SIMPLE HARMONIC MOTION

1. PERIODIC MOTION

When a body or a moving particle repeats its motion along a definite path after regular interval of time, its motion is said to be **Periodic Motion** and interval of time is called **time period** or harmonic motion period (T). The path of periodic motion may be linear, circular, elliptical or any other curve. For example, rotation of earth about the sun.

2. OSCILLATORY MOTION

'To and Fro' type of motion is called an **Oscillatory Motion**. It need not be periodic and need not have fixed extreme positions. For example, motion of pendulum of a wall clock.
The oscillatory motions in which energy is conserved are also periodic.
The force / torque (directed towards equilibrium point) acting in oscillatory motion is called restoring force / torque.
Damped oscillations are those in which energy is consumed due to some resistive forces and hence total mechanical energy decreases.

3. SIMPLE HARMONIC MOTION

If the restoring force/ torque acting on the body in oscillatory motion is directly proportional to the displacement of body/particle and is always directed towards equilibrium position then the motion is called simple Harmonic Motion (SHM). It is the simplest (easy to analyze) form of oscillatory motion.

3.1 TYPES OF SHM

(a) Linear SHM : When a particle undergoes to and fro motion about an equilibrium position, along a straight line. A and B are extreme positions. M is mean position. AM = MB = Amplitude

(b) Angular SHM : When a body/particle is free to rotate oscillate about a given axis on a curved path.

3.2 EQUATION OF SIMPLE HARMONIC MOTION (SHM) :

The necessary and sufficient condition for SHM is $F = -kx$
where k = positive constant for a SHM = Force constant
x = displacement from mean position.

or $m\dfrac{d^2x}{dt^2} = -kx$

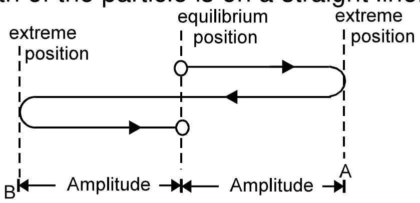
$\Rightarrow \dfrac{d^2x}{dt^2} + \dfrac{k}{m}x = 0$ [differential equation of SHM]

$\Rightarrow \dfrac{d^2x}{dt^2} + \omega^2 x = 0$ where $\omega = \sqrt{\dfrac{k}{m}}$

It's solution is $x = A\sin(\omega t + \phi)$

3.3 CHARACTERISTICS OF SHM

Note : In the figure shown, path of the particle is on a straight line.



(a) Displacement - It is defined as the distance of the particle from the mean position at that instant.

Displacement in SHM at time t is given by $x = A\sin(\omega t + \phi)$

(b) Amplitude : It is the maximum value of displacement of the particle from its equilibrium position.

Amplitude = $\frac{1}{2}$ [distance between extreme points or positions]

It depends on energy of the system.

(c) Angular Frequency (ω) : $\omega = \frac{2\pi}{T} = 2\pi f$ and its unit is rad/sec.

(d) Frequency (f) : Number of oscillations completed in unit time interval is called frequency of oscillations, $f = \frac{1}{T} = \frac{\omega}{2\pi}$, its units is sec^{-1} or Hz.

(e) Time period (T) : Smallest time interval after which the oscillatory motion gets repeated is called time period, $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Solved Examples

Example 1. For a particle performing SHM, equation of motion is given as $\frac{d^2x}{dt^2} + 4x = 0$. Find the time period.

Solution : $\frac{d^2x}{dt^2} = -4x$ $\quad \omega^2 = 4 \quad \omega = 2$

Time period ; $T = \frac{2\pi}{\omega} = \pi$

(f) Phase : The physical quantity which represents the state of motion of particle (eg. its position and direction of motion at any instant).

The argument $(\omega t + \phi)$ of sinusoidal function is called instantaneous phase of the motion.

(g) Phase constant (ϕ) : Constant ϕ in equation of SHM is called phase constant or initial phase. It depends on initial position and direction of velocity.

(h) Velocity(v) : Velocity at an instant is the rate of change of particle's position w.r.t time at that instant.

Let the displacement from mean position is given by

$$x = A \sin(\omega t + \phi)$$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}\left[A\sin(\omega t + \phi)\right]$

$v = A\omega \cos(\omega t + \phi)$ or, $v = \omega\sqrt{A^2 - x^2}$

At mean position (x = 0), velocity is maximum.

$v_{max} = \omega A$

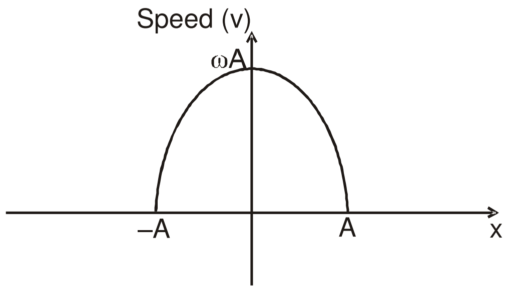
At extreme position (x = A), velocity is minimum.

v_{min} = zero

GRAPH OF SPEED (v) VS DISPLACEMENT (x):

$v = \omega\sqrt{A^2 - x^2}$ $\quad v^2 = \omega^2\left(A^2 - x^2\right)$

$v^2 + \omega^2x^2 = \omega^2A^2$ $\quad \frac{v^2}{\omega^2A^2} + \frac{x^2}{A^2} = 1$



GRAPH WOULD BE AN ELLIPSE

(i) Acceleration : Acceleration at an instant is the rate of change of particle's velocity w.r.t. time at that instant.

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}[A\omega \cos(\omega t + \phi)]$

$a = -\omega^2 A \sin(\omega t + \phi)$

$a = -\omega^2 x$

Note

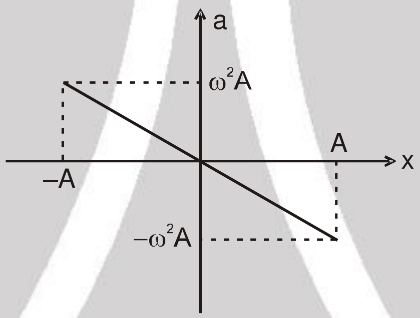
- Negative sign shows that acceleration is always directed towards the mean position.

  At mean position (x = 0), acceleration is minimum.

  a_{min} = zero

  At extreme position (x = A), acceleration is maximum.

  $a_{max} = \omega^2 A$



GRAPH OF ACCELERATION (A) VS DISPLACEMENT (x)

$a = -\omega^2 x$

Solved Examples

Example 2. The equation of particle executing simple harmonic motion is $x = (5\ m) \sin\left[(\pi s^{-1})t + \frac{\pi}{3}\right]$. Write down the amplitude, time period and maximum speed. Also find the velocity at t = 1 s.

Solution : Comparing with equation $x = A \sin(\omega t + \delta)$,

we see that the amplitude = 5 m,

and time period $= \frac{2\pi}{\omega} = \frac{2\pi}{\pi\ s^{-1}} = 2s$.

The maximum speed $= A\omega = 5\ m \times \pi\ s^{-1} = 5\pi$ m/s.

The velocity at time $t = \frac{dx}{dt} = A\omega\cos(\omega t + \delta)$

At t = 1 s,

$v = (5\ m)(\pi\ s^{-1}) \cos\left(\pi + \frac{\pi}{3}\right) = -\frac{5\pi}{2}$ m/s.

Example 3. A particle executing simple harmonic motion has angular frequency 6.28 s^{-1} and amplitude 10 cm. Find (a) the time period, (b) the maximum speed, (c) the maximum acceleration, (d) the speed when the displacement is 6 cm from the mean position, (e) the speed at t = 1/6 s assuming that the motion starts from rest at t = 0.

Solution : (a) Time period = $\frac{2\pi}{\omega} = \frac{2\pi}{6.28}$ s = 1 s.

(b) Maximum speed = $A\omega$ = (0.1 m) (6.28 s^{-1}) = 0.628 m/s.

(c) Maximum acceleration = $A\omega^2$

= (0.1 m) $(6.28\ s^{-1})^2$ = 4 m/s^2.

(d) $v = \omega\sqrt{A^2 - x^2} = (6.28\ s^{-1})\sqrt{(10cm)^2 - (6cm)^2}$ = 50.2 cm/s.

(e) At t = 0, the velocity is zero i.e., the particle is at an extreme. The equation for displacement may be written as

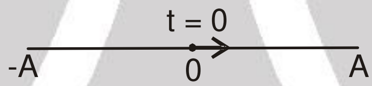
$x = A\cos\omega t$.

The velocity is $v = -A\,\omega\sin\omega t$.

At $t = \frac{1}{6}$ s, $v = -(0.1\ m)(6.28\ s^{-1})\sin\left(\frac{6.28}{6}\right)$

$= (-0.628\ m/s)\sin\frac{\pi}{3}$ = 54.4 cm/s.

Example 4. A particle starts from mean position and moves towards positive extreme as shown. Find the equation of the SHM. Amplitude of SHM is A.



Solution : General equation of SHM can be written as $x = A\sin(\omega t + \phi)$

At t = 0, x = 0

$\therefore$ 0 = A sinϕ $\therefore$ $\phi = 0, \pi$ $\phi \in [0, 2\pi)$

Also; at t = 0, v = +ve

$\therefore$ $A\omega\cos\phi$ = +ve

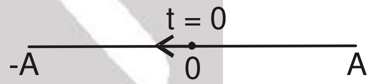
or, $\phi = 0$

Hence, if the particle is at mean position at t = 0 and is moving towards +ve extreme, then the equation of SHM is given by $x = A\sin\omega t$

Similarly

for

$\phi = \pi$



$\therefore$ equation of SHM is $x = A\sin(\omega t + \pi)$

or, $x = -A\sin\omega t$

Note :

- **If mean position is not at the origin, then we can replace x by $x - x_0$ and the eqn. becomes $x - x_0 = A\sin(\omega t + \phi)$, where x_0 is the position co-ordinate of the mean position.**

Example 5. A particle is performing SHM of amplitude "A" and time period "T". Find the time taken by the particle to go from 0 to A/2.

Solution : Let equation of SHM be $x = A\sin\omega t$

when x = 0, t = 0

when x = A/2 ; $A/2 = A\sin\omega t$

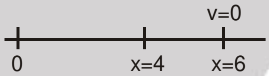
or $\sin\omega t = 1/2$ $\omega t = \pi/6$

$\frac{2\pi}{T}t = \pi/6$ $t = T/12$

Hence, time taken is T/12, where T is time period of SHM.

Example 6. A particle of mass 2 kg is moving on a straight line under the action of force F = (8 – 2x) N. It is released at rest from x = 6 m.
(a) Is the particle moving simple harmonically.
(b) Find the equilibrium position of the particle.
(c) Write the equation of motion of the particle.
(d) Find the time period of SHM.

Solution : $F = 8 - 2x$ or $F = -2(x - 4)$
at equilibrium position F = 0
$\Rightarrow$ x = 4 is equilibrium position
Hence the motion of particle is SHM with force constant 2 and equilibrium position x = 4.
(a) Yes, motion is SHM.
(b) Equilibrium position is x = 4



(c) At x = 6 m, particle is at rest i.e. it is one of the extreme position
Hence amplitude is A = 2 m and initially particle is at the extreme position.
$\therefore$ Equation of SHM can be written as

$x - 4 = 2\cos\omega t$, where $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{2}{2}} = 1$

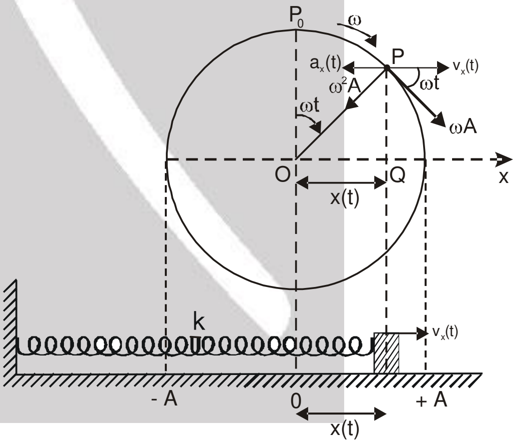
i.e. $x = 4 + 2\cos t$

(d) Time period, $T = \frac{2\pi}{\omega} = 2\pi$ sec.

4. SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle moving on a circle of radius A with a constant angular speed ω as shown in figure.
Suppose the particle is on the top of the circle (Y-axis) at t = 0. The radius OP makes an angle θ = ωt with the Y-axis at time t. Drop a perpendicular PQ on X-axis. The components of position vector, velocity vector and acceleration vector at time t on the X-axis are

$x(t) = A\sin\omega t$
$v_x(t) = A\omega\cos\omega t$
$a_x(t) = -\omega^2 A\sin\omega t$



Above equations show that the foot of perpendicular Q executes a simple harmonic motion on the X-axis. The amplitude is A and angular frequency is ω.
Similarly the foot of perpendicular on Y-axis will also execute SHM of amplitude A and angular frequency ω [$y(t) = A\cos\omega t$]. The phases of the two simple harmonic motions differ by π/2.

5. GRAPHICAL REPRESENTATION OF DISPLACEMENT, VELOCITY & ACCELERATION IN SHM

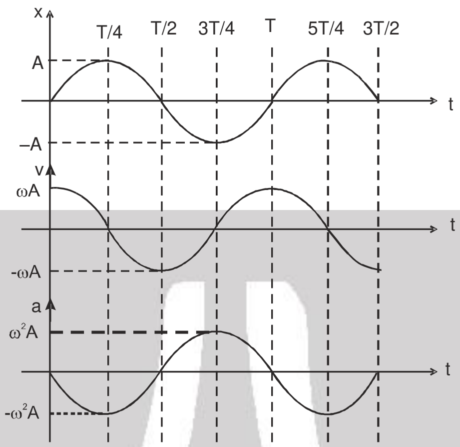
Displacement, $x = A\sin\omega t$

Velocity, $v = A\omega\cos\omega t = A\omega\sin(\omega t + \frac{\pi}{2})$ or $v = \sqrt{A^2 - x^2}\,\omega$

Acceleration, $a = -\omega^2 A\sin\omega t = \omega^2 A\sin(\omega t + \pi)$ or $a = -\omega^2 x$

Note : • $v = \omega\sqrt{A^2 - x^2}$
$a = -\omega^2 x$
These relations are true for any equation of x.

time, t	0	T/4	T/2	3T/4	T
displacement, x	0	A	0	– A	0
Velocity, v	Aω	0	– Aω	0	Aω
acceleration, a	0	– ω²A	0	ω²A	0



1. All the three quantities displacement, velocity and acceleration vary harmonically with time, having same period.
2. The velocity amplitude is ω times the displacement amplitude ($v_{max} = \omega A$).
3. The acceleration amplitude is ω^2 times the displacement amplitude ($a_{max} = \omega^2 A$).
4. In SHM, the velocity is ahead of displacement by a phase angle of $\frac{\pi}{2}$.
5. In SHM, the acceleration is ahead of velocity by a phase angle of $\frac{\pi}{2}$.

6. ENERGY OF SHM

6.1 Kinetic Energy (KE)

$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$ (as a function of x)

$= \frac{1}{2}mA^2\omega^2\cos^2(\omega t + \theta) = \frac{1}{2}KA^2\cos^2(\omega t + \theta)$ (as a function of t)

$KE_{max} = \frac{1}{2}kA^2$; $\langle KE \rangle_{0-T} = \frac{1}{4}kA^2$; $\langle KE \rangle_{0-A} = \frac{1}{3}kA^2$

Frequency of KE = 2 × (frequency of SHM)

6.2 Potential Energy (PE)

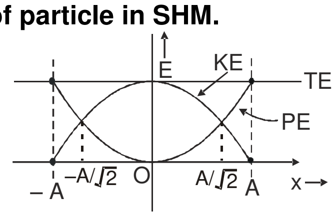
$\frac{1}{2}Kx^2$ (as a function of x) $= \frac{1}{2}kA^2\sin^2(\omega t + \theta)$ (as a function of time)

6.3 Total Mechanical Energy (TME)

Total mechanical energy = Kinetic energy + Potential energy $= \frac{1}{2}k(A^2 - x^2) + \frac{1}{2}Kx^2 = \frac{1}{2}KA^2$

Hence total mechanical energy is constant in SHM.

6.4 Graphical Variation of energy of particle in SHM.



Solved Examples

Example 7. A particle of mass 0.50 kg executes a simple harmonic motion under a force F = – (50 N/m)x. If it crosses the centre of oscillation with a speed of 10 m/s, find the amplitude of the motion.

Solution : The kinetic energy of the particle when it is at the centre of oscillation is

$$E = \frac{1}{2} mv^2 = \frac{1}{2} (0.50 \text{ kg}) (10 \text{ m/s})^2 = 25 \text{ J}.$$

The potential energy is zero here. At the maximum displacement x = A, the speed is zero and hence the kinetic energy is zero. The potential energy here is $\frac{1}{2}kA^2$. As there is no loss of energy,

$$\frac{1}{2}kA^2 = 25 \text{ J} \qquad \text{.............(i)}$$

The force on the particle is given by

F = – (50 N/m)x.

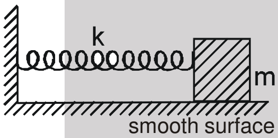
Thus, the spring constant is k = 50 N/m.

Equation (i) gives

$$\frac{1}{2} (50 \text{ N/m}) A^2 = 25 \text{ J} \quad \text{or,} \quad A = 1 \text{ m}.$$

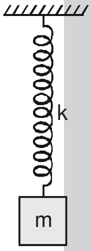
7. SPRING-MASS SYSTEM

(1)



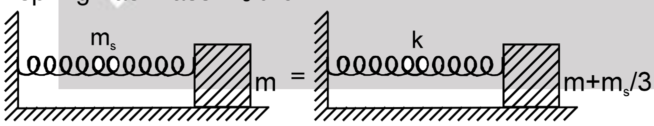
$$T = 2\pi\sqrt{\frac{m}{k}}$$

(2)



$$T = 2\pi\sqrt{\frac{m}{k}}$$

(3) If spring has mass m_s then



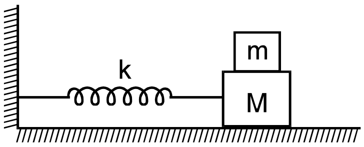
$$T = 2\pi\sqrt{\frac{m + \frac{m_s}{3}}{k}}$$

Solved Examples

Example 8. A particle of mass 200 g executes a simple harmonic motion. The restoring force is provided by a spring of spring constant 80 N/m. Find the time period.

Solution : The time period is $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{200 \times 10^{-3} \text{kg}}{80 \ \text{N/m}}} = 2\pi \times 0.05 \text{ s} = 0.31 \text{ s}.$

Example 9. The friction coefficient between the two blocks shown in figure is μ and the horizontal plane is smooth. (a) If the system is slightly displaced and released, find the time period. (b) Find the magnitude of the frictional force between the blocks when the displacement from the mean position is x. (c) What can be the maximum amplitude if the upper block does not slip relative to the lower block ?



Solution : (a) For small amplitude, the two blocks oscillate together. The angular frequency is

$\omega = \sqrt{\frac{k}{M+m}}$ and so the time period $T = 2\pi\sqrt{\frac{M+m}{k}}$.

(b) The acceleration of the blocks at displacement x from the mean position is

$a = -\omega^2 x = \left(\frac{-kx}{M+m}\right)$

The resultant force on the upper block is, therefore, $ma = \left(\frac{-mkx}{M+m}\right)$

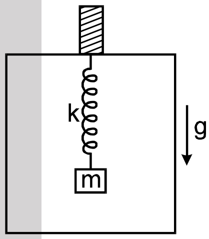
This force is provided by the friction of the lower block. Hence, the magnitude of the frictional force is $\left(\frac{mk\,|\,x\,|}{M+m}\right)$

(c) Maximum force of friction required for simple harmonic motion of the upper block is $\frac{mkA}{M+m}$ at the extreme positions. But the maximum frictional force can only be μ mg. Hence

$\frac{mk\ A}{M+m} = \mu\, mg$ or, $A = \frac{\mu(M+m)g}{k}$

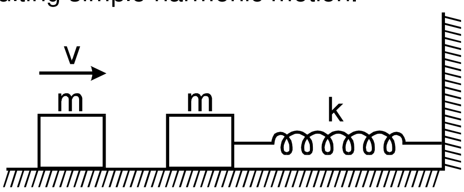
Example 10. A block of mass m is suspended from the ceiling of a stationary elevator through a spring of spring constant k and suddenly, the cable breaks and the elevator starts falling freely. Show that block now executes a simple harmonic motion of amplitude mg/k in the elevator.

