SOLUTIONS OF SEQUENCE & SERIES

EXERCISE - 1

PART-I

Section (A) :

A-1. (a + 2d) = 4a $\Rightarrow$ 3a = 2d.

We have given that a + 5d = 17 $\Rightarrow$ $a + 5\left(\frac{3a}{2}\right) = 17$

a = 2, d = 3
so series 2, 5, 8 ....

A-2. Given that; $T_p = \frac{p}{7} + 2$, then $S_p = \frac{1}{7}\sum p + \sum 2 = \frac{p(p+1)}{14} + 2p$.

taking p = 35 $S_{35} = 160$

A-3. 994 = 105 + (n –1) 7 $\Rightarrow$ 889 + 7 = 7n $\Rightarrow$ n = 128

A-4. First No. = 103 last No. = 791

No. of terms = 44 $s = \frac{44}{2}$ [103 + 791] = 22 [894] = 19668

A-5. $q = \frac{p}{2}[2A + (p-1)d] \Rightarrow \frac{2q}{p} = 2A + (p-1)d$ .....(i)

$p = \frac{p}{2}[2A + (q-1)d] \Rightarrow \frac{2q}{p} = 2A + (q-1)d$ .....(ii)

on subtracting equation (i) from (ii), we get

$\frac{2}{pq}(q^2 - p^2) = (p-q)\,d \Rightarrow d = \frac{-2}{pq}(p+q)$

$\therefore$ Sum of (p + q) terms is

$\frac{p+q}{2} = [2A + (p+q-1)\,d] = \frac{p+q}{2}[2A + (p-1)\,d + qd] = \frac{p+q}{2}\left[\frac{2q}{p} + q\left\{\frac{-2}{pq}(p+q)\right\}\right]$

$= \frac{p+q}{2}\left[\frac{2q}{p} - 2 - \frac{2q}{p}\right] = -(p+q)$ **Ans.**

A-6. Numbers are a – d, a, a + d $\Rightarrow$ s = a – d + a + a + d = 27 $\Rightarrow$ a = 9
$a(a^2 - d^2) = 504$ $\Rightarrow$ $9(81 - d^2) = 504$ $\Rightarrow$ $81 - d^2 = 56$
$d^2 = 25$ $\Rightarrow$ d = ± 5. Numbers are 4, 9, 14

A-7. $\because$ $(a - 3d)(a - d)(a + d)(a + 3d) + 16d^4 = (a^2 - 9d^2)(a^2 - d^2) + 16d^4$
$= a^4 - a^2d^2 - 9a^2d^2 + 9d^4 + 16d^4 = a^4 - 10a^2d^2 + 25d^4 = [a^2 - 5d^2]^2$
$= (a^2 - d^2 - 4d^2)^2 = ((a-d)(a+d) - (2d)^2)^2$
$\because$ (a – d), (a + d), 2d are integers. Hence Proved

A-8. (i) a, b, c are in A.P. a (ab + bc + ac), b(ab + bc + ac), c(ab + bc + ac) are in A.P.
$\Rightarrow$ $a^2(b + c)$, $b^2(c + a)$, $c^2(a + b)$ are in A.P.
(ii) b + c – a, a + c – b, a + b – c are in A.P
$\Rightarrow$ 2(a + c –b) = (b + c – a) (a + b – c) $\Rightarrow$ a + c = 2b $\Rightarrow$ a , b , c are in A.P

A-9. $\dfrac{(54-3)}{n+1} = d$; $d = \dfrac{51}{n+1}$

$\dfrac{A_8}{A_{n-2}} = \dfrac{3}{5} \Rightarrow \dfrac{3+8\dfrac{51}{n+1}}{3+(n-2)\dfrac{51}{n+1}} = \dfrac{3}{5} \Rightarrow \dfrac{3n+3+\ \ 408}{3n+3+51\ \ n-102} = \dfrac{3}{5}$

$\Rightarrow$ 15n + 2055 = 162n – 297 $\Rightarrow$ 147 n = 2352 ; n = 16

Section (B) :

B-1. Let the three terms be a, ar, ar^2 $\Rightarrow$ $ar^2 = a^2$ $\Rightarrow$ $a = r^2$ and ar = 8

$\Rightarrow$ $r^3 = 8$, r = 2 and a = 4 $\therefore$ $T_6 = 4(2)^5 = 128$

B-2. Let the Numbers are $\dfrac{a}{r}$, a, ar so $a^3 = 216$ $\Rightarrow$ a = 6 $\Rightarrow$ $\dfrac{a}{r}.a + a.ar + ar.\dfrac{a}{r} = 156$

$\Rightarrow$ $a^2(1 + r + \dfrac{1}{r}) = 156$ $\Rightarrow$ $(1 + r + \dfrac{1}{r}) =$ $\Rightarrow 1 + r + \dfrac{1}{r} = \dfrac{156}{36}$

$\Rightarrow$ r = 3 or 1/3. Numbers are 2, 6, 18 or 18, 6, 2

B-3. $\dfrac{a}{1-r} = 4$ $\Rightarrow$ $\dfrac{a^3}{1-r^3} = 192$ $\Rightarrow$ $\dfrac{(1-r)^3}{1-r^3} = \dfrac{192}{(4)^3}$ $\Rightarrow$ $\dfrac{(1-r)^2}{1+r+r^2} = 3$

$\Rightarrow$ $1 + r^2 - 2r = 3 + 3r + 3r^2$ $\Rightarrow$ $2r^2 + 5r + 2 = 0$ $\Rightarrow$ (2r + 1) (r + 2) = 0

r = –1/2, r = –2(rejected) When r = –1/2 , a = 6 so series is 6, –3, 3/2 .....

B-4. Let a – d , a, a + d $\Rightarrow$ 3a = 21 $\Rightarrow$ a =7

a – d, a –1, a + d + 1 are in G.P $\Rightarrow$ 7 – d, 6 , 8 + d are in G.P

$\Rightarrow$ 36 = (7– d) (8 + d) $\Rightarrow$ $36 = 56 - d - d^2$

$\Rightarrow$ $d^2 + d - 20 = 0$ $\Rightarrow$ d = –5 , 4

so Numbers are 3, 7, 11 $\Rightarrow$ 12, 7, 2

B-5. $\dfrac{T_q}{T_p} = \dfrac{T_r}{T_q}$ = common ratio; $\dfrac{a+(q-1)d}{a+(p-1)d} = \dfrac{a+(r-1)d}{a+(q-1)d}$ using dividendo

$\dfrac{(q-p)}{a+(p-1)d} = \dfrac{(r-q)}{a+(q-1)d}$ $\Rightarrow$ $\dfrac{T_q}{T_p} = \dfrac{r-q}{q-p} = \dfrac{q-r}{p-q}$

B-6. (i) Let b = ar

c = ar^2 and d = ar^3

So $a^2(1-r^2)$, $a^2(r^2)(1-r^2)$, $a^2r^4(1-r^2)$ these are in G.P.

So $(a^2 - b^2)$, $(b^2 - c^2)$, $(c^2 - d^2)$ are in G.P.

(ii) $\dfrac{1}{a^2+b^2}, \dfrac{1}{b^2+c^2}, \dfrac{1}{c^2+d^2} = \dfrac{1}{a^2(1+r^2)}, \dfrac{1}{a^2r^2(1+r^2)}, \dfrac{1}{a^2r^4(1+r^2)}$ are in G.P.

B-7. Common ratio of means = $\left(\dfrac{243}{2} \times \dfrac{3}{32}\right)^{1/6} = \dfrac{3}{2}$

$\Rightarrow$ means are 16, 24, 36, 54, 81

their sum is 211.

Section (C) :

C-1. $T_7 = \dfrac{1}{20}$ $\Rightarrow$ a + 6d = 20 ; $T_{13} = \dfrac{1}{38}$ $\Rightarrow$ a + 12d = 38

d = 3, a = 2 so $T_4 = \dfrac{1}{2+9} = \dfrac{1}{11}$

C-2. $1, A_1, A_2, A_3, \frac{1}{7}$

$\frac{1}{7} = 1 + 4\,.d$

$d = \frac{\frac{1}{7} - 1}{4} = \frac{-6}{28} = \frac{-3}{14}$

$A_1 = 1 - \frac{3}{14} = \frac{11}{14}$

$A_2 = 1 - \frac{6}{14} = \frac{18}{14}$

$A_3 = 1 - \frac{9}{14} = \frac{5}{14}$

so $\frac{14}{11}, \frac{14}{8}, \frac{14}{5}$ are three harmonic means

C-3. Let $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz} = k$

$P = \frac{a-x}{kx}, q = \frac{a-y}{kx}, r = \frac{a-z}{kz}$

$2\left(\frac{a-y}{ky}\right) = \frac{a-x}{kx} + \frac{a-z}{kz}$

$2\left(\frac{a}{y} - 1\right) = \frac{a}{x} - 1 + \frac{a}{z} - 1$

$\frac{2a}{y} = \frac{a}{x} + \frac{a}{z}$

$\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$

Hence x, y, z are in H.P.

C-4. a^2, b^2, c^2 are in A.P.
Let b + c, c + a, a + b are in H.P.

then $\frac{1}{b+c}\ \frac{1}{c+a}\ \frac{1}{a+b}$ are in A.P.

$\frac{2}{c+a} = \frac{1}{b+c} + \frac{1}{a+b}$

$2b^2 = a^2 + c^2$

hence a^2, b^2, c^2 are in A.P.

if a^2 , b^2, c^2 are in A.P. then b + c, c + a, a – b are in H.P.

C-5. $b = \frac{2ac}{a+c}$

$\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$

L.H.S. = $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{\frac{2ac}{a+c} - a} + \frac{1}{\frac{2ac}{a+c} - c} = \frac{(a+c)}{a(2c-a-c)} + \frac{(a+c)}{c(2a-a-c)}$

$= \frac{a+c}{a(c-a)} + \frac{(a+c)}{c(a-c)} = \frac{a+c}{(c-a)}\left[\frac{1}{a} - \frac{1}{c}\right] = \frac{a+c}{ac} = \frac{1}{a} + \frac{1}{c}$ = RHS

C-6. (i) $1 + \frac{2}{2} + \frac{3}{2^2} + \ldots\ldots\ldots.$ n terms

$T_n = \frac{n}{2^{n-1}}$

$S = 1 + \frac{2}{2} + \frac{3}{2^2} + \ldots\ldots\ldots\ldots + \frac{n}{2^{n-1}}$ ...(i)

$\frac{1}{2}S = \frac{1}{2} + \frac{2}{2^2} + \ldots\ldots\ldots + \frac{(n-1)}{2^{n-1}} + \frac{n}{2^n}$ ...(ii)

(i) – (ii) we get

$\frac{1}{2}S = [1 + \frac{1}{2} + \frac{1}{2^2} + \ldots\ldots\ldots + \frac{1}{2^{n-1}}] - \frac{n}{2^n}$

$\frac{1}{2}S = \frac{1.\left[1-\left(\frac{1}{2}\right)^n\right]}{1-\frac{1}{2}} - \frac{n}{2^n} \Rightarrow S = 4 - 4\left(\frac{1}{2}\right)^n - \frac{2n}{2^n}$; $S = 4 - \frac{2+n}{2^{n-1}}$

(ii) $S = 1 + \frac{3}{4} + \frac{7}{16} + \frac{15}{64} + \frac{31}{256} + \ldots\ldots\ldots\infty$ ..... (i)

$\frac{1}{4}S = \frac{1}{4} + \frac{3}{16} + \frac{7}{64} + \ldots\ldots\ldots\infty$ ..... (ii)

(i) – (ii), we get

$\frac{3}{4}S = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\ldots\ldots\infty \Rightarrow \frac{3}{4}S = \frac{1}{1/2} \Rightarrow S = \frac{8}{3}$

C-7. $T_r = (2r + 1)\, 2^r$

$S = 3.2 + 5.2^2 + 7.2^3 + \ldots\ldots\ldots + (2n + 1)\, 2^n$ ...(i)

$2S = 3.2^2 + 5.2^3 + \ldots\ldots\ldots + (2n + 1)\, 2^n + (2n + 1)\, 2^{n-1}$ ...(ii)

(i) – (ii) we get

$-S = 3.2 + (2.2^2 + 2.2^3 + \ldots\ldots + 2.2^n) - (2n + 1)\, 2^{n+1} \Rightarrow -S = 6 + 8\,(2^{n-1} - 1) - (2n + 1)\, 2^{n+1}$

$S = 2 - 2^{n+2} + n.\, 2^{n+2} + 2^{n+1} \Rightarrow S = n.2^{n+2} - 2^{n+1} + 2$

Section (D) :

D-1. (i) $(x^2y + y^2z + z^2x)\,(xy^2 + yz^2 + zx^2) \geq 9x^2y^2z^2 \Rightarrow \frac{x^2y + y^2z + z^2x}{3} \geq (x^2y.\ y^2z.\ z^2x)^{1/3}$

$x^2y + y^2z + z^2x \geq 3xyz$ ......(i)

and $\frac{xy^2 + yz^2 + zx^2}{3} \geq (xy^2\ .\ yz^2\ .\ zx^2)^{1/3} \Rightarrow xy^2 + yz^2 + zx^2 \geq 3xyz$ ......(ii)

By (i) and (ii)

$\Rightarrow (x^2y + y^2z + z^2x)\,(xy^2 + yz^2 + zx^2) \geq 9x^2y^2z^2$

(ii) $(a + b)\,(b + c)\,(c + a) > abc$

$\frac{abc + b^2c + bc^2 + c^2a + a^2b + ab^2 + abc + a^2c}{8} \geq (abc.\ b^2c.\ bc^2.\ c^2a.\ a^2b.\ ab^2.\ abc.\ a^2c)$

$\Rightarrow (a + b)\,(b + c)\,(c + a) \geq 8abc \Rightarrow (a + b)\,(b + c)\,(c + a) > abc$

D-2. $\frac{x^{100}}{1 + x + x^2 + x^3 + \ldots.. + x^{200}}$

AM ≥ GM

$\frac{1 + x + x^2 + x^3 + \ldots.. + x^{200}}{201} \geq (1\ .\ x\ .\ x^2 \ldots.. x^{200})^{\frac{1}{201}} \Rightarrow \frac{1 + x + x^2 + x^3 + \ldots.. + x^{200}}{201} \geq \left(x^{\frac{201}{2}\,.\,200}\right)^{\frac{1}{201}}$

$\frac{1 + x + x^2 + x^3 + \ldots.. + x^{200}}{201} \geq x^{100} \Rightarrow \frac{x^{100}}{1 + x + x^2 + x^3 + \ldots.. + x^{200}} \leq \frac{1}{201}$

D-3. Let a and b are two numbers $\frac{2ab}{a+b} = \frac{16}{5}$ ..... (1)

$\frac{a+b}{2} = A$ and $\sqrt{ab} = G$

$\because$ $2A + G^2 = 26$ $\Rightarrow$ $(a + b) + ab = 26$ ..... (2)

$\Rightarrow$ $\frac{10\ \ ab}{16} + ab = 26$ $\Rightarrow$ $26\,ab = 26 \times 16 \Rightarrow$ $ab = 16$

$\therefore$ from (1), we get $a + b = 10$ So a, b are (2, 8) **Ans.**

D-4. $\frac{(a+b-c)+(a+c-b)+(b+c-a)}{3} \geq ((a + b - c)\ (c + a - b)\ (b + c - a))^{1/3}$

$(a+b+c) \geq 3((a + b - c)\ (c + a - b)\ (b + c - a))^{1/3}$

$(a+b+c)^3 \geq 27\ (a + b - c)\ (c + a - b)\ (b + c - a)$ Hence Proved

D-5. Using A.M. $\geq$ G.M.

$\frac{1+a_1+a_1^2}{3} \geq a_1 \Rightarrow 1 + a_1 + a_1^2 \geq 3a_1$

similarly $1 + a_2 + a_2^2 \geq 3a_2$

: : :

$1 + a_n + a_n^2 \geq 3a_n$

multiplying

$(1 + a_1 + a_1^2)\ (1 + a_2 + a_2^2)\ ....(1 + a_n + a_n^2) \geq 3^n(a_1\ a_2\ a_3 ...... a_n)$

Section (E) :

E-1. (i) $S = 1 + 5 + 13 + 29 + 61 + ...n$ terms ...(i)

$S = 1 + 5 + 13 + 29 + 61 + ...T_n$ ...(ii)

(i) – (ii) we get

$0 = 1+ [\ 4 + 8 + 16 + 32 + ...(n -1)\ \text{term}] - T_n$

$T_n = 1 + \frac{4\ \ (2^{n-1}-1)}{(2-1)} = 1+2^{n+1} - 4 = 2^{n+1} - 3$

$S_n = \sum T_n = \sum\ (2^{n+1} - 3) = (2^{n+2} - 4) - 3n = 2^{n+2} - 3n - 4$

(ii) $S = \frac{3}{9}\ \ 3 + 33 + 333 + 3333 + ... n$ term.

