^2 = 1$

16. $x^3 - px^2 + qx = 0$ ...(1)

$x(x^2 - px + q) = 0;$ $x = 0,$ $x^2 - px + q = 0 \begin{matrix} \nearrow \alpha \\ \searrow \alpha \end{matrix}$ ∴ 0, α, α are the roots of equation (1)

$2\alpha = p \Rightarrow \alpha = p/2$ ...(2) & $\alpha^2 = q$ ...(3)

Since α is the root of the equation $x^2 - ax + b = 0$ also,

$\therefore \alpha^2 - a\alpha + b = 0$

$q - \frac{a\ \ p}{2} + b = 0$ [using (2) & (3)]

$\Rightarrow ap = 2(b + q)$ $\Rightarrow 2 = \frac{ap}{q+b}.$

17. Given expression is $f(x, y) = x^3 - 3x^2y + \lambda xy^2 + \mu y^3$ .....(i)

since (x – y) is a factor of (i)

$\therefore x^3 - 3x^3 + \lambda x^3 + \mu x^3 = 0 \Rightarrow \lambda + \mu - 2 = 0$ .....(ii)

(y – 2x) is also a factor of (i)

$\therefore x^3 - 3x^2(2x) + \lambda x(4x^2) + \mu(8x^3) = 0$

$\Rightarrow 4\lambda + 8\mu - 5 = 0$ .....(iii)

Solving (ii) & (iii) we get $\lambda = \frac{11}{4}$ and $\mu = -\frac{3}{4}$

$\Rightarrow \frac{16\lambda}{11} + 4\mu = \frac{16}{11}\ \frac{11}{4} + 4\left(\frac{-3}{4}\right) = 4 - 3 = 1.$ **Ans.**

PART - III

1.

$p = 0$	$\Rightarrow 2x^2 - 4x - 0 = 0$	two roots	**(A) Correct**
$p = 1$	$\Rightarrow 0x^2 - (0)x + 0 = 0$	identity more than two roots	**(B) Not answer**
$p = 2$	$\Rightarrow 0x^2 - (-2)x + (-2) = 0 \Rightarrow x = +1$	one root	**(C) Correct**
$p = 4$	$\Rightarrow 6x^2 - 0x - 12 = 0$	two root	**(D) Correct**

2. **(A)** $S = \alpha^2 + \beta^2 = a^2 - 2b$; $P = \alpha^2\beta^2 = b^2$

$\therefore$ equation is $x^2 - (a^2 - 2b)x + b^2 = 0$

(B) $S = \frac{1}{\alpha} + \frac{1}{\beta} = -\frac{a}{b}$, $P = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{b}$

$\therefore$ $x^2 + \frac{a}{b}x + \frac{1}{b} = 0$

$\Rightarrow$ $bx^2 + ax + 1 = 0$

(C) $S = \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{a^2-2b}{b}$; $P = \frac{\alpha}{\beta} \cdot \frac{\beta}{\alpha} = 1$

$x^2 - \frac{a^2-2b}{b}x + 1 = 0 \Rightarrow bx^2 - (a^2 - 2b)x + b = 0$

(D) $S = \alpha + \beta - 2 = -a - 2$; $P = (\alpha - 1)(\beta - 1)$

$= \alpha\beta - (\alpha + \beta) + 1 = b + a + 1$

$\therefore$ equation is $x^2 + (a + 2)x + (a + b + 1) = 0$.

3. $ax^2 + bx + c = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix} \Rightarrow \alpha + \beta = -\frac{b}{a}$ **,** $\alpha\beta = \frac{c}{a} \Rightarrow Ax^2 + Bx + C = 0 \begin{matrix}\nearrow \alpha+\delta \\ \searrow \beta+\delta\end{matrix}$

$(\alpha + \delta) + (\beta + \delta) = -\frac{B}{A}$, $(\alpha + \delta)(\beta + \delta) = \frac{C}{A}$ $\because$ $|(\alpha + \delta) - (\beta + \delta)| = |(\alpha - \beta)|$

$\Rightarrow \sqrt{\frac{B^2}{A^2} - \frac{4C}{A}} = \sqrt{\frac{b^2}{a^2} - \frac{4c}{a}} \Rightarrow \frac{b^2-4ac}{a^2} = \frac{B^2-4AC}{A^2}$ Hence proved

4.✍ $4x^2 + 2x - 1 = 0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$

$\Rightarrow$ $4\alpha^2 + 2\alpha - 1 = 0$ ....(1)

Let $\beta = 4\alpha^3 - 3\alpha$

with the help of equation (1)

$\beta = \alpha\,[4\alpha^2 - 3] = \alpha[1 - 2\alpha - 3] = -2\alpha^2 - 2\alpha = -2\frac{(1-2\alpha)}{4} - 2\alpha$ [using (1)]

$\beta = -\alpha - 1/2$

$\alpha + \beta = -1/2$ which is given.

hence second root is $4\alpha^3 - 3\alpha$.

5. $x^2 + 3x + 1 = (x - \alpha)(x - \beta)$. Put $x = 2$ $\Rightarrow 11 = (2 - \alpha)(2 - \beta)$ **option (B)**

$\alpha^2 + 3\alpha + 1 = 0$, $\beta^2 + 3\beta + 1 = 0$

$\alpha^2 = -(3\alpha + 1)$, $\beta^2 = -(3\beta + 1)$

$\frac{\alpha^2}{3\alpha+1} = -1$, $\frac{\beta^2}{3\alpha+1} = -1 \Rightarrow \frac{\alpha^2}{3\alpha+1} + \frac{\beta^2}{3\alpha+1} = -2$ **option (C).**

$$\left(\frac{\alpha}{1+\beta}\right)^2 + \left(\frac{\beta}{1+\alpha}\right)^2 = \frac{\alpha^2}{1+2\beta+\beta^2} + \frac{\beta^2}{1+2\alpha+\alpha^2} = \frac{-(3\alpha+1)}{-\beta} + \frac{-(3\beta+1)}{-\alpha} = \frac{\alpha\,(3\alpha+1)+\beta(3\beta+1)}{\beta\alpha}$$

$$= \frac{3(\alpha^2+\beta^2)+(\alpha+\beta)}{1} = \frac{3\left((\alpha+\beta)^2-2\alpha\beta\right)+(-3)}{1} = 3\,(7) - 3 = 18.$$

6. Split 32 into sum of two primes 32 = 2 + 30 = 3 + 29 = 5 + 27 = 7 + 25 = 11 + 21 = 13 + 19.
32 = 2 + 30 = 3 + 29 = 5 + 27 = 7 + 25 = 11 + 21 = 13 + 19.

7.✍ $\alpha^2 - a(\alpha + 1) - b = 0$ ....(i)
$\beta^2 - a(\beta + 1) - b = 0$ ....(ii)
by (i) & (ii)

(A) $\dfrac{1}{\alpha^2 - a\alpha} + \dfrac{1}{\beta^2 - a\beta} - \dfrac{2}{a+b} = \dfrac{1}{a+b} + \dfrac{1}{a+b} - \dfrac{2}{a+b} = 0$ (hence A)

(B) $f(a) + a + b = -(a + b) + (a + b) = 0$ (hence B)
$f(b) + a + b = b^2 - ab - a - b \neq 0$

(D) $f\left(\dfrac{a}{2}\right) + \dfrac{a^2}{4} + a + b = \dfrac{a^2}{4} - a\left(\dfrac{a}{2} + 1\right) - b + \dfrac{a^2}{4} + a + b = 0$

8.✍ Let $(x) = x^3 + bx^2 + cx + d$
$b + c + d = 0$ ......(i)
$4b + 2c + d = -4$ ......(ii)
$9b + 3c + d = -18$ ......(iii)
by (i), (ii) and (iii) $b = -5$, $c = 11$, $d = -6 \Rightarrow$ $f(x) = x^3 - 5x^2 + 11x - 6$

Alter : $f(x) = (x - 1)(x - 2)(x - 3) + x^2 = x^3 - 5x^2 + 11x - 6 = x^3 - (x - 1)(5x - 6)$

$\Rightarrow$ $f(4) = (3)(2)(1) + 16 = 22$ $f\left(\dfrac{6}{5}\right) = \left(\dfrac{6}{5}\right)^3$

Now $f(x) = x^3 \Rightarrow x = 1$ or $\dfrac{6}{5}$

$f(0)\,f(1) = (-6)(1) < 0$ one root in (0, 1)

9. **Case-I** (i) $x > 1$ $p(x) = x^{25}(x^7 - 1) + x^{11}(x^7 - 1) + x^3(x - 1) + 1$ $p(x) > 0$ no root for $x \in (1, \infty)$
(ii) $0 < x < 1$ $p(x) = x^{32} + x^{18}(1 - x^7) + x^4(x - x^7) + (1 - x^3)$ $p(x) > 0$ not root for (0, 1)
(iii) $x = 1$; $P(x) = 1$
hence no real root for $x > 0$

Case-II : for $x < 0$ let $x = -\alpha$ is root $(\alpha > 0)$ $p(\alpha) = \alpha^{32} + \alpha^{25} + \alpha^{18} + \alpha^{11} + \alpha^4 + \alpha^3 + 1$ $p(\alpha) \neq 0$
Hence no negative rootAll roots are imaginary
$p(x) + p(-x) = 2(x^{32} + x^{18} + x^4 + 1) \neq 0\ \forall\ x \in R$ Hence imaginary roots.

