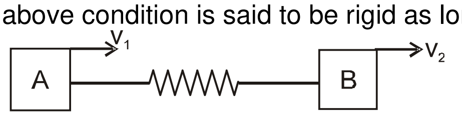
- *Imagination is more important than knowledge*
- *Everything should be made as simple as possible, but not simpler.* - *Albert Einstein*

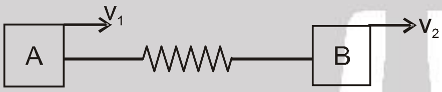
RIGID BODY DYNAMICS

1. RIGID BODY :

Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time). Remember, rigid body is a mathematical concept and any system which satisfies the above condition is said to be rigid as long as it satisfies it.

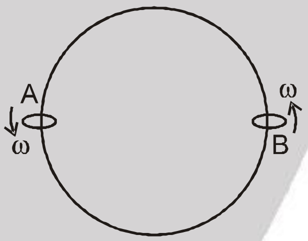
☞ $(v_1 = v_2)$


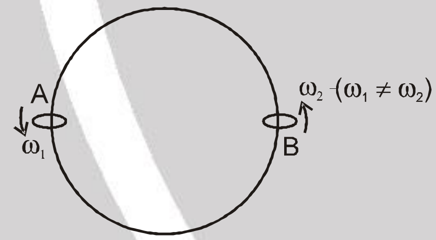
System behaves as a rigid body

$(v_1 \neq v_2)$


System behaves as a non-rigid body

☞ A & B are beads which move on a circular fixed ring

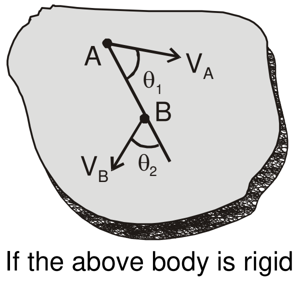


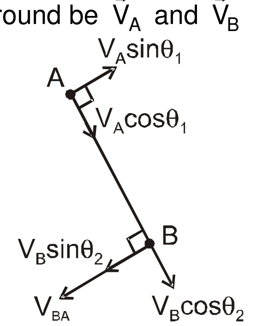


A + B is a rigid body system
but A + B + ring is non-rigid system

A + B is non-rigid system

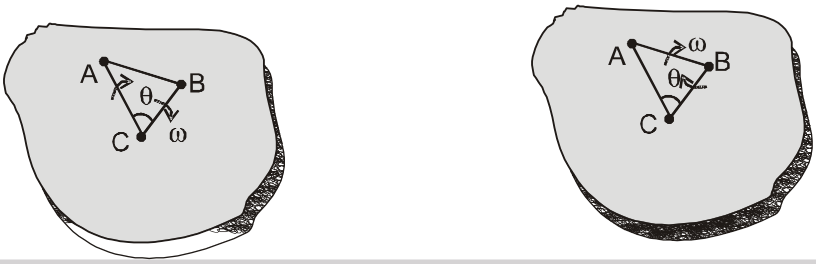
- If a system is rigid, since there is no change in the distance between any pair of particles of the system, shape and size of system remains constant. Hence we intuitively feel that while a stone or cricket ball are rigid bodies, a balloon or elastic string is non rigid.
  But any of the above system is rigid as long as relative distance does not change, whether it is a cricket ball or a balloon. But at the moment when the bat hits the cricket ball or if the balloon is squeezed, relative distance changes and now the system behaves like a non-rigid system.
- For every pair of particles in a rigid body, there is no velocity of separation or approach between the particles. i.e. any relative motion of a point B on a rigid body with respect to another point A on the rigid body will be perpendicular to line joining A to B, hence with respect to any particle A of a rigid body the motion of any other particle B of that rigid body is circular motion.
  Let velocities of A and B with respect ground be $\vec{V}_A$ and $\vec{V}_B$ respectively in the figure below.





If the above body is rigid $V_A \cos\theta_1 = V_B \cos\theta_2$ (velocity of approach / separation is zero)
V_{BA} = relative velocity of B with respect to A.
$V_{BA} = V_A \sin\theta_1 + V_B \sin\theta_2$ (which is perpendicular to line AB)
B will appear to move in a circle to an observer fixed at A.

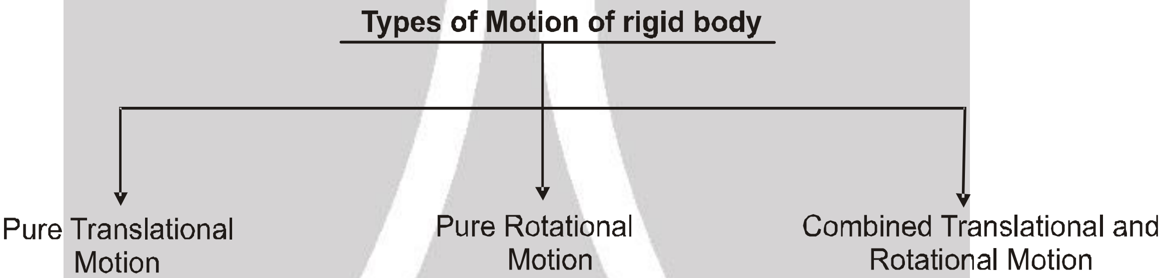
- W.r.t. any point of the rigid body the angular velocity of all other points of the that rigid body is same.
- Suppose A, B, C is a rigid system hence during any motion sides AB, BC and CA must rotate through the same angle. Hence all the sides rotate by the same rate.



(i) (ii)

From figure (i) angular velocity of A and B w.r.t. C is ω,

From figure (ii) angular velocity of A and C w.r.t. B is ω,



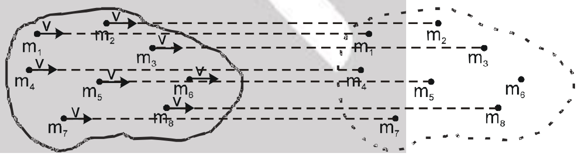
I. Pure Translational Motion :

A body is said to be in pure translational motion, if the displacement of each particle of the system is same during any time interval. During such a motion, all the particles have same displacement $(\vec{s})$, velocity $(\vec{v})$ and acceleration $(\vec{a})$ at an instant.

Consider a system of n particle of mass m_1, m_2, m_3, ... m_n under going pure translation then from above definition of translational motion

$\vec{a}_1 = \vec{a}_2 = \vec{a}_3 = ...\vec{a}_n = \vec{a}$ (say)

and $\vec{v}_1 = \vec{v}_2 = \vec{v}_3 = ...\vec{v}_n = \vec{v}$ (say)



From Newton's laws for a system.

$$\vec{F}_{ext} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + ...$$

$$\vec{F}_{ext} = M\vec{a}$$

Where M = Total mass of the body

$$\vec{P} = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + ...$$

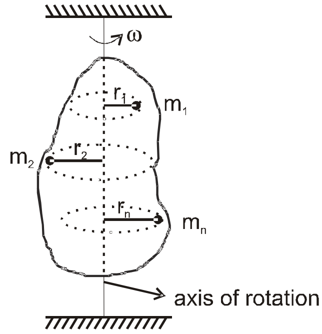
$$\vec{P} = M\vec{v}$$

Total Kinetic Energy of body = $\frac{1}{2}$ $m_1v_1^2 + m_2v_2^2 +$ .... = Mv^2

II. Pure Rotational Motion :

Figure shows a rigid body of arbitrary shape in rotation about a fixed axis, called the axis of rotation. Every point of the body moves in a circle whose center lies on the axis of rotation, and every point moves through the same angle during a particular time interval. Such a motion is called pure rotation.

We know that each particle has same angular velocity (since the body is rigid.)



so, $v_1 = \omega r_1$, $v_2 = \omega r_2$, $v_3 = \omega r_3$ ...... $v_n = \omega r_n$

Total Kinetic Energy = $\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + ...$

$= \frac{1}{2}[m_1 r_1^2 + m_2 r_2^2 + ...]\ \omega^2$

$= \frac{1}{2} I\omega^2$ Where $I = m_1 r_1^2 + m_2 r_2^2 + ...$ (is called moment of inertia)

ω = angular speed of body.

III. Combined Translational and Rotational Motion :

A body is said to be in combined translation and rotational motion if all point in the body rotates about an axis of rotation and the axis of rotation moves with respect to the ground. Any general motion of a rigid body can be viewed as a combined translational and rotational motion.

Solved Examples

Example 1. A body is moving down into a well through a rope passing over a fixed pulley of radius 10 cm. Assume that there is no slipping between rope & pulley. Calculate the angular velocity and angular acceleration of the pulley at an instant when the body is going down at a speed of 20 cm/s and has an acceleration of 4.0 m/s^2.

Solution : Since the rope does not slip on the pulley, the linear speed v of the rim of the pulley is same as the speed of the body.

The angular velocity of the pulley is then $\omega = v/r = \frac{20 cm/s}{10 cm} = 2$ rad/s

and the angular acceleration of the pulley is $\alpha = a/r = \frac{4.0\ m/s^2}{10\ cm} = 40$ rad/s^2.

Example 2. A disc rotates with a uniform angular acceleration of 2.0 rad/s^2 about its axis. If the disc starts from rest, how many revolutions will it make in the first 10 seconds?

Solution : The angular displacement in the first 10 seconds is given by

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}(2.0\ rad/s^2)\ (10s)^2 = 100$ rad.

As the wheel turns by 2π radian in each revolution, the number of revolutions in 10s is

$n = \frac{100}{2\pi} = 16.$

Example 3. The wheel of a motor, accelerated uniformly from rest, rotates through 5 radian during the first second. Calculate the angle rotated during the next second.

Solution : As the angular acceleration is constant, we have

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \frac{1}{2}\alpha t^2$

Thus, 5 rad = $\frac{1}{2}\alpha\ (1s)^2$

$\alpha = 10$ rad/s^2 or $\alpha = 10$ rad/s^2

The angle rotated during the first two seconds is = $\frac{1}{2} \times (10\ rad/s^2)\ (2s)^2 = 20$ rad

Thus, the angle rotated during the 2nd second is 20 rad – 5 rad = 15 rad

Example 4. Starting from rest, a fan takes four seconds to attain the maximum speed of 400 rpm (revolution per minute). Assuming uniform acceleration, calculate the time taken by the fan in attaining half the maximum speed.

Solution : Let the angular acceleration be α. According to the question,

400 rev/min $= 0 + \alpha\, 4$ ....(i)

Let t be the time taken in attaining the speed of 200 rev/min which is half the maximum.

Then, 200 rev/min $= 0 + \alpha t$ ....(ii)

Dividing (i) by (ii), we get,

$t = 2$ sec.

Example 5. The motor of an engine is rotating about its axis with an angular velocity of 120 rev/minute. It comes to rest in 10 s, after being switched off the engine. Assuming uniform angular deceleration, find the number of revolutions made by it before coming to rest.

Solution : The initial angular velocity = 120 rev/minute = (4π) rad/s.

Final angular velocity = 0.

Time interval = 10 s.

Let the angular acceleration be α. Using the equation $\omega = \omega_0 + \alpha t$, we obtain

$\alpha = (-4\pi/10)$ rad/s^2

The angle rotated by the motor during this motion is

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 = \left(4\pi \frac{\text{rad}}{\text{s}}\right)(10\text{s}) - \frac{1}{2}\left(\frac{4\pi}{10}\frac{\text{rad}}{\text{s}^2}\right)(10\text{s})^2$$

$= 20\pi$ rad $= 10$ revolutions.

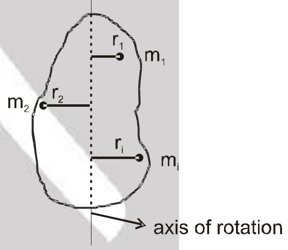
Hence the motor rotates through 10 revolutions before coming to rest.

2. MOMENT OF INERTIA (I) ABOUT AN AXIS :

(i) Moment of inertia of a system of n particles about an axis is defined as :

$I = m_1 r_1^2 + m_2 r_2^2 + ... + m_n r_n^2$

i.e. $I = \sum_{i=1}^{n} m_i r_i^2$



where, r_i = It is perpendicular distance of mass m_i from axis of rotation

SI units of Moment of Inertia is Kgm^2.

Moment of inertia is a scalar positive quantity.

(ii) For a continuous system :

$I = \int r^2 (dm)$

where dm = mass of a small element

r = perpendicular distance of the mass element dm from the axis

Moment of Inertia depends on :

(i) density of the material of body

(ii) shape & size of body

(iii) axis of rotation

In totality we can say that it depends upon distribution of mass relative to axis of rotation.

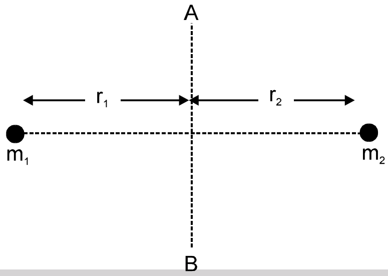
Note : Moment of inertia does not change if the mass :

(i) is shifted parallel to the axis of the rotation because r_i does not change.

(ii) is rotated about axis of rotation in a circular path because r_i does not change.

Solved Examples

Example 6. Two particles having masses m_1 & m_2 are situated in a plane perpendicular to line AB at a distance of r_1 and r_2 respectively as shown.



(i) Find the moment of inertia of the system about axis AB ?
(ii) Find the moment of inertia of the system about an axis passing through m_1 and perpendicular to the line joining m_1 and m_2 ?
(iii) Find the the moment of inertia of the system about an axis passing through m_1 and m_2?
(iv) Find moment of inertia about axis passing through centre of mass and perpendicular to line joining m_1 and m_2

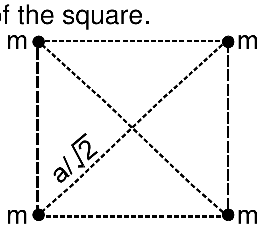
Solution : (i) Moment of inertia of particle on left is $I_1 = m_1r_1^2$.
Moment of Inertia of particle on right is $I_2 = m_2r_2^2$.
Moment of Inertia of the system about AB is $I = I_1 + I_2 = m_1r_2^2 + m_2r_2^2$
(ii) Moment of inertia of particle on left is $I_1 = 0$
Moment of Inertia of particle on right is $I_2 = m_2(r_1 + r_2)^2$.
Moment of Inertia of the system about AB is $I = I_1 + I_2 = 0 + m_2 (r_1 + r_2)^2$
(iii) Moment of inertia of particle on left is $I_1 = 0$
Moment of Inertia of particle on right is $I_2 = 0$
Moment of Inertia of the system about AB is $I = I_1 + I_2 = 0 + 0$
(iv) Centre of mass of system $r_{CM} = m_2\left(\frac{r_1 + r_2}{m_1 + m_2}\right)$ = Distance of centre mass from mass m_1

Distance of centre of mass from mass $m_2 = m_1\left(\frac{r_1 + r_2}{m_1 + m_2}\right)$

So moment of inertia about centre of mass = $I_{cm} = m_1\left(m_2\frac{r_1 + r_2}{m_1 + m_2}\right)^2 + m_2\left(m_1\frac{r_1 + r_2}{m_1 + m_2}\right)^2$

$$I_{CM} = \frac{m_1m_2}{m_1 + m_2}(r_1 + r_2)^2$$

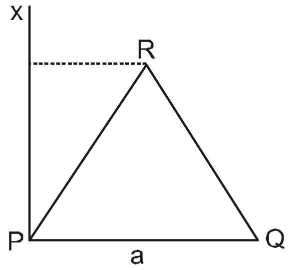
Example 7. Four particles each of mass m are kept at the four corners of a square of edge a. Find the moment of inertia of the system about a line perpendicular to the plane of the square and passing through the centre of the square.



Solution : The perpendicular distance of every particle from the given line is $a/\sqrt{2}$. The moment of inertia of one particle is, therefore, $m(a/\sqrt{2})^2 = \frac{1}{2}ma^2$. The moment of inertia of the system is, therefore,

$$4 \times \frac{1}{2}ma^2 = 2ma^2$$

Example 8. Three particles, each of mass m, are situated at the vertices of an equilateral triangle PQR of side a as shown in figure. Calculate the moment of inertia of the system about
(i) The line PX perpendicular to PQ in the plane of PQR.
(ii) One of the sides of the triangle PQR
(iii) About an axis passing through the centroid and perpendicular to plane of the triangle PQR.



Solution : (i) Perpendicular distance of P from PX = 0
Perpendicular distance of Q from PX = a
Perpendicular distance of R from PX = a/2
Thus, the moment of inertia of the particle at P = 0, of the particle at Q - ma^2, and of the particle at R = $m(a/2)^2$. The moment of inertia of the three-particle system about PX is

$$0 + ma^2 + m(a/2)^2 = \frac{5ma^2}{4}$$

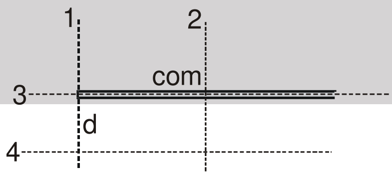
Note that the particles on the axis do not contribute to the moment of inertia.
(ii) Moment of inertia about the side PR = mass of particle Q × square of perpendicular distance of Q from side PR, $I_{PR} = m\left(\frac{\sqrt{3}}{2}a\right)^2 = \frac{3ma^2}{4}$

(iii) Distance of centroid from all the particle is $\frac{a}{\sqrt{3}}$, so moment of inertia about an axis and passing through the centroic perpendicular plane of triangle PQR = $I_R = 3m\left(\frac{a}{\sqrt{3}}\right)^2 = ma^2$

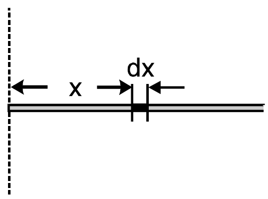
Example 9. Calculate the moment of inertia of a ring having mass M, radius R and having uniform mass distribution about an axis passing through the centre of ring and perpendicular to the plane of ring ?

Solution : $I = \int (dm)\ r^2$
Because each element is equally distanced from the axis so r = R
$= R^2 \int dm = MR^2$
$I = MR^2$
(Note : Answer will remain same even if the mass is nonuniformly distributed because $\int dm = M$ always.)

Example 10. Calculate the moment of inertia of a uniform rod of mass M and length ℓ about an axis 1, 2, 3 and 4.



Solution $(I_1) = \int (dm)\ r^2 = \int_0^{\ell} \left(\frac{M}{\ell}dx\right)\ x^2 = \frac{M\ell^2}{3}$

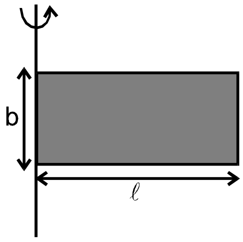


$(I_2) = \int (dm)\ r^2 = \int_{-\ell/2}^{\ell/2} \left(\frac{M}{\ell}dx\right)\ x^2 = \frac{M\ell^2}{12}$

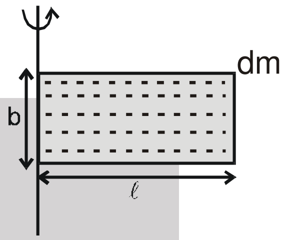
$(I_3) = 0$ (axis 3 passing through the axis of rod)

$(I_4) = d^2 \int (dm) = Md^2$

Example 11. Determined the moment of inertia of a uniform rectangular plate of mass, side 'b' and 'ℓ'' about an axis passing through the edge 'b' and in the plane of plate.

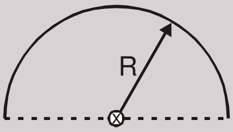


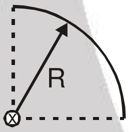
Solution : Each section of dm mass rod in the rectangular plate has moment of inertia about an axis passing through edge 'b' $dI = \dfrac{dm\ell^2}{3}$

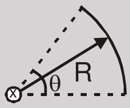


So $I = \int dI = \dfrac{\ell^2}{3}\int dm = \dfrac{M\ell^2}{3}$

Example 12. Find out the moment of Inertia of figures shown each having mass M, radius R and having uniform mass distribution about an axis passing through the centre and perpendicular to the plane ?

(a) 

(b) 

(c) 

Solution : MR^2 (infact M.I. of any part of mass M of a ring of radius R about axis passing through geometrical centre and perpendicular to the plane of the ring is = MR^2)

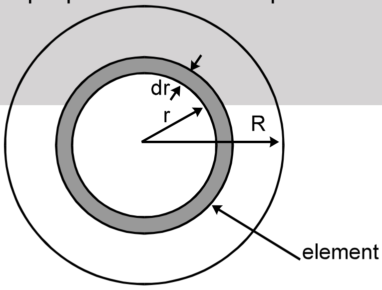
(iii) Moment of inertia of a large object can be calculated by integrating M.I.of an element of the object**:**

$I = \int dI_{element}$ where dI = moment of inertia of a small element.

Element chosen should be such that : either perpendicular distance of axis from each point of the element is same or the moment of inertia of the element about the axis of rotation is known.

Solved Examples

Example 13. Determine the moment of Inertia of a uniform disc having mass M, radius R about an axis passing through centre & perpendicular to the plane of disc ?



Solution : $I = \int dI_{ring}$

element - ring $dI = dmr^2$

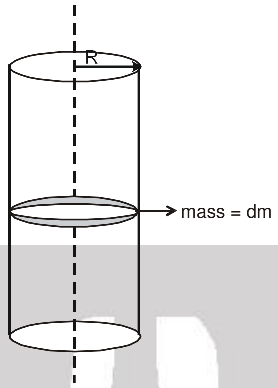
$dm = \dfrac{M}{\pi R^2} 2\pi r dr$ (here we have used the uniform mass distribution)

$\therefore \quad I = \int_0^R \dfrac{M}{\pi R^2}.(2\pi r dr).r^2 \quad \Rightarrow \quad I = \dfrac{MR^2}{2}$

Example 14. Calculate the moment of inertia of a uniform hollow cylinder of mass M, radius R and length ℓ about its axis.

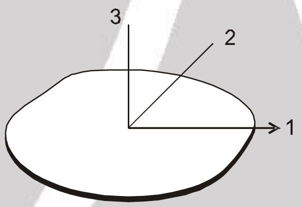
Solution : Moment of inertia of a uniform hollow cylinder is

$I = \int (dm)R^2 = mR^2$



3. TWO IMPORTANT THEOREMS ON MOMENT OF INERTIA

(i) Perpendicular Axis Theorem [Only applicable to plane laminar bodies (i.e. for 2 dimensional objects only)].



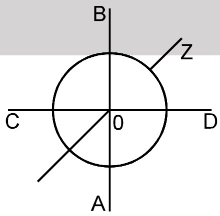
Body is in 1-2 plane

If axis 1 & 2 are in the plane of the body and perpendicular to each other.
Axis 3 in perpendicular to plane of 1 & 2.
Then, $I_3 = I_1 + I_2$

☞ The point of intersection of the three axis need not be center of mass, it can be any point in the plane of body which lies on the body or even outside it.

Solved Examples

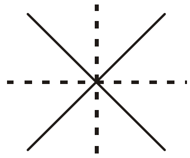
Example 15. Calculate the moment of inertia of a uniform disc of mass M and radius R about a diameter.



Solution : Let AB and CD be two mutually perpendicular diameters of the disc. Take them as X and Y-axes and the line perpendicular to the plane of the disc through the centre as the Z-axis. The moment of inertia of the ring about the Z-axis is $I = \frac{1}{2}MR^2$. As the disc is uniform, all of its diameters are equivalent and so $I_x = I_y$, From perpendicular axes theorem,

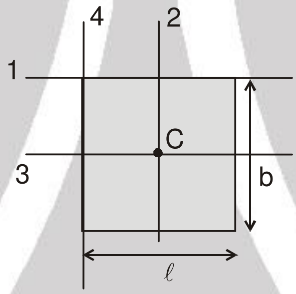
$I_z = I_x + I_y$. Hence $I_x = \frac{I_z}{2} = \frac{MR^2}{4}$

Example 16. Two uniform identical rods each of mass M and lengthare joined to form a cross as shown in figure. Find the moment of inertia of the cross about a bisector as shown dotted in the figure.



Solution : Consider the line perpendicular to the plane of the figure through the centre of the cross. The moment of inertia of each rod about this line is $\frac{M\ell^2}{12}$ and hence the moment of inertia of the cross is $\frac{M\ell^2}{6}$. The moment of inertia of the cross about the two bisector are equal by symmetry and according to the theorem of perpendicular axes, the moment of inertia of the cross about the bisector is $\frac{M\ell^2}{12}$.

Example 17. In the figure shown find moment of inertia of a plate having mass M, length ℓ and width b about axis 1,2,3 and 4. Assume that mass is uniformly distributed.



Solution : Moment of inertia of the plate about axis 1 (by taking rods perpendicular to axis 1)

$I_1 = Mb^2 / 3$

Moment of inertia of the plate about axis 2 (by taking rods perpendicular to axis 2)

$I_2 = M\ell^2 / 12$

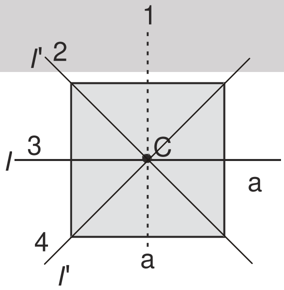
Moment of inertia of the plate about axis 3 (by taking rods perpendicular to axis 3)

$I_3 = Mb^2 / 12$

Moment of inertia of the plate about axis 4 (by taking rods perpendicular to axis 4)

$I_4 = M\ell^2 / 3$

Example 18. In the figure shown find the moment of inertia of square plate having mass m and sides a. About an axis 2 passing through point C (centre of mass) and in the plane of plate.



Solution : Using perpendicular axis theorems $I_C = I_4 + I_2 = 2I'$

Using perpendicular theorems $I_C = I_3 + I_1 = I + I = 2I$

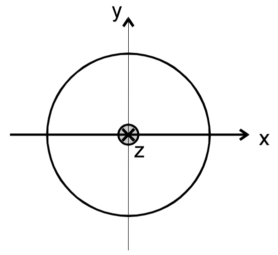
$2I' = 2I$

$I' = I$

$IC = 2I = \frac{ma^2}{6} \qquad \Rightarrow \qquad I' = \frac{ma^2}{12}$

Example 19. Find the moment of Inertia of a uniform disc of mass M and radius R about a diameter.

Solution : Consider x & y two mutually perpendicular diameters of the ring.



$I_x + I_y = I_z$

$I_x = I_y$ (due to symmetry)

$I_z = \frac{MR^2}{2}$

$I_x = I_y = \frac{MR^2}{4}$

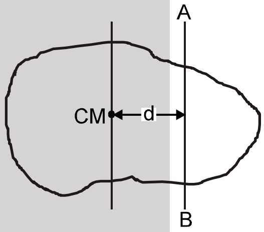
(ii) Parallel Axis Theorem (Applicable to planer as well as 3 dimensional objects):

If I_{AB} = Moment of Inertia of the object about axis AB

I_{cm} = Moment of Inertia of the object about an axis passing through centre of mass and parallel to axis AB

M = Total mass of object

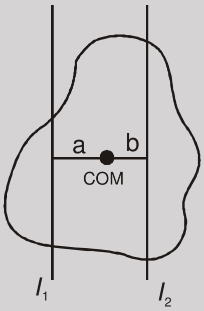
d = perpendicular distance between axis AB about which moment of Inertia is to be calculated & the one passing through the centre of mass and parallel to it.



$I_{AB} = I_{cm} + Md^2$

Solved Examples

Example 20. Find out relation between I_1 and I_2. I_1 and I_2 moment of inertia of a rigid body mass m about an axis as shown in figure.



Solution : Using parallel axis theorem

$I_1 = I_C + ma^2$ .....(1)

$I_2 = I_C + mb^2$ .....(2)

From (1) and (2) ; $I_1 - I_2 = m(a^2 - b^2)$

Example 21. Find the moment of inertia of a uniform sphere of mass m and radius R about a tangent if the spheres (i) solid (ii) hollow

Solution : (i) Using parallel axis theorem $I = I_{CM} + md^2$

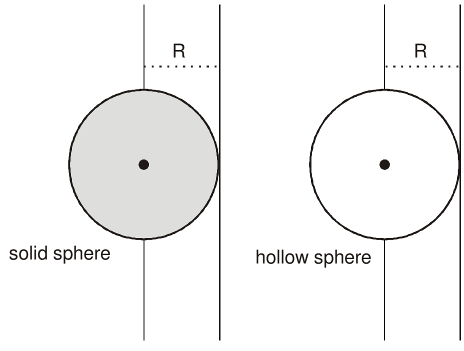
for solid sphere $I_{CM} = \frac{2}{5}mR^2$, d = R

$I = \frac{7}{5}mR^2$

(ii) Using parallel axis theorem $I = I_{CM} + md^2$

for hollow sphere $I_{CM} = \frac{2}{3}mR^2$, d = R

$I = \frac{5}{3}mR^2$



Example 22. Calculate the moment of inertia of a hollow cylinder of mass M and radius R about a line parallel to the axis of the cylinder and on the surface of the cylinder.

Solution : The moment of inertia of the cylinder about its axis = MR^2.

Using parallel axes theorem, $I = I_0 + MR^2 = MR^2 + MR^2 = 2\,MR^2$.

Similarly, the moment of inertia of a hollow sphere about a tangent is

$$\frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$

Example 23. Find out the moment of inertia of a semi circular disc about an axis passing through its centre of mass and perpendicular to the plane?

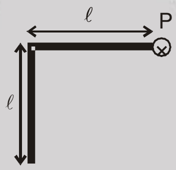
Solution : Moment of inertia of a semi circular disc about an axis passing through centre and perpendicular to plane of disc, $I = \frac{MR^2}{2}$

Using parallel axis theorem $I = I_{CM} + Md^2$, d is the perpendicular distance between two parallel axis passing through centre C and COM.

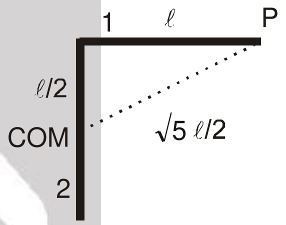
$$I = \frac{MR^2}{2},\ d = \frac{4R}{3\pi} \qquad \Rightarrow \qquad \frac{MR^2}{2} = I_{CM} + M\left(\frac{4R}{3\pi}\right)^2$$

$$I_{CM} = \left[\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2\right]$$

Example 24. Find the moment of inertia of the two uniform joint rods having mass m each about point P as shown in figure. Using parallel axis theorem.



Solution : Moment of inertia of rod 1 about axis P , $I_1 = \frac{m\ell^2}{3}$



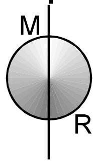
Moment of inertia of rod 2 about axis P, $I_2 = \frac{m\ell^2}{12} + m\left(\sqrt{5}\,\frac{\ell}{2}\right)^2$

So moment of inertia of a system about axis P ,

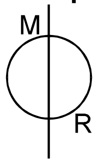
$$I = I_1 + I_2 = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\left(\sqrt{5}\,\frac{\ell}{2}\right)^2$$

$$I = \frac{m\ell^2}{3}$$

List of some useful formule :

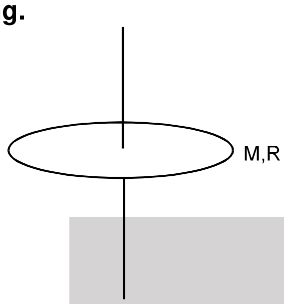
Object	Moment of Inertia
Solid Sphere 	$\frac{2}{5}MR^2$ (Uniform)

Hollow Sphere



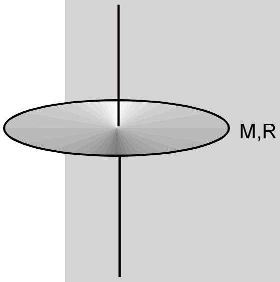
$\frac{2}{3}MR^2$ (Uniform)

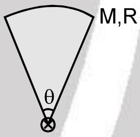
Ring.



MR^2 (Uniform or Non Uniform)

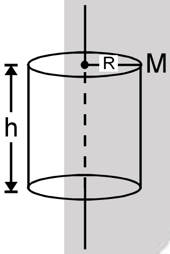
Disc

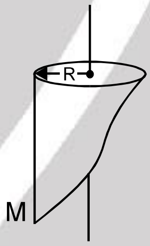




$\frac{MR^2}{2}$ (Uniform)

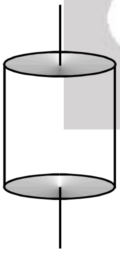
Hollow cylinder





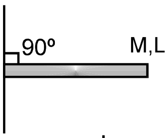
MR^2 (Uniform or Non Uniform)

Solid cylinder

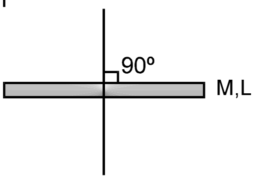


$\frac{MR^2}{2}$ (Uniform)

Thin rod

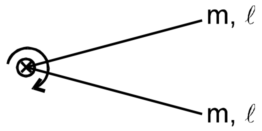


$\frac{ML^2}{3}$ (Uniform)



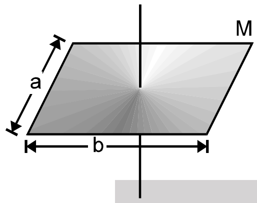
$\frac{ML^2}{12}$ (Uniform)

Two thin rod



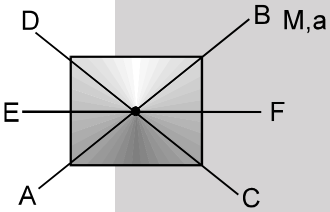
$\frac{2m\ell^2}{3}$ (Uniform)

Rectangular Plate



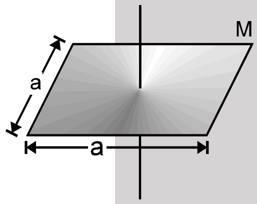
$I = \frac{M(a^2+b^2)}{12}$ (Uniform)

Square Plate



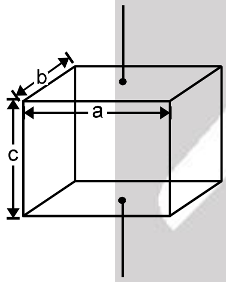
$I_{AB} = I_{CD} = I_{DF} = \frac{Ma^2}{12}$ (Uniform)

Square Plate



$\frac{Ma^2}{6}$ (Uniform)

Cuboid



$\frac{M(a^2+b^2)}{12}$ (Uniform)

4. RADIUS OF GYRATION :

As a measure of the way in which the mass of rigid body is distributed with respect to the axis of rotation, we define a new parameter, the radius of gyration (K). It is related to the moment of inertia and total mass of the body.

$I = MK^2$

where I = Moment of Inertia of a body

M = Mass of a body

K = Radius of gyration

$K = \sqrt{\frac{I}{M}}$

☞ Length K is the geometrical property of the body and axis of rotation.

S.I. Unit of K is meter.

Solved Examples

Example 25. Find the radius of gyration of a solid uniform sphere of radius R about its tangent.

Solution : $I = \frac{2}{5}mR^2 + mR^2 = \frac{7}{5}mR^2 = mK^2 \quad \Rightarrow K = \sqrt{\frac{7}{5}}R$

Example 26. Find the radius of gyration of a hollow uniform sphere of radius R about its tangent.

