SOLUTIONS OF ELECTROSTATICS

EXERCISE-1

PART - I

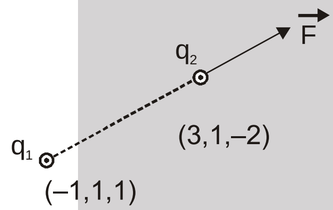
SECTION (A)

A-1. $F = \frac{Kq_1q_2}{x^2}$ = 5400 N, attractive.

$F = \frac{Kq_1q_2}{x^2}$ = 5400 N,

A-2. (a) Distance between the two charges = $x = \sqrt{[3-(-1)]^2 + (1-1)^2 + (-2-1)^2}$ = 5 m

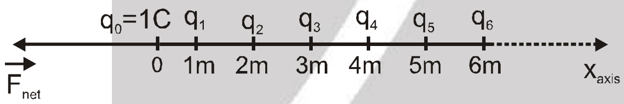
$\therefore$ Force, $|\vec{F}| = \frac{kq_1 \; q_2}{x^2} = \frac{9\times10^9 \times 20\times10^{-6} \times 25\times10^{-6}}{(5)^2}$ = 0.18 N



(b)

Unit vector in direction of $\vec{F}$ is $\frac{[\{3-(-1)\}\hat{i} + 0\hat{j} - 3\hat{k}]}{5} = \frac{4\hat{i} - 3\hat{k}}{5}$

A-3.



$q_1 = 1 \times 10^{-6}$C, $q_2 = 8 \times 10^{-6}$C, $q_3 = 27 \times 10^{-6}$C, ...$q_{20} = 20^3 \times 10^{-6}$ C

$\therefore \; |\vec{F}_{net}| = |\vec{F}_1| + |\vec{F}_2|$ + ...+ $|\vec{F}_{20}|$

$= \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2}$ + ...+ $\frac{kq_0q_{20}}{r_{20}^2}$

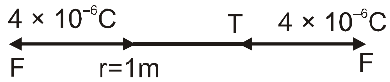
$= kq_0\left[\frac{q_1}{r_1^2} + \frac{q_2}{r_2^2} + \text{...} + \frac{q_{20}}{r_{20}^2}\right]$

$= 9 \times 10^9 \times 1 \left[\frac{1\times10^{-6}}{(1)^2} + \frac{8\times10^{-6}}{(2)^2} + \text{..} + \frac{20^3\times10^{-6}}{(20)^2}\right]$

= 9000 [1 + 2 + 3 + ...+ 20]

= 1890000 = 1.89×10^6 N

A.4. (i)



; $T = F = \frac{kq_1q_2}{r^2} = \frac{9\times10^9 \times (4\times10^{-6})^2}{(1)^2}$ = 0.144 N

(ii) $a = \frac{F}{m} = \frac{144\times10^{-3}}{24\times10^{-3}}$ m/s^2 = 6 m/s^2

A-5. (i) Let charges on the spheres are + q_1 and – q_2. Initially

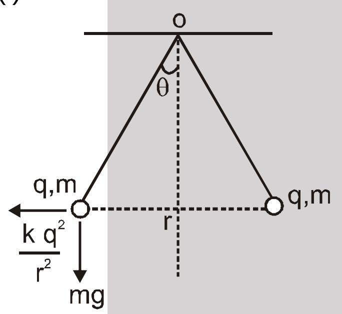
$\therefore$ Force $F_1 = \dfrac{kq_1q_2}{(0.5)^2} = 0.108$ N ......(i)

(ii) After connecting with conducting wire charge on each sphere becomes: $\dfrac{q_1 - q_2}{2}$

$\therefore$ $F_2 = \dfrac{k\left(\dfrac{q_1 - q_2}{2}\right)^2}{(0.5)^2} = 0.036$ N .....(ii)

(iii) on solving (i) and (ii) ; q_1 and q_2 are $\pm 1 \times 10^{-6}$ C and $\mp$ 3×10^{-6} C.

A-6. (i)

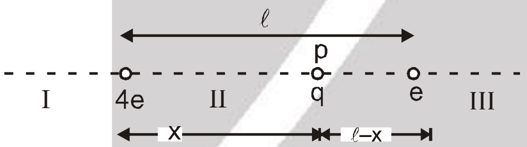


From the given diagram,

$\tan\theta = \dfrac{kq^2}{r^2mg} = \dfrac{9 \times 10^9 \times 10^{-18}}{(0.1 \times 10^{-3}) \times 10 \times 9 \times 10^{-4}} = \dfrac{1}{100}$

$\therefore$ $\theta = \tan^{-1}\left(\dfrac{1}{100}\right) = 0.6^{o}$

A-7.



The charge 'q' cannot be placed in the region I and III for it to be in equilibrium [whether it is positive or negative]

Only region is II, where equilibrium can be attained.

Let charge q is placed at distance 'x' from 4e.

Equating the forces, we get.

$\dfrac{kq(4e)}{x^2} = \dfrac{kq(e)}{(\ell - x)^2}$

or $4 = \left(\dfrac{x}{\ell - x}\right)^2$ or $\dfrac{x}{(\ell - x)} = 2$

$\therefore$ $x = 2\ell - 2x$ or $x = \dfrac{2\ell}{3}$

$\therefore$ Charge q has to be placed at distance $\dfrac{'2\ell'}{3}$ from 4e

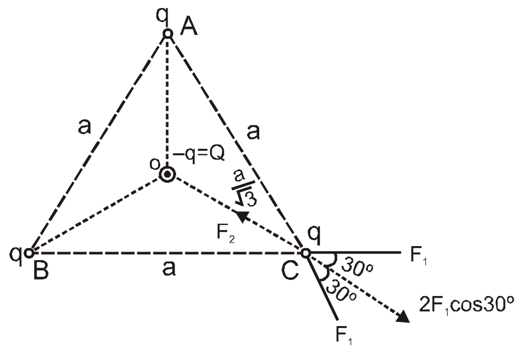
If q is +ve, then on displacing slightly from point P the charge will return back to P

$\therefore$ Stable equilibrium

If q is negative, then on displacing slightly from P, charge will be attracted towards the charge towards which it is displaced.

$\therefore$ Unstable equilibrium

A-8.



Equating the forces on charge q placed at point 'C' we see,

$$F_1 = \frac{kq.q}{a^2} = \frac{kq^2}{a^2}$$

$$\therefore \quad 2F_1 \cos 30^o = 2 \times \frac{kq^2}{a^2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\ kq^2}{a^2}$$

$$F_2 = \frac{k(q)(q)}{\left(\frac{a}{\sqrt{3}}\right)^2} = \frac{3kq^2}{a^2}$$

$\because$ F_2 is stronger than $2F_1 \cos 30^o$, so charge 'q' at C is attracted towards 'O'

Ans. All 3 charges move towards centre 'O'.