$S = [9 + 99 + 999 + ...n\ \text{term}] = \frac{3}{9}\ [(10 - 1) + (10^2 - 1) + (10^3 - 1) + ...n\ \text{term}]$

$= \frac{3}{9}\left[\frac{10(10^n-1)}{10-1} - n\right] = \frac{3}{81}\ [10^{n+1} - 9n - 10]$

E-2. Let $S = \frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{15}{16}+$ ...... to n term $= \left(1-\frac{1}{2}\right)+\left(1-\frac{1}{4}\right)+\left(1-\frac{1}{8}\right)+\left(1-\frac{1}{16}\right)$ + ... n term

$= (1 + 1 + 1 + ...n\ \text{times}) - \left(\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+....n\ \ \text{term}\right)$

$= n - \frac{\frac{1}{2}\left(1-\left(\frac{1}{2}\right)^n\right)}{\left(1-\frac{1}{2}\right)} = n - 1 + \frac{1}{2n} = \frac{n\,.\,2^n - 2^n + 1}{2^n}$

E-3. (i) $T_n = 3^n - 2^n$; $S_n = \sum 3^n - \sum 2^n$ $S_n = \frac{3^{n+1}-3}{2} - \frac{2^{n+1}-2}{1}$; $S_n = \frac{1}{2}\left(3^{n+1}+1\right) - 2^{n+1}$

(ii) $\sum_{n=2}^{k} t_n = \sum_{n=2}^{k} n(n+2) = \frac{1}{6}k(k+1)(2k+7)$

(iii) clearly n^6 term of the given series is negative or positive accordingly as n is even or odd respectively

(a) n is even

$1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + ... + (n-1)^2 - n^2 = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + ...((n-1)^2 - n^2)$

$= (1-2)(1-2) + (3-4)(3+4) + (5-6)(5+6) + ... + ((n-1) - (n))(n-1+n) = -\frac{n(n+1)}{2}$

(b) n is odd $(1^2 - 2^2) + (3^2 - 4^2) + ... \{(n-2)^2 - (n-1)^2\} + n^2$

$= (1-2)(1+2) + (3-4)(3+4) + ... + [(n-2) - (n-1)][(n-2) + (n-1)] + n^2$

$= -(1 + 2 + 3 + 4 + ... + (n-2) + (n-1)) + n^2 = \frac{(n-1)(n-1+1)}{2} + n^2 = \frac{n(n+1)}{2}$

(iv) $\sum_{r=1}^{10}(3r+7)^2 = 6265$ (v) $S_n = \sum_{r=1}^{n} I(r) = n(2n^2 + 9n + 13) \Rightarrow I(r) = S_r - S_{r-1}$

$= r(2r^2 + 9r + 13) - (r-1)(2(r-1)^2 + 9(r-1) + 13) = 6r^2 + 12r + 6 - 6(r+1)^2 \Rightarrow \sqrt{I(r)} = \sqrt{6}(x+1)$

$\Rightarrow \sum_{r=1}^{n} I(r) = \sqrt{6}\sum_{r=1}^{n}(r+1) = \sqrt{6}\left(\frac{n^2+3n}{2}\right) = \sqrt{\frac{3}{2}}(n^2+3n)$

E-4. (i) $S = \frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} + ...$n terms; $T_n = \frac{1}{(2n-1)(2n+1)(2n+3)}$

$\therefore$ $T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$; $T_1 = \frac{1}{4}\left[\frac{1}{1.3} - \frac{1}{3.5}\right]$, $T_2 = \frac{1}{4}\left[\frac{1}{3.5} - \frac{1}{5.7}\right]$,

$T_3 = \frac{1}{4}\left[\frac{1}{5.7} - \frac{1}{7.9}\right] \vdots$ $T_n = \frac{1}{4}\left[\frac{1}{(2n-1)(2n+1)} - \frac{1}{(2n+1)(2n+3)}\right]$ sum of all terms gives S_n

$\Rightarrow$ $S_n = \frac{1}{4}\left[\frac{1}{3} - \frac{1}{(2n+1)(2n+3)}\right]$

(ii) $1.3.2^2 + 2.4.3^2 + 3.5.4^2 + ...$ n terms

$T_n = n(n+2)(n+1)^2 = n(n+1)(n+2)(n+3-2)$

$T_n = n(n+1)(n+2)(n+3) - 2(n)(n+1)(n+2)$

$S_n = S_1 - 2S_2$

$S_1 = \sum_{r=1}^{n} r(r+1)(r+2)(r+3) = \sum_{r=1}^{n}\frac{1}{5}[r(r+1)(r+2)(r+3)(r+4) - (r-1)r(r+1)(r+2)(r+3)$

$= \frac{1}{5}[1.2.3.4.5 - 0] + \frac{1}{5}[2.3.4.5.6 - 1.2.3.4.5] + \frac{1}{5}[3.4.5.6.7 - 2.3.4.5.6] + ... + \frac{1}{5}[n(n+1)$
$(n+2)(n+3)(n+4) - (n-1)n(n+1)(n+2)(n+3)]$

$\therefore$ $S_1 = \frac{1}{5}[n(n+1)(n+2)(n+3)(n+4)]$

Now $S_2 = \sum_{r=1}^{n} r(r+1)(r+2) = \frac{1}{4}\sum_{r=1}^{n}[r(r+1)(r+2)(r+3) - (r-1)r(r+1)(r+2)]$

$= \frac{1}{4}[1.2.3.4 - 0] + \frac{1}{4}[2.3.4.5 - 1.2.3.4] + \frac{1}{4}[3.4.5.6 - 2.3.4.5]$

$......... + \frac{1}{4}[n(n+1)(n+2)(n+3) - (n-1)n(n+1)(n+2)]$

$\therefore$ $S_2 = \frac{1}{4}[n(n+1)(n+2)(n+3)]$

$\therefore$ $S_n = \left[\frac{n(n+1)(n+2)(n+3)(n+4)}{5}\right] - \frac{2}{4}[n(n+1)(n+2)(n+3)]$

$= n(n+1)(n+2)(n+3)\left[\frac{n+4}{5} - \frac{1}{2}\right] = \frac{1}{10}n(n+1)(n+2)(n+3)(2n+3)$

PART - II

Section (A) :

A-1. $S = \frac{2p+1}{2}$ $[2(p^2 + 1) + 2p] = (2p + 1)(p^2 + 1 + p) = 2p^3 + 3p^2 + 3p + 1 = p^3 + (p + 1)^3$

A-2. $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225 \quad \Rightarrow \quad 3(a_1 + a_{24}) = 225$
(sum of terms equidistant from beginning and end are equal) $a_1 + a_{24} = 75$

Now $a_1 + a_2 + \dots\dots + a_{23} + a_{24} = \frac{24}{2}[a_1 + a_{24}] = 12 \times 75 = 900$

A-3. 2, 5, 8 ....
$a = 2, d = 3 \quad \Rightarrow \quad S_{2n} = n(4 + (2n - 1)3] = n(6n + 1) \Rightarrow$ 57, 59, 61,........
$S_n = [2 \times 57 + (n - 1)2] = n[57 + n - 1] = n(56 + n)$
$n(6n + 1) = n(56 + n) \quad \Rightarrow \quad 5n = 55 \quad \Rightarrow \quad n = 11.$

A-4. Sum of the integer divided by 2 = $2 + 4 + \dots\dots\dots + 98 + 100 = \frac{50}{2}[2.2 + (50 - 1)2] = 50[51] = 2550$

Sum of the integer divided by 5 = $5 + 10 + \dots\dots\dots 95 + 100 = \frac{20}{2}[5 + 100] = 1050$

Sum of the integer divided by 10 $\Rightarrow \frac{10}{2}[10 + 100] = 550$

Sum of the integers divided by 5 or 10 = 2550 + 1050 – 550 = 3050

A-5. $A_1^2 - A_2^2 + A_3^2 - A_4^2 + A_5^2 - A_6^2 = -d(A_1 + A_2 + \dots + A_6) = -\left(\frac{b-a}{7}\right)(3(b + a)) = 3\left(\frac{a^2 - b^2}{7}\right)$ = Prime

$\Rightarrow a = 4, b = 3$

Section (B) :

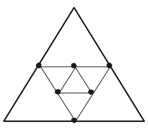
B-1. $T_3 = 4$
$T_1 . T_2 . T_3 . T_4 . T_5 = a^5 . r^{1+2+3+4} = a^5 . r^{10} = (ar^2)^5 = 4^5$

B-2. $S = \frac{a}{1-r} \Rightarrow r = \frac{S-a}{S}$; $S' = \frac{a[1-r^n]}{1-r} = S\left[1 - \left(\frac{S-a}{S}\right)^n\right]$

B-3. $a_1 = 2; a_{n+1} = \frac{a_n}{3}; a_2 = \frac{a_1}{3} = \frac{2}{3}; a_3 = \frac{a_2}{3} = \frac{2}{3^2}$

$a_1 + a_2 + \dots\dots\dots + a_{20} = 2 + \frac{2}{3} + \frac{2}{3^2} + \dots\dots = \frac{2 \; . \left[1 - \left(\frac{1}{3}\right)^{20}\right]}{1 - \frac{1}{3}} = 3\left(1 - \frac{1}{3^{20}}\right)$

B-4. $\alpha + \beta = 3$, $\alpha\beta = a$; $\gamma + \delta = 12$, $\gamma\delta = b$
$\alpha, \beta, \gamma, \delta$ are in G.P. Let r be the common ratio so $\alpha(1 + r) = 3$
$\alpha r^2(1 + r) = 12 \quad \Rightarrow \quad r^2 = 4 \quad \Rightarrow \quad r = 2$
so $\alpha = 1 \quad \Rightarrow \quad$ so a = 2, b = 32 **Ans**

B-5.  $= 3[24 + 12 + 6 + \dots\dots\infty] = 3\frac{24}{1 - \frac{1}{2}} = 144$

B-6. If a, G_1, G_2, G_3, b are in G.P. with common ratio equal to 'r' then $G_1 -$ a, $G_2 - G_1$, $G_3 - G_2$, $b - G_3$ are also in G.P. with same common ratio $\Rightarrow \frac{G_3 - G_2}{G_2 - G_1} = r = 2 \Rightarrow \frac{b}{a} = r^4 = 16$

Section (C) :

C-1. $T_3 = \frac{1}{3}$, $T_6 = \frac{1}{5}$. $T_n = \frac{3}{203}$

then 3rd , 6th term of A.P. series are 3, 6, $\frac{203}{3}$

$a + 2d = 3 \Rightarrow a = 5d = 5$

$d = \frac{2}{3}$, $a = \frac{5}{3}$

$a + (n-1)d = \frac{203}{3} \Rightarrow \frac{5}{3} + (n-1) = \frac{203}{3}$

$(n-1)^2 = 198$
n = 100

C-2. a, b, c are in H.P., then $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P. $S = \frac{b+a}{b-a} + \frac{b+c}{b-c} = \frac{\frac{1}{a}+\frac{1}{b}}{\frac{1}{a}-\frac{1}{b}} + \frac{\frac{1}{c}+\frac{1}{b}}{\frac{1}{c}-\frac{1}{b}}$

Let $\frac{1}{a} - \frac{1}{b} = \frac{1}{b} - \frac{1}{c} = d$

$S = \frac{\left(\frac{1}{a}+\frac{1}{b}\right) - \left(\frac{1}{c}+\frac{1}{b}\right)}{d} = \frac{\left(\frac{1}{a}-\frac{1}{c}\right)}{d} = \frac{2d}{d} = 2$

C-3. $x^3 - 11x^2 + 36x - 36 = 0$
if roots are in H.P. , then roots of new equation
$\frac{1}{x^3} - \frac{11}{x^2} + \frac{36}{x} - 36 = 0$ are in A.P.
$36x^3 + 36x^2 - 11x + 1 = 0$
$36x^3 - 36x^2 + 11x - 1 = 0$
Let the roots be α, β, γ
$\alpha + \beta + \gamma = 1$

$3\beta = 1$ $(2\beta, = \alpha + \gamma)$ $\beta = \frac{1}{3}$

so middle roots in 3.

C-4. a, b, c, d $\rightarrow$; $\frac{a}{abcd}, \frac{b}{abcd}, \frac{c}{abcd}, \frac{d}{abcd} \rightarrow$

$\frac{1}{bcd}, \frac{1}{acd}, \frac{1}{abd}, \frac{1}{abc} \rightarrow$; $\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd} \rightarrow$

abc, abd, acd, bcd $\rightarrow$

C-5. $3 + \frac{1}{4}(3+d) + \frac{1}{4^2}(3+2d) + \ldots\ldots\ldots\ldots + \infty = 8$

$S = 3 + (3+d) + (3+2d) + \ldots\ldots\ldots\ldots + \infty$ ...... (i)

$\frac{1}{4} S = \frac{3}{4} + \frac{1}{4^2}(3 + d) + \ldots\ldots\ldots\infty$ ...... (ii)

(i) – (ii) we get

$\frac{3}{4} S = 3 + \frac{1}{4} d + \frac{1}{4^2} d + \ldots\ldots\ldots \infty$; $\frac{3}{4} S = 3 + \frac{\frac{1}{4}d}{1 - \frac{1}{4}}$

$\frac{3}{4}S = 3 + \frac{d}{3}$; $S = \frac{12}{3} + \frac{4}{9}$ d = 8 = 4 + $\frac{4}{9}$ d = 8 $\Rightarrow \frac{4}{9}$ d = 4 $\Rightarrow$ d = 9 **Ans**

C-6. $n\left(\frac{a+b}{2}\right) = n\left(\frac{a+b}{2ab}\right) \Rightarrow ab = 1$

C-7. If first and last term of A.P. and H.P. are same the product of x terms begining in A.P. and kth term from end in H.P. is constant and equal = first term × last term
$a_7 h_{24} + a_{14} h_{17}$ = ab + ab = 2ab = 2(25) (2) = 100

C-8. Let a, b, c in G.P. then $b^2 = ac$ then a + b, 2b, b + c in HP
$\frac{1}{a+b}, \frac{1}{2b}, \frac{1}{b+c}$ in AP $\frac{2}{2b} = \frac{1}{a+b} + \frac{1}{b+c}$
(a + b) (b + c) = (a + c + 2b) b $\Rightarrow$ ab + b^2 + ac + bc = ab + bc + $2b^2$
$\therefore$ $b^2 = ac$
So statement (1) and (2) is true

C-9. $S = \frac{3^{10}}{\left(1-\frac{1}{6}\right)} + \frac{3^{10}\left(\frac{1}{6}\right)}{\left(1-\frac{1}{6}\right)^2} = \frac{6^2.3^{10}}{5^2} \Rightarrow \left(\frac{25}{36}\right)S = 3^{10}$

C-10. $S = \frac{3}{2} + \frac{15}{2^2} + \frac{35}{2^3} + \frac{63}{2^4} + ...\infty$

$\frac{1}{2}S = \frac{3}{2^2} + \frac{15}{2^3} + \frac{35}{24} + ...\infty$

$\frac{S}{2} = \frac{3}{2^2} + \frac{12}{2^2} + \frac{20}{2^3} + ...\infty$

again use same concept S = 23

Section (D) :

D-1. $x \in R$
$5^{1+x} + 5^{1-x}$, a/2 , $5^{2x} + 5^{-2x}$ are in A.P
$a = (5^{2x} + 5^{-2x}) + (5^{1+x} + 5^{1-x})$ $\Rightarrow$ $a = (5^{2x} + 5^{-2x}) + 5(5^x + 5^{-x}) = (5^x - 5^{-x})^2 + 2 + 5(5^{x/2} - 5^{-x/2})^2 + 10$
$a = 12 + (5^x - 5^{-x})^2 + 5(5^{x/2} - 5^{-x/2})^2 \Rightarrow$ $a \ge 12$

D-2. AM = A = $\frac{a+b+c}{3}$; GM = G = $(abc)^{1/3}$

HM = H = $\frac{3abc}{ab+bc+ca} = \frac{3G^3}{ab+bc+ca}$.

