SOLUTIONS OF FUNDAMENTALS OF MATHEMATICS-I

EXERCISE # 1

PART–1

Section (A)

A-1. Collection of all intelligent women in Jalandhar is not a set as it is not a well defined collection. It is not possible to decide logically which woman is to be included in the collection and which is not to be included.

A-2. 2, 3, 5 and 7 are the only positive primes less than 10.

A-3. Obvious

A-4. (i) $x^2 - 1 = 0 \quad x = \pm 1$
(ii) $x^2 + 1 = 0 \quad x = \pm i \quad x \in \phi$
(iii) $x^2 - 9 = 0 \quad x = \pm 3$
(iv) $x^2 - x - 2 = 0, \quad x = 2, -1$

A-5. $P(A) = \{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\} = \{\phi, \{\phi\}, \{\{\phi\}\}, A\}$

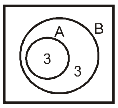
Section (B)

B-1. $A = \{1, 2, 3\}$
$B = \{3, 4\}$
$C = \{4, 5, 6\}$
$B \cap C = \{4\}$
$B \cup C = \{3, 4, 5, 6\}$
$A \cup (B \cap C) = \{1, 2, 3, 4\}$
$A - (B \cap C) = \{1, 2, 3\}$
$(B \cup C) - A = \{4, 5, 6\}$

B-2. Obvious

B-3. $bN \cap . cN$
(+ve integral multiple of b) $\cap$ (+ve integral multiple of c)
since b & c are relatively primes $= b\, c\, N$
$\therefore \quad d = bc$

B-4.



for minimum $n(A \cup B)$, the set A is subset of B and for maximum $n(A \cup B)$, the sets A and B are disjoint set.
also $\quad n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow \quad$ minimum $n(A \cap B) = 0$, maximum $n(A \cap B) = 3$
$\Rightarrow \quad$ minimum value of $n(A \cup B) = 3 + 6 - 3 = 6$ or maximum value of $n(A \cup B) = 3 + 6 - 0 = 9$

Section (C)

C-1. $n(A^c \cap B^c) = n[(\{A \cup B)^c] = n(U) - n(A \cup B) = n(U) - [n(A) + n(B) - n(A \cap B)]$
$= 700 - [200 + 300 - 100] = 300.$

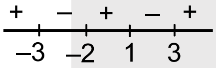
C-2. Let number of newspapers is x. As every newspaper is read by 60 students
Since, every students reads 5 newspapers
$\therefore \quad 60x = 300(5) \quad \Rightarrow \quad x = 25.$

C-3. n (A) = 40% of 10,000 = 4,000
n(B) = 20% of 10,000 = 2,000
n(C) = 10% of 10,000 = 1,000
$n(A \cap B)$ = 5% of 10,000 = 500
$n(B \cap C)$ = 3% of 10,000 = 300
$n(C \cap A)$ = 4% of 10,000 = 400
$n(A \cap B \cap C)$ = 2% of 10,000 = 200
$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$
$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$
$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$
$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300.$

C-4. n(A) = 21, n(B) = 26, n(C) = 29
$n(A \cap B) = 14$, $n(A \cap C) = 12$, $m(B \cap C) = 13$, $n(A \cap B \cap C) = 8$
$n(C \cap A' \cap B') = n(C \cap \overline{A \cup B}) = n(C) - n((C \cap A) \cup (C \cap B))$
$n(C) - [n(C \cap A) + n(C \cap B) - n(A \cap B \cap C)]$
$29 - [12 + 13 - 8] = 12$
$n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C) = 14 - 8 = 6$

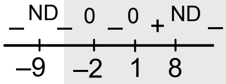
Section (D)

D-1. (i)



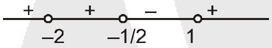
$\Rightarrow x \in (-3,-2) \cup (1,3)$

(ii)

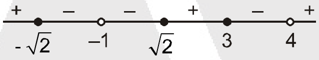


$x \in \{-2\} \cup [1,8)$

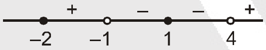
(iii) $\dfrac{(x+2)^2}{(2x+1)(x-1)} > 0$



(iv) $\dfrac{(x-\sqrt{2})(x+\sqrt{2})(x-3)^3}{(x+1)^2(x-4)} \le 0$



(v) $\dfrac{(x+2)(x-1)^2}{(x+1)(x-4)} \le 0 \quad \Rightarrow$



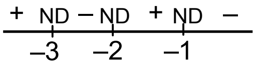
D-2. (i) $\dfrac{7x-5}{8x+3} - 4 > 0$

(ii) $\dfrac{14x}{x+1} - \left(\dfrac{9x-30}{x-4}\right) < 0 \quad \Rightarrow \quad \dfrac{14x^2 - 56x - 9x^2 - 9x + 30x + 30}{(x+1)(x-4)} < 0$

$\Rightarrow \quad \dfrac{5x^2 - 35x + 30}{(x+1)(x+4)} < 0 \quad \Rightarrow \quad \dfrac{(x-6)(x-1)}{(x+1)(x+4)} < 0$

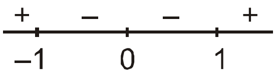
$\Rightarrow$ $\mathbf{x \in (-6, -1) \cup (1, 4)}$

(iii) $\dfrac{-12x^2 - 12}{(x+1)(x+2)(x+3)} \le 0$



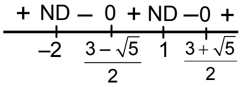
$x \in (-3,-2) \cup (-1,\infty)$

(iv) $\dfrac{x^2 + 2 + 2x^2 - 2}{x^2 - 1} < 0 \quad \Rightarrow \quad \dfrac{3x^2}{(x-1)(x+1)} < 0$



$\Rightarrow \quad x \in (-1,0) \cup (0,1)$

D-3. (i)



$x \in \left(-2, \frac{3-\sqrt{5}}{2}\right] \cup \left(1, \frac{3+\sqrt{5}}{2}\right]$

(ii) $2x^2 - 3x - 459 > x^2 + 1$

$\Rightarrow \quad x^2 - 3x - 460 > 0 \quad \Rightarrow \quad x \in (-\infty, -20) \cup (23, \infty)$

(iii) $x^2 - 5x + 12 > 3(x^2 - 4x + 5)$ (since $x^2 - 4x + 5 = (x-2)^2 + 1 > 0$)

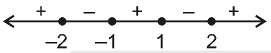
$\Rightarrow \quad 2x^2 - 7x + 3 < 0 \quad \Rightarrow \quad 2x^2 - 6x - x + 3 < 0$

$\Rightarrow \quad 2x(x-3) - 1(x-3) < 0 \quad \Rightarrow \quad x \in \left(\frac{1}{2}, 3\right)$

(iv) $x^4 + x^2 + 1$ is always positive सदैव धनात्मक

$\Rightarrow \quad \frac{1}{(x-5)(x+1)} > 0$

D-4. (i) $(x^2 - 1)(x^2 - 4) \le 0 \quad \Rightarrow \quad (x-1)(x+1)(x+2)(x-2) \le 0$



$x \in [-2, -1] \cup [1, 2]$

(ii) $(x^2 - 9)(x^2 + 7) \le 0 \quad \Rightarrow \quad (x-3)(x+3) \le 0$

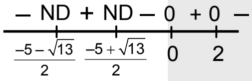
$x \in [-3, 3]$

D-5. $\frac{x-1-7x-14}{x+2} < 0$ and $\frac{x-1-x-2}{x+2} > 0$

$x \in (-\infty, -\frac{5}{2}) \cup (-2, \infty)$ and $x \in (-\infty, -2) \Rightarrow x \in (-\infty, -\frac{5}{2})$

(i) $x \in (-\infty, -\frac{5}{2})$ (ii) $x^2 \in (\frac{25}{4}, \infty)$ (iii) $\frac{1}{x} \in \left(-\frac{2}{5}, 0\right)$

D-6.



Number of positive integer satisfying the inequality equal to 2 (which are 1 and 2)

Section (E)

E-1. (i) $\log_{10}(\log_{10}5 + 2\log_{10}2) + (\log_{10}2)^2 = (\log_{10}5 + \log_{10}2)^2 = (\log_{10}10)^2 = 1$

(ii) $2^{\log_{\sqrt{5}} 5} + 7^{\log_3 9} + -5^{\log_2 8} = 2^2 + 7^2 - 5^3 = 53 - 125 = -72$

(iii) $$\left(5^{\log_5 7} + \frac{1}{\left(\log_{10}\left(\frac{1}{0.1}\right)\right)}\right)^{1/3} = (7+1)^{1/3} = 2$$

(iv) $\log_{3/4} \log_2\left((8)^{1/2}\right)^{1/2} = \log_{3/4} \log_2 (2)^{3/4} = 1$

(v) $\left(\frac{1}{49}\right)^{1+\log_7^2} = (7^{-2})^{\log_7 7 + \log_7 2} = (7^{-2})^{\log_7^{14}} = 7^{\log_7 (14)^{-2}} = \frac{1}{196}$ & $5^{-\log_{1/5} 7} = 5^{\log_5 7} = 7$

$\therefore \ 7 + \frac{1}{196}$

(vi) $7^{\log_3 5} + 3^{\log_5 7} - 5^{\log_3 7} - 7^{\log_5 3} = 7^{\log_3 5} + 3^{\log_5 7} - 7^{\log_3 5} - 3^{\log_5 7}$

{using property $a^{\log_c b} = b^{\log_c a}$ }= 0

E-2. (i) $\sqrt{2} > 1$ & $\sqrt{3} > 1$, hence positive
(ii) Number & base on oppsite sides of unity, hence negative
(iii) Number & base on same sides of unity, hence positive
(iv) Number & base on same sides of unity, hence positive
(v) Number & base on same sides of unity, hence positive
(vi) Number & base on oppsite sides of unity, hence negative
(vii) Number & base on same sides of unity, hence positive
(viii) Number & base on oppsite sides of unity, hence negative
(ix) Number & base on oppsite sides of unity, hence negative

E-3. (i) $\log_{10}\left(\frac{3}{4}\right) = \log_{10}3 - 2\log_{10}2 = b - 2a$

(ii) $\log_{10}2 + 3\log_{10}3 = a + 3b$

(iii) $\frac{2\log_{10}3}{\log_{10}2} + \frac{3\log_{10}2}{\log_{10}3} = \frac{2b}{a} + \frac{3a}{b} = \frac{2b^2 + a^2}{ab}$

(iv) $2\left(\frac{4\log_{10}2 + 2\log_{10}3}{2\log_{10}3 + \log_{10}5}\right) = \frac{4(2a+b)}{1-a+2b}$

E-4. (i) $n = 75600$

Now $4\log_n 2 + 3\log_n 3 + 2\log_n 5 + \log_n 7 = \log_n(2^4.3^3.5^2.7^1) = \log_n(75600) = 1$

(ii) $x = 4^{3^2} = 4^3 = 64$

$y = (2^4)^1 = 16$, $z = 3^{2^1} = 9$

sum = 64 + 16 + 9 = 89

(iii) $a_n = \frac{1}{\log_n 2002} = \log_{2002} n$

$b = a_2 + a_3 + a_4 + a_5 = \log_{2002} 2 + \log_{2002} 3 + \log_{2002} 4 + \log_{2002} 5 = \log_{2002} (2.3.4.5) = \log_{2002} 120$

$c = a_{10} + a_{11} + a_{12} + a_{13} + a_{14} = \log_{2002}10 + \log_{2002}11 + \log_{2002}12 + \log_{2002}13 + \log_{2002}14$

$= \log_{2002}(10.11.12.13.14) = \log_{2002}240240$

Now $b - c = \log_{2002}120 - \log_{2002}240240 = \log_{2002}10 + =$

E-5. Let $\log_2 7 = \frac{p}{q}$, where p & q are coprime numbers.