Solution : When the elevator is stationary, the spring is stretched to support the block. If the extension is x, the tension is kx which should balance the weight of the block.
Thus, x = mg/k. As the cable breaks, the elevator starts falling with acceleration 'g'. We shall work in the frame of reference of the elevator. Then we have to use a pseudo force mg upward on the block. This force will 'balance' the weight.



Thus, the block is subjected to a net force kx by the spring when it is at a distance x from the position of unstretched spring. Hence, its motion in the elevator is simple harmonic with its mean position corresponding to the unstretched spring. Initially, the spring is stretched by x = mg/k, where the velocity of the block (with respect to the elevator) is zero. Thus, the amplitude of the resulting simple harmonic motion is mg/k.

Example 11. The left block in figure collides inelastically with the right block and sticks to it. Find the amplitude of the resulting simple harmonic motion.



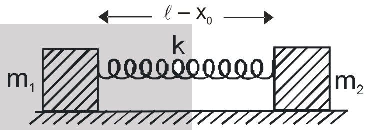
Solution : Assuming the collision to last for a small interval only, we can apply the principle of conservation of momentum. The common velocity after the collision is $\frac{v}{2}$.

The kinetic energy = $\frac{1}{2}$ (2m) $\left(\frac{v}{2}\right)^2$ = $\frac{1}{4}$ mv^2.

This is also the total energy of vibration as the spring is unstretched at this moment. If the amplitude is A, the total energy can also be written as $\frac{1}{2}$ kA^2. Thus,

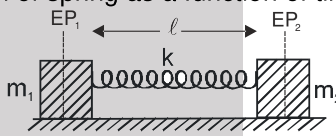
$$\frac{1}{2}kA^2 = \frac{1}{4}mv^2, \text{ giving } A = \sqrt{\frac{m}{2k}}\,v.$$

Example 12. Two blocks of mass m_1 and m_2 are connected with a spring of natural length l and spring constant k. The system is lying on a smooth horizontal surface. Initially spring is compressed by x_0 as shown in figure.

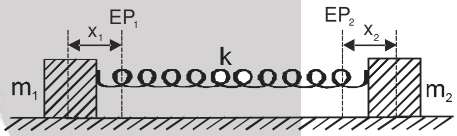


Show that the two blocks will perform SHM about their equilibrium position. Also (a) find the time period, (b) find amplitude of each block and (c) length of spring as a function of time.

Solution : (a) Here both the blocks will be in equilibrium at the same time when spring is in its natural length. Let EP_1 and EP_2 be equilibrium positions of block A and B as shown in figure.



Let at any time during oscillations, blocks are at a distance of x_1 and x_2 from their equilibrium positions. As no external force is acting on the spring block system



$\therefore$ $(m_1 + m_2)\Delta x_{cm} = m_1x_1 - m_2x_2 = 0$ or $m_1x_1 = m_2x_2$

For 1st particle, force equation can be written as

$k(x_1 + x_2) = -m_1\frac{d^2x_1}{dt^2}$ or, $k(x_1 + \frac{m_1}{m_2}x_1) = -m_1a_1$

or, $a_1 = \frac{k(m_1+m_2)}{m_1m_2}-x_1$ $\therefore$ $\omega^2 = \frac{k(m_1+m_2)}{m_1m_2}$

Hence, $T = 2\pi\sqrt{\frac{m_1m_2}{k(m_1+m_2)}} = 2\pi\sqrt{\frac{\mu}{K}}$

where $\mu = \frac{m_1m_2}{(m_1+m_2)}$ which is known as reduced mass **Ans (a)**

Similarly time period of 2nd particle can be found. Both will be having the same time period.

(b) Let the amplitude of blocks be A_1 and A_2.

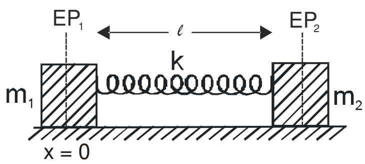
$m_1A_1 = m_2A_2$

By energy conservation; $\frac{1}{2}k(A_1 + A_2)^2 = \frac{1}{2}k\,x_0^2$ or, $A_1 + A_2 = x_0$

or, $A_1 + A_2 = x_0$ or, $A_1 + \frac{m_1}{m_2}A_1 = x_0$

or, $A_1 = \frac{m_2x_0}{m_1+m_2}$ Similarly, $A_2 = \frac{m_1x_0}{m_1+m_2}$

(c) Consider equilibrium position of 1st particle as origin, i.e. x = 0. x co-ordinate of particles can be written as



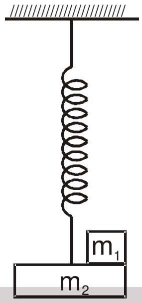
$x_1 = A_1\cos\omega t$ and $x_2 = \ell - A_2\cos\omega t$

Hence, length of spring at time t can be written as;

length = $x_2 - x_1 = \ell - (A_1 + A_2)\cos\omega t$

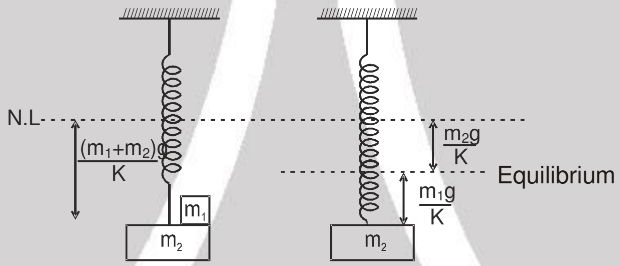
Solved Examples

Example 13. The system is in equilibrium and at rest. Now mass m_1 is removed from m_2. Find the time period and amplitude of resultant motion. Spring constant is K.



Solution : Initial extension in the spring $x = \frac{(m_1 + m_2)g}{K}$

Now, if we remove m_1 equilibrium position (E.P.) of m_2 will be $\frac{m_2 g}{K}$ below natural length of spring.



At the initial position, since velocity is zero i.e. it is the extreme position.

Hence Amplitude = $\frac{m_1 g}{K}$

Time period = $2\pi\sqrt{\frac{m_2}{K}}$

Since only block of mass m_2 is oscillating

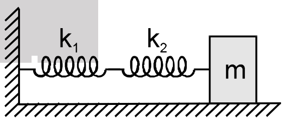
8. COMBINATION OF SPRINGS

8.1 Series Combination :

Total displacement $x = x_1 + x_2$

Tension in both springs $= k_1 x_1 = k_2 x_2$

$\therefore$ Equivalent spring constant in series combination K_{eq} is given by :



$1/k_{eq} = 1/k_1 + 1/k_2 \quad \Rightarrow \quad T = 2\pi\sqrt{\frac{m}{k_{eq}}}$

Note :

- In series combination, tension is same in all the springs & extension will be different. (If k is same then deformation is also same)
- In series combination, extension of springs will be reciprocal of its spring constant.
- Spring constant of spring is reciprocal of its natural length

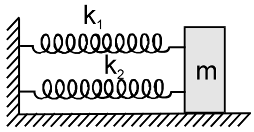
  $\therefore$ $k \propto 1/\ell$

  $\therefore$ $k_1 \ell_1 = k_2 \ell_2 = k_3 \ell_3$
- If a spring is cut in 'n' pieces then spring constant of one piece will be nk.

8.2 Parallel combination :

Extension is same for both springs but force acting will be different.

Force acting on the system = F



$\therefore$ $F = -(k_1 x + k_2 x)$ $\Rightarrow$ $F = -(k_1 + k_2)x$ $\Rightarrow$ $F = -k_{eq}x$

$\therefore$ $k_{eq} = k_1 + k_2$ $\Rightarrow$ $T = 2\pi\sqrt{\dfrac{m}{k_{eq}}}$

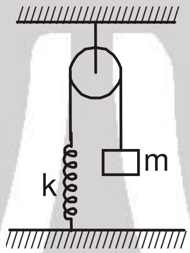
9. METHOD'S TO DETERMINE TIME PERIOD, ANGULAR FREQUENCY IN S.H.M.

(a) Force / torque method

(b) Energy method

Solved Examples

Example 14. The string, the spring and the pulley shown in figure are light. Find the time period of the mass m.



Solution : **(a) Force Method :**

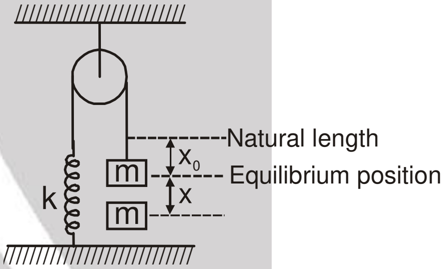
Let in equilibrium position of the block, extension in spring is x_0.

$\therefore$ $kx_0 = mg$ .....(1)

Now if we displace the block by x in the downward direction, net force on the block towards mean position is

$F = k(x + x_0) - mg = kx$ using (1)

Hence the net force is acting towards mean position and is also proportional to x.



So, the particle will perform S.H.M. and its time period would be $T = 2\pi\sqrt{\dfrac{m}{k}}$

(b) Energy Method :

Let gravitational potential energy is to be zero at the level of the block when spring is in its natural length.

Now at a distance x below that level, let speed of the block be v.

Since total mechanical energy is conserved in S.H.M.

$\therefore$ $-mgx + \dfrac{1}{2}kx^2 + \dfrac{1}{2}mv^2 = \text{constant}$

Differentiating w.r.t. time, we get

$-mgv + kxv + mva = 0$

where a is acceleration.

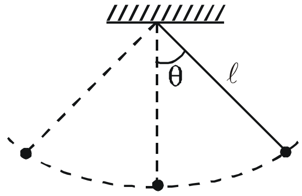
$\therefore$ $F = ma = -kx + mg$ or $F = -k\left(x - \dfrac{mg}{k}\right)$

This shows that for the motion, force constant is k and equilibrium position is $x = \dfrac{mg}{k}$.

So, the particle will perform S.H.M. and its time period would be $T = 2\pi\sqrt{\dfrac{m}{k}}$

10. SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum



Time period of a simple pendulum $T = 2\pi\sqrt{\frac{\ell}{g}}$

(some times we can take $g = \pi^2$ for making calculation simple)

Note :

- If angular amplitude of simple pendulum is more, then time period

  $T = 2\pi\sqrt{\frac{\ell}{g}}\left(1+\frac{\theta_0^2}{16}\right)$ **(Not in JEE, For other exams)**

  where θ_0 is in radians.
- General formula for time period of simple pendulum when ℓ is comparable to radius of Earth R.

  $T = 2\pi\sqrt{\frac{1}{g\left(\frac{1}{R}+\frac{1}{\ell}\right)}}$ where, R = Radius of the earth

- Time period of simple pendulum of infinite length is maximum and is given by : $T = 2\pi\sqrt{\frac{R}{g}}$ = 84.6 min

  (Where R is radius of earth)
- Time period of seconds pendulum is 2 sec and ℓ = 0.993 m.
- Simple pendulum performs angular S.H.M. but due to small angular displacement, it is considered as linear S.H.M.
- If time period of clock based on simple pendulum increases then clock will be slow but if time period decrease then clock will be fast.
- If g remains constant & $\Delta\ell$ is change in length, then $\frac{\Delta T}{T}\times 100 = \frac{1}{2}\frac{\Delta\ell}{\ell}\times 100$
- If ℓ remain constant & Δg is change in acceleration then, $\frac{\Delta T}{T}\times 100 = -\frac{1}{2}\frac{\Delta g}{g}\times 100$
- If $\Delta\ell$ is change in length & Δg is change in acceleration due to gravity then,

  $\frac{\Delta T}{T}\times 100 = \left[\frac{1}{2}\frac{\Delta\ell}{\ell} - \frac{1}{2}\frac{\Delta g}{g}\right]\times 100$

Solved Examples

Example 15 A simple pendulum of length 40 cm oscillates with an angular amplitude of 0.04 rad. Find (a) the time period, (b) the linear amplitude of the bob, (c) the speed of the bob when the string makes 0.02 rad with the vertical and (d) the angular acceleration when the bob is in momentary rest. Take g = 10 m/s^2.

Solution : (a) The angular frequency is $\omega = \sqrt{g/\ell} = \sqrt{\frac{10 m/s^2}{0.4 m}} = 5\ s^{-1}$

the time period is $\frac{2\pi}{\omega} = \frac{2\pi}{5s^{-1}}$ = 1.26 s.

(b) Linear amplitude = 40 cm × 0.04 = 1.6 cm

(c) Angular speed at displacement 0.02 rad is

$\Omega = (5\ s^{-1})\sqrt{(0.04)^2-(0.02)^2}$ rad = 0.17 rad/s.

where speed of the bob at this instant = (40 cm) × 0.175^{-1} = 6.8 cm/s.

(d) At momentary rest, the bob is in extreme position.

Thus, the angular acceleration α = (0.04 rad) (25 s^{-2}) = 1 rad/s^2.

10.1 Time Period of Simple Pendulum in accelerating Reference Frame :

$T = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$ where

$g_{eff.}$ = Effective acceleration in accelerating reference system = $|\vec{g} - \vec{a}|$, at mean position

$\vec{a}$ = acceleration of the point of suspension w.r.t. ground.

Condition for applying this formula: $|\vec{g} - \vec{a}|$ = constant

Also $g_{eff} = \frac{\text{Net tension in string}}{\text{mass of bob}}$ at mean position

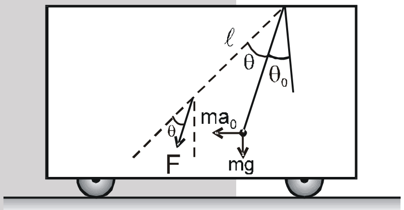
Solved Examples

Example 16. A simple pendulum is suspended from the ceiling of a car accelerating uniformly on a horizontal road. If the acceleration is a_0 and the length of the pendulum is ℓ, find the time period of small oscillations about the mean position.

Solution : We shall work in the car frame. As it is accelerated with respect to the road, we shall have to apply a pseudo force ma_0 on the bob of mass m.

For mean position, the acceleration of the bob with respect to the car should be zero. If θ_0 be the angle made by the string with the vertical, the tension, weight and the pseudo force will add to zero in this position.

Hence, resultant of mg and ma_0 (say $F = m\sqrt{g^2 + a_0^2}$) has to be along the string.



$\therefore \quad \tan\theta_0 = \frac{ma_0}{mg} = \frac{a_0}{g}$

Now, suppose the string is further deflected by an angle θ as shown in figure.

Now, restoring torque can be given by

$(F \sin\theta) = \ell - m\ell^2\alpha$

Substituting F and using $\sin\theta \simeq \theta$, for small θ.

$(m\sqrt{g^2 + a_0^2})\ell\,\theta = -m\ell^2\alpha$

or, $\alpha = \frac{\sqrt{g^2 + a_0^2}}{\ell} - \theta$ so; $\omega^2 = \frac{\sqrt{g^2 + a_0^2}}{\ell}$

This is an equation of simple harmonic motion with time period $T = \frac{2\pi}{\omega} = 2\pi\frac{\sqrt{\ell}}{(g^2 + a_0^2)^{1/4}}$

10.2 If forces other then $m\vec{g}$ acts then :

$T = 2\pi\sqrt{\frac{\ell}{g_{eff.}}}$ where $g_{eff.} = \left|\vec{g} + \frac{\vec{F}}{m}\right|$

$\vec{F}$ = constant force acting on 'm'.

Solved Examples

Example 17. A simple pendulum of length 'ℓ' and having bob of mass 'm' is doing angular SHM inside water. A constant buoyant force equal to half the weight of the bob is acting on the ball. Find the time period of oscillations?

Solution : Here $g_{eff.} = g - \frac{mg/2}{m}$ = g/2. Hence $T = 2\pi\sqrt{\frac{2\ell}{g}}$

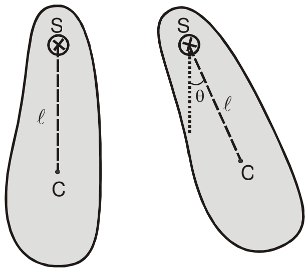
11. COMPOUND PENDULUM / PHYSICAL PENDULUM

When a rigid body is suspended from an axis and made to oscillate about that then it is called compound pendulum.

C = Position of center of mass

S = Point of suspension

ℓ = Distance between point of suspension and center of mass (it remains constant during motion)



For small angular displacement "θ" from mean position

The restoring torque is given by

$\tau = -\,mg\ell\sin\theta$

$\tau = -\,mg\ell\theta \quad \because$ for small θ, $\sin\theta \simeq \theta$

or $I\alpha = -\,mg\ell\theta$ where, I = Moment of inertia about point of suspension.

or $\alpha = \frac{mg\ell}{I} - \theta$ or $\omega^2 = \frac{mg\ell}{I}$

Time period, $T = 2\pi\sqrt{\frac{I}{mg\ell}}$ $\quad I = I_{CM} + m\ell^2$

Where I_{CM} = moment of inertia relative to the axis which passes from the center of mass & parallel to the axis of oscillation.

$T = 2\pi\sqrt{\frac{I_{CM} + m\ell^2}{mg\ell}}$

where $I_{CM} = mk^2$

k = gyration radius (about axis passing from centre of mass)

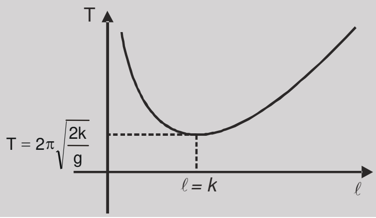
$T = 2\pi\sqrt{\frac{mk^2 + m\ell^2}{mg\ell}}$ $\quad T = 2\pi\sqrt{\frac{k^2 + \ell^2}{\ell g}} = 2\pi\sqrt{\frac{L_{eq}}{g}}$

$L_{eq} = \frac{k^2}{\ell} + \ell$ = equivalent length of simple pendulum ;

T is minimum when ℓ = k.

$T_{min} = 2\pi\sqrt{\frac{2k}{g}}$

Graph of T vs ℓ



Solved Examples

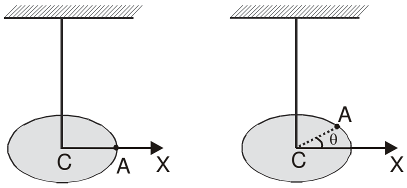
Example 18. A uniform rod of length 1.00 m is suspended through an end and is set into oscillation with small amplitude under gravity. Find the time period of oscillation. (g = 10 m/s^2)

Solution : For small amplitude the angular motion is nearly simple harmonic and the time period is given by

$$T = 2\pi\sqrt{\frac{I}{mg(\ell/2)}} = 2\pi\sqrt{\frac{(m\ell^2/3)}{mg(\ell/2)}} = 2\pi\sqrt{\frac{2\ell}{3g}} = 2\pi\sqrt{\frac{2\times 1.00\,m}{3\times 10\,m/s^2}} = \frac{2\pi}{\sqrt{15}}\ s.$$

12. TORSIONAL PENDULUM

In torsional pendulum, an extended object is suspended at the centre by a light torsion wire. A torsion wire is essentially inextensible, but is free to twist about its axis. When the lower end of the wire is rotated by a slight amount, the wire applies a restoring torque causing the body to oscillate (rotate) about vertical wire, when released.