Equation whose roots are a,b,c $\Rightarrow$ $x^3 - (a + b + c)x^2 + (\Sigma ab)x - abc = 0$

$\Rightarrow x^3 - 3Ax^2 + \frac{3G^3}{H}.x - G^3 = 0$ **Ans**

D-3. a + b + c + d = 2 $\Rightarrow$ a, b, c, d > 0
$\frac{(a+b)+(c+d)}{2} \ge \sqrt{(a+b)\ (c+d)}$ $\Rightarrow$ $1 \ge \sqrt{(a+b)\ (c+d)} \ge 0$
$\Rightarrow$ $0 \le (a + b)(c + d) \le 1$ $\Rightarrow$ $0 \le M \le 1$

D-4. Taking A.M. and G.M. of number, $\frac{a}{2}, \frac{a}{2}, \frac{b}{3}, \frac{b}{3}, \frac{b}{3}, \frac{c}{2}, \frac{c}{2}$

we get A.M. ≥ G.M. $\dfrac{2.\frac{a}{2}+3.\frac{b}{3}+2.\frac{c}{2}}{7} \ge \left(\left(\frac{a}{2}\right)^2\left(\frac{b}{3}\right)^3\left(\frac{c}{2}\right)^2\right)^{1/7}$

or $\frac{3}{7} \ge \left(\frac{a^2b^3c^2}{2^2.3^3.2^2}\right)^{1/7}$ or $\frac{3^7}{7^7} \ge \frac{a^2b^3c^2}{2^4.3^3}$ or $a^2b^3c^2 \le \frac{3^{10}.2^4}{7^7}$

$\therefore$ Greatest value of $a^2\,b^3\,c^2 = \frac{3^{10}.2^4}{7^7}$

D-5. $P = \frac{a+b}{2}$, $Q = \sqrt{ab}$; $P - Q = \frac{a+b-2\sqrt{ab}}{2} = \frac{\left(\sqrt{a}-\sqrt{b}\right)^2}{2}$

Section (E) :

E-1 $H_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$

$1 + \frac{3}{2} + \frac{5}{3} + \ldots\ldots + \frac{2n-1}{n} = (2-1) + \left(2-\frac{1}{2}\right) + \left(2-\frac{1}{3}\right) + \ldots\ldots + \left(2-\frac{1}{n}\right) = 2n - H_n$

E-2. $S = 1 + 2 + 4 + 7 + 11 + 16 \ldots\ldots T_n$ ...(i)

$S = 1 + 2 + 4 + 7 + 11 \ldots\ldots T_n$ ...(ii)

(i) – (ii) we get

$O = 1+(1+2+3+4+5 \ldots\ldots (n-1) \text{ term}) - T_n \Rightarrow T_n = 1+\frac{(n-1)\,n}{2} = \frac{n^2}{2}-\frac{n}{2}+1$

$\therefore$ General term $= T_n = an^2 + bn + c$ here a = 1/2, b= –1/2, c = 1

$S_n = \sum T_n = \frac{n(n+1)\,(2n+1)}{12} - \frac{n(n+1)}{4} + n$

$S_{30} = \frac{30.\ 31.\ 61}{12} - \frac{30.\ 31}{4} + 30 = 4727.5 - 232.5 + 30 = 4525$ **Ans (D)**

E-3. $\sum_{r=1}^{n}\frac{1}{\sqrt{a+r\,x}+\sqrt{a+(r-1)\,x}}$; $\sum_{r=1}^{n}\frac{\sqrt{(a+rx)}-\sqrt{a+(r-1)\,x}}{(a+rx)-(a+(r-1)\,x)}$;

$= \frac{1}{x}\left[(\sqrt{a+x}-\sqrt{a+0.x})+(\sqrt{a+2x}-\sqrt{a+x})+(\sqrt{a+3x}-\sqrt{a+2x}) + \ldots + (\sqrt{a+nx}-\sqrt{a+(n-1)\,x})\right]$

$= \frac{1}{x}\left[\sqrt{a+nx}-\sqrt{a}\right] = \frac{n}{\sqrt{a}+\sqrt{a+nx}}$ **Ans**

E-4. $\sum_{r=1}^{10}(2r-1)r^2 = \sum_{r=1}^{10}2r^3 - \sum_{r=1}^{10}r^2 = 6050 - 385 = 5665$

PART - III

1. **(A)** $1x^{50} - (1 + 3 + 5 + \ldots + 99)x^{49} + (\quad)x^{48} \ldots \Rightarrow$ coefficient $x^{49} = -50^2 = -2500$

(B) $\frac{S_{2n}}{S_n} = \frac{\frac{2n}{2}\left[2a+(2n-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = 3 = \frac{2a+(2n-1)d}{2a+(n-1)d} = \frac{3}{2}$ $\therefore$ $d = \frac{2a}{n+1}$

Now $\frac{S_{3n}}{S_n} = \frac{\frac{3n}{2}\left[2a+(3n-1)d\right]}{\frac{n}{2}\left[2a+(n-1)d\right]} = \frac{3\left[2a+(3n-1)\frac{2a}{n+1}\right]}{\left[2a+(n-1)\frac{2a}{n+1}\right]} = \frac{3\left[(n+1)+(3n-1)\right]}{(n+1)+(n-1)} = \frac{3.4n}{2n} = 6$

(C) $S = \sum_{r=2}^{\infty}\left(\frac{1}{r^2-1}\right)$; $S = \frac{1}{2}\sum_{r=2}^{\infty}\left(\frac{1}{r-1}-\frac{1}{r+1}\right)$

$S = \frac{1}{2}\left[\left(\frac{1}{1}-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+\left(\frac{1}{4}-\frac{1}{6}\right)+\ldots\ldots\infty\right] = \frac{1}{2}\left[1+\frac{1}{2}\right] = \frac{3}{4}$

(D) $\ell, \ell r, \ell r^2$ $\therefore$ $\ell^3 r^3 = 27$ (volume) $\Rightarrow$ $\ell r = 3$

surface area

$2(\ell \,.\, \ell r + \ell r \,.\, \ell r^2 + \ell r^2\, \ell) = 78 \quad \Rightarrow \quad \ell^2 (r + r^2 + r^3) = 39 \Rightarrow \ell^2 \left(\frac{3}{\ell} + \frac{3^2}{\ell^2} + \frac{3^3}{\ell^3}\right) = 39$

$3\ell + 3^2 + \frac{3^3}{\ell} = 39 \quad \Rightarrow \quad \ell + 3 + \frac{9}{\ell} = 13$

$\therefore \quad \ell^2 - 10\ell + 9 = 0 \quad \Rightarrow \quad \ell = 1, 9 \quad \Rightarrow \quad \ell r = 3$ and $\Rightarrow \ell > \ell r$

$\therefore \quad r = \frac{1}{3} \qquad \therefore \quad \ell = 9$

2. (A) a, x, y, z, b in A.P. & a, x, y, z, b in H.P.

$\frac{1}{b}, \frac{1}{z}, \frac{1}{y}, \frac{1}{x}, \frac{1}{a}$ in A.P. ; $a, \frac{ab}{z}, \frac{ab}{y}, \frac{ab}{x}, b$ in A.P,

$\Rightarrow \quad ab = xz = y^2 = zx = ba$

$\Rightarrow \quad xz.y^2 .zx = (ab)\,(ab)\,(ab)$

$\Rightarrow \quad (xyz)\,(xyz) = a^3 b^3$

$ab = 3 \quad \Rightarrow \quad a = 1, b = 3$

or $\quad a = 3, b = 1$

(B) $S_\infty = 2^{1/4} .4^{1/8} .8^{1/16}\ldots\ldots\ldots\ldots\infty$; $S_\infty = 2^{1/4} .2^{2/8} .2^{3/16}\ldots\ldots\ldots\ldots\infty$

$S_\infty = 2^{1/4 + 2/8 + 3/16 \ldots\ldots\infty} = 2^{S'_\infty}$

Let $S'_\infty = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} \ldots\ldots\ldots$

$S'_n = \frac{1}{4} + \frac{2}{8} + \frac{3}{16} \ldots\ldots\ldots \frac{n}{2^{n+1}}$ ...(i)

$\frac{S'_n}{2} = \frac{1}{8} + \frac{2}{16} + \ldots\ldots + \frac{n-1}{2^{n+1}} + \frac{n}{2^{n+2}}$ ...(ii)

(i) – (ii) we get

$\frac{S'_n}{2} = \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots\ldots\ldots \frac{1}{2^{n+1}} - \frac{n}{2^{n+2}}$

$\Rightarrow \quad \frac{S'_n}{2} = \frac{1}{4}\left(\frac{1-(1/2)^n}{1-1/2}\right) - \frac{n}{2^{n+2}}$

$S'_n = \frac{2.2}{4}\left(1 - \left(\frac{1}{2}\right)^n\right) - \frac{2n}{2^{n+2}} \Rightarrow S'_\infty = 1$

$\therefore \quad S_\infty = 2^{S'_\infty} = 2$

(C) x,y,z are in A.P $y = \frac{x+z}{2}$,

or 2y = x + z

(x + 2y – z) (x + z + z – x) (z + x –y) = (x + (x + z) – z) (x + z + z – x) (2y–y) = 2x .2z .y = 4xyz

$\therefore \quad k = 4$

(D) $d = \frac{31-1}{m+1} = \frac{30}{m+1}$; $\frac{A_7}{A_{m-1}} = \frac{1+7\frac{30}{m+1}}{1+(m-1)\frac{30}{m+1}} = \frac{5}{9}$

$\Rightarrow \frac{m+211}{31 \quad m-29} = \frac{5}{9}$

$\Rightarrow$ 146 m = 2044 $\qquad \Rightarrow$ m =14

$\therefore \quad \frac{m}{7} = 2$

EXERCISE # 2
PART - I

1. $\frac{x_1}{x_1+1} = \frac{x_2}{x_2+3} = \frac{x_3}{x_3+5} = ... = \frac{x_{2013}}{x_{2013}+4025} = \frac{1}{\lambda}$

$\Rightarrow \quad x_1 = \frac{1}{\lambda-1}, x_2 = \frac{3}{\lambda-1}, x_3 = \frac{5}{\lambda-1}, ..., x_{2013} = \frac{4025}{\lambda-1}$

$\Rightarrow \quad x_1, x_2, x_3, ..., x_{2013}$ are in A.P. with common difference = $\frac{2}{\lambda-1}$ = d

$x_1, x_2, x_3, ..., x_{2013} = \frac{2}{\lambda-1}$ = d

2. 2b = a + c and $b^2 = \pm$ ac

case-I

if b^2 = ac and a + c + b = $\frac{3}{2}$ $\Rightarrow$ b = $\frac{1}{2}$

a + c = 1 $\Rightarrow$ ac = $\frac{1}{4}$ $\Rightarrow$ (1 – c) c = $\frac{1}{4}$

$c^2 - c + \frac{1}{4} = 0$ $\Rightarrow$ c = $\frac{1}{2}$ $\Rightarrow$ a = $\frac{1}{2}$

a = b = c so not valid

case-II

b^2 = – ac and b = $\frac{1}{2}$; a + c = 1 $\Rightarrow$ ac = – $\frac{1}{4}$

(1 – c) c = – $\frac{1}{4}$ $\Rightarrow$ $c^2 - c - \frac{1}{4} = 0$

$\Rightarrow$ c = $\frac{1 \pm \sqrt{1+1}}{2} = \frac{1 \pm \sqrt{2}}{2}$

c = $\frac{1+\sqrt{2}}{2}$ $\Rightarrow$ a = $\frac{1-\sqrt{2}}{2}$

3. $S_1 + S_2 + S_3 + ... + S_p$

$\Rightarrow \quad S_1 = \frac{n}{2}$ [2.1 + (n – 1) 1] $S_2 = \frac{n}{2}$ [2.2 + (n – 1) 3]

$S_3 = \frac{n}{2}$ [2.3 + (n – 1) 5]

⋮

⋮

$S_p = \frac{n}{2}$ [2.p + (n – 1) (2p – 1)]

So $S_1 + S_2 + ... + S_p = \frac{n}{2}$ [2(1 + 2 + ... p) + (n – 1) (1 + 3 + 5 + ... (2p – 1))]

$= \frac{n}{2}\left[2\ .\ \frac{p\ (p+1)}{2} + (n-1)\ \ p^2\right] = \frac{n\ \ p}{2}\ \ (n\,p + 1)$ **Ans.**

4. $\frac{ar^{p-1}(r^n-1)}{r-1} = k\ \frac{ar^{q-1}(r^n-1)}{r-1}$. ; $r^{p-1} = k.\ r^{q-1}$; $k = r^{p-q}$

5. $45^2 = 2025$ & $46^2 = 2116$

$\Rightarrow$ there are 45 squares $\leq$ 2056

which are left only from sequence of possible integers since 2056 = 2011 + 45

$\therefore$ 2011^{th} term = 2056

6. $x = \frac{1}{1-a}$, $y = \frac{1}{1-b}$, $z = \frac{1}{1-c}$ $\Rightarrow$ a, b, c are in A.P.

$\Rightarrow$ 1 – a, 1 – b, 1 – c are also in A.P. $\Rightarrow$ $\frac{1}{1-a}, \frac{1}{1-b}, \frac{1}{1-c}$ are in H.P.

7. $f(k)\ \sum_{r=1}^{n}(a_r - a_k) = \sum_{r=1}^{n}a_r - \sum_{r=1}^{n}a_k$ $f(k) = \lambda - na_k$ $f(i) = \lambda - na_i$ $\frac{a_i}{f(i)} = \frac{a_i}{\lambda - na_i} = \frac{1}{\frac{\lambda}{a_i} - n}$ $<a_i>$ in A.P.

$\Rightarrow < \frac{1}{a_i} >$ in A.P. $\Rightarrow < \frac{\lambda}{a_i} - n >$ in A.P. $\Rightarrow < \frac{1}{\frac{\lambda}{a_i} - n} >$ in H.P. $\Rightarrow < \frac{a_i}{f(i)} >$ in A.P.

8. $a_1 \cdot a_2 \cdot a_3 \cdot ....a_n = c \Rightarrow \frac{a_1 + a_2 + a_3 + ...2a_n}{n} \geq (a_1a_2a_3\ ...\ 2a_n)^{1/n} \geq (2c)^{1/n}$

$\Rightarrow$ $a_1 + a_2 + a_3 + ...2a_n \geq n(2\,c)^{1/n}$

9. $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$n terms $= \frac{n(n+1)^2}{2}$, when n is even

$1^2 + 2.\ 2^2 + 3^2 + ...2.\ n^2 = n\ \frac{(n+1)^2}{2}$ $\Rightarrow$ when n is odd n + 1 is even

$1^2 + 2.\ 2^2 + 3^2 + ...\ n^2 + 2.\ (n + 1)^2 = (n + 1)\ \frac{(n+2)^2}{2}$

$1^2 + 2.\ 2^2 + 3^2 + ...\ n^2 = (n + 1)\left[\frac{(n+2)^2}{2} - 2(n+1)\right] = \frac{(n+1)\ \ n^2}{2}$

10. $S_n - S_{n-2} = 2$ $\Rightarrow$ $T_n + T_{n-1} = 2$

Also $T_n - T_{n-1} = 2$; $T_n + T_{n-1} = \left(\frac{1}{n^2} + 1\right)\ T_{n-1} = 2 \Rightarrow T_{n-1} = \frac{2}{1 + \frac{1}{n^2}} = \frac{2n^2}{1+n^2}$ So $T_m = \frac{2(m+1)^2}{1+(m+1)^2}$

11. If $1^2 + 2^2 + 3^2 + ... + 2003^2 = (2003)\ (4007)\ (334)$

(1) (2003) + (2) (2002) + (3) (2001) + ... + (2003) (1) = (2003) (334) (x)

$\Rightarrow \sum_{r=1}^{2003} r\ \ (2003 - r + 1) = (2003)\ (334)\ (x)$ $\Rightarrow$ $2004.\ \sum_{r=1}^{2003} r - \sum_{r=1}^{2003} r^2 = (2003)\ (334)\ (x)$

$\Rightarrow$ $2004\left(\frac{2003\ \ .\ \ 2004}{2}\right)$. – 2003. (4007). 334 = (2003) (334) (x)

$\Rightarrow$ x = 2005 **Ans.**

12. $\sum_{r=1}^{n} t_r = S_n$ $\Rightarrow$ $\sum_{r=1}^{n-1} t_r = S_{n-1} \Rightarrow t_n = S_n - S_{n-1} = \frac{n(n+1)(n+2)}{2}$

$$\sum_{r=1}^{n}\frac{1}{t_r} = \sum_{r=1}^{n}\frac{2}{r(r+1)(r+2)} = \sum\left(\frac{1}{n(n+1)}-\frac{1}{(n+1)(n+2)}\right) = -\left(\frac{1}{(n+1)(n+2)}-\frac{1}{2}\right)$$

13. $\because$ $I = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots \infty = \frac{\pi^2}{6}$

Let $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = A$

$\because$ $I = \left(\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots\ldots\infty\right) + \frac{1}{2^2}\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\ldots\ldots\infty\right)$

$\Rightarrow$ $I = A + \frac{I}{4}$ $\Rightarrow A = \frac{3I}{4} = \frac{3}{4} \times \frac{\pi^2}{6}$ $\Rightarrow A = \frac{\pi^2}{8}$

14. $= (-1^2 + 2^2 + 3^2 + 4^2 - 5^2 + 6^2 + 7^2 + 8^2 \ldots\ldots) + (1^2 + 2^2 - 3^2 - 4^2 + 5^2 + 6^2 - 7^2 - 8^2 \ldots\ldots) - 4(6^2 + 14^2 + \ldots)$
$= 2[2^2 - 6^2 + 10^2 - 14^2 + \ldots\ldots]$ {2n terms }
$= 2[(2-6)(2+6) + (10-14)(10+14)\ldots\ldots]$
$= 2 \times (-4)[2 + 6 + 10 + 14 \ldots\ldots]$ {2n terms }
$= 2 \times (-4) \times 2[1 + 3 + 5 + 7 \ldots\ldots]$ {2n terms }
$= -16(2n)^2 = -64n^2 = -(8n)^2$

PART - II

1. Let first installment be = 'a' and the common difference of the A.P. be 'd'

So $a + (a + d) + (a + 2d) + \ldots\ldots + (a + 39d) = 3600$

$\Rightarrow$ $\frac{40}{2}[2a + 39d] = 3600$

$\Rightarrow$ $2a + 39d = 180$ ..... (1)

and $\frac{30}{2}[2a + 29d] = 2400$

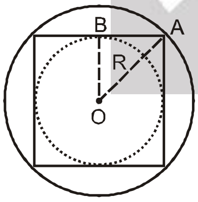
$\Rightarrow$ $2a + 29d = 160$ ..... (2)

By equations (1) & (2), we get

$d = 2$ and $a = 51$ **Ans.**

2. Area $= A_1 = \pi R^2$ $\Rightarrow$ $OB = \frac{R}{\sqrt{2}}$

So Area $A_2 = \pi\left(\frac{R^2}{2}\right)$. So $\lim_{n\to\infty}\left((\pi R^2)+\frac{\pi R^2}{2}+\frac{\pi R^2}{4}+\ldots\infty\right) = \pi R^2 \frac{1}{\left(1-\frac{1}{2}\right)} = 2\pi R^2$



Now sum of areas of the squares $= 2R^2 + \frac{2R^2}{2} + \frac{2R^2}{4} + \ldots\ldots \infty = \frac{2R^2}{1-\frac{1}{2}} = 4R^2$

3. $a + 6d = 9$; $T_1 T_2 T_7 = a(a + d)(a + 6d) = 9a(a + d) = 9(9 - 6d)(9 - 5d)$

$\because$ $T_7 = a + 6d = 9.$

Let $A = T_1 T_2 T_7$

$\frac{dA}{d(d)} = 9[-45 - 54 + 60d] = 0$ $\Rightarrow$ $60d = 99$

$\Rightarrow$ $d = \frac{33}{20}$ Ans.