(b) The charge 'Q' at centre 'O' is already in equilibrium

Now, for each charge to be in equilibrium let us consider equilibriun (rest) for charge 'q' at 'C'

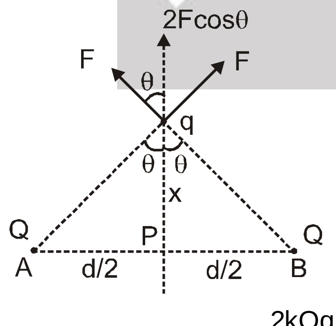
Equating forces: $2F_1 \cos 30^o = F_2$

$$\therefore \frac{\sqrt{3}kq^2}{a^2} = \frac{kQ^2}{\left(\frac{a}{\sqrt{3}}\right)^2} \qquad \therefore \quad \frac{\sqrt{3}kq^2}{a^2} = \frac{3kQq}{a^2} \qquad \therefore \quad Q = \frac{q}{\sqrt{3}}$$

value of Q should be –ve for F_2 to be attractive

$$\therefore \quad Q = -\frac{q}{\sqrt{3}}$$ **Ans.**

A-9.



$$F_{max} = 2\ F\cos\theta = \frac{2kQq}{\left(\sqrt{x^2 + \frac{d^2}{4}}\right)^2} . \frac{x}{\left(\sqrt{x^2 + \frac{d^2}{4}}\right)} = 2\ kQq\ \frac{x}{\left(x^2 + \frac{d^2}{4}\right)^{3/2}}$$

for F_{net} to be max. $\frac{dF_{max}}{dx} = 0$ or $2kQq \frac{d}{dx}\left[\frac{x}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}\right] = 0$

or $$\frac{\left(x^2+\frac{d^2}{4}\right)^{3/2}[1] - x.\frac{3}{2}.\left[x^2+\frac{d^2}{4}\right]^{1/2}.[2x]}{\left(x^2+\frac{d^2}{4}\right)^{3}} = 0$$

or $x^2 + \frac{d^2}{4} - 3x^2 = 0$ or $2x^2 = \frac{d^2}{4}$ $\therefore$ $x = \frac{d}{2\sqrt{2}}$

Ans : $x = \frac{d}{2\sqrt{2}}$

Value of F_{max} $\Rightarrow$ $(F_{net})_{max} = 2kQq \frac{x}{\left(x^2+\frac{d^2}{4}\right)^{3/2}}$ Put the value of x

$\therefore$ $$(F_{net})_{max} = 2kqQ \frac{\frac{d}{2\sqrt{2}}}{\left[\frac{d^2}{8}+\frac{d^2}{4}\right]^{3/2}} = \frac{2kQq.\frac{d}{2\sqrt{2}}}{\left(\frac{3d^2}{8}\right)^{3/2}} = \frac{2kQq}{3\sqrt{3}}.\frac{1}{\left(\frac{d^2}{8}\right)} = \frac{16\ \ k\ \ Qq}{3\sqrt{3}\ \ d^2}$$

$$(F_{net})_{max} = \frac{4Qq}{3\sqrt{3}\pi\varepsilon_0 d^2}$$

Section (B)

B-1. By definition; $E = \frac{F}{q} = \frac{25\times10^{-3}}{5\times10^{-6}} = 5 \times 10^3$ N/C

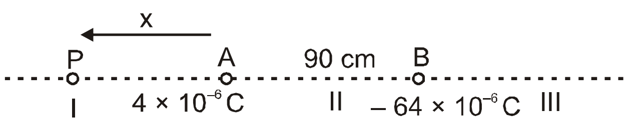
B-2. Let final velocity is $V_x\hat{i} + V_y\hat{j}$

then तो $V_x = v_0$. and व $t = \ell/v_0$.

$V_y = a_y t = \frac{eE\ell}{mv_0}$; where $\left(a_y = \frac{eE}{m}\right)$

$\therefore$ $\tan\theta = \frac{V_y}{V_x} = \frac{eE\ell}{mv_0^2} = \frac{1.6\times10^{-19}\times91\times10^{-6}\times1}{9.1\times10^{-31}\times16\times10^{6}} = 1$ $\Rightarrow$ $\theta = 45^o$

B-3.



The electric field cannot be zero in regions II and III

It can be zero only in regions 'I'.

(i) Let electric field is zero at point 'P' in I at distance 'x' from point A.

$\therefore$ Equating electric fields at point P, due to both charges [x in cm]

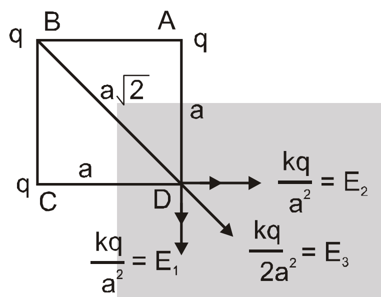
$\therefore$ $$\frac{k(4\times10^{-6})}{x^2} = \frac{k(64\times10^{-6})}{(90+x)^2}$$

$\therefore \quad \left[\frac{90+x}{x}\right]^2 = 16$

$\therefore \quad \frac{90+x}{x} = 4$ or x = 30 cm

$\therefore$ At distance x = 30 cm from A along BA.

B-4.

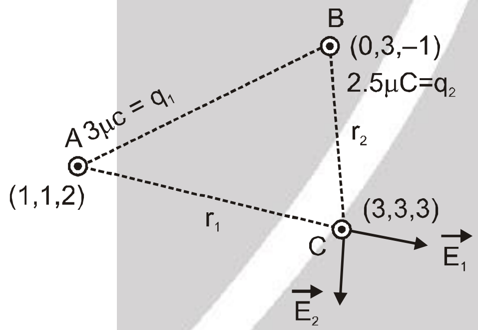


Let E_1, E_2 and E_3 be the electric fields at point D due to charges at points A, C and B respectively.

$\therefore \quad E_{net} = \frac{kq}{2a^2} + \frac{\sqrt{2}\ kq}{a^2} = \left[\sqrt{2} + \frac{1}{2}\right]\frac{kq}{a^2}$

along the line BD

B-5.



$r_1 = \sqrt{(3-1)^2 + (3-1)^2 + (3-2)^2} = \sqrt{2^2 + 2^2 + 1^2} = 3$

$r_2 = \sqrt{(3-0)^2 + (3-3)^2 + [3-(-1)]^2} = \sqrt{9+0+16} = 5$

$\therefore \quad \vec{E}_1 = \frac{kq_1}{r_1^3}\vec{r}_1 = \frac{9\times10^9\times3\times10^{-6}}{(3)^3} . [(3-1)\hat{i} + (3-1)\hat{j} + (3-2)\hat{k}$

$= 10^3 [2\hat{i} + 2\hat{j} + \hat{k}]$

$\vec{E}_2 = \frac{kq_2}{r_2^3}\vec{r}_2 = \frac{9\times10^9\times2.5\times10^{-6}}{(5)^3} . [(3-0)\hat{i} + (3-3)\hat{j} + \{(3-(-1)\}\hat{k}]$

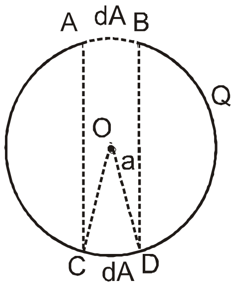
$= \frac{9\times2.5}{125} \times 10^3 [3\hat{i} + 0\hat{j} + 4\hat{k}]$

$= \frac{9}{50} \times 1000 [3\hat{i} + 4\hat{k}] = 180 [3\hat{i} + 4\hat{k}]$

$\therefore \quad \vec{E}_{net} = \vec{E}_1 + \vec{E}_2$

$= [2000\hat{i} + 2000\hat{j} + 1000\hat{k}] + [540\hat{i} + 720\hat{k}] = [2540\hat{i} + 2000\hat{j} + 1720\hat{k}]$ N/C.