10. $x^2 + px + q = 0 \begin{smallmatrix}\nearrow \alpha\\ \searrow \beta\end{smallmatrix} \Rightarrow \alpha + \beta = -p$, $\alpha\beta = q$ and $p^2 - 4q > 0 \Rightarrow x^2 - rx + s = 0 \begin{smallmatrix}\nearrow \alpha^4\\ \searrow \beta^4\end{smallmatrix}$ .....(1)

Now $\alpha^4 + \beta^4 = r \Rightarrow \alpha^4 + \beta^4 = r$, $(\alpha\beta)^4 = s = q^4$ $\therefore$ $(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2 = r$
$\Rightarrow [(\alpha + \beta)^2 - 2\alpha\beta]^2 - 2\alpha^2\beta^2 = r \Rightarrow (p^2 - 2q)^2 - 2q^2 = r \Rightarrow (p^2 - 2q)^2 = 2q^2 + r > 0$ .....(2)
Now, for $x^2 - 4qx + 2q^2 - r = 0$ $\Rightarrow$
$D = 16q^2 - 4(2q^2 - r)$ by equation (2) $= 8q^2 + 4r = 4(2q^2 + r) > 0 \Rightarrow D > 0$ two real and distinct roots
Product of roots $= 2q^2 - r = 2q^2 - [(p^2 - 2q)^2 - 2q^2] = 4q^2 - (p^2 - 2q)^2 = -p^2(p^2 - 4q) < 0$ from (1)
So product of roots is – ve. hence roots are opposite in sign

11. $ax^3 + bx^2 + cx + d = 0 \begin{smallmatrix}\nearrow \alpha\\ \rightarrow \beta\\ \searrow \gamma\end{smallmatrix}$

Let $ax^3 + bx^2 + cx + d \equiv (x^2 + x + 1)(Ax + B)$
Roots of $x^2 + x + 1 = 0$ are imaginary, Let these are α, β So the third root 'γ' will be real.

$\alpha + \beta + \gamma = \dfrac{-b}{a} \Rightarrow -1 + \gamma = \dfrac{-b}{a} \Rightarrow \gamma = \dfrac{a - b}{a}$

Also $\alpha\beta\gamma = \dfrac{-d}{a}$. But $\alpha\beta = 1$

$\therefore$ $\gamma = \dfrac{-d}{a}$

$\therefore$ Ans are (A) & (D).

12. If $-5 + i\beta$ is a root then other root is $-5 - i\beta$ and $\gamma = 0$

$\Rightarrow$ roots are $-5 + i\beta, -5 - i\beta, -5$

Product of roots $(25 + \beta^2)(-5) = -860$; $25 + \beta^2 = 172$; $\beta^2 = 147$; $\beta = \pm 7\sqrt{3}$

$\therefore$ roots are $-5 + 7i\sqrt{3}, -5 - 7i\sqrt{3}, -5$

and $c = -5(-5 + 7i\beta) - 5(-5 - 7i\sqrt{3}) + (-5 + 7i\sqrt{3})(-5 - 7i\sqrt{3})$

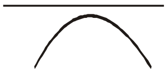
$c = 50 + (250 + 147) = 222.$

13. $f(x) > 0\ \forall x \in R$ or $f(x) < 0\ \forall x \in R$ hence $D < 0$

its graph can be

(i)

or



(ii)

(A) $f(1) > 0$ graph (i) will be possible

so $f(x) > 0\ \forall x \in R$

(B) $f(-1) < 0$ graph (ii) will be possible so $f(x) < 0\ \forall x \in R$

(C) $f\left(-\frac{1}{2}\right) > 0$ so $f(x) < 0\ \forall x \in R$

so not possible

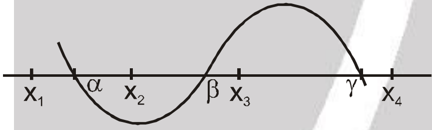
(D) $a > 0 \quad c > 0 \quad$ (graph (i))

$a < 0 \quad c < 0 \quad$ (graph (ii))

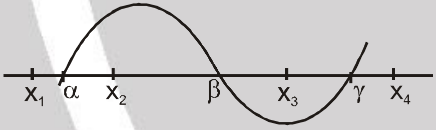
in both cases $ac > 0$

14. $f(\alpha) = f(\beta) = \mathbf{f(\gamma) = 0}$

hence f(x) has three real roots α, β, γ possible graphs of f(x) are



or



$\alpha \in (x_1\ x_2)$, $\beta \in (x_2\ x_3)$ and $\gamma \in (x_3\ x_4)$ or $\alpha \in (x_1\ x_3)$, $\beta \in (x_2\ x_3)$ and $\gamma \in (x_2\ x_4)$

hence A and D are correct

B is wrong as $\beta \notin (x_3, x_4)$

C is wrong as $\beta \notin (x_1, x_2)$

15. only A and C are correct as in these graphs

$f(\alpha) = f(\beta) = f(\gamma) = f'(x_1) = f'(x_2) = 0$

In option B $f(\alpha) < 0$ and $f(\beta) > 0$ (can't be equal).

In option D $f(\alpha) > 0$ and $f(\beta) < 0$ (can't be equal).

16. $f(x) = \frac{3}{x-2} + \frac{4}{x-3} + \frac{5}{x-4}$ $\quad \left.\begin{array}{ll} \because & f(2^+) \to \infty \\ \text{and} & f(3^-) \to -\infty \end{array}\right\} \Rightarrow$ f(x) = 0 has exactly one root in (2, 3).

again $\left.\begin{array}{ll} \because & f(3^+) \to \infty \\ \text{and} & f(4^-) \to -\infty \end{array}\right\}$

$\Rightarrow$ f(x) = 0 has exactly one root in (3, 4).

17. $\because$ D of $x^2 + 4x + 5 = 0$ is less than zero

$\Rightarrow$ both the roots are imaginary

$\Rightarrow$ both the roots of quadratic are same

$\Rightarrow$ $b^2 - 4ac < 0$ & $\frac{a}{1} = \frac{b}{4} = \frac{c}{5} = k$

$\Rightarrow$ $a = k, b = 4k, c = 5k.$

18. $x^2 + abx + c = 0 <^{\alpha}_{\beta}$ ...(1) $\alpha + \beta = -ab,\ \alpha\beta = c$

$x^2 + acx + b = 0 <^{\alpha}_{\delta}$ ...(2) $\alpha + \delta = -ac,\ \alpha\delta = b$

$\alpha^2 + ab\,\alpha + c = 0$

$\alpha^2 + ac\,\alpha + b = 0$

$$\frac{\alpha^2}{ab^2 - ac^2} = \frac{\alpha}{c - b} = \frac{1}{a\ \ c - ab} \Rightarrow \alpha^2 = \frac{a\ \ (b^2 - c^2)}{a(c - b)} = -(b + c)$$

$$\&\ \ \alpha = \frac{c - b}{a(c - b)} = \frac{1}{a}$$

$\therefore$ common root, $\alpha = \dfrac{1}{a}$

$\therefore\ -(b + c) = \dfrac{1}{a^2} \quad \Rightarrow \quad a^2(b + c) = -1$

Product of the roots of equation (1) & (2) gives

$\beta \times \dfrac{1}{a} = c \Rightarrow \beta = ac \quad \& \quad \delta \times \dfrac{1}{a} = b \Rightarrow \delta = ab.$

$\therefore$ equation having roots β, δ is (β, δ)

$x^2 - a(b + c)x + a^2bc = 0$

$\Rightarrow \quad a(b + c)x^2 - a^2(b + c)^2x + a.(b + c)a^2bc = 0$

$a(b + c)x^2 + (b + c)x - abc = 0.$

19. **S_1 :** $2x^2 + 3x + 1 = 0$

$\because\ D = 9 - 4 \times 1 \times 1 = 1$

Which is perfect square of a rational number

$\therefore$ roots will be rational.

S_2 : $\because$ Let $f(x) = (x - a)(x - c) + 2(x - b)(x - d)$

$\because \quad f(a) > 0 \quad f(b) < 0 \quad f(c) < 0\ f(d) > 0$

$\therefore$ two real and distinct roots.

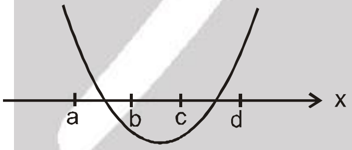
S_3 : $x^2 + 3x + 5 = 0$ .....(i) and $ax^2 + bx + c = 0$ .....(ii)

for equation (i) , $D < 0$

$\therefore$ Roots are imaginary and they occur in conjugate pair

$\therefore$ Roots of equation (i) and (ii) will be identical

$\Rightarrow \quad \dfrac{a}{1} = \dfrac{b}{3} = \dfrac{c}{5} = \lambda,\ (\lambda \in N) \Rightarrow \quad a = \lambda, \quad b = 3\lambda,\ c = 5\lambda \Rightarrow a + b + c = 9\lambda$ $\therefore$ least value is 9.



20. $x^2 + ax + 12 = 0$ .....(1)

$x^2 + bx + 15 = 0$ .....(2)

$x^2 + (a + b)x + 36 = 0$ .....(3)

(1) + (2) – (3) gives $x^2 - 9 = 0 \quad \Rightarrow \quad x = \pm 3$ given that common root will be +ve

so $x = 3$ put in equation (3) $9 + 3(a + b) + 36 = 0 \quad \Rightarrow \quad a + b = -15$

by equation (1) $9 + 3a + 12 = 0 \Rightarrow a = -7\ \&\ b = -8$

21. $4x^3 + 3x + 2c = (4x + 2c)(x^2 + \lambda x + 1)$

compairing co-efficents $\Rightarrow \quad c = 1$ and $\lambda = -\dfrac{1}{2}$ or $c = -1$ and $\lambda = \dfrac{1}{2}$

$\Rightarrow \quad c + \lambda = \dfrac{1}{2}$ or $-\dfrac{1}{2}$

PART - IV :

1.✍ $x^2 + 2xy + 2y^2 + 4y + 7 = (x + y)^2 + (y + 2)^2 + 3 \geq 0 + 0 + 3$ ∴ Least value = 3.