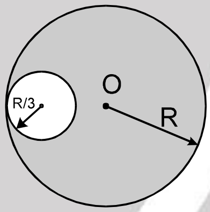
Solution : Moment of inertia of a hollow sphere about a tangent, $I = \frac{5}{3}MR^2$

$MK^2 = \frac{5}{3}MR^2 \quad \Rightarrow \quad K = \sqrt{\frac{5}{3}}R$

5. MOMENT OF INERTIA OF BODIES WITH CUT :

Solved Examples

Example 27 A uniform disc of radius R has a round disc of radius R/3 cut as shown in Fig. .The mass of the remaining (shaded) portion of the disc equals M. Find the moment of inertia of such a disc relative to the axis passing through geometrical centre of original disc and perpendicular to the plane of the disc.



Solution : Let the mass per unit area of the material of disc be σ. Now the empty space can be considered as having density $-\sigma$ and σ.

Now $\quad I_0 = I_\sigma + I_{-\sigma}$

$I_\sigma = (\sigma \pi R^2)R^2/2$ = M.I. of σ about o

$I_{-\sigma} = \frac{-\sigma\pi(R/3)^2(R/3)^2}{2} + [-\sigma\pi(R/3)^2]\,(2R/3)^2$ = M.I. of $-\sigma$ about o

$\therefore \quad I_0 = \frac{4}{9}MR^2$ **Ans.**

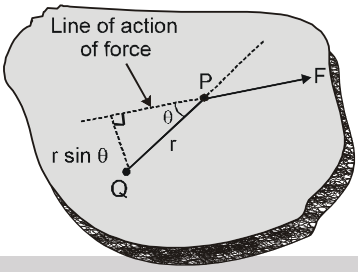
Example 28. Find the moment of inertia of a uniform disc of radius R_1 having an empty symmetric annular region of radius R_2 in between, about an axis passing through geometrical centre and perpendicular to the disc.

Solution : $\rho = \frac{M}{\pi\ (R_1^2 - R_2^2)} \quad \Rightarrow \quad I = \rho \times \left(\frac{\pi R_1^4 - \pi R_2^4}{2}\right)$

$I = \frac{M\left({R_1}^2 + {R_2}^2\right)}{2}$ **Ans.**

6. TORQUE :

Torque represents the capability of a force to produce change in the rotational motion of the body.



6.1 Torque about a point :

Torque of force $\vec{F}$ about a point $\vec{\tau} = \vec{r} \times \vec{F}$

Where $\vec{F}$ = force applied

P = point of application of force

Q = Point about which we want to calculate the torque.

$\vec{r}$ = position vector of the point of application of force w.r.t. the point about which we want to determine the torque.

$|\vec{\tau}|$ = r F sinθ = $r_\perp F = rF_\perp$

Where θ = angle between the direction of force and the position vector of P wrt. Q.

$r_\perp$ = r sin θ = perpendicular distance of line of action of force from point Q ,it is also called force arm.

$F_\perp$ = F sin θ = component of $\vec{F}$ perpendicular to $\vec{r}$

SI unit of torque is Nm

☞ Torque is a vector quantity and its direction is determined using right hand thumb rule and its always perpendicular to the plane of rotation of the body.

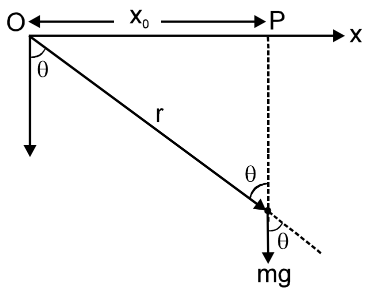
Solved Examples

Example 29. A particle of mass M is released in vertical plane from a point P at x = x_0 on the x-axis it falls vertically along the y-axis. Find the torque τ acting on the particle at a time t about origin ?

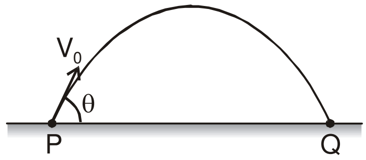
Solution : Torque is produced by the force of gravity.

$\vec{\tau}$ = r F sin θ $\hat{k}$

or $\tau = r_\perp F = x_0 mg = r\, mg\, \frac{x_0}{r} = mgx_0\, \hat{k}$



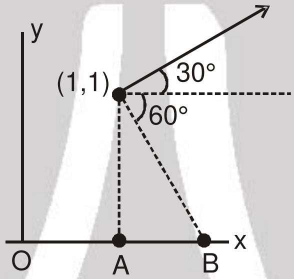
Example 30. A particle having mass m is projected with a velocity v_0 from a point P on a horizontal ground making an angle θ with horizontal. Find out the torque about the point of projection acting on the particle when it is at its maximum height ?



Solution : $\tau = rF\sin\theta = \frac{R}{2}\ mg = \frac{v_0^2 \sin 2\theta}{2g}\ mg$

$\tau = \frac{mv_0^2 \sin 2\theta}{2}$

Example 31. Find the torque about point O and A.



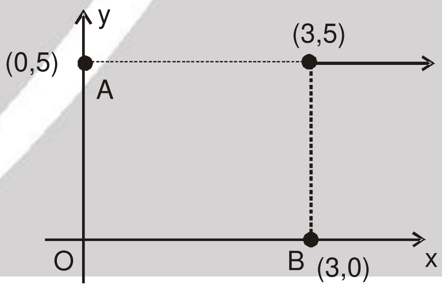
Solution : Torque about point O, $\vec{\tau} = \vec{r}_0 \times \vec{F}$, $\vec{r}_0 = \hat{i} + \hat{j}$, $\vec{F} = 5\sqrt{3}\hat{i} + 5\hat{j}$

$\vec{\tau} = (\hat{i} + \hat{j}) \times (5\sqrt{3}\hat{i} + 5\hat{j}) = 5(1 - \sqrt{3})\hat{k}$

Torque about point A , $\vec{\tau} = \vec{r}_a \times \vec{F}$, $\vec{r}_a = \hat{j}$, $\vec{F} = 5\sqrt{3}\hat{i} + 5\hat{j}$

$\vec{\tau} = \hat{j} \times (5\sqrt{3}\hat{i} + 5\hat{j}) = 5(-\sqrt{3})\hat{k}$

Example 32. Find out torque about point A, O and B.



Solution : Torque about point A , $\vec{\tau}_A = \vec{r}_A \times \vec{F}$, $\vec{r}_A = 3\hat{i}$, $\vec{F} = 10\hat{i}$

$\vec{\tau}_A = 3\hat{i} \times 10\hat{i} = 0$

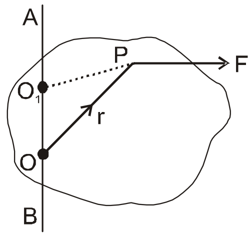
Torque about point B , $\vec{\tau}_B = \vec{r}_B \times \vec{F}$, $\vec{r}_B = 5\hat{j}$, $\vec{F} = 10\hat{i}$

$\vec{\tau}_B = 5\hat{j} \times 10\hat{i} = -50\hat{k}$

Torque about point O , $\vec{\tau}_O = \vec{r}_O \times \vec{F}$, $\vec{r}_O = 3\hat{i} + 5\hat{j}$, $\vec{F} = 10\hat{i}$

$\vec{\tau}_O = (3\hat{i} + 5\hat{j}) \times 10\hat{i} = -50\hat{k}$

6.2 Torque about an axis :



The torque of a force $\vec{F}$ about an axis AB is defined as the component of torque of $\vec{F}$ about any point O on the axis AB, along the axis AB.

In the given figure torque of $\vec{F}$ about O is $\vec{\tau}_0 = \vec{r} \times \vec{F}$

The torque of $\vec{F}$ about AB, τ_{AB} is component of $\vec{\tau}_0$ along line AB.

There are four cases of torque of a force about an axis.:

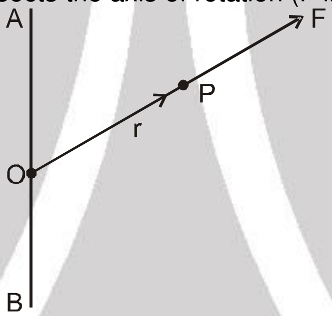
Case I : Force is parallel to the axis of rotation, $\vec{F} \parallel \overrightarrow{AB}$

AB is the axis of rotation about which torque is required

$\vec{r} \times \vec{F}$ is perpendicular to $\vec{F}$, but $\vec{F} \parallel \overrightarrow{AB}$, hence $\vec{r} \times \vec{F}$ is perpendicular to $\overrightarrow{AB}$.

The component of $\vec{r} \times \vec{F}$ along $\overrightarrow{AB}$ is, therefore, zero.

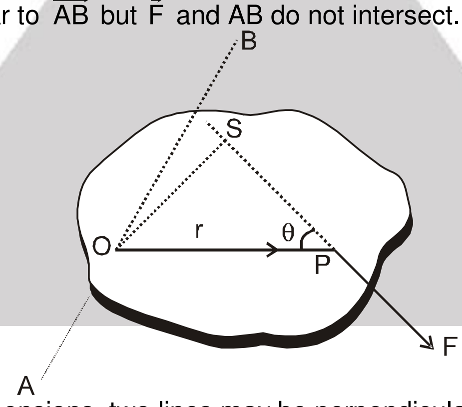
Case II : The line of force intersects the axis of rotation (F intersect AB)



$\vec{F}$ intersects AB along $\vec{r}$ then $\vec{F}$ and $\vec{r}$ are along the same line. The torque about O is $\vec{r} \times \vec{F} = 0$.

Hence component this torque along line AB is also zero.

Case III : $\vec{F}$ perpendicular to $\overrightarrow{AB}$ but $\vec{F}$ and AB do not intersect.



In the three dimensions, two lines may be perpendicular without intersecting each other. Two nonparallel and nonintersecting lines are called skew lines.

Figure shows the plane through the point of application of force P that is perpendicular to the axis of rotation AB. Suppose the plane intersects the axis at the point O. The force F is in this plane (since F is perpendicular to AB). Taking the origin at O,

Torque = $\vec{r} \times \vec{F} = \overrightarrow{OP} \times \vec{F}$.

Thus, torque = rF sin θ = F(OS)

where OS is the perpendicular from O to the line of action of the force $\vec{F}$. The line OS is also perpendicular to the axis of rotation. It is thus the length of the common perpendicular to the force and the axis of rotation.

The direction of $\vec{\tau} = \overrightarrow{OP} \times \vec{F}$ is along the axis AB because $\overrightarrow{AB} \perp \overrightarrow{OP}$ and $\overrightarrow{AB} \perp \vec{F}$. The torque about AB is, therefore, equal to the magnitude of $\vec{\tau}$ that is F.(OS).

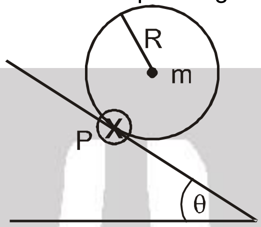
☞ Thus, the torque of F about AB = magnitude of the force F × length of the common perpendicular to the force and the axis. The common perpendicular OS is called the lever arm or moment arm of this torque.

Case IV : $\vec{F}$ and $\overrightarrow{AB}$ are skew but not perpendicular.

Here we resolve $\vec{F}$ into two components, one is parallel to axis and other is perpendicular to axis. Torque of the parallel part is zero and that of the perpendicular part may be found, by using the result of **case (III)**.

Solved Examples

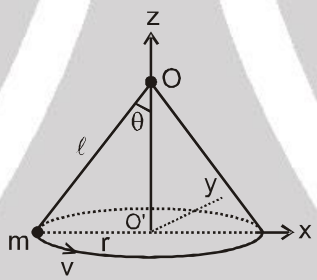
Example 33. Find the torque of weight about the axis passing through point P.



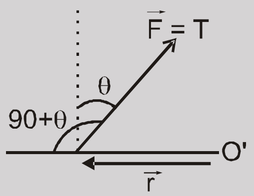
Solution : $\vec{\tau} = \vec{r} \times \vec{F}$, $\vec{r} = R$, $\vec{F} = mg\sin\theta$

r and F both are at perpendicular so torque about point P = mgRsinθ

Example 34. A bob of mass m is suspended at point O by string of length ℓ. Bob is moving in a horizontal circle find out (i) torque of gravity and tension about point O and O'. (ii) Net torque about axis OO' .



Solution :



(i) Torque about point O

Torque of tension (T), $\tau_{ten} = 0$ (tension is passing through point O)

Torque of gravity $\tau_{mg} = \ell mg\sin\theta$

Torque about point O'

Torque of gravity $\tau_{mg} = mgr$ $\quad r = \ell\sin\theta$

Torque of tension $\tau_{mg} = \ell mg\sin\theta$ (along negative $\hat{j}$)

Torque of tension $\tau_{ten} = Tr\sin(90+\theta)$ $\quad (T\cos\theta = mg)$

$\tau_{ten} = Tr\cos\theta$

$\tau_{ten} = \dfrac{mg}{\cos\theta}(\ell\sin\theta)\cos\theta = mg\ell\sin\theta$ (along positive $\hat{j}$)

(ii) Torque about axis OO'

Torque of gravity about axis OO' $\tau_{mg} = 0$ (force mg parallel to axis OO')

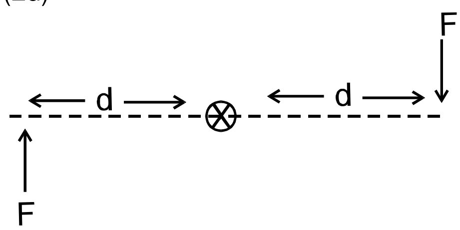
Torque of tension about axis OO' $\tau_{ten} = 0$ (force T is passing through the axis OO')

Net torque about axis OO' $\tau_{net} = 0$

6.3 Force Couple :

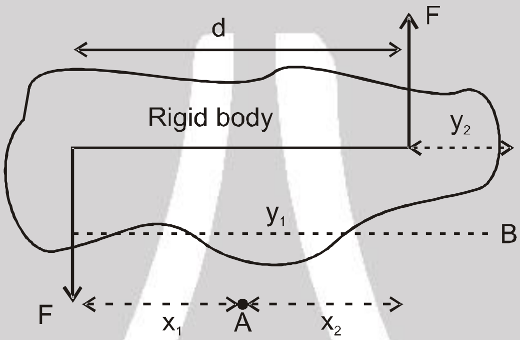
A pair of forces each of same magnitude and acting in opposite direction is called a force couple. Torque due to couple = Magnitude of one force × distance between their lines of action.

Magnitude of torque = τ = F (2d)



☞ A couple does not exert a net force on an object even though it exerts a torque.

☞ Net torque due to a force couple is same about any point.



Torque about A = $x_1F + x_2F = F(x_1 + x_2) = Fd$

Torque about B = $y_1F - y_2F = F(y_1 - y_2) = Fd$

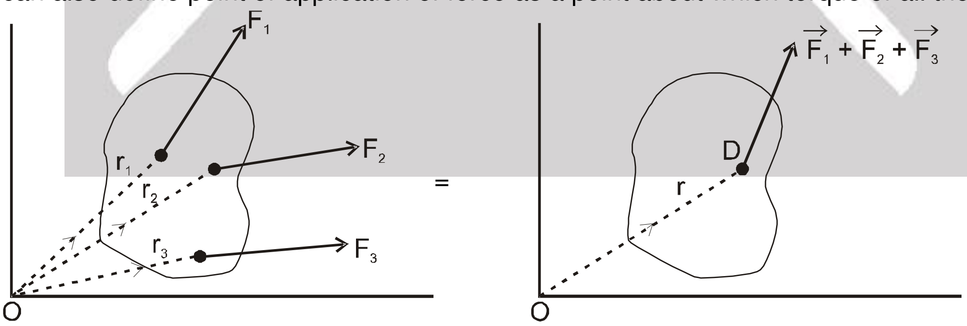
☞ If net force acting on a system is zero, torque is same about any point.

☞ A consequence is that, if $F_{net} = 0$ and $\tau_{net} = 0$ about one point, then $\tau_{net} = 0$ about any point.

6.4 Point of Application of Force :

Point of Application of force is the point at which, if net force is assumed to be acting, then it will produce same translational as well as rotational effect, as was produced earlier.

We can also define point of application of force as a point about which torque of all the forces is zero.

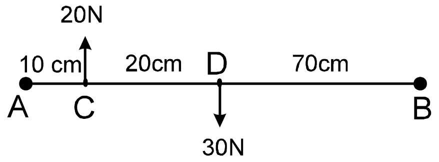


Consider three forces $\vec{F}_1, \vec{F}_2, \vec{F}_3$ acting on a body if D is point of application of force then torque of $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ acting at a point D about O is same as the original torque about O

$$\left[\vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3\right] = \vec{r} \times (\vec{F}_1 + \vec{F}_2 + \vec{F}_3)$$

Solved Examples

Example 35. Determine the point of application of force, when forces of 20 N & 30 N are acting on the rod as shown in figure.

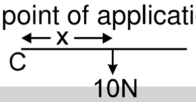


Solution : Net force acting on the rod $F_{rel} = 10N$

Nett torque acting on the rod about point C

$\tau_C = (20 \times 0) + (30 \times 20) = 600$ clockwise

Let the point of application be at a distance x from point C



$600 = 10\,x \Rightarrow x = 60$ cm

$\therefore$ 70 cm from A is point of Application

Note : (i) Point of application of gravitational force is known as the centre of gravity.

(ii) Centre of gravity coincides with the centre of mass if value of is assumed to be constant.

(iii) Concept of point of application of force is imaginary, as in some cases it can lie outside the body.

6.5 Rotation about a fixed axis :

If I_{Hinge} = moment of inertia about the axis of rotation (since this axis passes through the hinge, hence the name I_{Hinge}).

$\vec{\tau}_{ext})$ = resultant external torque acting on the body about axis of rotation

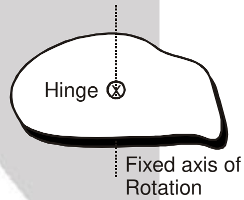
α = angular acceleration of the body.

$\vec{\tau}_{ext})_{Hinge} = I_{Hinge}\,\vec{\alpha}$

Rotational Kinetic Energy $= \frac{1}{2}.I.\omega^2$

$\vec{P} = M\vec{v}_{CM}$

$\vec{F}_{external} = M\,\vec{a}_{CM}$

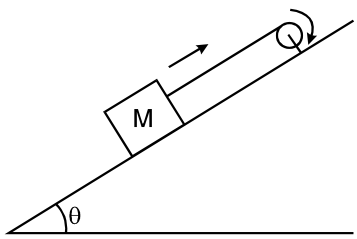


Net external force acting on the body has two component tangential and centripetal.

$\Rightarrow F_C = ma_C = m\frac{v^2}{r_{CM}} = m\omega^2 r_{CM}$ $\Rightarrow F_t = ma_t = m\alpha\, r_{CM}$

Solved Examples

Example 36. A pulley having radius r and moment of inertia I about its axis is fixed at the top of an inclined plane of inclination θ as shown in figure. A string is wrapped round the pulley and its free end supports a block of mass m which can slide on the plane. Initially, the pulley is rotating at a speed ω_0 in a direction such that the block slides up the plane. Calculate the distance moved by the block before stopping ?



Solution : Suppose the deceleration of the block is a. The linear deceleration of the rim of the pulley is also a. The angular deceleration of the pulley is α = a/r. If the tension in the string is T, the equations of motion are as follows :

$$mg\sin\theta - T = ma \quad \text{and} \quad Tr = I\alpha = Ia/r.$$

Eliminating T from these equations,

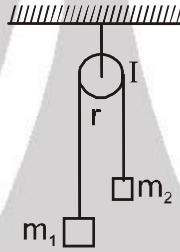
$$mg\sin\theta - I\frac{a}{r^2} = ma$$

giving, $a = \dfrac{mg\ \ r^2\sin\theta}{I+mr^2}$

The initial velocity of the block up the incline is v = ω_0 r. Thus, the distance moved by the block before stopping is

$$x = \frac{v^2}{2a} = \frac{\omega_0^2 r^2(I+mr^2)}{2m\ \ r^2\sin\theta} = \frac{(I+mr^2)\omega_0^2}{2m\ \ g\sin\theta}$$

Example 37. The pulley shown in figure has a moment of inertia I about its axis and its radius is r. Calculate the magnitude of the acceleration of the two blocks. Assume that the string is light and does not slip on the pulley.



Solution : Suppose the tension in the left string is T_1 and that in the right string in T_2. Suppose the block of mass m_1 goes down with an acceleration α and the other block moves up with the same acceleration. This is also the tangential acceleration of the rim of the wheel as the string does not slip over the rim. The angular acceleration of the wheel is, therefore, α = a/r. The equations of motion for the mass m_1, the mass m_2 and the pulley are as follows :

$m_1g - T_1 = m_1a$ ......(i)

$T_2 - m_2g = m_2a$ ......(ii)

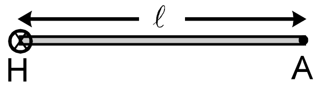
$T_1r - T_2r = I\alpha = I\alpha/r$ ......(iii)

Putting T_1 and T_2 from (i) and (ii) into (iii),

$$[(m_1g - a) - m_2(g + a)]\,r = I\frac{a}{r}$$

which gives $a = \dfrac{(m_1 - m_2)gr^3}{I + (m_1 + m_2)r^2}$.

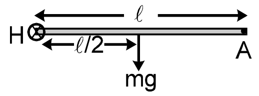
Example 38. A uniform rod of mass m and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H.



(i) Find angular acceleration α of the rod just after it is released from initial horizontal position from rest ?
(ii) Calculate the acceleration (tangential and radial) of point A at this moment.
(iii) Calculate net hinge force acting at this moment.
(iv) Find α and ω when rod becomes vertical.
(v) Find hinge force when rod become vertical.

Solution : (i) $\tau_H = I_H\ \alpha$

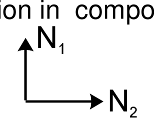
$mg.\frac{\ell}{2} = \frac{m\ell^2}{3}\alpha \quad \Rightarrow \quad \alpha = \frac{3g}{2\ell}$



(ii) $a_{tA} = \alpha\ell = \frac{3g}{2\ell}.\ell = \frac{3g}{2}$

$a_{CA} = \omega^2 r = 0.\ell = 0$ ($\omega = 0$ just after release)

(iii) Suppose hinge exerts normal reaction in component form as shown



In vertical direction

$F_{ext} = ma_{CM}$

$\Rightarrow \; mg - N_1 = m.\frac{3g}{4}$ (we get the value of a_{CM} from previous example)

$\Rightarrow \; N_1 = \frac{mg}{4}$

In horizontal direction

$F_{ext} = ma_{CM} \Rightarrow \quad N_2 = 0$ (a_{CM} in horizontal = 0 as $\omega = 0$ just after release).

(vi) Torque = 0 when rod becomes vertical.

so $\alpha = 0$

using energy conservation $\frac{mg\ell}{2} = \frac{1}{2}I\omega^2 \quad \left(I = \frac{m\ell^2}{3}\right)$

$\omega = \sqrt{\frac{3g}{\ell}}$

(v) When rod becomes vertical

$\alpha = 0, \; \omega = \sqrt{\frac{3g}{\ell}}$

$F_H - mg = \frac{m\omega^2\ell}{2}$

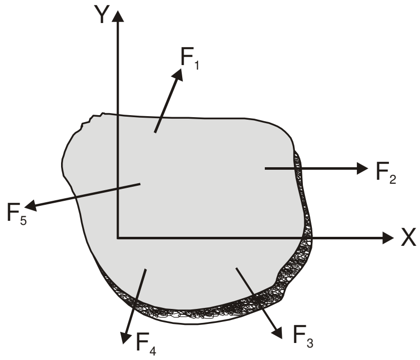
$F_H = \frac{5mg}{2}$ **Ans.**

7. EQUILIBRIUM

A system is in mechanical equilibrium if it is in translational as well as rotational equilibrium.

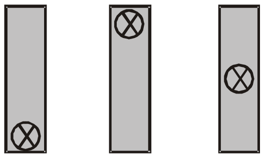
For this : $\overrightarrow{F_{net}} = 0$

$\overrightarrow{\tau_{net}} = 0$ (about every point)



From (6.3), if $\overrightarrow{F_{net}} = 0$ then $\overrightarrow{\tau_{net}}$ is same about every point

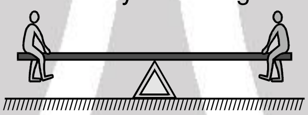
Hence necessary and sufficient condition for equilibrium is $\overrightarrow{F_{net}} = 0$, $\overrightarrow{\tau_{net}} = 0$ about any one point, which we can choose as per our convenience. ($\overrightarrow{\tau_{net}}$ will automatically be zero about every point)

☞ 

The equilibrium of a body is called **stable** if the body tries to regain its equilibrium position after being slightly displaced and released. It is called **unstable** if it gets further displaced after being slightly displaced and released. If it can stay in equilibrium even after being slightly displaced and released, it is said to be in **neutral** equilibrium.

Solved Examples

Example 39. Two boys weighing 20 kg and 25 kg are trying to balance a seesaw of total length 4m, with the fulcrum at the centre. If one of the boys is sitting at an end, where should the other sit ?



Solution : It is clear that the 20 kg kid should sit at the end and the 25 kg kid should sit closer to the centre. Suppose his distance from the centre is x. As the boys are in equilibrium, the normal force between a boy and the seesaw equals the weight of that boy. Considering the rotational equilibrium of the seesaw, the torque of the forces acting on it should add to zero. The forces are

(a) (25 kg) g downward by the 25 kg boy,

(b) (20 kg) g downward by the 20 kg boy,

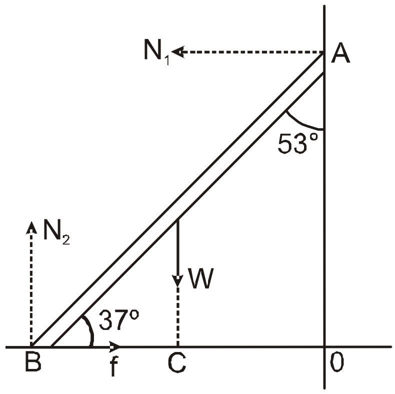
(c) weight of the seesaw and

(d) the normal force by the fulcrum.

Taking torques about the fulcrum, (25 kg)g x = (20 kg)g (2 m) or x = 1.6 m.

Example 40. A uniform rod of mass m = 15 kg leans against a smooth vertical wall making an angle θ = 37° with horizontal. The other ends rests on a rough horizontal floor. Calculate the normal force and the friction force that the floor exerts on the rod. [Take g = 10 m/s^2]

Solution : The forces acting on the rod are shown in figure. They are



(a) Its weight W,

(b) normal force N_1 by the vertical wall,

(c) normal force N_2 by the floor and

(d) frictional force f by the floor.

Taking horizontal and vertical components,

N_1 = f ....(i)

and N_2 = mg ....(ii)

Taking torque about B,

N_1(AO) = mg(CB)

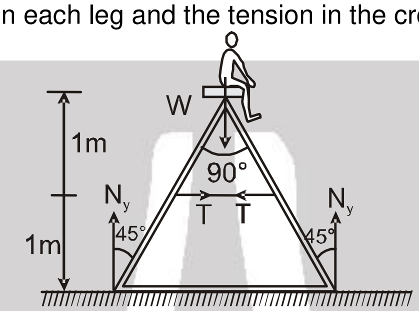
or, $N_1(AB)\cos\theta = mg\,\frac{AB}{2}\sin\theta$ or $N_1\frac{3}{5} = \frac{W}{2}\frac{4}{5}$

or, $N_1 = \frac{2}{3}W$ ....(iii)

The normal force by the floor is $N_2 = W = (15\text{ kg})(10\text{ m/s}^2) = 150$ N.

The frictional force is $f = N_1 = \frac{2}{3}W = 100$ N.

Example 41. The ladder shown in figure has negligible mass and rests on a smooth floor. A crossbar connects the two legs of the ladder at the centre as shown in figure. The angle between the two legs is 90°. The person sitting on the ladder has a mass of 60 kg. Calculate the contact forces exerted by the floor on each leg and the tension in the crossbar. [Take g = 10m/s^2]



Solution : The forces acting on different parts are shown in figure. Consider the vertical equilibrium of "the ladder plus the person" system. The forces acting on this system are its weight (60 kg)g and the contact force $N_y + N_y = 2N_y$ due to the floor. Thus

$2N_y = (60\text{ kg})\,g$ or $N_y = (30\text{ kg})(10\text{ m/s}^2) = 300$ N.

Next consider the equilibrium of the left leg of the ladder. Taking torques of the forces acting on it about the upper end,

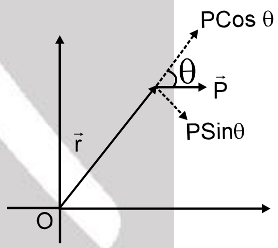
$N_y(2m)\tan 45° = T(1\text{ m})$ or $T = N_y\,2 = (300\text{ N}) \times 2 = 600$ N.

8. ANGULAR MOMENTUM ($\vec{L}$)

8. 1. Angular momentum of a particle about a point.

$\vec{L} = \vec{r} \times \vec{P} \Rightarrow L = rp\sin\theta$

or $|\vec{L}| = r_\perp \times P$ or $|\vec{L}| = P_\perp \times r$



Where $\vec{P}$ = momentum of particle

$\vec{r}$ = position of vector of particle with respect to point O about which angular momentum is to be calculated.

θ = angle between vectors $\vec{r}$ & $\vec{P}$

$r_\perp$ = perpendicular distance of line of motion of particle from point O.

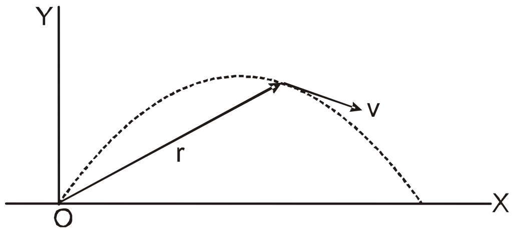
$P_\perp$ = component of momentum perpendicular to $\vec{r}$.

SI unit of angular momentum is kgm^2/sec.

Solved Examples

Example 42. A particle of mass m is projected at time t = 0 from a point O with a speed u at an angle of 45° to the horizontal. Calculate the magnitude and the direction of the angular momentum of the particle about the point O at time t = u/g.

Solution : Let us take the origin at P, X-axis along the horizontal and Y-axis along the vertically upward direction as shown in figure. For horizontal motion during the time 0 to t,



$v_x = u\cos 45° = u/\sqrt{2}$

and $x = v_x t = \frac{u}{\sqrt{2}}\cdot\frac{u}{g} = \frac{u^2}{\sqrt{2}g}$

For vertical motion, $v_y = u \sin 45° = \frac{u}{\sqrt{2}} - u = \frac{(1-\sqrt{2})}{\sqrt{2}} u$ and $y = (u \sin 45°)\, t - \frac{1}{2} g t^2$

$= \frac{u^2}{\sqrt{2}g} - \frac{u^2}{2g} = \frac{u^2}{2g}(\sqrt{2} - 1).$

The angular momentum of the particle at time t about the origin is $L = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v}$

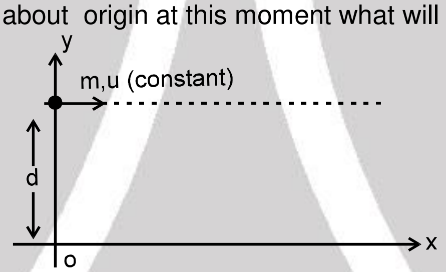
$= m(\hat{i}x + \hat{j}y) \times (\hat{i}v_x + \hat{j}v_y) = m(\hat{k}xv_y - \hat{k}yv_x)$

$= m\hat{k}\left[\left(\frac{u^2}{\sqrt{2}\ g}\right)\frac{u}{\sqrt{2}}(1-\sqrt{2}) - \frac{u^2}{2g}(\sqrt{2}-1)\frac{u^2}{\sqrt{2}}\right] = -\hat{k}\ \frac{mu^3}{2\sqrt{2}g}$.

Thus, the angular momentum of the particle is $\frac{mu^3}{2\sqrt{2}g}$ in the negative Z-direction i.e., perpendicular to the plane of motion, going into the plane.

Solved Examples

Example 43. A particle of mass 'm' starts moving from point (o,d) with a constant velocity $u\hat{i}$. Find out its angular momentum about origin at this moment what will be the answer at the later time?



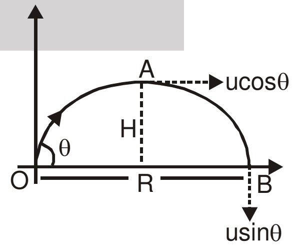
Solution : $\vec{L} = -m\,d\,u\ \hat{k}$.

Example 44. A particle of mass 'm' is projected on horizontal ground with an initial velocity of u making an angle θ with horizontal. Find out the angular momentum of particle about the point of projection when .

(i) it just starts its motion
(ii) it is at highest point of path.
(iii) it just strikes the ground.

Answer : (i) O ; (ii) mu $\cos\theta \frac{u^2 \sin^2\theta}{2g}$; (iii) mu $\sin\theta \frac{u^2 \sin 2\theta}{g}$

Solution : (i) Angular momentum about point O is zero.



(ii) Angular momentum about point A.

$\vec{L} = \vec{r} \times \vec{p}$

$L = H \times mu \cos\theta$

$L = mu \cos\theta\ \frac{u^2 \sin^2\theta}{2g}$ **Ans.**

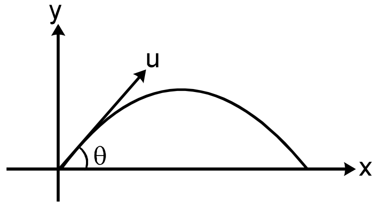
(iii) Angular momentum about point B.

$L = R \times mu \sin\theta$

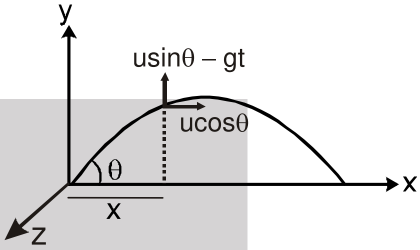
$mu \sin\theta \frac{u^2 \sin 2\theta}{g}$ **Ans.**

Example 45. A particle of mass'm' is projected on horizontal ground with an initial velocity of u making an angle θ with horizontal. Find out the angular momentum at any time t of particle p about :

(i) y axis (ii) z-axis



Solution : (i) velocity components are parallel to the y-axis. so,
L = 0



(ii) $\tau = \frac{dL}{dt}$ $-1/2$ mu cos θ. gt^2

$$-mgx = \frac{dL}{dt} - mgx\ dt = dL$$

$$\int_0^t -mgx\ \ dt = \int_0^L dL$$

angular momentum about the z-axis is :

$L = -1/2$ mu cos θ. gt^2 **Ans.**

8.2 Angular momentum of a rigid body rotating about fixed axis :

Angular momentum of a rigid body about the fixed axis AB is

$L_{AB} = L_1 + L_2 + L_3 + ... + L_n$

$L_1 = m_1 r_1\omega r_1$, $L_2 = m_2 r_2\omega r_2$, $L_3 = m_3 r_3\omega r_3$, $L_n = m_n r_n\omega r_n$

$L_{AB} = m_1 r_1\omega r_1 + m_2 r_2\omega r_2 + m_3 r_3\omega r_3$ .... $+ m_n r_n\omega r_n$

$$L_{AB} = \sum_{n=1}^{n=n} m_n \ (r_n)^2 \times \omega \qquad \left[\sum_{n=1}^{n=n} m_n \ (r_n)^2 = I_H\right]$$

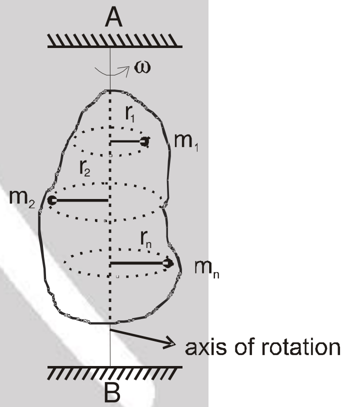
$L_{AB} = I_H\ \omega$

$L_H = I_H\ \omega$

L_H = Angular momentum of object about axis of rotation.