The restoring torque produced is given by

$\tau = -C\theta$ where, C = Torsional constant

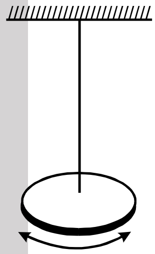
or $I\alpha = -C\theta$ where, I = Moment of inertia about the vertical axis.

or $\alpha = -\frac{C}{I}\theta$ $\therefore$ Time Period, $T = 2\pi\sqrt{\frac{I}{C}}$

Solved Examples

Example 19. A uniform disc of radius 5.0 cm and mass 200 g is fixed at its centre to a metal wire, the other end of which is fixed to a ceiling. The hanging disc is rotated about the wire through an angle and is released. If the disc makes torsional oscillations with time period 0.20 s, find the torsional constant of the wire.

Solution : The situation is shown in figure. The moment of inertia of the disc about the wire is



$$I = \frac{mr^2}{2} = \frac{(0.200\text{kg})(5.0\times10^{-2}\text{m})^2}{2} = 2.5 \times 10^{-4}\ \text{kg - m}^2.$$

The time period is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \text{ or } C = \frac{4\pi^2 I}{T^2} = \frac{4\pi^2(2.5\times10^{-4}\text{kg}-\text{m}^2)}{(0.20\ \ \text{s})^2} = 0.25\frac{\text{kg}-\text{m}^2}{\text{s}^2}.$$

13. SUPERPOSITION OF TWO SHM'S

13.1 In same direction and of same frequency.

$x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \theta)$, then resultant displacement $x = x_1 + x_2 = A_1 \sin \omega t + A_2 \sin (\omega t + \theta) = A \sin (\omega t + \phi)$

where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta}$ & $\phi = \tan^{-1}\left[\frac{A_2 \sin\theta}{A_1 + A_2\cos\theta}\right]$

If $\theta = 0$, both SHM's are in phase and $A = A_1 + A_2$

If $\theta = \pi$, both SHM's are out of phase and $A = |A_1 - A_2|$

The resultant amplitude due to superposition of two or more than two SHM's of this case can also be found by phasor diagram also.

13.2 In same direction but are of different frequencies.

$x_1 = A_1 \sin \omega_1 t$

$x_2 = A_2 \sin \omega_2 t$

then resultant displacement $x = x_1 + x_2 = A_1 \sin \omega_1 t + A_2 \sin \omega_2 t$ This resultant motion is not SHM.

13.3 In two perpendicular directions.

$x = A \sin \omega t$; $y = B \sin (\omega t + \theta)$

Case (i) : If $\theta = 0$ or π then $y = \pm (B/A)\, x$. So path will be straight line & resultant displacement will be

$r = (x^2 + y^2)^{1/2} = (A^2 + B^2)^{1/2} \sin\omega t$ $\sqrt{A^2 + B^2}$

which is equation of SHM having amplitude

Case (ii) : If $\theta = \frac{\pi}{2}$ then. $x = A \sin \omega t$; $y = B \sin (\omega t + \pi/2) = B \cos \omega t$

so, resultant will be $\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$. i.e. equation of an ellipse and if A = B, then superposition will be an equation of circle. This resultant motion is not SHM.

13.4 Superposition of SHM's along the same direction (using phasor diagram)

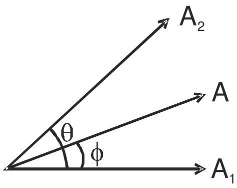
If two or more SHM's are along the same line, their resultant can be obtained by vector addition by making phasor diagram.

1. Amplitude of SHM is taken as length (magnitude) of vector.

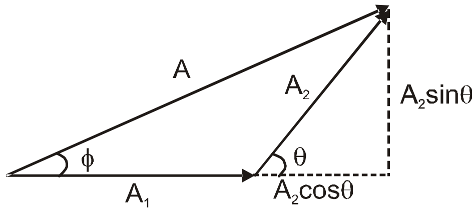
2. Phase difference between the vectors is taken as the angle between these vectors. The magnitude of resultant vector gives resultant amplitude of SHM and angle of resultant vector gives phase constant of resultant SHM.

For example : $x_1 = A_1 \sin \omega t$

$x_2 = A_2 \sin (\omega t + \theta)$



Phasor Diagram



Phasor diagram

If equation of resultant SHM is taken as $x = A \sin (\omega t + \phi)$

$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\theta} \Rightarrow \qquad \tan\phi = \dfrac{A_2 \sin\theta}{A_1 + A_2\cos\theta}$

Solved Examples

Example 20. Find the amplitude of the simple harmonic motion obtained by combining the motions $x_1 = (2.0 \text{ cm}) \sin\omega t$ and $x_2 = (2.0 \text{ cm}) \sin (\omega t + \pi/3)$.

Solution : The two equations given represent simple harmonic motions along X-axis with amplitudes $A_1 = 2.0$ cm and $A_2 = 2.0$ cm. The phase difference between the two simple harmonic motions is $\pi/3$. The resultant simple harmonic motion will have an amplitude A given by

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\delta} = \sqrt{(2.0\text{cm})^2 + (2.0\text{cm})^2 + 2(2.0\text{cm})^2\cos\frac{\pi}{3}} = 3.5 \text{ cm}$$

Example 21. $x_1 = 3 \sin \omega t \quad ; \quad x_2 = 4 \cos \omega t$

Find (i) amplitude of resultant SHM. (ii) equation of the resultant SHM.

Solution : First write all SHM's in terms of sine functions with positive amplitude. Keep "ωt" with positive sign.

$\therefore \quad x_1 = 3 \sin \omega t$

$x_2 = 4 \sin (\omega t + \pi/2)$

$$A = \sqrt{3^2 + 4^2 + 2\times3\times4\cos\frac{\pi}{2}} = \sqrt{9+16} = \sqrt{25} = 5$$

$$\tan\phi = \frac{4\sin\frac{\pi}{2}}{3 + 4\cos\frac{\pi}{2}} = \frac{4}{3} \qquad \phi = 53^\circ \qquad \text{equation } x = 5 \sin (\omega t + 53^\circ)$$

Example 22 $x_1 = 5 \sin (\omega t + 30^\circ)$

$x_2 = 10 \cos (\omega t)$.

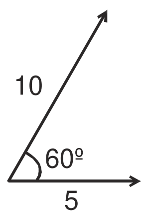
Find amplitude of resultant SHM.

Solution : $x_1 = 5 \sin (\omega t + 30^\circ)$

$x_2 = 10 \sin (\omega t + \frac{\pi}{2})$

$$A = \sqrt{5^2 + 10^2 + 2\times5\times10\cos 60^\circ}$$

$$= \sqrt{25+100+50} = \sqrt{175} = 5\sqrt{7}$$



Phasor Diagram

Example 23 A particle is subjected to two simple harmonic motions $x_1 = A_1 \sin \omega t$ and $x_2 = A_2 \sin (\omega t + \pi/3)$. Find (a) the displacement at t = 0, (b) the maximum speed of the particle and (c) the maximum acceleration of the particle.

Solution : (a) At t = 0, $x_1 = A_1 \sin \omega t = 0$ and $x_2 = A_2 \sin(\omega t + \pi/3) = A_2 \sin(\pi/3) = \frac{A_2\sqrt{3}}{2}$.

Thus, the resultant displacement at t = 0 is $x = x_1 + x_2 = A_2 \frac{\sqrt{3}}{2}$

(b) The resultant of the two motions is a simple harmonic motion of the same angular frequency ω. The amplitude of the resultant motion is

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\pi/3)} = \sqrt{A_1^2 + A_2^2 + A_1A_2}$$

The maximum speed is $u_{max} = A\,\omega = \omega\sqrt{A_1^2 + A_2^2 + A_1A_2}$

(c) The maximum acceleration is $a_{max} = A\,\omega^2 = \omega^2\sqrt{A_1^2 + A_2^2 + A_1A_2}$.

Example 24. A particle is subjected to two simple harmonic motions in the same direction having equal amplitudes and equal frequency. If the resultant amplitude is equal to the amplitude of the individual motions, find the phase difference between the individual motions.

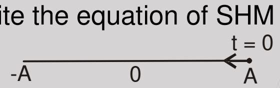
Solution : Let the amplitudes of the individual motions be A each. The resultant amplitude is also A. If the phase difference between the two motions is δ, $A = \sqrt{A^2 + A^2 + 2A.\ A.\cos\delta}$

$$A = A\sqrt{2(1+\cos\delta)} = 2A\cos\frac{\delta}{2} \quad ; \quad \cos\frac{\delta}{2} = \frac{1}{2} \quad \Rightarrow \quad \delta = 2\pi/3.$$

Solved Miscellaneous Problems

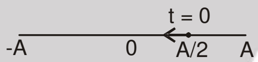
Problem 1. Write the equation of SHM for the situations shown below:

(a)



(b)

(c)



Solution : **(a)** At t = 0, x = +A

$x = A\sin(\omega t + \phi)$; $A = A\sin(\phi)$

$\phi = \pi/2$

$x = A\sin\left(\omega t + \frac{\pi}{2}\right) = A\cos(\omega t)$

(b) At t = 0, x = – A

$x = A\sin(\omega t + \phi)$; $-A = A\sin\phi$

$\phi = \frac{3\pi}{2}$; $x = A\sin\left(\omega t + \frac{3\pi}{2}\right)$

$x = -A\cos(\omega t)$

(c) At t = 0, $x = \frac{A}{2}$; $x = A\sin(\omega t + \phi)$

$\frac{A}{2} = A\sin(\omega t + \phi)$; $\frac{1}{2} = \sin\phi \Rightarrow \phi = 30°$, 150

Particle is moving towards the mean position and in negative direction.

velocity $v = A\omega\cos(\omega t + \phi)$

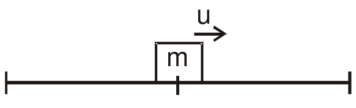
At t = 0, v = –ve

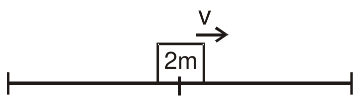
$v = A\omega\cos\phi$ hence $\phi = 150°$

$x = A\sin(\omega t + 150º)$ **Ans.** (**a**) $x = A\cos\omega t$; **(b)** $x = -A\cos\omega t$; **(c)** $x = A\sin(\omega t + 150º)$

Problem 2. Block A of mass m is performing SHM of amplitude a. Another block B of mass m is gently placed on A when it passes through mean position and B sticks to A. Find the time period and amplitude of new SHM.

Solution :





Time period of mass m = $2\pi\sqrt{\frac{m}{K}}$

Time period of mass 2m = $2\pi\sqrt{\frac{2m}{K}}$

At mean position, Kinetic energy = Total Energy

For mass m : $\frac{1}{2}mu^2 = \frac{1}{2}m\,\omega^2 a^2$ .....(1)

For mass 2m : $\frac{1}{2}2mv^2 = \frac{1}{2}2m\,A^2\left(\frac{\omega}{\sqrt{2}}\right)^2$ .....(2)

By Conservation of momentum mu = 2mv $\Rightarrow$ v = $\frac{u}{2}$

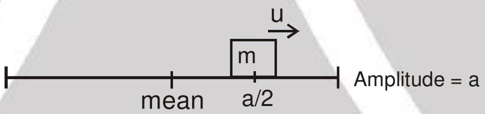
$\therefore \quad \frac{1}{2}2m\left(\frac{u}{2}\right)^2 = \frac{1}{2}2mA^2\left(\frac{\omega}{\sqrt{2}}\right)^2$ .....(3)

Divide equation (1) & (3) ; $4 = \frac{2a^2}{A^2}$

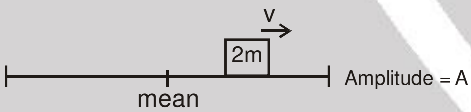
New Amplitude $A = \frac{a}{\sqrt{2}}$ **Ans.** $T = 2\pi\sqrt{\frac{2m}{K}}$ Amplitude = $\frac{a}{\sqrt{2}}$

Problem 3. Repeat the above problem assuming B is placed on A at a distance $\frac{a}{2}$ from mean position.

Solution : $T = 2\pi\sqrt{\frac{m}{K}}$



$T = 2\pi\sqrt{\frac{2m}{K}}$



By conservation of momentum mu = 2mv $\Rightarrow$ v = $\frac{u}{2}$

Kinetic Energies at $\frac{a}{2}$

For mass m : $\frac{1}{2}mu^2 = \frac{1}{2}m\omega^2\left[a^2 - \left(\frac{a}{2}\right)^2\right]$ .....(1)

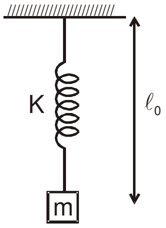
For mass 2m : $\frac{1}{2}2mv^2 = \frac{1}{2}2m\left(\frac{\omega}{\sqrt{2}}\right)^2\left[A^2 - \left(\frac{a}{2}\right)^2\right]$ .....(2)

$\frac{1}{2}2m\,\frac{u^2}{4} = \frac{1}{2}2m\,\frac{\omega^2}{2}\left[A^2 - \frac{a^2}{4}\right]$ .....(3)

Dividing equation (1) & (3) ; $2 = \frac{a^2 - \frac{a^2}{4}}{A^2 - \frac{a^2}{4}}$

New Amplitude $A = \sqrt{\frac{5}{8}}\,a$ **Ans.** $T = 2\pi\sqrt{\frac{2m}{K}}$, Amplitude = $a\sqrt{\frac{5}{8}}$

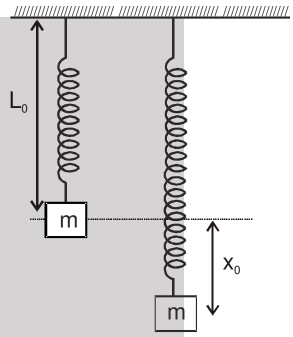
Problem 4. The block is allowed to fall, slowly from the position where spring is in its natural length. Find, maximum extension in the string.



Solution : Since the block falls slowly from rest the maximum extension occurs when $mg = Kx_0$

$x_0 = \frac{mg}{K}$ is maximum extension **Ans.** $\frac{mg}{K}$

Problem 5. In the above problem if block is released from there, what would be maximum extension.



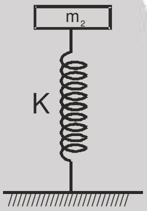
Solution : Let x_0 = maximum extension

applying conservation of energy $mgx_o = \frac{1}{2}Kx_0^2$

The velocity at the point of maximum extension is zero.

$x_0 = \frac{2mg}{K}$ is maximum extension **Ans.** $\frac{2mg}{K}$

Problem 6. Block of mass m_2 is in equilibrium as shown in figure. Another block of mass m_1 is kept gently on m_2. Find the time period of oscillation and amplitude.

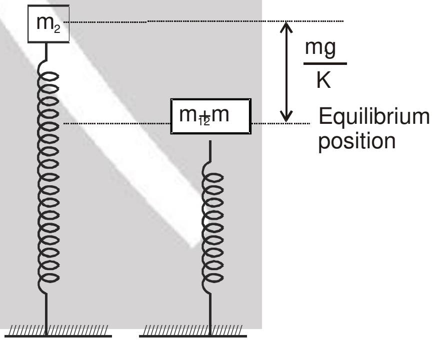


Solution : Time period $T = 2\pi\sqrt{\frac{m_1+m_2}{K}}$.

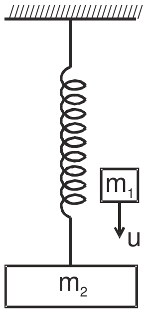
At initial position since velocity is zero it is the extreme position.

Amplitude $A = \frac{m_1 g}{K}$

Ans. $T = 2\pi\sqrt{\frac{m_1+m_2}{K}}$ Amplitude = $\frac{m_1 g}{K}$



Problem 7. Block of mass m_2 is in equilibrium and at rest. The mass m_1 moving with velocity u vertically downwards collides with m_2 and sticks to it. Find the energy of oscillation.



Solution : Let the velocity of m_1 & m_2 be v after collision

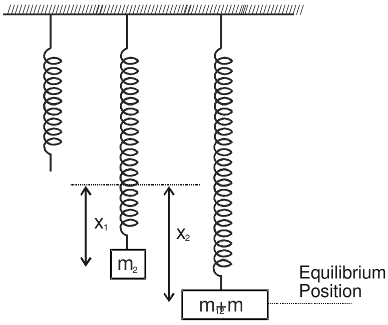
By conservation of momentum $m_1u = (m_1 + m_2)v$

$$v = \frac{m_1 u}{m_1 + m_2}$$

Hence, $KE = \frac{1}{2}(m_1+m_2)v^2 = \frac{1}{2}\frac{m_1^2}{m_1+m_2}u^2$

$m_2g = Kx_1 \Rightarrow (m_1 + m_2)g = Kx_2$

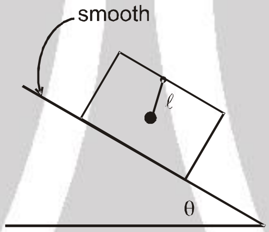
$PE = \frac{1}{2}K(x_2 - x_1)^2 = \frac{1}{2}K\left(\frac{m_1 g}{K}\right)^2 \Rightarrow PE = \frac{1}{2}\frac{m_1^2 g^2}{K}$



Therefore energy of oscillation is E = KE + PE

$$E = \frac{1}{2}\frac{m_1^2u^2}{m_1+m_2} + \frac{m_1^2g^2}{2K} \Rightarrow = \frac{1}{2}\left[\frac{m_1^2u^2}{m_1+m_2} + \frac{m_1^2g^2}{K}\right]$$ **Ans.**

Problem 8. A box is placed on a smooth inclined plane and it is free to move. A simple pendulum is attached in the block. Find its time period.



Solution : Let F_p = Pseudo force

For equilibrium position

$T\sin\alpha + F_P = mg\sin\theta$

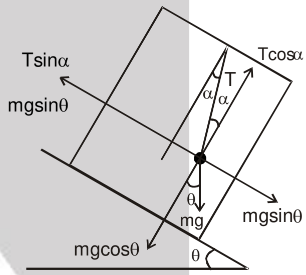
$T\sin\alpha + ma = mg\sin\theta$

since $a = g\sin\theta$

$T\sin\alpha + mg\sin\theta = mg\sin\theta$

$T\sin\alpha = 0$

$\therefore \alpha = 0$

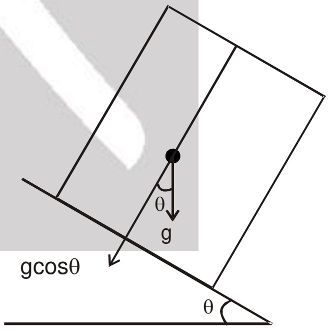


Hence in equilibrium position the string is perpendicular to incline plane.

Therefore effective 'g' is $g_{eff} = g\cos\theta$

Time period $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$

$T = 2\pi\sqrt{\frac{\ell}{g\cos\theta}}$



Problem 9. $x_1 = 5\sin\omega t$; $x_2 = 5\sin(\omega t + 53^\circ)$; $x_3 = -10\cos\omega t$. Find amplitude of resultant SHM

Solution : $x_1 = 5\sin\omega t$

$x_2 = 5\sin(\omega t + 53^\circ)$

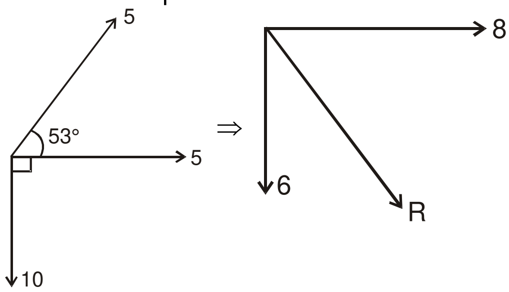
$x_3 = -10\cos\omega t$

we can write $x_3 = 10\sin(\omega t + 270^\circ)$

Finding the resultant amplitude by vector notation.

Resultant Amplitude

$|R| = \sqrt{8^2 + 6^2} = 10$ **Ans.**



HANDOUT SIMPLE HARMONIC MOTION

DAMPED SIMPLE HARMONIC MOTION

We know that the motion of a simple pendulum, swinging in air, dies out eventually. Why does it happen ? This is because the air drag and the friction at the support oppose the motion of the pendulum and dissipate its energy gradually. The pendulum is said to execute damped oscillations.