2.✍ $P(x) = 4x^2 + 6x + 4 = 4\left(x + \frac{3}{4}\right)^2 + \frac{7}{4} \Rightarrow P(x) = \geq \frac{7}{4}$

$Q(y) = 4y^2 - 12y + 25 = 4\left(y - \frac{3}{2}\right)^2 + 16 \Rightarrow Q(y) \geq 16$

$\Rightarrow$ p(x).Q(y) ≥ 28 but it is given P(x).Q(y) = 28 p(x).Q(y) ≥ 28 P(x).Q(y) = 28

$\Rightarrow P(x) = \frac{7}{4}$ & $Q(y) = 16$

$\Rightarrow x = \frac{-3}{4}$, $y = \frac{3}{2}$; $11y - 26x = 11 \times \frac{3}{2} - 26\,\frac{(-3)}{4} = \frac{33}{2} + \frac{39}{2} = \frac{72}{2} = 36$. **Ans.**

(3 & 4)

Let the coordinates of A(α, 0), B(2α, 0), C(0, 2α). Now $y = x^2 + bx + c$ passes through C(0, 2α)

∴ given equation of curve reduces to $y = x^2 + bx + 2\alpha$. Now it also passes through A & B

∴ $0 = \alpha^2 + b\alpha + 2\alpha \Rightarrow 0 = \alpha + b + 2$ ..... (i)

& $0 = 4\alpha^2 + 2\alpha b + 2\alpha \Rightarrow 0 = 2\alpha + b + 1$ ..... (ii)

On solving (i) & (ii) for α & b we get α = 1, b = – 3

∴ given curve is $y = x^2 - 3x + 2$

3. roots of y = 0 are {2, 1}

4. $(\alpha + \beta) \Rightarrow 3$ (∵ α = 2, β = 1) $\Rightarrow \alpha - \beta) \Rightarrow 1$

∴ equation whose roots are 3, 1 is $x^2 - 4x + 3 = 0$

(5 to 7)

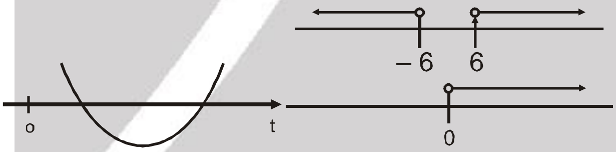
$x^4 - \lambda x^2 + 9 = 0 \Rightarrow x^2 = t \geq 0 \Rightarrow f(t) = t^2 - \lambda t + 9 = 0$

5. given equation has four real & distinct roots

$D > 0 \Rightarrow \lambda^2 - 36 > 0 \quad \frac{-b}{2a} > 0 \Rightarrow \frac{\lambda}{2} > 0 \Rightarrow \lambda > 0$

$f(0) > 0 \Rightarrow 9 > 0$

∴ $\lambda \in (6,\infty)$



6. Equation has no real roots.

case-I $D \geq 0 \Rightarrow \lambda^2 - 36 \geq 0 \quad \frac{-b}{2a} < 0$

$\Rightarrow \lambda < 0 \quad f(0) > 0$

$\Rightarrow 9 > 0.$

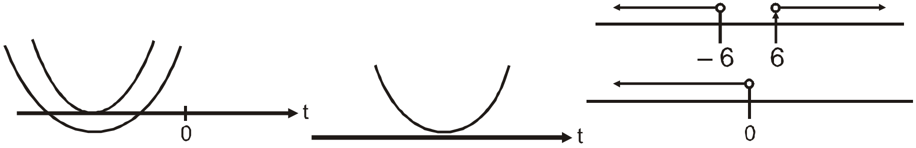
∴ $\lambda \in (-\infty, -6]$

case-II D < 0

$\Rightarrow \lambda^2 - 36 < 0$

$\Rightarrow \lambda \in (-6, 6)$

union of both cases gives ∪ $\lambda \in (-\infty, 6)$

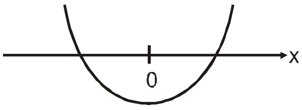


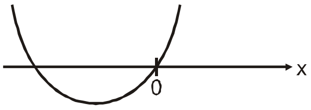
7. Equation has only two real roots

case-I $f(0) < 0$ $9 < 0$

which is false case-II $f(0) = 0$ and $\frac{-b}{2a} < 0$

∴ No solution ∴ Final answer is ϕ





8.✍ Divide by $x^2 \Rightarrow x^2 - 10x + 26 - \frac{10}{x} + \frac{1}{x^2} = 0 \Rightarrow x^2 + \frac{1}{x^2} - 10\left(x + \frac{1}{x}\right) + 26 = 0$

$t = x + \frac{1}{x} \Rightarrow t^2 - 2 = x^2 + \frac{1}{x^2} \Rightarrow t^2 - 2 - 10t + 26 = 0 \Rightarrow t^2 - 10t + 24 = 0 < \begin{matrix}4\\6\end{matrix}$

$t = 4 \quad x + \frac{1}{x} = 4 \Rightarrow x^2 - 4x + 1 = 0 \Rightarrow x = 2 \pm \sqrt{3}$

$t = 6 \quad x + \frac{1}{x} = 6 \Rightarrow x^2 - 6x + 1 = 0 \Rightarrow x = 3 \pm 2\sqrt{2}$.

9.✍ By trail x = 1 is a root divide by x – 1 x = 1,

$$\begin{array}{c|cccccc} 1 & 1 & -5 & 9 & -9 & 5 & -1 \\ & \times & 1 & -4 & 5 & -4 & 1 \\ \hline & 1 & -4 & 5 & -4 & 1 & 0 \end{array}$$

$(x - 1)(x^4 - 4x^3 + 5x^2 - 4x + 1) = 0 \Rightarrow x = 1$ or $x^4 - 4x^3 + 5x^2 - 4x + 1 = 0$

$x^2 - 4x + 5 - \frac{4}{x} + \frac{1}{x^2} = 0 \Rightarrow t = x + \frac{1}{x} \Rightarrow t^2 = x^2 + \frac{1}{x^2} + 2$

$t^2 - 2 - 4t + 5 = 0 \Rightarrow t^2 - 4t + 3 = 0 < \begin{matrix}1\\3\end{matrix} \Rightarrow x + \frac{1}{x} = 1, x + \frac{1}{x} = 3$

$x^2 - x + 1 = 0, \quad x^2 - 3x + 1 = 0 \Rightarrow x = \frac{1 \pm i\sqrt{3}}{2}, x = \frac{3 \pm \sqrt{5}}{2}$

∴ roots $1, \frac{1 \pm i\sqrt{3}}{2}, \frac{3 \pm \sqrt{5}}{2}$.

10.✍ Divide by $x^3 \Rightarrow x^3 - 4x + \frac{4}{x} - \frac{1}{x^3} = 0; x^3 - \frac{1}{x^3} - 4\left(x - \frac{1}{x}\right) = 0$

Put $t = x - \frac{1}{x} \Rightarrow t^3 = x^3 - 3x^2 \frac{1}{x} + 3x \frac{1}{x^2} - \frac{1}{x^3} = x^3 - 3\left(x - \frac{1}{x}\right) - \frac{1}{x^3}$

$t^3 + 3t = x^3 - \frac{1}{x^3}$

Put in equation above $t^3 + 3t - 4t = 0$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$t^3 + 3t - 4t = 0$

$\Rightarrow t^3 - t = 0$

$\Rightarrow t = 0, 1, -1$

$x - \frac{1}{x} = 0, x - \frac{1}{x} = 1, x - \frac{1}{x} = -1; x = \pm 1, x^2 - x - 1 = 0,$

$x^2 + x - 1 = 0$

$x = \pm 1, x = \frac{1 \pm \sqrt{5}}{2}, x = \frac{-1 \pm \sqrt{5}}{2}$.

EXERCISE # 3

PART - I

1. (i) $x^2 - 8kx + 16(k^2 - k + 1) = 0$ $\therefore$ $D = 64\,(k^2 - (k^2 - k + 1)) = 64\,(k - 1) > 0$

$\Rightarrow$ $k > 1$ ....(1)

(ii) $\frac{b}{2a} - > 4$ $\Rightarrow$ $\frac{8k}{2} > 4$ $\Rightarrow$ $k > 1$ ....(2)

(iii) $f(4) \ge 0$

$\Rightarrow$ $16 - 32k + 16\,(k^2 - k + 1) \ge 0$ $\Rightarrow$ $k^2 - 3k + 2 \ge 0$

$\Rightarrow$ $(k - 2)\,(k - 1) \ge 0$ $\Rightarrow$ $k \le 1$ or $k \ge 2$ ....(3)

$(1) \cap (2) \cap (3)$. Hence $k = 2$

2. Product = 1

Sum = $\frac{\alpha^2+\beta^2}{\alpha\beta} = \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$

Since $\alpha^3 + \beta^3 = q$ $\Rightarrow$ $-\,p\,(\alpha^2 + \beta^2 - \alpha\beta) = q$

$((\alpha + \beta)^2 - 3\alpha\beta\,) = -\,\frac{q}{p}$ $\Rightarrow$ $p^2 + \frac{q}{p} = 3\alpha\beta$

Hence sum = $\dfrac{\left\{p^2 - \frac{2}{3}\left(\frac{p^3+q}{p}\right)\right\}\ 3p}{(p^3+q)} = \dfrac{p^3-2q}{p^3+q}$

so the equation $x^2 - \left(\dfrac{p^3-2q}{p^3+q}\right) x + 1 = 0$ $\Rightarrow$ $(p^3 + q)\,x^2 - (p^3 - 2q)x + (p^3 + q) = 0$