4. Let AP has 2n terms

Sum of odd term = 24 $\Rightarrow$ $\frac{n}{2}[a_1 + a_{2n-1}] = 24$ ..... (1)

and sum of even terms = 30 $\Rightarrow$ $\frac{n}{2}[a_2 + a_{2n}] = 30$ ..... (2)

and $a_{2n} = a_1 + \frac{21}{2}$

$a_1 + (2n-1)d = a_1 + \frac{21}{2} \Rightarrow (2n-1)\,d = \frac{21}{2}$ ..... (3)

By equation (1) & (2)

$a_1 + a_{2n-1} = \frac{48}{n}$ and $a_2 + a_{2n} = \frac{60}{n}$

So $a_1 + (n-1)d = \frac{24}{n}$ and $a_1 + nd = \frac{30}{n}$ So $d = \frac{6}{n}$

Now $(2n-1)\frac{6}{n} = \frac{21}{2} \Rightarrow n = 4,\ d = \frac{6}{4} = \frac{3}{2}$

So no. of terms = 2n = 8 and $a_1 = 3/2$. Numbers are $\frac{3}{2}$, 3, $\frac{9}{2}$ ....

5. AP(1, 3) = {1, 4, 7, 10, 13, 16……}
AP(3, 5) = {3, 8, 13, 18, ……}
AP(5, 7) = {5, 12, 19, 26, 33, ……}
AP(1, 3) $\cap$ AP(3, 5) = AP(13, 15) = {13, 28, 43, 58, 73, 88, 103, ……}
AP(1, 3) $\cap$ AP(3, 5) $\cap$ AP(5, 7) = AP{103, 105)

6. $\log_2 x + \log_2(\sqrt{x}) + \log_2 (x)^{1/4} + \log_2 (x)^{1/8} + \ldots = 4 \Rightarrow \log_2 x\ \frac{1}{2} + \log_2 x + \frac{1}{4}\log_2 x + \ldots = 4$

$\Rightarrow \frac{\log_2 x}{1-\frac{1}{2}} = 4 \Rightarrow \log_2 x = 2 \Rightarrow x = 4$

$$\frac{\left(\frac{x^2+3x+2}{x+2}\right) + 3x - \frac{x(x^3+1)}{(x+1)(x^2-x+1)}\log_2 8}{(x-1)(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 2)} = \frac{x+1}{x-1}$$

7. $\alpha + \gamma = \frac{4}{A}$, $\alpha\gamma = \frac{1}{A}$ and $\beta + \delta = \frac{6}{B}$, $\beta\delta = \frac{1}{B}$

Since $\alpha, \beta, \gamma, \delta$ are in H.P. $\beta = \frac{2\alpha\gamma}{\alpha+\gamma} = \frac{1}{2}$ is root of $Bx^2 - 6x + 1 = 0 \Rightarrow B = 8$

similarly $\gamma = \frac{2\beta\delta}{\beta+\delta} = \frac{1}{3}$ is root of $Ax^2 - 4x + 1 = 0 \Rightarrow A = 3$

8. ar^2, $3a.ar^3$, ar are in A.P. d = 1/8
$3a^2r^3 - ar^2 = 1/8$ .....(1) ; $ar = a^2 + 2.1/8$ ......(2)

from (2) a = $(-)^2 = \frac{1}{4r(1-r)}$ ......(3)

from (1) & (3) $r = \frac{1}{2}$; r = –2 but $0 < r < 1$

$r = \frac{1}{2} \Rightarrow a = 1 \Rightarrow 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ .... sum = $\frac{1}{1-\frac{1}{2}} = 2$

9. a, b, c are in G.P. $\Rightarrow$ $b^2 = ac$ $\Rightarrow$ (a – b), (c – a), (b – c) are in H.P.

So $\frac{1}{a-b}, \frac{1}{c-a}, \frac{1}{b-c}$ are in AP. Let a, b, c are $\frac{b}{r}$, b, br

So $\frac{1}{\frac{b}{r}-b}, \frac{1}{br-\frac{b}{r}}, \frac{1}{b-br}$ are in AP. So $\frac{2}{br-\frac{b}{r}} = \frac{1}{\frac{b}{r}-b} + \frac{1}{b-br}$

$\Rightarrow \frac{2r}{r^2-1} = \frac{r}{1-r} + \frac{1}{1-r}$ $\Rightarrow -\frac{2r}{(r+1)} = (1+r)$ $\Rightarrow (1+r)^2 = -2r$

$\Rightarrow r^2 + 1 + 4r = 0$ $\Rightarrow \frac{c}{a} + 1 + \frac{4b}{a} = 0$ $\Rightarrow a + 4b + c = 0$

10. $a, a_1, a_2 ...., a_{2n}, b$ are in AP and $a, g_1, g_2, ...g_{2n}, b$ are in GP and $h = \frac{2ab}{a+b}$

$\therefore \frac{a_1+a_{2n}}{g_1 g_{2n}} + \frac{a_2+a_{2n-1}}{g_2 g_{2n-1}} + ... + \frac{a_n+a_{n+1}}{g_n g_{n+1}} = \frac{a+b}{ab} + \frac{a+b}{ab} + ... + \frac{a+b}{ab} = 2n\left(\frac{a+b}{2ab}\right) = \frac{2n}{h}$

11. $\frac{a+b}{2} = 6$ $G^2 + 3H = 48$ $\Rightarrow ab + 3\frac{2ab}{a+b} = 48$ $\Rightarrow ab + \frac{3ab}{6} = 48$

$\Rightarrow \frac{3}{2}ab = 48$ $\Rightarrow ab = 32$ $\Rightarrow a = 4, b = 8.$

12. $S = \frac{5}{13} + \frac{55}{(13)^2} + \frac{555}{(13)^3} + ...\infty$; $S = \frac{5}{9}\left[\frac{(10-1)}{13} + \frac{10^2-1}{(13)^2} + \frac{10^3-1}{(13)^3} + ...\infty\right]$

$= \frac{5}{9}\left[\frac{\frac{10}{13}}{1-\frac{10}{13}} - \left(\frac{\frac{1}{13}}{1-\frac{1}{13}}\right)\right] = \frac{5}{9}\left[\frac{10}{3} - \frac{1}{12}\right] = \frac{5}{9}\left[\frac{39}{12}\right] = \frac{65}{36}$ **Ans**

13. $S = 1^2 - \frac{2^2}{5} + \frac{3^2}{5^2} - \frac{4^2}{5^3} + \frac{5^2}{5^4} - \frac{6^2}{5^5} + ...\infty$ ...(i)

$-\frac{1}{5}S = \frac{1}{5} - \frac{2^2}{5^2} + \frac{3^2}{5^3} - \frac{4^2}{5^4} + \frac{5^2}{5^5} - + ...\infty$ ...(ii)

(i) – (ii) we get

$\frac{6}{5}S = 1 - \frac{3}{5} + \frac{5}{5^2} - \frac{7}{5^3} + \frac{9}{5^4} - \frac{11}{5^5} + ...\infty$ ...(iii)

$-\frac{6}{25}S = -\frac{1}{5} + \frac{3}{5^2} - \frac{5}{5^3} + \frac{7}{5^4} - \frac{9}{5^5} + ...\infty$ ...(iv)

(iii) – (iv) we get

$\frac{6s}{5}\left(\frac{6}{5}\right) = 1 - \frac{2}{5} + \frac{2}{5^2} - \frac{2}{5^3} + \frac{2}{5^4} - ....\infty$

$\frac{36}{25}S = 1 - \frac{2}{5}\left[\frac{1}{1+\frac{1}{5}}\right]$; $\frac{36}{25}S = 1 - \frac{2}{5}\left(\frac{5}{6}\right) = \frac{2}{3}$; $S = \frac{25}{36} \times \frac{2}{3} = \frac{25}{54}$ **Ans**

14. $x_1 + x_2 + x_3 + ...+ x_{50} = 50$

AM $\geq$ HM

$$\frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{1}{\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + ... + \frac{1}{x_{50}}}{50}}$$

$$\Rightarrow \frac{x_1 + x_2 + ... + x_{50}}{50} \geq \frac{50}{\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}}}$$

$$\Rightarrow \frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} \geq 50$$

so min value of $\frac{1}{x_1} + \frac{1}{x_2} + ... + \frac{1}{x_{50}} = 50$

15. $\frac{(a_1 + a_2) + (a_3 + a_4)}{2} \geq \sqrt{(a_1 + a_2)(a_3 + a_4)}$

$\Rightarrow (a_1 + a_2)(a_3 + a_4) \leq \frac{225}{4}$

16. $\frac{S_3(1 + 8\ S_1)}{S_2^2} = \frac{\left[\frac{n\ \ (n+1)}{2}\right]^2 \left[1 + \frac{8n(n+1)}{2}\right]}{\left[\frac{n(n+1)\ \ (2n+1)}{6}\right]^2} = \frac{\left[1 + 4n\ \ (n+1)\right]\ 9}{(2n+1)^2} = 9$ **Ans**

17. $T_n = \frac{n}{1 + n^2 + n^4} = \frac{1}{2}\left[\frac{(2n)}{(1+n+n^2)(1-n+n^2)}\right]$; $T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2} - \frac{1}{1+n+n^2}\right]$

$T_1 = \frac{1}{2}\left[\frac{1}{1} - \frac{1}{3}\right]$, ; $T_2 = \frac{1}{2}\left[\frac{1}{3} - \frac{1}{7}\right]$, $T_3 = \frac{1}{2}\left[\frac{1}{7} - \frac{1}{13}\right]$,

⋮

$T_n = \frac{1}{2}\left[\frac{1}{1-n+n^2} - \frac{1}{1+n+n^2}\right]$

$S_n = \sum T_n = \frac{1}{2}\left[1 - \frac{1}{1+n+n^2}\right] = \frac{n+n^2}{2(1+n+n^2)}$

PART - III

1. Let a, a + d, a + 2d .... are Interior angles

∵ sum of interior angles = (n – 2) π , where n is the number of sides

∴ a = 120°, d = 5° ⇒ $\frac{n}{2}$ [240° + (n – 1) 5°] = (n – 2) 180°

⇒ $n^2 = 25n - 144$ ⇒ n = 16, 9 but $n \neq 16$

because if n = 16, then an interior angle will be 180° which is not possible. So n = 9

2. 1, $\log_y x$, $\log_z y$, $-15 \log_x z$ are in AP. Let commen diff. is d.

$\log_y x = 1 + d \Rightarrow x = (y)^{1+d}$; $\log_z y = 1 + 2d \Rightarrow y = (z)^{1+2d}$

$-15 \log_x z = 1 + 3d \Rightarrow z = x^{\left(\frac{1+3d}{-15}\right)}$

So $x = (y)^{1+d} = ((z)^{1+2d})^{1+d} \Rightarrow x = (x)^{\left(\frac{1+3d}{-15}\right)(1+d)(1+2d)}$

So (1 + d) (1 + 2d) (1 + 3d) = –15

So $d = -2 \Rightarrow x = (y)^{-1} \Rightarrow y = (z)^{-3} \Rightarrow z = (x)^{1/3} \Rightarrow z^3 = x$. **Ans.**

3. (D) $a_1 + 4a_2 + 6a_3 - 4a_4 + a_5 = 0 \Rightarrow a - 4(a + d) + 6(a + 2d) - 4(a + 3d) + (a + 4d) = 0 - 0 = 0$

Like wise we can check other options

4. $\frac{1}{16}$ a, b are in G.P. hence $a^2 = \frac{b}{16}$ or $16a^2 = b$ ......(1)

a, b, $\frac{1}{6}$ are in H.P. hence, $b = \dfrac{2a\frac{1}{6}}{a+\frac{1}{6}} = \dfrac{2a}{6a+1}$ ....(2)

From (1) and (2)

$16a^2 = \dfrac{2a}{6a+1} \Rightarrow$ $2a = \left(8a - \dfrac{1}{6a+1}\right) = 0 \Rightarrow$ $48a^2 + 8a - 1 = 0$ (a 0)

$\Rightarrow$ $(4a + 1)(12a - 1) = 0$ $\therefore$ $a = -\frac{1}{4}, \frac{1}{12}$

when $a = \frac{-1}{4}$, then from (1) ; $b = 16\left(-\frac{1}{4}\right)^2 = 1 \Rightarrow$ when $a = \frac{1}{12}$ then from (1) $\Rightarrow$ $b = 16\left(\frac{1}{12}\right)^2 = \frac{1}{9}$

therefore $a = -\frac{1}{4}$, $b = 1$ or $a = \frac{1}{12}$, $b = \frac{1}{9}$

5. We have 1111.....1 (91 digits) = $10^{90} + 10^{89} + ... + 10^2 + 10^1 + 10^0$

$= \dfrac{10^{91}-1}{10-1} = \dfrac{(10^{91}-1)}{10-1} \times \left(\dfrac{10^7-1}{10^7-1}\right) = \dfrac{10^{91}-1}{10^7-1}\left(\dfrac{10^7-1}{10-1}\right)$

$= (10^{84} + 10^{77} + 10^{70} + ... + 1)(10^6 + 10^5 + ... + 1)$

Thus 111.... 1 (91 digits) is not a prime number

6. $a + b + c = 25 \Rightarrow$ $2a = 2 + b$ $\Rightarrow$ $c^2 = 18b$ $\Rightarrow \frac{1}{2}\left(2 + \frac{c^2}{18}\right) + \frac{c^2}{18} + c = 25$

$\Rightarrow$ $c = 12, -24 \Rightarrow$ $c \neq -24$ $\Rightarrow$ $b = \frac{c^2}{18} = 8$ $\Rightarrow$ $a = 5$

7. $\because \dfrac{a}{1-r} = 4$ and $ar = \frac{3}{4}$

$\therefore \dfrac{3/4r}{1-r} = 4$ $\Rightarrow$ $\dfrac{3}{4r} = 4 - 4r$

$16r^2 - 16r + 3 = 0$ $\Rightarrow$ $16r^2 - 12r - 4r + 3 = 0$

$4r(4r - 3) - 1(4r - 3) = 0$ $\Rightarrow$ $(4r - 3)(4r - 1) = 0$ $\Rightarrow$ $r = \frac{1}{4}, \frac{3}{4}$

8. $\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2} + ...\right) + (1 + (\sqrt{2}) + (2\sqrt{2} - 1) + ...)$ $\Rightarrow$ $r = 1/\sqrt{2}$ $d = \sqrt{2} - 1$

9. $a_{k-1}[a_{k-2} + ak] = 2a_ka_{k-2}$

$a_{k-1} = \dfrac{2a_ka_{k-2}}{a_k + a_{k-2}} \Rightarrow \dfrac{1}{a_{k-2}}, \dfrac{1}{a_{k-1}}, \dfrac{1}{a_k}$ are in A.P. $\Rightarrow$ A_{k-2}, A_{k-1}, A_k are in A.P.

Now $\dfrac{S_{2p}}{S_p} = \dfrac{\frac{2p}{2}(2a + (2p-1)d)}{\frac{p}{2}(2a + (p-1)d)} \Rightarrow$ for independent of p

$2a - d = 0$ $\Rightarrow$ $d = 2a$ or $d = 0$

if $d = 2a$ $\Rightarrow$ $A_1 = 1, d = 2$

$A_{2016} = 1 + 2015 \times 2 = 4031$

$\Rightarrow$ $a_{2016} = \dfrac{1}{A_{2016}} = \dfrac{1}{4031}$

If $d = 0$, $A_1 = 1 = A_2 = A_3 .... = A_{2016}$

10. $\dfrac{a_{k+1}}{a_k}$ is constant $\therefore$ G.P.

$a_n > a_m$ for $n > m$ $\therefore$ increasing G.P.