B-6.



Small part 'AB' of hollow sphere of area 'dA' is cut off. Now, electric field at centre 'O' of system will only be due to part CD of area 'dA' directly in front of it. [rest is cancelled out]

Let charge on CD $\Rightarrow$ dQ

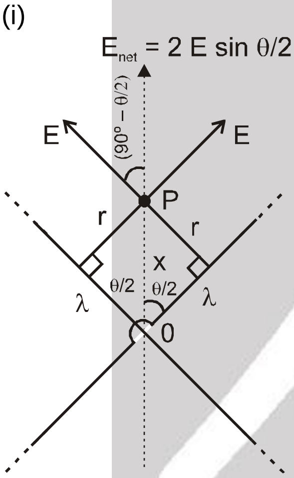
$\therefore \quad dQ = \dfrac{Q}{(4\pi a^2)} \cdot (dA)$

E_O = Electric field at O = $\dfrac{k(dQ)}{(a^2)} = \dfrac{1}{4\pi\varepsilon_0} \dfrac{Q}{4\pi a^2} dA \cdot \dfrac{1}{a^2}$

$\therefore \quad E_O = \dfrac{QdA}{16\pi^2\varepsilon_0 a^4}$ **Ans.**

B-7.

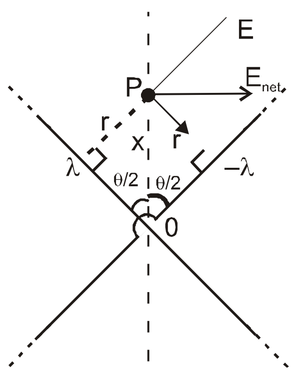
(i)



As shown in figure

$E_{net} = 2E \sin\theta/2 = 2\left[\dfrac{2k\lambda}{r}\right]\dfrac{r}{x} = \dfrac{4k\lambda}{x}$ along the line OP.

(ii)



As shown in figure

$E_{net} = 2E\cos\theta/2$

$$2 = \left[\frac{2k\lambda}{r}\right]\left(\cos\frac{\theta}{2}\right) \quad = \frac{4k\lambda}{r} . \sin\theta/2 . \cot\theta/2$$

$$= \frac{4k\lambda}{r} . \frac{r}{x} . \cot\theta/2$$

$$= \frac{4k\lambda}{x} \cot\theta/2$$; perpendicular to OP as shown.

B-8. (1) Time period in the absence of field

$$T = 2\pi\sqrt{\frac{\ell}{g}} = \frac{50}{30}$$

(2) When electric field is switched on;

$$g_{eft} = g - \frac{qE}{m} = 10 - \frac{6\times10^{-6}\times5\times10^{4}}{60\times10^{-3}} = 5 \text{ m/s}^2$$

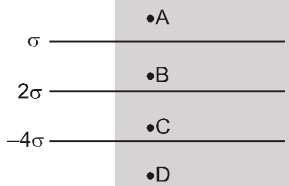
(3) $\therefore$ New time period T' = $2\pi\sqrt{\frac{\ell}{5}} = \frac{5\sqrt{10}}{3\sqrt{5}} = \frac{5\sqrt{2}}{3}$

(4) Let time taken to complete 60 oscillations is t_0.

$$\therefore \quad T' = \frac{t_0}{60} = \frac{5\sqrt{2}}{3}$$

or $\quad t_0 = T' \times 60 = 100\sqrt{2} \approx 141\text{s}$

B-9.

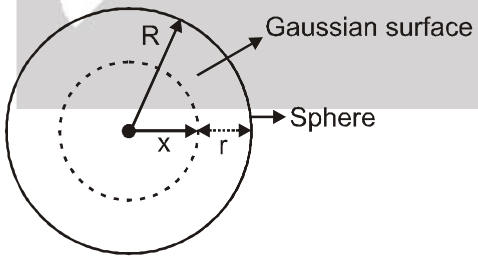


$$\vec{E}_A = \frac{\sigma}{2\varepsilon_0}\hat{j} + \frac{2\sigma}{2\varepsilon_0}\hat{j} - \frac{4\sigma}{2\varepsilon_0}\hat{j}; \qquad \vec{E}_B = -\frac{\sigma}{2\varepsilon_0}\hat{j} + \frac{2\sigma}{2\varepsilon_0}\hat{j} - \frac{4\sigma}{2\varepsilon_0}\hat{j}$$

$$\vec{E}_C = -\frac{\sigma}{2\varepsilon_0}\hat{j} - \frac{2\sigma}{2\varepsilon_0}\hat{j} - \frac{4\sigma}{2\varepsilon_0}\hat{j} \qquad \vec{E}_D = -\frac{\sigma}{2\varepsilon_0}\hat{j} - \frac{2\sigma}{2\varepsilon_0}\hat{j} + \frac{4\sigma}{2\varepsilon_0}\hat{j}$$

$$\therefore \quad \vec{E}_A = -\frac{\sigma}{2\varepsilon_0}\hat{j}\ ; \qquad \vec{E}_B = -\frac{3\sigma}{2\varepsilon_0}\hat{j}; \quad \vec{E}_C = \frac{-7\sigma}{2\varepsilon_0}; \quad \vec{E}_D = \frac{\sigma}{2\varepsilon_0}\hat{j}.$$

B-10. (i)



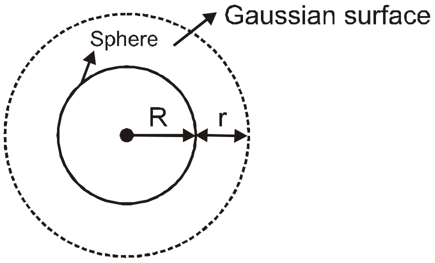
Let us construct a Gaussian surface at distance r from surface of sphere inside it (as shown dotted in figure)

$\therefore \quad r = R - x \qquad$ or $\qquad x = R - r$

Using Gauss law $\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$; $\therefore \qquad E.\,4\pi x^2 = \frac{\rho.\frac{4}{3}\pi\ x^3}{\varepsilon_0}$

$\therefore \quad E = \frac{\rho x}{3\varepsilon_0} \qquad$ or $\qquad E = \frac{\rho(R - r)}{3\varepsilon_0}$

(ii)

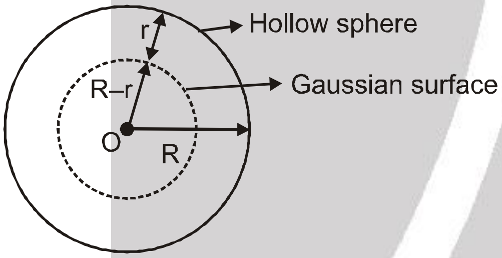


Using Gauss law,

$$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad E.4\pi\,(R + r)^2 = \frac{\rho.\frac{4}{3}\pi R^3}{\varepsilon_0};$$

$\therefore \qquad E = \dfrac{\rho R^3}{3\varepsilon_0 (R + r)^2}$

B-11.