3. $x^2 - 6x - 2 = 0$ having roots α and β $\Rightarrow$ $\alpha^2 - 6\alpha - 2 = 0$

$\Rightarrow$ $\alpha^{10} - 6\alpha^9 - 2\alpha^8 = 0$ $\Rightarrow$ $\alpha^{10} - 2\alpha^8 = 6\alpha^9$ .... (i)

similarly $\beta^{10} - 2\beta^8 = 6\beta^9$ .... (ii)

by (i) and (ii)

$(\alpha^{10} - \beta^{10}) - 2(\alpha^8 - \beta^8) = 6\,(\alpha^9 - \beta^9)$ $\Rightarrow$ $a_{10} - 2a_8 = 6a_9$ $\Rightarrow$ $\frac{a_{10}-2a_8}{2a_9} = 3$

Aliter

$$\frac{\alpha^{10}-\beta^{10}-2(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)} = \frac{\alpha^{10}-\beta^{10}+\alpha\beta(\alpha^8-\beta^8)}{2(\alpha^9-\beta^9)} = \frac{\alpha^9(\alpha+\beta)-\beta^9(\alpha+\beta)}{2(\alpha^9-\beta^9)} = \frac{\alpha+\beta}{2} = \frac{6}{2} = 3$$

4. $x^2 + bx - 1 = 0$

$x^2 + x + b = 0$

$\frac{x^2}{b^2+1} = \frac{x}{-1-b} = \frac{1}{1-b}$ $\Rightarrow$ $x = \frac{b^2+1}{-(b+1)} = \frac{-(b+1)}{1-b}$ $\Rightarrow$ $(b^2+1)(1-b) = (b+1)^2$

$\Rightarrow$ $b^2 - b^3 + 1 - b = b^2 + 2b + 1$ $\Rightarrow$ $b^3 + 3b = 0 \Rightarrow b = 0;\ b^2 = -3 \Rightarrow b = 0 \pm \sqrt{3}\ \ i$,

5. p(x) will be of the form $ax^2 + c$. Since it has purely imaginary roots only.

Since p(x) is zero at imaginary values while $ax^2 + c$ takes real value only at real 'x', no root is real.

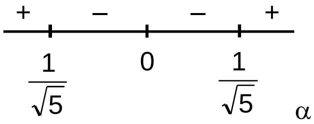
Also $p(p(x)) = 0$

$\Rightarrow$ p(x) is purely imaginary $\Rightarrow$ $ax^2 + c$ = purely imaginary

Hence x can not be purely imaginary since x^2 will be negative in that case and $ax^2 + c$ will be real.

Thus .(D) is correct.

6. $(x_1 + x_2)^2 - 4x_1x_2 < 1 \quad \frac{1}{\alpha^2} - 4 < 1 \Rightarrow 5 - \frac{1}{\alpha^2} > 0 \quad \frac{5\alpha^2 - 1}{\alpha^2} > 0$



$\alpha \in \left(-\infty, -\frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \infty\right)$ ...(1)

$D > 0 \quad 1 - 4\alpha^2 > 0 \quad \alpha \in \left(-\frac{1}{2}, \frac{1}{2}\right)$ ...(2)

(1) & (2) $\alpha \in \left(-\frac{1}{2}, \frac{1}{\sqrt{5}}\right) \cup \left(\frac{1}{\sqrt{5}}, \frac{1}{\sqrt{2}}\right)$

7. $x^2 - 2x\sec\theta + 1 = 0 \Rightarrow x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} \Rightarrow x = \sec\theta + \tan\theta,\ \sec\theta - \tan\theta \Rightarrow \alpha_1 = \sec\theta - \tan\theta$

now $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2} \quad \Rightarrow x = -\tan\theta \pm \sec\theta \Rightarrow \alpha_2 = (\sec\theta - \tan\theta)$

$\Rightarrow \quad \beta_2 = -(\sec\theta + \tan\theta)$

$\therefore \quad \alpha_1 + \beta_2 = -2\tan\theta$

Alt : (i) $x^2 - 2x\sec\theta + 1 = 0 \quad x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2} = \sec\theta \pm \tan\theta$

$\alpha_1 = \sec\theta - \tan\theta \qquad \beta_1 = \sec\theta + \tan\theta$

(ii) $x^2 + 2x\tan\theta - 1 = 0 \Rightarrow x = \frac{-\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$

$x = -\tan\theta \pm \sec\theta \quad \alpha_2 = -\tan\theta + \sin\theta \qquad \beta_2 = -\tan\theta - \sec\theta \quad \alpha_1 + \beta_2 = -2\tan\theta$

8. As α and β are roots of equation $x^2 - x - 1 = 0$, we get : $\alpha^2 - \alpha - 1 = 0 \Rightarrow \alpha^2 = \alpha + 1$
$\beta^2 - \beta - 1 = 0 \quad \Rightarrow \beta^2 = \beta + 1$
$\therefore a_{11} + a_{10} = p\alpha^{11} + q\beta^{11} + p\alpha^{10} + q\beta^{10} = p\alpha^{10}(\alpha+1) + q\beta^{10}(\beta+1) = p\alpha^{10} \times \alpha^2 + q\beta^{10} \times \beta^2 = p\alpha^{12} + q\beta^{12} = a_{12}$

9. $a_{n+2} = a_{n+1} + a_n \quad a_4 = a_3 + a_2 = 3a_1 + 2a_0 = 3p\alpha + 3q\beta + 2(p + q)$

As $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$, we get $a_4 = 3p\left(\frac{1+\sqrt{5}}{2}\right) + 3q\left(\frac{1-\sqrt{5}}{2}\right) + 2p + 2q = 28$

$\Rightarrow \left(\frac{3p}{2} + \frac{3q}{2} + 2p + 2q - 28\right) = 0$ …….(i)

and $\frac{3p}{2} - \frac{3q}{2} = 0$ …………….(ii)

$\Rightarrow$ p = q (from (ii)) $\Rightarrow$ 7p = 28 (from (i) and (ii))
$\Rightarrow$ p = 4 $\Rightarrow$ q = 4 $\Rightarrow$ p + 2q = 12

10. (A) $b_n = a_{n+1} + a_{n-1} = \frac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta} + \frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta} = \frac{\alpha^{n-1}(\alpha^2+1)-\beta^{n-1}(\beta^2+1)}{\alpha-\beta}$

$= \frac{\alpha^{n-1}(\alpha+2)-\beta^{n-1}(\beta+2)}{\alpha-\beta} = \frac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right)-\beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$

$= \frac{\sqrt{5}\alpha^{n-1}\left(\frac{\sqrt{5}+1}{2}\right)-\sqrt{5}\,\beta^{n-1}\left(\frac{\sqrt{5}-1}{2}\right)}{\alpha-\beta} = \frac{\sqrt{5}(\alpha^n+\beta^n)}{\alpha-\beta} = \alpha^n + \beta^n \quad \because \quad \alpha-\beta = \sqrt{5}$

(B) $\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \sum\left(\frac{\alpha}{10}\right)^n + \sum\left(\frac{\beta}{10}\right)^n = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{\alpha}{10-\alpha} + \frac{\beta}{10-\beta}$

$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta} = \frac{10+2}{89} = \frac{12}{89}$

(C) $\sum_{n=1}^{\infty}\frac{a_n}{10^n} = \sum\frac{\alpha^n-\beta^n}{(\alpha-\beta)10^n} = = \frac{1}{\alpha-\beta}\left(\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} - \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right) \frac{1}{\alpha-\beta}\left(\frac{\alpha}{10-\alpha} - \frac{\beta}{10-\beta}\right)$

$= \frac{1}{\alpha-\beta}.\frac{(10(\alpha-\beta)-\alpha\beta+\alpha\beta)}{100-10(\alpha+\beta)+\alpha\beta} = \frac{10}{89}$ Option (C) is correct.

(D) $a_1 + a_2 + ...a_n = \Sigma a_i = \frac{\Sigma\alpha^i - \Sigma\beta^i}{\alpha-\beta} = \frac{\frac{\alpha(1-\alpha^n)}{(1-\alpha)} - \frac{\beta(1-\beta^n)}{(1-\beta)}}{\alpha-\beta}$

$= \frac{(\alpha+1)(1-\alpha^n)-(\beta+1)(1-\beta^n)}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \frac{\alpha^2-\alpha^{n+2}-\beta^2+\beta^{n+2}}{(1-\alpha)(1-\beta)(\alpha-\beta)} = \frac{\sqrt{5}+\beta^{n+2}-\alpha^{n+2}}{\beta-\alpha} = -1 + a_{n+2}$

PART - II

---

1. Let the correct equation be $ax^2 + bx + c = 0$ $ax^2 + bx + c = 0$

now sachin's equation $\Rightarrow$ $ax^2 + bx + c' = 0 \begin{matrix}\nearrow 4\\ \searrow 3\end{matrix}$

Rahul's equation $\Rightarrow$ $ax^2 + b'x + c = 0 \begin{matrix}\nearrow 3\\ \searrow 2\end{matrix}$

$-\frac{b}{a} = 7$ .... (i) $\frac{c}{a} = 6$ .... (ii)

from (i) and (ii)

correct equation is $x^2 - 7x + 6 = 0$ roots are 6 and 1. $x^2 - 7x + 6 = 0$

2. $P(x) = 0 \Rightarrow$ $f(x) = g(x) \Rightarrow$ $ax^2 + bx + c = a_1x^2 + b_1x + C, \Rightarrow (a - a_1)x^2 + (b - b_1)x + (c - c_1) = 0.$