$a_1 + a_n = 66$ $\qquad$ $a_2a_{n-1} = 128$

$a + ar^{n-1} = 66$ $\qquad$ $a.ar^{n-1} = 128$ .... (2)

$a(1 + r^{n-1}) = 66$ .... (1)

$a(66 - a) = 128 \Rightarrow a^2 - 66a + 128 = 0$

$(a - 2)(a - 64) = 0 \Rightarrow a = 2, a = 64$

$\therefore \quad r^{n-1} = 32$

$$\sum_{i=1}^{n} a_i = 126 \Rightarrow \frac{a(r^n - 1)}{r-1} = 126 \Rightarrow \frac{2(32r-1)}{r-1} = 126$$

$\Rightarrow \quad 64r = 126 + 124 \Rightarrow n = 6$

11. **Case - I**

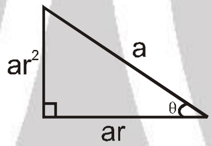
$r > 1$

$a^2 + a^2r^2 = a^2r^4 \quad \Rightarrow \quad r^4 - r^2 - 1 = 0$

$$r^2 = \frac{\sqrt{5}+1}{2}\ ;\ r = \sqrt{\frac{\sqrt{5}+1}{2}}$$

$$\text{tangent of smallest angle} = \tan\theta = \frac{1}{r} = \sqrt{\left(\frac{2}{\sqrt{5}+1}\right)}$$

Case - II



$0 < r < 1$

$a^2 = a^2r^2 + a^2r^4$

$\Rightarrow \quad r^4 + r^2 - 1 = 0$

$$r^2 = \frac{\sqrt{5}-1}{2}\ ;\ r = \sqrt{\frac{\sqrt{5}-1}{2}}$$

$$\text{tangent of smallest angle} = \tan\theta = r = \sqrt{\frac{\sqrt{5}-1}{2}}$$

12. b_1, b_2, b_3 are in G.P.

$\therefore \quad b_3 > 4b_2 - 3b_1$

$\Rightarrow \quad r^2 > 4r - 3$

$\Rightarrow \quad r^2 - 4r + 3 > 0$

$\Rightarrow \quad (r - 1)(r - 3) > 0$

So $\quad 0 < r < 1$ and $r > 3$

13. Let a = 1, then s_1 = 2017. If a 1 then $s = \frac{a^{2017} - 1}{a-1}$.

but $a^{2017} = 2a - 1$, therefore, $S_2 = \frac{2(a-1)}{a-1} = 2$

14. H.P. is 10, 12, 15, 20, 30, 60

a = 15, b = 30, c = 60

A.P. is 15, 20, 25, ..., 55, 60

sum of all term of A.P. is 10/2 (15 + 60) = 375

15. $2x = a + b$ ..... (1)

$y^2 = ab$ ..... (2)

$z = \frac{2\ \ ab}{a+b}$ ..... (3)

$x = y + 2$ ..... (4)

and $\quad a = 5z$ ..... (5)

$z = \frac{2\ \ y^2}{2x} \quad \Rightarrow \quad y^2 = xz$

$\because$ $x = y + 2$

$\therefore$ $\frac{a+b}{2} = \sqrt{ab} + 2$..... (6)

and $a = 5\,\frac{(2ab)}{a+b}$

$\Rightarrow$ $(a + b) = 10\,b$

$\Rightarrow$ $a = 9b$ ..... (7)

$\frac{9b+b}{2} = \sqrt{b\ .\ 9b} + 2$

$\Rightarrow$ $5b = 3b + 2$

$\Rightarrow$ $b = 1$

So $a = 9$ $\Rightarrow$ $x > y > z$

16. Obvious

17. (A) $\because$ equal numbers are not always in A.P., G.P. and in H.P.
for example 0, 0, 0, ...

(B) $\frac{a-b}{b-c} = \frac{a}{c}$ $\Rightarrow ac - bc = ba - a\,c$ $\Rightarrow$ $2ac - bc = ab$ $\Rightarrow$ $b = \frac{2\ ac}{a+c}$

consider

$a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ in A.P. $\Rightarrow$ $b - a = c - b$ $\Rightarrow$ $2b = a + c$

So statemet is false.

(C) Let numbers are a, b

a, G_1, G_2, b or $A = \frac{a+b}{2}$

$r = \left(\frac{b}{a}\right)^{\frac{1}{3}}$; $G_1 = a\left(\frac{b}{a}\right)^{\frac{1}{3}}$, $G_2 = a.\left(\frac{b}{a}\right)^{\frac{2}{3}}$

$\therefore$ $\frac{G_1^3 + G_2^3}{G_1G_2} = \frac{a^3.\frac{b}{a} + a^3.\frac{b^2}{a^2}}{a^2.\frac{b}{a}} = \frac{a^2b + ab^2}{ab} = a + b = 2A$

(D) Let $T_{k+1} = ar^k$ and $T'_{k+1} = br^k$. Since $T''_{k+1} = ar^k + br^k = (a + b)\,r^k$,

$\therefore$ T''_{k+1} is general term of a G.P.

18. $\frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{2}{1}$ $\Rightarrow$ $\frac{a+b}{2\sqrt{ab}} = \frac{2}{1}$ use compendendo and dividendo rule

$\Rightarrow$ $\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{3}{1}$ $\Rightarrow$ $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{\sqrt{3}}{1} \Rightarrow \frac{2\sqrt{a}}{2\sqrt{b}} = \frac{\sqrt{3}+1}{\sqrt{3}-1}$

$\Rightarrow$ $\frac{a}{b} = \frac{3+1+2\sqrt{3}}{3+1-2\sqrt{3}} \Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}} = \frac{(2+\sqrt{3})\ (2+\sqrt{3})}{4-3} = 7 + 4\sqrt{3}$ **Ans**

19. $\sum_{r=1}^{n} r\,(r + 1)\,(2r + 3) = an^4 + bn^3 + cn^2 + dn + e$

$\sum_{r=1}^{n} (r^2+r)\,(2r + 3) = \sum_{r=1}^{n} (2r^3 + 5r^2 + 3r) = 2.\,\frac{n^2(n+1)^2}{4} + 5\,\frac{n(n+1)\ (2n+1)}{6} + 3\,\frac{n(n+1)}{2}$

$= \frac{n(n+1)}{2} \left[n(n+1) \; + \frac{5}{3}(2n+1)+3 \right] = \frac{n(n+1)}{2} \left[\frac{6 \; (n^2+n) \; +10(2n+1)+18}{6} \right] = \frac{n(n+1)}{12}$

$[6n^2 + 26n + 28] = \frac{1}{12} \; [6n^4 + 26n^3 + 28n^2 + 6n^3 + 26n^2 + 28n] = \frac{1}{12} \; [6n^4 + 32n^3 + 54n^2 + 28n]$

$a = \frac{6}{12}$; $b = \frac{32}{12}$; $c = \frac{54}{12}$; $d = \frac{28}{12} = \frac{7}{3}$; e = 0 so a + c = b + d

$b - \frac{2}{3} = \frac{32}{12} - \frac{2}{3} = \frac{24}{12}$; $c - 1 = \frac{42}{12}$

so a, b–2/3, c–1 are in A.P & $\frac{c}{a} = \frac{54}{6} = 9$ is an integer

20. Roots are $\alpha_1, \alpha_2, \alpha_3, \alpha_4$. ; A.M. = G.M. = 2.

Hence, all the roots are equal.

21. $6a = (a_3 - a_2) - (a_2 - a_1)$

$\Rightarrow \quad a = \frac{a_1 + a_3 - 2a_2}{6}$

PART - IV

1. $g(n) - f(n) = \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} = \frac{n(n+1)}{2}\left(\frac{2n+1}{3} - 1\right) = \frac{n(n+1)}{2} \; \frac{2n-2}{3}$

$= \frac{n(n+1)(n-1)}{3} = \frac{(n-1) \; n \; (n+1)}{3}$

for n = 2 $\frac{(n-1) \; n \; (n+1)}{3} = \frac{1.2.3}{3}$ which is divisible by 2 but not by 2^2

$\therefore$ greatest even integer which divides $\frac{(n-1) \; n \; (n+1)}{3}$,

for every $n \in N$, $n \geq 2$, is 2

2. $f(n) + 3g(n) + h(n) = \frac{n(n+1)}{2} + \frac{n(n+1)(2n+1)}{2} + \left(\frac{n(n+1)}{2}\right)^2$

$= \frac{n(n+1)}{2}\left(1 + 2n + 1 + \frac{n(n+1)}{2}\right) = (1 + 2 + 3 + ... + n) \left(2n + 2 + \frac{n(n+1)}{2}\right)$

$\Rightarrow$ for all $n \in N$

Sol. **(3, 4)**

Let 1st term be a. and common difference is 2;

$T_{2n+1} = a + 4n = A$ (say) $r = \frac{1}{2}$

Middle term of AP = T_{n+1};

Middle term of GP = T_{3n+1}

$T_{n+1} = a + 2n \quad \Rightarrow \quad T_{3n+1} = A. \, r^n = \frac{(a+4n)}{2^n} \; (a + 2n) = \frac{a+4n}{2^n}$

$\Rightarrow \quad 2^n a + 2n2^n = a + 4n$

$a = \frac{4n - 2n \;.\; 2^n}{2^n - 1} \Rightarrow T_{2n+1} = a + 4n = \frac{4n-2n \;.\; 2^n}{2^n-1} + 4n = \frac{2n \;.\; 2^n}{2^n - 1} = \frac{2^{n+1}n}{2^n-1}$

$T_{3n+1} = \frac{a+4n}{2^n} = \frac{2^{n+1}n}{2^n(2^n-1)} = \frac{2n}{2^n-1}$

Sol. (5 to 7)

Let first term is 'a'

(5) $a(1-r)^2 = 36$

$\Rightarrow$ r can be 2, 3, 4, 7, –1, –2, –5

(6) $\frac{a}{1-r} = \frac{7}{3} \Rightarrow r = \frac{7-3a}{7}$

$\Rightarrow$ a can be 1, 2, 3, 4 and p can be 4, 1, –2, –5

(7) $ar^{n-1}(1-r)^6 = ar^{n-1}(1-r)^2$

$\Rightarrow (1-r)^4 = 1 \quad \Rightarrow r = 2$

EXERCISE # 3

PART - I

1. **(C)**

$S_n = cn^2$; $S_{n-1} = c(n-1)^2 = cn^2 + c - 2cn$

$T_n = 2cn - c$; $T_n^2 = (2cn - c)^2 = 4c^2n^2 + c^2 - 4c^2n$

Sum $= \sum T_n^2 = \frac{4c^2 \;.\; n(n+1)(2n+1)}{6} + nc^2 - 2c^2n(n+1)$

$= \frac{2c^2n(n+1)(2n+1) + 3nc^2 - 6c^2n(n+1)}{3} = \frac{nc^2[4n^2+6n+2+3-6n-6]}{3} = \frac{nc^2(4n^2-1)}{3}$

2. $\sum_{k=2}^{100} \left|(k^2-3k+1)\;\; S_k\right|$

for k = 2 $\quad |(k^2 - 3k + 1) S_k| = 1$

$\sum_{k=3}^{100} \left|\frac{k-1}{(k-2)\;!} - \frac{k-1+1}{(k-1)\;!}\right|$

$\sum_{k=3}^{100} \frac{1}{(k-3)\;!} + \frac{1}{(k-2)\;!} - \frac{1}{(k-2)\;!} - \frac{1}{(k-1)\;!}$

$\sum_{k=3}^{100} \left(\frac{1}{(k-3)\;!} - \frac{1}{(k-1)\;!}\right)$

$S = 1 + \left(1 - \frac{1}{2!}\right) + \left(\frac{1}{1\;!} - \frac{1}{3!}\right) + \left(\frac{1}{2!} - \frac{1}{4!}\right) + \left(\frac{1}{3!} - \frac{1}{5!}\right) + \left(\frac{1}{4!} - \frac{1}{6!}\right) + ... \left(\frac{1}{94!} - \frac{1}{96!}\right)$

$+ \left(\frac{1}{95!} - \frac{1}{97!}\right) + \left(\frac{1}{96!} - \frac{1}{98!}\right) + \left(\frac{1}{97!} - \frac{1}{99\;!}\right) = 2 - \frac{1}{98!} - \frac{1}{99!}$

$\therefore \quad E = \frac{100^2}{100!} + 3 - \frac{1}{98!} - \frac{1}{99.98!} = \frac{100^2}{100!} + 3 - \frac{100}{99!} = \frac{100^2}{100.99!} + 3 - \frac{100}{99!} = 3$

3. $a_1 = 15$

$\frac{a_k + a_{k-2}}{2} = a_{k-1}$ for k = 3, 4,....,11 $\Rightarrow$ $\quad a_1, a_2, ..., a_{11}$ are in AP $a_1 = a = 15$

$\frac{a_1^2 + a_2^2 + ... + a_n^2}{11} = 90 \quad \Rightarrow \quad \frac{(15)^2 + (15+d)^2 + ... + (15+10d)^2}{11} = 90$

$\Rightarrow \quad 9d^2 + 30d + 27 = 0 \quad \Rightarrow \quad d = -3 \text{ or } -\frac{9}{7}$

Since $27 - 2a_2 > 0 \Rightarrow a_2 < \frac{27}{2} \Rightarrow \quad d = -3 \quad \frac{a_1 + a_2 + ... + a_{11}}{11} = \frac{11}{2}\ \frac{[30 + 10(-3)]}{11} = 0$

4. $\frac{S_m}{S_n} = \frac{S_{5n}}{S_n} = \frac{\frac{5n}{2}[6 + (5n-1)d]}{\frac{n}{2}[6 + (n-1)d]} = \frac{5[(6-d) + 5nd]}{[(6-d) + nd]}$; $d = 6$ or $d = 0$

Now if d = 0 then $a_2 = 3$ else $a_2 = 9$ for single choice more appropriate choice is 9, but in principal, question seems to have an error.

$\therefore \quad a_2 = 3 + 6 = 9$

5. A.M. $\geq$ G.M.

$$\frac{\frac{1}{a^5} + \frac{1}{a^4} + \frac{1}{a^3} + \frac{1}{a^3} + \frac{1}{a^3} + 1 + a^8 + a^{10}}{8} \geq \left(\frac{1}{a^5} \cdot \frac{1}{a^4} \cdot \frac{1}{a^3} \cdot \frac{1}{a^3} \cdot \frac{1}{a^3} .1.a^8.a^{10}\right)^{1/8}$$

$\Rightarrow \quad \frac{1}{a^5} + \frac{1}{a^4} + \frac{3}{a^3} + 1 + a^8 + a^{10} \geq 8(1)^{1/8}$

$\Rightarrow$ minimum value of $\frac{1}{a^5} + \frac{1}{a^4} + \frac{3}{a^3} + 1 + a^3 + a^{10} = 8$, at a = 1

6. Corresponding A.P. $\frac{1}{5}$,..................... $\frac{1}{25}$ (20th term)

$\frac{1}{25} = \frac{1}{5} + 19\,d \quad \Rightarrow \quad d = \frac{1}{19}\left(\frac{-4}{25}\right) = -\frac{4}{19 \times 25} \quad a_n < 0$

$\frac{1}{5} - \frac{4}{19 \times 25} \times (n-1) < 0 \quad \Rightarrow \quad \frac{19 \times 5}{4} < n - 1 \Rightarrow n > 24.75$

7.* $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2 = -1^2 - 2^2 + 3^2 + 4^2 - 5^2 - 6^2 + 7^2 + 8^2 + ...$

$= (3^2 - 1) + (4^2 - 2^2) + (7^2 - 5^2) + (8^2 - 6^2)...... = 2\ [\underbrace{4 + 6 + 12 + 14 + 20 + 22 + ...}_{2n \text{ terms}}]$

$= 2[(4 + 12 + 20 ......) + (6 + 14 + 22 ....)]$

n terms n terms

$= 2\left[\frac{n}{2}(4 \times 2 + (n-1)8)\frac{n}{2}(2 \times 6 + (n+1)8)\right] = 2[n(4 + 4n - 4) + n(6 + 4n - 4)]$

$= 2(4n^2 + (4n + 2)n) = 2(8n^2 + 2n) = 4n(4n + 1)$

(A) 1056 = 32 × 33 n = 8

(B) 1088 = 32 × 34

(C) 1120 = 32 × 35

(D) 1332 = 36 × 37 n = 9

8. Numbers removed are k and k + 1. now $\frac{n(n+1)}{2} - k - (k + 1) = 1224$

$\Rightarrow n^2 + n - 4k = 2450 \quad \Rightarrow n^2 + n - 2450 = 4k \Rightarrow (n + 50)(n - 49) = 4k \Rightarrow n > 49$

Alternative

$\therefore$ To satisfy this equation n should be of the form of (4p + 1) or (4p + 2) taking n = 50

$\Rightarrow \quad 4k = 100 \quad \Rightarrow \quad k = 25$

$\therefore \quad k - 20 = 5$

Now if we take n = 53 $\Rightarrow$ k = 103 $\Rightarrow$ n < k

so not possible. Hence n $\geq$ 53 will not be possible.

9. Let b = ar, c = ar² $\Rightarrow$ r is Integers. Also $\frac{a+ar+ar^2}{3}$ = ar + 2 $\Rightarrow$ a + ar² = 2ar + 6 $\Rightarrow$ a(r – 1)² = 6

$\Rightarrow$ r must be 2 and a = 6. Thus $\frac{a^2+a-14}{a+1} = \frac{36+6-14}{7}$ = 4 Ans.