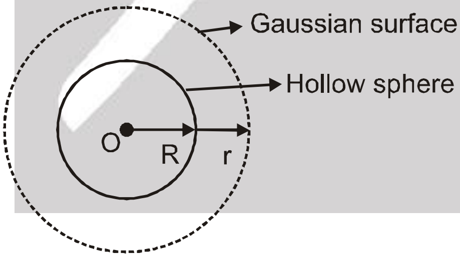


$\because$ Complete charge is on the surface of sphere and no charge is present inside it. So by Gauss's law.

$E.\ 4\pi(R - r)^2 = \dfrac{q_{en}}{\epsilon_0} = 0$

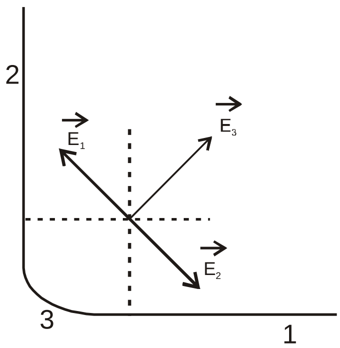
$\therefore \qquad E = 0$

(ii)



Note : Solution is same as that of Question B - 10 part (ii)

B-12.



here $\vec{E}_1$ = field due to wire 1 (Infinitely long)

$\vec{E}_2$ = field due to wire 2

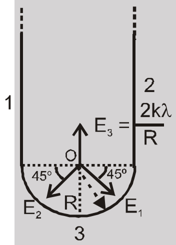
$\vec{E}_3$ = field due to wire 3

So, Net Electric field :

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$\because$ $\vec{E}_1 = -\vec{E}_2$

$\therefore$ $\vec{E}_{net} = \vec{E}_3 = \dfrac{2k\left(\dfrac{\lambda\pi R}{2}\right)\sin\dfrac{\pi}{4}}{\dfrac{\pi}{2}R^2} = \dfrac{\sqrt{2}\lambda}{4\pi\varepsilon_o R}$



Also here,

$\vec{E}_1$ = Electric field due to wire 1.

$\vec{E}_2$ = Electric field due to wire 2.

$\vec{E}_3$ = Electric field due to wire 3 (Ring).

$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

In X axis : $E_1 \cos 45^o - E_2 \cos 45^o = 0$ $\{E_1 = E_2\}$

In Y axis : $E_1 \sin 45^o + E_2 \sin 45^o - \dfrac{2k\lambda}{R} = \dfrac{k\lambda}{R} + \dfrac{k\lambda}{R} - \dfrac{2k\lambda}{R} = 0$

$\Rightarrow$ Electric field at centre = 0

Section (C)

C-1. (speed) v = const

∞ (q)-------------------------(q) P

$V_\infty = 0$ $V_P = 1000$ V

(i) $W_{ext} = q(V_P - V_\infty) = 20 \times 10^{-6} \times 1000 = 20$ mJ

(ii) $W_{elec} = q(V_\infty - V_P) = -20$ mJ $= -\Delta U$

(iii) $W_{ext} = \Delta U + \Delta k$ $= q\Delta V + \Delta K$ $= 20 + 10 = 30$ mJ

(iv) $W_{elec} = q(V_\infty - V_P) = -20$ mJ $= -\Delta U$

(v) P $(q)^{V=0}$--------------------$(q)^{V = (k.E = ?)}$

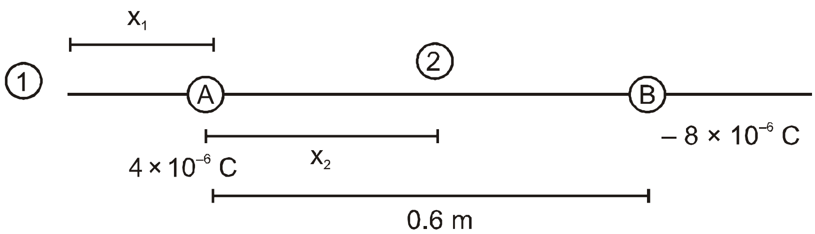
$V_P = 1000$ V ∞

$W_{elec} = \Delta K$ since no external force.

$\therefore W_{elec.} = -\Delta U = -q\Delta V = -q(V_\infty - V_p) = -30 \times 10^{-6}(0 - 1000) = 30$ mJ $= \Delta K = K_\infty - 0$

$\therefore$ $K_\infty = 30$ mJ

C-2.



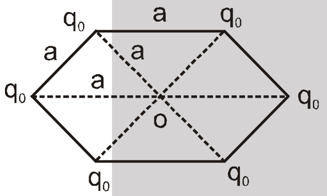
At Pt. (1) : $\frac{kq_A}{x_1} + \frac{kq_B}{0.6 + x_1} = 0$ or $\left(\frac{4}{x_1} - \frac{8}{0.6 + x_1} = 0\right)$

$\therefore$ $x_1 = 0.6$ m to the left of A.

At pt (2): $\frac{kq_A}{x_2} + \frac{kq_B}{0.6 - x_2}$ or $\frac{4}{x_2} = \frac{8}{0.6 - x_2}$

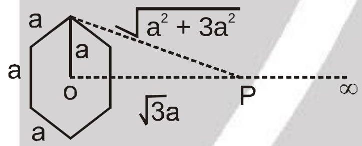
$\therefore$ $0.6 - x_2 = 2x_2$ or $x_2 = 0.2$ m to the right of A.

C-3.



(i) $W_{ext} = q\Delta V = q\,(V_0 - V_\infty) = q\left[\frac{6kq_0}{a} - 0\right] = \frac{6kqq_0}{a}$

(ii)



$W_{ext} = q\Delta V = q\,(V_p - V_\infty) = q\left[\frac{6\ \ kq_0}{\sqrt{a^2 + 3a^2}} - 0\right] = \frac{3kqq_0}{a}$

(iii) No, since work is done against conservative force without change in kinectic energy, so work is path independent.

C-4. $\therefore$ $W_{ext} = U_B - U_A$

$\therefore$ $20 = q\,(V_B - V_A)$

$\therefore$ $V_B - V_A = \frac{20}{0.05} = 400$ V

C-5. $\because$ Work is path independent, so no use of point C.

$\therefore$ $W = U_B - U_A$

$= q\left(\frac{kq_0}{r_B} - \frac{kq_0}{r_A}\right)$; where, r_A & r_B are positions of point A & B w.r.t origin.