I_H = Moment of Inertia of rigid, body about axis of rotation.

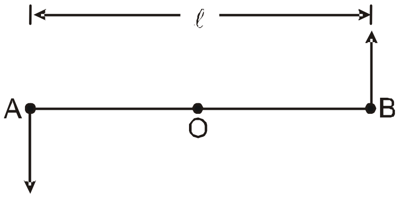
ω = angular velocity of the object.



Solved Examples

Example 46. Two particles balls A and B, each of mass m, are attached rigidly to the ends of a light rod of length ℓ. The system rotates about the perpendicular bisector of the rod at an angular speed ω. Calculate the angular momentum of the individual particles and of the system about the axis of rotation.

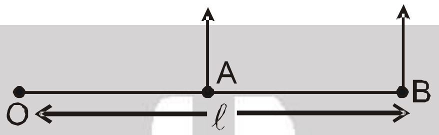
Solution : Consider the situation shown in figure. The velocity of the particle A with respect to the centre O is $v = \frac{\omega\ell}{2}$. The angular momentum of the particle with respect to the axis is $L_1 = mvr = m\left(\frac{\omega\ \ell}{2}\right)\left(\frac{\ell}{2}\right) = \frac{1}{4}m\omega\ell^2$. The same the angular momentum L_2 of the second particle. The angular momentum of the system is equal to sum of these two angular momentum i.e., $L = 1/2\ m\omega\ell^2$.



Example 47. Two small balls of mass m each are attached to a light rod of length ℓ, one at its centre and the other at a free end. The rod is fixed at the other end and is rotated in horizontal plane at an angular speed ω. Calculate the angular momentum of the ball at the end with respect to the ball at the centre.

Solution : The situation is shown in figure. The velocity of the ball A with respect to the fixed end O is $v_A = \omega(\ell/2)$ and that of B with respect to O is $v_B = \omega\ell$. Hence the velocity of B with respect to A is $v_B - v_A = \omega(\ell/2)$. The angular momentum of B with respect to A is, therefore,

$$L = mvr = m\omega\left(\frac{\ell}{2}\right)\frac{\ell}{2} = \frac{1}{4}m\omega\ell^2$$



along the direction perpendicular to the plane of rotation.

Example 48. A uniform circular ring of mass 400 g and radius 10 cm is rotated about one of its diameter at an angular speed of 20 rad/s. Find the kinetic energy of the ring and its angular momentum about the axis of rotation.

Solution : The moment of inertia of the circular ring about its diameter is

$$I = \frac{1}{2}Mr^2 = \frac{1}{2}(0.400\ kg)(0.10\ m)^2 = 2 \times 10^{-3}\ kg\text{-}m^2.$$

The kinetic energy is $K = I\omega^2 = (2 \times 10^{-3}\ kg - m^2)(400\ rad^2/s^2) = 0.4$ J

and the angular momentum about the axis of rotation is

$L = I\omega = (2 \times 10^{-3}\ kg\text{-}m^2)(20\ rad/s) = 0.04\ kg\text{-}m^2/s = 0.04$ J-s.

8.3 Conservation of Angular Momentum

Newton's 2nd law in rotation : $\vec{\tau} = \frac{d\vec{L}}{dt}$

where $\vec{\tau}$ and $\vec{L}$ are about the same axis.

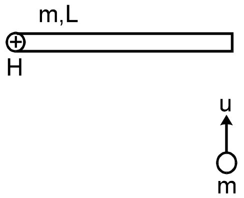
☞ Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about the axis of rotation. Even if net angular momentum is not constant, one of its component of an angular momentum about an axis remains constant if component of torque about that axis is zero

Impulse of Torque : $\int \tau dt = \Delta J$

$\Delta J \rightarrow$ Charge in angular momentum.

Solved Example

Example 49. A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod in-elastically at its free end. Find out the angular velocity of the rod just after collision ?

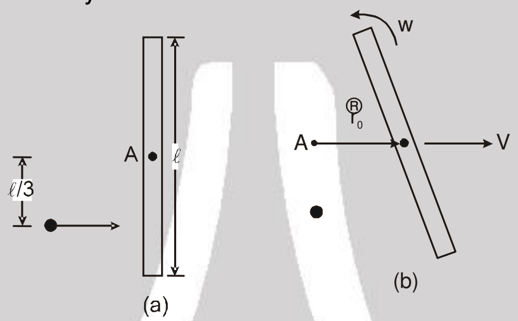


Solution : Angular momentum is conserved about H because no external force is present in horizontal plane which is producing torque about H.

$$mul = \left(\frac{m\ell^2}{3} + m\ell^2\right)\omega \qquad \Rightarrow \qquad \omega = \frac{3a}{4\ell}$$

Example 50. A uniform rod of mass m_1 and length ℓ lies on a frictionless horizontal plane. A particle of mass m_2 moving at a speed v_0 perpendicular to the length of the rod strikes it at a distance $\ell/3$ from the centre and stops after the collision. Calculate (a) the velocity of the centre of the rod and (b) the angular velocity of the rod about its centre just after the collision.

Solution : The situation is shown in figure. Consider the rod and the particle together as the system. As there is no external resultant force, the linear momentum of the system will remains constant. Also there is no resultant external torque on the system and so the angular momentum of the system about the any line will remain constant. Suppose the velocity of the centre of the rod is V and the angular velocity about the centre is ω.



(a) The linear momentum before the collision is mv and that after the collision is MV.

Thus, $m_2v_0 = m_1V$, or $V = \left(\frac{m_2}{m_1}\right)v_0$

(b) Let A be the centre of the rod when it is at rest. Let AB be the line perpendicular to the plane of the figure. Consider the angular momentum of "the rod plus the particle" system about AB. Initially the rod is at rest. The angular momentum of the particle about AB is

$L = m_2v_0\ (\ell/3)$

After the collision, the particle comes to rest. The angular momentum of the rod about A is

$\vec{L} = \vec{L}_{cm} + m_1\vec{r}_0 \times \vec{V}$

As $\vec{r}_0 \parallel \vec{V}$, $\vec{r}_0 \times \vec{V} = 0$

Thus, $\vec{L} = \vec{L}_{cm}$

Hence the angular momentum of the rod about AB is $L = I\omega = \frac{m_1\ell^2}{12}\omega$. Thus, $\frac{m_2v\ell}{3} = \frac{m_1\ell^2}{12}\omega$

or, $\omega = \frac{4m_2v_0}{m_1\ell}$

9. COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY :

The general motion of a rigid body can be thought of as a sum of two independent motions. A translation of some point of the body plus a rotation about this point. A most convenient choice of the point is the centre of mass of the body as it greatly simplifies the calculations.

☞ Consider a fan inside a train, and an observer A on the platform.

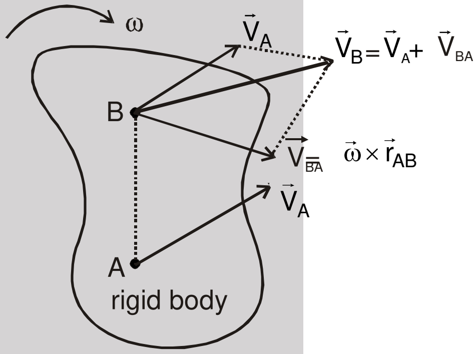
It the fan is switched off while the train moves, the motion of fan is pure translation as each point on the fan undergoes same translation in any time interval.

It fan is switched on while the train is at rest the motion of fan is pure rotation about is axle ; as each point on the axle is at rest, while other points revolve about it with equal angular velocity.

If the fan is switched on while the train is moving, the motion of fan to the observer on the platform is neither pure translation nor pure rotation. This motion is an example of general motion of a rigid body.
Now if there is an observer B inside the train, the motion of fan will appear to him as pure rotation.
Hence we can see that the general motion of fan w.r.t. observer A can be resolved into pure rotation of fan as observed by observer B plus pure translation of observer B (w.r.t. observer A)
Such a resolution of general motion of a rigid body into pure rotation & pure translation is not restricted to just the fan inside the train, but is possible for motion of any rigid system.

9.1 Kinematics of general motion of a rigid body :

For a rigid body as earlier stated value of angular displacement (θ), angular velocity (ω), angular acceleration (α) is same for all points on the rigid body about any other point on the rigid body.
Hence if we know velocity of any one point (say A) on the rigid body and angular velocity of any point on the rigid body about any other point on the rigid body (say ω), velocity of each point on the rigid body can be calculated.



since distance AB is fixed $\vec{V}_{BA} \perp \vec{AB}$

we know that $\omega = \dfrac{V_{BA\perp}}{r_{BA}}$

$V_{BA\perp} = V_{BA} = \omega r_{BA}$

in vector form $\vec{V}_{BA} = \vec{\omega} \times \vec{r}_{BA}$

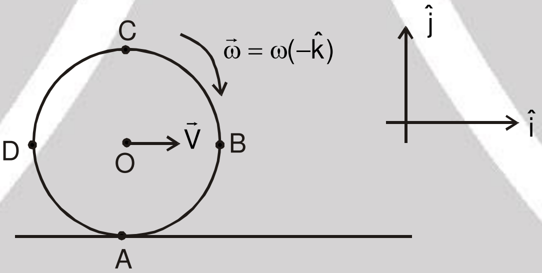
Now from relative velocity : $\vec{V}_{BA} = \vec{V}_B - \vec{V}_A$

$\vec{V}_B = \vec{V}_A + \vec{V}_{BA} \Rightarrow \vec{V}_B = \vec{V}_A + \vec{\omega} \times \vec{r}_{BA}$

similarly $\vec{a}_B = \vec{a}_A + \vec{\alpha} \times \vec{r}_{BA}$ [for any rigid system]

Solved Examples

Example 51. Consider the general motion of a wheel (radius r) which can be view on pure translation of its center O (with the velocity v) and pure rotation about O (with angular velocity ω)



Find out $\vec{v}_{AO}, \vec{v}_{BO}, \vec{v}_{CO}, \vec{v}_{DO}$ and $\vec{v}_A, \vec{v}_B, \vec{v}_C, \vec{v}_D$

Solution : $\vec{v}_{AO} = \left(\vec{\omega} \times \vec{r}_{AO}\right)$

$\vec{v}_{AO} = \left(\omega(-\hat{k}) \times \vec{OA}\right)$

$\vec{v}_{AO} = \left(\omega(-\hat{k}) \times r(-\hat{j})\right)$

$\vec{v}_{AO} = -\omega r\hat{i}$

similarly $\vec{v}_{BO} = \omega r(-\hat{j})$

$\vec{v}_{CO} = \omega r(\hat{i})$; $\vec{v}_{DO} = \omega r(\hat{j})$

$\vec{v}_A = \vec{v}_O + \vec{v}_{AO} = v\hat{i} - \omega r\hat{i}$

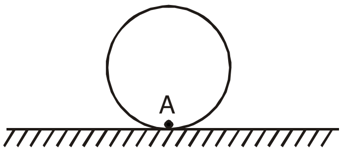
Similarly $\vec{v}_B = \vec{v}_O + \vec{v}_{BO} = v\hat{i} - \omega r\hat{j}$

$\vec{v}_C = \vec{v}_O + \vec{v}_{CO} = v\hat{i} + \omega r\hat{i}$

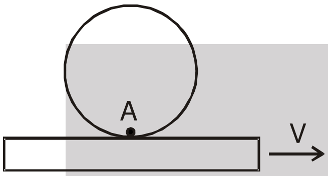
$\vec{v}_D = \vec{v}_O + \vec{v}_{DO} = v\hat{i} + \omega r\hat{j}$

9.2 Pure Rolling (or rolling without sliding) :

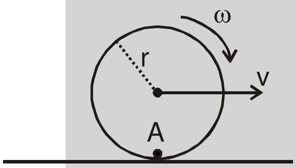
Pure rolling is a special case of general rotation of a rigid body with circular cross section (e.g. wheel, disc, ring, sphere) moving on some surface. Here, there is no relative motion between the rolling body and the surface of contact, at the point of contact

☞ 

Here contact point is A & contact surface is horizontal ground. For pure rolling velocity of A w.r.t. ground = 0 $\Rightarrow V_A = 0$

☞ 

From above figure, for pure rolling, velocity of A w.r.t. to plank is zero $\Rightarrow V_A = V$.

☞ 

From above figure for, pure rolling, velocity of A w.r.t. ground is zero.

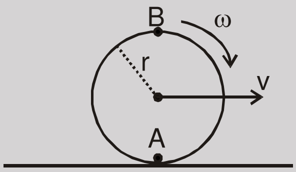
$\Rightarrow$ $v - \omega r = 0$

$v = \omega r$

Similarly $a = \alpha r$

Solved Examples

Example 52. A wheel of radius r rolls (rolling without sleeping) on a level road as shown in figure.



Find out velocity of point A and B

Solution : Contact surface is in rest for pure rolling velocity of point A is zero.

so $v = \omega r$

velocity of point B = $v + \omega r = 2v$

9.3 Dynamics of general motion of a rigid body :

This motion can be viewed as translation of centre of mass and rotation about an axis passing through centre of mass

If I_{CM} = Moment of inertia about this axis passing through COM

τ_{cm} = Net torque about this axis passing through COM

$\vec{a}_{CM}$ = Acceleration of COM

$\vec{v}_{CM}$ = Velocity of COM

$\vec{F}_{ext}$ = Net external force acting on the system.

$\vec{P}_{system}$ = Linear momentum of system.

$\vec{L}_{CM}$ = Angular momentum about centre of mass.

$\vec{r}_{CM}$ = Position vector of COM w.r.t. point A.

then (i) $\vec{\tau}_{cm} = I_{cm} \vec{\alpha}$

(ii) $\vec{F}_{ext} = M\vec{a}_{cm}$

(iii) $\vec{P}_{system} = M\vec{v}_{cm}$

(vi) Total K.E.= $\frac{1}{2}Mv_{cm}^2 + \frac{1}{2}I_{cm}\omega^2$

(v) $\vec{L}_{CM} = I_{CM}\vec{\omega}$

(vi) Angular momentum about point A = $\vec{L}$ about C.M. + $\vec{L}$ of C.M. about A

$\vec{L}_A = I_{cm} \vec{\omega} + \vec{r}_{cm} \times M\vec{v}_{cm}$

☞ $\frac{dL_A}{dt} = \frac{d}{dt}(I_{cm}\vec{\omega} + \vec{r}_{cm} \times M\vec{v}_{cm}) \neq I_A \frac{d\vec{\omega}}{dt}$. Notice that torque equation can be applied to a rigid body in a general motion only and only about an axis through centre of mass.

Solved Examples

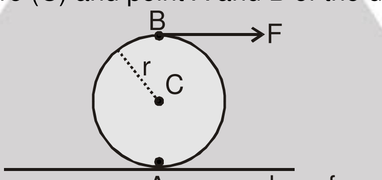
Example 53. A uniform sphere of mass 200 g rolls without slipping on a plane surface so that its centre moves at a speed of 2.00 cm/s. Find its kinetic energy.

Solution : As the sphere rolls without slipping on the plane surface, its angular speed about the centre is $\omega = \frac{v_{cm}}{r}$. The kinetic energy is

$$K = \frac{1}{2} I_{cm}\omega^2 + \frac{1}{2} Mv_{cm}^2 = \frac{1}{2}.\frac{2}{5}Mr^2\omega^2 + \frac{1}{2}Mv_{cm}^2$$

$$= \frac{1}{5} Mv_{cm}^2 + \frac{1}{2}Mv_{cm}^2 = \frac{7}{10}Mv_{cm}^2 = \frac{7}{10}(0.200 \text{ kg})(0.02 \text{ m/s})^2 = 5.6 \times 10^{-5} \text{ J}.$$

Example 54. A constant force F acts tangentially at the highest point of a uniform disc of mass m kept on a rough horizontal surface as shown in figure. If the disc rolls without slipping, calculate the acceleration of the centre (C) and point A and B of the disc.



Solution : The situation is shown in figure. As the force F rotates the disc, the point of contact has a tendency to slip towards left so that the static friction on the disc will act towards right. Let r be the radius of the disc and a be the linear acceleration of the centre of the disc. The angular acceleration about the centre of the disc is $\alpha = a/r$, as there is no slipping.

For the linear motion of the centre,

$F + f = ma$ ......(i)

and for the rotational motion about the centre,

$Fr - f\,r = I\,\alpha = \left(\frac{1}{2}mr^2\right)\left(\frac{a}{r}\right)$ or, $F - f = \frac{1}{2}ma$ ......(ii)

From (i) and (ii),

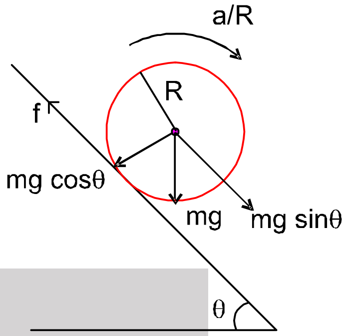
$2F = \frac{3}{2}ma$ or $a = \frac{4F}{3m}$

Acceleration of point A is zero.

Acceleration of point B is $2a = 2\left(\frac{4F}{3m}\right) = \left(\frac{8F}{3m}\right)$ **Ans.**

Example 55. A circular rigid body of mass m, radius R and radius of gyration (k) rolls without slipping on an inclined plane of a inclination θ. Find the linear acceleration of the rigid body and force of friction on it. What must be the minimum value of coefficient of friction so that rigid body may roll without sliding?

Solution : If a is the acceleration of the centre of mass of the rigid body and f the force of friction between sphere and the plane, the equation of translatory and rotatory motion of the rigid body will be.



$mg\sin\theta - f = ma$ (Translatory motion)

$fR = I\alpha$ (Rotatory motion)

$$f = \frac{I\alpha}{R}$$

$I = mk^2$, due to pure rolling $a = \alpha R$

$$mg\sin\theta - \frac{I\alpha}{R} = m\alpha R$$

$$mg\sin\theta = m\alpha R + \frac{I\alpha}{R}$$

$$mg\sin\theta = m\alpha R + \frac{mk^2\alpha}{R}$$

$$mg\sin\theta = ma + \frac{mk^2\alpha}{R}$$

$$mg\sin\theta = a\left[\frac{R^2+k^2}{R^2}\right]$$

$$a = \frac{g\sin\theta}{\left[\frac{R^2+k^2}{R^2}\right]} \quad ; \quad a = \frac{g\sin\theta}{\left(1+\frac{k^2}{R^2}\right)}$$

$$f = \frac{I\alpha}{R}$$

$$f = \frac{mk^2a}{R^2} \Rightarrow \frac{mg\ k^2\sin\theta}{R^2+k^2}$$

$f \le \mu N$

$$\frac{mk^2}{R^2}a \le \mu \le mg\cos\theta$$

$$R^2\frac{k^2}{R^2} \times \frac{g\sin\theta}{(k^2+R^2)} \le \mu g\cos\theta$$

$$\mu \ge \frac{\tan\theta}{\left[1+\frac{R^2}{k^2}\right]} \Rightarrow \mu_{min} = \frac{\tan\theta}{\left[1+\frac{R^2}{k^2}\right]}$$

Note : From above example if rigid bodies are solid cylinder, hollow cylinder, solid sphere and hollow sphere.

(1) Increasing order of acceleration.

$a_{\text{solid sphere}} > a_{\text{hollow sphere}} > a_{\text{solid cylinder}} > a_{\text{hollow cylinder}}$

(2) Increasing order of required friction force for pure rolling.

$f_{\text{hollow cylinder}} > f_{\text{hollow sphere}} > f_{\text{solid cylinder}} > f_{\text{solid sphere}}$

(3) Increasing order of required minimum friction coefficient for pure rolling.

$\mu_{\text{hollow cylinder}} > \mu_{\text{hollow sphere}} > \mu_{\text{solid cylinder}} > \mu_{\text{solid sphere}}$

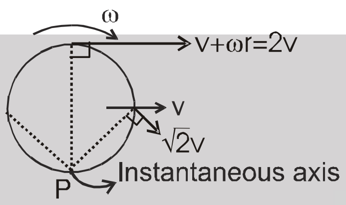
9.4 Instantaneous axis of rotation :

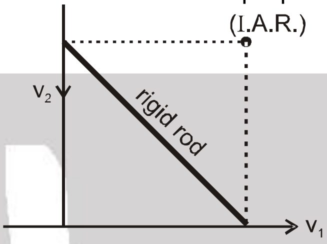
It is the axis about which the combined translational and rotational motion appears as pure rotational motion.

The combined effect of translation of centre of mass and rotation about an axis through the centre of mass is equivalent to a pure rotation with the same angular speed about a stationary axis ; this axis is called instantaneous axis of rotation. It is defined for an instant and its position changes with time.

eg. In pure rolling the point of contact with the surface is the instantaneous axis of rotation.

Geometrical construction of instantaneous axis of rotation (I.A.R). Draw velocity vector at any two points on the rigid body. The I.A.R. is the point of intersection of the perpendicular drawn on them.





☞ In case of pure rolling the lower point is instantaneously axis of rotation.
The motion of body in pure rolling can therefore by analysed as pure rotation about this axis.
Consequently

$\tau_P = I_P\alpha$

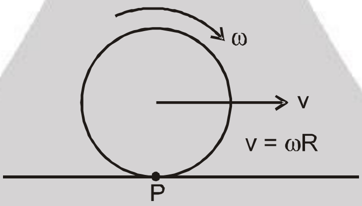
$\alpha_P = I_P\omega$

K.E. = $1/2\ I_P\ \omega^2$

Where I_P is moment of inertial instantaneous axis of rotation passing through P.

Solved Examples

Example 56.



Prove that kinetic energy = $1/2\ I_P\ \omega^2$

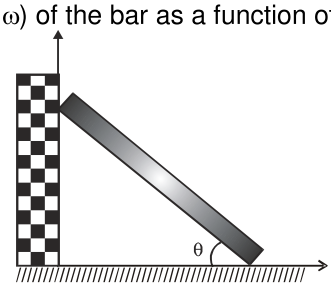
Solution : K.E. = $\frac{1}{2} I_{cm}\ \omega^2 + \frac{1}{2} Mv_{cm}^2 = \frac{1}{2} I_{cm}\ \omega^2 + \frac{1}{2} M\omega^2R^2$

$\frac{1}{2}(I_{cm} + MR^2)\ \omega^2$

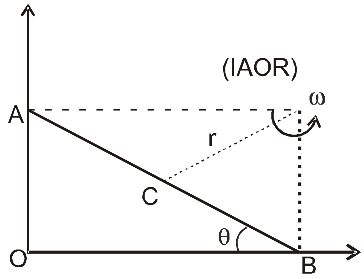
$\frac{1}{2}\left(I_{contact\ point}\right)\ \omega^2$

☞ Notice that pure rolling of uniform object equation of torque can also be applied about the contact point.

Example 57. A uniform bar of length ℓ and mass m stands vertically touching a vertical wall (y-axis). When slightly displaced, its lower end begins to slide along the floor (x-axis). Obtain an expression for the angular velocity (ω) of the bar as a function of θ. Neglect friction everywhere.



Solution : The position of instantaneous axis of rotation (IAOR) is shown in figure.



$$C = \left(\frac{\ell}{2}\cos\theta,\ \frac{\ell}{2}\sin\theta\right)$$

$r = \frac{\ell}{2}$ = half of the diagonal

All surfaces are smooth. Therefore, mechanical energy will remain conserved.

$\therefore$ Decrease in gravitational potential energy of bar = increase in rotational kinetic energy of bar about IAOR.

$$\therefore \quad mg\,\frac{\ell}{2}(1 - \sin\theta) = \frac{1}{2}I\omega^2 \quad ....(1)$$

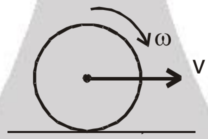
Here, $I = \frac{m\ell^2}{12} + mr^2$ (about IAOR) or $I = \frac{m\ell^2}{12} + \frac{m\ell^2}{4} = \frac{m\ell^2}{3}$

Substituting in Eq. (1), we have

$$mg\,\frac{\ell}{2}(1 - \sin\theta) = \frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 \quad \text{or} \quad \omega = \sqrt{\frac{3g\ (1-\sin\theta)}{\ell}}$$ **Ans.**

The nature of friction in the following cases assume body is perfectly rigid

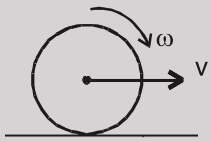
(i) $v = \omega R$



smooth surface

No friction and pure rolling.

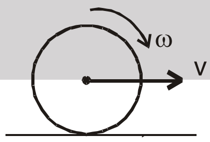
(ii) $v = \omega R$



rough surface

No friction and pure rolling (If the body is not perfectly rigid, then there is a small friction acting in this case which is called rolling friction)

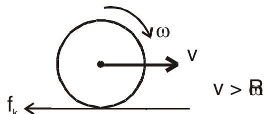
(iii) $v > \omega R$ or $v < \omega R$



smooth surface

No friction force but not pure rolling.

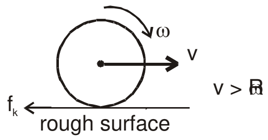
(iv) $v > \omega R$



rough surface

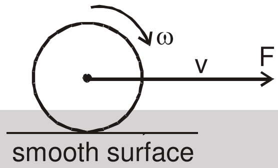
There is Relative Motion at point of contact so Kinetic Friction, $f_k = \mu N$ will act in backward direction. This kinetic friction decrease v and increase ω, so after some time $v = \omega R$ and pure rolling will resume like in case (ii).

(v) $v < \omega R$



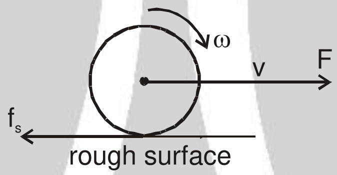
There is Relative Motion at point of contact so Kinetic Friction, $f_k = \mu N$ will act in forward direction. This kinetic friction increase v and decrease ω, so after some time $v = \omega R$ and pure rolling will resume like in case (ii).

(vi) $v = \omega R$ (initial)



No friction and no pure rolling.

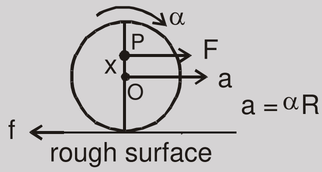
(vii) $v = \omega R$ (initial)



Static friction whose value can be lie between zero and $\mu_s N$ will act in backward direction. If coefficient of friction is sufficiently high, then f_s compensates for increasing v due to F by increasing ω and body may continue in pure rolling with increases v as well as ω.

Solved Examples

Example 58. A rigid body of mass m and radius r rolls without slipping on a rough surface. A force is acting on a rigid body x distance from the centre as shown in figure. Find the value of x so that static friction is zero.



Solution : Torque about centre of mass

$Fx = I_{cm}\alpha$ .....(1)

$F = ma$ .....(2)

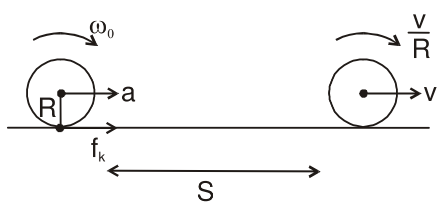
From eqn. (1) & (2)

$max = I_{cm}\,\alpha \qquad (a = \alpha R)\ ; \quad x = \dfrac{I_{cm}}{mR}$

Note : For pure rolling if any friction is required then friction force will be statics friction. It may be zero, backward direction or forward direction depending on value of x. If F below the point P then friction force will act in backward direction or above the point P friction force will act in forward direction.

Example 59. A cylinder is given angular velocity ω_0 and kept on a horizontal rough surface the initial velocity is zero. Find out distance travelled by the cylinder before it performs pure rolling and work done by friction force

Solution : $\mu Mg\, R = \dfrac{MR^2\alpha}{2}$



$\alpha = \dfrac{2\mu g}{R}$ .....(1)

Initial velocity u = 0

$v^2 = u^2 + 2as$

$v_2 = 2as$ .....(2)

$f_K = Ma$

$\mu Mg = Ma$

$a = \mu g$ .....(3)

$\omega = \omega_0 - \alpha t$

from equation (1) $\omega = \omega_0 - \frac{2\mu g}{R}t$

$v = u + at$

from equation (3) $v = \mu g t$

$\omega = \omega_0 - \frac{2v}{R}$

$\omega = \omega_0 - 2\omega$

$\omega = \frac{\omega_0}{3}$

from equation (2)

$$\left(\frac{\omega_0 R}{3}\right)^2 = (2as) = 2\mu gs$$

$$s = \left(\frac{\omega_0^2 R^2}{18 \mu g}\right)$$

work done by the friction force $w = (-f_k R\, d\theta + f_k \Delta s)$

$$-\mu mg R \Delta\theta + \frac{\mu mg \times \omega_0^2 R^2}{18 \mu g}$$

$$\Delta\theta = \omega_0 \times t - \frac{1}{2}\alpha t^2 = \omega_0 \times \left(\frac{\omega_0 R}{3\mu g}\right) - \frac{1}{2} \times \frac{2\mu g}{R}\left(\frac{\omega_0 R}{3\mu g}\right)^2$$

$$\frac{\omega_0^2 R}{3\mu g} - \frac{\omega_0^2 R}{9\mu g}$$

$$\frac{2\omega_0^2 R}{9\mu g}$$

$$-\mu mg \times R\, \frac{2\omega_0^2 R}{9\mu g} + \mu mg \times \frac{\omega_0^2 R^2}{18\mu g}$$

$$-\frac{2m\omega_0^2 R^2}{9} + \frac{m\omega_0^2 R^2}{18}$$

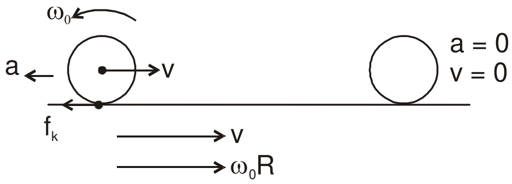
$$\frac{-3m\omega_0^2 R^2}{18} = -\frac{m\omega_0^2 R^2}{6}$$

Alternative Solution

Using work energy theorm $w_g + w_a + w_{f_k} = \Delta K$

$$w_{f_k} = \left[\frac{1}{2}m\left(\frac{\omega_0 R}{3}\right)^2 + \frac{1}{2}\frac{mR^2}{2}\times\left(\frac{\omega_0}{3}\right)^2\right] - \left[\frac{1}{2}\frac{mR^2}{2}\times\omega_0^2\right] = \left(-\frac{m\omega_0^2 R^2}{6}\right)$$

Example 60. A hollow sphere is projected horizontally along a rough surface with speed v and angular velocity ω_0 find out the ratio. So that the sphere stope moving after some time.



Solution : Torque about lowest point of sphere.

$f_k \times R = I\alpha$

$\mu mg \times R = \frac{2}{3}mR^2\alpha$

$\alpha = \frac{3\mu g}{2R}$ angular acceleration in opposition direction of angular velocity.

$\omega = \omega_0 - \alpha t$ (final angular velocity $\omega = 0$)

$\omega_0 = \frac{3\mu g}{2R} \times t$

$t = \frac{\omega_0 \times 2R}{3\mu g}$

acceleration '$a = \mu g$'

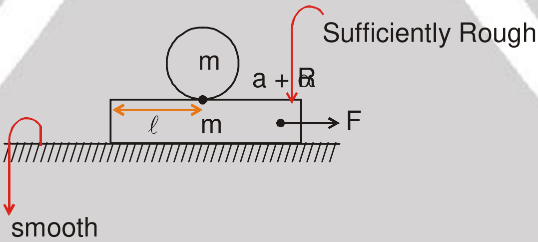
$v_f = v - at$ (final velocity $v_f = 0$)

$v = \mu g \times t$; $t = \frac{v}{\mu g}$

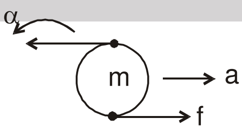
To stop the sphere time at which v & ω are zero, should be same.

$\frac{v}{\mu g} = \frac{2\omega_0 R}{3\mu g} = \frac{v}{\omega_0} = \frac{2R}{3}$

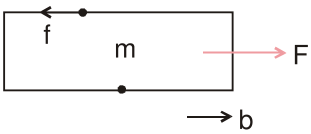
9.5 Rolling on moving surface



Friction on the plate backward or on cylinder friction forward so cylinder move forward.



Because of pure rolling static friction f.



$fR = \frac{mR^2}{2}\alpha$

$\alpha = \frac{2f}{mR}$

$f = ma$

$F - f = mb$

$F = m(a + b)$

$a = \frac{\alpha R}{2}$

At contact point

$b = a + \alpha R$

$b = \frac{3\alpha R}{2}$

$b = 3a$

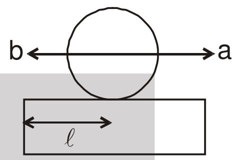
$F = 4ma$

$a = \frac{F}{4m}$

$b = \frac{3F}{4m}$

w.r.t. plate distance is covered = ℓ

and acceleration w.r.t. plate (b – a)



$\ell = \frac{1}{2}(b - a)\,t^2$

$\ell = \frac{1}{2} \times 2at^2 = t = \sqrt{\frac{a \times \ell}{F}} = 2\sqrt{\frac{m\ell}{F}}$

Solved Examples

Example 61. A solid sphere is released from rest from the top of an incline of inclination θ and length. If the sphere rolls without slipping, what will be its speed when it reaches the bottom ?

Solution : Let the mass of the sphere be m and its radius r. Suppose the linear speed of the sphere when it reaches the bottom is v. As the sphere rolls without slipping, its angular speed about its axis is $\omega = v/r$. The kinetic energy at the bottom will be

$$K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\omega^2 + \frac{1}{2}mv^2 = \frac{1}{5}mv^2 + \frac{1}{2}mv^2 = \frac{7}{10}\,mv^2$$

This should be equal to the loss of potential energy mg $\sin\theta$. Thus,

$$\frac{7}{10}mv^2 = mg\ell\sin\theta \quad \text{or} \quad v = \sqrt{\frac{10}{7}g\ell\sin\theta}$$

Example 62. There are two cylinders of radii R_1 and R_2 having moments of inertia I_1 and I_2 about their respective axes as shown in figure. Initially, the cylinders rotate about their axes with angular speed ω_1 and ω_2 as shown in the figure. The cylinders are moved closed to touch each other keeping the axes parallel. The cylinders first slip over each other at the contact but the slipping finally ceases due to the friction between them. Calculate the angular speeds of the cylinders after the slipping ceases.