It has only one solution x = – 1

$\Rightarrow$ $b - b_1 = a - a_1 + c - c_1$ .... (1)

vertex (–1, 0) $\Rightarrow$ $\frac{b-b_1}{2(a-a_1)} = -1$

$\Rightarrow$ $b - b_1 = 2(a - a_1)$ .... (2)

$\Rightarrow$ $f(-2) - g(-2) = 2 \Rightarrow 4a - 2b + c - 4a_1 + 2b_1 - c_1 = 2$

$\Rightarrow$ $4(a - a_1) - 2(b - b_1) + (c - c_1) = 2$ .... (3)

by (1), (2) and (3) $(a - a_1) = (c - c_1) = \frac{1}{2}$ $(b - b_1) = 2$

NowP(2) = f(2) – g(2) $= 4(a - a_1) + 2(b - b_1) + (c - c_1) = 8 + 8 + 2 = 18$

3. Let $e^{\sin x} = t \Rightarrow t^2 - 4t - 1 = 0$

$\Rightarrow \quad t = \frac{4 \pm \sqrt{16+4}}{2}$

$\Rightarrow \quad t = e^{\sin x} = 2 \pm \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5}$, $e^{\sin x} = 2 + \sqrt{5}$

$\Rightarrow \quad e^{\sin x} = 2 - \sqrt{5} < 0,$

$\Rightarrow \quad \sin x = ln(2 + \sqrt{5}) > 1$ so rejected so rejected hence no solution

4. $x^2 + 2x + 3 = 0$ ...(i)
$ax^2 + bx + c = 0$ ...(ii)
Since equation (i) has imaginary roots.
So equation (ii) will also have both roots same as (i).

Thus $\frac{a}{1} = \frac{b}{2} = \frac{c}{3} \Rightarrow$ a = λ, b = 2λ, c = 3λ. Hence 1 : 2 : 3

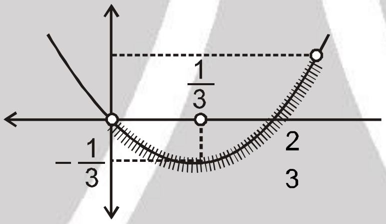
5. $a^2 = 3\{x\}^2 - 2\{x\}$ [$x - [x] = \{x\}$]
Let {x} = t ∴ $t \in (0, 1)$ As x is not an integer

$\therefore \quad a^2 = 3t^2 - 2t \quad f(t) = 3t\left(t - \frac{2}{3}\right) \Rightarrow a^2 = 3t\left(t - \frac{2}{3}\right)$

Clearly by graph $-\frac{2}{3} \le a^2 < 1$

∴ $a \in (-1, 1) - \{0\}$ (As $x \neq$ integer) **Ans.** (3)

Note : It should have been given that the solution exists else answer will be $a \in R - \{0\}$



6. $px^2 + qx + r = 0 <^{\alpha}_{\beta}$; p, q, r → A.P. ; 2q = p + r $\frac{1}{\alpha} + \frac{1}{\beta} = 4$; $\frac{\alpha+\beta}{\alpha\beta} = 4 \Rightarrow \frac{-q}{r} = 4$

q = – 4r ..... (i)
∴ –8r = p + r
p = – 9r ..... (ii)

$|\alpha - \beta| = \sqrt{(\alpha+\beta)^2 - 4\alpha\beta} = \sqrt{\frac{q^2}{p^2} - \frac{4r}{p}}$ by (i) and (ii)

$= \frac{\sqrt{q^2 - 4pr}}{|p|} = \frac{\sqrt{16r^2 + 36r^2}}{|-9r|} = \frac{2\sqrt{13}}{9}$

7.✍ $x^2 - 6x - 2 = 0 \; a_n = \alpha^n - \beta^n$

$$\frac{a_{10} - 2a_8}{2a_9} = \frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)} = \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)} = \frac{6\alpha^9 - 6\beta^9}{2(\alpha^9 - \beta^9)} \; \frac{\alpha+\beta}{2} = \frac{6}{2} = 3$$ Ans. (3)

8. For rational roots D must be perfect squareD = $121 - 24\alpha = k^2$ for $121 - 24\alpha$ to be perfect square α must be equal to 3, 4, 5 (observation) so number of possible values of α is 3.

9.✍ Let roots are α & β now $\lambda + \frac{\lambda}{\lambda} = 1 \Rightarrow \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = 1 \Rightarrow \alpha^2 + \beta^2 = \alpha\beta \; (\alpha+\beta)^2 = 3\alpha\beta \; \left(\frac{-m(m-4)}{3m^2}\right)^2 = 3.\frac{2}{3m^2}$

$m^2 - 8m - 2 = 0 \quad m = 4 \pm 3\sqrt{2}$ so least value of $m = 4 - 3\sqrt{2}$

HIGH LEVEL PROBLEMS (HLP)

1. $$\alpha_1^3\,\frac{(x-\alpha_2)(x-\alpha_3)......(x-\alpha_n)}{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)......(\alpha-\alpha_n)}+\frac{(x-\alpha_1)(x-\alpha_3)......(x-\alpha_n)}{(\alpha_2-\alpha_1)(\alpha_2-\alpha_3)......(\alpha_2-\alpha_n)}\,\alpha_2^3+..........$$

$$........+\frac{(x-\alpha_1)(x-\alpha_2)......(x-\alpha_{n-1})}{(\alpha_n-\alpha_1)(\alpha_n-\alpha_2)......(\alpha_n-\alpha_{n-1})}\,\alpha_n^3-x^3=0$$

$\Rightarrow$ α_1, α_2 ....α_n are roots of above relation whose degree appeared as (n–1)

$\Rightarrow$ above relation is indentify

2. $a^2x^2 + (b^2 + a^2 - c^2)\,x + b^2 = 0$ ......(1)

$\because$ $a + b > c \Rightarrow a + b - c > 0$ ......(2)

and $|a - b| < c \Rightarrow$ $a - b - c < 0$ ....(3)

and $a - b + c > 0$ ....(4)

Discriminant of equation (1) i.e. $D = (b^2 + a^2 - c^2)^2 - 4a^2b^2 = (b^2 + a^2 - c^2 - 2ab)\,(b^2 + a^2 - c^2 + 2ab)$

$= \{(a - b)^2 - c^2\}\,\{(a + b)^2 - c^2\}$

$= (a - b + c)\,(a - b - c)\,(a + b + c)\,(a + b - c) < 0$ (using (2), (3), (4))

$D < 0$

$\therefore$ roots are not real.

3. $$\frac{1}{x-1}-\frac{4}{x-2}+\frac{4}{x-3}-\frac{1}{x-4}<\frac{1}{30}$$

$$\Rightarrow\quad \frac{-3}{x^2-5x+4}+\frac{4}{x^2-5x+6}<\frac{1}{30}$$

Let $x^2 - 5x = y$

$$\Rightarrow\quad \frac{4}{y+6}-\frac{3}{y+4}<\frac{1}{30}$$

$$\Rightarrow\quad \frac{y^2-20y+84}{(y+6)(y+4)}>0$$

$\Rightarrow$ $y \in (-\infty, -6) \cup (-4, 6) \cup (14, \infty)$

Now (i) if $y \in (-\infty, -6)$

$\Rightarrow$ $x^2 - 5x < -6.$

$\Rightarrow$ $x \in (2, 3)$

(ii) if $y \in (-4, 6)$ $\Rightarrow$ $-4 < x^2 - 5x < 6$

$\Rightarrow$ $x \in (-1, 1) \cup (4, 6)$

(iii) if $y \in (14, \infty)$ $\Rightarrow$ $x^2 - 5x > 14$

$\Rightarrow$ $x \in (-\infty, -2) \cup (7, \infty)$

$\therefore$ final answer is $x \in (-\infty, -2) \cup (-1, 1) \cup (2, 3) \cup (4, 6) \cup (7, \infty)$

4. Let the three numbers in G.P. be a, ar, ar^2

$\because$ $a + b + c = xb$

$\frac{a}{b} + 1 + \frac{c}{b} = x$ $\because$ $b = ar,\ c = ar^2$

$\Rightarrow$ $r^2 + (1 - x)r + 1 = 0$ ....(1)

r is real $\therefore$ for (1) $D \ge 0$

$\Rightarrow$ $(1 - x)^2 - 4 \ge 0$

$\Rightarrow$ $x^2 - 2x - 3 \ge 0$

$\Rightarrow$ $x \le -1$ or $x \ge 3$

Note: If we put $x = -1$ and $x = 3$ in (1) we get $r = -1$ and $r = 1$ respectively which is not possible because in both cases the three numbers will not be distinct therefore $x < -1$ or $x > 3$

5. $V_n + V_{n-3} = (\alpha^n + \beta^n) + (\alpha^{n-3} + \beta^{n-3}) = \alpha^{n-3}(\alpha^3 + 1) + \beta^{n-3}(\beta^3 + 1)$

$\because \quad \alpha^2 + \alpha - 1 = 0$

$$\begin{array}{l} \alpha^2+\alpha-1\,\big)\;\overline{\alpha^3+1\qquad}\big(\;\alpha-1 \\ \qquad\qquad\quad \alpha^3+\alpha^2-\alpha \\ \qquad\qquad\quad -\quad\; -\quad\; + \\ \qquad\qquad\quad \overline{-\alpha^2+\alpha+1\quad} \\ \qquad\qquad\quad -\alpha^2-\alpha+1 \\ \qquad\qquad\quad +\quad\; +\quad\; - \\ \qquad\qquad\quad \overline{\quad 2\alpha\qquad\qquad} \end{array}$$