$r_A = 0.03$ m, $r_B = 0.04$ m

$= kq\,q_0\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = -9 \times 10^9 \times 2 \times 10^{-9} \times 8 \times 10^{-3} \cdot \left[\frac{1}{0.04} - \frac{1}{0.03}\right] = 1.2$ J

C-6. (a) $\vec{E} = \dfrac{kQ}{\left|\vec{r} - \vec{r}_0\right|^3}\ (\vec{r} - \vec{r}_0) = \dfrac{9 \times 10^9 \times 50 \times 10^{-6}}{(\sqrt{6^2 + 8^2})^3}(6\hat{i} - 8\hat{j})$

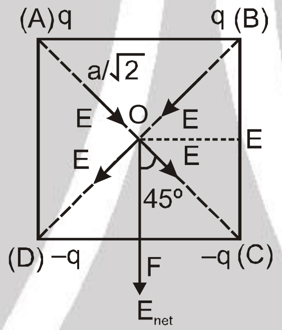
$\therefore\ \vec{E} = 450\ \ (6\hat{i} - 8\hat{j})V/m$ or $|\vec{E}|$ = 4500 V/m = 4.5 kV/m

(b) $W_{ext} = U_f - U_i = \dfrac{kqQ}{r_2} - \dfrac{kqQ}{r_1}$ (where r_1 and r_2 distances of (8, 6) & (4, 3) from charge Q)

$W_{ext} = 9 \times 10^9 \times 10 \times 10^{-6} \times 50 \times 10^{-6}\left[\dfrac{1}{2} - \dfrac{1}{\sqrt{45}}\right] = 4.5\left[\dfrac{1}{2} - \dfrac{1}{\sqrt{45}}\right]$ = 1.579 J (approx)

C-7. (i) Potential at the point O = $2\left(\dfrac{Kq}{\frac{a}{\sqrt{2}}}\right) - 2\left(\dfrac{Kq}{\frac{a}{\sqrt{2}}}\right) = 0$ V

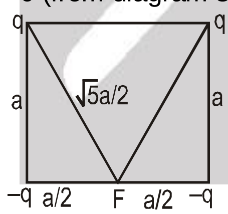
(ii) $\rightarrow$ Electric field at point O :



As shown from figure. E_{net} (at O) = E_0 = 4 E cos 45º (where, E = Electric field due to individual charge at point O)

$\therefore\ \ E_0 = \dfrac{4.kq}{\left(\frac{a}{\sqrt{2}}\right)^2}.\dfrac{1}{\sqrt{2}} = \dfrac{4\sqrt{2}kQq}{a^2}$ (in shown direction)

(iii) $V_E = 0$ (from diagram symmetry)



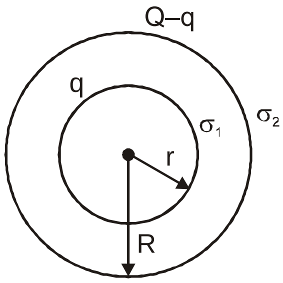
$$\dfrac{2kq}{\left(\frac{\sqrt{5}a}{2}\right)} - \dfrac{2kq}{a/2} = \dfrac{4kq}{a}\left[\dfrac{1}{\sqrt{5}} - 1\right]$$

$\therefore\ \ W_{ext}(O \rightarrow E) = Q(V_E - V_0) = Q(0 - 0) = 0$ J

$W_{ext}(O \rightarrow F) = Q(V_F - V_0) = Q\left[\dfrac{4kq}{a}\left(\dfrac{1}{\sqrt{5}} - 1\right) - 0\right]$ J

$= \dfrac{4kqQ}{a}\left[\dfrac{1}{\sqrt{5}} - 1\right]$ J

C-8.



(i) Let the charge on inner sphere = q

$\therefore$ Charge on outer sphere = Q – q

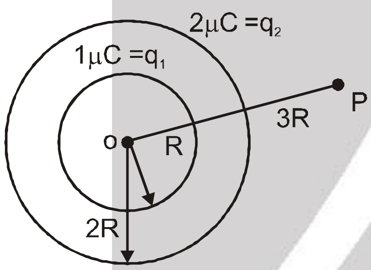
(ii) $\therefore \sigma_1 = \sigma_2$ $\qquad \therefore \qquad \dfrac{q}{4\pi r^2} = \dfrac{Q-q}{4\pi R^2}$

or $\dfrac{q}{r^2} = \dfrac{Q-q}{R^2} \qquad \Rightarrow \qquad q = \dfrac{Qr^2}{R^2+r^2}, \qquad Q - q = \dfrac{QR^2}{R^2+r^2}$

$\therefore$ Potential at centre O $\Rightarrow$ $V_0 = \dfrac{kq}{r} + \dfrac{k(Q-q)}{R}$

$$R\left[\frac{q}{r} + \frac{Q-q}{R}\right] = \left[\frac{Qr}{R^2+r^2} + \frac{QR}{R^2+r^2}\right] = \frac{QR}{\left(R^2+r^2\right)}\,(R+r)$$

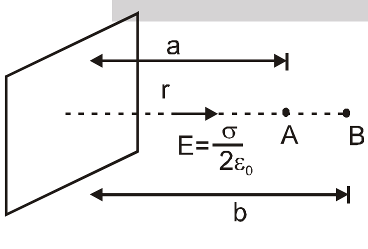
C-9.



$\because \quad V_p = 9000$ V $\quad \therefore \quad \dfrac{kq_1}{3R} + \dfrac{kq_2}{3R} = 9000$ V or $\quad \dfrac{k}{3R}$. [1μ c + 2μc] = 9000

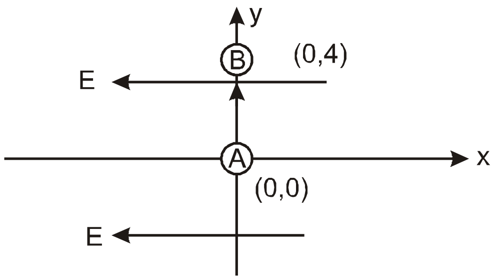
$\therefore \quad \dfrac{9\times10^9 \quad (3\times10^{-6})}{3R} = 9000 \qquad$ or $\qquad$ R = 1 m

C-10.



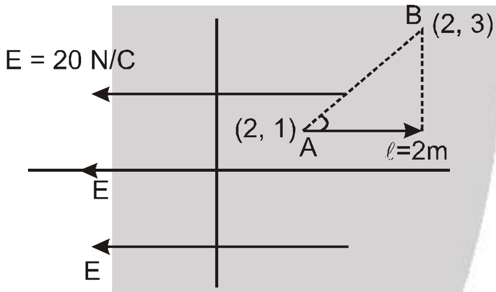
$W_{ext} = q_0\,(V_B - V_A) = q_0(-E\,(b-a))\,; \qquad \therefore \qquad W_{ext} = q_0\left(-\dfrac{\sigma}{2\varepsilon_0}\quad(b-a)\right) = \dfrac{q_0\sigma}{2\,\epsilon_0}\,(a-b)$

C-11.