$\Rightarrow$ $7 = 2^{p/q}$ $\Rightarrow$ $7^q = 2^p$

$\because$ 7^q is an odd number

while 2^p is an even number

$\therefore$ this is not possible & $\log_2 7$ is an irrational number

E-6. $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b} = k$ (let)

$\log_a = k(b - c)$ $\Rightarrow$ $a \log a = ka (b - c)$

$\log_b = k(c - a)$ $\Rightarrow$ $b \log b = kb (c - a)$

$\log_c = k(a - b)$ $\Rightarrow$ $c \log c = kc (a - b)$

$\therefore$ $\log a^a + \log b^b + \log c^c = k (ab - ac + bc - ab + ca - bc)$

$\Rightarrow$ $\log a^a b^b c^c = 0$ $\Rightarrow$ $a^a b^b c^c = 1$

Section (F)

F-1. (i) $\log_x(4x - 3) = 2 \Rightarrow 4x - 3 = x^2 \Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x - 1)(x - 3) = 0 \Rightarrow x = 1, x = 3$

But $4x - 3 > 0 \Rightarrow x > \frac{3}{4}$ and $x > 0, x \neq 1$

Ans. x = 3

(ii) $\log_2\log_3(x^2 - 1) = 0 \Rightarrow \log_3(x^2 - 1) = 2^0 = 1 \Rightarrow (x^2 - 1) = 3^1$

$\Rightarrow x^2 - 1 = 3 \Rightarrow x^2 = 4 \Rightarrow x = \pm 2$

both are satisfied

(iii) $x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0 \quad x = 3, x = -1$

but $x > 0$

Ans. $x = 3$

(iv) $\log_4(\log_2 x) + \log_2(\log_4 x) = 2 \Rightarrow \frac{1}{2}\log_2(\log_2 x) + \log_2 [\frac{1}{2}\log_2 x] = 2$

$\frac{1}{2}\log_2(\log_2 x) + \log_2(\log_2 x) = 2 \Rightarrow \frac{3}{2}\log_2(\log_2 x) = 3$

$\log_2(\log_2 x) = 2 \quad \Rightarrow \log_2 x = 4 \Rightarrow x = 2^4 = 16$

(v) $\log_3\left(\log_9 x + \frac{1}{2} + 9^x\right) = 2x \Rightarrow \log_9 x + \frac{1}{2} + 9^x = 3^{2x} \Rightarrow \log_9 x + \frac{1}{2} + 9^x = 9^x$

$\Rightarrow \log_9 x = -\frac{1}{2} \Rightarrow x = 9^{-1/2} \Rightarrow x = \frac{1}{3}$

(vi) $2 \log_4 (4 - x) = 4 - \log_2 (-2 - x)$

(i) $4 - x > 0 \Rightarrow x < 4$

(ii) $-2 - x > 0 \Rightarrow x < -2$

(iii) $\log_2 (4 - x) = 4 - \log_2 (-2 - x) \Rightarrow \log_2 (4 - x)(-2 - x) = 4$

$\Rightarrow (4 - x)(-2 - x) = 16 \Rightarrow -8 - 2x + x^2 = 16$

$\Rightarrow x^2 - 2x - 24 = 0 \Rightarrow (x - 6)(x + 4) = 0$

$x = 6$ (not possible) , $x = -4.$

(viii) $x^{\frac{1}{2}\log_{\sqrt{x}}(x^2-x)} = 3\log_{3^2}(2^2) = 2$

$\Rightarrow (\sqrt{x})^{\log_{\sqrt{x}}(x^2-x)} = 2$

$\Rightarrow x^2 - x = 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow (x - 2)(x + 1) = 0$

$\Rightarrow x = 2, x = -1$ but $x > 0$ and $x^2 - x = 0 \Rightarrow x(x - 1) > 0$

$x \in (-\infty, 0) \cup (1, \infty)$

F-2. (i) $t^2 - 2t - 5 = 0$

sum of roots = 2

$\log_3 x_1 + \log_3 x_2 = 2 \Rightarrow \log_3(x_1x_2) = 2 \Rightarrow x_1x_2 = 3^2 = 9$

(ii) $(2^x)^2 - 7(2^x) + 6 = 0$

$\Rightarrow t^2 - 7t + 6 = 0$

$t = 1, t = 6$

Roots $2^x = 1, 2^x = 6$

$\Rightarrow x = 1, x = \log_2 6$

Product of roots = $(1)(\log_2 6) = \log_2 6$

(iii) $x^{\log_{10} x+2} = 10^{\log_{10} x+2}$

$x = 10$ or $\log_{10}x + 2 = 0 \Rightarrow x = 10^{-2} = \frac{1}{100}$

(iv) $x^{\frac{\log x+5}{3}} = 10^{5+\log x}$; $\left(\frac{\log x + 5}{3}\right)\log x = 5 + \log x$

$\log^2 x + 2\log x - 15 = 0$; $(\log x + 5)(\log x - 3) = 0$

$\log x = -5, \log x = 3$; $x = 10^{-5}$, $x = 10^3$.

Section (G) :

G-1. (i) $\log_{\frac{5}{8}}\left(2x^2 - x - \frac{3}{8}\right) \ge 1 \Rightarrow 2x^2 - x - \frac{3}{8} \le \frac{5}{8} \Rightarrow 16x^2 - 8x - 8 \le 0$

$\Rightarrow 2x^2 - x - 1 \le 0 \Rightarrow (2x + 1)(x - 1) \le 0$

$x \in \left[-\frac{1}{2}, 1\right]$ ......(i)

also $2x^2 - x - \frac{3}{8} > 0 \Rightarrow 16x^2 - 8x - 3 > 0$

$\Rightarrow \quad 16x^2 - 12x + 4x - 3 > 0 \Rightarrow \quad (4x - 3)(4x + 1) > 0$

$\Rightarrow \quad x \in \left(-\infty, \frac{-1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$ ....(ii)

(i) $\cap$ (ii) $\Rightarrow \quad x \in \left[\frac{-1}{2}, -\frac{-1}{4}\right) \cup \left(\frac{3}{4}, 1\right]$

(ii) $x^2 - 5x + 6 > 0 \Rightarrow \quad (x - 3)(x - 2) > 0$

$\Rightarrow \quad x \in (-\infty, 2) \cup (3, \infty)$ ......(i)

and $\quad x^2 - 5x + 6 < \left(\frac{1}{2}\right)^{-1}$

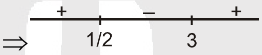
$\Rightarrow \quad x^2 - 5x + 4 < 0 \Rightarrow \quad (x - 1)(x - 4) < 0 \quad \Rightarrow \quad x \in (1, 4)$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow \quad x \in (1, 2) \cup (3, 4)$

(iii) $\log_7\left(\frac{2x-6}{2x-1}\right) > 0 \quad \Rightarrow \frac{2x-6}{2x-1} > 7^0 \quad \Rightarrow \frac{2x-6}{2x-1} - 1 > 0$

$\Rightarrow \frac{-5}{2x-1} > 0 \quad \Rightarrow \frac{1}{2x-1} < 0 \quad \Rightarrow x \in \left(-\infty, \frac{1}{2}\right)$

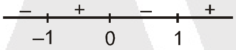
and $\quad \frac{2x-6}{2x-1} > 0 \quad \Rightarrow$



Ans. $\left(-\infty, \frac{1}{2}\right)$

(iv) $2 - x < \frac{2}{x+1} \quad \Rightarrow \quad \frac{2}{x+1} + x - 2 > 0 \quad \Rightarrow \frac{2 + x^2 - x - 2}{x+1} > 0$

$\Rightarrow \quad \frac{x(x-1)}{x+1} > 0 \quad \Rightarrow$



and $\quad 2 - x > 0 \quad \Rightarrow \quad x < 2 \quad$ and $\quad \frac{2}{x+1} > 0 \quad \Rightarrow \quad x > -1$

Ans. $(-1, 0) \cup (1, 2)$

(v) $2^2.2^x - 4^x \le \left(\frac{1}{3}\right)^{-2} \quad \Rightarrow \quad 4.2^x - (2^x)^2 \le 9$

Let $\quad 2^x = t \quad \Rightarrow \quad t^2 - 4t + 9 \ge 0$ always true

$2^{x+2} - 4^x > 0 \quad \Rightarrow \quad 2^x.(4 - 2^x) > 0$

$4 - 2^x > 0 \quad \Rightarrow \quad 2^x < 2^x$

$\Rightarrow \quad x < 2 \quad \Rightarrow \quad x \in (-\infty, 2)$

(vi) $\log_x(4x - 3) \ge 2$

Case-I $\quad 0 < x < 1$ and $4x - 3 > 0 \Rightarrow \quad x > \frac{3}{4}$

then $\quad 4x - 3 \le x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \ge 0 \Rightarrow \quad (x - 3)(x - 1) \ge 0$