$\alpha^3 + 1 = (\alpha^2 + \alpha - 1)(\alpha - 1) + 2\alpha = 2\alpha$

also $\beta^3 + 1 = 2\beta \quad \Rightarrow \quad V_n + V_{n-3} = \alpha^{n-3}(2\alpha) + \beta^{n-3}(2\beta) = 2[\alpha^{n-2} + \beta^{n-2}] = 2V_{n-2}$

$V_1 = \alpha + \beta = -1$; $V_2 = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2(-1) = 3$

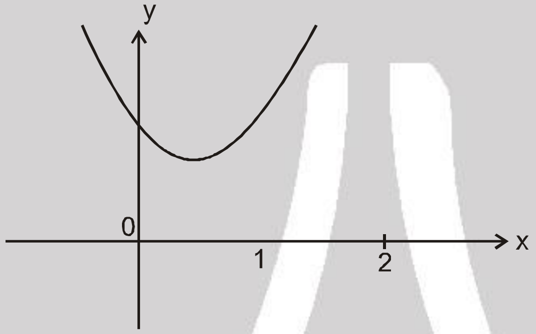
$V_n = 2V_{n-2} - V_{n-3} \Rightarrow \quad V_7 = 2V_5 - V_4 = 2[2V_3 - V_2] - (2V_2 - V_1) = 4V_3 - 2V_2 - 2V_2 + V_1$

$= 4[2V_1 - V_0] - 4V_2 + V_1 = 9V_1 - 4V_0 - 4V_2 = 9\{[-1]\} - 4[2] - 4[3] = -9 - 8 - 12 = -29$

6. $f(x) = x^2 - (m - 3)x + m > 0 \ \forall x \in [1, 2]$. Here $D = (m - 3)^2 - 4m = m^2 - 10m + 9 = (m - 1)(m - 9)$

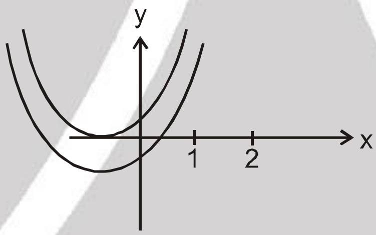
All possible graphs are

Case 1 :



$D < 0 \Rightarrow \qquad m \in (1, 9)$

Case 2 :



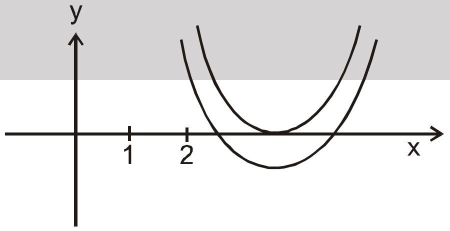
(i) $f(1) > 0 \quad \Rightarrow \quad 4 > 0$ always true

(ii) $\frac{b}{2a} - \ < 1 \ \Rightarrow \quad m < 5$

(iii) $D \geq 0 \quad \Rightarrow \quad m \in (-\infty, 1]\ \ [9, \infty)$

$\therefore \quad$ (i) $\cap$ (ii) $\cap$ (iii), we get $m \in (-\infty, 1]$

Case 3 :



(i) $f(2) > 0 \quad \Rightarrow \quad m < 10$

(ii) $\frac{-\ b}{2a} > 2 \ \Rightarrow \quad m > 7$

(iii) $D \geq 0 \quad \Rightarrow \quad m \in (-\infty, 1] \cup [9, \infty)$

$\therefore \quad$ (i) $\cap$ (ii) $\cap$ (iii), we get $m \in [9, 10)$

Now final Answer is (Case 1) $\cup$ (Case 2) $\cup$ (Case 3) we get $m \in (-\infty, 10)$

7. Let $f(x) = ax^2 + (a - 2)x - 2$ $\because$ $f(0) = -2$ and $f(-1) = 0$

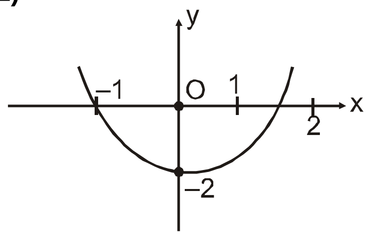
Since the quadratic expression is negative for exactly two integral values

$\Rightarrow$ $f(1) < 0$ and $f(2) \geq 0$

$\Rightarrow$ $a + a - 2 - 2 < 0$ and $4a + 2a - 4 - 2 \geq 0$

$\Rightarrow$ $a < 2$ and $a \geq 1$

$\therefore$ **$a \in [1, 2)$**



8. **(i)** when $x < a \Rightarrow x^2 + 2a(x - a) - 3a^2 = 0 \Rightarrow (x + a)^2 = 6a^2$

$x = -a \pm \sqrt{6}a = -a\ (1+\sqrt{6}), -a$. Since $a \leq 0$, then $x = -a(1-\sqrt{6})$

when $x \geq a$ then $x^2 - 2a(x - a) - 3a^2 = 0 \Rightarrow x = a \pm \sqrt{2}a = a((1+\sqrt{2}), a(1-\sqrt{2})$

since $a \leq 0$, $x \geq a$

$\therefore$ $x = a(1-\sqrt{2})$ Hence $x = a(\sqrt{6}-1), a(1-\sqrt{2})$

(iii) $\left(x+\frac{1}{x}\right)^3 + \left(x+\frac{1}{x}\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)\left(\left(x+\frac{1}{x}\right)^2+1\right) = 0 \Rightarrow \left(x+\frac{1}{x}\right)\left(x^2+\frac{1}{x^2}+3\right) = 0$

$\Rightarrow$ No real value of x. $\therefore$ Number of real roots = 0

9. $\because$ α, β are the roots of $x^2 - 34x + 1 = 0$ $\Rightarrow$ $\alpha + \beta = 34$ and $\alpha\beta = 1$

$\because$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} - 2(\alpha\beta)^{1/4}$ $\because$ $\alpha\beta = 1$

$\therefore$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = \sqrt{\alpha} + \sqrt{\beta} - 2$ ....(1)

$\because$ $\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = \alpha + \beta + 2\sqrt{\alpha\beta}$ $\because$ $\alpha + \beta = 34$ and $\alpha\beta = 1$

$\therefore$ $\left(\sqrt{\alpha} + \sqrt{\beta}\right)^2 = 36$ $\because$ we consider the principal value

$\therefore$ $\sqrt{\alpha} + \sqrt{\beta} = 6$ put in (1), we get. $\Rightarrow$ $\left(\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}}\right)^2 = 4$

$\therefore$ $\alpha^{\frac{1}{4}} - \beta^{\frac{1}{4}} = \pm 2$ **Ans.**

10. The equation can be rewritten as $\left(\frac{x^2+x+2}{x^2+x+1}\right)^2 - (a-3)\left(\frac{x^2+x+2}{x^2+x+1}\right) + (a-4) = 0$

Let $\frac{x^2+x+2}{x^2+x+1} = t$ or $t = 1 + \frac{1}{x^2+x+1}$ since $x^2 + x + 1 = \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$

Therefore $(x^2 + x + 1) \geq \frac{3}{4} \Rightarrow t \in \left(1, \frac{7}{3}\right]$ Now given equation reduces to $t^2 - (a - 3)t + (a - 4) = 0$

Atleast one root of the equation must lie in $\left(1, \frac{7}{3}\right]$ Now, $t = \frac{(a-3) \pm \sqrt{(a-3)^2 - 4(a-4)}}{2} \Rightarrow t = a - 4, 1$

For one root to lie in $\left(1,\frac{7}{3}\right]$ we must have $1 < a - 4 \le \frac{7}{3} \Rightarrow 5 < a \le \frac{19}{3}$

11. Let roots be α, β $\Rightarrow$ $\alpha\beta = -14(q^2 + 1)$. Clearly, $q^2 + 1$ is not multiple of 7

$\because$ α, β are integers clearly one of α or β is multiple of 7

$\because$ $\alpha + \beta = -7$; which is possible if α, β are both multiple of 7. Hence α, β are not integers.

12. Let $x^2 = t \ge 0$, for only real solution. Again let $f(t) = t^2 - (a^2 - 5a + 6)\, t - (a^2 - 3a + 2)$

$f(0) \ge 0 \Rightarrow a^2 - 3a + 2 \le 0$ $a \in [1, 2]$ – $b/2a \ge 0 \Rightarrow a^2 - 5a + 6 \ge 0$ $\Rightarrow$ $(a - 2)(a - 3) \ge 0$ $\Rightarrow a \in (-\infty, 2] \cup [3, \infty)$

So possible 'a' from above two conditions are a = 1, 2. Now condition for $D = ((a - 2)(a - 3))^2 + 4(a - 1)(a - 2) \ge 0$ is also satisfied by these two possible values of a. So required value of 'a' are 1, 2

13. α, β are the roots of $a_1x^2 + b_1x + c_1 = 0$ $\Rightarrow$ $\alpha + \beta = -\frac{b_1}{a_1}$ and $\alpha\beta = \frac{c_1}{a_1}$

$$1 + \alpha + \beta + \alpha\beta = \frac{c_1 - b_1 + a_1}{a_1} \Rightarrow (1+\alpha)(1+\beta) = \frac{a_1 - b_1 + c_1}{a_1} \qquad \text{....................(1)}$$

Similarly $(1+\beta)(1+\gamma) = \frac{a_2 - b_2 + c_2}{a_2}$ ....(2)

$(1+\gamma)(1+\alpha) = \frac{a_3 - b_3 + c_3}{a_3}$ ....(3)