In damped oscillations, the energy of the system is dissipated continuously; but, for small damping, the oscillations remain approximately periodic. The dissipating forces are generally the frictional forces. To understand the effect of such external forces on the motion of an oscillator. Let us consider a system as shown in figure below, here a block of mass m connected to an elastic spring of spring constant k oscillates vertically. If the block is pushed down a little and released, its angular frequency of oscillation is ω ($\omega = \sqrt{\frac{k}{m}}$).However, in practice, the surrounding medium (air) will exert a damping force on the motion of the block and the mechanical energy of the block-spring system will decrease. The energy loss will appear as heat of the surrounding medium (and the block also)

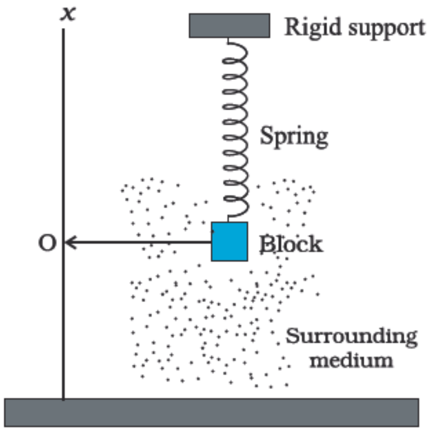


Figure 1: The viscous surrounding medium exerts a damping force on an oscillating spring, eventually bringing it to rest.

The damping force depends on the nature of the surrounding medium. If the block is immersed in a liquid, the magnitude of damping will be much greater and the dissipation of energy much faster. The damping force is generally proportional to velocity of the bob. [Remember Stokes' Law] and acts opposite to the direction of velocity. If the damping force is denoted by F_d, we have

$$\vec{F}_d = -b\vec{V}$$

where the positive constant b depends on characteristics of the medium (viscosity, for example) and the size and shape of the block, etc. This equation is usually valid only for small velocity. When the mass m is attached to the spring and released, the spring will elongate a little and the mass will settle at some height. This position, shown by O in Figure 1, is the equilibrium position of the mass. If the mass is pulled down or pushed up a little, the restoring force on the block due to the spring is $\vec{F}_s = -k\vec{x}$ where $\vec{x}$ is the displacement of the mass from its equilibrium position. Thus, the total force acting on the mass at any time t, is $\vec{F} = -k\vec{x} - b\vec{v}$. If a(t) is the acceleration of mass at time t, then by Newton's Law of Motion applied along the direction of motion, we have m a(t) = –kx(t) – b v(t)

Here we have dropped the vector notation because we are discussing one-dimensional motion. Using the first and second derivatives of x (t) for v (t) and a (t) respectively, we have

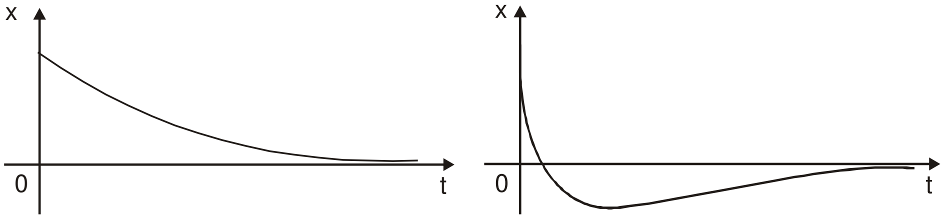
$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0 \qquad .....(1)$$

Equation 1 is a second-order linear differential equation and its auxiliary equation is $mr^2 + br + k = 0$. The roots are

$$r_1 = \frac{-b + \sqrt{b^2 - 4mk}}{2m} \qquad r_2 = \frac{-b - \sqrt{b^2 - 4mk}}{2m} \qquad .....(2)$$

We need to discuss three cases.

CASE I : $b^2 - 4mk > 0$ (**overdamping**)



In this case r_1 and r_2 are distinct real roots and $x = b_1e^{r_1t} + b_2e^{r_2t}$

Since b, m, and k are all positive, we have $\sqrt{b^2 - 4mk} < b$, so the roots r_1 and r_2 given by Equation 2 must both be negative. This shows that $x \to 0$ as $t \to \infty$. Typical graphs of x as a function of t are shown in above figure. Now Notice that oscillations do not occur. (It's possible for the mass to pass through the equilibrium position once but only once.) This is because $b^2 > 4mk$ means that there is a strong damping force (high-viscosity oil or grease compared with a weak spring or small mass.

CASE II : $b^2 - 4mk = 0$ (**critical damping**)

This case corresponds to equal roots

$$r_1 = r_2 = -\frac{b}{2m}$$

and the solution is given by

$$x = (b_1 + b_2t)e^{-(b/2m)t}$$

It is similar to Case I, and typical graphs resemble those in above figure, but the damping is just sufficient to suppress vibrations. Any decrease in the viscosity of the fluid leads to the vibrations of the following case.

CASE III : $b^2 - 4mk < 0$ (**underdamping**)

Here the roots are complex:

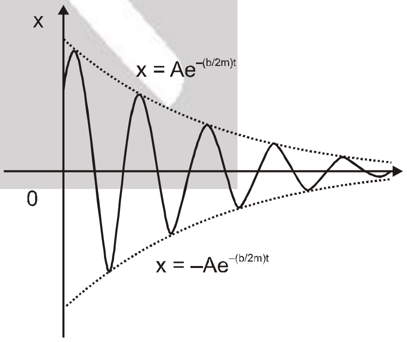
$$\left.\begin{matrix} r_1 \\ r_2 \end{matrix}\right\} = -\frac{b}{2m} \pm \omega' i$$

Where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ .....(3)

The solution is given by

$$x = e^{-(b/2m)t}(b_1 \cos \omega't + b_2 \sin \omega't) = Ae^{-(b/2m)t}\cos(\omega't+\phi)$$

We see that there are oscillations the are damped by the factor $e^{-(b/2m)t}$. Since $b > 0$ and $m > 0$, we have $-(b/2m) < 0$ so $e^{-(c/2m)t} \to 0$ as $t \to \infty$. This implies that $x \to 0$ as $t \to \infty$; that is, the motion decays to 0 as time increases. A graph is shown in figure.



Now the mechanical energy of the undamped oscillator is $1/2\ kA^2$. For a damped oscillator, the amplitude is not constant but depends on time. For small damping, we may use the same expression but regard the amplitude as $A\ e^{-bt/2m}$.

$$E(t) = \frac{1}{2}kA^2e^{-bt/m}$$ .....(4)

Equation shows that the total energy of the system decreases exponentially with time. Note that small damping means that is $\left(\frac{b}{2m}\right)$ much less than ω. Of course, as expected, if we put $b = 0$, all equations of a damped oscillator in this section reduce to the corresponding equations of an undamped oscillator.

Quality Factor or Q value:

Practical applications require consideration of the **quality** of the oscillator, *Q,* which specifies the ratio of total energy, E, to the energy loss, ΔE, over one complete oscillation period, T:

$$Q = 2\pi \frac{E}{|\Delta E|}$$

$$Q = \frac{\omega'}{2\omega_\gamma}$$

where $\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$ and $\omega_\gamma = \frac{b}{2m}$

In the limit of zero damping, the oscillator experiences no energy loss, and $Q \to \infty$. In the limit of small damping, the quality of the oscillator can be approximated by

$$Q \approx \frac{\omega}{2\omega_\gamma}$$

Combining these two results provides a handy formula for the energy loss during a complete oscillation period of weakly damped motion:

$$|\Delta E| = E\frac{2\pi}{Q} = E\frac{4\pi\omega_\gamma}{\omega} \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

Example 1. For the damped oscillator shown in Figure the mass m of the block is 200 g, k = 90 N m^{-1} and the damping constant b is 40 g s^{-1}. Calculate (a) the period of oscillation, (b) time taken for its amplitude of vibrations to drop to half of its initial value and (c) the time taken for its mechanical energy to drop to half its initial value.

Answer : (a) The time period T from Equation. is given by

$$T = \frac{2\pi}{\omega'} = \frac{2\pi}{\sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}} = 0.3\,\text{sec}$$

(b) Now, from equation, the time, $T_{1/2}$ for the amplitude drop half of its initial value is given by

$$A(t) = Ae^{-bt/2m}$$

$$\frac{A}{2} = Ae^{-\frac{bT_{1/2}}{2m}} \Rightarrow T_{1/2} = \frac{2m\ell n2}{b} = 6.93 \text{ s}$$

(c) For calculating the time, $t_{1/2}$, for its mechanical energy to drop to half its initial value we make use of equation from this equation we have.

$$E(t_{1/2}) = \frac{E(0)}{2} \Rightarrow E(0)e^{-bt/m} = \frac{E(0)}{2}$$

$$t_{1/2} = \frac{(\ell n2)m}{b} = \frac{6.93}{2}$$

This is just half of the decay period for amplitude. This is not surprising, because, according to Equation 4, energy depends on the square of the amplitude. Notice that there is a factor of 2 in the exponents of the two exponentials.

Example 2. Show that the system $\frac{d^2x}{dt^2} + \frac{dx}{dt} + 3x = 0$ is underdamped, find its damped angular frequency

Solution : m = 1, b = 1, k = 3

$b^2 < 4mk$

So, the system is underdamped

$$\omega_d = \sqrt{\left(\frac{k}{m} - \frac{b^2}{4m^2}\right)} = \frac{\sqrt{11}}{2}\ s^{-1}$$

FORCED OSCILLATIONS AND RESONANCE

When a system (such as a simple pendulum or a block attached to a spring) is displaced from its equilibrium position and released, it oscillates with its natural frequency ω, and the oscillations are called free oscillations. All free oscillations eventually die out because of the ever present damping forces. However, an external agency can maintain these oscillations. These are called force or driven oscillations. We consider the

case when the external force is itself periodic, with a frequency ω_d called the driven frequency. A most important fact of forced periodic oscillations is that the system oscillates not with its natural frequency ω, but at the frequency ω_d of the external agency; the free oscillations die out due to damping. A most familiar example of forced oscillation is when a child in a garden swing periodically presses his feet against the ground (or someone else periodically gives the child a push) to maintain the oscillations. Suppose an external force F(t) of amplitude F_0 that varies periodically with time is applied to a damped oscillator. Such a force can be represented as,

$$F(t) = F_o \cos \omega_d t$$

The motion of a particle under the combined action of a linear restoring force, damping force and a time dependent driving force represented by Equation is given by,

$$m\, a(t) = -k\, x(t) - b\nu(t) + F_o \cos \omega_d t$$

Substituting d^2x/dt^2 for acceleration in Equation and rearranging it, we get

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_o \cos \omega_d t \qquad .....(5)$$

This is the equation of an oscillator of mass m on which a periodic force of (angular) frequency ω_d is applied. The oscillator initially oscillates with its natural frequency ω. When we apply the external periodic force, the

oscillations with the natural frequency die out, and then the body oscillates with the (angular) frequency of the external periodic force. Its displacement, after the natural oscillations die out, is given by

$$x(t) = A \cos (\omega_d t + \phi) \qquad .....(6)$$

where t is the time measured from the moment when we apply the periodic force. The amplitude A is a function of the forced frequency ω_d and the natural frequency ω. Analysis shows that it is given by

$$A = \frac{F_0}{\sqrt{\left(m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2\right)}} \qquad .....(7)$$

and $\quad \tan\phi = \dfrac{-v_0}{\omega_d x_0}$

where m is the mass of the particle and v_0 and x_0 are the velocity and the displacement of the particle at time t = 0, which is the moment when we apply the periodic force. Equation (7) shows that the amplitude of the forced oscillator depends on the (angular) frequency of the driving force. We can see a different behaviour

of the oscillator when ω_d is far from ω and when it is close to ω. We consider these two cases.

(a) Small Damping, Driving Frequency far from Natural Frequency : In this case, $\omega_d b$ will be much smaller than $m(\omega^2 - \omega_d^2)$, and we can neglect that term. Then Equation. (7) reduces to

$$A = \frac{F_0}{m\left(\omega^2 - \omega_d^2\right)} \qquad .....(8)$$

Figure (3) shows the dependence of the displacement amplitude of an oscillator on the angular frequency of the driving force for different amounts of damping present in the system. It may be noted that in all the cases the amplitude is greatest when $\omega_d / \omega = 1$. The curves in this figure show that smaller the damping, the taller and narrower is the resonance peak.

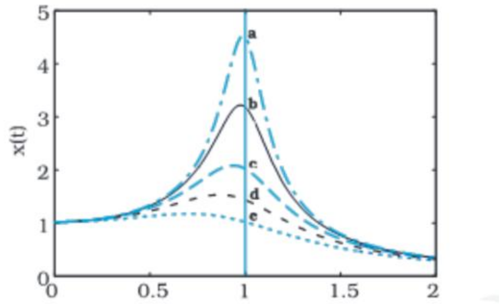


Figure. 3 The graphs illustrate equation (7). The resonant amplitude($\omega = \omega_d$) decreases with increasing damping.

If we go on changing the driving frequency, the amplitude tends to infinity when it equals the natural frequency. But this is the ideal case of zero damping, a case which never arises in a real system as the damping is never perfectly zero. You must have experienced in a swing that when the timing of your push exactly matches with the time period of the swing, your swing gets the maximum amplitude. This amplitude is large, but not infinity, because there is always some damping in your swing. This will become clear in the case (b).

(b) Driving Frequency Close to Natural Frequency : If ω_d is very close to ω, m $(\omega^2 - \omega_d^2)$ would be much less than $\omega_d b$, for any reasonable value of b, then Equation. (7) reduces to

$$A = \frac{F_0}{\omega_d b}$$

This makes it clear that the maximum possible amplitude for a given driving frequency is governed by the driving frequency and the damping, and is never infinity. The phenomenon of increase in amplitude when the driving force is close to the natural frequency of the oscillator is called resonance. In our daily life we encounter phenomena which involve resonance. Your experience with swings is a good example of resonance. You might have realised that the skill in swinging to greater heights lies in the synchronization of the rhythm of pushing against the ground with the natural frequency of the swing.

Example 3. A particle of mass m is attached to a spring (of spring constant k) and has a natural angular frequency ω_0. An external force F(t) proportional to cos ωt ($\omega \neq \omega_0$) is applied to the oscillator. The time displacement of the oscillator will be proportional to :

(1) $\frac{m}{\omega_0^2 - \omega^2}$ (2) $\frac{1}{m\ (\omega_0^2 - \omega^2)}$ (3) $\frac{1}{m\ (\omega_0^2 + \omega^2)}$ (4) $\frac{m}{\omega_0^2 + \omega^2}$

Solution : $x(t) = A\cos(\omega_d t + \phi_0)$

where $A = \frac{F_0}{\sqrt{m^2\left(\omega^2 - \omega_d^2\right)^2 + \omega_d^2 b^2}}$

for small damping

$A \approx \frac{F_0}{[m(\omega^2 - \omega_d^2)]}$ $\Rightarrow$ $x\ (t) = \left[\frac{F_0}{m(\omega^2 - \omega_d^2)}\right]\cos(\omega_d + \phi_0)$ Ans. (2)

Example. 4 In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force, then :

(1) $\omega_1 = \omega_2$ (2) $\omega_1 > \omega_2$

(3) $\omega_1 < \omega_2$ when damping is small and $\omega_1 > \omega_2$ when damping is large

(4) $\omega_1 < \omega_2$

Solution : Amplitude and energy both are maximum at resonance, when driving frequency is equal to the natural frequency of oscillation. Hence $\omega_1 = \omega_2 = \omega_0$

Ans. (1)

Example 5. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = τ s, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average life time of the pendulum is (assuming damping is small) in seconds :

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

Solution: $A(t) = A_0 e^{-bt/2m}$

$\frac{A_0}{e} = A_0 e^{-bt/2m} \Rightarrow \tau = \frac{2m}{b}$

In the given question $\frac{b}{m}$ is given as b. $\left(\because a = \frac{-kx}{m} - bv\right)$ **Ans (4)**

Example 6. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals.

(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

Solutions : $A(t) = A_0 e^{-bt/2m}$ Given $A(5) = 0.9 A_0$

$0.9A_0 = A_0 e^{-b(5)/2m}$ $A(15) = \alpha A_0$

$\alpha A_0 = A_0 e^{-b(15)/2m} \Rightarrow \alpha = (0.9)^3 = 0.729$ **Ans (3)**

Example 7. Determine whether the system is underdamped, overdamped or critically damped.

(i) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 3x = 0$ (ii) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 4x = 0$

Solution : $m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$

If $b^2 < 4mk$ then the system is underdamped

If $b^2 > 4mk$ then the system is overdamped

If $b^2 = 4mk$ then the system is critically damped

(i) overdamped (ii) Critically damped

Exercise-1

✍ Marked Questions can be used as Revision Questions.

PART - I : SUBJECTIVE QUESTIONS

Section (A) : Equation of SHM

A-1.✍ The equation of a particle executing SHM is $x = (5m)\sin\left[(\pi s^{-1})t + \frac{\pi}{6}\right]$. Write down the amplitude, initial phase constant, time period and maximum speed.

A-2. A particle having mass 10 g oscillates according to the equation x = (2.0 cm) sin [(100 s^{-1}) t+ π/6]. Find
(a) the amplitude, the time period and the force constant
(b) the position, the velocity and the acceleration at t = 0.

A-3.✍ A simple harmonic motion has an amplitude A and time period T. Find the time required by it to travel directly from
(a) x = 0 to x = A/2 (b) x = 0 to x = $\frac{A}{\sqrt{2}}$ (c) x = A to x = A/2 (d) x = $-\frac{A}{\sqrt{2}}$ to x = $\frac{A}{\sqrt{2}}$
(e) x = $\frac{A}{\sqrt{2}}$ to x = A.

A-4. A particle is executing SHM with amplitude A and has maximum velocity v_0. Find its speed when it is located at distance of A/2 from mean position.

A-5. A particle executes simple harmonic motion with an amplitude of 10 cm and time period 6 s. At t = 0 it is at position x = 5 cm from mean postion and going towards positive x-direction. Write the equation for the displacement x at time t. Find the magnitude of the acceleration of the particle at t = 4s.

A-6.✍ A particle is executing SHM. Find the positions of the particle where its speed is 8 cm/s, If maximum magnitudes of its velocity and acceleration are 10 cm/s and 50 cm/s^2 respectively.

Section (B) : Energy

B-1.✍ A particle performing SHM with amplitude 10 cm. At What distance from mean position the kinetic energy of the particle is thrice of its potential energy?

B-2.✍ An object of mass 0.2 kg executes simple harmonic oscillations along the x–axis with a frequency of (25 / π) Hz. At the position x = 0.04m, the object has kinetic energy of 0.5 J and potential energy 0.4 J. Find the amplitude of oscillations **[1994 ; 2M]**

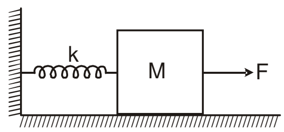
Section (C) : Spring mass system

C-1. A spring mass system has a time period of 2 second. What should be the spring constant of spring if the mass of the block is 10 grams ?

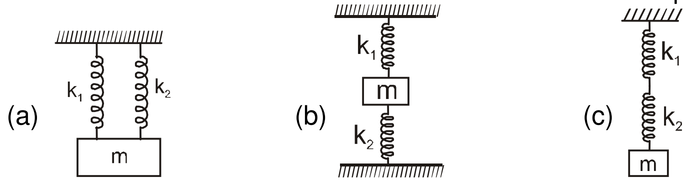
C-2.✍ A body of mass 2 kg suspended through a vertical spring executes simple harmonic motion of period 4s. If the oscillations are stopped and the body hangs in equilibrium, find the potential energy stored in the spring.

C-3. A vertical spring-mass system with lower end of spring is fixed, made to undergo small oscillations. If the spring is stretched by 25cm, energy stored in the spring is 5J .Find the mass of the block if it makes 5 oscillations each second.