$\Rightarrow \quad x \in \left(\frac{3}{4}, 1\right)$

Case -II $x > 1, 4x - 3 > 0 \quad \Rightarrow \quad x > \frac{3}{4}$

then $4x - 3 \ge x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \le 0$

$\Rightarrow \quad (x - 3)(x - 1) \le 0 \Rightarrow \quad x \in (1,3]$

Ans. $\quad x \in \left(\frac{3}{4}, 1\right) \cup (1, 3]$

G-2. $\log_{\frac{1}{5}} \dfrac{4x+6}{x} \geq 0$

$\dfrac{4x+6}{x} > 0 \quad \Rightarrow \quad x \in \left(-\infty, -\dfrac{3}{2}\right) \cup (0, \infty) \quad$ ....(i)

& $\quad \dfrac{4x+6}{x} \leq 1 \quad \Rightarrow \quad \dfrac{x+2}{x} \leq 0 \quad \Rightarrow \quad x \in (-\infty, -2] \cup (0, \infty)$ ....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in \left[-2, \dfrac{-3}{2}\right)$

G-3 (i) $\quad -2 \leq \log_{.5} x \leq 1$

$x \in [(0.5)^1, (0.5)^{-2}] \quad \Rightarrow \quad x \in \left[\dfrac{1}{2}, 4\right]$

(ii) $\quad 15^x - 25\ 3^x - 9.5^x + 225 > 0 \qquad (5^x - 25)(3^x - 9) \geq 0 \qquad \Rightarrow x \in R$

(iii) $\quad 8\left(\dfrac{3^{x-2}}{3^x - 2^x}\right) > 1 + \left(\dfrac{2}{3}\right)^x \quad \Rightarrow \quad \dfrac{8}{9}\left(\dfrac{\left(\frac{3}{2}\right)^x}{\left(\frac{3}{2}\right)^x - 1}\right) > 1 + \left(\dfrac{2}{3}\right)^x$

Let $\left(\dfrac{3}{2}\right)^x = t$ then $\quad \dfrac{8t}{9(t-1)} > \dfrac{t+1}{t} \Rightarrow \dfrac{8t^2 - 9(t^2-1)}{9t(t-1)} > 0$

$\Rightarrow \quad \dfrac{(t-3)(t+3)}{t(t-1)} < 0 \Rightarrow t \in (-3,0) \cup (1, 3)$

$\Rightarrow \quad \left(\dfrac{3}{2}\right)^x \in (1,3) \Rightarrow x \in (0, \log_{\frac{3}{2}} 3)$

G-4. (i) $\quad \log_x(4x - 3) \geq 2$

Case -I : When $x > 1 \quad \Rightarrow \quad 4x - 3 \geq x^2 \quad \Rightarrow \quad x^2 - 4x + 3 \leq 0$

$\Rightarrow \quad (x - 1)(x - 3) \leq 0$

$x \in (1, 3]$ ....(i)

Case-II : When $0 < x < 1 \quad \Rightarrow \quad 4x - 3 \leq x^2$

and $4x - 3 > 0 \quad \Rightarrow \quad (x - 1)(x - 3) \geq 0$

$x > 3/4 \qquad x \in \left(\dfrac{3}{4}, 1\right)$ ......(ii) $\qquad$ Ans. $\quad$ (i) $\cup$ (ii)

(ii) $\quad \log_{3x^2+1} 2 < \dfrac{1}{2}$

$3x^2 + 1 > 1 \quad \Rightarrow \quad x^2 > 0 \quad \Rightarrow \quad x \in R - \{0\}$

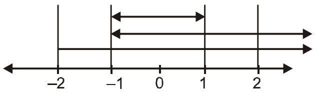
$2 < (3x^2 + 1)^{1/2} \quad \Rightarrow \quad 3x^2 + 1 > 4 \quad \Rightarrow \quad (x - 1)(x + 1) > 0$

$\Rightarrow \quad x \in (-\infty, -1) \cup (1, \infty)$

(iii) $\quad \log_{x^2} (x + 2) < 1 \qquad x + 2 > 0 \quad x > -2$

Case -I : when $0 < x^2 < 1 \qquad x \in (-1, 0) \cup (0, 1)$

then $x + 2 > x^2 \Rightarrow \quad x^2 - x - 2 < 0$

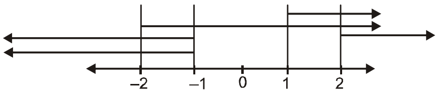


$x \in (-1, 1) - \{0\}$

Case -II : $x^2 > 1 \qquad |x| > 1$

$x + 2 < x^2 \quad \Rightarrow \quad x^2 - x - 2 > 0$

$x \in (-2, -1) \cup (2, \infty)$



Hence, $\quad x \in (-2, -1) \cup (-1, 0) \cup (0, 1) \cup (2, \infty)$

PART - II

Section (A) :

A-1. Since, intelligency is not defined for students in a class so set of intelligent students in a class is not well defined collection.

A-2. $x^2 = 16 \Rightarrow \quad x = \pm 4$
$2x = 6 \quad x = 3$
No common value of x

A-3. $A = \{-2, -1, 0, 1, 2\}$
No. of subsets $= 2^n = 2^5 = 32$

A-4. Obvious

A-5. $P(A) = \{\phi, \{7\}, \{10\}, \{11\}, \{7, 10\}, \{7, 11\}, \{10, 11\}, \{7, 10, 11\}\}$
Number of subsets $= 2^n = 2^8 = 256$

A-6. Between any two real numbers there lie infinitely many real numbers.

Section (B) :

B-1. $A = [x : x \in R, -1 < x < 1]$
$B = [x : x \in R : x \leq 0 \text{ or } x \geq 2]$
$\therefore \quad A \cup B = R - D$, where $D = [x : x \in R, 1 \leq x < 2]$

B-2. $A \cap B = \{3, 4, 10\}$
$A \cap C = \{4\}$
$(A \cap B) \cup (A \cap C) = \{3, 4, 10\}$

B-3. Obviously $A - (B \cup C)$

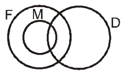
B-4. $B' = U - B = \{1, 2, 3, 4, 5, 8, 9, 10\}$
$A \cap B' = \{1, 2, 5\} = A$

B-5. $A = \{5, 9, 13, 17, 21\}$ and $B = \{3, 6, 9, 12, 15, 18, 21, 24\}$
$A - B = \{5, 13, 17\}$
$A - (A - B) = \{9, 21\}$

B-6. Let $A \cup B = A \cap B$
Now, $x \in A \Rightarrow x \in A \cup B \quad (\because A \subset A \cup B)$
$\Rightarrow x \in A \cap B \quad (\because A \cup B = A \cap B)$
$\Rightarrow x \in B$
Similarly, $x \in B$ implies $x \in A \quad \therefore A = B$
Conversly, let $A = B$
$\therefore A \cup B = A \cup A = A = A \cap A = A \cap B$
$\therefore A \cup B = A \cap B$

B-7. 1. $(N \cup B) \cap Z = (N \cap Z) \cup (B \cap Z) = N \cup (B \cap Z)$
2. $A = \{3, 6, 9, 12, 15, 18, 21, 24\}$

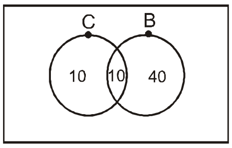
B-8.



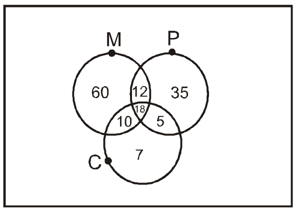
$M \equiv$ Mother; $F \equiv$ Female; $D =$ Doctor

Section (C) :

C-1. (i) $A \cup B \underline{>} A \cup B$ (ii) $A \cap B \underline{<} A \cup B$ (iii) $A \cap B = A \cup B$ not always



C-2. P = 10 + 10 + 40 = 60 %



C-3. Number of students offered maths alone = 60

n (M) = 100

n(P) = 70

n (C) = 40

$n (M \cap P) = 30$

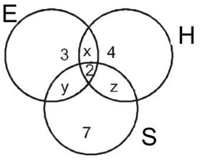
$n (M \cap P) = 28$

$n (P \cap C) = 23$

$n (M \cap P \cap C) = 18$

C-4. x+ y = 10; x + z = 9; y + z = 11 $\Rightarrow$x + y + z = 15

x = 4, y = 6, z = 5



Section (D) :

D-1. $$\frac{x^2(x^2-3x+2)}{x^2-x-30}\ge 0 \quad \Rightarrow \quad \frac{x^2(x-1)(x-2)}{(x+5)(x-6)}\ge 0$$

$x \ne -5, 6$

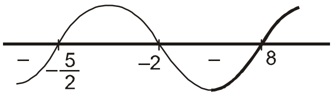
$x \in (-\infty, -5) \cup [1, 2] \cup (6, \infty) \cup \{0\}$

D-2. $x \in (-\infty, -9) \cup (-9, -3) \cup [-1, 0) \cup (0, 2) [4, 6)$

so +ve integral solution

D-3. $$\frac{x^2-1}{2x+5} - 3 < 0 \quad \Rightarrow \frac{x^2-1-6x-15}{2x+5} < 0 \quad \Rightarrow \frac{x^2-6x-16}{2x+5} < 0 \quad \Rightarrow \frac{x^2-8x+2x-16}{\left(x+\frac{5}{2}\right)} < 0$$

$$\Rightarrow \frac{x(x-8)+2(x-8)}{x+\frac{5}{2}} < 0 \Rightarrow \frac{(x-8)(x-2)}{x+\frac{5}{2}} < 0$$



$x \in (-\infty, -5/2) \cup (-2, 8)$

D-4. $$\frac{x^2-1}{x+3} \ge 0$$ $x^2 - 5x + 2 \le 0.$

$\Rightarrow \quad x \in (-3, -1] \cup [1, \infty) \quad \Rightarrow \quad x \in \left[\frac{5-\sqrt{17}}{2}, \frac{5+\sqrt{17}}{2}\right]$

so the common solution is $x \in \left[1, \frac{5+\sqrt{17}}{2}\right]$

D-5. $2x - 1 \le x^2 + 3 \le x - 1$

$x^2 + 3 \le x - 1 \quad \Rightarrow \quad x^2 - x + 4 \le 0$ which is not true for $x \in R$

D-6. $x^2 + 9 < (x + 3)^2 < 8x + 25$

$\Rightarrow \quad (x + 3)^2 > x^2 + 9 \Rightarrow \quad x > 0 \quad$ ....(i)

and $\quad (x + 3)^2 < 8x + 25$

$\Rightarrow \quad x^2 - 2x - 16 < 0$

$\Rightarrow \quad x \in (1 - \sqrt{17}\,,\ 1 + \sqrt{17}\,) \quad$ .....(ii)

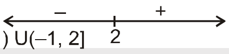
(i) $\cap$ (ii) $\Rightarrow x \in (0,\ 1 + \sqrt{17}\,)$