10. $\frac{S_7}{S_{11}} = \frac{6}{11}$ $\frac{\frac{7}{2}[2a+6d]}{\frac{11}{2}[2a+10d]} = \frac{6}{11}$ Given 130 < a + 6d < 140 $\frac{7(a+3d)}{11(a+5d)} = \frac{6}{11}$

7a + 21d = 6a + 30d $\Rightarrow$ 130 < 15d < 140 a = 9d Hence d = 9 a = 81

Alternative :

Let the AP be a, a + d, a + 2d, ...where a , d $\in$ N Given $\frac{S_7}{S_{11}} = \frac{6}{11}$ and 130 < a + 6d < 140..... (2)

$\Rightarrow \frac{\frac{7}{2}\{2a+6d\}}{\frac{11}{2}\{2a+10d\}} = \frac{6}{11} \Rightarrow \frac{14a+42d}{22a+110d} = \frac{6}{11}$ $\Rightarrow$ 154a + 462d = 132a + 660d $\Rightarrow$ 22a = 198d $\Rightarrow$ a = $\frac{99d}{11}$ = 9d

(2) $\Rightarrow$ $\therefore$ 130 < 9d + 6d < 140 $\Rightarrow$ 8.6 < d < 9.3

$\therefore$ d = 9

11. $4\alpha x^2 + \frac{1}{x} \geq 1 \Rightarrow y = 4\alpha x^2 + \frac{1}{x} \Rightarrow y' = \frac{dy}{dx} = 8\alpha x - \frac{1}{x^2} = 0 \Rightarrow x = \left(\frac{1}{8\alpha}\right)^{1/3}$

$\Rightarrow f(x) = \frac{4\alpha x^3+1}{x} = \frac{1/2+1}{1/8(8\alpha)^{1/3}} \Rightarrow \frac{3}{2}\left(\frac{1}{8\alpha}\right)^{1/3} \Rightarrow \alpha^{1/3} \geq 1/3 \Rightarrow \alpha \geq \frac{1}{27}$

12. $\log_e b_1, \log_e b_2, \log_e b_3, \ldots\ldots \log_e b_{101}$ are in A.P. $b_1, b_2, b_3, \ldots\ldots\ldots, b_{101}$ are in G.P.

Given : $\log_e(b_2) - \log_e(b_1) = \log_e(2) \Rightarrow \frac{b_2}{b_1}$ = 2 = r (common ratio of G.P. $a_1, a_2, a_3, \ldots\ldots\ldots a_{101}$ are in A.P.

$a_1 = b_1$ = a $b_1 + b_2 + b_3 + \ldots\ldots\ldots b_{51}$ = t, S = $a_1 + a_2 + \ldots\ldots + a_{51}$

t = sum of 51 terms of G.P. = $b_1 = b_1\frac{(r^{51}-1)}{r-1} = \frac{a(2^{51}-1)}{2-1}$ a(2⁵¹ –1)

s = sum of 51 terms of A.P. = $\frac{51}{2}$ [2a₁ + (n–1)d] = $\frac{51}{2}$ (2a + 50d)

Given $a_{51} = b_{51}$ a + 50d = a(2)⁵⁰ 50d = a(2⁵⁰ – 1)

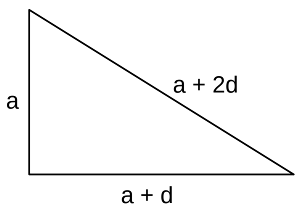
Hence s = $a\frac{51}{2}$ [2⁵⁰ + 1] $\Rightarrow$ s = $a\left(51.2^{49} + \frac{51}{2}\right)$

$s = 2\left(4.2^{49} + 47.2^{49} + \frac{51}{2}\right) \Rightarrow s = a\left((2^{51}-1) + 47.2^{49} + \frac{53}{2}\right)$ s – t = a $\left(47.2^{49} + \frac{53}{2}\right)$

Clearly s > t

$a_{101} = a_1 + 100d = a + 2a.2^{50} - 2a = a(2^{51} - 1)$ $b_{101} = b_1 r^{100} = a.2^{100}$ Hence $b_{101} > a_{101}$

13.



$\frac{1}{2}$a(a + d) = 24 $\Rightarrow$ a(a + d) = 48 ....(1)

a² + (a + d)² = (a + 2d)² $\Rightarrow$ 3d² + 2ad – a² = 0 (3d – a)(a + d) = 0

$\Rightarrow$ 3d = a ($\because$ a + d $\neq$ 0) $\Rightarrow$ d = 2 a = 6 so smallest side = 6

14. P = {1, 6, 11, ...}

Q = {9, 16, 23,.........}

Common terms: 16, 51, 86

$t_p = 16 + (p - 1)35 = 35p - 19 \leq 10086 \Rightarrow p \leq 288.7$

$\therefore$ n (P $\cup$ Q) = n(P) + n (Q) – n (P $\cap$ Q) = 2018 + 2018 – 288 = 3748

15. First series is {1, 4, 7, 10, 13,……..}

Second series is {2, 7, 12, 17, …….}

Third series is {3, 10, 17, 24, ………}

See the least number in the third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5.

$\Rightarrow$ 52 is the least number of third series which leaves remainder 1 on dividing by 3 and leaves remainder 2 on dividing by 5

Now, A = 52

D is L.C. M. of (3, 5, 7) = 105 $\Rightarrow$ A + D = 52 + 105 = 157

PART - II

1. $a_1 + a_2 + ... + a_N$ = 4500 notes

$a_1 + a_2 + ... + a_{10}$ = 150 × 10 = 1500 notes = 4500 – 1500 = 3000 notes

$a_{11} + a_{12} + ... + a_n = 3000 \Rightarrow 148 + 146 ...... = 3000$

$[2 \times 148 + (n - 10 - 1)(-2)] = 3000 \Rightarrow n = 34, 135$

$a_{34} = 148 + (34 - 1)(-2) = 148 - 66 = 82$

$a_{135} = 148 + (135 - 1)(-2) = 148 - 268 = -120 < 0$. so answer in 34 minutes is taken

Hence correct option is (1)

2. a = Rs. 200 ; d = Rs. 40 $\Rightarrow$ savings in first two months = Rs. 400

remained savings = 200 + 240 + 280 + ... upto n terms

$= \frac{n}{2}[400 + (n-1)\ 40] = 11040 - 400 \Rightarrow 200n + 20n^2 - 20n = 10640$

$20n^2 + 180n - 10640 = 0 \Rightarrow n^2 + 9n - 532 = 0$

$(n + 28)(n - 19) = 0 \Rightarrow n = 19$

$\therefore$ no. of months = 19 + 2 = 21.

3. Let A.P. be a, a + d, a + 2d, ...

$a_2 + a_4 + ... + a_{200} = \alpha \Rightarrow \frac{100}{2}[2(a + d) + (100 - 1)d] = \alpha$ .... (i)

and $a_1 + a_3 + a_5 + ... + a_{199} = \beta \Rightarrow \frac{100}{2}[2a + (100 - 1)d] = \beta$ .....(ii)

on solving (i) and (ii) $d = \frac{\alpha - \beta}{100}$

4. $\frac{7}{10} + \frac{77}{100} + \frac{777}{10^3} + ...+$ up to 20 terms $= 7\left[\frac{1}{10} + \frac{11}{100} + \frac{111}{10^3} + \text{.....up to 20 terms}\right]$

$= \frac{7}{9}\left[\frac{9}{10} + \frac{99}{100} + \frac{999}{1000} + \text{.....up to 20 terms}\right] =$

$\frac{7}{9}\left[\left(1 - \frac{1}{10}\right) + \left(1 - \frac{1}{10^2}\right) + \left(1 - \frac{1}{10^3}\right) + \text{.....up to 20 terms}\right]$

$$= \frac{7}{9}\left[20 - \frac{\frac{1}{10}\left(1-\left(\frac{1}{10}\right)^{20}\right)}{1-\frac{1}{10}}\right] = \frac{7}{9}\left[20 - \frac{1}{9}\left(1-\left(\frac{1}{10}\right)^{20}\right)\right] = \frac{7}{9}\left[\frac{179}{9} + \frac{1}{9}\left(\frac{1}{10}\right)^{20}\right] = \frac{7}{81}\left[179 + (10)^{-20}\right]$$

5. Let $s = (10)^9 + 2(11)^1 (10)^8 + 3(11)^2(10)^7 + \ldots\ldots + 10(11)^9$

$\therefore s\,\frac{11}{10} = (11)^1 (10)^8 + 2(11)^2 (10)^7 + \ldots\ldots + 9(11)^9 + (11)^{10}$

substract $\left(1-\frac{11}{10}\right) s = (10)^9 + (11)^1 (10)^8 + (11)^2 (10)^7 + \ldots\ldots + (11)^9 - (11)^{10}$

$\Rightarrow -\frac{1}{10} s = \frac{10^9\left\{1-\left(\frac{11}{10}\right)^{10}\right\}}{1-\frac{11}{10}} - (11)^{10} - \frac{1}{10} s = 10^9 \frac{\{10^{10} - 11^{10}\}}{10^{10}} \times \frac{10}{-1} - (11)^{10}$

$\Rightarrow -\frac{1}{10} s = -10^{10} + 11^{10} - 11^{10}$ $\quad\therefore\quad s = 10^{11}$

$\therefore$ given $10^{11} = k(10)^9$ $\quad\therefore\quad k = 100$

6. a $\quad a_1 \quad ar^2$ G. P. ; a $\quad$ 2ar $\quad ar^2$ A. P.

$L_1 ar = a + ar^2$; $4r = 1 + r^2$

$r^2 - 4r + 1 = 0$; $r = \frac{4 \mp 2\sqrt{3}}{2} = 2 + \sqrt{3}$, $\quad 2 - \sqrt{3}$

But $r > 1$ $\quad r = 2 + \sqrt{3}$

7. $M = \frac{\ell + n}{2}$

ℓ, G_1, G_2, G_3, n are in G.P.

$r = \left(\frac{n}{\ell}\right)^{\frac{1}{4}}$

$G_1 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{4}}$ $\quad G_2 = \ell\left(\frac{n}{\ell}\right)^{\frac{1}{2}}$ $\quad G_3 = \ell\left(\frac{n}{\ell}\right)^{\frac{3}{4}}$

$G_1^4 + 2G_2^4 + G_3^4$

$= \ell^4 \times \frac{n}{\ell} + 2\ell^4 \frac{n^2}{\ell^2} + \ell^4 \times \frac{n^3}{\ell^3}$

$= \ell^3 n + 2\ell^2 n^2 + \ell n^3$

$= n\ell(\ell^2 + 2n\ell + n^2)$

$= n\ell(\ell + n)^2$

$= 4m^2 n\ell$

8. $T_n = \frac{\frac{n^2(n+1)^2}{4}}{n^2}$

$T_n = \frac{1}{4}(n+1)^2$

$T_n = \frac{1}{4}[n^2 + 2n + 1]$

$S_n = \sum_{n=1}^{n} T_n$

$S_n = \frac{1}{4}\left[\frac{n(n+1)\ (2n+1)}{6} + n(n+1)\ + n\right]$

n = 9

$S_9 = \frac{1}{4}\left[\frac{9\times10\times19}{6} + 9\times10+9\right] = \frac{1}{4}[285 + 90 + 9] = \frac{384}{4} = 96.$

9. a + d, a + 4d, a + 8d $\rightarrow$ G.P

$\therefore \quad (a + 4d)^2 = a^2 + 9ad + 8d^2$

$\Rightarrow \quad 8d^2 = ad \qquad \Rightarrow a = 8d$

$\therefore \quad$ 9d, 12d, 16d $\rightarrow$ G.P. common ratio $r = \frac{12}{9} = \frac{4}{3}$

10. $\left(\frac{8}{5}\right)^2 + \left(\frac{12}{5}\right)^2 + \left(\frac{16}{5}\right)^2 + \left(\frac{20}{5}\right)^2 + \left(\frac{24}{5}\right)^2 + \ldots \frac{8^2}{5^2} + \frac{12^2}{5^2} + \frac{16^2}{5^2} + \frac{20^2}{5^2} + \frac{24^2}{5^2} + \ldots..$

$T_n = \frac{(4n+4)^2}{5^2} \quad S_n = \frac{1}{5^2}\sum_{n=1}^{10} 16(n+1)^2 = \frac{16}{25}\sum_{n=1}^{10}(n^2+2n+1)$

$= \frac{16}{25}\left[\frac{10\times11\times21}{6} + \frac{2\times10\times11}{2} + 10\right] = \frac{16}{25}\times505 = \frac{16}{5}\,m \Rightarrow m = 101$

11. $225a^2 + 9b^2 + 25c^2 - 75ac - 45ab - 15bc = 0$

$(15a)^2 + (3b)^2 + (5c)^2 - (15a)(3b) - (3b)(5c) - (15a)(5c) = 0$

$\frac{1}{2}[(15a - 3b)^2 + (3b - 5c)^2 + (5c - 15a)^2] = 0$

15a = 3b, 3b = 5c, 5c = 15a

5a = b, 3b = 5c, c = 3a

$\frac{a}{1} = \frac{b}{5} = \frac{c}{3} = \lambda$

$a = \lambda,\ b = 5\lambda,\ c = 3\lambda$

a, c, b are in AP

b, c, a are in AP

12. $f(x) = ax^2 + bx + c$

$f(x + y) = f(x) + f(y) + xy$

$a(x + y)^2 + b(x+ y) + c = ax^2 + bx + c + ay^2 + by + c + xy$

$2axy = c + xy \qquad \forall x, y\in R$

$(2a - 1)xy - c = 0 \qquad \forall\ x, y\in R$

$\Rightarrow \quad c = 0,\ a = \frac{1}{2} \quad a + b + c = 3$

$\frac{1}{2} + b + 0 = 3\ b = \frac{5}{2}$

$\therefore f(x) = \frac{1}{2}x^2 + \frac{5}{2}x \quad \sum_{n=1}^{10} f(n) = \frac{1}{2}\sum_{n=1}^{10} n^2 + \frac{5}{2}\sum_{n=1}^{10} n \Rightarrow \frac{1}{2}\times\frac{10\times11\times21}{6} + \frac{5}{2}\times\frac{10\times11}{2} = 330$

13. The given quadratic equation is

$nx^2 + x(1 + 3 + 5 + \ldots\ldots.+(2n-1)) + (1.2 + 2.3 + \ldots\ldots.+ (n-1).n) - 10n = 0$

$\Rightarrow nx^2 + x(n^2) + \frac{n(n^2-1)}{3} - 10n = 0 \qquad \Rightarrow x^2 + x(n) + \frac{(n^2-1)}{3} - 10 = 0 \;<^{\alpha}_{\beta}$

$(\alpha - \beta)^2 = 1 \qquad \Rightarrow (\alpha + \beta)^2 - 4\alpha\beta = 1 \qquad \Rightarrow n^2 - 4\left(\frac{n^2-1}{3} - 10\right) = 1 \Rightarrow n = 11$

14. $a_1 + a_5 + a_9 + a_{13} + \ldots\ldots.+ a_{49} = 416$

$a_1 + (a_1+ 4d) + (a_1+ 8d) + (a_1+ 12d) + \ldots\ldots..+ (a_1+ 48d) = 416$

$13\,a_1 + 4d(1+2+3+\ldots\ldots+12) = 416$

$13\,a_1 + \frac{4d \times 12 \times 13}{2} = 416$

$13a_1 + 24 \times 13d = 416$

$a_1 + 24d = 32 \quad a_9 + a_{43} = 66 \quad a_1 + 8d + a_1 + 42d = 66$

$2a_1 + 50d = 66 \quad a_1 + 25d = 33 \quad d = 1 \quad a_1 = 8$

$a_1^2 + (a_1 + d)^2 + (a_1 + 2d)^2 + \ldots\ldots + (a_1 + 16d)^2 = 140\ m$

$17a_1^2 + d^2(1^2 + 2^2 + \ldots + 16^2) + 2a_1d\,(1+2+3+\ldots\ldots+16) = 140m$

$17 \times 64 + \frac{16 \times 17 \times 33}{6} + \frac{2 \times 8 \times 1 \times 16 \times 17}{2} = 140\ m$

$17 \times 64 + 8 \times 11 \times 17 + 8 \times 11 \times 17 = 140\ m$

$17 \times 16 + 22 \times 17 + 2 \times 16 \times 17 = 35\ m$

$272 + 374 + 544 = 35\ m$

$1190 = 35\ m \Rightarrow m = 34$

15. $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + \ldots$

$A = 1^2 + 2.2^2 + \ldots + 2.20^2$

$= (1^2 + 2^2 + \ldots + 20^2) + (2^2 + 4^2 + \ldots + 20^2)$

$= \frac{20.21.41}{6} + 4.\frac{10.11.21}{6} = \frac{20.21}{6}\{41 + 22\} = 70 \times 63 = 4410$

$B = 1^2 + 2.2^2 + \ldots + 2.40^2$

$= (1^2 + 2^2 + \ldots + 40^2) + (2^2 + 4^2 + \ldots + 40^2)$

$= \frac{40.41.81}{6} + \frac{4.20.21.41}{6} = \frac{40.41}{6}(81 + 42) = \frac{40.41}{6} \times 123$

$= 20\,(41)^2 = 33620$

$B - 2A = 100\lambda \Rightarrow \lambda = \frac{33620 - 8820}{100} = \frac{24800}{100} = 248$

16. This series can be written as

$\frac{3(1^2)}{3} + \frac{6(1^2 + 2^2)}{5} + \frac{9(1^2 + 2^2 + 3^2)}{7} + \frac{12(1^2 + 2^2 + 3^2 + 4^2)}{9} + \ldots$