Multiplying (1), (2) & (3), we get $(1+\alpha)^2(1+\beta)^2(1+\gamma)^2 = \frac{(a_1 - b_1 + c_1)}{a_1}\frac{(a_2 - b_2 + c_2)}{a_2}\frac{(a_3 - b_3 + c_3)}{a_3}$

$$\Rightarrow (1+\alpha)(1+\beta)(1+\gamma) = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2}$$

$$\Rightarrow 1 + (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \alpha\gamma) + \alpha\beta\gamma = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2}$$

$$\Rightarrow (\alpha + \beta + \gamma) + (\alpha\beta + \beta\gamma + \alpha\gamma) + \alpha\beta\gamma = \left\{\prod_{i=1}^{3}\left(\frac{a_i - b_i + c_i}{a_i}\right)\right\}^{1/2} - 1$$

14. On putting the value of p, q and r in given equation we have

$2x^3 - (a_1 + a_2 \text{ ....} + a_6)x^2 + (a_1a_3 + a_3a_5 \text{ ....} + a_6a_2)x - (a_1a_3a_5 + a_2a_4a_6) = 0$

$\left\{x^3 - (a_1 + a_3 + a_5)x^2 + (a_1a_3 + a_3a_5 + a_5a_1)x - a_1a_3a_5\right\} +$

$\left\{x^3 - (a_2 + a_4 + a_6)x^2 + (a_2a_4 + a_4a_6 + a_6a_2)x - a_2a_4a_6\right\} = 0$

$\Rightarrow (x - a_1)(x - a_3)(x - a_5) + (x - a_2)(x - a_4)(x - a_6) = 0$

Let $f(x) = (x - a_1)(x - a_3)(x - a_5) + (x - a_2)(x - a_4)(x - a_6)$

Now $f(a_1) = (a_1 - a_2)(a_1 - a_4)(a_1 - a_6) > 0$; $f(a_2) = (a_2 - a_1)(a_2 - a_3)(a_2 - a_5) < 0$

$f(a_3) = (a_3 - a_2)(a_3 - a_4)(a_3 - a_6) < 0$; $f(a_4) = (a_4 - a_1)(a_4 - a_3)(a_4 - a_5) > 0$

$f(a_5) = (a_5 - a_2)(a_5 - a_4)(a_5 - a_6) > 0$; $f(a_6) = (a_6 - a_1)(a_6 - a_3)(a_6 - a_5) < 0$

From above results it is clear that there are three real roots lying in the intervals (a_1, a_2), (a_3,a_4) and (a_5, a_6)

15. Let A_1, A_2 are the roots of $ax^2 + bx + c = 0$, then $(A_1 - A_2)^2 = (A_1 + A_2)^2 - 4A_1A_2 = \frac{b^2}{a^2} - \frac{4c}{a} = \frac{b^2 - 4ac}{a^2}$

Using same result for $x^2 + 2bx + c = 0 \Rightarrow \left\{\left(\beta + \cos^2\alpha\right) - \left(\beta + \sin^2\alpha\right)\right\}^2 = 4b^2 - 4c$

$\Rightarrow \quad \left(\cos^2\alpha - \sin^2\alpha\right)^2 = 4b^2 - 4c \Rightarrow \cos^2 2\alpha = 4(b^2 - c)$ ....(i)

Similarly for $X^2 + 2BX + C = 0 \Rightarrow \left\{\left(\gamma + \cos^4\alpha\right) - \left(\gamma + \sin^4\alpha\right)\right\}^2 = \frac{4B^2 - 4C}{1} = 4B^2 - 4C$

$\Rightarrow \quad \left\{\left(\cos^2\alpha - \sin^2\alpha\right)\left(\cos^2\alpha + \sin^2\alpha\right)\right\}^2 = 4(B^2 - C)$

$\Rightarrow \quad \cos^2 2\alpha = 4(B^2 - C)$ ....(ii)

$\therefore$ from (i) & (ii)

$B^2 - C = b^2 - c$; $B^2 - b^2 = C - c$ Hence Proved.

16. $(x^2 + x)^2 + a(x^2 + x) + 4 = 0$. Let $x^2 + x = t$ then $x^2 + x - t = 0 \quad \forall\ x \in R$

$D \geq 0 \Rightarrow 1 + 4t \geq 0 \Rightarrow t \in \left[-\frac{1}{4}, \infty\right)$ ....(1)

Now $f(t) = t^2 + at + 4 = 0$

(i) all four real and distinct roots

(A) $D > 0$

(B) $f(-1/4) > 0$

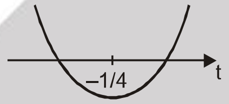
(C) $-\frac{b}{2a} > -\frac{1}{4}$

(A) $D > 0 \Rightarrow a^2 - 16 > 0 \Rightarrow |a| > 4$

(B) $f(-1/4) = \frac{1}{16} - \frac{a}{4} + 4 > 0 \Rightarrow a < 65/4$

(C) $-\frac{b}{2a} = -\frac{a}{2} > -\frac{1}{4} \Rightarrow a < \frac{1}{2} \Rightarrow a \in (-\infty, -4)$

(ii) Two real roots which are distinct



$f(-1/4) < 0 \Rightarrow a > 65/4 \Rightarrow a \in (65/4, \infty)$

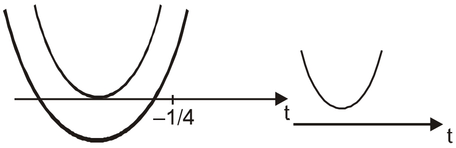
(iii) all four roots are imaginary

Case-(i) (A) $D \geq 0 \Rightarrow |a| \geq 4$

(B) $f(-1/4) > 0 \Rightarrow a < \frac{65}{4}$

(C) $-\frac{b}{2a} < -\frac{1}{4} \Rightarrow a > \frac{1}{2}$

$\Rightarrow a \in \left[4, \dfrac{65}{4}\right)$



case- (i) case- (ii)

Case-(ii) D < 0 $\Rightarrow a \in (-4, 4)$ ...(i)

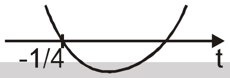
taking union of both conditions mentioned by graph of case-(i) and case-(ii) $a \in \left(-4\ ,\ \dfrac{65}{4}\right)$

(iv) four real roots in which two are equal

(A) $D > 0$ (B) $f(-1/4) = 0$ (C) $-\dfrac{b}{2a} > -\dfrac{1}{4}$

(A) $|a| > 4$ (B) $a = 65/4$ (C) $a < \dfrac{1}{2}$

No common solution $\therefore$ $a \in \phi$



17. $(x^2 + bx + c).P(x) = 3x^4 + 18x^2 + 75$ $\Rightarrow$ $(x^2 + bx + c).Q(x) = 3x^4 + 4x^2 + 28x + 5$

equation (i) – (ii) $(x^2 + bx + c)P(x) - Q(x) = 14x^2 - 28x + 70 = 14(x^2 - 2x + 5)$

$x^2 + bx + c = x^2 - 2x + 5$. hence $f(x) = x^2 - 2x + 5$.

18. Because discrminent of $x^2 + (2-a)x + 3 = 0$ is negative so

$x^2 + (2-a)x + 3 = 0$ has both imaginary roots

$\Rightarrow$ both the equation have two common roots

Now, roots of equation $x^2 + (2-a)x + 3 = 0$ are roots of $(ax^4 + bx^3 + x^2 + (3-a)x + 3) - (x^2 + (2-a)x + 3) = 0$

$\Rightarrow$ roots of equation $x^2 + (2-a)x + 3 = 0$ are roots of $ax^4 + bx^3 + x = 0$

$\Rightarrow$ roots of equation $x^2 + (2-a)x + 3 = 0$ are roots of $ax^3 + bx^2 + 1 = 0$

$\Rightarrow$ roots of equation $x^2 + (2-a)x + 3 = 0$ are roots of $(ax^3 + bx^2 + 1) - (ax(x^2 + (2-a)x + 3)) = 0$

$\Rightarrow$ roots of equation $x^2 + (2-a)x + 3 = 0$ and $(b - 2a + a^2)x^2 - 3ax + 1 = 0$ are same

$\Rightarrow$ $\dfrac{b - 2a + a^2}{1} = \dfrac{2-a}{-3a} = \dfrac{3}{1}$ $\Rightarrow a = -\dfrac{1}{4}$ and $b = -\dfrac{11}{48}$

$\Rightarrow$ $|a + 12b| = 3$

19. $\alpha\beta = \alpha^2\beta^2$ ......(1) $\alpha + \beta = \alpha^2 + \beta^2$ ......(2)

From (1), $\alpha\beta = 0$ or $\alpha\beta = 1$

Case 1 : If $\alpha\beta = 0$ then we have following possibilities

(i) If $\alpha = 0$ then from (2), $\beta = 0$ or $\beta = 1$

(ii) If $\beta = 0$ then from (2), $\alpha = 0$ or $\alpha = 1$

$\therefore$ from (i) and (ii) we can say that roots are $\alpha = 0, \beta = 0$ or $\alpha = 0, \beta = 1$

$\therefore$ Required quadratic equations are $x^2 = 0$ or $x^2 - x = 0$

Case 2 : If $\alpha\beta = 1$ then from (2) we get $\alpha + \beta = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ $\because \alpha\beta = 1$

$\therefore$ $\alpha + \beta = -1, 2$ So the required quadratic equations are $x^2 + x + 1 = 0$

and $x^2 - 2x + 1 = 0$ $\Rightarrow$ four equations are possible.