Solution : When slipping ceases, the linear speeds of the points of contact of the two cylinders will be equal. If ω'_1 and ω'_2 be the respective angular speeds, we have

$\omega'_1 R_1$ and $\omega'_2 R_2$ ....(i)

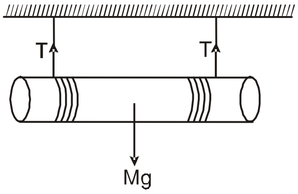
The change in the angular speed is brought about by the frictional force which acts as long as the slipping exists. If this force f acts for a time t, the torque on the first cylinder is fR_1 and that on the second is fR_2. Assuming $\omega_1 > \omega_2$ the corresponding angular impulses are – fR_1t and fR_2t, We, there fore, have

$-f R_1 t = I_1 (\omega'_1 - \omega_1)$ and $fR_2 t = I_2 (\omega'_2 - \omega_2)$

or, $-\frac{I_1}{R_1}(\omega'_1 - \omega_1) = \frac{I_2}{R_2}(\omega'_2 - \omega_2)$ ....(ii)

Solving (i) and (ii) $\omega'_1 = \frac{I_1\ \omega_1\ R_2 + I_2\ \omega_2\ R_1}{I_2\ R_1^2\ + I_1\ R_2^2} R_2$ and $\omega'_2 = \frac{I_1\ \omega_1\ R_2 + I_2\ \omega_2\ R_1}{I_2\ R_1^2\ + I_1\ R_2^2} R_1$

Example 63. A hollow cylinder of mass m is suspended through two light strings rapped around it as shown in figure. Calculate (a) the tension T in the string and (b) the speed of the cylinder as it falls through a distance ℓ.



Solution : The portion of the strings between the ceiling and the cylinder is at rest. Hence the points of the cylinder where the strings leave it are at rest. The cylinder is thus rolling without slipping on the strings. Suppose the centre of the cylinder falls with an acceleration a. The angular acceleration of the cylinder about its axis is α = a/R, as the cylinder does not slip over the strings.

The equation of motion for the centre of mass of the cylinder is

$$Mg - 2T = Ma \qquad ....(i)$$

and for the motion about the centre of mass, it is

$$2\,Tr = (Mr^2\alpha) = Mra \text{ or } \qquad 2T = Ma.$$

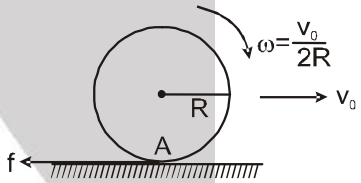
From (i) and (ii),

$$a = \frac{g}{2} \text{ and } T = \frac{Mg}{4}$$

As the centre of the cylinder starts moving from rest, the velocity after it has fallen through a distance ℓ is given by

$$v^2 = 2\left(\frac{g}{2}\right)\ell \qquad \text{or} \qquad v = \sqrt{g\ell}$$

Example 64. A hollow sphere of mass M and radius R as shown in figure slips on a rough horizontal plane. At some instant it has linear velocity v_0 and angular velocity about the centre $\frac{v_0}{2R}$ as shown in figure. Calculate the linear velocity after the sphere starts pure rolling.



Solution : Velocity of the centre = v_0 and the angular velocity about the centre = $\frac{v_0}{2R}$. Thus $v_0 > \omega_0 R$. The sphere slips forward and thus the friction by the plane on the sphere will act backward. As the friction is kinetic, its value is µN = µMg and the sphere will be decelerated by a_{cm} = f/M. Hence,

$$v(t) = v_0 - \frac{f}{M}t \qquad ....(i)$$

This friction will also have a torque Γ' = fr about the centre. This torque is clockwise and in the direction of ω_0. Hence the angular acceleration about the centre will be

$$\alpha = f\,\frac{R}{(2/3)MR^2} = \frac{3f}{2MR}$$

and the clockwise angular velocity at time t will be $\omega(t) = \omega_0 + \frac{3f}{2MR}t = \frac{v_0}{2R} + \frac{3f}{2MR}t.$

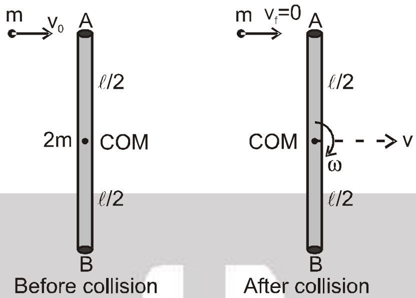
Pure rolling starts when $v(t) = R\omega(t)$ i.e., $v(t) = \frac{v_0}{2} + \frac{3f}{2M}t.$ ....(ii)

Eliminating t from (i) and (ii), $\frac{3}{2}v(t) + v(t) = \frac{3}{2}v_0 + \frac{v_0}{2}$ or $v(t) = \frac{2}{5} \times 2v_0 = \frac{4}{5}v_0.$

Thus, the sphere rolls with linear velocity $4v_0/5$ in the forward direction.

Example 65. A rod AB of mass 2m and length ℓ is lying on a horizontal frictinless surface. A particle of mass m traveling along the surface hits the end 'A' of the rod with a velocity v_0 in a direction perpendicular to AB. The collisin is elastic. After the collision the particle comes to rest. Find out after collision
(a) Velocity of centre of mass of rod (b) Angular velocity.

Solution : (a) Let just after collision lthe sped of COM of rod is v and angular velocity about COM is ω.



External force on the system (rod + mass) in horizontal plane is zero
Apply conservation of linear momentum in x direction

$$mv_0 = 2mv \quad ....(1)$$

Net torque on the system about any point is zero
Apply conservation of angular momentum about COM of rod.

$$mv_0 \frac{\ell}{2} = I\omega \quad \Rightarrow \quad mv_0 \frac{\ell}{2} = \frac{2m\ell^2}{12}\omega$$

$$mv_0 = m\omega\frac{\ell}{3} \quad ....(2)$$

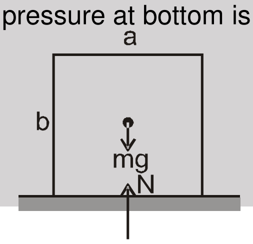
From eq (1) velocity of centre of mass $v = \frac{v_0}{2}$

From eq (2) anuglar velocity $\omega = \frac{3v_0}{\ell}$

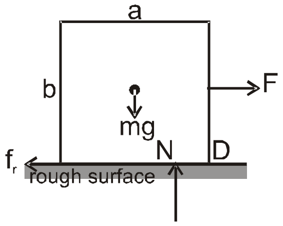
10. TOPPLING :

In many situations an external force is applied to a body to cause it to slide along a surface. In certain cases, the body may tip over before sliding ensues. This is known as topping.

(1) There is a no horizontal force so pressure at bottom is uniform and normal is colinear with mg.



(2) If a force is applied at COM, pressure is not uniform Normal shifts right so that torque of N can counter balance torque of friction.



$F_{max} = f_r$
$N = mg$
$f_r \cdot b/2 = N \cdot a/2 \Rightarrow f_r = Na/b = mg\, a/b$, $F_{max} = mg\, a/b$

(3) If surface is not sufficiently rough and the body slides before F is increased to F_{max} = mg a/b then body will slide before toppling. Once body starts sliding friciton becomes constant and hence no topping. This is the case if

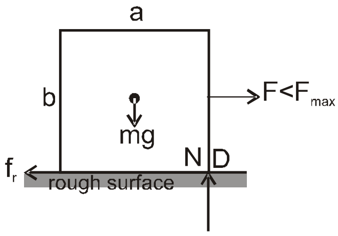
$F_{max} > f_{limit}$

$\Rightarrow$ mg a/b > μmg

μ < a/b

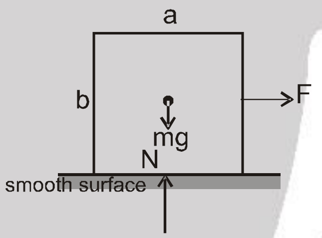
Condition for toppling when $\mu \geq$ a/b in this case body will topple if F > mg a/b

but if μ < a/b, body will not topple any value of F applied a COM



Solved Examples

Example 66.



Find out minimum value of F for toppling

Solution : Never topple

Example 67. A uniform cube of side 'a' and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point directly below the centre of the face, at a height $\frac{a}{4}$ above the base.

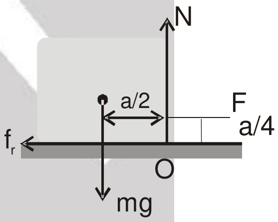
(i) What is the minimum value of F for which the cube begins to tip about an edge?

(ii) What is the minimum value of μ_s so that toppling occures.

(iii) If $\mu = \mu_{min}$, find minimum force for topping.

(iv) Minimum μ_s so that F_{min} can cause toppling.

Solution : (i) In the limiting case normal reaction will pass through O. The cube will tip about O if torque of Fabout O exceeds the torque of mg.



Hence, F $\left(\frac{a}{4}\right) >$ mg $\left(\frac{a}{2}\right)$ or F > 2 mg

therefore, minimum value of F is 2 mg

(ii) In this case since it is not acting at COM, toppling can occur even after body started slinding because increasing the the torque of F about COM.hence μ_{min} = 0,

(iii) Now body is sliding before toppling, O is not I.A.R., torque equation can not be applied across it. It can now be applied about COM.

$F \times \frac{a}{4} = N \times \frac{a}{2}$ .....(1)

N = mg .....(2)

from (1) and (2)

F = 2 mg

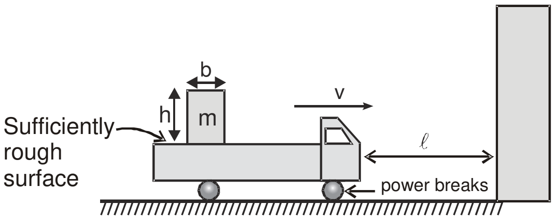
(iv) F > 2 mg .... (1) (from sol. (i))

N = mg ....(2)

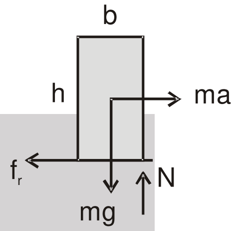
F = μ_sN = $\mu_s$mg .... (3) from (1) and (2)

μ_s = 2

Example 68. Find minimum value of ℓ so that truck can avoid the dead end, without toppling the block kept on it.



Solution : $ma\frac{h}{2} \le mg\frac{b}{2} \Rightarrow a \le \frac{b}{h}g$



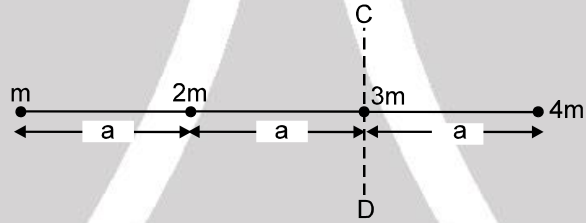
Final velocity of truck is zero. So that

$0 = v^2 - 2(\frac{b}{h}g)\,\ell$

$\ell = \frac{h}{2b}\frac{v^2}{g}$

Self Practice Problems

Problem 1. Four point masses are connected by a massless rod as shown in figure. Find out the moment of inertia of the system about axis CD ?



Solution : $I_1 = m(2a)^2$

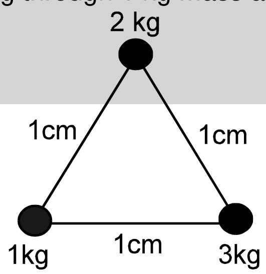
$I_2 = 2ma^2$

$I_3 = 0$

$I_4 = 4ma^2$

$I_{CD} = I_1 + I_2 + I_3 + I_4 = 10\ ma^2$ **Ans.**

Problem 2. Three point masses are located at the corners of an equilibrium triangle of side 1 cm. Masses are of 1, 2 & 3kg respectively and kept as shown in figure. Calculate the moment of Inertia of system about an axis passing through 1 kg mass and perpendicular to the plane of triangle?



Solution : Moment of inertia of 2 kg mass about an axis passing through 1 kg mass

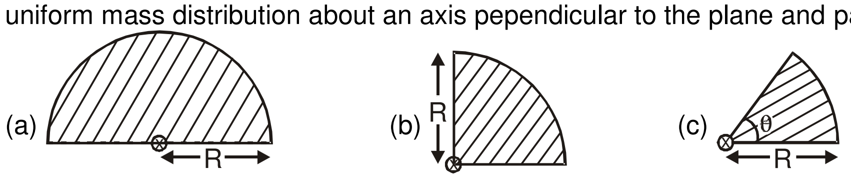
$I_1 = 2 \times (1\times10^{-2})^2 = 2\times10^{-4}$

Moment of inertia of 3 kg mass about an axis passing through 1 kg mass

$I_2 = 3 \times (1\times10^{-2})^2 = 3\times10^{-4}$

$I = I_1 + I_2 = 5 \times 10^{-4}$ kgm^2

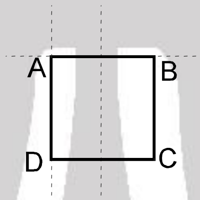
Problem 3. Calculate the moment of Inertia of figure shown each having mass M, radius R and having uniform mass distribution about an axis pependicular to the plane and passing through centre?



Solution : $dI = dm\,\dfrac{R^2}{2}$

$I = \int dI = \dfrac{R^2}{2}\int dm = \dfrac{MR^2}{2}$

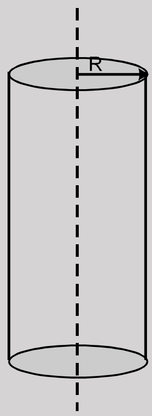
Problem 4. Find the moment of inertia of the uniform square plate of side 'a' and mass M about the axis AB.



Solution : $dI = dm\,\dfrac{a^2}{3}$

$I = \int dI = \dfrac{a^2}{3}\int dm = \dfrac{Ma^2}{3}$

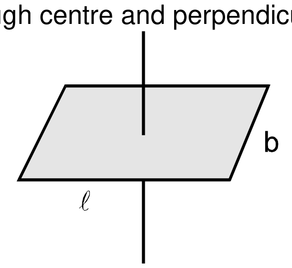
Problem 5. Calculate the moment of inertia of a uniform solid cylinder of mass M, radius R and length ℓ about its axis.



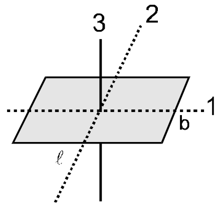
Solution : Each segment of cylinder is solid disc so $\int dI = \int dm\dfrac{R^2}{2}$

$I = \dfrac{MR^2}{2}$ **Ans.**

Problem 6. Find the moment of inertia of a uniform rectangular plate of mass M, edges of length 'ℓ' and 'b' about its axis passing through centre and perpendicular to it.



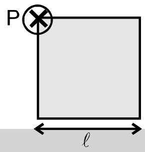
Solution : Using perpendicular axis theorem $I_3 = I_1 + I_2$



$I_1 = \frac{Mb^2}{12}$

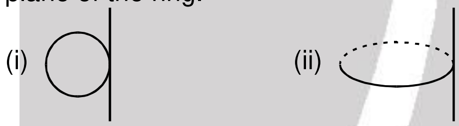
$I_2 = \frac{M\ell^2}{12}$; $I_3 = \frac{M(\ell^2 + b^2)}{12}$

Problem 7. Find the moment of inertia of a uniform square plate of mass M, edge of length 'ℓ' about its axis passing through P and perpendicular to it.



Solution : $I_P = \frac{M\ell^2}{6} + \frac{M\ell^2}{2} = \frac{2M\ell^2}{3}$

Problem 8. Find out the moment of inertia of a ring having uniform mass distribution of mass M & radius R about an axis which is tangent to the ring and (i) in the plane of the ring (ii) perpendicular to the plane of the ring.



Solution : **(i)** Moment of inertia about an axis passing through centre of ring and plane of the ring $I_1 = \frac{MR^2}{2}$

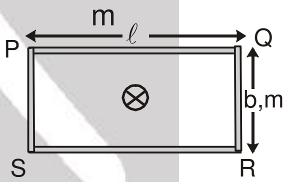
Using parallel axis theorem $I' = I_1 + MR^2 = \frac{3MR^2}{2}$

(ii) Moment of inertia about an axis passing through centre of ring and perpendicular to palne of the ring

$I_C = MR^2$

Using parallel axis theorem $I'' = I_C + MR^2 = 2MR^2$

Problem 9. Calculate the moment of inertia of a rectangular frame formed by uniform rods having mass m each as shown in figure about an axis passing through its centre and perpendicular to the plane of frame ? Also find moment of inertia about an axis passing through PQ ?



Solution : (i) Moment of inertia about an axis passing through its centre and perpendicular to the plane of frame

$I_C = I_1 + I_2 + I_3 + I_4$

$I_1 = I_3$, $I_2 = I_4$

$I_C = 2I_1 + 2I_2$

$I_1 = \frac{m\ell^2}{12} + m\left(\frac{b}{2}\right)^2 \Rightarrow I_2 = \frac{mb^2}{12} + m\left(\frac{\ell}{2}\right)^2$ so, $I_C = \frac{2m}{3}(\ell^2 + b^2)$

(ii) M.I. about axis PQ of rod PQ $I_1 = 0$

M.I. about axis PQ of rod PS $I_2 = \frac{mb^2}{2}$

M.I. about axis PQ of rod QR $I_3 = \frac{mb^2}{2}$

M.I. about axis PQ of rod SR $I_4 = mb^2$

$I = I_1 + I_2 + I_3 + I_4 = \frac{5mb^2}{3}$

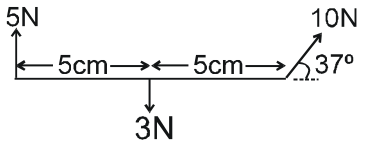
Problem 10. In the previous question, during the motion of particle from P to Q. Torque of gravitational force about P is :

(A) increasing (B) decreasing

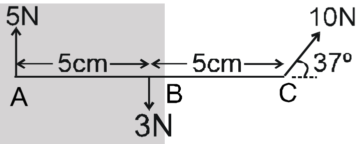
(C) remains constant (D) first increasing then decreasing

Solution : Increasing because distance from point P is increasing.

Problem 11. Determine the point of application of force, when forces are acting on the rod as shown in figure.



Solution : Torque of B about A τ_1 = 3N × 5 = 15N cm (clockwise)



Torque of C about A τ_2 = 6N × 10 = 60 N cm (anticlockwise)

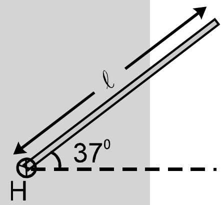
Resultant force perpendicular to the rod F = 8 N

$\tau_1 + \tau_2 = F x$ (x = distance from point A)

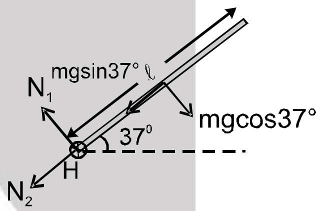
$-15 + 60 = 8x$

$x = 45/8 = 5.625$ cm

Problem 12. A uniform rod of mass m and length ℓ can rotate in vertical plane about a smooth horizontal axis hinged at point H. Find angular acceleration α of the rod just after it is released from initial position making an angle of 37° with horizontal from rest? Find force exerted by the hinge just after the rod is released from rest.



Solution : Torque about hing = $\tau_H = I\alpha$



$$mg\cos 37 \frac{\ell}{2} = \frac{m\ell^2}{3}\alpha$$

$\alpha = 6g / 5\ell$

$$a_t = \alpha\frac{\ell}{2} = \frac{3g}{5}$$

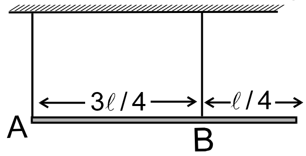
$mg\cos 37 - N_1 = ma_t$

$$N_1 = \frac{mg}{5}$$

angular velocity of rod is zero. so $N_2 = mg\sin 37° = 3mg/5$

$$N = \sqrt{N_1^2 + N_2^2} = \sqrt{\left(\frac{mg}{5}\right)^2 + \left(\frac{3mg}{5}\right)^2} = \frac{mg\sqrt{10}}{5}$$

Problem 13. A uniform rod of length, mass m is hung from two strings of equal length from a ceiling as shown in figure. Determine the tensions in the strings ?



Solution : $T_A + T_B = mg$ ....(i)

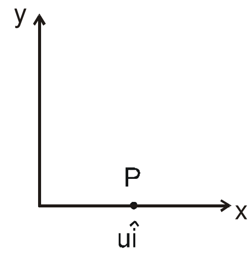
Torque about point A is zero

$$\text{So, } T_B \times \frac{3\ell}{4} = mg\frac{\ell}{2} \quad \text{............(ii)}$$

From eq. (i) & (ii),

$T_A = mg/3$, $T_B = 2mg/3$

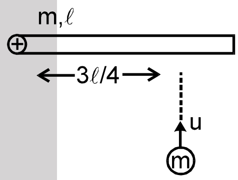
Problem 14. A particle of mass m starts moving from origin with a constant velocity $u\hat{i}$ find out its angular momentum about origin at this moment. What will be the answer later on? What will be the answer if the speed increases.



Solution : $\vec{L} = \vec{r} \times \vec{p}$

$\vec{L} = r\hat{i} \times mu\hat{i} = 0$

Problem 15. A uniform rod of mass m and length ℓ can rotate freely on a smooth horizontal plane about a vertical axis hinged at point H. A point mass having same mass m coming with an initial speed u perpendicular to the rod, strikes the rod and sticks to it at a distance of $3\ell/4$ from hinge point. Find out the angular velocity of the rod just after collision ?

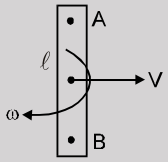


Solution : Angular Momentum about hinge

$L_i = L_f$

$$mu\left(\frac{3\ell}{4}\right) = \left(\frac{m\ell^2}{3} + m\left(\frac{3\ell}{4}\right)^2\right)\omega \qquad \omega = \frac{36u}{43\ell}$$

Problem 16. Uniform & smooth Rod of length ℓ is moving with a velocity of centre v and angular velocity ω on smooth horizontal surface. Findout velocity of point A and B.



Solution : velocity of point A w.r.t. center is $\omega\frac{\ell}{2}$

velocity of point A w.r.t. ground $V_A = V + \omega\frac{\ell}{2}$

velocity of point B w.r.t. center is $-\omega\frac{\ell}{2}$

velocity of point B w.r.t. ground $V_A = V - \omega\frac{\ell}{2}$

Exercise-1

✍ **Marked Questions can be used as Revision Questions.**

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Kinematics

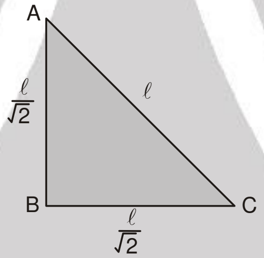
A-1. A uniform disk rotating with constant angular acceleration covers 50 revolutions in the first five seconds after the start. Calculate the angular acceleration and the angular velocity at the end of five seconds.

A-2. A body rotating with 20 rad/s is acted upon by a uniform torque providing it an angular deceleration of 2 rad/s^2. At which time will the body have kinetic energy same as the initial value if the torque acts continuously ?

Section (B) : Moment of inertia

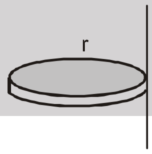
B-1. Calculate the moment of inertia of a uniform square plate of mass M and side L about one of its diagonals, with the help of its moment of inertia about its centre of mass.

B-2.✍ A uniform triangular plate of mass M whose vertices are ABC has lengths ℓ, $\frac{\ell}{\sqrt{2}}$ and $\frac{\ell}{\sqrt{2}}$ as shown in figure. Find the moment of inertia of this plate about an axis passing through point B and perpendicular to the plane of the plate.



B-3. Find the moment of inertia of a uniform half-disc about an axis perpendicular to the plane and passing through its centre of mass. Mass of this disc is M and radius is R.

B-4. Calculate the radius of gyration of a uniform circular disk of radius r and thickness t about a line perpendicular to the plane of this disk and tangent to the disk as shown in figure.



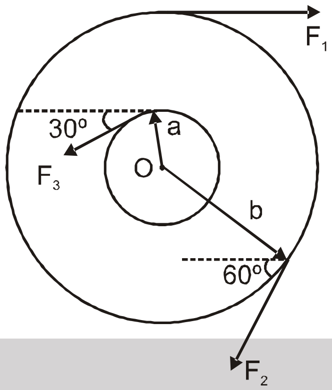
Section (C) : Torque

C-1. Two forces $\vec{F}_1 = 2\hat{i} - 5\hat{j} - 6\hat{k}$ and $\vec{F}_2 = -\hat{i} + 2\hat{j} - \hat{k}$ are acting on a body at the points (1, 1, 0) and (0, 1, 2) respectively. Find torque acting on the body about point (–1, 0, 1).

C-2. A simple pendulum having bob of mass m and length ℓ is pulled aside to make an angle θ with the vertical. Find the magnitude of the torque of the weight of the bob about the point of suspension. At which position its torque is zero? At which θ it is maximum?

C-3. A particle having mass m is projected with a speed v at an angle α with horizontal ground. Find the torque of the weight of the particle about the point of projection when the particle (a) is at the highest point. (b) reaches the ground.

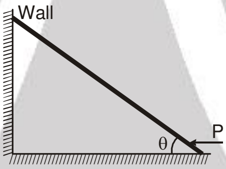
C-4. Calculate the net torque on the system about the point O as shown in figure if F_1 = 11 N, F_2 = 9 N, F_3 = 10 N, a = 10 cm and b = 20 cm. (All the forces along the tangent.)



Section (D) : Rotational Equilibrium

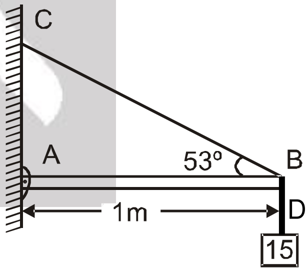
D-1. A uniform metre stick having mass 400 g is suspended from the fixed supports through two vertical light strings of equal lengths fixed at the ends. A small object of mass 100 g is put on the stick at a distance of 60 cm from the left end. Calculate the tensions in the two strings. (g = 10 m/s²)

D-2. Assuming frictionless contacts, determine the magnitude of external horizontal force P applied at the lower end for equilibrium of the rod as shown in figure. The rod is uniform and its mass is ' m'.



D-3. A uniform ladder having length 10.0 m and mass 24 kg is resting against a vertical wall making an angle of 53° with it. The vertical wall is smooth but the ground surface is rough. A painter weighing 75 kg climbs up the ladder. If he stays on the ladder at a point 2 m from the upper end, what will be the normal force and the force of friction on the ladder by the ground? What should be the minimum coefficient of friction between ground and ladder for the painter to work safely? (g = 10 m/s²)

D-4. In the system as shown in figure, AB is a uniform rod of mass 10 kg and BC is a light string which is connected between wall and rod, in vertical plane. There is block of mass 15 kg connected at B with a light string. [Take g = 10 m/s²] (BC and BD are two different strings)
If whole of the system is in equilibrium then find
(i) Tension in the string BC
(ii) Hinge force exerted on beam at point A

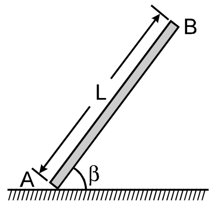


Section (E) : Rotation about fixed axis ($\tau_H = I_H\alpha$)

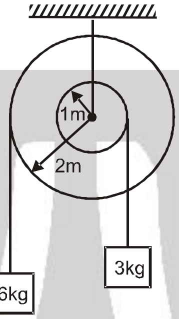
E-1. A rod of negligible mass having length ℓ = 2 m is pivoted at its centre and two masses of m_1 = 6 kg and m_2 = 3 kg are hung from the ends as shown in figure.

(a) Find the initial angular acceleration of the rod if it is horizontal initially.
(b) If the rod is uniform and has a mass of m_3 = 3 kg.
(i) Find the initial angular acceleration of the rod.
(ii) Find the tension in the supports to the blocks of mass 3 kg and 6 kg (g = 10 m/s²).

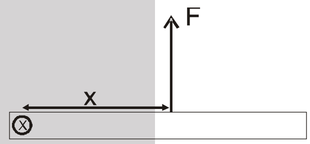
E-2. The uniform rod AB of mass m is released from rest when $\beta = 60^{\circ}$. Assuming that the friction force between end A and the surface is large enough to prevent sliding, determine (for the instant just after release)
(a) The angular acceleration of the rod
(b) The normal reaction and the friction force at A.



E-3. The moment of inertia of the pulley system as shown in the figure is 3 kg – m^2. The radii of bigger and smaller pulleys are 2m and 1m respectively. As the system is released from rest, find the angular acceleration of the pulley system. (Assume that there is no slipping between string & pulley and string is light) [Take g = 10 m/s^2]

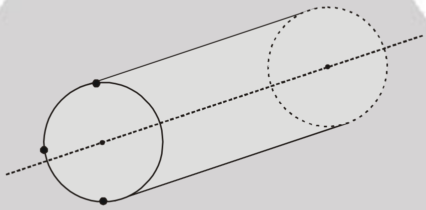


E-4. A uniform thin rod of length L is hinged about one of its ends and is free to rotate about the hinge without friction. Neglect the effect of gravity. A force F is applied at a distance x from the hinge on the rod such that force is always perpendicular to the rod. Find the normal reaction at the hinge as function of 'x', at the initial instant when the angular velocity of rod is zero.

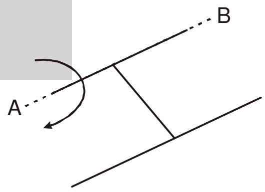


Section (F) : Rotation about Fixed Axis (Energy conservation)

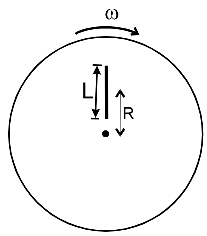
F-1. A solid cylinder of mass M = 1kg & radius R = 0.5m is pivoted at its centre & has three particles of mass m = 0.1kg mounted at its perimeter in the vertical plane as shown in the figure. The system is initially at rest. Find the angular speed of the cylinder, when it has swung through 90_0 in anticlockwise direction. [Take g = 10 m/s^2]



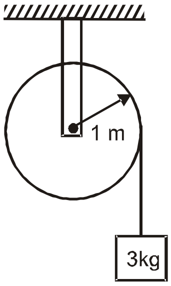
F-2. A rigid body is made of three identical uniform thin rods each of length L fastened together in the form of letter H. The body is free to rotate about a fixed horizontal axis AB that passes through one of the legs of the H. The body is allowed to fall from rest from a position in which the plane of H is horizontal. What is the angular speed of the body, when the plane of H is vertical.



F-3. A uniform rod of mass m and length L lies radially on a disc rotating with angular speed ω in a horizontal plane about its axis. The rod does not slip on the disc and the centre of the rod is at a distance R from the centre of the disc. Find out the kinetic energy of the rod.



F-4. The moment of inertia of the pulley system as shown in figure is 3 kgm^2. Its radius is 1m. The system is released from rest find the linear velocity of the block, when it has descended through 40 cm. (Assume that there is no slipping between string & pulley and string is light) [Take g = 10 m/s^2]

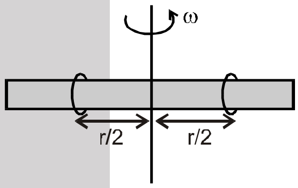


Section (G) : Angular Momentum & its conservation

G-1. A particle having mass 2 kg is moving with velocity $(2\hat{i}+3\hat{j})$m/s. Find angular momentum of the particle about origin when it is at (1, 1, 0).

G-2. A particle having mass 2 kg is moving along straight line 3x + 4y = 5 with speed 8m/s. Find angular momentum of the particle about origin. x and y are in meters.

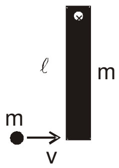
G-3. Two beads (each of mass m) can move freely in a frictionless wire whose rotational inertia with respect to the vertical axis is I. The system is rotated with an angular velocity ω_0 when the beads are at a distance r/2 from the axis. What is the angular velocity of the system when the beads are at a distance r from the axis ? **[JEE - 1990]**



G-4. A system consists of two identical small balls of mass 2 kg each connected to the two ends of a 1 m long light rod. The system is rotating about a fixed axis through the centre of the rod and perpendicular to it at an angular speed of 9 rad/s. An impulsive force of average magnitude 10 N acts on one of the masses in the direction of its velocity for 0.20 s. Calculate the new angular velocity of the system.

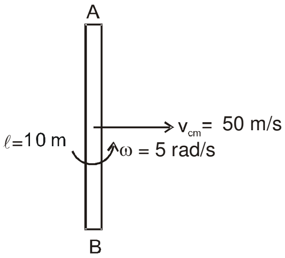
G-5. A uniform round board of mass M and radius R is placed on a fixed smooth horizontal plane and is free to rotate about a fixed axis which passes through its centre. A man of mass m is standing on the point marked A on the circumference of the board. At first the board & the man are at rest. The man starts moving along the rim of the board at constant speed v_o relative to the board. Find the angle of board's rotation when the man passes his starting point on the disc first time.

G-6. A point object of mass m moving horizontally hits the lower end of the uniform thin rod of length ℓ and mass m and sticks to it. The rod is resting on a horizontal, frictionless surface and pivoted at the other end as shown in figure. Find out angular velocity of the system just after collision.

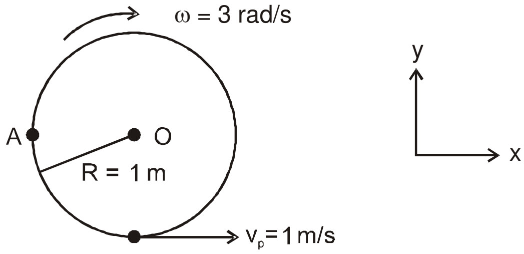


Section (H) : Combined Translational & Rotation Motion (Kinematics)

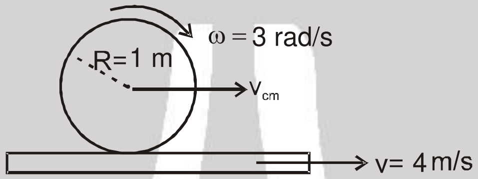
H-1 The centre of mass of a uniform rod of length 10 meter is moving with a translational velocity of 50 m/sec. on a frictionless horizontal surface as shown in the figure and the rod rotates about its centre of mass with an angular velocity of 5 radian/sec. Find out V_A and V_B



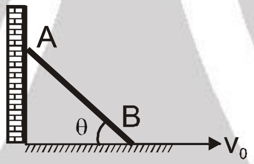
H-2 A ring of radius 1 m. performs combined translational and rotational motion on a frictionless horizontal surface with an angular velocity of 3 rad/sec as shown in the figure. Find out velocity of its centre and point A if the velocity of the lowest point V_P is 1 m/sec.



H-3 A plank is moving with a velocity of 4 m/sec. A disc of radius 1 m rolls without slipping on it with an angular velocity of 3 rad/sec as shown in figure. Find out the velocity of centre of the disc.

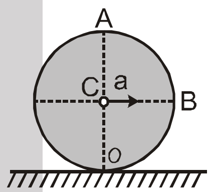


H-4 The end B of uniform rod AB which makes angle θ with the floor is being pulled with a velocity v_0 as shown. Taking the length of the rod as ℓ, calculate the following at the instant when $\theta = 37º$



(a) The velocity of end A (b) The angular velocity of rod (c) Velocity of CM of the rod.