$T_r = \frac{3r}{(2r+1)}(1^2 + 2^2 + 3^2 + \ldots r^2)$

$T_r = \frac{3r}{(2r+1)} \frac{r(r+1)(2r+1)}{6}$

$T_r = \frac{1}{2} r^2(r + 1)$

sum of n term $\sum_{i=1} T_r = \frac{1}{2}(\Sigma r^3 + \Sigma r^2) = \frac{1}{2}\left[\left(\frac{n(n+1)}{2}\right)^2 + \frac{n(n+1)(2n+1)}{6}\right]$

hence sum of 15 term $= \frac{1}{2}\left[\left(\frac{15\ .\ 16}{2}\right)^2 + \frac{15.16.31}{6}\right] = \frac{1}{2}[14400 + 1240] = 7820$

17. $5, 5r, 5r^2$ sides of triangle,

$5 + 5r > 5r^2$ .... (1)

$5 + 5r^2 > 5r$ ....(2)

$5r + 5r^2 > 5$ ....(3)

from (1) $r^2 - r - 1 < 0$, (1)

$\left[r - \left(\frac{1+\sqrt{5}}{2}\right)\right]\left[r - \left(\frac{1-\sqrt{5}}{2}\right)\right] < 0 \quad r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ ...(4)

from (2) , (2)

$r^2 - r + 1 > 0 \Rightarrow r \in R$ ....(5)

from (3), (3) $r^2 + r - 1 > 0$ so, $\left(r + \frac{1+\sqrt{5}}{2}\right)\left(r + \frac{1-\sqrt{5}}{2}\right) > 0 \; r \in \left(-\infty, -\frac{1+\sqrt{5}}{2}\right) \cup \left(-\frac{1-\sqrt{5}}{2}, \infty\right)$ ...(6)

from (4), (5), (6), $r \in \left(\frac{-1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$ now check options

18. Natural numbers between 100 & 200.
101, 102, ..., 199.
Either divide by 7 or divide by 13.
(sum of numbers (divide by 7) + (sum of number divide by 13) – (sum of number of divide by 91)

$$\sum_{r=1}^{14}(98+7r) + \sum_{r=1}^{8}(91+13r) - (182) = \left(98 \times 14 + 7.\frac{14\times15}{2}\right) + \left((91\times8) + 13\times\frac{8\times9}{2}\right) - 182$$

= 1372 + 735 + 728 + 468 – 182 = 3121

19. $a_1, a_2, a_3, \ldots\ldots a_{50}$ in A.P.
$a_6 = 2 \quad \therefore \quad a_1 + 5d = 2$
$a_1 a_4 a_5 = a_1(a_1 + 3d)(a_1 + 4d)$
$= a_1(2 - 2d)(2 - d)$
$= -2((5d - 2)(d - 1)(d - 2))$
$= -2(5d^3 - 17d^2 + 16d - 4)$

$\frac{dA}{d(d)} = -2(15d^2 - 34d + 16)$

$= -2(5d - 8)(3d - 2)$

(sign scheme: – , +, – with critical points 2/3, 8/5)

Maximum occurs at $d = \frac{8}{5}$

20. Let $b = ar$, $c = ar^2$
Hence $3a + 15ar^2 = 14ar$
$15r^2 - 14r + 3 = 0$
$15r^2 - 9r - 5r + 3 = 0$
$(3r - 1)(5r - 3) = 0$

$r = \frac{1}{3}, \frac{3}{5} \quad \Rightarrow \quad r = \frac{1}{3}$

AP is $3a, \frac{7}{3}a, \frac{5}{3}a, a, \ldots\ldots\ldots$

$\Rightarrow$ 4^{th} terms is a

21. $\frac{(49)^{126} - 1}{48} = \frac{((49)^{63} + 1)(49^{63} - 1)}{48}$

22. $f(x) = \left(\frac{2^{1-x} + 2^{1+x} + 3^x + 3^{-x}}{2}\right)$

Using AM ≥ GM
$f(x) \geq 3$

23. Let GP is $a, ar, ar^2 \ldots\ldots\ldots$

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots a_{201} = 200 \Rightarrow \frac{ar^2(r^{200} - 1)}{r^2 - 1} = 200$ ....(1)

$$\sum_{n=1}^{100} a_{2n} = a_2 + a_4 + \ldots\ldots. a_{200} = 100 = \frac{ar(r^{200}-1)}{r^2-1} = 100 \qquad \ldots.(2)$$

Form (1) and (2) r = 2

add both

$\Rightarrow a_2 + a_3 + \ldots\ldots\ldots.. a_{200} + a_{201} = 300 \quad \Rightarrow \quad r(a_1 + \ldots\ldots\ldots\ldots a_{200}) = 300$

$$\sum_{n=1}^{200} a_n = \frac{300}{r} = 150$$

24. $y = 1 + \cos^2\theta + \cos^4\theta + \ldots\ldots.$

$\Rightarrow y = \frac{1}{1-\cos^2\theta} \Rightarrow \frac{1}{y} = \sin^2\theta$

$x = \frac{1}{1-(-\tan^2\theta)} = \frac{1}{\sec^2\theta} \quad \Rightarrow \cos^2\theta = x \quad \Rightarrow \frac{1}{y} + x = 1 \quad \Rightarrow y(1-x) = 1$

HIGH LEVEL PROBLEMS (HLP)

PART - I

1. $T_m \equiv a + (m-1)\,d = \sqrt{2}$ ....(1)

$T_n \equiv a + (n-1)d = \sqrt{3}$ ....(2)

$T_p \equiv a + (p-1)\,d = \sqrt{5}$ ....(3)

$\frac{(2)-(1)}{(3)-(2)} \Rightarrow \frac{n-m}{p-n} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$. Not possible.

2. Let a denotes first term and d common difference of the A.P. then a_k, the k^{th} term of the A.P.

$a_k = a + (k-1)\,d;\ a_1 + a_2 + a_3 + \ldots. + a_m = a_{m+1} + a_{m+2} + \ldots.+ a_{m+n}$

$2S_m = S_{m+n};\ 2.\ \frac{m}{2}\,[2a + (m-1)\,d] = \frac{m+n}{2}\,[2a + (m+n-1)\,d]$

$\frac{2m}{m+n} = \frac{2a+(m+n-1)\,d}{2a+(m-1)\,d} = 1 + \frac{nd}{2a+(m-1)\,d} \Rightarrow \frac{nd}{2a+(m-1)\,d} = \frac{m-n}{m+n}$ ......(i)

similarly $\frac{pd}{2a+(m-1)d} = \frac{m-p}{m+p}$ ......(ii) divide (i) by (ii)

$\frac{n}{p} = \frac{m-n}{m+n}\cdot\frac{m+p}{m-p} \Rightarrow \frac{(m+n)(m-p)}{p} = \frac{(m+p)(m-n)}{n} \Rightarrow \frac{(m+n)(m-p)}{mp} = \frac{(m+p)(m-n)}{mn}$

$(m+n)\left(\frac{1}{m}-\frac{1}{p}\right) = (m+p)\left(\frac{1}{m}-\frac{1}{n}\right)$ Hence Proved

3. $a = A + (p-1)\,d \quad \Rightarrow \quad d = \frac{a-b}{p-q} \qquad b = A + (q-1)\,d$

$S = \frac{p+q}{2}\,[2A + (p+q-1)\,d] = \frac{p+q}{2}\,[A + (p-1)\,d + A + (q-1)d + d]$

$= \frac{p+q}{2}\,[a + b + d] = \frac{p+q}{2}\left[a+b+\frac{a-b}{p-q}\right]$

4. Given $\frac{p}{2}\,[2a + (p-1)\,d] = 0.$ so $2a + (p-1)d = 0 \quad \Rightarrow \quad d = -\frac{2a}{(p-1)}$

Now sum of next q terms are = sum of (p + q) terms of this A.P.

$= \frac{p+q}{2}\,[2a + (p+q-1)\,d] = \frac{p+q}{2}\,[2a + (p-1)d + qd] = \frac{p+q}{2}.\,q.\,d = -\frac{a\,(p+q)\,q}{p-1}$

5. $\frac{a+be^y}{a-be^y}+1 = \frac{b+c\,e^y}{b-c\,e^y}+1 = \frac{c+d\,e^y}{c-d\,e^y}+1; \qquad \frac{2a}{a-b\,e^y} = \frac{2b}{a-c\,e^y} = \frac{2c}{c-d\,e^y}$

$1 - \frac{b}{a} e^y = 1 - \frac{c}{b} e^y = 1 - \frac{d}{c} e^y \Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \Rightarrow$ a, b, c, d are in G.P.

6. Let first term of AP = a and common difference = d

$\therefore \quad \frac{10}{2}[2a + 9d] = 155 \Rightarrow .2a + 9d = 31$

GP first term = a′ and common ratio = r a′ + a′r = 9 ⇒ a′(1 + r) = 9 given a = r and a′ = d

$\Rightarrow \quad 2r + 9a' = 31 \Rightarrow 2r + 9.\frac{9}{1+r} = 31 \Rightarrow \quad 2r^2 - 29r + 50 = 0 \Rightarrow r = 2, 25/2.$

(i) if r = 2, So a = 2 ⇒ a′ = 3 = d

(ii) if $r = \frac{25}{2} \Rightarrow a = \frac{25}{2} \Rightarrow a' = \frac{9}{\frac{27}{2}} = \frac{2}{3} = d$

So the series 3, 6, 12.........second $\frac{2}{3}, \frac{25}{3}, \frac{625}{6}$

7. (i) 1, (2, 3), (4, 5, 6, 7), (8, 9, ...,15) ....

Number of terms in n^{th} group = 2^{n-1} ; 1^{st} term in n^{th} group = 2^{n-1}

So, Sum of terms in n^{th} group $\frac{2^{n-1}}{2} = [2.2^{n-1} + (2^{n-1} - 1)\,1] = 2^{n-2}\,[2^n + 2^{n-1} - 1]$

(ii) (1) , (2, 3, 4), (5, 6, 7, 8, 9) ; 1^{st} term in n^{th} group let T_n

1   2   5   10

 1   3   5

  2   2

$T_n = a + bn + cn^2 \Rightarrow T_1 = a + b + c = 1$ .... (i)

$T_2 = a + 2b + 4c = 2$ .... (ii)

and $T_3 = a + 3b + 9c = 5$ .... (iii)

a = 2, b = – 2, c = 1   On solving (i), (ii) and (iii), we get

So I^{st} term is $T_n = (2 - 2n + n^2)$. Number of terms in n^{th} group = (2n – 1)

$\therefore$ sum of terms in n^{th} group $\frac{2n-1}{2} = [2\,(2 - 2n + n^2) + 2n - 2]$

$= (2n - 1)\,[n^2 - n + 1] = 2n^3 - 2n^2 + 2n - n^2 + n - 1 = n^3 + (n - 1)^3$

8. $a, A_1, A_2, b \rightarrow$ A.P. $\Rightarrow A_1 + A_2 = a + b.$

$a, G_1, G_2, b \rightarrow$ G.P. $\Rightarrow G_1 G_2 = ab$

$a, H_1, H_2, b \rightarrow$ H.P. $\Rightarrow \frac{ab}{a+b} = \frac{H_1 \; H_2}{H_1 + H_2}$

$\Rightarrow \frac{A_1 + A_2}{H_1 + H_2} = \frac{G_1 \; G_2}{H_1 \; H_2} \Rightarrow \frac{1}{H_1} = \frac{1}{a} + d \quad \frac{1}{b} = \frac{1}{a} + 3d \Rightarrow \frac{a - b}{3 \; a \; b} = d$

$\therefore \frac{1}{H_1} = \frac{a + 2b}{3ab} \quad \frac{1}{H_2} = \frac{2a + b}{3 \; ab} \quad \frac{1}{H_1 H_2} = \frac{(a + 2b) \; (2a + b)}{9a^2b^2} \Rightarrow \frac{G_1 G_2}{H_1 \; H_2} = \frac{(a + 2b) \; (2a + b)}{9 \; ab}$

9. $T_k = k.2^{n+1-k}$ and $S_n = \left(\frac{n+1}{4}\right)\left(2^{n+1} - n - 2\right)$

Now, $S_n = \sum_{k=1}^{n} k.2^{n+1-k} = 2^{n+1} \sum_{k=1}^{n} k \; . \; 2^{-k}$; $S_n = 2^{n+1}. \; 2 \; .\left[1 - \frac{1}{2^n} - \frac{n}{2^{n+1}}\right]$ (sum of A.G.P.)

$\left(\frac{n+1}{4}\right)\left(2^{n+1} - n - 2\right) = 2.\,[2^{n+1} - 2 - n] \Rightarrow \frac{n+1}{4} = 2 \Rightarrow n = 7$

10. Let G_m be the geometric mean of $G_1, G_2, \ldots\ldots\ldots, G_n \Rightarrow G_m = (G_1 \cdot G_2 \ldots . G_n)^{1/n}$

$= [(a_1)(a_1 a_1 r)^{1/2}.(a_1.a_1 r.a_1 r^2)^{1/3} \ldots (a_1.a_1 r.a_1 r^2 \ldots a_1 r^{n-1})^{1/n}]^{1/n}$

where r is the common ratio of G.P. $a_1, a_2, ...,a_n$. = [($a_1.a_1$...n times) ;त१४२. $r^{3/3}$. $r^{6/4}$.... $r^{\frac{(n-1)n}{2n}}$)]$^{1/n}$

$\Rightarrow \quad a_1 . \left[r^{\frac{1}{2}\left[\frac{(n-1)\,n}{2}\right]} \right]^{1/n} = a_1 \left[r^{\frac{n-1}{4}} \right]$. Now, $\quad A_n = \frac{a_1 + a_2 + ... + a_n}{n} = \frac{a_1(1-r^n)}{n\ \ (1-r)}$

and $\quad H_n = \frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} ... + \frac{1}{a_n}\right)} = \frac{n}{\frac{1}{a_1}\ \left(1 + \frac{1}{r} + ... + \frac{1}{r^{n-1}}\right)} = \frac{a_1\ \ n(1-r)\ \ r^{n-1}}{1-r^n}$

Again $\quad A_n.H_n = \frac{a_1(1-r^n)}{n\ \ (1-r)} \times \frac{a_1 n\ \ (1-r)\ \ r^{n-1}}{(1-r^n)} = a_1^2\ \ r^{n-1} \quad \Rightarrow \prod_{k=1}^{n} A_k H_k = \prod_{k=1}^{n} (a_1^2\ \ r^{k-1})$

$= (a_1^2.\ \ a_1^2 ...\ \ n\ \ \text{times}) \times (r^0.r^1.r^2 ...r^{n-1}) = a_1^{2n}.r^{1+2+....(n-1)} = a_1^{2n}\ \ r^{\frac{n(n-1)}{2}} = [a_1\ \ r^{\frac{n-1}{4}}]^{2n}$

$= [G_m]^{2n} \Rightarrow \quad G_m = \left[\prod_{k=1}^{n} A_K H_K \right]^{1/2n} \quad \Rightarrow \quad G_m = (A_1 A_2 ... A_n.H_1 H_2 ....H_n)^{1/2n}$

11. $2b = a + c$ ..... (1)

$q = \frac{2\ \ pr}{p+r}$ ..... (2)

and $b^2 q^2 = acrp$ ..... (3)

So $\left(\frac{a+c}{2}\right)^2 \left(\frac{2\ \ pr}{p+r}\right)^2 = acrp \quad \Rightarrow \frac{(p+r)^2}{pr} = \frac{(a+c)^2}{ac}$

$\Rightarrow \quad \frac{p}{r} + \frac{r}{p} + 2 = \frac{a}{c} + \frac{c}{a} + 2 \quad \Rightarrow \quad \frac{p}{r} + \frac{r}{p} = \frac{a}{c} + \frac{c}{a}$ **Ans.**

12. $\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2} \quad \Rightarrow \quad \alpha + \beta = \frac{\alpha^2 + \beta^2}{\alpha^2 \beta^2} \Rightarrow \quad -\frac{b}{a} = \frac{b^2 - 2ac}{c^2}$

$\Rightarrow \quad -bc^2 = ab^2 - 2a^2c \quad \Rightarrow \quad ab^2 + bc^2 = 2a^2c \quad \Rightarrow \quad \frac{b}{c} + \frac{c}{a} = \frac{2a}{b}$

So $\frac{c}{a}, \frac{a}{b}, \frac{b}{c}$ are in A.P. $\quad \Rightarrow \quad \frac{a}{c}, \frac{b}{a}, \frac{c}{b}$ are in H.P.