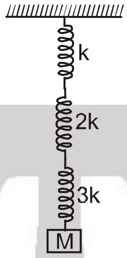
C-4. A spring mass system is shown in figure, spring is initially unstretched. A man starts pulling the block with constant force F. Find
(a) The amplitude and the time period of motion of the block
(b) The K.E. of the block at mean position
(c) The energy stored in the spring when the block passes through the mean position



C-5. Three spring mass systems are shown in figure. Assuming gravity free space, find the time period of oscillations in each case. What should be the answer if space is not gravity free ?



C-6. Spring mass system is shown in figure. Find the elastic potential energy stored in each spring when block is at its mean position. Also find the time period of vertical oscillations. The system is in vertical plane.



Section (D) : Simple Pendulum

D-1. Find the length of seconds pendulum at a place where $g = \pi^2$ m/s^2.

D-2. Instantaneous angle (in radian) between string of a simple pendulum and vertical is given by $\theta = \frac{\pi}{180}\sin 2\pi t$. Find the length of the pendulum if $g = \pi^2$ m/s^2

D-3. A pendulum clock is accurate at a place where g = 9.8 m/s^2. Find the value of g at another place where clock becomes slow by 24 seconds in a day (24 hrs).

D-4. A pendulum is suspended in a lift and its period of oscillation is T_0 when the lift is stationary.
(i) What will be the period T of oscillation of pendulum, if the lift begins to accelerate downwards with an acceleration equal to $\frac{3g}{4}$?
(ii) What must be the acceleration of the lift for the period of oscillation of the pendulum to be $\frac{T_0}{2}$?

Section (E) : Compound Pendulum & Torsional Pendulum

E-1. Compound pendulums are made of :
(a) A rod of length ℓ suspended through a point located at distance $\ell/4$ from centre of rod.
(b) A ring of mass m and radius r suspended through a point on its periphery.
(c) A uniform square plate of edge a suspended through a corner.
(d) A uniform disc of mass m and radius r suspended through a point r/2 away from the centre.
Find the time period in each case.

E-2. Two compound pendulums are made of :
(a) A disc of radius r and
(b) A uniform rod of length L. Find the minimum possible time period and distance between centre and point of suspension in each case.

Section (F) : Superposition of SHM

F-1. A particle is subjected to two SHM's simultaneously $X_1 = a_1\sin\omega t$ and $X_2 = a_2\sin(\omega t + \phi)$ Where a_1 = 3.0 cm, a_2 = 4.0 cm. Find resultant amplitude if the phase difference ϕ has values (a) 0° (b) 60° (c) 90°.

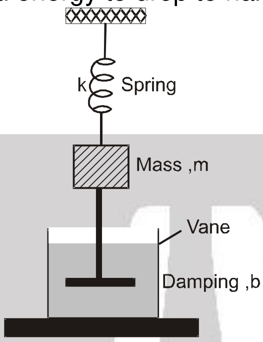
F-2. A particle is subjected to three SHM's in same direction simultaneously each having equal amplitude a and equal time period. The phase of the second motion is 30^0 ahead of the first and the phase of the third motion is 30° ahead of the second. Find the amplitude of the resultant motion.

F-3. A particle simultaneously participates in two mutually perpendicular oscillations $x = \sin \pi t$ & $y = 2\cos 2\pi t$. Write the equation of trajectory of the particle.

Section (G) : For JEE-Main

G-1. In forced oscillation of a particle, the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force. What is the relation between ω_1 and ω_2 ?

G-2. For the damped oscillator shown in Figure, the mass of the block is 200 g, k = 80 Nm^{-1} and the damping constant b is 40 gs^{-1}. Calculate
(a) The period of oscillation,
(b) Time taken for its amplitude of vibrations to drop to half of its initial value
(c) The time for the mechanical energy to drop to half initial value.



PART - II : ONLY ONE OPTION CORRECT TYPE

Section (A) : Equation of SHM

A-1. According to a scientists, he applied a force $F = -cx^{1/3}$ on a particle and the particle is performing SHM. No other force acted on the particle. He refuses to tell whether c is a constant or not. Assume that he had worked only with positive x then :
(A) as x increases c also increases
(B) as x increases c decreases
(C) as x increases c remains constant
(D) the motion cannot be SHM

A-2. A particle performing SHM takes time equal to T (time period of SHM) in consecutive appearances at a particular point. This point is :
(A) An extreme position
(B) The mean position
(C) Between positive extreme and mean position
(D) Between negative extreme and mean position

A-3. A particle executing linear SHM. Its time period is equal to the smallest time interval in which particle acquires a particular velocity $\vec{v}$, the magnitude of $\vec{v}$ may be :
(A) Zero
(B) V_{max}
(C) $\frac{V_{max}}{2}$
(D) $\frac{V_{max}}{\sqrt{2}}$

A-4. If $\vec{F}$ is force vector, $\vec{v}$ is velocity vector, $\vec{a}$ vector is acceleration vector and $\vec{r}$ vector is displacement vector with respect to mean position than which of the following quantities are always non-negative in a simple harmonic motion along a straight line?
(A) $\vec{F}.\vec{a}$
(B) $\vec{v}.\vec{r}$
(C) $\vec{a}.\vec{r}$
(D) $\vec{F}.\vec{r}$

A-5. Two SHM's are represented by y = a sin ($\omega t - \phi$) and y = b cos ($\omega t - \phi$). The phase difference between the two is :
(A) $\frac{\pi}{2}$
(B) $\frac{\pi}{4}$
(C) $\frac{\pi}{6}$
(D) $\frac{3\pi}{4}$

A-6. How long after the beginning of motion is the displacement of a harmonically oscillating particle equal to one half its amplitude if the period is 24s and particle starts from rest.
(A) 12s
(B) 2s
(C) 4s
(D) 6s

A-7. The magnitude of average acceleration in half time period from equilibrium position in a simple harmonic motion is
(A) $\frac{2A\omega^2}{\pi}$
(B) $\frac{A\omega^2}{2\pi}$
(C) $\frac{A\omega^2}{\sqrt{2}\pi}$
(D) Zero

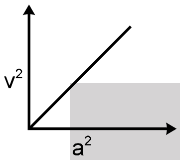
A-8. A particle performing SHM on the y axis according to equation y = A + B sinωt. Its amplitude is :

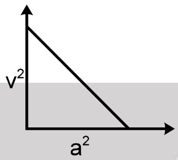
(A) A (B) B (C) A + B (D) $\sqrt{A^2 + B^2}$

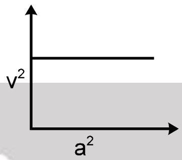
A-9. Two particles execute S.H.M. of same amplitude and frequency along the same straight line from same mean position. They cross one another without collision, when going in opposite directions, each time their displacement from mean position is half of their amplitudes. The phase-difference between them is

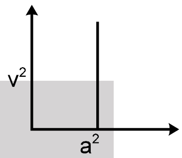
(A) 0° (B) 120° (C) 180° (D) 135°

A-10. A mass M is performing linear simple harmonic motion, then correct graph for acceleration a and corresponding linear velocity v is

(A) 

(B) 

(C) 

(D) 

Section (B) : Energy

B-1. A body executing SHM passes through its equilibrium. At this instant, it has

(A) maximum potential energy (B) maximum kinetic energy
(C) minimum kinetic energy (D) maximum acceleration

B-2. The K.E. and P.E of a particle executing SHM with amplitude A will be equal when its displacement is

(A) $\sqrt{2}A$ (B) $\frac{A}{2}$ (C) $\frac{A}{\sqrt{2}}$ (D) $\sqrt{\frac{2}{3}}A$

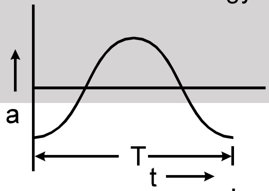
B-3. A point particle of mass 0.1 kg is executing S.H.M. of amplitude of 0.1 m. When the particle passes through the mean position, its kinetic energy is 8×10^{-3}J. The equation of motion of this particle when the initial phase of oscillation is 45° can be given by **[I.I.T. 1991]**

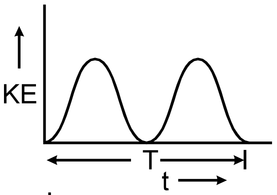
(A) $0.1\cos\left(4t + \frac{\pi}{4}\right)$ (B) $0.1\sin\left(4t + \frac{\pi}{4}\right)$ (C) $0.4\sin\left(t + \frac{\pi}{4}\right)$ (D) $0.2\sin\left(\frac{\pi}{2} + 2t\right)$

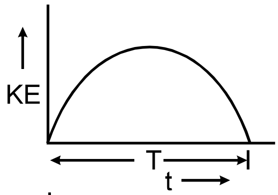
B-4. For a particle performing SHM :

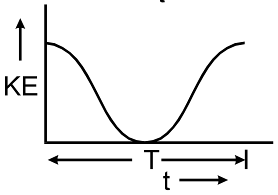
(A) The kinetic energy is never equal to the potential energy
(B) the kinetic energy is always equal to the potential energy
(C) The average kinetic energy in one time period is equal to the average potential in this period
(D) The average kinetic energy in any time interval is equal to average potential energy in that interval

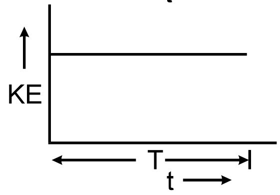
B-5. Acceleration a versus time t graph of a body in SHM is given by a curve shown below. T is the time period. Then corresponding graph between kinetic energy KE and time t is correctly represented by



(A) 

(B) 

(C) 

(D) 

B-6. A particle performs S.H.M. of amplitude A along a straight line. When it is at a distance $\frac{\sqrt{3}}{2}$ A from mean position, its kinetic energy gets increased by an amount $\frac{1}{2}m\omega^2A^2$ due to an impulsive force. Then its new amplitude becomes:

(A) $\frac{\sqrt{5}}{2}$ A (B) $\frac{\sqrt{3}}{2}$ A (C) $\sqrt{2}$ A (D) $\sqrt{5}$ A

Section (C) : Spring mass system

C-1. Two spring mass systems have equal mass and spring constant k_1 and k_2.If the maximum velocities in two systems are equal then ratio of amplitude of 1st to that of 2nd is :

(A) $\sqrt{k_1/k_2}$ (B) k_1/k_2 (C) k_2/k_1 (D) $\sqrt{k_2/k_1}$

C-2. A toy car of mass m is having two similar rubber ribbons attached to it as shown in the figure. The force constant of each rubber ribbon is k and surface is frictionless. The car is displaced from mean position by x cm and released. At the mean position the ribbons are undeformed. Vibration period is

(A) $2\pi\sqrt{\frac{m(2k)}{k^2}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{m(2k)}{k^2}}$ (C) $2\pi\sqrt{\frac{m}{k}}$ (D) $2\pi\sqrt{\frac{m}{k+k}}$

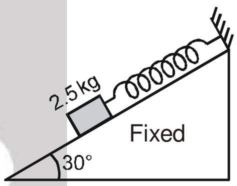
C-3. A mass of 1 kg attached to the bottom of a spring has a certain frequency of vibration. The following mass has to be added to it in order to reduce the frequency by half : **[REE – 1988]**

(A) 1 kg (B) 2 kg (C) 3 kg (D) 4 kg

C-4. A ball of mass m kg hangs from a spring of spring constant k. The ball oscillates with a period of T seconds. If the ball is removed, the spring is shortened (with respect to length in mean position) by

(A) $\frac{gT^2}{(2\pi)^2}$ metre (B) $\frac{3T^2g}{(2\pi)^2}$ metre (C) $\frac{Tm}{k}$ metre (D) $\frac{Tk}{m}$ metre

C-5. A smooth inclined plane having angle of inclination 30° with horizontal has a mass 2.5 kg held by a spring which is fixed at the upper end as shown in figure. If the mass is taken 2.5 cm up along the surface of the inclined plane, the tension in the spring reduces to zero. If the mass is then released, the angular frequency of oscillation in radian per second is

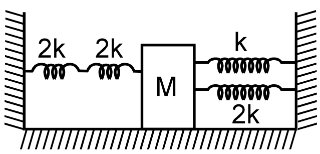


(A) 0.707 (B) 7.07 (C) 1.414 (D) 14.14

C-6. A particle executes simple harmonic motion under the restoring force provided by a spring. The time period is T. If the spring is divided in two equal parts and one part is used to continue the simple harmonic motion, the time period will

(A) remain T (B) become 2T (C) become T/2 (D) become T/$\sqrt{2}$

C-7. Four massless springs whose force constants are 2k, 2k, k and 2k respectively are attached to a mass M kept on a frictionless plane (as shown in figure). If the mass M is displaced in the horizontal direction, then the frequency of the system. **[JEE 1990]**



(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{7M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{7k}{M}}$

C-8. The total mechanical energy of a particle of mass m executing SHM with the help of a spring is $E = (1/2)m\omega^2A^2$. If the particle is replaced by another particle of mass m/2 while the amplitude A remains same. New mechanical energy will be :

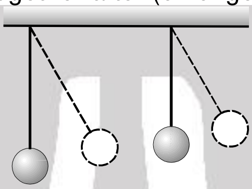
(A) $\sqrt{2}$ E (B) 2E (D) E/2 (D) E

Section (D) : Simple Pendulum

D-1. Two pendulums begin to swing simultaneously. The first pendulum makes 9 full oscillations when the other makes 7. Find the ratio of length of the two pendulums.

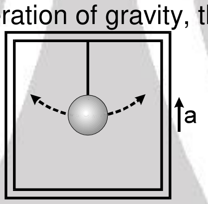
(A) $\frac{49}{81}$ (B) $\frac{7}{9}$ (C) $\frac{50}{81}$ (D) $\frac{1}{2}$

D-2. Two pendulums at rest start swinging together. Their lengths are respectively 1.44 m and 1 m. They will again start swinging in same phase together after (of longer pendulum) :



(A) 1 vibration (B) 3 vibrations (C) 4 vibrations (D) 5 vibrations

D-3. A scientist measures the time period of a simple pendulum as T in a lift at rest. If the lift moves up with acceleration as one fourth of the acceleration of gravity, the new time period is



(A) $\frac{T}{4}$ (B) 4 T (C) $\frac{2}{\sqrt{5}}$ T (D) $\frac{\sqrt{5}}{2}$ T

D-4. A simple pendulum has some time period T. What will be the percentage change in its time period if its amplitude is decreased by 5%?

(A) 6 % (B) 3 % (C) 1.5 % (D) 0 %

D-5. A simple pendulum with length ℓ and bob of mass m executes SHM of small amplitude A. The maximum tension in the string will be

(A) $mg(1 + A/\ell)$ (B) $mg(1 + A/\ell)^2$ (C) $mg[1 + (A/\ell)^2]$ (D) 2 mg

Section (E) : Compound Pendulum & Torsional Pendulum

E-1. A 25 kg uniform solid sphere with a 20 cm radius is suspended by a vertical wire such that the point of suspension is vertically above the centre of the sphere. A torque of 0.10 N-m is required to rotate the sphere through an angle of 1.0 rad and then the orientation is maintained. If the sphere is then released, its time period of the oscillation will be :

(A) π second (B) $\sqrt{2}\,\pi$ second (C) 2π second (D) 4π second

E-2. A metre stick swinging about its one end oscillates with frequency f_0. If the bottom half of the stick was cut off, then its new oscillation frequency will be :

(A) f_0 (B) $\sqrt{2}\,f_0$ (C) $2f_0$ (D) $2\,f_0$

Section (F) : Superposition of SHM

F-1. When two mutually perpendicular simple harmonic motions of same frequency, amplitude and phase are superimposed

(A) the resulting motion is uniform circular motion.

(B) the resulting motion is a linear simple harmonic motion along a straight line inclined equally to the straight lines of motion of component ones.

(C) the resulting motion is an elliptical motion, symmetrical about the lines of motion of the components.

(D) the two S.H.M. will cancel each other.

F-2. The position of a particle in motion is given by y = Csinωt + Dcosωt w.r.t. origin. Then motion of the particle is:

(A) SHM with amplitude C+D
(B) SHM with amplitude $\sqrt{C^2 + D^2}$
(C) SHM with amplitude $\frac{(C+D)}{2}$
(D) not SHM

F-3. A simple harmonic motion is given by y = 5 (sin 3πt + $\sqrt{3}$ cos 3πt). What is the amplitude of motion if y is in m ?

(A) 100 cm (B) 5 m (C) 200 cm (D) 1000 cm

F-4. The position vector of a particle moving in x-y plane is given by $\vec{r} = (A\sin\omega t)\hat{i} + (A\cos\omega t)\hat{j}$ then motion of the particle is :

(A) SHM (B) on a circle (C) on a straight line (D) with constant acceleration

Section (G) : For JEE Main

G-1. When an oscillator completes 100 oscillations its ampliutde reduced to $\frac{1}{3}$ of initial value. What will be its amplitude, when it completes 200 oscillations : **[AIPMT 2002]**

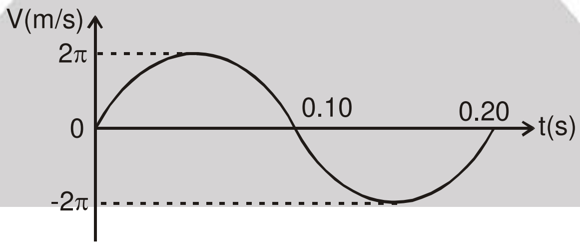
(A) $\frac{1}{8}$ (B) $\frac{2}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{9}$

G-2. The damping force on an oscillator is directly proportional to the velocity. The units of the constant of proportionality are :

(A) $kgms^{-1}$ (B) $kgms^{-2}$ (C) kgs^{-1} (D) kgs

PART - III : MATCH THE COLUMN

1. A simple harmonic oscillator consists of a block attached to a spring with k = 200 N/m. The block slides on a frictionless horizontal surface, with equilibrium point x = 0. A graph of the block's velocity v as a function of time t is shown. Correctly match the required information in the left column with the values given in the right column. (Use π^2 = 10)



Left Column	Right Column
(A) The block's mass in kg	(p) – 0.20
(B) The block's displacement at t = 0 in metres	(q) – 200
(C) The block's acceleration at t = 0.10 s in m/s^2	(r) 0.20
(D) The block's maximum kinetic energy in Joule	(s) 4.0

2.✍ In the column-I, a system is described in each option and corresponding time period is given in the column-II. Suitably match them.