Noumber of integers = 5

D-7. $$\frac{2}{x^2 - x + 1} - \frac{1}{x+1} - \frac{2x-1}{x^3+1} \ge 0$$

$$\frac{2x + 2 - x^2 + x - 1}{x^3 + 1} - \frac{(2x-1)}{x^2+1} \ge 0 = \frac{3x + 1 - x^2 - 2x + 1}{x^3+1} \ge 0 = \frac{-x^2 + x + 2}{x^3+1} \ge 0 = \frac{x^2 - x - 2}{x^3+1} \le 0$$

$$= \frac{(x+1)(x-2)}{(x+1)(x^2 - x + 1)} \le 0 \quad \Rightarrow \frac{(x-2)}{(x^2 - x + 1)} \le 0 \quad (-\infty, -1)\ \text{U}(-1, 2]$$



required value of x, {0, 1, 2}

Section (E) :

E-1. $a^4\, b^5 = 1 \Rightarrow$ log w.r.t. $a \to 4 + 5\log_a b = \log_a 1 \Rightarrow \log_a b = -\frac{4}{5}$

Now $\log_a(a^5 b^4) = 5 + 4\log_a b = 5 + 4\left(-\frac{4}{5}\right) = \frac{25-16}{5} = \frac{9}{5}$

E-2. $$\frac{1}{1 + \log_b a + \log_b c} + \frac{1}{1 + \log_c a + \log_c b} + \frac{1}{1 + \log_a b + \log_a c} = \frac{1}{\log_b abc} + \frac{1}{\log_c abc} + \frac{1}{\log_a abc}$$

$= \log_{abc} b + \log_{abc} c + \log_{abc} a = \log_{abc} abc = 1$

E-3. $$\frac{1}{\log_{\sqrt{bc}} abc} + \frac{1}{\log_{\sqrt{ca}} abc} + \frac{1}{\log_{\sqrt{ab}} abc} = \log_{abc}\sqrt{bc} + \log_{abc}\sqrt{ca} + \log_{abc}\sqrt{ab} = \log_{abc} abc = 1$$

E-4. Obvious

E-5. $$y = \frac{2^{\log_{2^{1/4}} a} - 3^{\log_{27}\left(a^2+1\right)^3} - 2a}{7^{4\log_{49} a} - a - 1} \qquad \Rightarrow \qquad 2^{\log_{2^{1/4}} a} = 2^{4\log_2 a} = a^4$$

$$3^{\log_{27}(a^2+1)^3} = 3^{\log_3(a^2+1)} = a^2 + 1 \qquad \Rightarrow \qquad 7^{4\log_{49} a} = 7^{2\log_7 a} = a^2$$

$$\therefore \qquad y = \frac{a^4 - (a^2 + 1 + 2a)}{a^2 - a - 1} = \frac{a^4 - (a+1)^2}{a^2 - a - 1} = a^2 + a + 1$$

E-6. $\log_a(ab) = x \quad \Rightarrow \quad 1 + \log_a b = x \quad \Rightarrow \quad \log_a b = x - 1 \quad \Rightarrow \quad \log_b a = \frac{1}{x-1}$

Now $\log_b(ab) = 1 + \log_b a = 1 + \frac{1}{x-1} = \frac{x-1+1}{x-1} = \frac{x}{x-1}$

E-7. $\log_p(\log_q(\log_r x)) = 0 \quad \Rightarrow \quad \log_q(\log_r x) = b \quad \Rightarrow \quad \log_r x = q \quad \Rightarrow \quad x = r^q$ ......(i)

and $\log_q(\log_r(\log_p x)) = 0$

$\Rightarrow \quad \log_r(\log_p x) = 1 \quad \Rightarrow \quad \log_p x = r \quad \Rightarrow \quad x = p^r$ ......(ii)

from (i) and (ii) $p^r = r^q$

$\Rightarrow \quad p = r^{q/r}$

E-8. $\log_{10}\pi$ is quantity lie between 0 to 1.

E-9. $\log_{10}(\log_2 3\, .\, \log_3 4\, .\, \log_4 5 ....\log_{1023} 1024) = \log_{10}(\log_2 1024) = \log_{10}(\log_2 2^{10}) = \log_{10}(10) = 1$

Section (F) :

F-1. $2\log_{10}x - \log_{10}(2x - 75) = 2 \quad \Rightarrow \quad \dfrac{x^2}{2x-75} = 10^2 = 100$

$\Rightarrow \quad x^2 - 200x + 7500 = 0 \quad \Rightarrow \quad x = 50,\ x = 150$

sum = 200

F-2. Take $\log_5 x = t$

on solving we get $x = 1/625$ & $x = 5$.

$\log_5 x = t$

$x = 1/625$ & $x = 5$.

F-3. Take $\log_x 5 = t$ on solving we get $x = \sqrt[3]{5}$ and 5.

$\log_x 5 = t$

F-4. $\log_p \log_p (p)^{\frac{1}{p^n}} = \log_p\left(\dfrac{1}{p}\right)^n = -\log_p p^n = -n$ independent of p.

F-5. $\log_x \log_{18}\left(\sqrt{2}+2\sqrt{2}\right) = \dfrac{1}{3} \quad \Rightarrow \quad \log_x \log_{18}\left(\sqrt{18}\right) = \dfrac{1}{3} \quad \Rightarrow \quad \log_x \dfrac{1}{2} = \dfrac{1}{3}$

$\Rightarrow \quad x^{1/3} = \dfrac{1}{2} \quad \Rightarrow \quad x = \dfrac{1}{8} \quad \Rightarrow \quad 1000\,x = 125.$

F-6. $\sqrt{\log_{10}(-x)} = \log_{10}|x| \quad \Rightarrow \quad -x > 0 \quad \Rightarrow \quad x < 0$

$\therefore \quad |x| = -x \quad \Rightarrow \quad \sqrt{\log_{10}(-x)} = \log_{10}(-x)$

$\log_{10}(-x)\left(\log_{10}(-x) - 1\right) = 0$

$\log_{10}(-x) = 0 \qquad \log_{10}(-x) = 1$

$\Rightarrow \quad -x = 1 \qquad -x = 10$

$x = -1 \qquad x = -10.$

F-7. Clearly Domain is $x > 0$ and $x \neq 1$

Section (G) :

G-1. $\log_{\sin\frac{\pi}{3}}(x^2 - 3x + 2) \geq 2 \quad \Rightarrow x^2 - 3x + 2 \leq \left(\dfrac{\sqrt{3}}{2}\right)^2 \Rightarrow 4x^2 - 12x + 8 \leq 3$

$\Rightarrow \quad 4x^2 - 12x + 5 \leq 0 \quad \Rightarrow \quad (2x - 5)(2x - 1) \leq 0 \quad \Rightarrow \quad x \in \left[\dfrac{1}{2}, \dfrac{5}{2}\right]$

But domain $x^2 - 3x + 2 > 0 \quad \Rightarrow \quad (x - 1)(x - 2) > 0 \quad \Rightarrow \quad x \in (-\infty, 1) \cup (2, \infty)$

Hence $x \in \left[\dfrac{1}{2}, 1\right) \cup \left(2, \dfrac{5}{2}\right]$

G-2. $\log_{0.3}(x - 1) < \log_{0.09}(x - 1) \quad ; \quad \log_{0.3}(x - 1) < \dfrac{\log_{0.3}(x-1)}{2}$

$\Rightarrow \quad \log_{0.3}(x - 1) < 0 \Rightarrow \quad x - 1 > 1 \quad \Rightarrow \quad x > 2$

G-3. $2 - \log_2(x^2 + 3x) \geq 0 \quad \Rightarrow \quad \log_2(x^2 + 3x) \leq 2$

$x^2 + 3x > 0 \quad \Rightarrow \quad x \in (-\infty, -3) \cup (0, \infty)$ .....(i)

and $\quad x^2 + 3x \leq 4$

$\Rightarrow \quad (x - 1)(x + 4) \leq 0 \quad \Rightarrow \quad x \in [-4, 1]$ .....(ii)

(i) $\cap$ (ii) $\Rightarrow x \in [-4, -3) \cup (0, 1]$

G-4. $\log_{0.5}\log_5(x^2 - 4) > \log_{0.5} 1; \qquad \log_{0.5}\log_5(x^2 - 4) > 0$

$\Rightarrow \quad x^2 - 4 > 0 \quad \Rightarrow \quad x \in (-\infty, -2) \cup (2, \infty)$ ....(i)

$\log_5(x^2 - 4) > 0 \quad \Rightarrow \quad x^2 - 5 > 0$

$\Rightarrow \quad x \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}, \infty)$ ....(ii)

$\log_5(x^2 - 4) < 1$

$\Rightarrow \quad x^2 - 9 < 0 \quad \Rightarrow \quad x \in (-3, 3)$ ....(iii)

(i) $\cap$ (ii) $\cap$ (iii) $\quad \Rightarrow \quad x \in (-3, \sqrt{5}) \cup (\sqrt{5}, 3)$

G-5. $\left(\frac{1}{2}\right)^{x^2-2x} < \left(\frac{1}{2}\right)^{2} \Rightarrow x^2-2x > 2 \Rightarrow x^2-2x-2 > 0 \Rightarrow x \in (-\infty, 2-\sqrt{3}) \cup (2+\sqrt{3}, \infty)$

G-6. $\log_2 (4^x - 2.2^x + 17) > 5$

$4^x - 2.2^x + 17 > 0$

$(2^x)^2 - 2.2^x + 17 > 0 \Rightarrow \forall x \in R$ and $4^x - 2.2^x + 17 > 32$

$\Rightarrow (2^x)^2 - 2.2^x - 15 > 0 \Rightarrow (2^x + 3)(2^x - 5) > 0 \Rightarrow 2^x < -3$ or $2^x > 5$

$\Rightarrow x \in \phi$ or $x > \log_2 5 \Rightarrow x \in (\log_2 5, \infty)$

G-7. $\log_{1-x}(x-2) \geq -1$

$x > 2$ ....(1)

(i) When $0 < 1-x < 1 \Rightarrow 0 < x < 1$. So no common range comes out.

(ii) When $1-x > 1 \Rightarrow x < 0$ but $x > 2$

here, also no common range comes out. , hence no solution. Finally, no solution

---

PART - III

1. (A) The set $\{3^{2n} - 8n - 1 : n \in N\}$ contains 0 and every element of this set is a multiple of 64.

(B) $2^{3n} - 1$ is always divisible by 7.