20. Let $y = Q(x) \Rightarrow$ $\sqrt{y+2} = x$ $\Rightarrow$ $P\left(\sqrt{y+2}\right) = 0$

$(y+2)^{\frac{5}{2}} = -y - 3.$ $\Rightarrow$ $y^5 + 10y^4 + 40y^3 + 79y^2 + 74y + 23 = 0.$

(i) $\displaystyle\prod_{i=1}^{5} Q(\alpha_i) = -\frac{\text{constant term}}{\text{co-efficient of } y^5} = -23$

(ii) $$\sum_{i=1}^{5} Q(\alpha_i) = -\frac{\text{coefficent of } y^4}{\text{coefficent of } y^5} = -10$$

(iii) $$\sum\sum_{1\le i<j\le 5} Q(\alpha_i)\, Q(\alpha_j) = \frac{\text{coefficent of } y^3}{\text{coefficent of } y^5} = 40$$

(iv) $$\sum_{i=1}^{5} Q^2(\alpha_i) = \left(\sum_{i=1}^{5} Q(\alpha_i)\right)^2 - 2\sum\sum_{1\le i<j\le 5} Q(\alpha_i)Q(\alpha_j) = (-10)^2 - 2(40) = 100 - 80 = 20$$

21. $(abc^2)\, x^2 + x\,(3a^2c + b^2c) - (6a^2 + ab - 2b^2) = 0$ for roots to be rational D should be perfect square

$D = (3a^2c + b^2c)^2 + 4[abc^2]\,(6a^2 + ab - 2b^2) = c^2\,[9a^4 + b^4 + 10a^2b^2 + 24a^3b - 8ab^3]$

$= c^2\,(3a^2 - b^2 + 4ab)^2 = [c(3a^2 - b^2 + 4ab)]^2$

22. $\Delta = (a^2 + b^2 + c^2)^2 - 4(a^2b^2 + b^2c^2 + c^2a^2) = a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 - 2c^2a^2$

$= a^4 + b^4 + c^4 - 2a^2b^2 - 2b^2c^2 + 2c^2a^2 - 4c^2a^2 = (a^2 + c^2 - b^2)^2 - 4c^2a^2$

$= (a^2 + c^2 - b^2 - 2ac)\,(a^2 + c^2 - b^2 - 2ac) = [(a - c)^2 - b^2][(a + c)^2 - b^2]$

$a + c > b$ and $|a - c| < b$ (a, b, c are sides of a Δ)

$\therefore$ $(a - c)^2 < b^2$ and $(a + c)^2 > b^2$ $\Rightarrow$ $\Delta = -$ve roots are imaginary.

23. $ax^2 + bx + c = 0$ has roots x_1

and $-ax^2 + bx + c = 0$ has root x_2

and $0 < x_1 < x_2$

Let $f(x) = ax^2 + 2bx + 2c = 0$

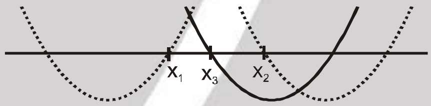
$\therefore$ $f(x_1)\, f(x_2) < 0$

L.H.S. = $(ax_1^2 + 2bx_1 + 2c)\,(ax_2^2 + 2bx_2 + 2c)$

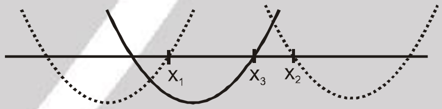
$[ax_1^2 + 2(-ax_1^2)]\,[ax_2^2 + 2(ax_2^2)]$

$(-ax_1^2)\,(3ax_2^2) = -\,3a^2x_1^2\,x_2^2 < 0$

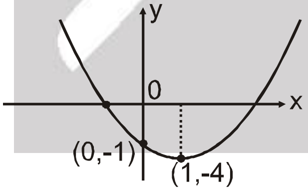
$\therefore$ $f(x_1)\, f(x_2) < 0$ Hence proved.



or



24. Clearly the graph of $y = x^4 - 4x - 1$ is



$\therefore$ no of positive real roots = 1

Aliter

$y = x^4 - 4x - 1;\ \frac{dy}{dx} = 4x^3 - 4;\ \frac{d^2y}{dx^2} = 12x^2$ when $\frac{dy}{dx} = 0$, then $x = 1$

$\left.\frac{d^2y}{dx^2}\right|_{x=1} = 12 > 0$. so $x = 1$ is a minima point so by graph, number of positive real roots = 1

25. Since the given equation has distinct roots

$\therefore$ $D > 0$ $\Rightarrow$ $16 + 4(1 - k^2) > 0$ $\Rightarrow$ $k^2 < 5$, also $k \neq -1$

$\because$ If k = –1we will get only one solution, but we want two solutions

$\therefore$ $k^2 < 5, k \neq -1$

26. Since $\alpha, \beta = \frac{-\ b \pm \Delta}{2a}$ and $\Delta^2 = b^2 - 4ac$ either $2a\alpha + \Delta = -b$ and $2a\beta - \Delta = -b$

or $2a\alpha + \Delta = -b + 2\Delta$ and $2a\beta - \Delta = -b - 2\Delta$

$\therefore$ Sum of roots of required equation = – 2b

And product of roots = b^2 or $b^2 - 4\Delta^2 = -3b^2 + 16ac$

$\therefore$ Required equation is either $x^2 + 2bx + b^2 = 0$ or $x^2 + 2bx - 3b^2 + 16ac = 0$

27. Given equation can be Expressed as

$\pi^e(x - \pi)(x - \pi - e) + e^\pi(x - e)(x - \pi - e) + (\pi^\pi + e^e)(x - e)(x - \pi) = 0$

Let $f(x) = \pi^e(x - \pi)(x - \pi - e) + e^\pi(x - e)(x - \pi - e) + (\pi^\pi + e^e)(x - e)(x - \pi)$

$\Rightarrow$ $f(e) = \pi^e(e - \pi)(-\pi) > 0$

and $f(\pi) = e^\pi(\pi - e)(-e) < 0$; hence given equation has a real root in (e, π)

again $f(\pi + e) = (\pi^\pi + e^e)\,\pi.\,e > 0$

$\because$ $\pi + e > \pi$, It concluds it has a real root in $(\pi, \pi + e)$

also $\because$ $\pi - e < e$

Hence f(x) has two real roots in $(\pi - e, \pi + e)$

28. Call roots as $-2\alpha, \alpha + i\beta, \alpha - i\beta$

Sum of roots $0 = -a$ ....(1)

Sum of products taken two at a time

$-2\alpha(\alpha + i\beta) - 2\alpha(\alpha - i\beta) + \alpha^2 + \beta^2 = b$ ....(2)

Product of roots

$-2\alpha(\alpha^2 + \beta^2) = 316$ ....(3)

$\Rightarrow$ $\alpha(\alpha^2 + \beta^2) = -2 \times 79$

Case-I : $\alpha = -1, \alpha^2 + \beta^2 = 158$

$\Rightarrow$ $\beta^2 = 157$ not multiple of 3

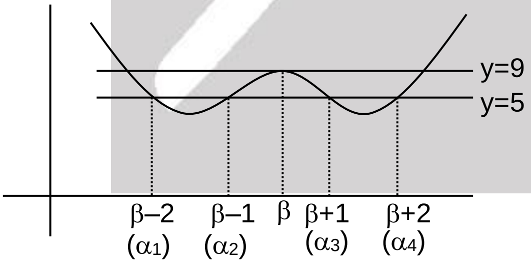
Rejected

Case-II : $\alpha = -2, \alpha^2 + \beta^2 = 79$ $\Rightarrow$ $\beta^2 = 75 = 3 \times 25$ $\Rightarrow$ $\beta = \pm 5\sqrt{3}$

$\therefore$ Roots are $4, -2 \pm 5\sqrt{3}$ $\Rightarrow$ $-a = 4 + (-2 + 5\sqrt{3}) + (-2 - 5\sqrt{3}) = 0$

$b = 4(-2 + 5\sqrt{3}) + (-2 + 5\sqrt{3})(-2 - 5\sqrt{3}) + 4(-2 - 5\sqrt{3}) = -16 + 4 + 75 = 63.$

29_.



$f(x) - 5 = a(x - \alpha_1)(x - \alpha_2)(x - \alpha_3)(x - \alpha_4)$

Let at $x = \beta$, $f(\beta) = 9$ then

$f(\beta) - 5 = a(\beta - \alpha_1)(\beta - \alpha_2)(\beta - \alpha_3)(\beta - \alpha_4) = 9 - 5 = 4$

$\Rightarrow$ $\beta - \alpha_1 = 2, \beta - \alpha_3 = -1, \beta - \alpha_2 = 1, \beta - \alpha_4 = -2$ and $a = 1$

$\Rightarrow$ $\alpha_1 = \beta - 2, \alpha_4 = \beta + 2, \alpha_2 = \beta - 1, \alpha_3 = \beta + 1$

$\Rightarrow$ $f(x) = (x - \beta + 2)(x - \beta - 2)(x - \beta - 1)(x - \beta + 1) + 5$

$\Rightarrow$ $f(x) = ((x - \beta)^2 - 4)((x - \beta)^2 - 1) + 5$

$\Rightarrow$ $f(x) = (x - \beta)^4 - 5(x - \beta)^2 + 9$

$\Rightarrow$ $f(x + \beta) = x^4 - 5x^2 + 9$

$\Rightarrow$ $f(\beta) = 9$ and $f'(x + \beta) = 4x^3 - 10x$

$\Rightarrow$ $f'(\beta) = 0$

30. $(xy - 7)^2 = x^2 + y^2 \Rightarrow$ $(xy - 6)^2 + 13 = (x + y)^2$ $\Rightarrow (x + y - xy + 6)(x + y + xy - 6) = 13$

Case-I $x + y + xy - 6 = 13$; $x + y - xy + 6 = 1$

On solving $(x, y) \equiv (4, 3), (3, 4)$

Case-II $x + y + xy - 6 = 1$; $x + y - xy + 6 = 13$

On solving $(x,) \neq (0, 7), (7, 0)$

In all other cases negative solutions are obtained

hence solution set is (3, 4) (4, 3), (7, 0), (0, 7)

$\therefore$ Sum of all possible values of x is $3 + 4 + 7 + 0 = 14$. **Ans.**