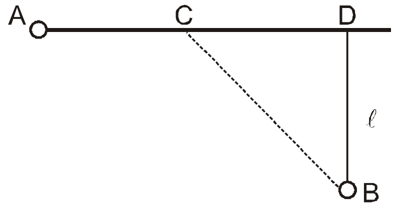
High Level Problems (HLP)

1. From point A located on a highway (Fig.) one has to get by car as soon as possible to point B located in the field at a distance ℓ from the highway. It is known that the car moves in the field η times slower than on the highway. At what distance from point D one must turn off the highway?

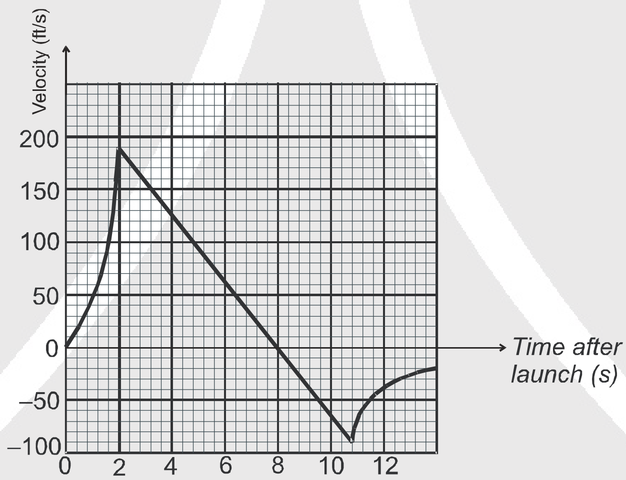


2. The velocity of a particle moving in the positive direction of the x axis varies as $v = \alpha\sqrt{x}$ where α is a positive constant. Assuming that at the moment t = 0 the particle was located at the point x = 0, find:

(a) the time dependence of the velocity and the acceleration of the particle;

(b) the mean velocity of the particle averaged over the time that the particle takes to cover the first s metres of the path.

3. When a model rocket is launched, the propellant burns for a few seconds, accelerating the rocket upward. After burnout, the rocket moves upward for a while and then begins to fall. A parachute opens shortly after the rocket starts down. The parachute slows the rocket to keep it from breaking when it lands.The figure here shows velocity data from the flight of the model rocket. Use the data to answer the following.



(a) How fast was the rocket climbing when the engine stopped?

(b) For how many seconds did the engine burn?

(c) When did the rocket reach its highest point? What was its velocity then?

(d) When did the parachute open up? How fast was the rocket falling then?

(e) How long did the rocket fall before the parachute opened?

(f) When was the rocket's acceleration greatest?

(g) When was the acceleration constant? What was its value then (to the nearest integer)?

4. A particle of mass 10^{-2} kg is moving along the x–axis under the influence of a force $F(x) = -\frac{K}{2x^2}$ where $K = 10^{-2}$ N m^2. At time t = 0 it is at x = 1.0 m and its velocity is v = 0. Find

(i) its velocity when it reaches x = 0.50 m

(ii) the time at which it reaches x = 0.25 m. **[JEE 1998, 8]**

5. A parachutist jumps from height 100 m. He wants to reach at ground with zero velocity. For this purpose he switches on a parachute propeller after falling freely for certain height. Given that after the parachute propeller is switched on total acceleration of the man varies with velocity as a = –2v, where v is the instantaneous velocity of the man. Find the time after falling freely man should switch on parachute propeller for this purpose. (Use g = $10 m/s^2$).

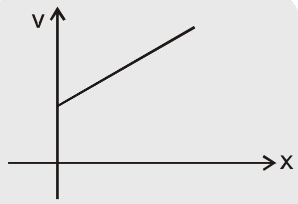
6. Position (in m) of a particle moving on a straight line varies with time (in sec) as $x = t^3/3 - 3t^2+8t+4$ (m). Consider the motion of the particle from t = 0 to t = 5 sec. s_1 is the total distance travelled and s_2 is the distance travelled during retardation. Find s_1/s_2.