(C) $3^{2n} - 1$ is always divisible by 8.

(D) $2^{2n} - 7n - 1$ is always divisible by 49 and $2^{3n} - 7n - 1 = 0$ for n = 1.

2. (A) $a = 3 \quad ((\sqrt{7}+1) - (\sqrt{7}-1))$

$a = 3(2) = 6$

$b = \sqrt{(42)(30)+36} = 6\sqrt{7 \times 5 + 1} = 6 \times 6 = 36$

$\log_a b = \log_6 36 = 2$

(B) $a = (\sqrt{3}+1) - (\sqrt{3}-1) = 2$

$b = (3+\sqrt{2}) - (3-\sqrt{2}) = 2\sqrt{2}$

$\log_a b = \log_2(2\sqrt{2}) = \log_2(2^{3/2}) = 3/2$

(C) $a\sqrt{3+2\sqrt{2}} = (\sqrt{2}+1)$ = , $b = \sqrt{3-2\sqrt{2}} = (\sqrt{2}-1) = \frac{1}{\sqrt{2}+1}$

$\log_a b = \log_{\sqrt{2}+1}(\sqrt{2}+1)^{-1} = -1$

(D) $a = \sqrt{7+\sqrt{7^2-1}} = \sqrt{7+\sqrt{48}} = \sqrt{7+4\sqrt{3}} = (2+\sqrt{3})$

$b = 2-\sqrt{3} = \frac{1}{2+\sqrt{3}} = (2+\sqrt{3})^{-1}$ Now $\log_a b = \log_{2+\sqrt{3}}(2+\sqrt{3})^{-1}$ $\log_{2+\sqrt{3}} = -1$

(E) First 20 prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, ...

zero are formed by multiple of 2 and 5 that is one times 2 and one times 5 there fore one zero at end of product of first 20 prime numbers.

(F) $2^{2x} - 3^{2y} = 55$, x, y $\in I \Rightarrow 4^x - 9^y = 55$

only x = 3, y = 1 satisfy

There fore number of solution is one set (x, y) g.e. (3, 1)

3. (A) x = 0.363636......

100x = 36.363636.......

99x = 36

$\Rightarrow x = \frac{36}{99} = \frac{4}{11}$ sum of numerator and denominator is 4 + 11 = 15

(C) $\frac{1}{\log_a 8} + \frac{1}{\log_b 8} = \frac{1}{\log_a 8.\log_b 8} \Rightarrow \log_b 8 + \log_a 8 = 1$ (given $\log_a b = 3$)

$\Rightarrow 4\log_b 8 = 1 \qquad \log_b 8 = 3\log_8 a$ ....(1)

$\Rightarrow \log_b 8 = 1/4 \qquad \log_a 8 = 3\log_b 8$

$8 = b^{1/4} \Rightarrow b = 8^4$

$\log_8(8^4) = 3\log_8 a \Rightarrow \log_8 a = \frac{4}{3} \Rightarrow a = (8^{4/3}) = (2^3) = 2^4 = 16$

4. (A) $\text{Antilog}_{27}(0.\bar{6}) = x$ ⇒ $0.\bar{6} = \log_{27}x = \dfrac{2}{3}$ ⇒ $x = (27)^{2/3} = (3^3)^{2/3} = 9$

(B) Since $2^{10} < 2008 < 2^{11}$ ⇒ $\log_2(2^{10}) < \log_2 2008 < \log_2(2^{11}) \Rightarrow 10 < \log_2 2008 < 11$

(C) $\log_e 2.\log_b 625 = \log_{10}16.\log_e 10$ ⇒ $\log_e 2.\log_b 265 = \log_e 16$ ⇒ $\log_b 625 = \log_2 16 = 4$

⇒ $625 = b^4$

(D) $x = \left(\dfrac{5}{6}\right)^{100} = \left(\dfrac{10}{2\times 6}\right)^{100} = \left(\dfrac{10}{2^2.3}\right)^{100}$

$\log_{10}x = 10(1 - 2\log 2 - \log 3) = 100(1 - 2(0.3010) - 0.4771) = 100(-0.0791) = 7.91$

EXERCISE–2

PART - I

1.✍ $A_1 \cup A_2 \cup A_3$ is the smallest element containing subset of all we set A_1, A_2 and A_3

2. 1. $((A \cap B) \cup C)' \cap B')'$

$= (A \cap B) \cup C) \cup B$

$= (A \cap B) \cup B \cup C$

$= B \cup C \neq B \cap C$

2. $(A' \cap B') \cap (A \cup B \cup C')$

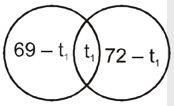
$= (A \cup B)' \cap ((A \cup B) \cup C')$

$= \phi \cup ((A \cup B)' \cap C')$

$= ((A \cup B) \cup C)'$

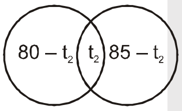
$= (A \cup (B \cup C))'$

3.✍

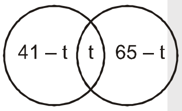


$70 + 72 - t_1 = 100$

$t_1 = 42\%$ ⇒ min. in $P \cap C = 42\%$



$t_2 = 85\% - 20\% = 65\%$ ⇒ min. $M \cap E = 65\%$



$t = 42 - 35 = 7\%$

min. in $((P \cap C) \cap (M \cap E)) = 7\%$

4. $X \cap (Y \cup X)' = X \cap (Y' \cap X') = X \cap X' \cap Y' = \phi$

⇒ Statement – 1 true.

$X \Delta Y = (X \sim Y) \cup (Y \sim X) = (X \cup Y) \sim (X \cap Y)$ ⇒ number of element in $X \Delta Y = m - n$.

⇒ Statement-2 is true but does explain statement-1

5. $\dfrac{6x^2 - 5x - 3}{x^2 - 2x + 6} - 4 \le 0$ ⇒ $\dfrac{2x^2 + 3x - 27}{x^2 - 2x + 6} \le 0$

denominator $x^2 - 2x + 6 > 0$ $\forall$ $x \in R(\because D < 0)$

then $2x^2 + 3x - 27 \le 0$ ⇒ $(2x + 9)(x - 3) \le 0$

$-\dfrac{9}{2} \le x \le 3$ ⇒ $0 \le x^2 \le \dfrac{81}{4}$

$(4x^2)_{max} = 4\left(-\dfrac{9}{2}\right)^2 = 81$ ⇒ $(4x^2)_{min} = 4(0) = 0$

6. $x^2 - 16 \ge 0$

$\therefore$ $(x - 4)(x + 4) \ge 0$

$\therefore$ $x \in (-\infty, -4] \cup [4, \infty)$ ....(1)

Now $\dfrac{(x^2+2)(\sqrt{x^2-16})}{(x^4+2)(x-3)(x+3)} \le 0$

$x \in (-3, 3)$ ....(2)

By (1) and (2) $x \in \{-4, 4\}$

7. $b = a^2$, $c = b^2$, $\dfrac{c}{a} = 3^3$ $\Rightarrow$ $c = 27a$

$\Rightarrow$ $b^2 = 27a$

$\Rightarrow$ $a^4 = 27a$

$\Rightarrow$ $a = 3, a > 0$

$c = 81, b = 9$

$\therefore$ $a + b + c = 3 + 9 + 81 = 93$

8. $x = \left(-\log_3 5\right)\left(\log_{5^3}(7^3)\right)\left(\log_{7^2}(3^6)\right)$

$x = -\dfrac{3}{3}.\dfrac{6}{2}\log_3 5 \,.\, \log_5 7 \,.\, \log_7 3$ and और $y = 25^{\left(3\log_{17^2} 11\right)\left(\log_{28} 17^{1/2}\right)\left(\log_{(11)^3}(28)^2\right)}$

$y = 25^{\frac{3}{2}.\frac{1}{2}.\frac{2}{3}(\log_{17} 11).(\log_{28} 17)(\log_{11} 28)}$

$y = 5$

$\therefore x^2 + y^2 = (-3)^2 + 5^2 = 34$

9. $\dfrac{(x+1)+3x-x\log_2 8}{(x-1)(1)} = \dfrac{(x+2)+3x-3x}{(x-1)} = \dfrac{x+1}{x-1}$

10. $a^{(\log_3 7)^2} = \left(a^{\log_3 7}\right)^{\log_3 7} = 27^{\log_3 7} = 27^{\log_3 7} = 7^3 = 343$

$b^{(\log_7 11)^2} = \left(b^{\log_7 11}\right)^{\log_7 11} = 49^{\log_7 11} = 11^{\log_7 49} = 121$

$c^{(\log_{11} 25)^2} = \left(c^{\log_{11} 25}\right)^{\log_{11} 25} = \left(\sqrt{11}\right)^{\log_{11} 25} = 25^{\log_{11}\sqrt{11}} = 5$

hence the sum is 343 + 121 + 5 = 469

11. For $0 < x < y < 1$

$f(x) = x(\alpha - x) = -x^2 + x$ $f(x) < f(1)$

so f(x) should be increasing in (0, 1) and one root is 0 so vertex should be ≥ 1

$\Rightarrow$ $\dfrac{-\alpha}{-2} \ge 1, \alpha \ge 2$

12. $\dfrac{\sqrt{(x-8)(2-x)}}{\log_{0.3}\left(\dfrac{10}{7}(\log_2 5 - 1)\right)} \ge 0$

For $\sqrt{(x-8)(2-x)}$ to be defined

(i) $(x-8)(2-x) \ge 0$

$(x-2)(x-8) \le 0$ $\Rightarrow$ $2 \le x \le 8$

Now Let say $y = \log_{0.3}\dfrac{10}{7}(\log_2 5 - \log_2 2) = \log_{0.3}\dfrac{10}{7}(\log_2 5/2)$

Let $y < 0$ (assume) then $\log_{0.3}\dfrac{10}{7}(\log_2 5/2) < 0$

$\Rightarrow$ $\dfrac{10}{7}\log_2 5/2 > 1$ $\Rightarrow$ $\log_2 5/2 > \dfrac{7}{10}$ $\Rightarrow$ $\dfrac{5}{2} > 2^{(7/10)}$ which is true

So $y < 0$

so denominator is – ve and numerator is +ve, so inequality is not satisfied,

thus $\sqrt{(x-8)(2-x)} = 0$

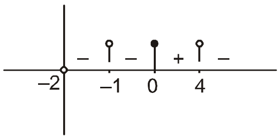
$x = 2, 8$ .....(i)