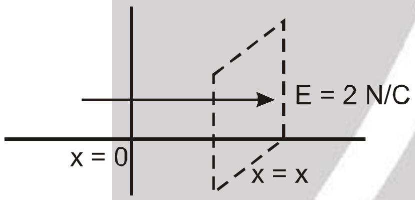
(a) $\therefore \quad \vec{E} \perp \vec{r} \quad \therefore$ A and B at same potential $\quad \therefore \quad V_B - V_A = 0$

(b)



$\therefore$ Against the field potential rises $\quad \therefore \quad V_B - V_A = E\ell = 20 \times 2 = 40$ V

C-12.



(a) $V_{(x,y,z)} - V_{(0,0,0)} = -Ex \quad \therefore \quad V - 0 = -8x$

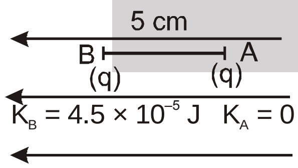
(b) $160 = -8x \quad \therefore \quad x = -20$

This equation represents a plane & all points on this plane have potential of 160 V

(c) $V - 80 = -8x \quad \therefore \quad V = 80 - 8x$

(d) $V_\infty - V_0 = -8x$, (where $x = \infty$) $\quad \therefore \quad 0 - V_0 = -8(\infty) \quad \Rightarrow \quad V_0 = \infty$

C-13.



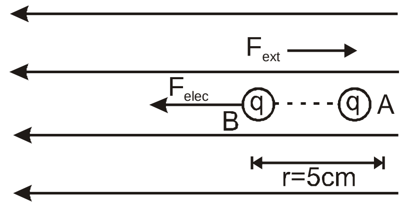
(a) $W_{elec} = \Delta K = 4.5 \times 10^{-5}$ J.

(b) $W_{elec} = qEr = 4.5 \times 10^{-5}$

$\therefore \quad E = \dfrac{4.5 \times 10^{-5}}{3 \times 10^{-9} \times 5 \times 10^{-2}} = 3 \times 10^5$ N/C

(c) $V_A - V_B = Er = 3.0 \times 10^5 \times 0.05 = 1.5 \times 10^4$ V

C-14.



(a) $W_{ext} + W_{elec} = \Delta K$, $\therefore$ $9 \times 10^{-5} + W_{elec} = 4.5 \times 10^{-5}$ $\therefore W_{elec} = -4.5 \times 10^{-5}$ J

(b) $|W_{elec}| = qEr = 4.5 \times 10^{-5}$ J $\therefore$ $E = \dfrac{4.5 \times 10^{-5}}{3 \times 10^{-9} \times 5 \times 10^{-2}} = 3 \times 10^5$ N/C

(c) $V_B - V_A = -Er = -3 \times 10^5 \times 5 \times 10^{-2} = -1.5 \times 10^4$ V

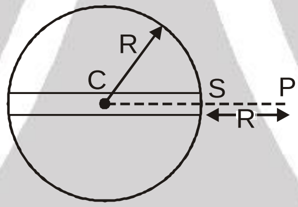
SECTION (D)

D-1. P.E. = qV = 2e × 5V = 10 eV.

D-2. $P.E = q_0V = q_0 \times \dfrac{6kq}{a} = \dfrac{6kq\ q_0}{a}$

D-3. (1) Let velocity at the surface of sphere is v_s. so, by conservation of mechanical energy between point P & S :

$U_P + K_p = U_s + K_s$



$\Rightarrow \quad -\dfrac{kQq}{2R} + 0 = -\dfrac{KQq}{R} + \dfrac{1}{2}mv_s^2$

or $\quad \dfrac{1}{2}mv_s^2 = \dfrac{1}{2}\dfrac{kQq}{R}$; $\quad v_s = \sqrt{\dfrac{kQq}{mR}}$ m/sec

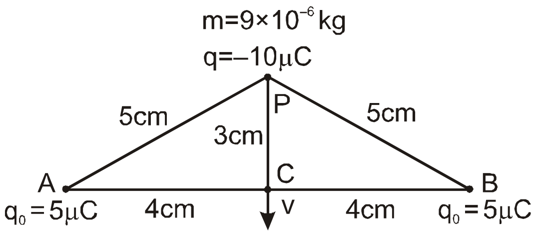
(2) Let velocity at the centre of sphire is v_e. So, by conservation of mech. energy between P & C.

$U_P + K_P = K_C + U_C$

$\Rightarrow \quad -\dfrac{kQq}{2R} + 0 = \dfrac{1}{2}mv_c^2 - \dfrac{3\ kQq}{2R}$

or $\quad \dfrac{1}{2}mv_c^2 = \dfrac{kQq}{R}$ $\therefore$ $v_c = \sqrt{\dfrac{2kQq}{mR}}$.

D-4.



By C.O.M.E., $U_p + K_p = K_c + U_c$

$\Rightarrow \quad q\left[\dfrac{kq_0}{5 \times 10^{-2}} \times 2\right] + 0 = \dfrac{1}{2}mv^2 + q\left[\dfrac{kq_0}{4 \times 10^{-2}} \times 2\right]$

or $\frac{1}{2}mv^2 = 2\,K\,q_0q\left[\frac{100}{5}-\frac{100}{4}\right]$

$\frac{1}{2}\times 9\times 10^{-6}\times v^2 = 2\times 9\times 10^9\times 5\times 10^{-6}\times 10\times 10^{-6}\times 5.$

or $v^2 = 4\times 25\times 10^4.$

$\therefore$ $v = 1000$ m/sec.

D-5. (a) Let minimum distance is r_0 by

$\therefore$ by cons. of ME between ∞ and point P :

$U_\infty + K_\infty = U_p + K_p$ or $0 + K = \frac{kQq}{r_0} + 0$ $\therefore$ $r_0 = \frac{Qq}{4\pi\varepsilon_0 K}$ ....(i)

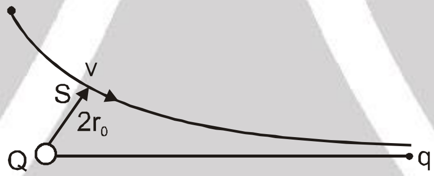
(b) As shown in diagram, Let particle approach closest distance point S from ∞ at speed v.

$\therefore$ By C.O.M.E. between ∞ & S.

$U_\infty + K_\infty = U_s + K_s$ or $0 + K = \frac{kQq}{2r_0} + \frac{1}{2}mV^2$ ....(ii)

Putting r_0 from (i) in (ii)

$K = \frac{K}{2}+\frac{1}{2}mv^2$ $\therefore$ $v = \sqrt{\frac{K}{m}}$



E-1. By energy conservation between initial and final state of the system

$W + (K.E)_1 + (P.E)_1 = (K.E)_2 + (P.E)_2$

$W + 0 + \frac{kq_1q_2}{r_1} = 0 + \frac{kq_1q_2}{r_2}$

or, $W = kq_1q_2\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$

$W = 9\times 10^9\times 15\times 10^{-6}\times 10\times 10^{-6}\left[\frac{1}{.15}-\frac{1}{.30}\right]$

$\therefore$ W = 4.5 Joule

E-2. The electrostatic potential energy of the system

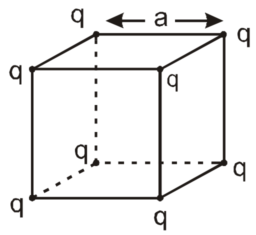
$U = \frac{k(q)\ (2q)}{0.1}+\frac{k(2q)\ (-4q)}{0.1}+\frac{k(q)\ (-4q)}{0.1}$

or $U = \frac{kq^2}{0.1}\ [-10]$

$\Rightarrow$ $U = \frac{9\times 10^9\times (10^{-7})^2\times(-10)}{(0.1)}$

$U = -9.0\times 10^{-3}$ Joule

E-3.