13. $b = \frac{2ac}{a+c}$; $c^2 = bd$, $d = \frac{c+e}{2}$; $\quad ab + bc = 2ac$

$c = \frac{ab}{2a-b}$; $\quad d = \frac{1}{2}\left(\frac{ab}{2a-b} + e\right)$

from $c^2 = bd$ $\left(\frac{ab}{2a-b}\right)^2 = \frac{b}{2}\left(\frac{ab}{2a-b} + e\right) \quad \Rightarrow \quad \frac{a^2b^2}{(2a-b)^2} = \frac{b}{2}\left(\frac{ab + 2ae - be}{2a-b}\right)$

$\Rightarrow \quad \frac{2a^2b}{(2a-b)^2} - \frac{ab}{(2a-b)} = e \quad \Rightarrow \quad e(2a-b)^2 = 2a^2b - ab(2a-b) \quad \Rightarrow \quad e(2a-b)^2 = ab^2$

14. $x + y + z = 15$ ....(i)

a, x, y, z, b are in AP

Suppose d is common difference $d = \frac{b-a}{4}$

$\therefore \quad x = a + \frac{b-a}{4} = \frac{b+3a}{4}$, $y = \frac{2b+2a}{4}$ and $z = \frac{3b+a}{4}$

on substituting the values of X, Y and Z in (i), we get

$\Rightarrow \quad \frac{6a+6b}{4} = 15$

$a + b = 10$ ..... (ii)

$\because \quad \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{5}{3}$ ....(iii)

and a, x, y, z, b are in H.P.

$\therefore \quad \frac{1}{a}, \frac{1}{x}, \frac{1}{y}, \frac{1}{z}, \frac{1}{b}$ are in A.P.

$\therefore \quad \frac{1}{x} = \frac{1}{a} + \frac{\left(\frac{1}{b} - \frac{1}{a}\right)}{4}, \quad \frac{1}{y} = \frac{1}{a} + \frac{2}{4}\left(\frac{1}{b} - \frac{1}{a}\right)$ and $\frac{1}{z} = \frac{1}{a} + \frac{3}{4}\left(\frac{1}{b} - \frac{1}{a}\right)$

on substituting the value of $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ in (iii), we get

$\frac{3}{a} + \frac{6}{4}\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{5}{3} \Rightarrow \frac{3}{2a} + \frac{3}{2b} = \frac{5}{3} \Rightarrow \frac{1}{a} + \frac{1}{b} = \frac{10}{9}$ ..... (iv)

By equations (ii) & (iv), we get

a = 9, b = 1 or a = 1, b = 9

15. Let n HM's between a and c are $H_1, H_2, H_3, \ldots\ldots H_n$

$\therefore \quad \frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \ldots\ldots, \frac{1}{H_n}, \frac{1}{c}$ are in A.P.

suppose 'd' is common difference of this A.P.

$\therefore \quad \frac{1}{c} = \frac{1}{a} + (n+1)\, d \Rightarrow d = \frac{1}{(n+1)}\left(\frac{1}{c} - \frac{1}{a}\right)$

so $\frac{1}{H_1} = \frac{1}{a} + \frac{1}{(n+1)}\left(\frac{1}{c} - \frac{1}{a}\right) = \frac{nc+a}{(n+1)\ ca} \Rightarrow H_1 = \frac{(n+1)\ ca}{nc+a}$

and $\frac{1}{H_n} = \frac{1}{a} + \frac{n}{(n+1)}\left(\frac{1}{c} - \frac{1}{a}\right) = \frac{c+na}{(n+1)\ ca} \Rightarrow H_n = \frac{(n+1)\ ca}{c+na}$

$\therefore \quad H_1 - H_n = ac(n+1)\left[\frac{1}{nc+a} - \frac{1}{c+na}\right] = ac(n+1)\left[\frac{(a-c)\ (n-1)}{(nc+a)\ (c+na)}\right]$

$H_1 - H_n = \frac{ac\ (n^2-1)\ (a-c)}{n^2ac + n(a^2+c^2) + ac}$ ....(i)

$\because$ n is a root of $x^2(1 - ac) - x(a^2 + c^2) - (1 + ac) = 0$ we have $n^2(1 - ac) - n(a^2 + c^2) - (1 + ac) = 0$

$\Rightarrow \quad n^2ac + n(a^2 + c^2) + ac = (n^2 - 1)$ ....(ii)

$\therefore$ from (i) and (ii), we get So $H_1 - H_n = ac(a - c)$ Ans.

16. (i) $b^2c^2 + c^2a^2 + a^2b^2 \geq abc\,(a + b + c) \Rightarrow (bc - ca)^2 + (ab - bc)^2 + (ca - ab)^2 \geq 0$

$\therefore \quad b^2c^2 + c^2a^2 + a^2b^2 \geq abc\,(a + b + c)$

If a, b, x, y are real number and x, y > 0, then $\frac{a^2}{x} + \frac{b^2}{y} \geq \frac{(a+b)^2}{x+y}$ ....(i)

as on solving it we have $(ay - bx)^2 \geq 0$

Similarly, we can extend the inequality to three pairs of numbers

i.e. $\frac{a^2}{x} + \frac{b^2}{y} + \frac{c^2}{z} \geq \frac{(a+b+c)^2}{x+y+z}$

Now we use this result to following questions

(ii) As $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} = \frac{a^2}{ab+ac} + \frac{b^2}{bc+ba} + \frac{c^2}{ca+cb} \geq \frac{(a+b+c)^2}{2(ab+bc+ca)}$

Now, we have to prove that $\frac{(a+b+c)^2}{2(ab+bc+ac)} \geq \frac{3}{2}$

or $a^2 + b^2 + c^2 \geq ab + bc + ca \Rightarrow (a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$

(iii) L.H.S. = $\frac{(\sqrt{2})^2}{a+b} + \frac{(\sqrt{2})^2}{b+c} + \frac{(\sqrt{2})^2}{c+a} \geq \frac{(3\sqrt{2})^2}{2(a+b+c)}$ L.H.S. = $\frac{(\sqrt{2})^2}{a+b} + \frac{(\sqrt{2})^2}{b+c} + \frac{(\sqrt{2})^2}{c+a} \geq \frac{9}{a+b+c}$

17. A.M. = G.M.

$\therefore$ $x_1 = x_2 = x_3 = x_4 = 2$.

18. Let $\sqrt{a_1} = b_1$; $\sqrt{a_2 - 1} = b_2$; $\sqrt{a_3 - 2} = b_3$; .... $\sqrt{a_n - (n-1)} = b_n$

$\therefore$ $b_1 + b_2 + ... + b_n = \frac{1}{2}\,[b_1^2 + (b_2^2 + 1) + (b_3^2 + 2) + ... + (b_n^2 + (n - 1))] - \frac{n(n-3)}{4}$

$\therefore$ $\Sigma b_1 = \frac{1}{2}\,[(b_1^2 + b_2^2 + b_3^2 + ...+ b_n^2) + (1 + 2 + 3 + ...+ (n - 1)] - \frac{n(n-3)}{4}$

$\Rightarrow$ $2\Sigma b_1 = \Sigma b_1^2 + \frac{n(n-1)}{2} - \frac{n(n-3)}{2}$ $\Rightarrow$ $2\Sigma b_1 = \Sigma b_1^2 + n$

$\therefore$ $\Sigma b_1^2 - 2\Sigma b_1 + \Sigma 1 = 0$ $\Rightarrow$ $\sum_{i=1}^{n}(b_1 - 1)^2 = 0$ $b_1 - 1 = 0$ $\Rightarrow$ $b_1^2 = a_1 = 1$

$b_2 - 1 = 0$ $\Rightarrow$ $b_2^2 = a_2 - 1 = 1$ $\Rightarrow$ $a_2 = 2$

$b_3 - 1 = 0$ $\Rightarrow$ $b_3^2 = a_3 - 2 = 1$ $\Rightarrow$ $a_2 = 3$ and soon

hence $a_n = n$ $\therefore$ $\sum_{i=1}^{100} a_1 = 1 + 2 + 3 + ... + 100 = 5050$

19. $\sum_{i=1}^{n-1} (a_i + 3a_{i+1})^2 \le 0 \Rightarrow \frac{a_{i+1}}{a_i} = -\frac{-1}{3}$ G.P. $8, \frac{-8}{3}, \frac{8}{9}, \frac{-8}{27}, \frac{8}{81}$

Sum $8\left(1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \frac{1}{81}\right) \Rightarrow \frac{8}{81}\,(81 - 27 + 9 - 3 + 1) = \frac{488}{81}$

20. $a_n = (x)^{\frac{1}{2^n}} + (y)^{\frac{1}{2^n}}$; $b_n = (x)^{\frac{1}{2^n}} - (y)^{\frac{1}{2^n}}$, $n \in N$; $a_1 . a_2 . a_3$ ...... a_n

$= (x^{1/2} + y^{1/2})\left(x^{\frac{1}{2^2}} + y^{\frac{1}{2^2}}\right)$ .... $\left(x^{\frac{1}{2^n}} + y^{\frac{1}{2^n}}\right) \times \frac{(x^{\frac{1}{2^n}} - y^{\frac{1}{2^n}})}{b_n} = \frac{x - y}{b_n}$

21. $2a_{i+1} = a_i + a_{i+2}$

$$\sum_{i=1}^{10} \frac{a_i \quad a_{i+1} \quad a_{i+2}}{2a_{i+1}} = \frac{1}{2}\sum_{i=1}^{10} a_i a_{i+2} = \frac{1}{2}\sum_{i=1}^{10} i(i+2) = \frac{1}{2}\left[\frac{10 \times 11 \times 21}{6} + \frac{2 \times 10(11)}{2}\right] = \frac{1}{2}[385 + 110] = \frac{495}{2}$$

ss

22. $\frac{n}{1.2.3} + \frac{n-1}{2.3.4} + ... + \frac{1}{n(n+1)(n+2)}$; $S_n = \sum_{r=1}^{n}\frac{n+1}{r(r+1)(r+2)} - \sum_{r=1}^{n}\frac{1}{(r+1)(r+2)}$

Now $\sum_{r=1}^{n}\frac{1}{r\ \ (r+1)(r+2)} = \sum_{r=1}^{n}\frac{1}{2\ \ (r+1)}\left[\frac{1}{r} - \frac{1}{r+2}\right] = \frac{1}{2}\left[\sum_{r=1}^{n}\frac{1}{r(r+1)} - \sum_{r=1}^{n}\frac{1}{(r+1)(r+2)}\right]$

$= \frac{1}{2}\left[\sum_{r=1}^{n}\left(\frac{1}{r} - \frac{1}{r+1}\right) - \sum_{r=1}^{n}\left(\frac{1}{r+1} - \frac{1}{r+2}\right)\right] = \frac{1}{2}\left[\left(1 - \frac{1}{n+1}\right) - \left(\frac{1}{2} - \frac{1}{n+2}\right)\right]$

$= \frac{1}{2}\left[\frac{n}{n+1} - \frac{n}{2(n+2)}\right] = \frac{n}{2}\left[\frac{1}{n+1} - \frac{1}{2(n+2)}\right]$

Now $S_n = (n+1)\,\frac{n}{2}\left[\frac{1}{n+1} - \frac{1}{2(n+2)}\right] - \left(\frac{1}{2} - \frac{1}{n+2}\right)$

$= \frac{n}{2} - \frac{n(n+1)}{4(n+2)} - \frac{n}{2(n+2)} = \frac{2n^2 + 4n - n^2 - n - 2n}{4(n+2)} = \frac{n^2 + n}{4(n+2)} = \frac{n(n+1)}{4(n+2)}$

23. $T_n = \frac{3^n \times 5^n}{(5^n - 3^n)(5^{n+1} - 3^{n+1})}$; $T_n = \frac{1}{2}\left(\frac{3^n}{5^n - 3^n} - \frac{3^{n+1}}{5^{n+1} - 3^{n+1}}\right)$

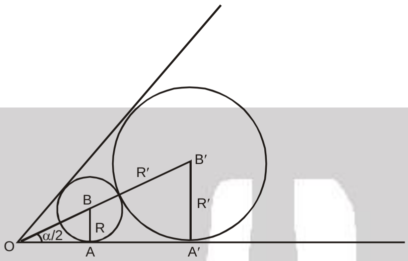
$T_1 = \frac{1}{2}\left(\frac{3}{5 - 3} - \frac{3^2}{5^2 - 3^2}\right)$; $T_2 = \frac{1}{2}\left(\frac{3^2}{5^2 - 3^2} - \frac{3^3}{5^3 - 3^3}\right)$

⋮ ⋮ ⋮

$$T_n = \frac{1}{2}\left(\frac{3^n}{5^n - 3^n} - \frac{3^{n+1}}{5^{n+1} - 3^{n+1}}\right)$$

$$S_n = \frac{1}{2}\left[\frac{3}{2} - \frac{3^{n+1}}{5^{n+1} - 3^{n+1}}\right] ; S_\infty = \frac{3}{4}$$

24. Radius of first circle = R Let Radius of second circle be = R'



$$\frac{OB}{BA} = \text{cosec } \frac{\alpha}{2} \qquad \therefore \qquad OB = R \text{ cosec} \frac{\alpha}{2}$$

Now $\frac{OB'}{A'\ B'} = \text{cosec } \frac{\alpha}{2} \quad \Rightarrow \quad OB' = R' \text{ cosec } \frac{\alpha}{2} = OB + R + R'$

$\Rightarrow R + R \text{ cosec } \frac{\alpha}{2} + R' = R' \text{ cosec} \frac{\alpha}{2} \Rightarrow R(1 + \text{cosec } \alpha/2) = R' (\text{cosec } \alpha/2 - 1) \Rightarrow R' = R \left(\frac{1 + \sin\alpha/2}{1 - \sin\alpha/2}\right)$

if radius of the third circle be R'', then similarly $R'' = R \left(\frac{1 + \sin\alpha/2}{1 - \sin\alpha/2}\right)^2$

So R + R' + R'' + ...n terms $= R \left[\frac{\left(\frac{1 + \sin\ \ \alpha/2}{1 - \sin\alpha/2}\right)^n - 1}{\left(\frac{1 + \sin\alpha/2}{1 - \sin\alpha/2}\right) - 1}\right] = R \left(\frac{1 - \sin\alpha/2}{2\sin\alpha/2}\right)\left[\left(\frac{1 + \sin\alpha/2}{1 - \sin\alpha/2}\right)^n - 1\right]$

25. Given $(abc)^{2/3} = \frac{(a + b + c)}{3} \cdot \frac{3}{\frac{1}{a} + \frac{1}{b} + \frac{1}{c}} \Rightarrow (ab + bc + ac)^3 = abc\ (a + b + c)^3$ .....(i)

Now consider the polynomial $p(x) = x^3 + mx^2 + nx + p$, with roots a, b, c, then have $a + b + c = -m; ab + bc + ca = n$; $abc = -p$ using these values equation (i) becomes $n^3 = m^3p$ ......(ii)

Hence, if $m \neq 0$, then equation p(x) = 0 can be written as $x^3 + mx^2 + nx + \frac{n^3}{m^3} = 0$

or $m^3x^3 + m^4 x^2 + nm^3x + n^3 = 0 \Rightarrow (mx + n)(m^2 x^2 + (m^3 - mn)x + n^2) = 0$

It follows that one of the roots of p(x) = 0 is $x_1 = -\frac{n}{m}$ and other two satisfy the condition $x_2x_3 = \frac{n^2}{m^2}$

$\Rightarrow \quad x_1^2 = x_2\, x_3$

Thus the roots are the terms of a geometric sequence. If should be noted that m, n ≠ 0 as in this case $x^3 + p = 0$ cannot have three real roots.

26. $t_n = S_n - S_{n-1} = \frac{n}{6}(2n^2 + 9n + 13) - \frac{(n-1)}{6}\{2(n-1)^2 + 9(n-1) + 13\}$

$= \frac{1}{6}[2n^3 + 9n^2 + 13n - 2(n-1)^3 - 9(n-1)^2 - 13(n-1)] = (n+1)^2 \sum_{r=1}^{\infty} \frac{1}{r\ .\ (r+1)} = \sum_{r=1}^{\infty}\left(\frac{1}{r} - \frac{1}{r+1}\right) = 1$

27. $\alpha + \beta = \frac{-\ b}{a}$, $\alpha\beta = \frac{c}{a} \Rightarrow (\alpha + \beta), \alpha^2 + \beta^2, \ \alpha^3 + \beta^3$ are in G.P then $(\alpha^2 + \beta^2)^2 = (\alpha + \beta)(\alpha^3 + \beta^3)$

$\Rightarrow 2\alpha^2\beta^2 = \alpha\beta^3 + \beta\alpha^3 \Rightarrow \alpha\beta[\alpha^2 + \beta^2 - 2\alpha\beta] = 0$ so $\alpha\beta(\alpha - \beta)^2 = 0 \Rightarrow \frac{c}{a} \cdot \frac{b^2 - 4ac}{a^2} = 0, a \neq 0 \Rightarrow c \cdot \Delta = 0$

28. $S_n = (1 + 2T_n)(1 - T_n) \Rightarrow T_1 = (1 + 2T_1)(1 - T_1)$

$T_1 = 1 - T_1 + 2T_1 - 2T_1^2 \Rightarrow 2T_1^2 = 1 \Rightarrow T_1 = \frac{1}{\sqrt{2}}$

$S_2 = T_1 + T_2 = (1 + 2T_2)(1 - T_2) \Rightarrow T_1 + T_2 = 1 - T_2 + 2T_2 - 2T_2^2$

$T_1 = 1 - 2T_2^2 \Rightarrow 2T_2^2 = 1 - \frac{1}{\sqrt{2}}$

$T_2^2 = \frac{\sqrt{2} - 1}{2\sqrt{2}} \Rightarrow T_2^2 = \frac{2 - \sqrt{2}}{4} \Rightarrow a = 4, b = 2 \Rightarrow a + b = 6$