Now $2^{x-3} > 31$

$\Rightarrow$ $(x - 3) > \log_2 31 \Rightarrow x > 3 + \log_2 2^{4.9}$ (approx)

$\Rightarrow$ $x > 7.9 \Rightarrow x = 8$

13✍. $$\frac{4^x\left(\left(\frac{3}{4}\right)^x - 1\right)\ell n\,(x+2)}{(x-4)\,(x+1)} \le 0$$



14. $\sqrt{\log_4\{\log_3\{\log_2(x^2-2x+a)\}\}}$

for defined $\log_4\log_3\log_2(x^2 - 2x + a) \ge 0$

$\Rightarrow$ $\log_3\log_2(x^2 - 2x + a) \ge 1 \Rightarrow$ $\log_2(x^2 - 2x + a) \ge 3$

$\Rightarrow$ $x^2 - 2x + a \ge 8$ $\Rightarrow$ $x^2 - 2x + (a - 8) \ge 0$

$\Rightarrow$ $D \le 0$

$4 - 4(a - 8) \le 0$ $\Rightarrow$ $1 - a + 8 \le 0$

$\Rightarrow$ $a \ge 9$

15. domain $x > -\frac{3}{2}$

$\log_{2x+3}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

$\log_{(2x+3)}(2x + 3)(3x + 7) = 4 - \log_{3x+7}(2x + 3)^2$

$1 + \log_{2x+3}(3x + 7) = 4 - 2\log_{3x+7}(2x + 3)$

$\log_{2x+3}(3x + 7) = y$

$y + \frac{2}{y} - 3 = 0$ $\Rightarrow$ $y = 1$ or $y = 2$ $\Rightarrow$ $x = -2, , \frac{-1}{4} - 4$

$x \ne -2, -4$ so $x = -\frac{1}{4}$

PART - II

1.✍ $n(A \cup B) = 280$

Now $n(A' \cap B') = n(A \cup B)' = 2009 - n(A \cup B) = 2009 - 280 = 1729 = 12^3 + 1^3 = 10^3 + 9^3$

2.✍ $n(A - B) = 1681 - 1075 = 606 = 4 + 2 \times 301 = 4 + 2 \times 7 \times 43 = (2)\ 2 + 2 \times 7 \times 43$

3. For $A \cap B$

$x^3 + (x-1)^3 = 1$ $\Rightarrow$ $x^3 + x^3 - 3x^2 + 3x - 1 = 1$ $\Rightarrow$ $2x^3 - 3x^2 + 3x - 2 = 0 \Rightarrow (x-1)(2x^2 - x + 2) = 0$

$\Rightarrow$ $x = 1$ $\Rightarrow$ $y = 0$ $\Rightarrow$ $(x, y) = (1, 0)$

For $A \cap C$

$x^3 + (1 - x)^3 = 1 \Rightarrow x^3 + 1 - 3x + 3x^2 - x^3 = 1$ $\Rightarrow$ $x^2 - x = 0 \Rightarrow$ $x = 0, 1 \Rightarrow (x, y) = (0, 1)\ (1, 0)$

4. $n(M) = 23,\ n(P) = 24,\ n(C) = 19$

$n(M \cap P) = 12,\ n(M \cap C) = 9,\ n(P \cap C) = 7$

$n(M \cap P \cap C) = 4$

$n\,(M \cap P' \cap C') = n[M \cap (P \cup C)'] = n(M) - n(M \cap (P \cup C)) = n(M) - n[(M \cap P) \cup (M \cap C)]$

$= n(M) - n(M \cap P) - n(M \cap C) + n(M \cap P \cap C) = 23 - 12 - 9 + 4 = 27 - 21 = 6$

$n(P \cap M' \cap C) = n[P \cap (M \cup C)'\,]$

$= n(P) - n[P \cap (M \cup C) = n(P) - n\,[P \cap M) \cup (P \cap C)] = n(P) - n(P \cap M) - n(P \cap C) + n(P \cap M \cap C)$

$= 24 - 12 - 7 + 4 = 9$

$n(C \cap M' \cap P') = n(C) - n(C \cap P) - n(C \cap M) + n(C \cap P \cap M)$

$= 19 - 7 - 9 + 4 = 23 - 16 = 7$

5. $c(a-b) = a(b-c) \quad \Rightarrow \quad ac - bc = ab - ac \quad \Rightarrow \quad 2ac = ab + bc$

$\Rightarrow \quad \frac{2ac}{b} = a + c \quad \Rightarrow \quad \frac{2ac}{a+c} = b$

Now $\frac{\log(a+c)+\log(a+c-2b)}{\log(a-c)} = \frac{\log(a+c)+\log\left(a+c-\frac{4ac}{a+c}\right)}{\log(a-c)}$

$= \frac{\log(a+c)+2\log(a-c)-\log(a+c)}{\log(a-c)} = 2$

6. $\log_b a \,.\, \log_c a + \log_a b \,.\, \log_c b + \log_a c \,.\, \log_b c = 3$

$\Rightarrow \quad \frac{(\log a)^2}{\log b.\log c} + \frac{(\log b)^2}{\log a.\log c} + \frac{(\log c)^2}{\log a.\log b} = 3$

$\Rightarrow \quad (\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a.\log b.\log c$

$\Rightarrow \quad \log a + \log b + \log c = 0$ ($\because$ a, b, c are distinct)

$\Rightarrow \quad \log abc = 0 \quad \Rightarrow \quad abc = 1$

7. $A = \log_{16}4 = \frac{1}{2}$

$B = \log_3 9 = 2$

$\therefore \quad 4^{1/2} + 9^2 = 10^{\log_x 83} \quad \Rightarrow \quad 83 = 83^{\log_x 10} \quad \Rightarrow \quad x = 10$

8. $b^2 = a^3 = k \quad \Rightarrow \quad b = k^{1/2}, a = k^{1/3}$ and $d^4 = c^5 = \lambda \quad \Rightarrow \quad c = \lambda^{1/5}, d = \lambda^{1/4}$

Now $a - c = 9 \quad \Rightarrow \quad k^{1/3} - \lambda^{1/5} = 9$

Let $k^{1/3} = 25, \quad \lambda^{1/5} = 16 \quad \Rightarrow \quad k = 25^3 = 5^6$ and $\lambda = 16^5 = 2^{20}$

$\Rightarrow \quad b = (5^6)^{1/2} = 125, d = (2^{20})^{1/4} = 32$

Now $\quad a - c = 9 \quad \Rightarrow \quad k^{1/3} - \lambda^{1/5} = 9$

let $\quad k^{1/3} = 25, \lambda^{1/5} = 16$

$\Rightarrow \quad b = (5^6)^{1/2} = 125, d = (2^{20})^{1/4} = 32$

Now $b - d = 125 - 32 = 93$

$\frac{b+d}{a+c} = \frac{157}{41}$ = **03.829**

9. $\log_{10}(2x^2 - 21x + 50) = 2$

(i) $\quad 2x^2 - 21x + 50 = 100$

$\Rightarrow 2x^2 - 21x - 50 = 0 \quad \Rightarrow \quad x = -2, \frac{25}{2}$

10. Domain $x - 1 > 0$ and $x + 1 > 0$ and $y - x > 0$

$x > 1 \qquad x > -1 \qquad x < 7$

$\Rightarrow \quad x \in (1, 7)$ ....(i)

$-\log_2(x-1) - \log_2(x+1) = 1 + \log_{2^{-\frac{1}{2}}}(7-x)$

$-\log_2(x^2-1) + \log_2(7-x)^2 = 1 \quad ; \quad \log_2 \frac{(7-x)^2}{x^2-1} = 1 \Rightarrow \frac{(7-x)^2}{x^2-1} = 2$

$\Rightarrow \quad x^2 + 14x - 51 = 0 \quad ; \quad (x+17)(x-3) = 0$

$x = 3$ or $x = -17$ (rejected) ; $x = 3$

11. $\log^2_{10} x + \log_{10}x^2 = \log^2_{10} 2 - 1 \quad ; \quad \log^2_{10} x + 2\log_{10}x + 1 = \log^2_{10} 2$

$\Rightarrow \quad (\log_{10}x + 1)^2 = \log_{10}^2 2$

$\log_{10}x + 1 = \pm \log_{10} 2 \quad ; \quad x = \frac{1}{20}$ and $\frac{1}{5}$

12. $\frac{2009}{2010}x = (2009)^{\log_x(2010)}$

$\Rightarrow$ Taking log both sides w.r.t. x

$\log_x 2009 + 1 - \log_x 2010 = \log_x 2010.\log_x 2009 \quad \Rightarrow \frac{\ell n2009}{\ell nx} + 1 - \frac{\ell n2010}{\ell nx} = \frac{\ell n2009\ell n2010}{(\ell nx)^2}$

$\Rightarrow \quad \ell n\left(\frac{2009}{2010}\right).\ell nx + (\ell nx)^2 = \ell n2009\ \ell n2010$

Let $\ell nx = t$, $t^2 + \ell n\left(\frac{2009}{2010}\right)x - \ell n2009\ \ell n2010 = 0$

sum of roots $= -\ell n\left(\frac{2009}{2010}\right) \quad \Rightarrow \quad \ell nx_1 + \ell nx_2 = -\ell n\left(\frac{2009}{2010}\right) = \ell n\left(\frac{2010}{2009}\right)$

$\Rightarrow \quad x_1x_2 = \frac{2010}{2009}$

m = 2010, n = 2009 $\quad \Rightarrow \quad$ m – n = 1

13. $(\log x)^2 - \log x - 2 \geq 0$

$x > 0$ ....(i)

$(\log x - 2)(\log x + 1) \geq 0$

$\Rightarrow \quad \log x \leq -1$ or $\log x \geq 2$

$\Rightarrow \quad x \leq \frac{1}{10}$ or $x \geq 100$ ....(ii)

(i) ∩ (ii) $\quad \Rightarrow \quad x \in \left(0, \frac{1}{10}\right] \cup [100, \infty)$.