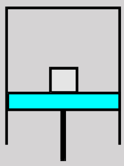
Column-I	Column-II
(A) A simple pendulum of length 'ℓ' oscillating with small amplitude in a lift moving down with retardation g/2.	(p) $T = 2\pi\sqrt{\frac{2\ell}{3g}}$
(B) A block attached to an end of a vertical spring, whose other end is fixed to the ceiling of a stationary lift, stretches the spring by length 'ℓ' in equilibrium. It's time period when lift moves up with an acceleration g/2 is	(q) $T = 2\pi\sqrt{\frac{\ell}{g}}$
(C) The time period of small oscillation of a uniform rod of length 'ℓ' smoothly hinged at one end. The rod oscillates in vertical plane.	(r) $T = 2\pi\sqrt{\frac{2\ell}{g}}$
(D) A cubical block of edge 'ℓ' and specific gravity 1/2 is in equilibrium with some volume inside water filled in a large fixed container. Neglect viscous forces and surface tension. The time period of small oscillations of the block in vertical direction is	(s) $T = 2\pi\sqrt{\frac{\ell}{2g}}$

Exercise-2

✍ **Marked Questions can be used as Revision Questions.**

PART - I : ONLY ONE OPTION CORRECT TYPE

1. A block of mass m is resting on a piston as shown in figure which is moving vertically with a SHM of period 1 s. The minimum amplitude of motion at which the block and piston separate is :

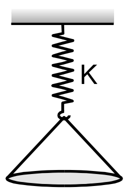


(A) 0.25 m (B) 0.52 m (C) 2.5 m (D) 0.15 m

2.✍ The potential energy of a particle of mass 'm' situated in a unidimensional potential field varies as $U(x) = U_0\,[1 - \cos ax]$, where U_0 and a are constants. The time period of small oscillations of the particle about the mean position :

(A) $2\pi\sqrt{\frac{m}{aU_0}}$ (B) $2\pi\sqrt{\frac{am}{U_0}}$ (C) $2\pi\sqrt{\frac{m}{a^2U_0}}$ (D) $2\pi\sqrt{\frac{a^2m}{U_0}}$

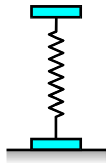
3.✍ A solid ball of mass m is made to fall from a height H on a pan suspended through a spring of spring constant K as shown in figure. If the ball does not rebound and the pan is massless, then amplitude of oscillation is



(A) $\frac{mg}{K}$ (B) $\frac{mg}{k}\left(1+\frac{2HK}{mg}\right)^{1/2}$

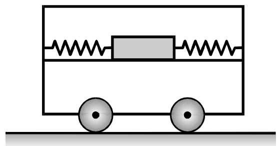
(C) $\frac{mg}{K}+\left(\frac{2HK}{mg}\right)^{1/2}$ (D) $\frac{mg}{K}\left[1+\left(1+\frac{2HK}{mg}\right)^{1/2}\right]$

4. Two plates of same mass are attached rigidly to the two ends of a spring as shown in figure. One of the plates rests on a horizontal surface and the other results a compression y of the spring when it is in equilibrium state. The further minimum compression required, so that after the force causing compression is removed the lower plate is lifted off the surface, will be :



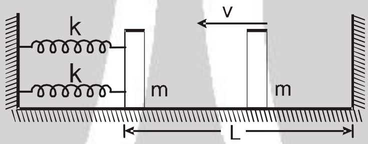
(A) 0.5 y (B) 3y (C) 2y (D) y

5. Two springs, each of spring constant k, are attached to a block of mass m as shown in the figure. The block can slide smoothly along a horizontal platform clamped to the opposite walls of the trolley of mass M. If the block is displaced by x cm and released, the period of oscillation is :



(A) $T = 2\pi\sqrt{\frac{Mm}{2k}}$ (B) $T = 2\pi\sqrt{\frac{(M+m)}{kmM}}$ (C) $T = 2\pi\sqrt{\frac{mM}{2k(M+m)}}$ (D) $T = 2\pi\,\frac{(M+m)^2}{k}$

6. The right block in figure moves at a speed V towards the left block placed in equilibrium. All the surfaces are smooth and all the collisions are elastic. Find the time period of periodic motion. Neglect the width of the blocks.



(A) $\pi\sqrt{\frac{m}{2k}} + \frac{2L}{v}$ (B) $\pi\sqrt{\frac{m}{2k}} + \frac{L}{v}$ (C) $\pi\sqrt{\frac{m}{2k}} - \frac{L}{v}$ (D) $\pi\sqrt{\frac{m}{k}} + \frac{L}{v}$

7. The bob in a simple pendulum of length ℓ is released at t = 0 from the position of small angular displacement θ_0. Linear displacement of the bob at any time t from the mean position is given by

(A) $\ell\theta_0 \cos\sqrt{\frac{g}{\ell}}t$ (B) $\ell\sqrt{\frac{g}{\ell}}\,t\cos\theta_0$ (C) $\ell g\sin\theta_0$ (D) $\ell\theta_0 \sin\sqrt{\frac{g}{\ell}}t$

8. The period of small oscillations of a simple pendulum of length ℓ if its point of suspension O moves a with a constant acceleration $\alpha = \alpha_1\hat{i} - \alpha_2\hat{j}$ with respect to earth is ($\hat{i}$ and $\hat{j}$ are unit vectors in horizontal and vertically upward directions respectively)

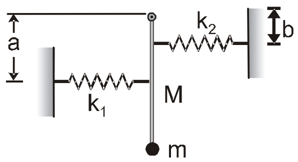
(A) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_2)^2 + \alpha_1^2\}^{1/2}}}$ (B) $T = 2\pi\sqrt{\frac{\ell}{\{(g-\alpha_1)^2 + \alpha_2^2\}^{1/2}}}$

(C) $T = 2\pi\sqrt{\frac{\ell}{g}}$ (D) $T = 2\pi\sqrt{\frac{\ell}{(g^2+\alpha_1^2)^{1/2}}}$

9. A simple pendulum ; a physical pendulum; a torsional pendulum and a spring–mass system, each of same frequency are taken to the Moon. If frequencies are measured on the moon, which system or systems will have it unchanged ?

(A) spring–mass system and torsional pendulum. (B) only spring–mass system.
(C) spring–mass system and physical pendulum. (D) None of these

10. A rod of mass M and length L is hinged at its one end and carries a particle of mass m at its lower end. A spring of force constant k_1 is installed at distance a from the hinge and another of force constant k_2 at a distance b as shown in the figure. If the whole arrangement rests on a smooth horizontal table top, the frequency of vibration is



(A) $\frac{1}{2\pi}\sqrt{\frac{k_1a^2 + k_2b^2}{L^2(m+\frac{M}{3})}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k_2+k_1}{M+m}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k_2 + k_1\frac{a^2}{b^2}}{4\frac{M}{3}+m}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{k_1 + \frac{k_2b^2}{a^2}}{\frac{4}{3}m+M}}$

11. A particle moves along the X-axis according to the equation x = 10 $\sin^3$ (πt). The amplitudes and frequencies of component SHMs are
(A) amplitude 30/4, 10/4 ; frequencies 3/2, 1/2 (B) amplitude 30/4, 10/4 ; frequencies 1/2, 3/2
(C) amplitude 10, 10 ; frequencies 1/2, 1/2 (D) amplitude 30/4, 10 ; frequencies 3/2, 2

12. The amplitude of a particle due to superposition of following S.H.Ms. Along the same line is
$X_1 = 2 \sin 50\pi t$; $X_2 = 10 \sin (50\pi t + 37^\circ)$
$X_3 = -4 \sin 50\pi t$; $X_4 = -12 \cos 50\pi t$
(A) $4\sqrt{2}$ (B) 4 (C) $6\sqrt{2}$ (D) none of these

13._ When a body is suspended from a fixed point by a spring, the angular frequency of its vertical oscillations is ω_1. When a different spring is used, the angular frequency is ω_2. The angular frequency of vertical oscillations when both the springs are used together in series is given by **[Olympiad (Stage-1) 2017]**

(A) $\omega = \left[\omega_1^2 + \omega_2^2\right]^{\frac{1}{2}}$ (B) $\omega = \left[\frac{\omega_1^2 + \omega_2^2}{2}\right]^{\frac{1}{2}}$ (C) $\omega = \left[\frac{\omega_1^2\omega_2^2}{\left(\omega_1^2 + \omega_2^2\right)}\right]^{\frac{1}{2}}$ (D) $\omega = \left[\frac{\omega_1^2\omega_2^2}{2\left(\omega_1^2 + \omega_2^2\right)}\right]^{\frac{1}{2}}$

14._ A particle performs simple harmonic motion at a frequency f. The frequency at which its kinetic energy varies is : **[Olympiad (Stage-1) 2017]**
(A) f (B) 2f (C) 4f (D) f/2

15._ A particle rests in equilibrium under two forces of repulsion whose centres are at distance of a and b from the particle. The forces vary as the cube of the distance. The forces per unit mass are k and k' respectively. If the particle be slightly displaced towards one of them the motion is simple harmonic with the time period equal to **[Olympiad (Stage-1) 2017]**

(A) $\frac{2\pi}{\sqrt{3\left(\frac{k}{a^3} + \frac{k'}{b^3}\right)}}$ (B) $\frac{2\pi}{\sqrt{\left(\frac{k}{a^3} + \frac{k'}{b^3}\right)}}$ (C) $\frac{2\pi}{\sqrt{\left(\frac{k}{a^4} + \frac{k'}{b^4}\right)}}$ (D) $\frac{2\pi}{\sqrt{3\left(\frac{k}{a} + \frac{k'}{b}\right)}}$

PART - II : SINGLE AND DOUBLE VALUE INTEGER TYPE

1. Two particles A and B are performing SHM along x and y-axis respectively with equal amplitude and frequency of 2 cm and 1 Hz respectively. Equilibrium positions of the particles A and B are at the co-ordinates (3, 0) and (0, 4) respectively. At t = 0, B is at its equilibrium position and moving towards the origin, while A is nearest to the origin and moving away from the origin. If the maximum and minimum distances between A and B is s_1 and s_2 then find $s_1 + s_2$ (in cm).

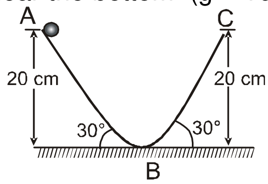
2. Two particles P and Q describe S.H.M. of same amplitude a, same frequency f along the same straight line from the same mean position. The maximum distance between the two particles is $a\sqrt{2}$. If the initial phase difference between the particles is π/N then find N:

3. A street car moves rectilinearly from station A (here car stops) to the next station B (here also car stops) with an acceleration varying according to the law f = a – bx, where a and b are positive constants and x is the distance from station A. If the maximum distance between the two stations is x = $\frac{Na}{b}$ then find N.

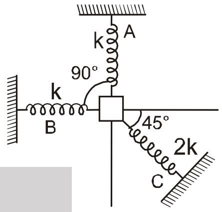
4. A particle is oscillating in a straight line about a centre O, with a force directed towards O. When at a distance 'x' from O, the force is mn^2x where 'm' is the mass and 'n' is a constant. The amplitude is a = 15 cm. When at a distance $\sqrt{3}\frac{a}{2}$ from O the particle receives a blow in the direction of motion which generates an extra velocity na. If the velocity is away from O at the time of blow and the new amplitude becomes $k\sqrt{3}$ cm, then find k.

5. Two particles P_1 and P_2 are performing SHM along the same line about the same mean position. Initially they are at their positive extreme positions. If the time period of each particle is 12 sec and the difference of their amplitudes is 12 cm then find the minimum time after which the separation between the particles become 6 cm.

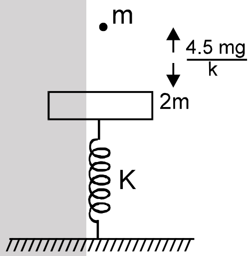
6. Assuming all the surfaces to be smooth, if the time period of motion of the ball is N × 10^{-1} sec then find N. Neglect the small effect of bend near the bottom. (g = 10m/s²)



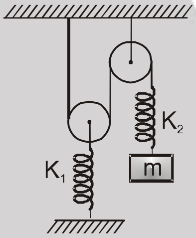
7. A block of mass m is attached to three springs A, B and C having force constants k, k and 2k respectively as shown in figure. If the block is slightly pushed against spring C. If the angular frequency of oscillations is $\sqrt{\frac{Nk}{m}}$, then find N. The system is placed on horizontal smooth surface.



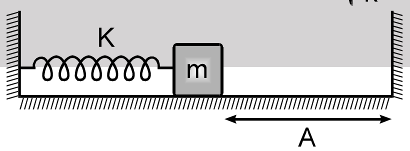
8. In the figure shown mass 2m is at rest and in equilibrium. A particle of mass m is released from height $\frac{4.5mg}{k}$ from plate. The particle sticks to the plate. Neglecting the duration of collision. Starting from the time when the particle sticks to plate to the time when the spring is in maximum compression for the first time is $2\pi\sqrt{\frac{m}{ak}}$ then find a.



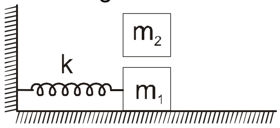
9. For given spring mass system, If the time period of small oscillations of block about its mean position is $\pi\sqrt{\frac{nm}{K}}$, then find n. Assume ideal conditions. The system is in vertical plane and take $K_1 = 2K$, $K_2 = K$.



10. In the figure shown the spring is relaxed and mass m is attached to the spring. The spring is compressed by 2 A and released at t = 0. Mass m collides with the wall and loses two third of its kinetic energy and returns. Starting from t = 0, find the time taken by it to come back to rest again (instant at which spring is again under maximum compression). Take $\sqrt{\frac{m}{k}} = \frac{12}{\pi}$



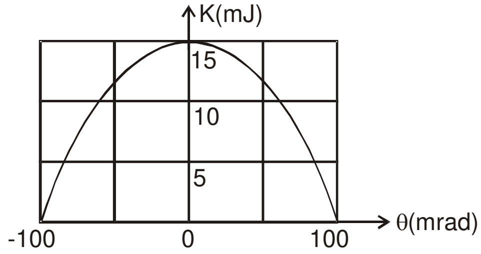
11. A block of mass 4kg attached with spring of spring constant 100 N/m is executing SHM of amplitude 0.1m on smooth horizontal surface as shown in figure. If another block of mass 5 kg is gently placed on it, at the instant it passes through the mean position and new amplitude of motion is n^{-1} meter then find n. Assuming that two blocks always move together.



12. The period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle which moves without friction down on inclined plane of inclination $\alpha = 60°$ is given by $\pi\sqrt{\frac{XL}{g}}$ then find X.

[I.I.T. (Scr.) 2000, 1/35]

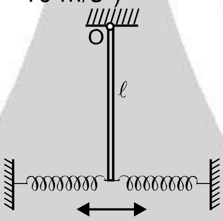
13. Figure shows the kinetic energy K of a simple pendulum versus its angle θ from the vertical. The pendulum bob has mass 0.2 kg. If the length of the pendulum is equal to n/g meter, then find n ($g = 10\ m/s^2$).



14. The bob of a simple pendulum executes simple harmonic motion in water with a period 't', while the period of oscillation of the bob is t_0 in air. Neglecting frictional force of water and given that the density of the bob is (4/3) × 1000 kg/m³. Find $\frac{t}{t_0}$. **[AIEEE 2004]**

15. A solid sphere of radius R is half submerged in a liquid of density ρ. The sphere is slightly pushed down and released, If the frequency of small oscillations is $\sqrt{\frac{3}{nR}}$, then find n. Take $\pi = \sqrt{g}$. **[JEE (Mains) 2004, 2/60]**

16. If the angular frequency of small oscillations of a thin uniform vertical rod of mass m and length ℓ hinged at the point O (Fig.) is $\sqrt{\frac{n}{\ell}}$, then find n. The force constant for each spring is K/2 and take $K = \frac{2mg}{\ell}$. The springs are of negligible mass. ($g = 10\ m/s^2$)



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

1. A particle moves on the X-axis according to the equation $x = x_0 \sin^2 \omega t$. The motion is simple harmonic

(A) with amplitude $x_0/2$ (B) with amplitude $2x_0$ (C) with time period $\frac{2\pi}{\omega}$ (D)with time period $\frac{\pi}{\omega}$

2. Which of the following functions represent SHM?

(A) $\sin 2\omega t$ (B) $\sin^2 \omega t$ (C) $\sin \omega t + 2 \cos \omega t$ (D) $\sin \omega t + \cos 2\omega t$

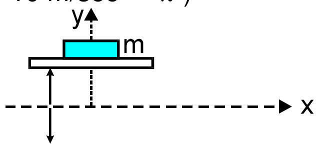
3. The speed v of a particle moving along a straight line, when it is at a distance (x) from a fixed point of the line is given by $v^2 = 108 - 9x^2$ (assuming mean position to have zero phase constant) (all quantities are in cgs units) :

(A) the motion is uniformly accelerated along the straight line
(B) the magnitude of the acceleration at a distance 3cm from the fixed point is 27 cm/s²
(C) the motion is simple harmonic about the given fixed point.
(D) the maximum displacement from the fixed point is 4 cm.

4. A horizontal plank has a rectangular block placed on it. The plank starts oscillating vertically and simple harmonically with an amplitude of 40 cm. The block just loses contact with the plank when the latter is at momentary rest. Then :

(A) the period of oscillation is $\left(\frac{2\pi}{5}\right)$ seconds
(B) the block weighs double its weight, when the plank is at one of the positions of momentary rest.
(C) the block weighs 0.5 times its weight on the plank halfway up
(D) the block weighs 1.5 times its weight on the plank halfway down
(E) the block weights its true weight on the plank when the latter moves fastest

5. As shown in figure a horizontal platform with a mass m placed on it is executing SHM along y-axis. If the amplitude of oscillation is 2.5 cm, the minimum period of the motion for the mass not to be detached from the platform is :(g = 10 m/sec^2 = π^2)



(A) $\frac{10}{\pi}$ s (B) $\frac{\pi}{10}$ s (C) $\frac{\pi}{\sqrt{10}}$ s (D) $\frac{1}{\sqrt{10}}$ s.

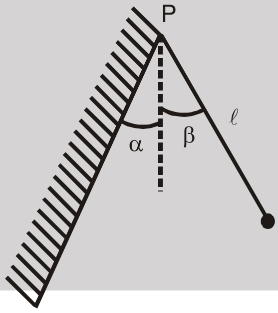
6. For a body executing SHM with amplitude A, time period T, maximum velocity v_{max} and phase constant zero, which of the following statements are correct for $0 \le t \le \frac{T}{4}$ (y is displacement from mean position) ?

(A) At y = (A/2), v > (v_{max}/2)
(B) for v = (v_{max}/2), y > (A/2)
(C) For t = (T/8), y > (A/2)
(D) For y = (A/2), t < (T/8)

7. The potential energy of a particle of mass 0.1 kg, moving along the x-axis, is given by U = 5x (x – 4) J, where x is in meters. It can be concluded that
(A) the particle is acted upon by a constant force
(B) the speed of the particle is maximum at x = 2 m
(C) the particle executes SHM
(D) the period of oscillation of the particle is (π/5) sec

8. A particle free to move along the x-axis has potential energy given by $U(x) = k[1 - e^{-x^2}]$ for $-\infty \le x \le +\infty$, where k is a positive constant of appropriate dimensions. Then select the incorrect options : **[JEE - 1999, 2/200]**
(A) at points away from the origin, the particle is in unstable equilibrium.
(B) for any finite non-zero value of x, there is a force directed away from the origin.
(C) if its total mechanical energy is k/2, it has its minimum kinetic energy at the origin.
(D) for small displacements from x = 0, the motion is simple harmonic.

9. A ball is hung vertically by a thread of length '' from a point 'P' of an inclined wall that makes an angle 'α' with the vertical. The thread with the ball is then deviated through a small angle 'β' ($\beta > \alpha$) and set free. Assuming the wall to be perfectly elastic, the period of such pendulum is/are



(A) $2\sqrt{\frac{\ell}{g}}\left[\sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$
(B) $2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \sin^{-1}\left(\frac{\alpha}{\beta}\right)\right]$
(C) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(\frac{\alpha}{\beta}\right)\right]$
(D) $2\sqrt{\frac{\ell}{g}}\left[\cos^{-1}\left(-\frac{\alpha}{\beta}\right)\right]$

10. If a SHM is given by y = (sin ωt + cos ωt) m, which of the following statements are true?
(A) The amplitude is 1m
(B) The amplitude is $\sqrt{2}$ m
(C) Time is considered from y = 1 m
(D) Time is considered from y = 0 m

11. The position of a particle at time t moving in x-y plane is given by $\vec{r} = (\hat{i} + 2\hat{j})$ A cos ωt. Then, the motion of the particle is :
(A) on a straight line (B) on an ellipse (C) periodic (D) SHM

12. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45^0, then, **[IIT- 1999, 3/100]**
(A) the resultant amplitude is $(1+\sqrt{2})a$
(B) the phase of the resultant motion relative to the first is 90^0.
(C) the energy associated with the resulting motion is $(3+2\sqrt{2})$ times the energy associated with any single motion.
(D) the resulting motion is not simple harmonic.

PART - IV : COMPREHENSION

Comprehension-1

A 2kg block hangs without vibrating at the bottom end of a spring with a force constant of 400 N/m. The top end of the spring is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of 5 m/s^2 when the acceleration suddenly ceases at time t = 0 and the car moves upward with constant speed. (g = 10 m/s^2)

1. What is the angular frequency of oscillation of the block after the acceleration ceases?
(A) $10\sqrt{2}$ rad/s (B) 20 rad/s (C) $20\sqrt{2}$ rad/s (D) 32 rad/s

2. The amplitude of the oscillations is
(A) 7.5 cm (B) 5 cm (C) 2.5 cm (D) 1 cm

3. The initial phase angle observed by an observer in the elevator, taking upward direction to be positive and positive extreme position to have π/2 phase, is equal to
(A) –π/4 rad (B) π/2 rad (C) π rad (D) 3π/2 rad

Comprehension-2

A particle of mass 'm' moves on a horizontal smooth line AB of length 'a' such that when particle is at any general point P on the line two forces act on it. A force $\frac{mg(AP)}{a}$ towards A and another force $\frac{2mg(BP)}{a}$ towards B.