H-5. A ball of radius R = 10.0 cm rolls without slipping on a horizontal plane so that its centre moves with constant acceleration a = 2.50 cm/s^2; t = 2.00 s after the beginning of motion its position corresponds to that shown in Fig. Find :
(a) the velocities of the points A, B and O
(b) the accelerations of these points.

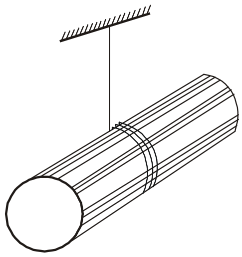


Section (I) : Combined translational & Rotational Motion (Dynamics)

I-1. A small solid cylinder is released from a point at a height h on a rough as track shown in figure. Assuming that it does not slip anywhere, calculates its linear speed when it rolls on the horizontal part of the track.

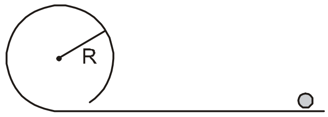
I-2. A uniform ball of mass 'm' rolls without sliding on a fixed horizontal surface. The velocity of the lowest point of the ball with respect to the centre of the ball is V. Find out the total kinetic energy of the ball.

I-3. A string is wrapped over the curved surface of a uniform solid cylinder and the free end is fixed with rigid support. The solid cylinder moves down, unwinding the string. Find the downward acceleration of the solid cylinder.

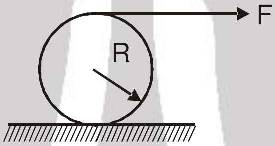


I-4. A uniform disk of mass m is released from rest from the rim of a fixed hemispherical bowl so that it rolls along the surface. If the rim of the hemisphere is kept horizontal, find the normal force exerted by the bowl on the disk when it reaches the bottom of the bowl.

I-5. There is a rough track, a portion of which is in the form of a cylinder of radius R as shown in the figure. Find the minimum linear speed of a uniform ring of radius r with which it should be set rolling without sliding on the horizontal part so that it can complete round the circle without sliding on the cylindrical part.



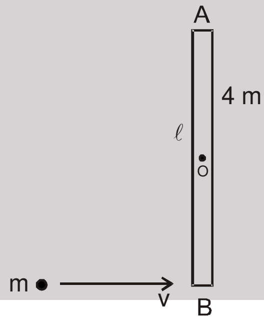
I-6. A uniform solid sphere of radius R is placed on a smooth horizontal surface. It is pulled by a by constant force acting along the tangent from the highest point. Calculate the distance travelled by the centre of mass of the solid sphere during the time it makes one full revolution.



I-7. A uniform hollow sphere of mass m = 1 kg is placed on a rough horizontal surface for which the coefficient of static friction between the surfaces in contact is $\mu = 2/5$. Find the maximum constant force which can be applied at the highest point in the horizontal direction so that the sphere can roll without slipping. (Take $g = 10\ m/s^2$)

Section (J) : Conservation of angular momentum (Combined translation & rotational motion)

J-1 A uniform rod of length ℓ and mass 4m lies on a frictionless horizontal surface on which it is free to move anyway. A ball of mass m moving with speed v as shown in figure.collides with the rod at one of the ends. If ball comes to rest immediately after collision then find out angular velocity ω of rod just after collistion.



J-2. A uniform rod having mass m_1 and length L lies on a smooth horizontal surface. A particle of mass m_2 moving with speed u on the horizontal surface strikes the free rod perpendicularly at an end and it sticks to the rod.

(a) Calculate the velocity of the com C of the system constituting "the rod plus the particle".

(b) Calculate the velocity of the particle with respect to C before the collision.

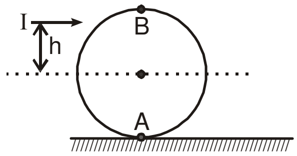
(c) Calculate the velocity of the rod with respect to C before the collision

(d) Calculate the angular momentum of the particle and of the rod about the com C before the collision.

(e) Calculate the moment of inertia of the rod plus particle about the vertical axis through the centre of mass C after the collision.

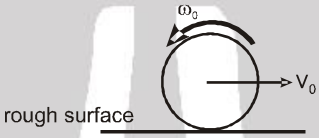
(f) Calculate the velocity of the com C and the angular velocity of the system about the centre of mass after the collision.

J-3. A uniform solid sphere is placed on a smooth horizontal surface. An impulse I is given horizontally to the sphere at a height h = 4R/5 above the centre line. m and R are mass and radius of sphere respectively.



(a) Find angular velocity of sphere & linear velocity of centre of mass of the sphere after impulse.
(b) Find the minimum time after which the highest point B will touch the ground,
(c) Find the displacement of the centre of mass during this interval.

J-4. A uniform disc of radius R = 0.2 m kept over a rough horizontal surface is given velocity v_0 and angular velocity ω_0. After some time its kinetic energy becomes zero. If v_0 = 10 m/s, find ω_0.



Section (K) : Toppling

K-1. A solid cubical block of mass m and side a slides down a rough inclined plane of inclination θ with a constant speed. Calculate the torque of the normal force acting on the block about its centre and the perpendicular distance 'x' from centre of mass at which it is acting.

PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Kinematics

A-1. A fan is running at 3000 rpm. It is switched off. It comes to rest by uniformly decreasing its angular speed in 10 seconds. The total number of revolutions in this period.
(A) 150 (B) 250 (C) 350 (D) 300

A-2. A block hangs from a string wrapped on a disc of radius 20 cm free to rotate about its axis which is fixed in a horizontal position. If the angular speed of the disc is 10 rad/s at some instant, with what speed is the block going down at that instant ?
(A) 4 m/s (B) 3 m/s (C) 2 m/s (D) 5 m/s

Section (B) : Moment of inertia

B-1. A uniform circular disc A of radius r is made from a copper plate of thickness t and another uniform circular disc B of radius 2r is made from a copper plate of thickness t/2. The relation between the moments of inertia I_A and I_B is
(A) $I_A > I_B$ (B) $I_A = I_B$
(C) $I_A < I_B$ (D) depends on the values of t and r.

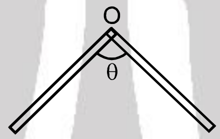
B-2. The moment of inertia of a non-uniform semicircular wire having mass m and radius r about a line perpendicular to the plane of the wire through the centre is
(A) mr^2 (B) $\frac{1}{2}mr^2$ (C) $\frac{1}{4}mr^2$ (D) $\frac{2}{5}mr^2$

B-3. Let I_A and I_B be the moments of inertia of two solid cylinders of identical geometrical shape and size about their axes, the first made of aluminium and the second of iron.
(A) $I_A < I_B$ (B) $I_A = I_B$ (C) $I_A > I_B$
(D) relation between I_A and I_B depends on the actual shapes of the bodies.

B-4. Let I_1 and I_2 be moments of inertia of a body about two axes 1 and 2 respectively, The axis 1 passes through the centre of mass of the body but axis 2 does not.
(A) $I_1 < I_2$ (B) If $I_1 < I_2$, the axes are parallel.
(C) If the axes are parallel, $I_1 < I_2$ (D) If the axes are not parallel, $I_1 \geq I_2$.

B-5. The moment of inertia of an elliptical disc of uniform mass distribution of mass 'm', semi major axis 'r', semi minor axis 'd' about its axis is :
(A) $= \frac{mr^2}{2}$ (B) $= \frac{md^2}{2}$ (C) $> \frac{mr^2}{2}$ (D) $< \frac{mr^2}{2}$

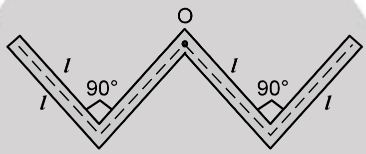
B-6. A unifrom thin rod of length L and mass M is bent at the middle point O as shown in figure. Consider an axis passing through its middle point O and perpendicular to the plane of the bent rod. Then moment of inertia about this axis is :



(A) $\frac{2}{3}$ mL^2 (B) $\frac{1}{3}$ mL^2 (C) $\frac{1}{12}$ mL^2 (D) dependent on θ

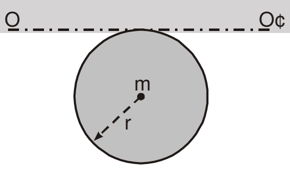
B-7. The moment of inertia of a uniform circular disc about its diameter is 200 gm cm^2. Then its moment of inertia about an axis passing through its center and perpendicular to its circular face is
(A) 100 gm cm^2 (B) 200 gm cm^2 (C) 400 gm cm^2 (D) 1000 gm cm^2

B-8. A thin uniform rod of length 4 *l*, mass 4m is bent at the points as shown in the fig. What is the moment of inertia of the rod about the axis passing point O & perpendicular to the plane of the paper.



(A) $\frac{m\ell^2}{3}$ (B) $\frac{10\,m\ell^2}{3}$ (C) $\frac{m\ell^2}{12}$ (D) $\frac{m\ell^2}{24}$

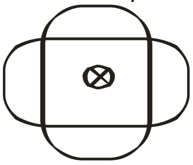
B-9. Moment of inertia of a uniform disc about the axis O O′ is:



(A) $\frac{3mr^2}{2}$ (B) $\frac{mr^2}{2}$ (C) $\frac{5mr^2}{2}$ (D) $\frac{5mr^2}{4}$

B-10. The moment of inertia of a hollow cubical box of mass M and side a about an axis passing through the centres of two opposite faces is equal to
(A) $\frac{5Ma^2}{3}$ (B) $\frac{5Ma^2}{6}$ (C) $\frac{5Ma^2}{12}$ (D) $\frac{5Ma^2}{18}$

B-11. A uniform thin rod of length (4a + 2πa) and of mass (4m + 2πm) is bent and fabricated to form a square surrounded by semicircles as shown in the figure. The moment of inertia of this frame about an axis passing through its centre and perpendicular to its plane is **[Olympiad 2014 (stage-1)]**



(A) $\frac{(4+2\pi)}{3}ma^2$ (B) $\frac{(4+\pi)}{2}ma^2$ (C) $\frac{(4+3\pi)}{3}ma^2$ (D) $\frac{ma^2\{10+3\pi\}}{3}$

Section (C) : Torque

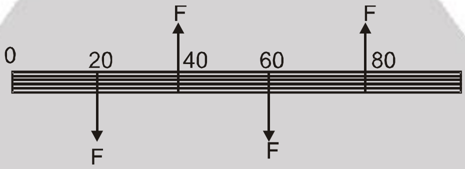
C-1. If a rigid body is subjected to two forces $\vec{F}_1 = 2\hat{i}+3\hat{j}+4\hat{k}$ acting at (3, 3, 4) and $\vec{F}_2 = -2\hat{i}-3\hat{j}-4\hat{k}$ acting at (1, 0, 0) then which of the following is (are) true? **[REE - 1994]**
(A) The body is in equilibrium.
(B) The body is under the influence of a torque only.
(C) The body is under the influence of a single force.
(D) The body is under the influence of a force together with a torque .

C-2. A force $\vec{F} = 4\hat{i} - 10\hat{j}$ acts on a body at a point having position vector $-5\hat{i} - 3\hat{j}$ relative to origin of co-ordinates on the axis of rotation. The torque acting on the body about the origin is :
(A) $38\hat{k}$ (B) $-25\hat{k}$ (C) $62\hat{k}$ (D) none of these

C-3. In case of torque of a couple if the axis is changed by displacing it parallel to itself, torque will :
(A) increase (B) decrease (C) remain constant (D) None of these

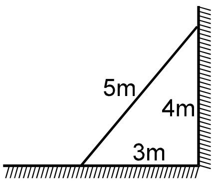
Section (D) : RotationAL Equilibrium

D-1. Four equal and parallel forces are acting on a rod (as shown in figure) in horizontal plane at distances of 20 cm, 40 cm, 60 cm and 80 cm respectively from one end of the rod. Under the influence of these forces the rod :



(A) is at rest (B) experiences a torque
(C) experiences a linear motion (D) experiences a torque and also a linear motion

D-2. A uniform ladder of length 5m is placed against the wall in vertical plane as shown in the figure. If coefficient of friction μ is the same for both the wall and the floor then minimum value of μ for it not to slip is



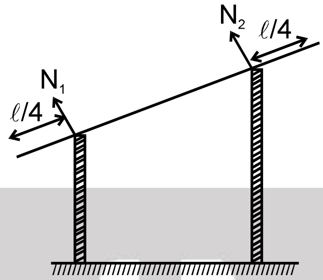
(A) μ = 1/2 (B) μ = 1/4 (C) μ = 1/3 (D) μ = 1/5

D-3 A rod of weight w is supported by two parallel knife edges A & B and is in equilibrium in a horizontal position. The knives are at a distance d from each other. The centre of mass of the rod is at a distance *x* from A. The normal reactions at A and B will be :
(A) N_A = 2w (1 – x/d), N_B = wx/d] (B) N_A = w (1 – x/d), N_B = wx/d]
(C) N_A = 2w (1 – x/d), N_B = 2wx/d] (D) N_A = w (2 – x/d), N_B = wx/d]

D-4 The beam and pans of a balance have negligible mass. An object weighs W_1 when placed in one pan and W_2 when placed in the other pan. The weight W of the object is :

(A) $\sqrt{W_1W_2}$ (B) $\sqrt{(W_1+W_2)}$ (C) $W_1^2 + W_2^2$ (D) $(W_1^{-1} + W_2^{-1})/2$

D-5. A uniform rod of length ℓ is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then



(A) $N_1 > N_2$ (B) $N_1 > N_2$
(C) $N_1 = N_2$ (D) N_1 and N_2 would be in the vertical directions.

Section (E) : Rotation about Fixed axis ($\tau_H = I_H\alpha$)

E-1. A uniform circular disc A of radius r is made from a metal plate of thickness t and another uniform circular disc B of radius 4r is made from the same metal plate of thickness t/4. If equal torques act on the discs A and B, initially both being at rest. At a later instant, the angular speeds of a point on the rim of A and another point on the rim of B are ω_A and ω_B respectively. We have

(A) $\omega_A > \omega_B$ (B) $\omega_A = \omega_B$ (C) $\omega_A < \omega_B$
(D) the relation depends on the actual magnitude of the torques.

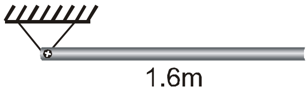
E-2. A body is rotating with constant angular velocity about a vertical axis fixed in an inertial frame. The net force on a particle of the body not on the axis is

(A) horizontal and skew with the axis (B) vertical
(C) horizontal and intersecting the axis (D) none of these.

E-3. One end of a uniform rod having mass m and length ℓ is hinged. The rod is placed on a smooth horizontal surface and rotates on it about the hinged end at a uniform angular velocity ω. The force exerted by the hinge on the rod has a horizontal component

(A) $m\omega^2\ell$ (B) zero (C) mg (D) $\frac{1}{2}m\omega^2\ell$

E-4. The uniform rod of mass 20 kg and length 1.6 m is pivoted at its end and swings freely in the vertical plane. Angular acceleration of rod just after the rod is released from rest in the horizontal position as shown in figure is



1.6m

(A) $\frac{15g}{16}$ (B) $\frac{17g}{16}$ (C) $\frac{16g}{15}$ (D) $\frac{g}{15}$

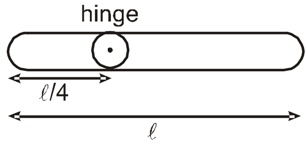
E-5. Two men support a uniform horizontal rod at its two ends. If one of them suddenly lets go, the force exerted by the rod on the other man just after this moment will:

(A) remain unaffected (B) increase (C) decrease
(D) become unequal to the force exerted by him on the rod.

Section (F) : Rotation about fixed axis (energy conservation)

F-1. A uniform metre stick is held vertically with one end on the floor and is allowed to fall. The speed of the other end when it hits the floor assuming that the end at the floor does not slip :
(A) $\sqrt{4g}$ (B) $\sqrt{3g}$ (C) $\sqrt{5g}$ (D) $\sqrt{g}$

F-2. A uniform rod is hinged as shown in the figure and is released from a horizontal position. The angular velocity of the rod as it passes the vertical position is: (axis is fixed, smooth and horizontal)



(A) $\sqrt{\frac{12g}{3\ell}}$ (B) $\sqrt{\frac{2g}{3\ell}}$ (C) $\sqrt{\frac{24g}{7\ell}}$ (D) $\sqrt{\frac{3g}{7\ell}}$

Section (G) : Angular Momentum & its conservation

G-1. A constant torque actingon a uniform circular wheel changes its angular momentum from A_0 to $4A_0$ in 4 sec. the magnitude of this torque is :
(A) $4A_0$ (B) A_0 (C) $3A_0/4$ (D) $12A_0$

G-2. A particle moves with a constant velocity parallel to the Y-axis. Its angular momentum about the origin.
(A) is zero (B) remains constant (C) goes on increasing (D) goes on decreasing.

G-3. A particle is projected at time t = 0 from a point P on the ground with a speed V_0, at an angle of 45° to the horizontal. What is the magnitude of the angular momentum of the particle about P at time $t = v_0/g$.
(A) $\frac{mv_0^2}{2\sqrt{2}g}$ (B) $\frac{mv_0^3}{\sqrt{2}g}$ (C) $\frac{mv_0^2}{\sqrt{2}g}$ (D) $\frac{mv_0^3}{2\sqrt{2}g}$

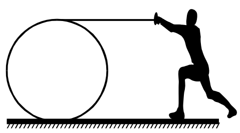
G-4. A uniform thin circular ring of mass 'M' and radius 'R' is rotating about its fixed axis passing through its centre perpendicular to its plane of rotation with a constant angular velocity ω. Two objects each of mass m, are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity. **[JEE - 1983]**
(A) $\frac{\omega M}{(M+m)}$ (B) $\frac{\omega M}{(M+2m)}$ (C) $\frac{\omega M}{(M-2m)}$ (D) $\frac{\omega(M+3m)}{M}$

G-5. A boy sitting firmly over a rotating stool has his arms folded. If he stretches his arms, his angular momentum about the axis of rotation
(A) increases (B) decreases (C) remains unchanged (D) doubles

Section (H) : Combined Translational + Rotational Motion (Kinematics)

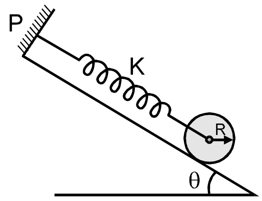
H-1. The centre of a disc rolling without slipping on a plane surface moves with speed u. A particle, on the lower half of the rim making an angle 60º with vertical, will be moving at speed
(A) zero (B) u (C) $\sqrt{2}u$ (D) 2u

H-2. A thin string is wrapped several times around a cylinder kept on a rough horizontal surface. A boy standing at a distance ℓ from the cylinder draws the string towards him as shown in figure. The cylinder rolls without slipping. The length of the string passed through the hand of the boy while the cylinder reaches his hand is



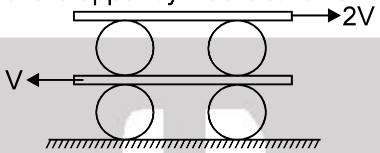
(A) ℓ (B) 2ℓ (C) 3ℓ (D) 4ℓ

H-3. A uniform cylinder of mass M and radius R rolls without slipping down a slope of angle θ to the horizontal. The cylinder is connected to a spring constant K while the other end of the spring is connected to a rigid support at P. The cylinder is released when the spring is unstretched. The maximum displacement of cylinder is



(A) $\frac{3}{4}\frac{Mg\sin\theta}{K}$ (B) $\frac{Mg\sin\theta}{K}$ (C) $\frac{2Mg\sin\theta}{K}$ (D) $\frac{4}{3}\frac{Mg\sin\theta}{K}$

H-4. A system of uniform cylinders and plates is shown in figure. All the cylinders are identical and there is no slipping at any contact. Velocity of lower & upper plate is V and 2V respectively as shown in figure. Then the ratio of angular speed of the upper cylinders to lower cylinders is

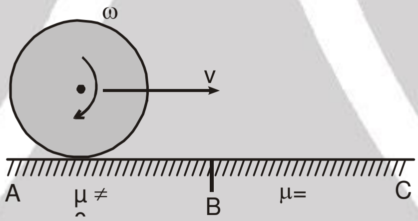


(A) 3 (B) 1/3 (C) 1 (D) none of these

H-5. When a person throws a meter stick it is found that the centre of the stick is moving with a speed of 10 m/s vertically upwards & left end of stick with a speed of 20 m/s vertically upwards. Then the angular speed of the stick is:
(A) 20 rad/ sec (B) 10 rad/sec (C) 30 rad/sec (D) none of these

Section (I): Combined translational & Rotational Motion (Dynamics)

I-1. As shown in the figure, a uniform disc of mass m is rolling without slipping with a angular velocity ω. The portion AB is rough and BC is smooth. When it crosses point B disc will be in :



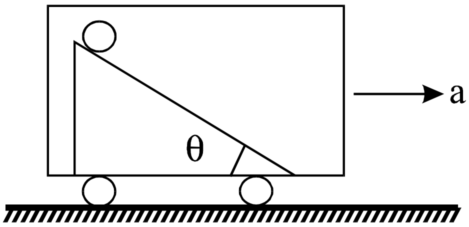
(A) translational motion only (B) pure rolling motion
(C) rotational motion only (D) none of these

I-2. A solid sphere, a hollow sphere and a ring, all having equal mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are equal but not sufficient to allow pure rolling. The greastest kinetic energy at the bottom of the incline will be achieved by
(A) the solid sphere (B) the hollow sphere (C) the ring
(D) all will achieve same kinetic energy.

I-3. A hollow sphere and a solid sphere having equal mass and equal radii are rolled down without slipping on a rough inclined plane.
(A) The two spheres reach the bottom simultaneously
(B) The hollow sphere reaches the bottom with lesser speed.
(C) The solid sphere reaches the bottom with greater kinetic energy
(D) The two spheres will reach the bottom with same linear momentum

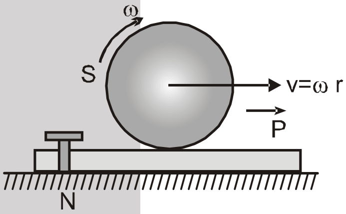
I-4. A solid sphere, a hollow sphere and a solid cylinder, all having equal mass and radius, are placed at the top of an incline and released. The friction coefficients between the objects and the incline are equal but not sufficient to allow pure rolling. Greatest time will be taken in reaching the bottom by
(A) the solid sphere (B) the hollow sphere (C) the solid cylinder (D) all will take same time.

I-5. A rough inclined plane fixed in a car accelerating on a horizontal road is shown in figure. The angle of incline θ is related to the acceleration a of the car as $a = g \tan\theta$. If a rigid sphere is set in pure rolling on the incline



(A) it will continue pure rolling
(B) Friction will act on it
(C) its angular velocity will increase
(D) its angular velocity will decrease.

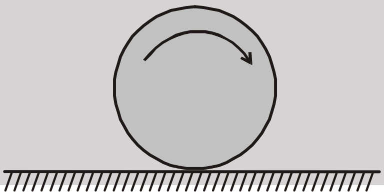
I-6. A sphere S rolls without slipping, moving with a constant speed on a plank P. The friction between the upper surface of P and the sphere is sufficient to prevent slipping, while the lower surface of P is smooth and rests on the ground. Initially, P is fixed to the ground by a pin N. If N is suddenly removed:



(A) S will begin to slip on P
(B) P will begin to move backwards
(C) the speed of S will decrease and its angular velocity will increase
(D) there will be no change in the motion of S and P will still be at rest.

I-7. A body is given translational velocity and kept on a surface that has sufficient friction. Then:
(A) body will move forward before pure rolling
(B) body will move backward before pure rolling
(C) body will start pure rolling immediately
(D) none of these

I-8. A body of mass m and radius r is rotated with angular velocity ω as shown in the figure & kept on a surface that has sufficient friction then the body will move :



(A) backward first and then move forward
(B) forward first and then move backward
(C) will always move forward
(D) none of these

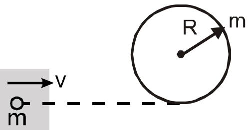
I-9. A body of mass m and radius R rolling horizontally without slipping at a speed v climbs a ramp to a height $\frac{3v^2}{4g}$. The rolling body can be **[Olympiad 2015 (stage-1)]**

(A) a sphere
(B) a circular ring
(C) a spherical shell
(D) a circular disc

Section (J) : Conservation of angular momentum (combined translation & rotational motion)

J-1. A sphere is released on a smooth inclined plane from the top. When it moves down its angular momentum is:
(A) conserved about every point
(B) conserved about the point of contact only
(C) conserved about the centre of the sphere only
(D) conserved about any point on a fixed line parallel to the inclined plane and passing through the centre of the ball.

J-2. A circular wooden loop of mass m and radius R rests flat on a horizontal frictionless surface. A bullet, also of mass m, and moving with a velocity V, strikes the loop and gets embedded in it. The thickness of the loop is much smaller than R. The angular velocity with which the system rotates just after the bullet strikes the loop is



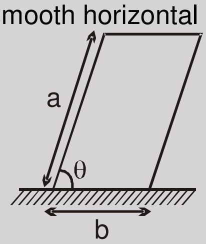
(A) $\frac{V}{4R}$ (B) $\frac{V}{3R}$ (C) $\frac{2V}{3R}$ (D) $\frac{3V}{4R}$

Section (K) : Toppling

K-1. A uniform cube of side a and mass m rests on a rough horizontal table. A horizontal force 'F' is applied normal to one of the faces at a point that is directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. The minimum value of 'F' for which the cube begins to tilt about the edge is (assume that the cube does not slide). **[JEE - 1984]**

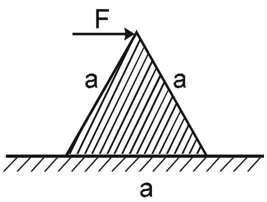
(A) $\frac{2}{3}mg$ (B) $\frac{4}{3}mg$ (C) $\frac{5}{4}mg$ (D) $\frac{1}{2}mg$

K-2. A homogenous block having its cross-section to be a parallelogram of sides 'a' and 'b' (as shown) is lying at rest and is in equilibrium on a smooth horizontal surface. Then for acute angle θ :



(A) $\cos\theta \le \frac{b}{a}$ (B) $\cos\theta \ge \frac{b}{a}$ (C) $\cos\theta < \frac{b}{a}$ (D) $\cos\theta > \frac{b}{a}$
(E) $\cos\theta > \frac{a}{b}$

K-3. An equilateral uniform prism of mass m rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the prism as shown in the figure. If the coefficient of friction is sufficiently high so that the prism does not slide before toppling, then the minimum force required to topple the prism is :

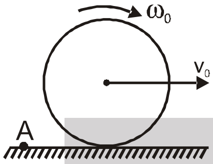


(A) $\frac{mg}{\sqrt{3}}$ (B) $\frac{mg}{4}$ (C) $\frac{\mu mg}{\sqrt{3}}$ (D) $\frac{\mu mg}{4}$

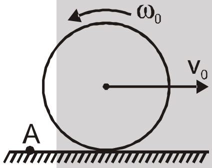
PART - III : MATCH THE COLUMN

1. In each situation of column-I, a uniform disc of mass m and radius R rolls on a rough fixed horizontal surface as shown in the figure. At t = 0 (initially) the angular velocity of disc is ω_o and velocity of centre of mass of disc is v_o (in horizontal direction). The relation between v_o and ω_o for each situation and also initial sense of rotation is given for each situation in column-I. Then match the statements in column-I with the corresponding results in column-II.

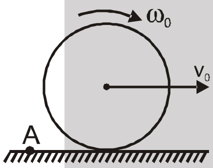
Column-I

(A) 

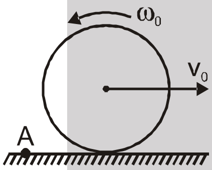
($v_o > R\ \omega_o$)

(B) 

($v_o > R\ \omega_o$)

(C) 

($v_o < R\ \omega_o$)

(D) 

($v_o < R\ \omega_o$)

Column-II

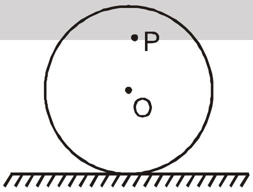
(p) The angular momentum of disc about point A (as shown in figure) remains conserved.

(q) The kinetic energy of disc after it starts rolling without slipping is less than its initial kinetic energy.

(r) In the duration disc rolls with slipping, the friction acts on disc towards left.

(s) In the duration disc rolls with slipping, the friction acts on disc for some time towards right and for some time towards left.

2. A uniform disc rolls without slipping on a rough horizontal surface with uniform angular velocity. Point O is the centre of disc and P is a point on disc as shown in the figure. In each situation of column I a statement is given and the corresponding results are given in column-II. Match the statements in column-I with the results in column-II.



Column I

(A) The velocity of point P on disc

(B) The acceleration of point P on disc

(C) The tangential acceleration of point P on disc

(D) The acceleration of point on disc which is in contact with rough horizontal surface

Column II

(p) Changes in magnitude with time.

(q) is always directed from that point (the point on disc given in column-I) towards centre of disc.

(r) is always zero.

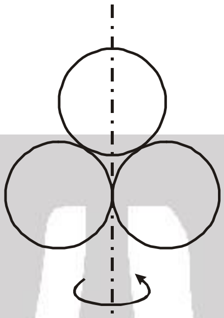
(s) is non-zero and remains constant in magnitude.

Exercise-2

Marked Questions can be used as Revision Questions.

PART - I : ONLY ONE OPTION CORRECT TYPE

1. Three rings each of mass m and radius r are so placed that they touch each other. The radius of gyration of the system about the axis as shown in the figure is :

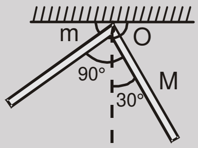


(A) $\sqrt{\frac{6}{5}}\ r$ (B) $\sqrt{\frac{5}{6}}\ r$ (C) $\sqrt{\frac{6}{7}}\ r$ (D) $\sqrt{\frac{7}{6}}\ r$

2. A hollow cylinder has mass M, outside radius R_2 and inside radius R_1. Its moment of inertia about an axis parallel to its symmetry axis and tangential to the outer surface is equal to :

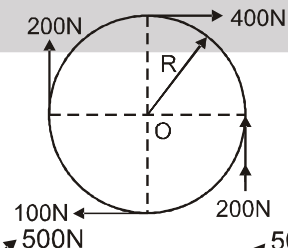
(A) $\frac{M}{2}(R_2^2 + R_1^2)$ (B) $\frac{M}{2}(R_2^2 - R_1^2)$ (C) $\frac{M}{4}(R_2 + R_1)^2$ (D) $\frac{M}{2}(3R_2^2 + R_1^2)$

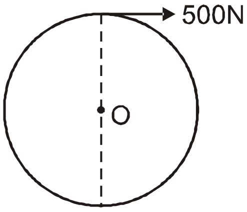
3. Two uniform rods of equal length but different masses are rigidly joined to form an L–shaped body, which is then pivoted about O as shown in the figure. If in equilibrium the body is in the shown configuration, ratio M/m will be:

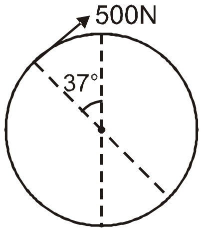


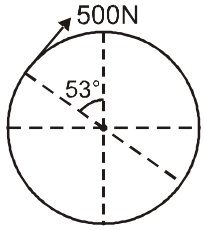
(A) 2 (B) 3 (C) $\sqrt{2}$ (D) $\sqrt{3}$

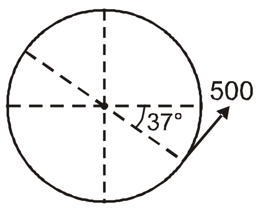
4. Four forces tangent to the circle of radius 'R' are acting on a wheel as shown in the figure. The resultant equivalent one force system will be :



(A)


(B)


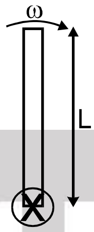
(C)


(D)


5. A uniform thin rod of mass 'm' and length L is held horizontally by two vertical strings attached to the two ends. One of the string is cut. Find the angular acceleration soon after it is cut :

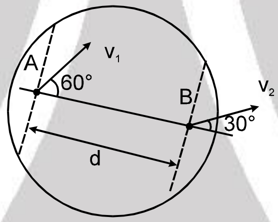
(A) $\frac{g}{2L}$ (B) $\frac{g}{L}$ (C) $\frac{3g}{2L}$ (D) $\frac{2g}{L}$

6. A uniform rod hinged at its one end is allowed to rotate in vertical plane. Rod is given an angular velocity ω in its vertical position as shown in figure. The value of ω for which the force exerted by the hinge on rod is zero in this position is :



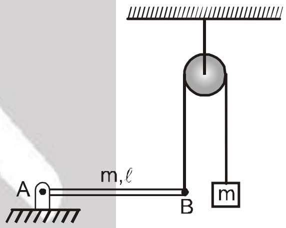
(A) $\sqrt{\frac{g}{L}}$ (B) $\sqrt{\frac{2g}{L}}$ (C) $\sqrt{\frac{g}{2L}}$ (D) $\sqrt{\frac{3g}{L}}$

7. Two points A & B on a disc have velocities v_1 & v_2 at some moment. Their directions make angles 60° and 30° respectively with the line of separation as shown in figure. The angular velocity of disc is :



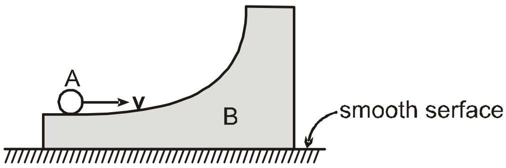
(A) $\frac{\sqrt{3}v_1}{d}$ (B) $\frac{v_2}{\sqrt{3}d}$ (C) $\frac{v_2 - v_1}{d}$ (D) $\frac{v_2}{d}$

8. Uniform rod AB is hinged at the end A in a horizontal position as shown in the figure (the hinge is frictionless, that is, it does not exert any friction force on the rod). The other end of the rod is connected to a block through a massless string as shown. The pulley is smooth and massless. Masses of the block and the rod are same and are equal to ' m '. Acceleration due to gravity is g. The tension in the thread, and angular acceleration of the rod just after release of block from this position



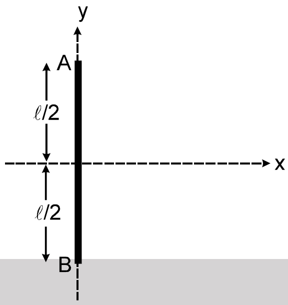
(A) $\frac{3mg}{8}, \frac{g}{8\ell}$ (B) $\frac{5mg}{8}, \frac{3g}{8\ell}$ (C) $\frac{mg}{8}, \frac{5g}{8\ell}$ (D) $\frac{7mg}{8}, \frac{7g}{8\ell}$

9. In the figure shown a ring A is rolling without sliding with a velocity v on the horizontal surface of the body B (of same mass as A). All surfaces are smooth. B has no initial velocity. What will be the maximum height (from initial position) reached by A on B.