14. $\frac{x+1}{x+2} \quad \Rightarrow \quad x \in (-\infty, -2) \cup (-1, \infty)$

and x + 3 > 0 $\quad \Rightarrow \quad$ x > –3

If $\quad \log_4\left(\frac{x+1}{x+2}\right) > \log_4(x+3)$

$\Rightarrow \quad \frac{x+1}{x+2} > x - 3 \quad \Rightarrow \quad \frac{x+1-(x+3)(x+2)}{(x+1)} > 0$

$\Rightarrow \quad \frac{x+1-x^2-5x-6}{x+1} > 0 \quad \Rightarrow \quad \frac{x^2+4x+5}{x+1} < 0$

$\Rightarrow \quad x > -1$

ans. (–1, ∞) $\quad \Rightarrow \quad$ –a = – 1 $\quad \Rightarrow \quad$ a = –1

If $\log_4\left(\frac{x+1}{x+2}\right) < \log_4(x+3)$

$\frac{x+1}{x+2} < x + 3$ then solution is (–3, –1)

15. $\log_{1/2}(x+5)^2 > \log_{1/2}(3x-1)^2$

$(x+5)^2 > 0 \quad \Rightarrow \quad x \in R - \{-5\}$ ....(i)

$(3x-1)^2 > 0 \quad \Rightarrow \quad x \in R - \left\{\frac{1}{3}\right\}$ ....(ii)

$(x+5)^2 < (3x-1)^2$

$\Rightarrow \quad 8x^2 - 16x - 24 > 0$

$\Rightarrow \quad x^2 - 2x - 3 > 0$

$\Rightarrow \quad (x-3)(x+1) > 0$

$\Rightarrow \quad x \in (-\infty, -1) \cup (3, \infty)$ ....(iii)

(i) ∩ (ii) ∩ (iii) gives

$(-\infty, -5) \cup (-5, -1) \cup (3, \infty)$

p = – 5, q = –5 , r = – 1, s = 3

PART - III

1.

so $a - 1 + a^2 - 2a - 1 = 18$

$a = 5, -4 \quad \therefore \quad a = 5$

2. $N = \dfrac{\log_3 135}{\log_{15} 3} - \dfrac{\log_3 5}{\log_{405} 3} = (\log_3 27 + \log_3 5) - (\log_3 15)\ \log_3 5.\log_3 405$

$= (3 + \log_3 5)\ (1 + \log_3 5) - \log_3 5\ \log_3(81 \times 5) = (3 + \log_3 5)\ (1 + \log_3 5) - \log_3 5(4 + \log_3 5) = 3$

3.✍ $(\log_5 x)^2 + \log_{5x} \dfrac{5}{x} = 1$

$\Rightarrow \quad (\log_5 x)^2 + \log_{5x} 5 - \log_{5x} x = 1$

$\Rightarrow \quad (\log_5 x)^2 + \dfrac{\log_5 5}{\log_5 5 + \log_5 x} - \dfrac{\log_5 x}{\log_5 5 + \log_5 x} = 1$

$\Rightarrow \quad (\log_5 x)^2 + \dfrac{1}{1 + \log_5 x} - \dfrac{\log_5 x}{1 + \log_5 x} = 1$

Let $\log_5 x = t$

$\therefore \quad t^2 + \dfrac{1}{1+t} - \dfrac{t}{1+t} = 1 \quad \Rightarrow \quad \dfrac{t^2(1+t) + 1 - t}{1+t} = 1 \quad \Rightarrow \quad t^3 + t^2 + 1 - t = 1 + t$

$t^3 + t^2 - 2t = 0 \quad ; \quad t(t^2 + t - 2) = 0 \quad ; \quad t(t - 1)(t + 2) = 0$

$t = 0, 1, -2$

$\therefore \quad \log_5 x = 0, 1, -2$

$\therefore \quad x = 1, 5, \dfrac{1}{25}$

4. $\log_{x^2} 16 + \log_{2x} 64 = 3 \quad \Rightarrow \quad 4\ \log_{x^2} 2\ + 6 \log_{2x} 2 = 3$

$\Rightarrow \quad \dfrac{4}{\log_2 x^2} + \dfrac{6}{\log_2 2x} = 3 \quad \Rightarrow \quad \dfrac{2}{\log_2 x} + \dfrac{6}{1 + \log_2 x} = 3$ but $\log_2 x = t$

$\therefore \quad \dfrac{2}{t} + \dfrac{6}{1+t} = 3 \quad \Rightarrow \quad 2 + 2t + 6t = 3t + 3t^2$

$\Rightarrow \quad 3t^2 - 5t - 2 = 0 \quad \Rightarrow \quad 3t^2 - 5t - 2 = 0 \ \Rightarrow \quad (3t + 1)(t - 2) = 0$

$\Rightarrow \quad t = -\dfrac{1}{3}, t = 2 \quad \Rightarrow \quad \log_2 x = -\dfrac{1}{3} \quad \log_2 x = 2$

$\Rightarrow \quad x = 2^{-1/3} \qquad x = 4 = \dfrac{1}{2^{1/3}}.$

5.✍ $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3} \quad \Rightarrow \quad (\log_3 x)^3 - \dfrac{9}{2}\ \log_3 x + 5 = \log_x 3\ \sqrt{3}$

$\Rightarrow \quad (\log_3 x)^2 - \dfrac{9}{2}\log_3 x + 5 = \dfrac{3}{2}\log_x 3$

Let $\log_3 x = t \quad \Rightarrow \quad t^2 - \dfrac{9}{2}t + 5 = \dfrac{3}{2t} \quad \Rightarrow \quad 2t^3 - 9t^2 + 10t - 3 = 0$

$t = 1$ satisfies it $\qquad 2t^3 - 9t^2 + 10t - 3 = 2t^2(t - 1) - 7t(t - 1) + 3(t - 1)$

$= (t - 1)\left(2t^2 - 7t + 3\right) = (t - 1)(2t - 1)(t - 3) \quad \Rightarrow \quad t = 1 \quad t = \dfrac{1}{2} \quad t = 3$

$\Rightarrow \quad \log_3 x = 1 \qquad \log_3 x = \dfrac{1}{2} \qquad \log_3 x = 3$

$\Rightarrow \quad x = 3 \qquad x = 3^{1/2} \qquad x = 27.$

6. $\log_3 x + \log_3 y = 2 + \log_3 2,$ $\log_{27}(x + y) = \frac{2}{3}$

$\Rightarrow$ $\log_3 xy = \log_3 9 + \log_3 2,$

$\Rightarrow$ $x + y = (27)^{2/3}$

$\Rightarrow$ $xy = 18\ ,$

$\Rightarrow$ $x + y = 9$

$\Rightarrow$ $x = 6$ or $x = 3$ $y = 3, y = 6$ as $x > 0, y > 0.$

7. $(\log_{10}8)x^2 - (\log_{10}5)x + x - 2\log_{10}2 = 0$

(A) sum of roots $= \frac{-(1-\log_{10}5)}{\log_{10}8} = \frac{-\log_{10}\left(\frac{10}{5}\right)}{\log_{10}8} = \frac{-\log_{10}2}{3\log_{10}2} = \frac{-1}{3}$ rational

(B) Product of roots $= \frac{-2\log_{10}2}{\log_{10}8} = \frac{-2}{3}$

(C) sum of cofficient $= \log_{10}8 - \log_{10}5 + 1 - \log_{10}4 = \log_{10}\left(\frac{8\times 10}{5\times 4}\right)$

$= \log_{10}4$ = irrational

(D) Dirscriminant $= (\log_{10}2)^2 - 4\log_{10}8(-2\log_{10}2) = (\log_{10}2)^2 + 24(\log_{10}2)^2$

$= 25(\log_{10}2)^2 = (5\log_{10}2)^2$ irrational.

8.✍ $x = a^b$

(D) If a is rational & b is rational then x may be rational

e.g. $= 2^2$

(C) $(\sqrt{2})^4$

(B) $(2)^{\log_2 3}$

(A) $(\sqrt{2})^{\log_2 9}$

9. $\log_2 3 > 1,\ \log_{12}10 < 1$ $\Rightarrow$ $\log_2 3 > \log_{12}10$

$\log_6 5 < 1,\ \log_7 8 > 1$ $\Rightarrow$ $\log_6 5 < \log_7 8$

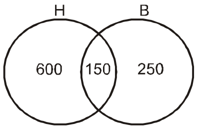
$\log_3 26 < 3, \log_2 9 > 3$ $\Rightarrow$ $\log_3 26 < \log_2 9$

$\log_{16}15 < 1, \log_{10}11 > 1$ $\Rightarrow$ $\log_{16}15 < \log_{10}11$

10. $\frac{1}{2} \le \log_{1/10} x \le 2 \Rightarrow$ $\frac{1}{100} \le x \le \frac{1}{\sqrt{10}}$

PART - IV

Sol. **(Q 1 to 3)**



$n(H \cup B) = n(H) + n(B) - n(H \cap B)$

$1000 = 750 + 400 - n(H \cap B) = 150$

Now $n(\text{only hindi}) = n(H) - n(H \cap B) = 750 - 150 = 600$

$n(\text{only bengali}) = n(B) - n(H \cap B)$

$400 - 150 = 250$

Sol. (Q 4 to 6) Let $\log_4 x = t$ $\Rightarrow$ $1 + t + 4(5 - 4t) = 3(5 - 4t)(1 + t)$

$21 - 15t = 15 + 3t - 12t^2$

$\Rightarrow$ $12t^2 - 18t + 6 = 0$ $\Rightarrow$ $2t^2 - 3t + 1 = 0$

$t = 1, t = 1/2$ $\Rightarrow$ $x = 4'$ or

$x = 4^{1/2} = 2$

A = sum of roots = 4 + 2 = 6

for B: $2^m = 3$, $3^n = 4$ $\Rightarrow$ $m = \log_2 3$, $\Rightarrow$ $n = \log_3 4$

$\Rightarrow$ Now $mn = \log_2 3.(\log_3 2^2) = 2$ B = 2

for C: $\frac{1-\log_3 x}{1+\log_3 x} + (\log_3 x)^2 = 1$ $\Rightarrow$ $1 - t + t^2(1 + t) = 1 + t$

$\Rightarrow$ $t^2 + t^3 - t = t$

$t^3 + t^2 - 2t = 0$ $\Rightarrow$ $t(t^2 + t - 2) = 0$ $\Rightarrow$ $t = 0, t = -2, t = 1$

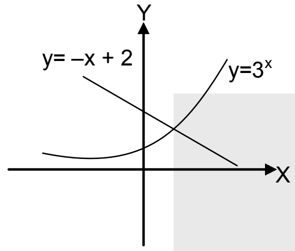
$x = 3^t$ $\Rightarrow$ $x = 3^{\circ} = 1$, $x = 3' = 3$ $x = 3^{-2} = 1/9$

sum of integral root = C = 1 + 3 = 4

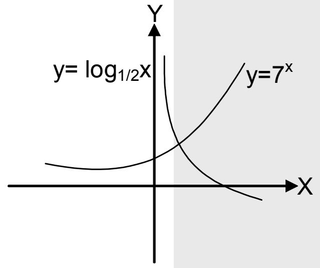
(4) A + B = 6 + 2 = 8 (5) B + C = 2 + 4 = 6

(6) A + C ÷ B = 6 + 4 ÷ 2 = 6 + 2 = 8

8.