Electrostatic potential energy of charge system

$u_{net} = \dfrac{u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8}{2}$

and by symmetry $u_1 = u_2 = u_3 = u_4 = u_5 = u_6 = u_7 = u_8$

so $u_{net} = 4\,u_1$.

(i) inital potential energy = $4\left[\dfrac{3kq^2}{a} + \dfrac{3kq^2}{(\sqrt{2}a)} + \dfrac{kq^2}{(\sqrt{3}a)}\right]$

$u_{initial} = \dfrac{4kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right]$

(ii) Potential energy of system when all sides of cube increase from a to 2a

$u_{final} = 4\left[\dfrac{3kq^2}{2a} + \dfrac{3kq^2}{2\sqrt{2}a} + \dfrac{kq^2}{2\sqrt{3}a}\right]$

$u_{final} = \dfrac{2kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right]$

Work done by external agent against electrostatic forces :

$W_{ext} = u_{final} - u_{initial} = -\dfrac{2kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right]$

work done by electrostatic forces

$W_{ele.} = u_{initial} - u_{final}$; $W_{ele.} = \dfrac{2kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right] = -W_{ext}$

(iii) By applying energy conservation between initial and final positions:

$(K.E)_{initial} + (P.E)_{initial} = (K.E)_{final} + (P.E)_{final}$

$0 + \dfrac{4kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right] = 8\left[\dfrac{1}{2}mv^2\right] + \dfrac{2kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right]$

$v = \sqrt{\dfrac{kq^2}{2ma}\left(3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right)}$

(iv) $\therefore$ At infinity P.E. of charge A = 0}

$\therefore$ By conservation of mechanical energy A & ∞

$U_A + K_A = U_\infty + K_\infty$

$\dfrac{kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right] + 0 = 0 + \dfrac{1}{2}mv^2$ or $v = \sqrt{\dfrac{2kq^2}{a}\left(3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right)}$

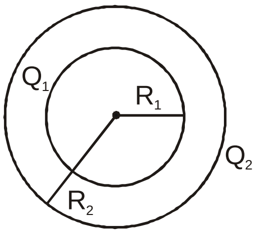
(v) By energy conservation

$(K.E)_{initial} + (P.E)_{initial} = (K.E)_{final} + (P.E)_{final}$

$0 + \dfrac{4kq^2}{a}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right] = 8\left[\dfrac{1}{2}mv^2\right] + 0$ $\therefore v = \sqrt{\dfrac{kq^2}{ma}}\left[3 + \dfrac{3}{\sqrt{2}} + \dfrac{1}{\sqrt{3}}\right]$ m/sec

SECTION (F)

F-1.



Self energy of a shell = $\dfrac{Q^2}{8\pi \in_0 R}$

Now total energy of system

$\Rightarrow$ (self energy)$_{R1}$ + (self energy)$_{R2}$ + (potential energy)$_{R1R2}$

$$= \frac{kQ_1^2}{2R_1} + \frac{kQ_2^2}{2R_2} + \frac{kQ_1Q_2}{(R_{external} = R_2)} = \frac{kQ_1^2}{2R_1} + \frac{kQ_2^2}{2R_2} + \frac{kQ_1Q_2}{R_2}$$

F-2 Work done by electric force = – [change in potential energy of the system]

$\Rightarrow$ $W_{ele} = -\Delta P.E = -[(P.E)_{final} - (P.E)_{initial}]$

$$W_{ele} = -\left[\left(\frac{kqq_0}{2R} + \frac{kq^2}{2(2R)}\right) - \left(\frac{kqq_0}{R} + \frac{kq^2}{2R}\right)\right]$$

$$W_{ele} = \frac{kq\ \ (q_0 + q/2)}{2R} \text{ or } W_{ele} = \frac{q\ \ (q_0 + q/2)}{8\pi\varepsilon_0 R}$$

(ii) Work done by external agent against electric forces = ΔP.E

$\therefore$ $W_{ext} = -W_{ele}$

or $W_{ext} = -\dfrac{q\ \ (q_0 + q/2)}{8\pi\varepsilon_0 R}$

F-3. By energy conservation :

$K.E_{initial} + (P.E)_{initial} = (K.E)_{final} + (P.E)_{final}$

$0 + \dfrac{kQQ}{d} = 2\,(K.E) + 0$; $\therefore$ $K.E = \dfrac{KQ^2}{2d} = \dfrac{Q^2}{8\pi\varepsilon_0 d}$

F-4. Eneryg stored out side the sphere = $\dfrac{kq^2}{2R} = U_0$; Total self energy of solid uniformly charged sphere $= \dfrac{3kq^2}{5R}$

$\therefore$ Self energy = $\dfrac{3}{5}\,[2U_0] = \dfrac{6U_0}{5}$

SECTION (G)

G-1. Given $\vec{E} = 2y\hat{i} + 2x\hat{j}$

& $dV = -\vec{E}\,.\,d\vec{r}$

$\therefore$ $dV = -(2y\hat{i} + 2x\hat{j}).\,(dx\hat{i} + dy\hat{j} + dz\hat{k})$

or $dV = -(2y\,dx + 2x\,dy)$

$\therefore$ $\int dV = -2\int (ydx + xdy) = -2\int d(xy)$

$V = -2xy + C$

G-2. $\therefore$ $\vec{E}(x,y,z) = -\frac{\partial}{\partial x}(V)\ \hat{i} - \frac{\partial}{\partial y}(V)\hat{j} - \frac{\partial}{\partial z}(V)\hat{k}$

$\therefore$ $\vec{E}(x,y,z) = -2xy\hat{i} - (x^2 + 2yz)\hat{j} - y^2\hat{k}$

G-3. $\because$ $\vec{E}(r) = -\vec{\nabla}V$ or $\vec{E}(r) = -\frac{\partial}{\partial r}(V)\hat{r}$

$\therefore$ $\vec{E}(r) = -\frac{\partial}{\partial r}(2r^2)\hat{r}$ $\Rightarrow$ $\vec{E}(r) = -4r\hat{r} = -4\vec{r}$

(i) Given $\vec{r} = \hat{i} - 2\hat{k}$ So, $\vec{E}(r) = -4(\hat{i} - 2\hat{k})$