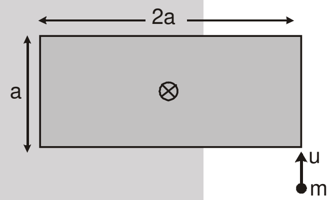
(A) $\frac{3v^2}{4g}$ (B) $\frac{v^2}{4g}$ (C) $\frac{v^2}{2g}$ (D) $\frac{v^2}{3g}$

10. A uniform rod of mass m, length ℓ is placed over a smooth horizontal surface along y-axis and is at rest as shown in figure. An impulsive force F is applied for a small time Δt along x-direction at point A after this rod moves freely. The x–coordinate of end A of the rod when the rod becomes parallel to x–axis for the first time is (initially the coordinate of centre of mass of the rod is (0, 0)) :



(A) $\frac{\pi\ell}{12}$ (B) $\frac{\ell}{2}\left(1+\frac{\pi}{12}\right)$ (C) $\frac{\ell}{2}\left(1-\frac{\pi}{6}\right)$ (D) $\frac{\ell}{2}\left(1+\frac{\pi}{6}\right)$

11. A uniform rectangular plate of mass m which is free to rotate about the smooth vertical hinge passing through the centre and perpendicular to the plate, is lying on a smooth horizontal surface. A particle of mass m moving with speed 'u' collides with the plate and sticks to it as shown in figure. The angular velocity of the plate after collision will be :

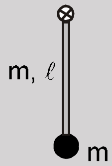


(A) $\frac{12}{5}\frac{u}{a}$ (B) $\frac{12}{19}\frac{u}{a}$ (C) $\frac{3}{2}\frac{u}{a}$ (D) $\frac{3}{5}\frac{u}{a}$

12. A rod can rotate about a fixed vertical axis. The mass is non-uniformly distributed along the length of the rod. A horizontal force of constant magnitude and always perpendicular to the rod is applied at the end. Which of the following quantity (after one rotation) will not depend on the information that through which end the axis passes ? (Assuming initial angular velocity to be zero)

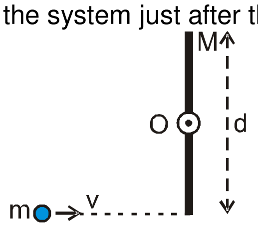
(A) angular momentum (B) kinetic energy (C) angular velocity (D) none of these

13. A particle is attached to the lower end of a uniform rod which is hinged at its other end as shown in the figure. The minimum speed given to the particle so that the rod performs circular motion in a vertical plane will be : [length of the rod is ℓ, consider masses of both rod and particle to be same]



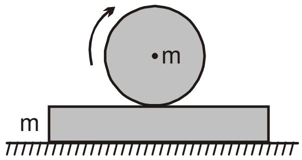
(A) $\sqrt{5g\ell}$ (B) $\sqrt{4g\ell}$ (C) $\sqrt{4.5g\ell}$ (D) none of these

14. A particle of mass m is moving horizontally at speed v perpendicular to a uniform rod of length d and mass M = 6m. The rod is hinged at centre O and can freely rotate in horizontal plane about a fixed vertical axis passing through its centre O. The hinge is frictionless.The particle strikes and sticks to the end of the rod. The angular speed of the system just after the collision :



(A) 2v/3d (B) 3v/2d (C) v/3d (D) 2v/d

15. A uniform sphere of mass ' m ' is given some angular velocity about a horizontal axis through its centre and gently placed on a plank of mass ' m '. The co-efficient of friction between the two is μ. The plank rests on a smooth horizontal surface. The initial acceleration of the centre of sphere relative to the plank will be :



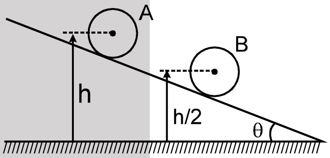
(A) zero (B) μg (C) $(7/5)\ \mu g$ (D) $2\ \mu g$

16. A rod of negligible mass and length ℓ is pivoted at the point $\ell/4$ distance from the left end as shown. A particle of mass m is fixed to its left end & another particle of mass 2 m is fixed to the right end. If the system is released from rest and after sometime becomes vertical, the speed v of the two masses and angular velocity at that instant is

(A) $\frac{1}{4}\sqrt{\frac{40g\ell}{19}}, \frac{3}{4}\sqrt{\frac{40g\ell}{19}}, \sqrt{\frac{40g}{19\ell}}$ (B) $\frac{1}{2}\sqrt{\frac{20g\ell}{19}}, \frac{3}{4}\sqrt{\frac{20g\ell}{19}}, \sqrt{\frac{20g}{19\ell}}$

(C) $\frac{1}{4}\sqrt{\frac{20g\ell}{19}}, \frac{3}{4}\sqrt{\frac{20g\ell}{19}}, \sqrt{\frac{20g}{19\ell}}$ (D) $\frac{1}{2}\sqrt{\frac{40g\ell}{19}}, \frac{1}{2}\sqrt{\frac{40g\ell}{19}}, \sqrt{\frac{40g}{19\ell}}$

17. Two identical balls A & B of mass m each are placed on a fixed wedge as shown in figure. Ball B is kept at rest and it is released just before two balls collide. Ball A rolls down without slipping on inclined plane & collides elastically with ball B. The kinetic energy of ball A just after the collision with ball B is (Neglect friction between A and B, also neglect the radius of the balls) :

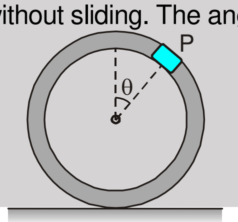


(A) $\frac{mgh}{7}$ (B) $\frac{mgh}{2}$ (C) $\frac{2mgh}{5}$ (D) $\frac{7mgh}{5}$

18. A solid uniform disc of mass m rolls without slipping down an inclined plane with an acceleration a. The frictional force on the disc due to surface of the plane is

(A) 2 ma (B) $\frac{3}{2}$ ma (C) ma (D) $\frac{1}{2}$ ma

19. A small block of mass 'm' is rigidly attached at 'P' to a ring of mass '3m' and radius 'r'. The system is released from rest at $\theta = 90°$ and rolls without sliding. The angular acceleration of ring just after release is

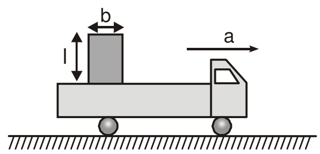


(A) $\frac{g}{4r}$ (B) $\frac{g}{8r}$ (C) $\frac{g}{3r}$ (D) $\frac{g}{2r}$

20. A uniform ring of radius R is given a back spin of angular velocity $V_0/2R$ and thrown on a horizontal rough surface with velocity of center to be V_0. The velocity of the centre of the ring when it starts pure rolling will be

(A) $V_0/2$ (B) $V_0/4$ (C) $3V_0/4$ (D) 0

21. A box of dimensions ℓ and b is kept on a truck moving with an acceleration a. If box does not slide, maximum acceleration for it to remain in equilibrium (w.r.t.truck) is :



(A) $\frac{g\ell}{b}$ (B) $\frac{gb}{\ell}$

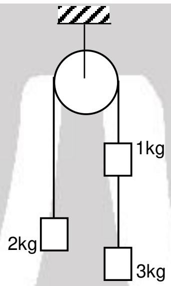
(C) g (D) none of these

22. If the positions of two like parallel forces on a light rod are interchanged, their resultant shifts by one–fourth of the distance between them then the ratio of their magnitude is:
(A) 1 : 2 (B) 2 : 3 (C) 3 : 4 (D) 3 : 5

23._ Consider two point masses m_1 and m_2 connected by alight rigid rod of length r_0. The moment of inertia of the system about an axis passing through their centre of mass and perpendicular to the rigid rod is given by **[Olympiad (Stage-1) 2017]**

(A) $\frac{m_1m_2}{2(m_1+m_2)}r_0^2$ (B) $\frac{m_1m_2}{m_1+m_2}r_0^2$ (C) $\frac{2m_1m_2}{m_1+m_2}r_0^2$ (D) $\frac{m_1^2+m_2^2}{m_1+m_2}r_0^2$

24._ In the following arrangement the pulley is assumed to be light and the string inextensible. The acceleration of the system can be determined by considering conservation of a certain physical quantity. The physical quantity conserved and the acceleration respectively, are **[Olympiad (Stage-1) 2017]**



(A) energy and g/3 (B) linear momentum and g/2
(C) angular momentum and g/3 (D) mass and g/2

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

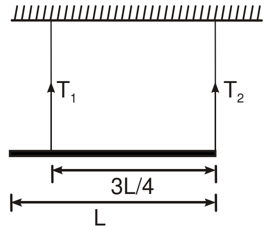
1. A wheel starting from rest is uniformly accelerated at 2 rad/s^2 for 20 seconds. It rotates uniformly for the next 20 seconds and is finally brought to rest in the next 20 seconds. Total angular displacement of the wheel is $n \times 10^2$ radian where n is.

2. Two steel balls of equal diameter are connected by a rigid bar of negligible weight as shown & are dropped in the horizontal position from height h above the heavy steel and brass base plates. If the coefficient of restitution between the ball & steel base is 0.6 & that between the other ball & the brass base is 0.4. The angular velocity of the bar immediately after rebound is $n \times 10^{-2}$ rad/s where n is : (Assume the two impacts are simultaneous.) (g = 9.8m/s^2)

3. Three identical uniform rods, each of length, are joined to form a rigid equilateral triangle. Its radius of gyration about an axis passing through a corner and perpendicular to the plane of the triangle is $\frac{\ell}{\sqrt{n}}$ where n is :

4. The moment of inertia of a thin uniform rod of mass m & length ℓ about an axis passing through one end & making angle $\theta = 45^o$ with its length is $\frac{m\ell^2}{n}$ where n is.

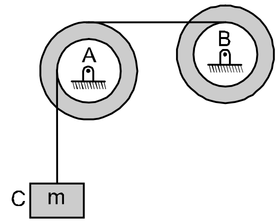
5. A uniform rod of mass m and length L is suspended with two massless strings as shown in the figure. If the rod is at rest in a horizontal position the ratio of tension in the two strings T_1/T_2 is:



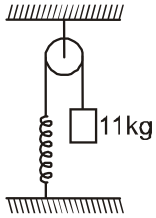
6. Two persons of equal height are carrying a long uniform wooden beam of length ℓ. They are at distance $\ell/4$ and $\ell/6$ from nearest ends of the rod. The ratio of normal reactions at their heads is n : 3 where n is :

7. A (i) ring and (ii) uniform disc both of radius R is given an angular velocity and then carefully placed on a horizontal surface such that its axis is vertical. If the coefficient of friction is μ for both cases then the ratio of the time taken by the ring and disk to come to rest is n : 3 where n is : (The pressure exerted by the disc and ring on the surface can be regarded as uniform).

8. Each of the double pulleys shown has a centroidal mass moment of inertia of 'mr²', inner radius r and an outer radius 2r. Assuming that the bearing friction of hinge at A and at B is equivalent to torque of magnitude $\frac{mgr}{4}$ then the tension (in N) in the string connecting the pulleys is : (m = 3kg, g = 10m/s^2, and r =0.1m)

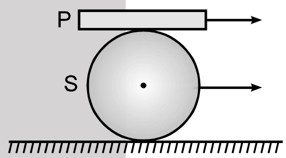


9. The pulley shown in figure has a radius of 10 cm and moment of inertia 0.1 kg-m^2. The string going over it attached at one end to a vertical spring of spring constant 100 N/m fixed from below, and supports a 11 kg mass at the other end. The system is released from rest with the spring at its natural length. Find the speed (in m/s) of the block when it has descended through 10 cm. (Take g = 10 m/s^2 and assume that the there is no slipping between string and pulley).



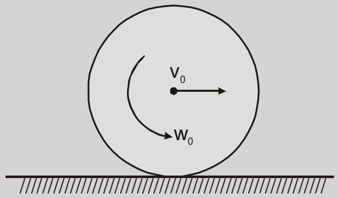
10. The angular momentum of a particle about origin is varying as L = $4\sqrt{2}$ t + 8 (SI units) when it moves along a straight line y = x – 4 (x, y in meters). The magnitude of force (in N) acting on the particle would be :

11. A plank P is placed on a solid cylinder S, which rolls on a horizontal surface. The two are of equal mass. There is no slipping at any of the surfaces in contact. The ratio of the kinetic energy of P to the kinetic energy of S is n : 3 where n is :

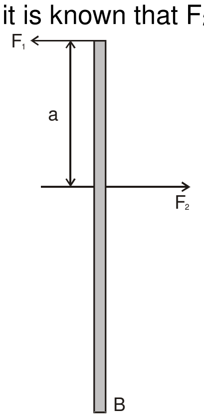


12. A uniform rod of mass m = 5.0 kg and length ℓ = 90 cm rests on a smooth horizontal surface. One of the ends of the rod is struck with the impulse J = 3.0 N-s in a horizontal direction perpendicular to the rod and removed. As a result of which the rod gets angular velocity and linear velocity instantaneously. The force (in N) with which one half of the rod will act on the other in the process of motion later on.

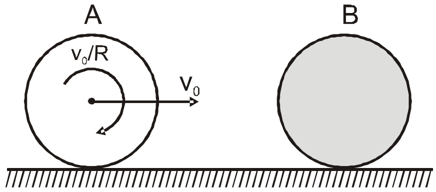
13. A uniform solid sphere of mass m and radius r is projected along a rough horizontal surface with the initial velocity v_0 and angular velocity ω_0 as shown in the figure. If the sphere finally comes to complete rest then $\frac{2\omega_0 r}{v_0}$ is equal to :



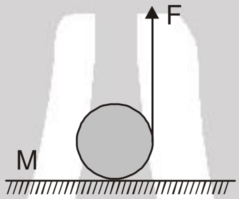
14. A thin uniform rod AB of mass m = 1.0 kg moves translationally with acceleration w = 2.0 m/s^2 due to two antiparallel forces F_1 and F_2 (Fig.). The distance between the points at which these forces are applied is equal to a = 20 cm. Besides, it is known that F_2 = 5.0.N. Find the length (in m) of the rod.



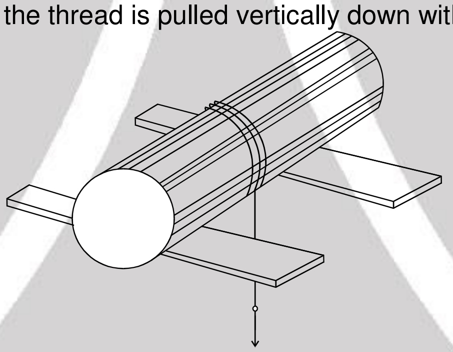
15. A hollow smooth uniform sphere A of mass 'm' rolls without sliding on a smooth horizontal surface. It collides elastically and headon with another stationary smooth solid sphere B of the same mass m and same radius. The ratio of kinetic energy of 'B' to that of 'A' just after the collision is 3 : n where n is :



16. The free end of the string wound on the surface of a solid cylinder of mass M = 1kg & radius R = $\frac{2}{3}$ m is pulled up by a force F as shown. If there is sufficient friction between cylinder & floor then the upper limit to the angular acceleration (in rad/s^2) of the cylinder for which it rolls without slipping is : (g = 10m/s^2)



17. A uniform solid cylinder of mass m = 1kg rests on two horizontal planks. A thread is wound on the cylinder. The hanging end of the thread is pulled vertically down with a constant force F.

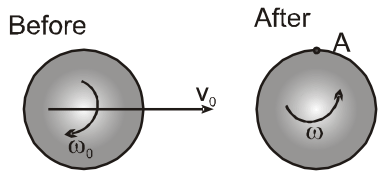


(a) Find the maximum magnitude of the force F(in N) which still does not bring about any sliding of the cylinder, if the coefficient of friction between the cylinder and the planks is equal to $\mu = \frac{1}{3}$.

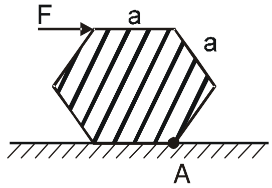
(b) The acceleration a_{max} of the axis of the cylinder rolling on the planks is $\frac{n}{3}$ m/s^2 where n is:

18. A cylinder rotating at an angular speed of 50 rev/s is brought in contact with an identical stationary cylinder. Because of the kinetic friction, torques act on the two cylinders, accelerating the stationary one and decelerating the moving one. If the common magnitude of the angular acceleration and deceleration be1 rev/s^2, then how much time (ins) will it take before the two cylinders have equal angular speed ?

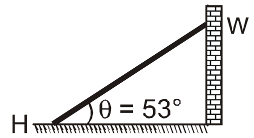
19. A uniform disc of mass M = 1kg, radius R = 1m is moving towards right on smooth horizontal surface with velocity v_0 = 20 m/s & having angular velocity ω_0 = 4 rad/s about the perpendicular axis outward the plane of disc passing through centre of disc. Suddenly top point of the disc gets hinged about a fixed smooth axis. The angular velocity (in rad/s) of disk about new rotation axis is:



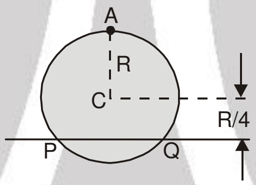
20. A regular hexagonal uniform block of mass $m = 4\sqrt{3}$ kg rests on a rough horizontal surface with coefficient of friction μ as shown in figure. A constant horizontal force is applied on the block as shown. If the coefficient of friction is sufficient to prevent slipping before toppling, then the minimum force(in N) required to topple the block about its corner A is:(g = 10m/s^2)



21. A uniform rod of length ℓ = 1m is kept as shown in the figure. H is a horizontal smooth surface and W is a vertical smooth wall. The rod is released from this position. The angular acceleration (in radian/sec^2) of the rod just after the release is :



22. A uniform circular disc has radius R & mass m. A particle also of mass m is fixed at a point A on the edge of the disc as shown in the figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. The linear speed of the particle as it reaches its lowest position is $\sqrt{ngR}$, where n is an integer. Find the value of n **[JEE - 1998' 8/200]**

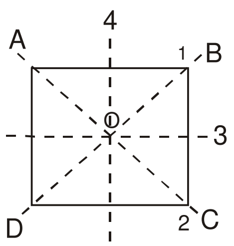


PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A rigid body is in pure rotation.
(A) You can find two points in the body in a plane perpendicular to the axis of rotation having same velocity.
(B) You can find two points in the body in a plane perpendicular to the axis of rotation having same acceleration.
(C) Speed of all the particles lying on the curved surface of a cylinder whose axis coincides with the axis of rotation is same.
(D) Angular speed of the body is same as seen from any point in the body.

2. A sphere is rotating uniformly about a fixed axis passing through its centre then:
(A) The particles on the surface of the sphere do not have any angular acceleration.
(B) The particles on the axis do not have any linear acceleration
(C) Different particles on the surface have same angular speeds.
(D) All the particles on the surface have same linear speed

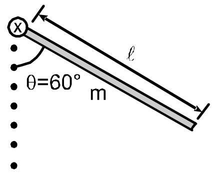
3. The moment of inertia of a thin uniform square plate ABCD of uniform thickness about an axis passing through the centre O and perpendicular to the plate is - **[REE - 1992]**



(A) $I_1 + I_2$ (B) $I_3 + I_4$ (C) $I_1 + I_3$ (D) $I_1 + I_2 + I_3 + I_4$

where I_1, I_2, I_3, and I_4 are respectively the moments of inertia about axes 1, 2, 3, and 4 which are in the plane of the plate.

4. In the figure shown a uniform rod of mass m and length ℓ is hinged. The rod is released when the rod makes angle $\theta = 60°$ with the vertical.



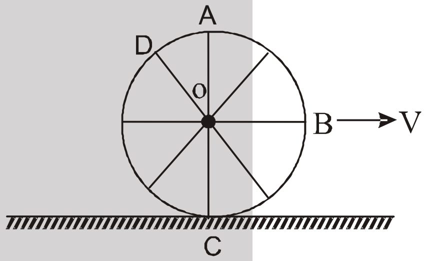
(A) The angular acceleration of the rod just after release is $\frac{3\sqrt{3}g}{4\ell}$

(B) The normal reaction due to the hinge just after release is $\frac{\sqrt{19}mg}{8}$

(C) The angular velocity of the rod at the instant it becomes vertical is $\sqrt{\frac{3g}{2\ell}}$

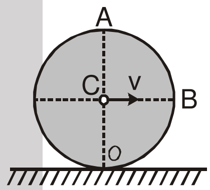
(D) The normal reaction due to the hinge at the instant the rod becomes vertical is $\frac{7}{4}$mg

5. Consider a disc rolling without slipping on a horizontal surface at a linear speed V as shown in figure



(A) the speed of the particle A is 2V
(B) the speed of B, C and D are all equal to V
(C) the speed of C is zero and speed of B is $\sqrt{2}$V
(D) the speed of O is less than the speed of B

6. A cylinder rolls without slipping over a horizontal plane with constant velocity. The radius of the cylinder is equal to r. At this moment



(A) The speed of B is $\sqrt{2}$ times the speed of A.
(B) The radius of curvature of trajectory traced out by A is 4r.
(C) The radius of curvature of trajectory traced out by B is $2\sqrt{2}$ r.
(D) The radius of curvature of trajectory traced out by C is r.

7. A uniform solid sphere is released from rest from the top of an inclined plane of inclination θ. Then choose the correct option(s).

(A) The minimum coefficient of friction between sphere and the incline to prevent slipping is $\frac{2}{7}\tan\theta$.

(B) The kinetic energy of solid sphere as it moves down a distance S on the incline is mgSsinθ if $\mu \geq \frac{2}{7}\tan\theta$.

(C) The magnitude of work done by friction on the solid sphere is less than μmgScosθ as it moves down a distance S on the incline if $\mu \geq \frac{2}{7}\tan\theta$.

(D) The magnitude of work done by friction on the solid sphere is equal to μmgScosθ as it moves down a distance S on the incline if $\mu \geq \frac{2}{7}\tan\theta$.

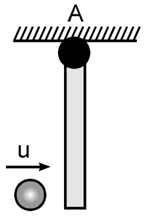
8. A uniform solid cylinder rolls without slipping on a rough horizontal floor, its centre of mass moving with a speed v. It makes an elastic collision with smooth vertical wall. After impact:
(A) its centre of mass will move with a speed *v* initially
(B) its motion will be rolling without slipping immediately
(C) its motion will be rolling with slipping initially and its rotational motion will stop momentarily at some instant
(D) its motion will be rolling without slipping only after some time.

9. If $\vec{\tau} \times \vec{L} = 0$ for a rigid body, where $\vec{\tau}$ = resultant torque & $\vec{L}$ = angular momentum about a point and both are non-zero. Then :

(A) $\vec{L}$ may be constant (B) $|\vec{L}|$ = constant (C) $|\vec{L}|$ may decrease (D) $|\vec{L}|$ may increase

10. In absence of external forces on a rigid system, which of the following quantities must remain constant?
(A) angular momentum
(B) linear momentum
(C) moment of inertia about fixed axis through any point on body
(D) kinetic energy

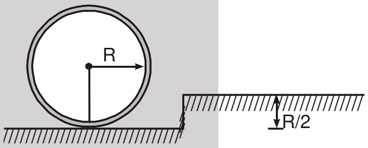
11. In the given figure a ball strikes a rod elastically and rod is smoothly hinged at point A. Then which of the statement(s) is/are correct for the collision?



(A) linear momentum of system (ball + rod) is conserved
(B) angular momentum of system about hinged point A is conserved
(C) initial KE of the system is equal to final KE of the system
(D) linear momentum of ball is conserved.

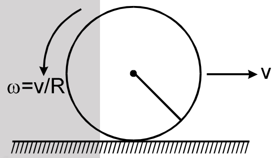
12. A horizontal disc rotates freely about a vertical fixed axis through its centre. A ring, having the same mass and radius as the disc, is now gently placed on the disc coaxially. After some time the two rotate with a common angular velocity:
(A) some friction exists between the disc and the ring before achieving common angular velocity
(B) the angular momentum of the 'disc plus ring' about axis of rotation is conserved
(C) the final common angular velocity is $2/3^{rd}$ of the initial angular velocity of the disc
(D) The final common angular velocity is $1/3^{rd}$ of the initial angular velocity of the disc

13. A wheel (to be considered as a ring) of mass m and radius R rolls without sliding on a horizontal surface with constant velocity v. It encounters a step of height R/2 at which it ascends without sliding.



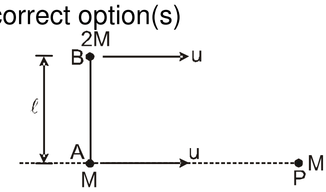
(A) the angular velocity of the ring just after it comes in contact with the step is 3v/4R
(B) the normal reaction due to the step on the wheel just after the impact is $\frac{mg}{2} - \frac{9\,mv^2}{16\,R}$
(C) the normal reaction due to the step on the wheel increases as the wheel ascends
(D) the friction will be absent during the ascent.

14. A hollow sphere is set into motion on a rough horizontal surface with a speed v in the forward direction and an rotational speed v/R in the anticlockwise direction as shown in figure. Find the translational speed of the sphere (a) when it stops rotating and (b) when slipping finally ceases and pure rolling starts.



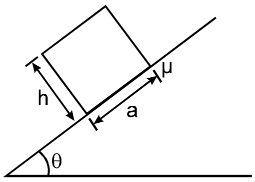
(A) The angular momentum of the sphere about its centre of mass is conserved.
(B) The speed of the sphere at the instant it stops rotating momentarily is v/3
(C) The speed of the sphere after pure rolling starts is v/5
(D) Work done by fricition upto pure rolling starts is zero.

15. Two small particles A and B of masses M and 2M respectively, are joined rigidly to the ends of a light rod of length ℓ as shown in the figure. The system translates on a smooth horizontal surface with a velocity u in a direction perpendicular to the rod. A particle P of mass M kept at rest on the surface sticks to the particle A as the particle P collides with it. Then choose the correct option(s)



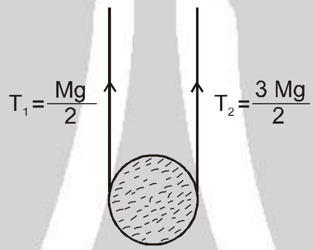
(A) The speed of particle A just after collision is $\frac{u}{2}$
(B) The speed of particle B just after collision is $\frac{u}{2}$
(C) The velocity of centre of mass of system A+B+P is $\frac{3u}{4}$
(D) The angular speed of the system A+B+P after collision is $\frac{u}{2\ell}$

16. A block with a square base measuring a × a and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will:



(A) topple before sliding if $\mu > \frac{a}{h}$
(B) topple before sliding if $\mu < \frac{a}{h}$
(C) slide before toppling if $\mu > \frac{a}{h}$
(D) slide before toppling if $\mu < \frac{a}{h}$

17. A uniform disc of mass M and radius R is lifted using a string as shown in the figure. Then,
[**Olympiad 2014 (stage-1)**]

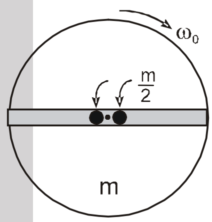


(A) its linear acceleration is g upward
(B) its linear acceleration is g downward
(C) its angular acceleration is $\frac{2g}{R}$.
(D) its rate of change of angular momentum is MgR.

PART - IV : COMPREHENSION

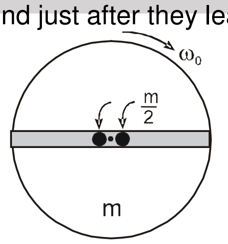
Comprehension-1

A uniform disc of mass 'm' and radius R is free to rotate in horizontal plane about a vertical smooth fixed axis passing through its centre. There is a smooth groove along the diameter of the disc and two small balls of mass $\frac{m}{2}$ each are placed in it on either side of the centre of the disc as shown in fig. The disc is given initial angular velocity ω_0 and released.



1. The angular speed of the disc when the balls reach the end of the disc is :
(A) $\frac{\omega_0}{2}$ (B) $\frac{\omega_0}{3}$ (C) $\frac{2\omega_0}{3}$ (D) $\frac{\omega_0}{4}$

2. The speed of each ball relative to ground just after they leave the disc is :

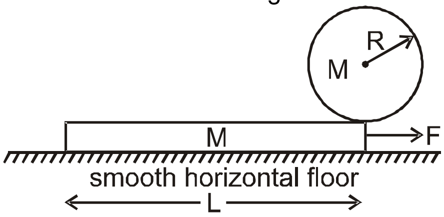


(A) $\frac{R\omega_0}{\sqrt{3}}$ (B) $\frac{R\omega_0}{\sqrt{2}}$ (C) $\frac{2R\omega_0}{3}$ (D) $\frac{R\omega_0}{3}$

3. The net work done by forces exerted by disc on one of the ball for the duration ball remains on the disc is
(A) $\frac{2mR^2\omega_0^2}{9}$ (B) $\frac{mR^2\omega_0^2}{18}$ (C) $\frac{mR^2\omega_0^2}{6}$ (D) $\frac{mR^2\omega_0^2}{9}$

Comprehension-2✍

A uniform disc of mass M and radius R initially stands vertically on the right end of a horizontal plank of mass M and length L, as shown in the figure. The plank rests on smooth horizontal floor and friction between disc and plank is sufficiently high such that disc rolls on plank without slipping. The plank is pulled to right with a constant horizontal force of magnitude F.



4.✍ The magnitude of acceleration of plank is

(A) $\frac{F}{8M}$ (B) $\frac{F}{4M}$ (C) $\frac{3F}{2M}$ (D) $\frac{3F}{4M}$

5.✍ The magnitude of angular acceleration of the disc is -

(A) $\frac{F}{4mR}$ (B) $\frac{F}{8mR}$ (C) $\frac{F}{2mR}$ (D) $\frac{3F}{2mR}$

6.✍ The distance travelled by centre of disc from its initial position till the left end of plank comes vertically below the centre of disc is

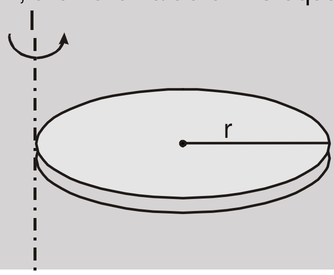
(A) $\frac{L}{2}$ (B) $\frac{L}{4}$ (C) $\frac{L}{8}$ (D) L

Exercise-3

✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

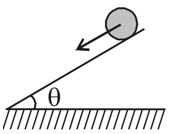
PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1.✍ A solid sphere of radius R has moment of inertia I about its geometrical axis. If it is melted into a disc of radius r and thickness t. If it's moment of inertia about the tangential axis (which is perpendicular to plane of the disc), is also equal to I, then the value of r is equal to : **[JEE 2006, 3/184]**



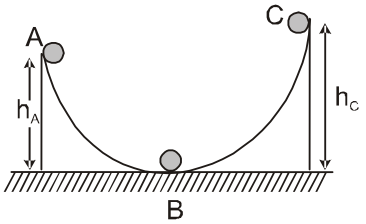
(A) $\frac{2}{\sqrt{15}}R$ (B) $\frac{2}{\sqrt{5}}R$ (C) $\frac{3}{\sqrt{15}}R$ (D) $\frac{\sqrt{3}}{\sqrt{15}}R$

2.* A solid sphere is in pure rolling motion on an inclined surface having inclination θ. **[JEE 2006, 5/184]**



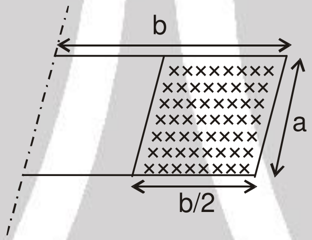
(A) frictional force acting on sphere is $f = \mu mg \cos\theta$.
(B) f is dissipative force.
(C) friction will increase its angular velocity and decreases its linear velocity.
(D) If θ decreases, friction will decrease.

3*. A ball moves over a fixed track as shown in the figure. From A to B the ball rolls without slipping. If surface BC is frictionless and K_A, K_B and K_C are kinetic energies of the ball at A, B and C respectively, then : **[JEE 2006, 5/184]**



(A) $h_A > h_C$; $K_B > K_C$ (B) $h_A > h_C$; $K_C > K_A$ (C) $h_A = h_C$; $K_B = K_C$ (D) $h_A < h_C$; $K_B > K_C$

4. A rectangular plate of mass M of dimensions (a × b) is hinged along one edge. The plate is maintained in horizontal position by colliding a ball of mass m, per unit area, elastically 100 times per second this ball is striking on the right half shaded region of the plate as shown in figure. Find the required speed of the ball (ball is colliding in only half part of the plate as shown). (It is given M = 3 kg, m = 0.01 kg, b = 2 m, a = 1 m, g = 10 m/s^2) **[JEE 2006, 6/184]**



Paragraph for Question Nos. 5 to 7

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[JEE-2007, 12/162]**

5. The ratio x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

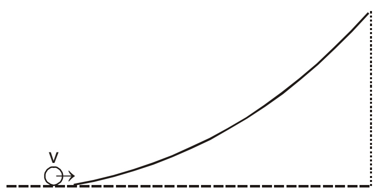
6. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{2t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

7. The loss of kinetic energy during the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

8. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches up to a maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[JEE-2007, 3/162]**



(A) ring (B) solid sphere (C) hollow sphere (D) disc

9. **STATEMENT-1 :** If there is no external torque on a body about its centre of mass, then the velocity of the center of mass remains constant. **[JEE-2007, 3/162]**

because

STATEMENT-2 : The linear momentum of an isolated system remains constant.

(A) Statement-1 is True, Statement-2 is True; Statement-2 **is** a correct explanation for Statement-1

(B) Statement-1 is True, Statement-2 is True; Statement-2 **is NOT** a correct explanation for Statement1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

10. **STATEMENT-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first. **[JEE-2008, 3/163]**

and

STATEMENT-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline.

(A) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is a correct explanation for STATEMENT-1

(B) STATEMENT-1 is True, STATEMENT-2 is True; STATEMENT-2 is NOT a correct explanation for STATEMENT-1

(C) STATEMENT-1 is True, STATEMENT-2 is False

(D) STATEMENT-1 is False, STATEMENT-2 is True.