9.



EXERCISE # 3

PART - I

1. Obvious

2. Obvious

3. $2 \log_2 \log_2 x + \log_{1/2} \log_2\left(2\sqrt{2}\,x\right) = 1$ $\Rightarrow$ $\log_2 (\log_2 x)^2 - \log_2 \log_2 \left(2\sqrt{2}\,x\right) = 1$

$\Rightarrow$ $\log_2 \frac{(\log_2 x)^2}{\log_2\left(2\sqrt{2}\,x\right)} = 1$ $\Rightarrow$ $\frac{(\log_2 x)^2}{\frac{3}{2} + \log_2 x} = 2$

let $\log_2 x = y$

$\therefore$ $y^2 - 2y - 3 = 0$ $\Rightarrow$ $(y - 3)(y + 1) = 0$

$\therefore$ $y = 3, -1$ $\Rightarrow$ $\log_2 x = 3, -1,$

but $\log_2 x > 0$

$\therefore$ $\log_2 x = -1$ is not possible $\Rightarrow$ $x = 8$

4. $\log_{3/4} \log_8 (x^2 + 7) + \log_{1/2} \log_{1/4} (x^2 + 7)^{-1} = -2$

$\Rightarrow$ $\log_{3/4} \frac{1}{3} \log_2 (x^2 + 7) - \log_2 \frac{\log_2(x^2+7)}{2} = -2$

let $\log_2 (x^2 + 7) = t$

$\Rightarrow \quad \log_{3/4}\frac{t}{3} - \log_2\frac{t}{2} + 2 = 0 \quad \Rightarrow \quad \log_{3/4}\frac{t}{3} + 1 - \left(\log_2\frac{t}{2} - 1\right) = 0$

$\Rightarrow \quad \log_{3/4}\frac{t}{4} = \log_2\frac{t}{4} \quad \Rightarrow \quad \frac{t}{4} = 1 \Rightarrow \quad t = 4$

$\therefore \quad \log_2 (x^2 + 7) = 4$

this gives $x = \pm 3$

5. $\frac{1}{2}\log_2(x - 1) = \log_2 (x - 3) \quad \Rightarrow \quad \sqrt{x-1} = x - 3$

$(x - 1) = x^2 - 6x + 9 \quad \Rightarrow \quad x^2 - 7x + 10 = 0$

$(x - 5)(x - 2) = 0$ but $x \neq 2$

$\therefore \quad x = 5$

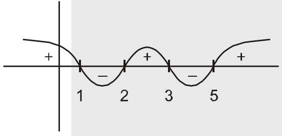
6. Given $f(x) = \frac{x^2-6x+5}{x^2-5x+6}$

(i) when $0 < f(x) < 1$ then $0 < \frac{x^2-6x+5}{(x^2-5x+6)} < 1$

So $\frac{(x-5)(x-1)}{(x-2)(x-3)} > 0$ and $\frac{x^2-6x+5}{x^2-5x+6} - 1 < 0 \quad \Rightarrow \quad \frac{(x+1)}{(x-2)(x-3)} > 0$

$\Rightarrow \quad x \in (-1, 1) \cup (5, 0)$

(ii) when $f(x) < 0$

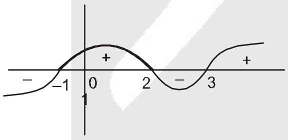


$\frac{(x-1)(x-5)}{(x-2)(x-3)} < 0$

$\Rightarrow \quad x \in (1, 2) \cup (3, 5)$

(iii) $f(x) > 0, x \in (-\infty, 1) \cup (2, 3) \cup (5, \infty)$

(iv) $f(x) < 1 \Rightarrow \quad \frac{x+1}{(x-2)(x-3)} > 0$



$x \in (-1, 2) \cup (3, \infty)$

(A) $-1 < x < 1$, f(x) satisfies p, q, s

(B) $1 < x < 2$, f(x) satisfies q, s

(C) $3 < x < 5$, f(x) satisfies q, s

(D) $x > 5$, f(x) satisfies p, r, s

7.✍ $(2x)^{\ell n2} = (3y)^{\ell n3}$

$\Rightarrow \ell n2 \; \ell n(2x) = \ell n3 \; \ell n(3y) = \ell n3 (\ell n3 + \ell ny)$ .... (1)

also $3^{\ell nx} = 2^{\ell ny} \quad \Rightarrow \quad \ell nx \; \ell n3 = \ell ny \; \ell n2$ .... (2)

by (1) $\Rightarrow \quad \ell n2 \; \ell n(2x) = \ell n3 (\ell n3 + \ell ny) \quad \Rightarrow \quad \ell n\, 2 \,.\, \ell n\,(2x) = \ell n3 \left\{\ell n3 + \frac{\ell nx\ell n3}{\ell n2}\right\}$

$\Rightarrow \quad \ell n^2 2 \; \ell n2x = \ell n^2 3 (\ell n2 + \ell nx) \quad \Rightarrow \left(\ell n^2 2 - \ell n^2 3\right) (\ell n2x) = 0 \quad \Rightarrow \ell n2x = 0 \Rightarrow x = \frac{1}{2}$

8.✍ Let $\sqrt{4 - \frac{1}{3\sqrt{2}}\sqrt{4 - \frac{1}{3\sqrt{2}}......}} = t \quad \Rightarrow \sqrt{4 - \frac{1}{3\sqrt{2}}t} = t \quad \Rightarrow 4 - \frac{1}{3\sqrt{2}}\; t = t^2 \Rightarrow$

$t^2 + \frac{1}{3\sqrt{2}}\ t - 4 = 0 \Rightarrow 3\sqrt{2}\,t^2 + t - 12\sqrt{2} = 0 \quad \Rightarrow \quad t = \frac{-1 \pm \sqrt{1 + 4 \times 3\sqrt{2} \times 12\sqrt{2}}}{2 \times 3\sqrt{2}} = \frac{-1 \pm 17}{2 \times 3\sqrt{2}}$

$t = \frac{16}{6\sqrt{2}}\ ,\ \frac{-18}{6\sqrt{2}} \quad \Rightarrow \quad t = \frac{8}{3\sqrt{2}}, \frac{-3}{\sqrt{2}}$ and $\frac{-3}{\sqrt{2}}$ is rejected

so $6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}} \times \frac{8}{3\sqrt{2}}\right) = 6 + \log_{3/2}\left(\frac{4}{9}\right) = 6 + \log_{3/2}\left(\left(\frac{2}{3}\right)^2\right) = 6 - 2 = 4$

9*. $3^x = 4^{x-1} \quad \Rightarrow \quad x = (x-1)\log_3 4 \quad \Rightarrow \quad x(1 - 2\log_3 2) = -2\log_3 2$

$x = \frac{2\log_3 2}{2\log_3 2 - 1}$ **Ans. (A)**

Again $x\log_2 3 = (x-1)\cdot 2 \Rightarrow x(\log_2 3 - 2) = -2 \Rightarrow x = \frac{2}{2 - \log_2 3}$ **Ans. (B)**

$x = \frac{1}{1 - \frac{1}{2}\log_2 3} = \frac{1}{1 - \log_4 3}$ **Ans.(C)**

10. $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\log_7 4}$

$(\log_2 9)^{2\log_{(\log_2 9)} 2}.(2) = 4.2 = 8$

PART - II

1.	**Obvious**	**2.**	**Obvious**	**3.**	**Obvious**	**4.**	**Obvious**
5.	**Obvious**	**6.**	**Obvious**	**7.**	**Obvious**	**8.**	**Obvious**

9. We have, $A \cup B = A \cup C \quad \Rightarrow \quad (A \cup B) \cap C = (A \cup C) \cap C$

$\Rightarrow (A \cap C) \cup (B \cap C) = C \quad [\because (A \cup C) \cap C = C]$

$\Rightarrow (A \cap B) \cup (B \cap C) = C \quad ...(i) \quad [\because A \cap C = A \cap B]$

Again, $A \cup B = A \cup C$

$\Rightarrow (A \cup B) \cap B = (A \cup C) \cap B \quad \Rightarrow \quad B = (A \cap B) \cup (C \cap B)$

$\Rightarrow (A \cap B) \cup (C \cap B) = B \quad \Rightarrow \quad (A \cap B) \cup (B \cap C) = B \quad ...(ii)$

From (i) and (ii), we get B = C

10. Every element has 3 options. Either set Y or set Z or none

so number of ordered pairs = 3^5

11. $X = \{0, 9, ... 4^n - 3n - 1\}$

$Y = \{0, 9, ..., 9(n-1)\}$

Now $4^n - 3n - 1 = (3+1)^n - 3n - 1$

$= 3^n + n.3^{n-1} + ... + {}^nC_2 .9.$

is a multiple of 9.

Also Y consists of all multiples of '9' from 0, 9,.......

Hence all values of X are subset of values of Y.

Thus $X \cup Y = Y$

12. $(x^2 - 5x + 5)^{x^2+4x-60} = 1$

$x^2 - 5x + 5 = 1$	$x^2 + 4x - 60 = 0$	$x^2 - 5x + 5 = -1$
$x^2 - 5x + 4 = 0$	$x = -10, x = 6$	$x^2 - 5x + 6 = 0$
$x = 1, x = 4$		$x = 2, 3$

at x=2, $x^2 + 4x - 60 = -48$

$\therefore$ x=2 is valid

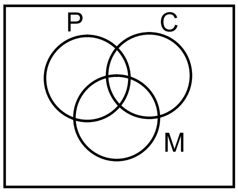
at x=3,$x^2 + 4x - 60 = -39$ (odd)

$\therefore$ x = 3 is invalid

x = 1, 2, 4, 6, –10

13. $n(P) = \left[\dfrac{140}{3}\right] = 46$

$n(C) = \left[\dfrac{140}{5}\right] = 28$



$n(M) = \left[\dfrac{140}{2}\right] = 70$

$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap M \cap C)$

$= 46 + 28 + 70 - \left[\dfrac{140}{15}\right] - \left[\dfrac{140}{10}\right] - \left[\dfrac{140}{6}\right] + \left[\dfrac{140}{30}\right]$

$= 144 - 9 - 14 - 23 + 4 = 102$

so required number of student = 140 – 102 = 38

14. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 25 + 7 - 3$

$= 29$