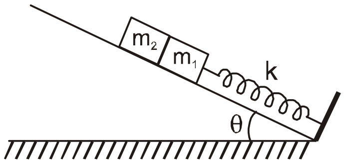
4. Find its time period when released from rest from mid-point of line AB.
(A) $T = 2\pi\sqrt{\frac{3a}{g}}$ (B) $T = 2\pi\sqrt{\frac{a}{2g}}$ (C) $T = 2\pi\sqrt{\frac{a}{g}}$ (D) $T = 2\pi\sqrt{\frac{a}{3g}}$

5. Find the minimum distance of the particle from B during the motion.
(A) $\frac{a}{6}$ (B) $\frac{a}{4}$ (C) $\frac{a}{3}$ (D) $\frac{a}{8}$

6. If the force acting towards A stops acting when the particle is nearest to B then find the velocity with which it crosses point B.
(A) $\frac{\sqrt{2ga}}{3}$ (B) $\frac{\sqrt{2ga}}{6}$ (C) $\frac{\sqrt{2ga}}{5}$ (D) $\frac{\sqrt{ga}}{3}$

Comprehension-3

Spring of spring constant k is attached with a block of mass m_1 as shown in figure. Another block of mass m_2 is placed against m_1 and both masses lie on smooth incline plane.



7. Find the compression in the spring when the system is in equilibrium.

(A) $\frac{(m_1+m_2)g\sin\theta}{2k}$ (B) $\frac{(m_1+m_2)g\sin\theta}{k}$

(C) $\frac{(m_1+m_2)g}{k}$ (D) $\frac{2(m_1+m_2)g\sin\theta}{k}$

8.✍ From the equilibrium position the blocks are pushed a further distance $\frac{2}{k}(m_1+m_2)g\sin\theta$ against the spring and released. Find the common speed of blocks when they separate.

(A) $\left(\sqrt{\frac{1}{3k}(m_1+m_2)}\right)g\sin\theta$ (B) $\left(\sqrt{\frac{2}{k}(m_1+m_2)}\right)g\sin\theta$

(C) $\left(\sqrt{\frac{3}{k}(m_1+m_2)}\right)g\sin\theta$ (D) $\left(\sqrt{\frac{1}{k}(m_1+m_2)}\right)g\sin\theta$

Exercise-3

✍ Marked Questions can be used as Revision Questions.
*** Marked Questions may have more than one correct option.**

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

1*.✍ Function $x = A\sin^2\omega t + B\cos^2\omega t + C\sin\omega t\cos\omega t$ represents SHM **[JEE 2006, 5/184,–1]**

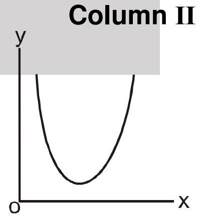
(A) for any value of A, B and C (except C = 0) (B) If A = – B, C = 2B, amplitude = $|B\sqrt{2}|$

(C) If A = B ; C = 0 (D) If A = B ; C = 2B, amplitude = |B|

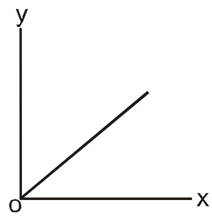
2.✍ **Column I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column II**. Match the set of parameters given in **Column I** with the graphs given in **Column II**. **[IIT-JEE 2008, 6/163]**

Column I **Column II**

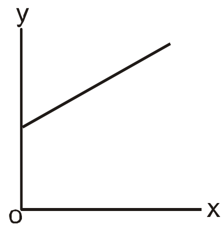
(A) Potential energy of a simple pendulum (y-axis) as a function of displacement (x-axis) (p)



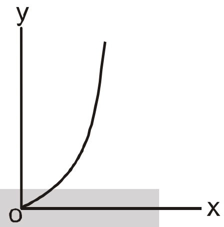
(B) Displacement (y-axis) as a function of time (x-axis) for a one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction. (q)



(C) Range of a projectile (y-axis) as a function of its velocity (x-axis) when projected at a fixed angle. (r)

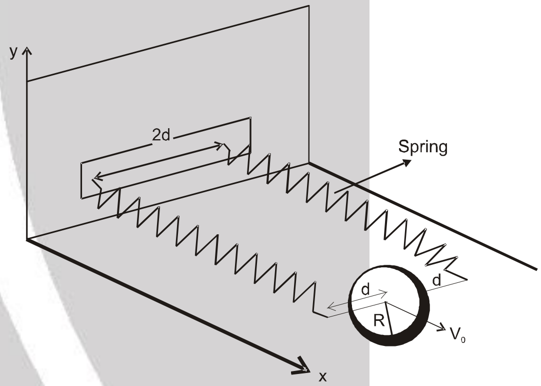


(D) The square of the time period (y-axis) of a simple pendulum as a function of its length (x-axis). (s)



Comprehension-1

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant k which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is L. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance L from the wall. The disk rolls without slipping with velocity $\vec{V}_0 = V_0\hat{i}$. The coefficient of friction is μ. Figure :



[JEE-2008, 3×4/163]

3. The net external force acting on the disk when its centre of mass is at displacement x with respect to its equilibrium position is

(A) –kx (B) –2kx (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

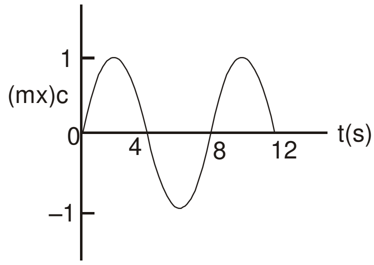
4. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

5. The maximum value of V_0 for which the disk will roll without slipping is

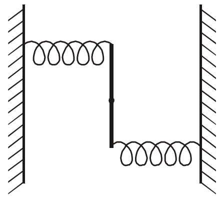
(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

6. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at t = 4/3 s is **[IIT-JEE 2009, 3/160, –1]**



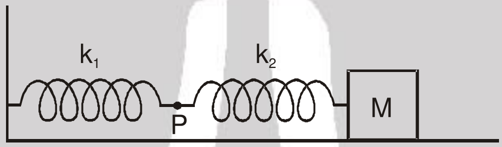
(A) $\frac{\sqrt{3}}{32}\pi^2$ cm/s² (B) $\frac{-\pi^2}{32}$ cm/s² (C) $\frac{\pi^2}{32}$ cm/s² (D) $\frac{\sqrt{3}}{32}\pi^2$ – cm/s²

7. A uniform rod of length L and mass M is pivoted at the centre. Its two ends are attached to two springs of equal spring constants k. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is : **[IIT-JEE 2009, 3/160, –1]**



(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

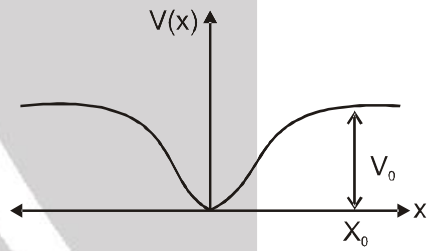
8. The mass M shown in the figure oscillates in simple harmonic motion with amplitude A. The amplitude of the point P is **[JEE 2009, 3/160, –1]**



(A) $\frac{k_1A}{k_2}$ (B) $\frac{k_2A}{k_1}$ (C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

Comprehension-2

When a particle of mass m moves on the x-axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of x = 0 in a way different from kx^2 and its total energy is such that the particle does not escape to infinity.



Consider a particle of mass m moving on the x-axis. Its potential energy is $V(x) = \alpha x^4$ ($\alpha > 0$) for |x| near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure) **[JEE 2010, 3×3/160, –1]**

9. If the total energy of the particle is E, it will perform periodic motion only if :

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

10. For periodic motion of small amplitude A, the time period T of this particle is proportional to :

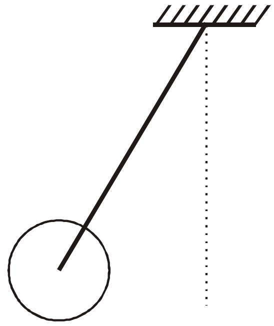
(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

11. The acceleration of this particle for $|x| > X_0$ is :

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$

(C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) zero

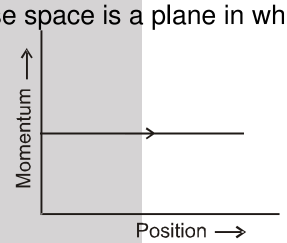
12*. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (<L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotate about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is (are) true? **[JEE 2011, 4/160]**



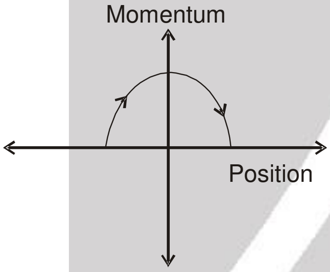
(A) Restoring torque in case A = Restoring torque in case B
(B) Restoring torque in case A **<** Restoring torque in case B
(C) Angular frequency for case A **>** Angular frequency for case B.
(D) Angular frequency for case A < Angular frequency for case B.

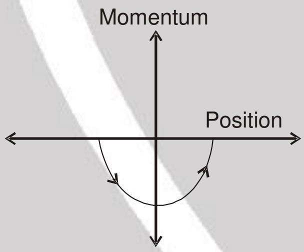
Comprehension-3

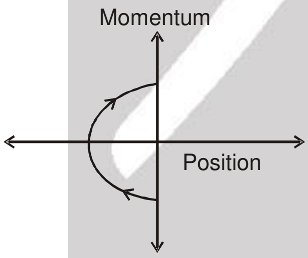
Phase space diagrams are useful tools in analyzing all kinds of dynamical problems. They are especially useful in studying the changes in motion as initial position and momentum are changed. Here we consider some simple dynamical systems in one-dimension. For such systems, phase space is a plane in which position is plotted along horizontal axis and momentum is plotted along vertical axis. The phase space diagram is x(t) vs. p(t) curve in this plane. The arrow on the curve indicates the time flow. For example, the phase space diagram for a particle moving with constant velocity is a straight line as shown in the figure. We use the sign convention in which position or momentum. Upwards (or to right) is positive and downwards (or to left) is negative. **[JEE 2011, 3×3/160, –1]**

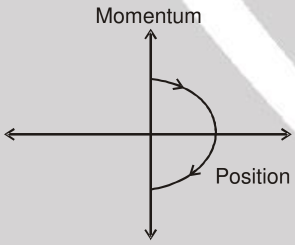


13. The phase space diagram for a ball thrown vertically up from ground is :

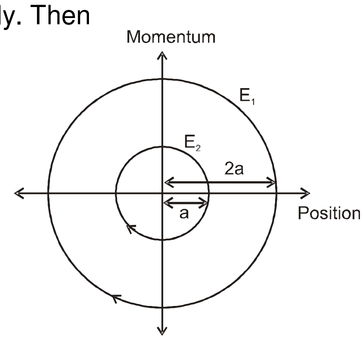
(A)


(B)


(C)


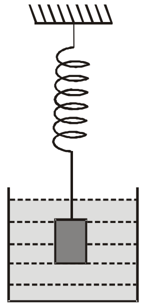
(D)


14. The phase space diagram for simple harmonic motion is a circle centered at the origin. In the figure, the two circles represent the same oscillator but for different initial conditions, and E_1 and E_2 are the total mechanical energies respectively. Then

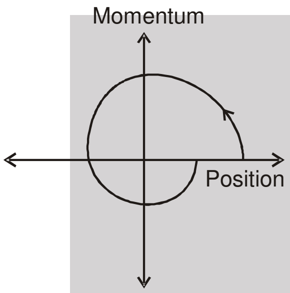


(A) $E_1 = \sqrt{2}E_2$ (B) $E_1 = 2E_2$ (C) $E_1 = 4E_2$ (D) $E_1 = 16E_2$

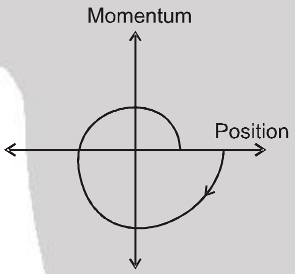
15. Consider the spring-mass system, with the mass submerged in water, as shown in the figure. The phase space diagram for one cycle of this system is :



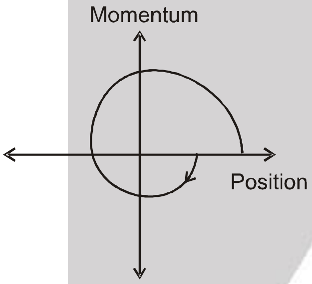
(A)



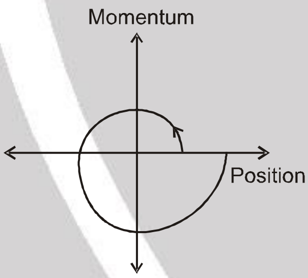
(B)



(C)



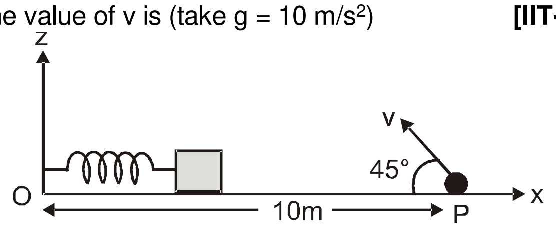
(D)



16. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A \sin \omega t$ and $x_2(t) = A \sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B \sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are **[JEE 2011, 3/160, –1]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

17. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45º as shown in the figure. Point P is at a horizontal distance of 10 cm from O. If the pebble hits the block at t = 1s, the value of v is (take g = 10 m/s²) **[IIT-JEE-2012, Paper-1; 3/70, –1]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

18*. A particle of mass m is attached to one end of a mass–less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collies elastically with a rigid wall. After this collision : **[JEE (Advanced) 2013, 4/60]**

(A) the speed of the particle when it returns to its equilibrium position is u_0 .

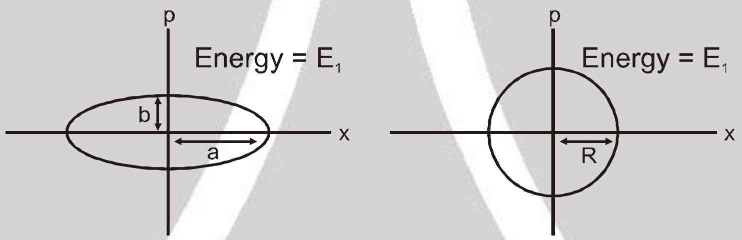
(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$.

(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$.

(D) the time at which the particle passes througout the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

19. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation(s) is (are) :

[JEE (Advanced) 2015 ; P-1, 4/88, –2]



(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

20. A particle of unit mass is moving along the x–axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

[JEE (Advanced) 2015 ; 8/88, –1]

Column-I

(A) $U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$

(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$

(C) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$

(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

Column-II

(P) the force acting on the particle is zero at x = a.

(Q) the force acting on the particle is zero at x = 0.

(R) the force acting on the particle is zero at x=–a.

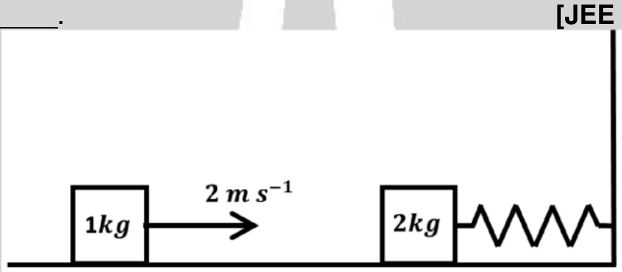
(S) The particle experiences an attractive force towards x = 0 in the region $|x| < a$

(T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point x = – a.

21*. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0 ; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (< M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M ? **[JEE (Advanced) 2016 ; P-2, 4/62, –2]**

(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged
(B) The final time period of oscillation in both the cases is same
(C) The total energy decreases in both the cases
(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases

22. A spring-block system is resting on a frictionless floor as shown in the figure. The spring constant is 2.0 Nm^{-1} and the mass of the block is 2.0 kg. Ignore the mass of the spring. Initially the spring is in an unstretched condition. Another block of mass 1.0 kg moving with a speed of 2.0 ms^{-1} collides elastically with the first block. The collision is such that the 2.0 kg block does not hit the wall. The distance, in metres, between the two blocks when the spring returns to its unstretched position for the first time after the collision is _________. **[JEE (Advanced) 2018, P-1, 3/60]**



PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

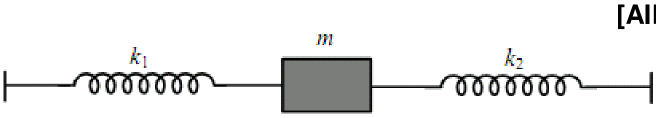
1. The maximum velocity of a particle, executing simple harmonic motion with an amplitude 7 mm, is 4.4 m/s. The period of oscillation is : **[AIEEE 2006 ; 3/165, –1]**
(1) 100 s (2) 0.01 s (3) 10 s (4) 0.1 s

2*. A coin is placed on a horizontal platform which undergoes vertical simple harmonic motion of angular frequency ω.The amplitude of oscillation is gradually increased. The coin will leave contact with the platform for the first time : **[AIEEE 2006 ; 3/165, –1]**
(1) at the highest position of the platform (2) at the mean position of the platform
(3) for an amplitude of $\frac{g}{\omega^2}$ (4) for an amplitude of $\frac{g^2}{\omega^2}$

3. The displacement of an object attached to a spring and executing simple harmonic motion is given by $x = 2 \times 10^{-2} \cos\pi t$ metres. The time at which the maximum speed first occurs is : **[AIEEE 2007 ; 3/120, –1]**
(1) 0.5 s (2) 0.75 s (3) 0.125 s (4) 0.25 s

4. A point mass oscillates along the x-axis according to the law $x = x_0 \cos(\omega t - \pi/4)$. If the acceleration of the particle is written as, $a = A \cos(\omega t + \delta)$, then : **[AIEEE 2007 ; 3/120, –1]**
(1) $A = x_0$, $\delta = -\pi/4$ (2) $A = x_0\omega^2$, $\delta = -\pi/4$ (3) $A = x_0\omega^2$, $\delta = -\pi/4$ (4) $A = x_0\omega^2$, $\delta = 3\pi/4$

5. Two springs, of force constants k_1 and k_2, are connected to a mass m as shown. The frequency of oscillation of mass is f. If both k_1 and k_2 are made four times their original values, the frequency of oscillation becomes: **[AIEEE 2007 ; 3/120, –1]**



(1) $f/2$ (2) $f/4$ (3) 4f (4) $2f$

6. A particle of mass m executes simple harmonic motion with amplitude a and frequency υ. The average kinetic energy during its motion from the position of equilibrium to the end is : **[AIEEE 2007; 3/120, –1]**

(1) $\pi^2 ma^2 \upsilon^2$ (2) $\frac{1}{4} ma^2 \upsilon^2$ (3) $4\pi^2 ma^2 \upsilon^2$ (4) $2\pi^2 ma^2\upsilon^2$

7.* If x, v and a denote the displacement, the velocity and the acceleration of a particle executing simple harmonic motion of time period T, then, which of the following does not change with time? **[AIEEE 2009 ; 3/120, –1]**

(1) $\frac{aT}{x}$ (2) $aT + 2\pi v$ (3) $\frac{aT}{v}$ (4) $a^2T^2 + 4\pi^2 v^2$

8. A mass M, attached to a horizontal spring, executes SHM with a amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is : **[AIEEE - 2011, 4/120, –1]**

(1) $\frac{M}{M+m}$ (2) $\frac{M+m}{M}$ (3) $\left(\frac{M}{M+m}\right)^{1/2}$ (4) $\left(\frac{M+m}{M}\right)^{1/2}$

9. If a spring of stiffness 'k' is cut into two parts 'A' and 'B' of length $\ell_A : \ell_B = 2 : 3$, then the stiffness of spring 'A' is given by : **[AIEEE 2011, 11 May; 4/120, –1]**

(1) $\frac{3k}{5}$ (2) $\frac{2k}{5}$ (3) k (4) $\frac{5k}{2}$.

10. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is : **[AIEEE - 2011, 4/120, –1]**

(1) $\pi/2$ (2) $\pi/3$ (3) $\pi/4$ (4) $\pi/6$

11. If a simple pendulum has significant amplitude (up to a factor of 1/e of original) only in the period between t = 0s to t = τs, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the averatge life time of the pendulum is (assuming damping is small) in seconds : **[AIEEE 2012 ; 4/120, –1]**

(1) $\frac{0.693}{b}$ (2) b (3) $\frac{1}{b}$ (4) $\frac{2}{b}$

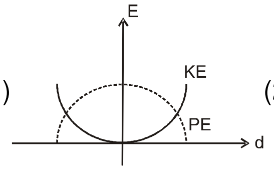
12. The amplitude of a damped oscillator decreases to 0.9 times its original magnitude is 5s. In another 10s it will decrease to α times its original magnitude, where α equals. **[JEE (Main) 2013, 4/120]**

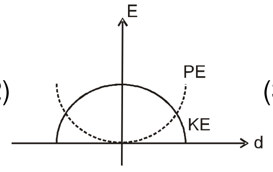
(1) 0.7 (2) 0.81 (3) 0.729 (4) 0.6

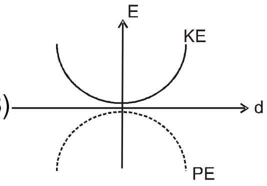
13. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE (Main) 2014 ; 4/120, –1]**

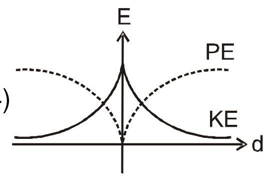
(1) amplitude of motion is 3a (2) time period of oscillations is 8τ
(3) amplitude of motion is 4a (4) time period of oscillations is 6τ

14. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ?
(graphs are schematic and not drawn to scale) **[JEE (Main) 2015 ; 4/120, –1]**

(1) 

(2) 

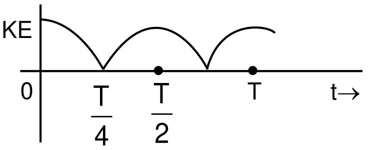
(3) 

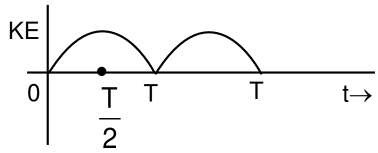
(4) 

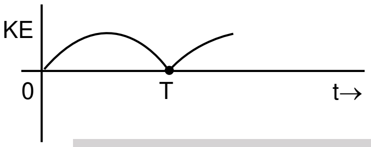
15. A particle performs simple harmonic motion with amplitude A. Its speed is trippled at the instant that it is at distance 2A/3 from equilibrium position. The new amplitude of the motion is. **[JEE (Main) 2016; 4/120, –1]**

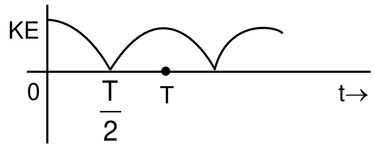
(1) 3A (2) $\sqrt{3}$ A (3) $\frac{7A}{3}$ (4) $\frac{A}{3}\sqrt{41}$

16. A particle is executing simple harmonic motion with a time period T. At time = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like : **[JEE (Main) 2017; 4/120, –1]**

(1) 

(2) 

(3) 

(4) 

17. A sliver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of siver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1} **[JEE (Main) 2018; 4/120, –1]**

(1) 2.2 N/m (2) 5.5 N/m (3) 6.4 N/m (4) 7.1 N/m

Answers

EXERCISE-1

PART - I

Section (A) :

A-1. Amplitude = 5 m, Phase constant = $\frac{\pi}{6}$, Time period = 2 s, Maximum speed = 5π m/s

A-2. (a) 2.0 cm, $\frac{\pi}{50}$ s = 0.063 s, 100 N/m

(b) 1.0 cm, $\sqrt{3}$ m/s, –100 m/s^2

A-3. (a) T/12, (b) T/8, (c) T/6, (d) T/4, (e) T/8

A-4. $\frac{\sqrt{3}v_0}{2}$

A-5. $x = (10\text{cm})\sin\left[\left(\frac{\pi}{3}s^{-1}\right)t + \frac{\pi}{6}\right], \frac{10}{9}\pi^2 \approx 11$ cm/s^2

A-6. $\pm\frac{6}{5}$ cm = ± 1.2 cm from the mean position

Section (B) :

B-1. ± 5 cm **B-2.** A = 0.06 m

Section (C) :

C-1. 0.1 N/m **C-2.** 40 J

C-3. $\frac{16}{10\pi^2}$ = 0.16 Kg

C-4. (a) $\frac{F}{k}$, $2\pi\sqrt{\frac{M}{k}}$, (b) $\frac{F^2}{2k}$ (c) $\frac{F^2}{2k}$

C-5. (a) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$; (b) $2\pi\sqrt{\frac{m}{k_1+k_2}}$, $k_{eq.} = k_1 + k_2$; (c) $2\pi\sqrt{\frac{m\,(k_1+k_2)}{k_1k_2}}$, $k_{eq.} = \frac{k_1k_2}{k_1+k_2}$

Answers will remian same

C-6. $\frac{M^2g^2}{2k}$, $\frac{M^2g^2}{4k}$ and $\frac{M^2g^2}{6k}$ from above,

time period = $2\pi\sqrt{\frac{11M}{6k}}$

Section (D) :

D-1. 1 m **D-2.** 0.25 m

D-3. $\left(\frac{3600}{3601}\right)^2 g = 9.794 m/s^2$

D-4. (i) $2T_0$ (ii) 3g upwards

Section (E) :

E-1. (a) T = $2\pi\sqrt{\frac{7\ell}{12g}}$ (b) $2\pi\sqrt{\frac{2r}{g}}$

(c) $2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$ (d) $2\pi\sqrt{\frac{3r}{2g}}$

E-2. (a) $2\pi\sqrt{\frac{r\sqrt{2}}{g}}$, $r/\sqrt{2}$ (b) $2\pi\sqrt{\frac{L}{\sqrt{3}g}}$, $\frac{L}{2\sqrt{3}}$

Section (F) :

F-1. (a) 7 cm (b) $\sqrt{37}$ cm = 6.1 cm (c) 5 cm

F-2. $a\sqrt{4+2\sqrt{3}}$ **F-3.** $2x^2 + \frac{y}{2} = 1$

Section (G) :

G-1. Both amplitude and energy of the particle can be maximum only in the case of resonance. For resonance to occur, $\omega_1 = \omega_2$

G-2. (a) 0.3 s(b) 6.93 s (c) 3.4 s

PART - II

Section (A) :

A-1.	(A)	**A-2.**	(A)	**A-3.**	(B)
A-4.	(A)	**A-5.**	(A)	**A-6.**	(C)
A-7.	(A)	**A-8.**	(B)	**A-9.**	(B)
A-10.	(B)				

Section (B) :

B-1.	(B)	**B-2.**	(C)	**B-3.**	(B)
B-4.	(C)	**B-5.**	(A)	**B-6.**	(C)

Section (C) :

C-1.	(D)	**C-2.**	(C)	**C-3.**	(C)
C-4.	(A)	**C-5.**	(D)	**C-6.**	(D)
C-7.	(B)	**C-8.**	(D)		

Section (D) :

D-1.	(A)	**D-2.**	(D)	**D-3.**	(C)
D-4.	(D)	**D-5.**	(C)		

Section (E) :

E-1.	(D)	**E-2.**	(B)

Section (F) :

F-1.	(B)	**F-2.**	(B)	**F-3.**	(D)
F-4.	(B)				

Section (G) :

G-1.	(D)	**G-2.**	(C)

PART - III

1. (A) $\rightarrow$ r, (B) $\rightarrow$ p, (C) $\rightarrow$ q, (D) $\rightarrow$ s

2. (A) $\rightarrow$ p ; (B) $\rightarrow$ q ; (C) $\rightarrow$ p ; (D) $\rightarrow$ s

EXERCISE-2

PART - I

1.	(A)	**2.**	(C)	**3.**	(B)
4.	(C)	**5.**	(C)	**6.**	(A)
7.	(A)	**8.**	(A)	**9.**	(A)
10.	(A)	**11.**	(B)	**12.**	(C)
13.	(C)	**14.**	(B)	**15.**	(D)

PART - II

1.	10	**2.**	2	**3.**	2
4.	15	**5.**	2	**6.**	16
7.	3	**8.**	3	**9.**	12
10.	17	**11.**	15	**12.**	8
13.	15	**14.**	2	**15.**	8
16.	75				

PART - III

1.	(AD)	**2.**	(ABC)	**3.**	(BC)
4.	(ABCDE)	**5.**	(BD)	**6.**	(ABCD)
7.	(BCD)	**8.**	(ABC)	**9.**	(BD)
10.	(BC)	**11.**	(ACD)	**12.**	(AC)

PART - IV

1.	(A)	**2.**	(C)	**3.**	(D)
4.	(D)	**5.**	(A)	**6.**	(B)
7.	(B)	**8.**	(C)		

EXERCISE-3

PART - I

1. (ABD)

2. (A)$\rightarrow$(p); (B) $\rightarrow$ (q, s) ; (C) $\rightarrow$ (s) ; (D) $\rightarrow$ (q)

3.	(D)	**4.**	(D)	**5.**	(C)
6.	(D)	**7.**	(C)	**8.**	(D)
9.	(C)	**10.**	(B)	**11.**	(D)
12.	(AD)	**13.**	(D)	**14.**	(C)
15.	(B)	**16.**	(B)	**17.**	(A)
18.	(AD)	**19.**	(BD)		

20. (A) $\rightarrow$ P,Q,R,T ; (B) $\rightarrow$ Q,S ; (C) $\rightarrow$ P,Q,R,S ; (D) $\rightarrow$ P,R,T

21. (ABD) **22.** 2.09

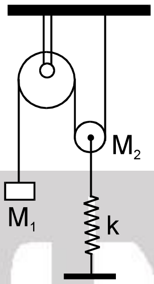
PART - II

1.	(2)	**2.**	(1,3)	**3.**	(1)
4.	(4)	**5.**	(4)	**6.**	(1)
7.	(1,4)	**8.**	(4)	**9.**	(4)
10.	(2)	**11.**	(4)	**12.**	(3)
13.	(4)	**14.**	(2)	**15.**	(3)
16.	(1)	**17.**	(4)		

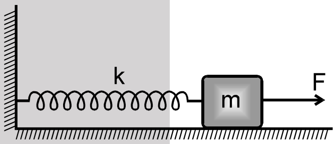
High Level Problems (HLP)

SUBJECTIVE QUESTIONS

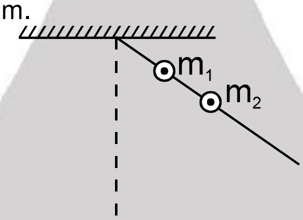
1. What would be the period of the free oscillations of the system shown here if mass M_1 is pulled down a little force constant of the spring is k, mass of fixed pulley is negligible and movable pulley is smooth



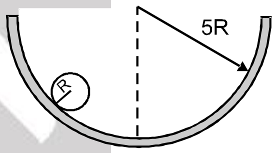
2. A constant force produces maximum velocity V on the block connected to the spring of force constant K as shown in the fig. When the force constant of spring becomes 4K, then find maximum velocity of the block. Assume that initially the spring is in relaxed state.



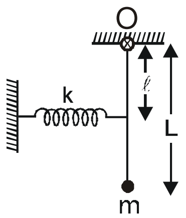
3 Two point masses m_1 and m_2 are fixed to a light rod hinged at one end. The masses are at distances ℓ_1 and ℓ_2 respectively from the hinge. Find the time period of oscillation (small amplitude) of this system in seconds if $m_1 = m_2$, $\ell_1 = 1$m, $\ell_2 = 3$m.



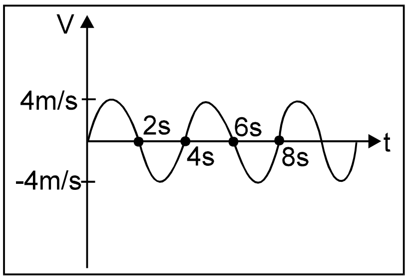
4. A solid sphere (radius = R) rolls without slipping in a cylindrical vessel (radius = 5R). Find the angular frequency of small oscillations of the sphere in s^{-1}. Take $R = \frac{1}{14}$ m and g = 10 m/s^2. (Axis of cylinder is fixed and horizontal).



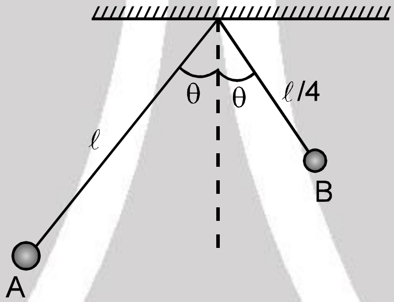
5. A particle of mass m is suspended at the lower end of a thin rod of negligible mass. The upper end of the rod is free to rotate in the plane of the page about a horizontal axis passing through the point O. The spring is undeformed when the rod is vertical as shown in fig. If the period of oscillation of the system is $\pi\sqrt{\frac{L}{n}}$, when it is slightly displaced from its mean position then find n. Take $k = \frac{9mgL}{\ell^2}$ and g = 10m/s^2.



6. If velocity of a particle moving along a straight line changes sinusoidally with time as shown in the given graph. Find the average speed over time interval t = 0 to t = 2 (2n – 1) seconds, n being any positive integer.



7. Two simple pendulums A and B having lengths ℓ and $\ell/4$ respectively are released from the position as shown in figure. Calculate the time after which the release of the two strings become parallel for the first time. Angle θ is very small.



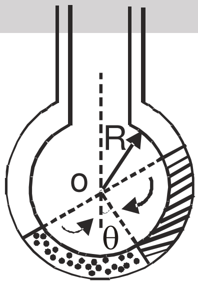
8. A particle of mass 'm' is moving in the x-y plane such that its x and y coordinate vary according to the law $x = a \sin \omega t$ and $y = a \cos \omega t$ where 'a' and 'ω' are positive constants and 't' is time. Find

(a) equation of the path. Name the trajectory (path)

(b) whether the particle moves in clockwise or anticlockwise direction

(c) magnitude of the force on the particle at any time t.

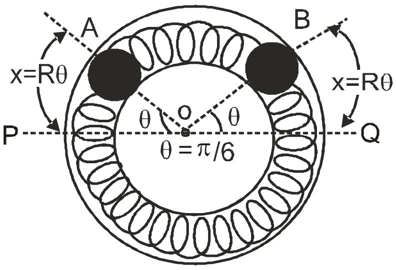
9. Two non–viscous, incompressible and immiscible liquids of densities ρ and 1.5 ρ are poured into the two limbs of a circular tube of radius R and small cross–section kept fixed in a vertical plane as shown in fig. Each liquid occupies one–fourth the circumference of the tube.



(a) Find the angle θ that the radius to the interface makes with the vertical in equilibrium position.

(b) If the whole liquid column is given a small displacement from its equilibrium position, show that the resulting oscillations are simple harmonic. Find the time period of these oscillations.

10. Two identical balls A and B, each of mass 0.1 kg, are attached to two identical mass less springs. The spring–mass system is constrained to move inside a rigid smooth pipe bent in the form of a circle as shown in the figure. The pipe is fixed in a horizontal plane. The centres of the balls can move in a circle of radius 0.06 m. Each spring has a natural length of 0.06π metre and spring constant 0.1 N/m. Initially, both the balls are displaced by an angle $\theta = \pi/6$ radian with respect to the diameter PQ of the circle (as shown in fig.) and released from rest.

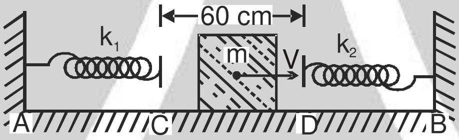


(i) Calculate the frequency of oscillation of ball B.

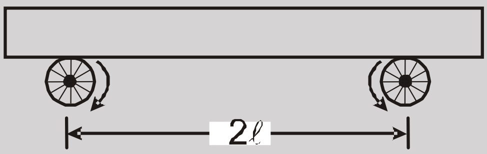
(ii) Find the speed of ball A when A and B are at the two ends of the diameter PQ.

(iii) What is the total energy of the system ? **[1993 ; 6M]**

11. Two light springs of force constant k_1 and k_2 and a block of mass m are in one line AB on a smooth horizontal table such that one end of each spring is fixed to rigid supports and the other end is free as shown in the figure. The distance CD between the free ends of the spring is 60 cm. If the block moves along AB with a velocity 120 cm/s in between the springs, calculate the period of oscillation of the block. (k_1 = 1.8 N/m, k_2 = 3.2 N/m, m = 200 g) **[1985 ; 6M]**



12. Two wheels which are rotated by some external source with constant angular velocity in opposite directions as shown in figure. A uniform plank of mass M is placed on it symmetrically. The friction co-efficient between each wheel and the plank is μ. Find the frequency of oscillations, when plank is slightly displaced along its length and released.



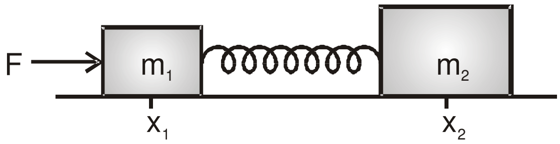
13. **The Cubic Potential :** Consider a particle of mass m moving in one dimension under the influence of potential energy $u(x) = \frac{m\omega^2x^2}{2} - \delta x - \frac{\alpha x^3}{3}$

Here ω, δ and a are real and positive.

(a) Sketch typical plots of u(x) and identify extrema if any.

(b) Consider the case where (in appropriate units) we have m = 1, $\omega = \sqrt{2}$, α = 1 and δ = 1/2. Sketch the potential energy u(x). If the total energy of the particle moving in this one-dimensional potential is E = 0 (in same units), discuss the motion of the particle in terms of allowed regions, boundedness and periodicity.

14. Two blocks of masses m_1 = 1.0 kg and m_2 = 2.0 kg are connected by a massless elastic spring and are at rest on a smooth horizontal surface with the spring at its natural length.



A horizontal force of constant magnitude F = 6.0 N is applied to the block m_1 for a certain time t in which m_1 suffers a displacement Δx_1 = 0.1 m and Δx_2 = 0.05 m. Kinetic energy of the system with respect to center of mass is 0.1 J. The force F is then withdrawn.

(a) Calculate t.

(b) Calculate the speed and the kinetic energy of the center of mass after the force is withdrawn.

(c) Calculate the energy stored in the system

HLP Answers

1. $T = 2\pi\sqrt{\frac{M_2 + 4M_1}{k}}$ **2.** V/2 **3.** π **4.** 5

5. 25 **6.** $\frac{8}{\pi}$ m/s **7.** $\frac{\pi}{3}\sqrt{\frac{\ell}{g}}$

8. (a) $x^2 + y^2 = a^2$, circle (b) The particle moves in clock wise sense.

(c)The magnitude of force = m $\sqrt{a_x^2 + a_y^2}$ = $m\omega^2 a$

9. (a) $\tan^{-1}\left(\frac{1}{5}\right)$ (b) $2\pi\sqrt{\frac{R}{6.11}}$

10. (i) f = $\frac{1}{2\pi}\sqrt{\frac{4 \times 0.1}{0.1}} = \frac{1}{\pi}$ Hz (ii) V = 0.0628 (iii) 3.9×10^{-4} J **11.** 2.82 s

12. $2\pi\sqrt{\frac{\ell}{\mu g}}$ **13.** (b) between x = 0 and x = $\frac{3-\sqrt{3}}{2}$. U is (–ve). So, K.E. is +ve.

14. (a) 0.26s (b) 0.52 ms^{-1}, 0.40 J (c) 0.20 J