11. If the resultant of all the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[JEE 2009, 4/160, –1]**

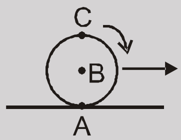
(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not changes in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

12*. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then, **[JEE 2009, 4/160, –1]**



(A) $\vec{V}_C - \vec{V}_A = 2(\vec{V}_B - \vec{V}_C)$

(B) $\vec{V}_C - \vec{V}_B = \vec{V}_B - \vec{V}_A$

(C) $|\vec{V}_C - \vec{V}_A| = 2\,|\vec{V}_B - \vec{V}_C|$

(D) $|\vec{V}_C - \vec{V}_A| = 4\,|\vec{V}_B|$

13. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0º. Then **[JEE 2009, 3/160, –1]**

(A) at θ = 30º, the block will start sliding down the plane

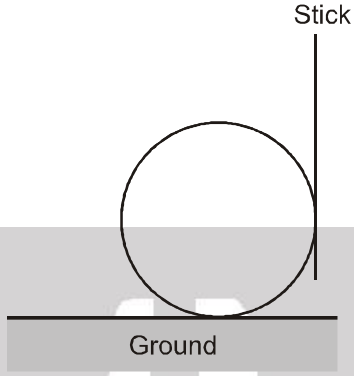
(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60º, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60º, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

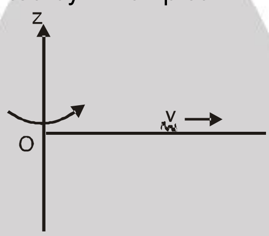
14. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a vertical stick as shown in the figure. The stick applies a normal force of 2 N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is

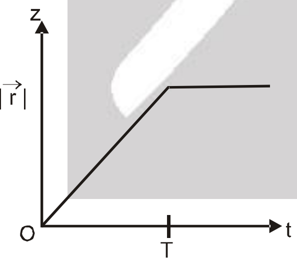
[JEE 2011, 4/160]

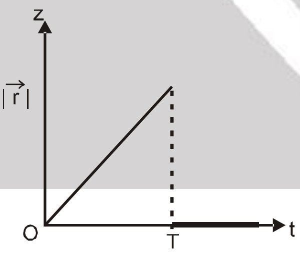


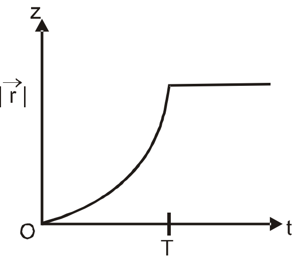
15. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4cm. The moment of inertia of the system about the diagonal of the square is N × 10^{-4} kg-m^2, then N is **[JEE 2011, 4/160]**

16. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time, t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about O, as a function of time is best represented by which plot ? **[IIT-JEE-2012, Paper-1; 3/70, –1]**



(A)


(B)


(C)


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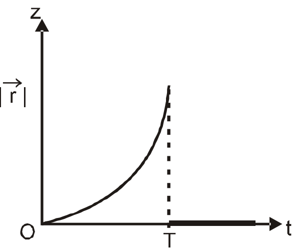
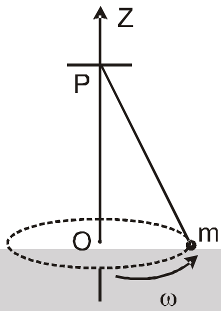


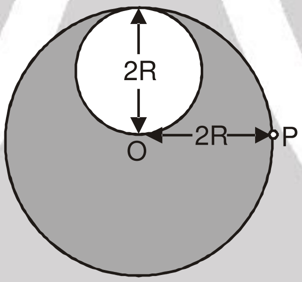
17. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion is the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted $\vec{L}_O$ by $\vec{L}_P$ and respectively, then **[IIT-JEE-2012, Paper-1; 3/70, –1]**



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time. (B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant.

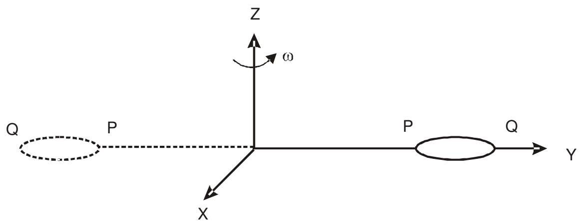
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time. (D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time.

18. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is : **[IIT-JEE-2012, Paper-1; 4/70]**

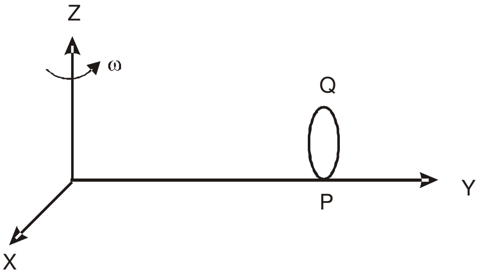


Paragraph for Q. No. 19-20

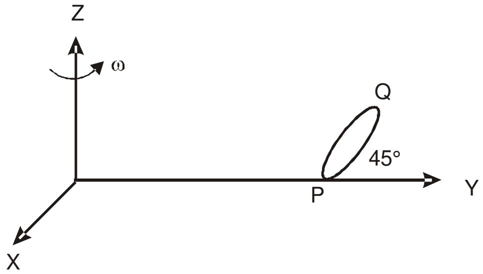
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantanneous axis passing through center of mass. These axes need not be stationary. Consider, for example, a thin uniform welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. Where disc-stick system is rotated about the origin ona horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both the motions have the same angular speed ω in the case.



Now consider two similar systems as shown in the figure: case (a) the disc with its face vertical and parallel to x-z plane; Case (b) the disc with its face making an angle of 45° with x-y plane its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and systems are rotated with constant angular speed ω about the z-axis.



Case (a)



Case (b)

19. Which of the following statement regarding the angular speed about the istantaneous axis (passing through the centre of mass) is correct? **[IIT-JEE-2012, Paper-2; 3/66, –1]**

(A) It is $\sqrt{2}\omega$ for both the cases
(B) it is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).
(C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b)
(D) It is ω for both the cases

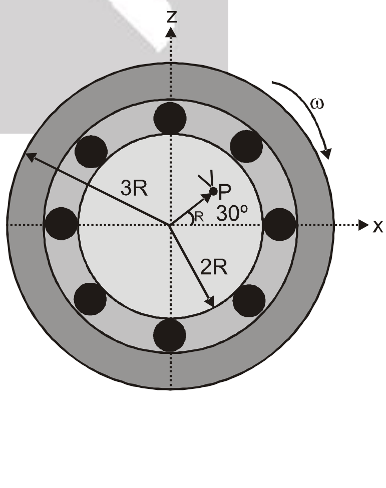
20. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct? **[IIT-JEE-2012, Paper-2; 3/66, –1]**

(A) It is vertical for both the cases (a) and (b).
(B) It is verticle for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b)
(C) It is horizontal ofr case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).
(D) It is vertical of case (a); and is at 45° to the x - z plane and is normal to the plane of the disc for case (b).

21. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane form the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement (s) is (are) correct?

(A) Both cylinders P and Q reach the ground at the same time **[IIT-JEE-2012, Paper-2; 4/66]**
(B) Cylinder P has larger linear acceleration than cylinder Q.
(C) Both cylinder reaches the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

22.* The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed $\omega/2$. The ring and disc are separated frictionaless ball bearings. The system is in the x-z plane. The point P on the inner disc is at distance R from the origin O, where OP makes an angle of 30º with the horizontal. Then with respect to the horizontal surface, **[IIT-JEE-2012, Paper-2; 4/66]**



(A) the point O has linear velocity $3R\omega\hat{i}$.

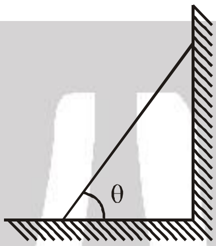
(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has linear velocity $\frac{13}{4}Rw\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) The point P has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)Rw\hat{i} + \frac{1}{4}Rw\hat{k}$.

23. A uniform circular disc of mass 50kg and radius 0.4 m is rotating with an angular velocity of 10 rad/s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is : **[JEE (Advanced) 2013; 4/60]**

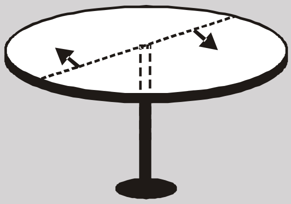
24.* In the figure, a ladder of mass m is shown leaning against a wall. It is in static equilibrium making an angle θ with the horizontal floor. The coefficient of friction between the wall and the ladder is μ_1 and that between the floor and the ladder is μ_2. The normal reaction of the wall on the ladder is N_1 and that of the floor is N_2. If the ladder is about to slip, then **[JEE (Advanced) 2014, P-1, 3/60]**



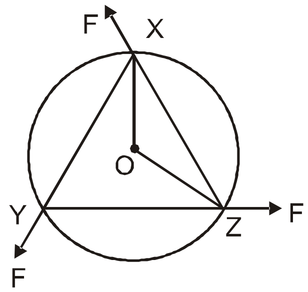
(A) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_2 \tan\theta = \frac{mg}{2}$ (B) $\mu_1 \neq 0$ $\mu_2 = 0$ and $N_1 \tan\theta = \frac{mg}{2}$

(C) $\mu_1 \neq 0$ $\mu_2 \neq 0$ and $N_2 = \frac{mg}{1+\mu_1\mu_2}$ (D) $\mu_1 = 0$ $\mu_2 \neq 0$ and $N_1 \tan\theta = \frac{mg}{2}$

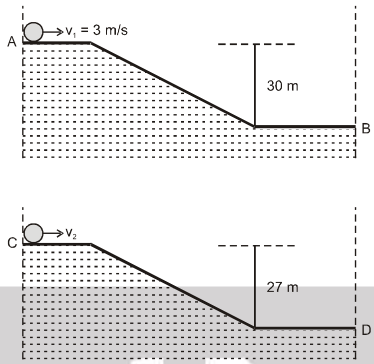
25. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platfrom at a distance 0.25 m from the centre on its either sides along its diameter (see figure).Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platfrom, the balls have horizontal speed of 9ms^{-1} with respect to the ground. The rotational speed of the platfrom in rad^{-1} after the balls leave the platform is **[JEE (Advanced)-2014,P-1, 3/60]**



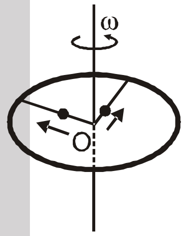
26. A uniform circular disc of mass 1.5 kg and radius 0.5m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is : **[JEE (Advanced)-2014,P-1, 3/60]**



27. Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2, respectively, and always remain in contact with ths surfaces. If they reach B and D with the same linear speed and v_1 = 3 m/s, then v_2 in m/s is (g = 10 m/s^2)
[JEE(Advanced) 2015 ; P-1, 4/88]



28. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass M/8 at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $8/9\omega$ and one of the masses is ata distance of $\frac{3}{5}R$ from O. At this instant the distance of the other mass from O is :



[JEE(Advanced) 2015 ; P-1, 4/88, –2]

(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

29. The dinsities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a contant. The moments of inertia of the individual spheres about axes passing through their centres are I_A and I_B, respectively, If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is : **[JEE(Advanced) 2015 ; P-2,4/88]**

30. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height *h* (< ℓ) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force *f* at the bottom of the stick are (g = 10 ms^{-2}) **[JEE (Advanced) 2016 ; P-1, 3/62, –1]**

(A) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$ (B) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3}N$ (C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3}N$ (D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3}N$

31.* The position vector $\vec{r}$ of particle of mass m is given by the following equation $\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$
Where α = 10/3 m s^{-3}, β = 5 m s^{-2} and m = 0.1 kg. At t = 1 s, which of the following statement(s) is (are) true about the particle. **[JEE (Advanced) 2016 ; P-1, 4/62, –2]**

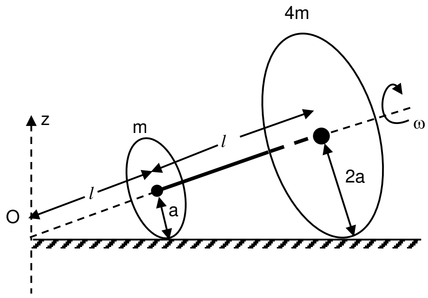
(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ N ms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$ N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -\frac{20}{3}\hat{k}$ Nm.

32.* Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, rigid rod of length $l = \sqrt{24}a$ through their centers. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement (s) is (are) true ?



(A) The magnitude of the z-component of $\vec{L}$ is 55 $ma^2\omega$. **[JEE (Advanced) 2016 ; P-2, 4/62, –2]**

(B) The magnitude of angular momentum of the assembly about its centre of mass is 17 $ma^2 \dfrac{\omega}{2}$.

(C) The magnitude of angular momentum of centre of mass of the assembly about the point O is $81ma^2\omega$.

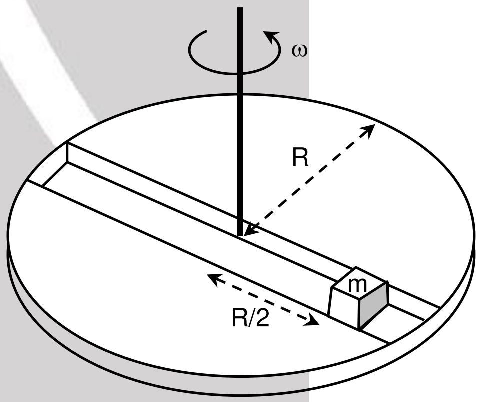
(D) The centre of mass of the assembly rotates about the z-axis with an angular speed of $\omega/5$.

Paragraph for Question Nos. 33 to 34

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is, $\vec{F}_{rot} = \vec{F}_{in} + 2m(\vec{v}_{rot} \times \vec{\omega}) + m(\vec{\omega} \times \vec{r}) \times \vec{\omega}$,

Where $\vec{v}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the center of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at t = 0 and is constrained to move only along the slot.



33. The distance r of the block at time t is **[JEE (Advanced) 2016 ; P-2, 3/62, –1]**

(A) $\dfrac{R}{2}\cos 2\omega t$ (B) $\dfrac{R}{4}\left(e^{2\omega t} + e^{-2\omega t}\right)$ (C) $\dfrac{R}{2}\cos\omega t$ (D) $\dfrac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$

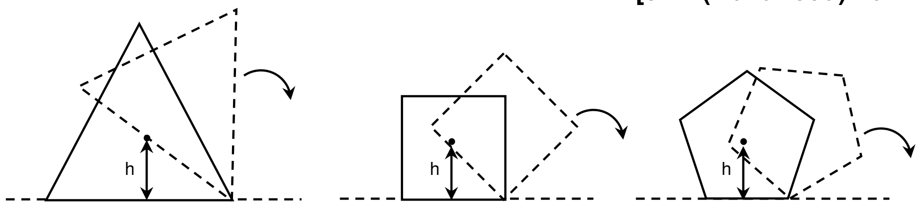
34. The net reaction of the disc on the block is : **[JEE (Advanced) 2016 ; P-2, 3/62, –1]**

(A) $m\omega^2 R\sin\omega t\,\hat{j} - mg\hat{k}$ (B) $-m\omega^2 R\cos\omega t\,\hat{j} - mg\hat{k}$

(C) $\dfrac{1}{2}m\omega^2 R\left(e^{\omega t} - e^{-\omega t}\right)\hat{j} + mg\hat{k}$ (D) $\dfrac{1}{2}m\omega^2 R\left(e^{2\omega t} - e^{-2\omega t}\right)\hat{j} + mg\hat{k}$

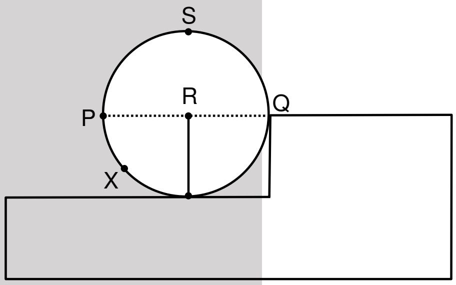
35. Consider regular polygons with number of sides n = 3, 4, 5 .... as shown in the figure. The centre of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as

[JEE (Advanced) 2017 ; P-2, 3/61, –1]



(A) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$ (B) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$ (C) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$ (D) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)} - 1\right)$

36. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ? **[JEE (Advanced) 2017 ; P-2, 4/61, –2]**



(A) If the force is applied normal to the circumference at point P then τ is zero

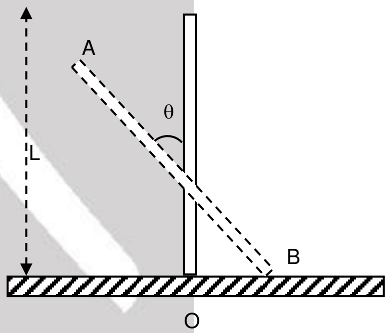
(B) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(C) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

(D) If the force is applied normal to the circumference at point X then τ is constant

37.* A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct ?

[JEE (Advanced) 2017 ; P-2, 4/61, –2]



(A) The trajectory of the point A is a parabola

(B) Instantaneous torque about the point in contact with the floor is proportional to sin θ.

(C) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to (1– cosθ)

(D) The midpoint of the bar will fall vertically downward.

Paragraph for Question Nos. 38 to 39

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure-1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring *rolls without slipping* on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure-2).

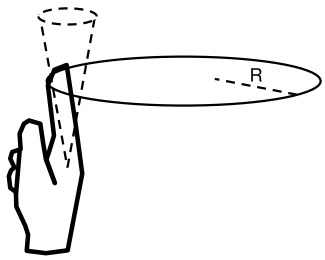


Figure-1

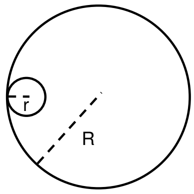


Figure-2

The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

38. The total kinetic energy of the ring is : **[JEE (Advanced) 2017 ; P-2, 3/61]**

(A) $\frac{1}{2}M\omega_0^2(R-r)^2$ (B) $\frac{3}{2}M\omega_0^2(R-r)^2$ (C) $M\omega_0^2R^2$ (D) $M\omega_0^2(R-r)^2$

39. The minimum value of ω_0 below which the ring will drop down is : **[JEE (Advanced) 2017 ; P-2, 3/61]**

(A) $\sqrt{\frac{g}{\mu(R-r)}}$ (B) $\sqrt{\frac{g}{2\mu(R-r)}}$ (C) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{\mu(R-r)}}$

40*. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where α = 1.0 Ns^{-1} and β = 1.0 N. The torque acting on the body about the origin at time t = 1.0s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE (Advanced) 2018 ; P-1, 4/60]**

(A) $|\tau| = \frac{1}{3}$Nm

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})$ms^{-1}

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}$m

41. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}$ s, then the height of the top of the inclined plane, in metres, is __________. Take g = 10 ms^{-2}. **[JEE (Advanced) 2018 ; P-1, 3/60]**

42. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq \beta$. In each case, the force acting on the particle is either zero or conservative. In List-II, five physical quantities of the particle are mentioned: $\vec{p}$ is the linear momentum, $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List-I with those quantities in List-II, which are **conserved for that path**. **[JEE (Advanced) 2018 ; P-2, 3/60, –1]**

List-I		**List-II**	
P.	$\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\hat{i} + \beta\sin\omega t\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\hat{i} + \sin\omega t\hat{j})$	3.	K
S.	$\vec{r}(t) = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$	4.	U
		5.	E

(A) P → 1, 2, 3, 4, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 5
(B) P → 1, 2, 3, 4, 5; Q → 3, 5; R → 2, 3, 4, 5; S → 2, 5
(C) P → 2, 3, 4; Q → 5; R → 1, 2, 4; S → 2, 5
(D) P → 1, 2, 3, 5; Q → 2, 5; R → 2, 3, 4, 5; S → 2, 5

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

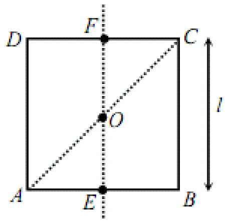
1. A thin circular ring of mass m and radius R is rotating about its axis with a constant angular velocity ω. Two objects each of mass M are attached gently to the opposite ends of a diameter of the ring. The ring now rotates with an angular velocity ω' : **[AIEEE-2006, 4½/180]**

(1) $\frac{\omega m}{(m+2M)}$ (2) $\frac{\omega(m+2M)}{m}$ (3) $\frac{\omega(m-2M)}{(m+2M)}$ (4) $\frac{\omega m}{(m+M)}$

2. Four point masses, each of value m, are placed at the corners of a square ABCD of side ℓ. The moment of inertia about an axis passing through A and parallel to BD is : **[AIEEE-2006, 4½/180]**

(1) $m\ell^2$ (2) $2m\ell^2$ (3) $\sqrt{3}\, m\ell^2$ (4) $3m\ell^2$

3. For the given uniform square lamina ABCD, whose centre is O, **[AIEEE 2007, 3/120]**



(1) $\sqrt{2} I_{AC} = I_{EF}$ (2) $I_{AD} = 2I_{EF}$ (3) $I_{AC} = I_{EF}$ (4) $I_{AC} = \sqrt{2} I_{EF}$

4. A round uniform body of radius R, mass M and moment of inertia I rolls down (without slipping) an inclined plane making an angle θ ? With the horizontal. Then its acceleration is : **[AIEEE 3/120 2007]**

(1) $\frac{g\sin\theta}{1+I/MR^2}$ (2) $\frac{g\sin\theta}{1+MR^2/I}$ (3) $\frac{g\sin\theta}{1-I/MR^2}$ (4) $\frac{g\sin\theta}{1-MR^2/I}$

5. Angular momentum of the particle rotating with a central force is constant due to : **[AIEEE 3/120 2007]**

(1) constant force (2) constant linear momentum
(3) zero torque (4) constant torque

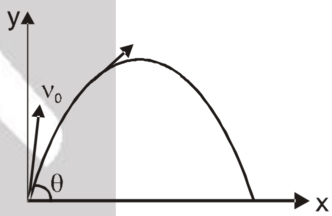
6. Consider a uniform square plate of side 'a' and mass 'm'. The moment of inertia of this plate about an axis perpendicular to its plane and passing through one of its corners is : **[AIEEE 3/105 2008]**

(1) $\frac{1}{12}ma^2$ (2) $\frac{7}{12}ma^2$ (3) $\frac{2}{3}ma^2$ (4) $\frac{5}{6}ma^2$

7. A thin uniform rod of length ℓ and mass m is swinging freely about a horizontal axis passing through its end. Its maximum angular speed is ω. Its centre of mass rises to a maximum height of : **[AIEEE 4/144 2009]**

(1) $\frac{1}{6}\frac{\ell\omega}{g}$ (2) $\frac{1}{2}\frac{\ell^2\omega^2}{g}$ (3) $\frac{1}{6}\frac{\ell^2\omega^2}{g}$ (4) $\frac{1}{3}\frac{\ell^2\omega^2}{g}$

8. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is **[AIEEE 4/144 2010]**



(1) $-mg\, v_0 t^2 \cos\theta\, \hat{j}$ (2) $mg\, v_0 t \cos\theta\, \hat{k}$

(3) $-\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{k}$ (4) $\frac{1}{2} mg\, v_0 t^2 \cos\theta\, \hat{i}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

9. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, the angular speed of the disc : **[AIEEE - 2011, 4/120, –1]**

(1) remains unchanged (2) continuously decreases
(3) continuously increases (4) first increases and then decreases

10. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is : **[AIEEE - 2011, 4/120, –1]**

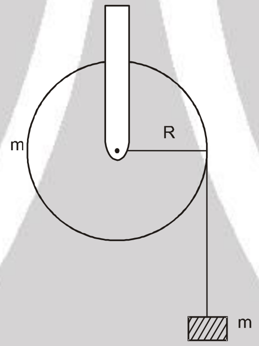
(1) $\frac{3}{2}g$ (2) g (3) $\frac{2}{3}g$ (4) $\frac{g}{3}$

11. A pulley of radius 2m is rotated about its axis by a force F = (20t – 5t^2) newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed, is :**[AIEEE - 2011, 4/120, –1]**
(1) less than 3 (2) more than 3 but less than 6
(3) more than 6 but less than 9 (4) more than 9

12. A particle of mass 'm' is projected with a velocity υ making an angle of 30º with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is : **[AIEEE 2011, 11 May; 4/120, –1]**
(1) zero (2) $\frac{m\upsilon^3}{\sqrt{2}g}$ (3) $\frac{\sqrt{3}}{16}\frac{m\upsilon^3}{g}$ (4) $\frac{\sqrt{3}}{2}\frac{m\upsilon^2}{g}$

13. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip ? **[AIEEE 2013, 4/120, –1]**
(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

14. A mass 'm' supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release ? **[JEE (Main) 2014, 4/120, –1]**

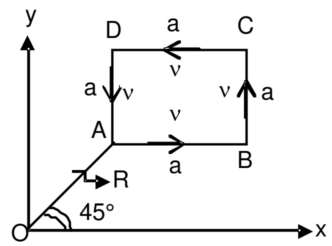


(1) $\frac{2g}{3}$ (2) $\frac{g}{2}$ (3) $\frac{5g}{6}$ (4) g

15. A bob of mass m attached to an inextensible string of length l is suspended from a vertical support. The bob rotates in a horizontal circle with a angular speed ω rad/s about the vertical. About the point of suspension :
(1) angular momentum is conserved. **[JEE (Main) 2014, 4/120, –1]**
(2) angular momentum changes in magnitude but not in direction
(3) angular momentum changes in direction but not in magnitude.
(4) angular momentum changes both in direction and magnitude.

16. Form a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is **[JEE (Main)-2015; 4/120, –1]**
(1) $\frac{MR^2}{32\sqrt{2}\pi}$ (2) $\frac{MR^2}{16\sqrt{2}\pi}$ (3) $\frac{4MR^2}{9\sqrt{3}\pi}$ (4) $\frac{4MR^2}{3\sqrt{3}\pi}$

17. A particle of mass m is moving along the side of square of side 'a' with a uniform speed ν in the x-y plane as shown in the figure :
Which of the following statements is false for the angular momentum about the origin ?

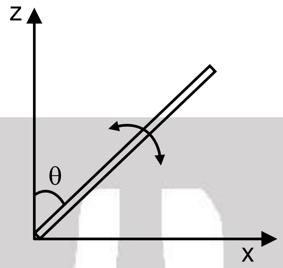


(1) $\vec{L} = m\nu\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D.
(2) $\vec{L} = m\nu\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C.
(3) $\vec{L} = \frac{m\nu}{\sqrt{2}}R\ \hat{k}$ when the particle is moving from D to A.
(4) $\vec{L} = -\frac{m\nu}{\sqrt{2}}R\ \hat{k}$ when the particle is moving from A to B.

18. The moment of inertia of a uniform cylinder of length l and radius R about its perpendicular bisector is I. What is the ratio l/R such that the moment of inertia is minimum ? **[JEE (Main) 2017 ; 4/120, –1]**

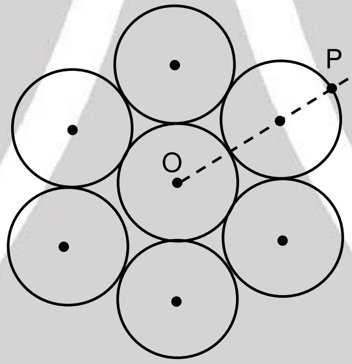
(1) $\frac{3}{\sqrt{2}}$ (2) $\sqrt{\frac{3}{2}}$ (3) $\frac{\sqrt{3}}{2}$ (4) 1

19. A slender uniform rod of mass M and length l is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is. **[JEE (Main) 2017; 4/120, –1]**



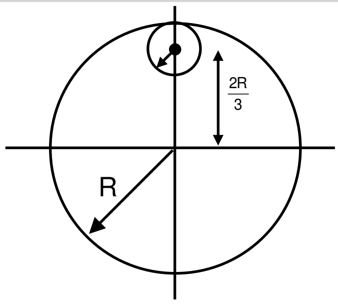
(1) $\frac{2g}{3l}\cos\theta$ (2) $\frac{3g}{2l}\sin\theta$ (3) $\frac{2g}{3l}\sin\theta$ (4) $\frac{3g}{2l}\cos\theta$

20. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE (Main) 2018; 4/120, –1]**



(1) $\frac{73}{2}MR^2$ (2) $\frac{181}{2}MR^2$ (3) $\frac{19}{2}MR^2$ (4) $\frac{55}{2}MR^2$

21. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE (Main) 2018; 4/120, –1]**



(1) 10 MR^2 (2) $\frac{37}{9}MR^2$ (3) 4 MR^2 (4) $\frac{40}{9}MR^2$

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. 4 rev/s^2, 20 rev/s **A-2.** 20 s

Section (B) :

B-1. $\frac{ML^2}{12}$ **B-2.** $\frac{M\ell^2}{6}$

B-3. $\frac{MR^2}{2} - M\left(\frac{4R}{3\pi}\right)^2$ **B-4.** (K = $\sqrt{\frac{3}{2}}$ r)

Section (C) :

C-1. $-14\hat{i} + 10\hat{j} - 9\hat{k}$

C-2. mgℓ sinθ, when the bob is at the lowest point, at θ = 90º.

C-3. (a) mv^2 sinα cosα perpendicular to the plane of motion
(b) 2mv^2 sinα cosα perpendicular to the plane of motion

C-4. 3N – m

Section (D) :

D-1. 2.4 N in the left string and 2.6 N in the right

D-2. P = $\frac{w}{2}$ cot θ or P = $\frac{mg}{2}$ cot θ

D-3. 990 N, 960 N, $\frac{32}{33}$

D-4. (i) T = 250 N (ii) F_H= 150 N ($\rightarrow$), F_V = 50 N ($\uparrow$)

Section (E) :

E-1. (a) $\frac{2g}{\ell}\frac{(m_1 - m_2)}{(m_1 + m_2)} = \frac{10}{3}$ rad/s^2

(b) (i) $\alpha' = \frac{2(m_1 - m_2)g}{\ell\left[m_1 + m_2 + \frac{m_3}{3}\right]}$ = 3 rad/s^2 ,

(ii) 42 N ; 39 N

E-2. (a) $\frac{3g}{4L}$ (cw)

(b) N = $\frac{13\ mg}{16}\uparrow$, F = $\left(\frac{3\sqrt{3}}{16}\right)$mg $\rightarrow$

E-3. 3 rad/s^2

E-4. N = F $\left(1 - \frac{3}{2}\frac{x}{\ell}\right)$

Section (F) :

F-1. $\omega = \sqrt{5}$ rad/s **F-2.** $\omega = \sqrt{\frac{9\ g}{4\ \ell}}$

F-3. $\frac{1}{2}m\omega^2\left(R^2 + \frac{L^2}{12}\right)$ **F-4.** 2 m/s

Section (G) :

G-1. $2\hat{k}$ kg m^2/s **G-2.** 16 kg m^2/s

G-3. $\frac{\left(I + \frac{mr^2}{2}\right)\omega_0}{I + 2mr^2}$ **G-4.** 10 rad/s

G-5. $\frac{4\ \pi\ m}{M + 2m}$ **G-6.** 3v / 4ℓ

Section (H) :

H-1 V_A = 25 m/s , V_B = 75 m/s

H-2 V_O = 4 m/sec $\hat{i}$, V_A = (4 $\hat{i}$ + 3 $\hat{j}$)m/sec

H-3 V_{CM} = 7 m/s.

H-4 (a) $\frac{4v_0}{3}$ (b) $\frac{5v_0}{3\ell}$

(c) $v_x = \frac{v_0}{2}$, $v_y = -\frac{2v_0}{3}$

H-5. (a) v_A = 2at = 10.0 cm/s,
$v_B = \sqrt{2}$ at = 7.1 cm/s, v_0 = 0 ;

(b) $a_A = 2a\sqrt{1 + \left(\frac{2t^2a}{R}\right)^2}$ = 5.6 cm/s^2,

a_B = a = 2.5 cm/s^2, a_0 = a^{2}t^2 / R = 2.5 cm/s^2

Section (I) :

I-1. $\sqrt{\frac{4gh}{3}}$ **I-2.** $\frac{7}{10}$mv^2

I-3. $\frac{2}{3}$g **I-4.** $\frac{7}{3}$mg

I-5. $\sqrt{3g(R - r)}$ **I-6.** 4πR/5

I-7. 5 μmg, 20 N

Section (J) :

J-1. ω = 3 v/2ℓ

J-2. (a) $\frac{m_2u}{m_1 + m_2}$ (b) $\frac{m_1u}{m_1 + m_2}$

(c) $-\frac{m_2u}{m_1 + m_2}$

(d) $\frac{m_1^2m_2uL}{2(m_1 + m_2)^2}$, $\frac{m_1m_2^2uL}{2\ (m_1 + m_2)^2}$

(e) $\frac{m_1(m_1 + 4m_2)L^2}{12\ (m_1 + m_2)}$

(f) $\frac{m_2u}{m_1 + m_2}$, $\frac{6m_2u}{(m_1 + 4m_2)L}$

J-3. (a) $\frac{I}{m}$, $\frac{2I}{mR}$ (b) $\frac{\pi mR}{2I}$ (c) $\frac{\pi R}{2}$

J-4. 100 rad/sec.

Section (K) :

K-1. $\frac{1}{2}$mg a sinθ , x = $\frac{a\tan\theta}{2}$

PART - II

Section (A) :

A-1.	(B)	A-2.	(C)

Section (B) :

B-1.	(C)	B-2.	(A)	B-3.	(A)
B-4.	(C)	B-5.	(D)	B-6.	(C)
B-7.	(C)	B-8.	(B)	B-9.	(D)
B-10.	(D)	B-11.	(D)		

Section (C) :

C-1.	(A)	C-2.	(C)	C-3.	(C)

Section (D) :

D-1.	(B)	D-2.	(C)	D-3	(B)
D-4	(A)	D-5.	(C)		

Section (E) :

E-1.	(A)	E-2.	(C)	E-3.	(D)
E-4.	(A)	E-5.	(C)		

Section (F) :

F-1.	(B)	F-2.	(C)

Section (G) :

G-1.	(C)	G-2.	(B)	G-3.	(D)
G-4.	(B)	G-5.	(C)		

Section (H) :

H-1.	(B)	H-2.	(B)	H-3.	(C)
H-4.	(A)	H-5.	(A)		

Section (I) :

I-1.	(B)	I-2.	(A)	I-3.	(B)
I-4.	(D)	I-5.	(A)	I-6.	(D)
I-7.	(A)	I-8.	(C)	I-9.	(D)

Section (J) :

J-1.	(D)	J-2.	(B)

Section (K) :

K-1.	(A)	K-2.	(A)	K-3.	(A)

PART - III

1. (A) $\rightarrow$p,q,r ; (B) $\rightarrow$ p,q,r ; (C)$\rightarrow$p,q;(D) $\rightarrow$p,q,r
2. (A) $\rightarrow$ p ; (B) $\rightarrow$ q, s;(C) $\rightarrow$p ;(D) $\rightarrow$q, s

EXERCISE-2

PART - I

1.	(D)	2.	(D)	3.	(D)
4.	(C)	5.	(C)	6.	(B)
7.	(D)	8.	(B)	9.	(B)
10.	(D)	11.	(D)	12.	(B)
13.	(C)	14.	(A)	15.	(D)
16.	(A)	17.	(A)	18.	(D)
19.	(B)	20.	(B)	21.	(B)
22.	(D)	23.	(B)	24.	(A)

PART - II

1.	16	2.	28	3.	2
4.	6	5.	2	6.	4
7.	4	8.	10	9.	1
10.	2	11.	8	12.	9
13.	5	14.	1	15.	2
16.	10	17.	(a) 10	(b) 20	
18.	25	19.	12	20.	20
21.	9	22.	5		

PART - III

1.	(CD)	2.	(ABC)	3.	(ABC)
4.	(ABCD)	5.	(ACD)	6.	(BC)
7.	(ABC)	8.	(ACD)	9.	(CD)
10.	(ABCD)	11.	(BC)	12.	(ABD)
13.	(ABC)	14.	(BC)	15.	(ACD)
16.	(AD)	17.	(ACD)		

PART - IV

1.	(B)	2.	(C)	3.	(D)
4.	(D)	5.	(C)	6.	(A)

EXERCISE-3

PART - I

1.	(A)	2.	(CD)	3.	(AB)
4.	10	5.	(C)	6.	(A)
7.	(B)	8.	(D)	9.	(D)
10.	(D)	11.	(A)	12.	(BC)
13.	(B)	14.	4	15.	9
16.	(B)	17.	(C)	18.	3
19.	(D)	20.	(A)	21.	(D)
22.	(AB)	23.	8	24.	(CD)
25.	4	26.	2	27.	7
28.	(D)	29.	6	30.	(D)
30.	(D)	31.	(ABD)	32.	(D)
33.	(D)	34.	(C)	35.	(D)
36.	(A)	37.	(BCD)	38.	(Bonus)
39.	(A)	40.	(AC)	41.	0.75
42.	(A)				

PART - II

1.	(1)	2.	(4)	3.	(3)
4.	(1)	5.	(3)	6.	(3)
7.	(3)	8.	(3)	9.	(4)
10.	(3)	11.	(2)	12.	(3)
13.	(3)	14.	(2)	15.	(3)
16.	(3)	17.	(1,3)	18.	(2)
19.	(2)	20.	(2)	21.	(3)

High Level Problems (HLP)

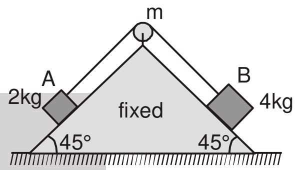
SUBJECTIVE QUESTIONS

1. Find the M.I. of a rod about (i) an axis perpendicular to the rod and passing through left end. (ii) An axis through its centre of mass and perpendicular to the length whose linear density varies as $\lambda = ax$ where a is a positive constant and 'x' is the position of an element of the rod relative to its left end. The length of the rod is ℓ.