7. The maximum possible acceleration of a train starting from the rest and moving on straight track is 10 m/s^2 and maximum possible retardation is 5 m/s^2. The maximum speed that train can achieve is 70 m/s. Minimum time in which the train can complete a journey of 1000m ending at rest is $\frac{347}{2\alpha}$ sec .Where α is an intger. Find α .

8. A train stopping at two stations 2 kms apart on a straight line takes 4 minutes for the journey. Assuming that its motion is first uniformly accelerated and then uniformly retarded. Prove that $\frac{1}{x} + \frac{1}{y} = 4$, where x and y' are the magnitude of the acceleration and retardation respectively in (km/min^2).

9. Two particles thrown from top of a tower with same speed in upward and downward directions simultaneously. If the first particle hits the ground in time t and second particle takes time t_0 after first one hitting the ground to hit the ground. Then find
(a) Maximum height attained by second particle above the tower.
(b) Height of tower

10. A particle moves along x-axis in positive direction. Its acceleration 'a' is given as a = cx + d, where x denotes the x-coordinate of particle, c and d are positive constants. For velocity-position graph of particle to be of type as shown in figure, find the value of speed (in m/s) of particle at x = 0.
Take c = 1 s^{-2} and d = 3 ms^{-2}



HLP Answers

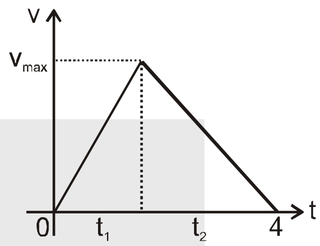
1. CD = $\dfrac{\ell}{\sqrt{\eta^2 - 1}}$ **2.** (a) v = $\alpha^2 t/2$, a = $\alpha^2/2$; (b) <v> = $\dfrac{\alpha\sqrt{s}}{2}$

3. (a) 190 ft/s (b) 2 s (c) 8 s, 0 ft/s (d) 54/5 = 10.8 s, 90 ft/s
(e) 14/5 = 2.8 s (f) Greatest acceleration happens 2 s after launch
(g) Constant acceleration between 2 and 54/5 = 10.8 s, – 32 ft/s^2.

4. (i) $\vec{V} = -1\,\hat{i}$ m/s (ii) t = $\dfrac{\pi}{3} + \dfrac{\sqrt{3}}{4}$ **5.** 4 sec **6.** 14/11

7. 7

8. Area of v-t curve is displacment which is equal to 2



$$\frac{1}{2} \times v_{max} \times 4 = 2$$

v_{max} = 1

Also $t_1 + t_2 = 4$

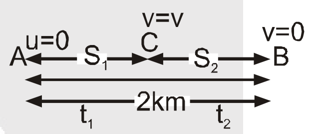
$$\frac{v_{max}}{x} + \frac{v_{max}}{y} = 4 \quad \Rightarrow \quad \frac{1}{x} + \frac{1}{y} = 4$$

Alter :

Given, $S_1 + S_2 = 2$ ....(i)

$t_1 + t_2 = 4$ ....(ii)

For motion from A to C:



From, V = u + at

V = 0 + xt,

t_1 = V/x

From $V^2 = u^2 + 2as$

$V^2 = 0 + 2xS_1 \Rightarrow S_1 = V^2/2x$

Similarly for motion from C to B, $t_2 = V/y$; $S_2 = V^2/2y$

From eqn.(i)

$$\frac{V^2}{2x} + \frac{V^2}{2y} = 2 \quad \Rightarrow \quad \frac{V^2}{2}\left(\frac{1}{x} + \frac{1}{y}\right) = 2 \qquad ....(iii)$$

From eqn. (ii)

$$V\left(\frac{1}{x} + \frac{1}{y}\right) = 4 \qquad ....(iv)$$

Solving (iii) & (iv) we get, $\dfrac{1}{x} + \dfrac{1}{y} = 4$

9. (a) $h_A = \dfrac{gt_0}{8}$ (b) $h_T = \dfrac{1}{2}$ gt (t + t_0) **10.** 3