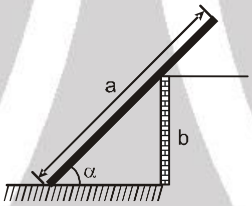
2. The pulley (uniform disc) shown in figure has, radius 10 cm and moment of inertia about its axis $I = 0.5$ kgm^2 (B and A both move)

(a) Assuming all the plane surfaces are smooth and there is no slipping between pulleyand string, calculate the acceleration of the mass 4kg.

(b) The friction coefficient between the block A and the plane below is $\mu = 0.5$ and the plane below the B block is frictionless.Assuming no slipping between pulley and string find accelaration of 4kg block



3. A uniform rod of length 'a' rests against a frictionless wall as shown in figure. Find the friction coefficient between the horizontal surface and the lower end if the minimum angle that the rod can make with the horizontal is α., without slipping.

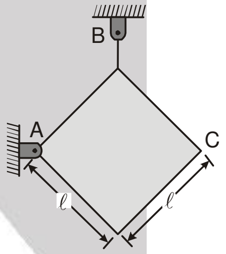


4. A uniform square plate of mass m is supported as shown. If the cable suddenly breaks, determine just after that moment;

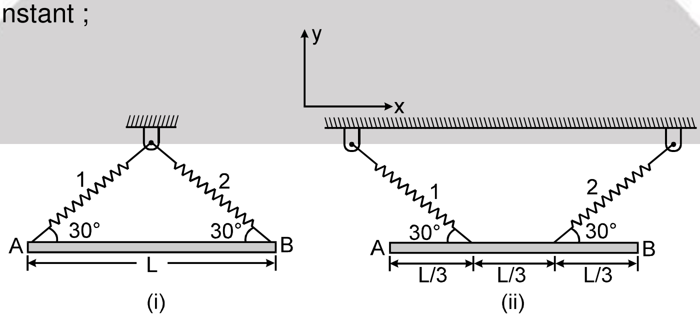
(a) The angular acceleration of the plate.

(b) The acceleration of corner C.

(c) The reaction at A.



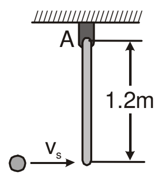
5. A uniform slender rod AB of mass m is suspended from two springs as shown. If spring 2 breaks, determine at that instant ;



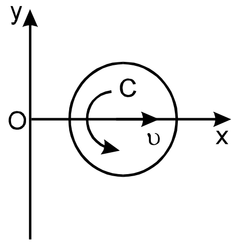
(a) The angular acceleration of the bar. (b) The acceleration of point A.

(c) The acceleration of point B.

6. A 2 kg sphere moving horizontally to the right with an initial velocity of 5 m/s strikes the lower end of an 8 kg rigid rod AB. The rod is suspended from a hinge at A and is initially at rest. Knowing that the coefficient of restitution between the rod and sphere is 0.80, determine the angular velocity of the rod and the velocity of the sphere immediately after the impact.



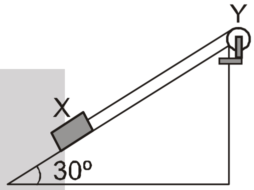
7. A rotating disc (figure) moves in the positive direction of the x-axis. Find the equation y (x) describing the position of the instantaneous axis of rotation, if at the initial moment the axis C of the disc was located at the point O after which it moved



(a) With a constant velocity v, while the disc started rotating counter clockwise with a constant angular acceleration β (the initial angular velocity is equal to zero);

(b) With a constant acceleration a (and the zero initial velocity), while the disc rotates counterclockwise with a constant angular velocity ω.

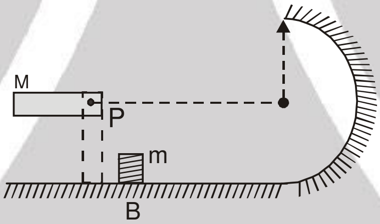
8. A block X of mass 0.5 kg is held by a long massless string on a fixed frictionless inclined plane inclined at 30º to the horizontal. The string is wound on a uniform solid cylindrical drum Y of mass 2 kg and radius 0.2 m as shown in figure. The drum is given an initial angular velocity such that block X starts moving up the plane. **[JEE - 1994]**



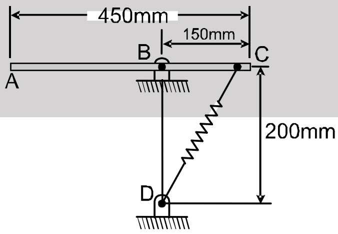
(a) Find the tension in the string during motion.

(b) At a certain instant of time the magnitude of the angular velocity of Y is 10 rad s^{-1}. Calculate the distance travelled by X from that instant of time until it comes to rest.

9. A rod of length R and mass M is free to rotate about a horizontal axis passing through hinge P as in figure. First it is taken aside such that it becomes horizontal and then released. At the lowest point the rod hits the small block B of mass m and stops. Find the ratio of masses such that the block B completes the circular track of radius R. Neglect any friction.



10. A 3 kg uniform rod rotates in a vertical plane about a smooth pivot at B. A spring of constant k = 300 N/m and of unstretched length 100 mm is attached to the rod as shown. Knowing that in the position shown the rod has an angular velocity of 4 rad/s clockwise, determine the angular velocity of the rod after it has rotated through. [g = 10 m/s^2]



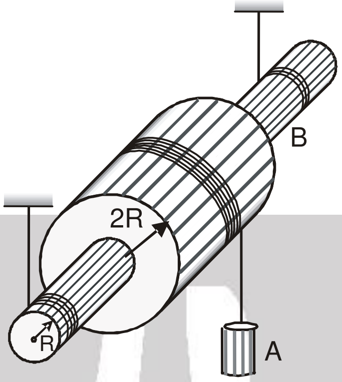
(D is vertically below B)

(a) 90º (b) 180º

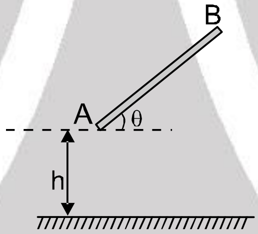
11. The angular momentum of a particle relative to a certain ' point O varies with time as $\vec{M} = \vec{a} + \vec{b}t^2$, where $\vec{a}$ and $\vec{b}$ are constant vectors, with $\vec{a} \perp \vec{b}$. Find the force moment N relative to the point O acting on the particle when the angle between the vectors N and M equals 45°.

12. A plank of mass m_1 with a uniform sphere of mass m_2 placed on it rests on a smooth horizontal plane. A constant horizontal force F is applied to the plank. With what accelerations will the plank and the centre of the sphere move provided there is no sliding between the plank and the sphere ?

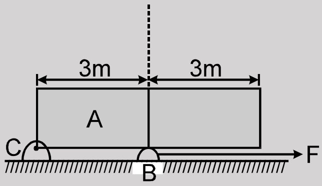
13. In the arrangement shown in the figure weight A possesses mass m, a pulley B possesses mass M. Also known are the moment of inertia I of the pulley relative to its axis and the radii of the pulley are R and 2R respectively. Consider the mass of the threads is negligible. Find the acceleration of weight A after the system is set free. (Assume no slipping takes place anywhere and axis of cylinder remains horizontal)



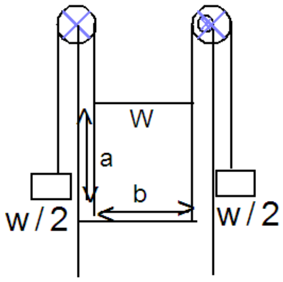
14. A uniform rod AB of length ℓ is released from rest with AB inclined at angle θ with horizontal. It collides elastically with smooth horizontal surface after falling through a height h. What is the height upto which the centre of mass of the rod rebounds after impact?



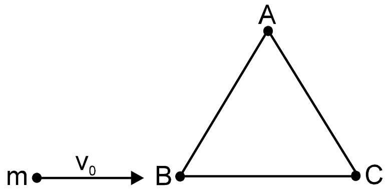
15. A uniform block A of mass 25 kg and length 6m is hinged at C and is supported by a small block B as shown in the Figure. A constant force F of magnitude 400N is applied to block B horizontally. What is the speed of B after it moves 1.5 m ? The mass of block B is 2.5 kg & the coeffcient of friction for all contact surfaces is 0.3. [Use $\ln(3/2)$ 0.41 and g = 10 ms^{-2}]



16. A window (of weight w) is supported by two strings passing over two smooth pulleys in the frame of the window in which window just fits in, the other ends of the string being attached to weights each equal to half the weight of the window. One thread breaks and the window moves down. Find acceleration of the window if μ is the coefficient of friction, and 'a' is the height and 'b' the breadth of the window.



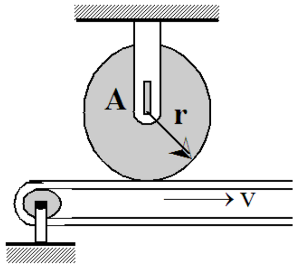
17. Three particles A, B, C of mass m each are joined to each other by massless rigid rods to form an equilateral triangle of side a. Another particle of mass m hits B with a velocity v_0 directed along BC as shown. The colliding particle stops immediately after impact.



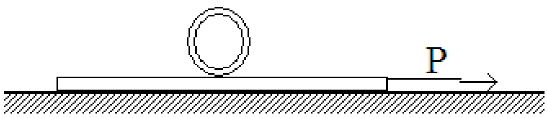
(a) Calculate the time required by the triangle ABC to complete half revolution in its subsequent motion.

(b) What is the net displacement of point B during this interval ?

18. Disk A has a mass of 4 kg and a radius r = 75 mm, it is at rest when it is placed in contact with the belt, which moves at a constant speed υ = 18 m/s. Knowing that μ_k = 0.25 between the disk and the belt, determine the number of revolutions executed by the disk before it reaches a constant angular velocity. (Assume that the normal reaction by the belt on the disc is equal to weight of the disc) .



19. A 160 mm diameter pipe of mass 6 kg rests on a 1.5 kg plate. The pipe and plate are initially at rest when a force P of magnitude 25 N is applied for 0.75 s. Knowing that μ_s = 0.25 & μ_k = 0.20 between the plate and both the pipe and the floor, determine ;

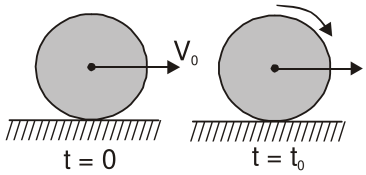


(a) whether the pipe slides with respect to the plate.

(b) the resulting velocities of the pipe and of the plate.

20. A uniform disc of mass m and radius R is rolling up a rough inclined plane, which makes an angle of 30º with the horizontal. If the coefficients of static and kinetic friction are each equal to μ and only the forces acting are gravitational, normal reaction and friction, then the magnitude of the frictional force acting on the disc is ______ and its direction is _____ (write 'up' or 'down') the inclined plane. **[JEE - 1997]**

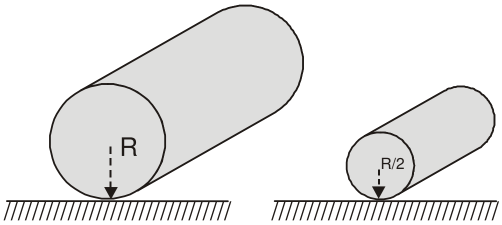
21. A uniform disc of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds, it acquires a purely rolling motion as shown in figure. **[JEE - 1997]**



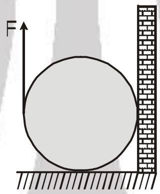
(a) Calculate the velocity of the centre of mass of the disc at t_0 .

(b) Assuming the coefficient of friction to be μ, calculate t_0. Also calculate the work done by the frictional force as a function of time & the total work done by it over a time t much longer than t_0.

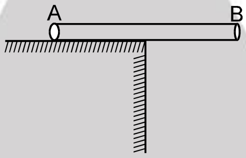
22. A carpet of mass 'M' made of inextensible material is rolled along its length in the form of a cylinder of radius 'R' and is kept on a rough floor. The carpet starts unrolling without sliding on the floor when a negligibly small push is given to it. Calculate the horizontal velocity of the axis of the cylindrical part of the carpet when its radius reduces to R/2. **[JEE - 1990]**



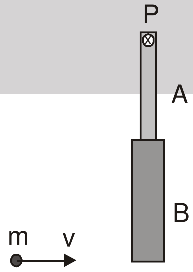
23. Figure shows a vertical force F that is applied tangentially to a uniform cylinder of weight W. The coefficient of static friction between the cylinder and all surfaces is 0.5. Find in terms of W, the maximum force that can be applied without causing the cylinder to rotate.



24. A drinking straw of mass 2m is placed on a smooth table orthogonally to the edge such that half of it extends beyond the table. A fly of mass m lands on the A end of the straw and walks along the straw untill it reaches the B end. It does not tip even when another fly gently lands on the top of the first one. Find the largest mass that the second fly can have. (Neglect the friction between straw and table).



25. Two uniform thin rods A & B of length 0.6 m each and of masses 0.01 kg & 0.02 kg respectively are rigidly joined, end to end. The combination is pivoted at the lighter end P as shown in figure such that it can freely rotate about the point P in a vertical plane. A small object of mass 0.05 kg moving horizontally hits the lower end of the combination and sticks to it. What should be the velocity of the object so that the system could just be raised to the horizontal position ? **[JEE - 1994]**



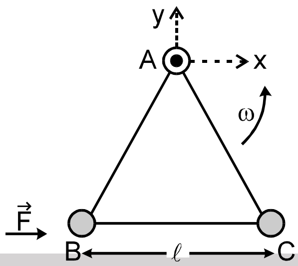
26. A uniform cube of side 'a' and mass m rests on a rough horizontal table. A horizontal force F is applied normal to one of the faces at a point directly above the centre of the face, at a height $\frac{3a}{4}$ above the base. (i) What is the minimum value of F for which the cube begins to tip about an edge?

(ii) What is the minimum value of μ_s so that toppling occures.

(iii) If $\mu = \mu_{min}$, find minimum force for topping.

(iv) Minimum μ_s so that F_{min} (as in part-(i)) can cause toppling.

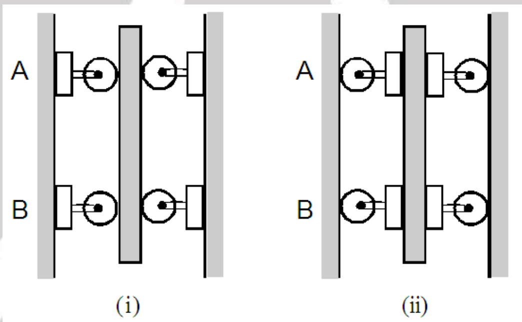
27. Three particles A, B and C each of mass m are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x - y plane) and is hinged to it at the point A so that it can move without friction about the vertical axis through A as shown in figure. The body is set into rotational motion on the table about A with a constant angular velocity ω



(a) Find the magnitude of the horizontal force exerted by the hinge on the body .

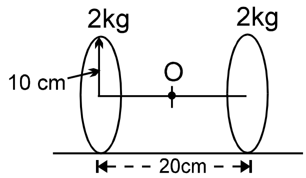
(b) At time T, when the side BC is parallel to the x – axis, a force F is applied on B along BC as shown. Obtain the x – component and the y – component of the force exerted by the hinge on the body, immediately after time T. **[JEE Mains 02, (1+4)/60]**

28. A bar of mass m is held as shown between 4 disks , each of mass M & radius r = 75 mm Determine the acceleration of the bar immediately after it has been released from rest, knowing that the normal forces exerted on the disks are sufficient to prevent any slipping and assuming that ;
In (i) case the discs are attached to the fixed support on wall. In (ii) case the discs are attached to the bar.



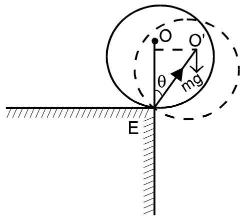
(a) m = 5 kg and M = 2 kg .
(b) the mass of M of the disks is negligible.
(c) the mass of m of the bar is negligible .

29. Two thin circular discs of mass 2 kg and radius 10 cm each are joined by a rigid massless rod of length 20 cm. The axis of the rod is along the perpendicular to the planes of the disk through their centres. This object is kept on a truck in such a way that the axis of the object is horizontal and perpendicular to the direction of motion of the truck. The friction with the floor of the truck is large enough, so that object can roll on the truck without slipping. Take x-axis as the direction of motion of the truck and z-axis as the vertically upward direction. If the truck has an acceleration of 9 m/s^2, calculate **[JEE - 1997' 5/100]**



(a) the force of friction on each disc.
(b) the magnitude & direction of frictional torque acting on each disk about the centre of mass O of the object. Express the torque in the vector form in terms of unit vectors $\hat{i}$, $\hat{j}$ & $\hat{k}$ along x, y & z direction.

30. A rectangular rigid fixed block has a long horizontal edge. A solid homogeneous cylinder of radius r is placed horizontally at rest with its length parallel to the edge such that the axis of the cylinder and the edge of the block are in the same vertical plane. There is sufficient friction present at the edge so that a very small displacement causes the cylinder to roll off the edge without slipping. Determine :

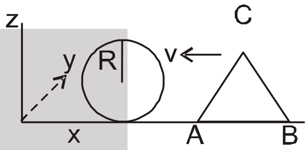


(a) The angle θ_c through which the cylinder rotates before it leaves contact with the edge.

(b) The speed of the centre of mass of the cylinder before leaving contact with the edge.

(c) The ratio of translational to rotational kinetic energies of the cylinder when its centre of mass is in horizontal line with the edge. **[JEE - 1995]**

31. A wedge of mass 'm' and triangular cross section (AB = BC = CA = 2R) is moving with a constant velocity-v $\hat{i}$ towards a sphere of radius R fixed on a smooth horizontal table as shown in the figure. The wedge makes an elastic collision with the fixed sphere and returns along the same path without any rotation. Neglect all friction and suppose that the wedge remains in contact with the sphere for a very short time Δt, during which the sphere exerts a constant force $\vec{F}$ on the wedge. The sphere is always fixed.



(a) Find the force $\vec{F}$ and also the normal force $\vec{N}$ exerted by the table on the wedge during the time Δt.

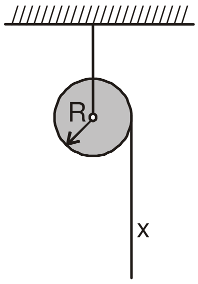
(b) Let 'h' denote the perpendicular distance between the centre of mass of the wedge and the line of action of force $\vec{F}$. Find the magnitude of the torque due to the normal force $\vec{N}$ about the centre of the wedge, during the time Δt. **[JEE - 1998, 8]**

32. The surface mass density (mass/area) of a circular disc of radius 'R' depends on the distance from the centre x given as, $\sigma(x) = \alpha + \beta x$. Where α and β are positive constant find its moment of inertia about the line perpendicular to the plane of the disc through its centre.

33. Calculate the moment of inertia of a uniform solid cone relative to its symmetry axis, if the mass of the cone is equal to m and the radius of its base to R.

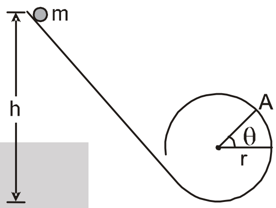
34. A force $\vec{F} = A\hat{i} + B\hat{j}$ is applied to a point whose radius vector relative to the origin of coordinates O is equal to $\vec{r} = a\hat{i} + b\hat{j}$, where a, b &, A, B are constants, and $\hat{i}$, $\hat{j}$ are the unit vectors of the x and y axes. Find the moment N (torque of $\vec{F}$) and the arm ℓ of the force relative to the point O.

35. A uniform cylinder of radius R and mass M can rotate freely about a stationary horizontal axis O Fig. A thin cord of length ℓ and mass m is wound on the cylinder in a single layer. Find the angular acceleration of the cylinder as a function of the length x of the hanging part of the cord. The wound part of the cord is supposed to have its centre of gravity on the cylinder axis.



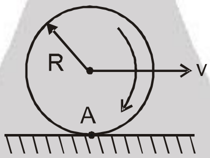
36. A vertically oriented uniform rod of mass M and length ℓ can rotate about a fixed horizontal smooth axis passing through its upper end. A horizontally flying bullet of mass m strikes the lower end of the rod and gets stuck in it; as a result, the rod swings through an angle $\alpha(\alpha < 90^\circ)$. Assuming that m << M, find :
(a) the velocity of the flying bullet ;
(b) the momentum increment in the system "bullet + rod" during the impact; what causes the change of that momentum ;
(c) at what distance x : from the upper end of the rod the bullet must strike for the momentum of the system "bullet-rod" to remain constant during the impact.

37. A small spherical ball of mass m is rolling without slipping down the loop track as shown in the figure. The ball is released from rest on the linear portion at a vertical height h from the lowest point. The circular part as shown in figure has a radius r. [g = 10 ms^2]

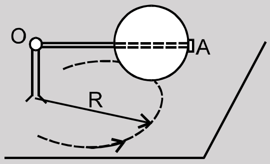


(a) Find the kinetic energy of the ball when it is at a point A where the radius make an angle θ with the horizontal
(b) Find the radial and the tangential accelerations of the centre when the ball isat A.
(c) Find the normal force and the frictional force acting on the ball if h = 50 cm, r = 10 cm, θ = 0 and m = 70 g.

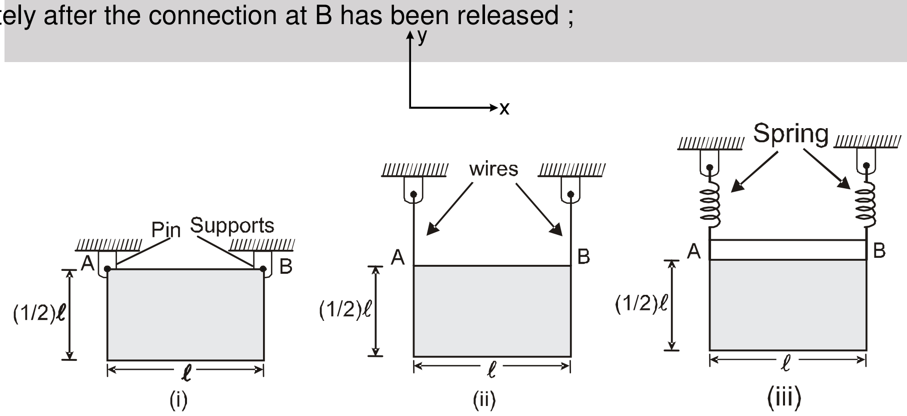
38. A point A is located on the rim of a wheel of radius R = 0.50 m which rolls without slipping along a horizontal surface with velocity v = 1.00 m/s as shown in figure. Find:
(a) the modulus and the direction of the acceleration vector of the point A ;
(b) the total distance s traversed by the point A between the two successive moments at which it touches the surface.



39. A uniform sphere of mass m and radius r rolls without sliding over a horizontal plane, rotating about a horizontal axle OA. In the process, the centre of the sphere moves with velocity along a circle of radius R Find the kinetic energy of the sphere.

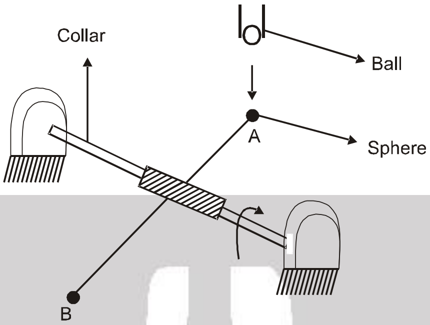


40. A uniform plate of mass 'm' is suspended in each of the ways shown. For each case determine immediately after the connection at B has been released ;



(a) The angular acceleration of the plate.
(b) The acceleration of its center of mass.

41. Figure (1) shows a mechanical system free of any dissipation. The two spheres (A and B) are each of equal mass m, and a uniform connecting rod AB of length 2r has mass 4m. The collar is massless. Right above the position of sphere A in Fig. (1) is a tunnel from which balls each of mass m fall vertically at suitable intervals. The falling balls cause the rods and attached spheres to rotate. Sphere B when reaches the position now occupied by sphere A, suffers a collision from another falling ball and so on. Just before striking, the falling ball has velocity υ. All collision are elastic and the spheres as well as the falling balls can be considered to be point masses. **[Olympiad_2011]**



(a) Find the angular velocity ω_{i+1} of the assembly in terms of {ω_i, υ, and r } after the i[th] ball has struck it.

ω_{i+1}

(b) The rotating assembly eventually assumes constant angular speed ω^*. Obtain ω^* in terms of υ and r by solving the equation obtained in part (a). Argue how a constant ω^* does not violate energy conservation.

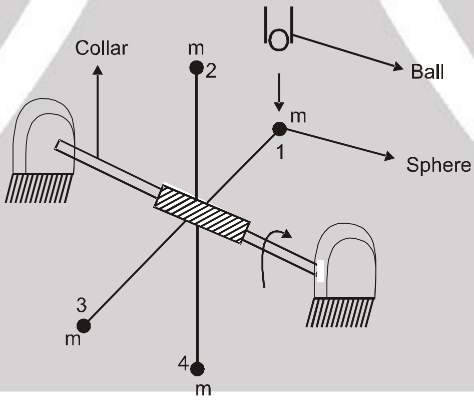
ω^* =

Argument

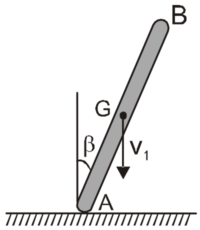
(c) Solve the expression obtained in part (a) to obtain ω_i in terms of {i,υ, and r} .

ω_i =

(d) If instead of a pair of spheres, we have two pairs of spheres as shown in figure below. What would be the new constant angular speed ω^* of the assembly (i.e. the answer corresponding to part (b).

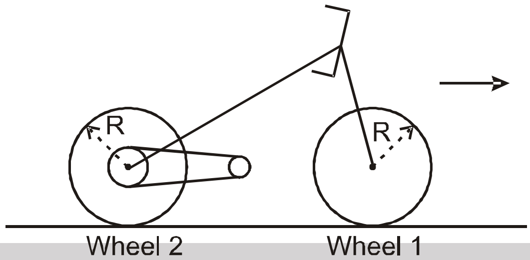


42. A rod of length L forming an angle β with the vertical strikes a frictionless floor at A with a vertical velocity v_1 and no angular velocity. Assuming that the impact at A is perfectly elastic, derive an expression for the angular velocity of the rod immediately after the impact.



43. Consider a bicyle in vertical position accelerating forward without slipping on a straight horizontal road. The combined mass of the bicycle and the rider is M and the magnitude of the accelerating torque applied on the rear wheel by the pedal and gear system is τ. The radius and the moment of inertia of each wheel is R and I (with respect to axis) respectively. The accelaration due to gravity is g. **[INPhO-2013]**

(a) Draw the free diagram of the system (bicycle and rider).



(b) Obtains the accelaration a in terms of the above mentioned quantities.

a =

(c) For simplicity assume that the centre of mass of the system is at height R from the gruond and equidistant at 2 R from the center of each of the wheels. Let μ be the coefficient of friction (both static and dynamic) between the wheels and the ground. Consider M >> I/R^2 and no slipping. Obtain the conditions for the maximum acceleration a_m of the bike.

a_m =

(d) For μ = 1.0 calculate a_m.

a_m =

HLP Answers

1. (i) $I = \frac{a\ell^4}{4}$ (ii) $\frac{a\,\ell^4}{36}$

2. (a) 0.25 m/s^2 (b) 0.125 m/s^2

3. $\frac{a\cos\alpha\sin^2\alpha}{2b - a\cos^2\alpha\sin\alpha}$

4. (a) $\frac{3g}{2\sqrt{2}\ell}$ (cw) (b) $\frac{3}{2}g\downarrow$ (c) $\frac{Mg}{4}\uparrow$

5. (i) (a) 3 g/L (cw) (b) $\left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)g = 1.323\,g \angle 49.1^\circ$ (c) $\left(\frac{\sqrt{3}}{2}\hat{i} - 2\hat{j}\right)g = 2.18\,g \angle -66.6^\circ$ (ii) (a) g/L (cw) (b) $-\left(\frac{\sqrt{3}}{2}\right)g\,\hat{i}$ (c) $\left(\frac{\sqrt{3}}{2}\hat{i} + \hat{j}\right)$–g=1.323g $\angle -130.9^\circ$

6. $\omega = \frac{45}{14} = 3.21$ rad/s (ccw), $v_s = \frac{1}{7} = 0.143$ m/s $\leftarrow$

7. (a) $y = \frac{v^2}{\beta x}$ (Hyperbola) ; (b) $y = \frac{\sqrt{2ax}}{\omega}$ (Parabola)]

8. (a) 1.633N (b) 1.224 m

9. $\frac{M}{m} = \sqrt{15}$

10. (a) $\frac{2}{3}\sqrt{86}$ rad/s (b) 4 rad/s

11. $N = 2b\sqrt{\frac{a}{b}}$

12. $w_1 = F/(m_1 + 2/7m_2)$; $w_2 = 2/7\, w_1$

13. w = 3g (M + 3m) / (M + 9m + I/R^2)

14. $H = \left(\frac{1-3\cos^2\theta}{1+3\cos^2\theta}\right)^2 h$; $h = \frac{49\,\pi\,\ell}{144}$

15. $\sqrt{323.4}$ m/s or 18.52 m/sec.

16. $A = \left(\frac{a - \mu b}{3a + \mu b}\right) g$

17. (a) $t = \frac{6a\pi}{\sqrt{3}\,v_0}$ (b) $s = \frac{a}{\sqrt{3}}\sqrt{1 + \left(2\pi + \sqrt{3}\right)^2}$

18. $\frac{216}{\pi}$

19. (a) pipe rolls without sliding (b) pipe : $\frac{5}{6}$ m/s $\rightarrow$, $\frac{125}{24}$ rad/s (ccw) ; plate : $\frac{5}{3}$ m/s $\rightarrow$

20. $\frac{mg}{6}$, up

21. (a) $v = \frac{2v_0}{3}$; $t_0 = \frac{v_0}{3\mu g}$ (b) $w = -\mu mg\,(v_0 t - \frac{3}{2}\mu g t^2)$; $-\frac{1}{6} m v_0^2$

22. $\upsilon = \sqrt{\frac{14gR}{3}}$ **23.** $\frac{3W}{8}$ **24.** 3m

25. 6.3 m/s **26.** (i) $\frac{2}{3}$ mg, (ii) $\mu_{min} = 0$, (iii) F = 2 mg], (iv) $\mu_s = \frac{2}{3}$

27. (a) $\sqrt{3}\, m\, \omega^2 \ell$ (b) $F_y = \sqrt{3}\, m\, \omega^2 \ell$ $F_x = -F/4$]

28. (i) (a) 5g/9 ↓ (b) g ↓ (c) 0 (ii) (a) $\frac{13g}{17}$ ↓ (b) g ↓ (c) $\frac{2g}{3}$ ↓

29. (a) 6 N (b) $\vec{\tau}_1 = 0.6\hat{k} - 0.6\hat{j}$, $\vec{\tau}_2 = -0.6\hat{k} - 0.6\hat{j}$

30. (a) $\theta = \cos^{-1}\frac{4}{7}$ (b) $v = \sqrt{\frac{4}{7}gr}$ (c) $\frac{k_T}{k_R} = 6$

31. (a) $\vec{F} = \frac{2mV}{\Delta t}\hat{i} - \frac{2mV}{\sqrt{3}\Delta t}\hat{k}$; $\vec{N} = \left(\frac{2mV}{\sqrt{3}\Delta t} + mg\right)\hat{k}$, (b) $\vec{\tau} = -\left(\frac{4mVh}{\sqrt{3}\Delta t}\right)\hat{j}$ **32.** $2\pi\left(\frac{\alpha R^4}{4} + \frac{\beta R^5}{5}\right)$

33. $I = 3/10\, mR^2$

34. $N = (aB - bA)\,\hat{k}$, where $\hat{k}$ is the unit vector of the z axis $\ell = |aB - bA| / \sqrt{A^2 + B^2}$

35. $\beta = 2mgx/R\ell(M + 2m)$

36. (a) $v = (M/m)\sqrt{2/3g\ell}\,\sin(\alpha/2)$; (b) $\Delta p = M\sqrt{1/6g\ell}\,\sin(\alpha/2)$; (c) $x \approx 2/3\ell$

37. (a) mg(h-r-r sinα), (b) $\frac{10}{7}g\left(\frac{h}{r} - 1 - \sin\alpha\right)$, $-\frac{5}{7}g\cos\alpha$ (c) 4N, 0.2N upward

38. (a) $\omega_A = \frac{v^2}{R} = 2.0\ m/s^2$, the vector ω_A is permanently directed to the centre of the wheel ;
(b) s = 8R = 4.0 m

39. $T = 7/10\, mv^2\,(1 + 2/7r^2/R^2)]$

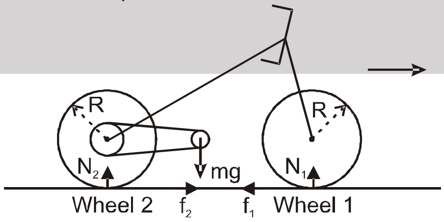
40. (i) (a) $\frac{1.2g}{\ell}$ (cw) (b) $-0.3(\hat{i} + 2\hat{j})$ g (ii) (a) $24g/17\ell$ (cw) (b) 12 g/17↓ (iii) $2.4g/\ell$ (cw) (b) 0.5 g↓

41. (a) $\omega_{i+1} = \frac{7}{13}\omega_i + \frac{6}{13}\frac{v}{r}$ (b) $\omega^* = \frac{v}{r}$ after this no further collision occurs

(c) $\omega_{i+1} = \frac{v}{r}\left(1 - \left(\frac{7}{13}\right)^i\right)$ (d) ω^* will remain same as in case b.

42. $\omega = \frac{v_1}{L}\frac{12\sin\beta}{3\sin^2\beta + 1}$ (cw)

43. (a) Here $f_1 f_2$ are frictional forces and N_1, N_2 are normal reactions



(b) $a = \frac{\tau}{MR^2 + 2I}R$ (c) $a \le \frac{\mu g/2}{(1 - \mu/4)}$ (d) $a_m = 2g/3$