(ii) $\vec{E}(r = 2) = -4.2.\hat{r}$ or $\vec{E}(r = 2) = -8\hat{r}$

G-4. $V_b - V_a = -\int_a^b \vec{E}.\vec{dr}$

$\therefore$ $V_{(3,3)} - V_{(0,0)} = -\int_{(0,0)}^{(3,3)} (10\hat{i} + 20\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$

or $V_{(3,3)} = -\int_0^3 10dx\ -\int_0^3 20dy = -30 - 60$

$\therefore$ $V_{(3,3)} = -90$ volt

G-5. $V_b - V_a = -\int_a^b \vec{E}.\vec{dr}$

$V_{(0,0)} - V_{(2,4)} = \int_{(2,4)}^{(0,0)} (20x\hat{i}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$

$\therefore$ $V_{(0,0)} = -\int_2^0 20x\,dx = \left[-10x^2\right]_2^0$

$V_{(0,0)} = 40$ Volt

G-6. $V(r) = -\int \vec{E}.\vec{dr}$

or $V(r) = -\int 2r^2 dr$

$\therefore$ $V(r) = -\frac{2r^2}{3} + C$

G-7. $V(x, y, z) = -\int \vec{E}.\vec{dr}$

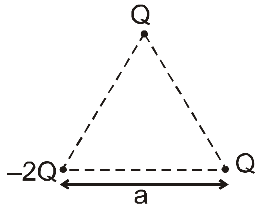
$V(x, y, z) = -\int (2x^2\hat{i} - 3y^2\hat{j}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$

or $V(x, y, z) = -\int 2x^2 dx + \int 3y^2 dy$

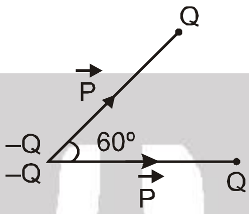
$\therefore$ $V(x, y, z) = -\frac{2x^3}{3} + y^3 + C$

SECTION (H)

H-1. (i) Given diagram is :



(ii) It can be shown in the form of dipoles as

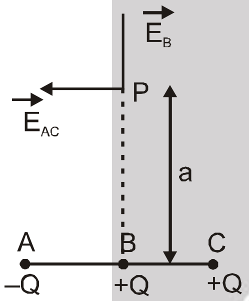


where, $\vec{P}$ = dipole moment of single dipole

So, net dipole moment is : $P_{net} = \sqrt{P^2 + P^2 + 2P.P \ \ \cos 60^o}$ or $P_{net} = \sqrt{3}P = \sqrt{3}Qa$

and direction is along the bisector of the angle at –2Q, towards the triangle

H-2.



a >> d so we can treat charge A and C as a dipole. So net electric filed at point P : $\vec{E}_{net} = \vec{E}_B + \vec{E}_{AC}$

or $\left|\vec{E}_{net}\right| = \sqrt{\left(\dfrac{kQ}{a^2}\right)^2 + \left(\dfrac{k(P)}{a^3}\right)^2}$ {dipole moment P = (2d) Q}

or $\left|\vec{E}_{net}\right| = \dfrac{1}{4\pi\varepsilon_0 a^3}\sqrt{Q^2a^2 + p^2}$

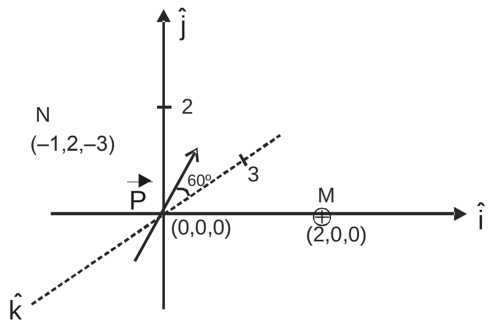
H-3. Work done by external agent = q [$V_{final} - V_{initial}$]

$\therefore \quad W_{ext} = q\left[\dfrac{kp\cos 45^o}{r^2} - \dfrac{kp\cos 135^o}{r^2}\right] = \dfrac{qp}{4\pi \in_0 r^2}\left[\dfrac{1}{\sqrt{2}} - \left(-\dfrac{1}{\sqrt{2}}\right)\right]$

$\therefore \quad W_{ext} = \dfrac{\sqrt{2}qp}{4\pi\varepsilon_0 r^2}$

H-4.

(i)



Given : $(\vec{P} = \hat{i} + \sqrt{3}\hat{j})$

$|\vec{P}| = \sqrt{1+3} = 2$

$\therefore$ Electric field at point M : $\rightarrow$

$|\vec{E}| = \frac{kP}{r^3}\sqrt{1+3\cos^2\theta}$

$= \frac{9\times10^9\times2}{(2)^3}\sqrt{1+3(\cos^2 60)}$

$= \frac{9}{4}\times10^9\times\sqrt{1+3/4} = \frac{\sqrt{7}k}{8}$

Potential at point M :

$V_m = \frac{kP\cos\theta}{r^2} = \frac{k\times2\times1/2}{(2)^2} = \frac{K}{4}$ s

(ii) $V = K\frac{\vec{p}.\vec{r}}{r^3} = K.\frac{(\hat{i}+\sqrt{3}\ \hat{j})\cdot(-\hat{i}\ +\sqrt{3}\ \hat{j}+0\hat{k})}{\left(\sqrt{(-1)^2+(\sqrt{3})^2+(0)^2}\right)^3}$

$\therefore$ $V = K.\frac{2}{8} = \frac{K}{4}$ ....(i)

& $\cos\theta = \frac{\vec{p}.\vec{r}}{pr} = \frac{(\hat{i}+\sqrt{3}\ \hat{j})\cdot(-\hat{i}\ +\sqrt{3}\ \hat{j}+0\hat{k})}{2\times2} = \frac{1}{2}$

$\therefore$ $E = \frac{KP}{r^3}\sqrt{3\cos^2\theta+1} = \frac{K\cdot2}{2^3}.\sqrt{\frac{3}{4}+1} = \frac{K\sqrt{7}}{8}$

H-5. Dipole moment of molecule of substance = 10^{-29} C-m

$|\vec{E}|$ applied = 10^6 Vm^{-1}

Change of angle of electric field = 60º

No of molecules in one mole $\Rightarrow$ n = 6.023×10^{23}

$\therefore$ Amount of heat released in aligning the dipoles along new direction

$\Rightarrow +\Delta U = \Delta W_{ext}$

$= +[U_f - U_i]$

$= n[-PE\cos60^\circ - (-PE)]$

$= n\left(\frac{-PE}{2}+PE\right) = n\frac{PE}{2}$

$= \frac{6.023\times10^{23}\times10^{-29}\times10^6}{2} = \frac{6.023}{2}$ = 3.0115 J

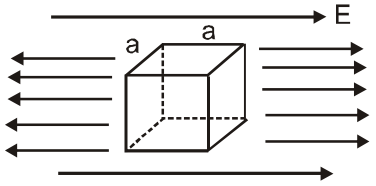
$\therefore$ $\frac{6.023}{2}$ = 3.0115 J (approx)

SECTION (I)

I-1. $\phi = \vec{E} \cdot \vec{A}$

$= (2\hat{i} - 10\hat{j} + 5\hat{k}) \cdot (10\hat{k}) = 50$ N m^2/c

I-2.

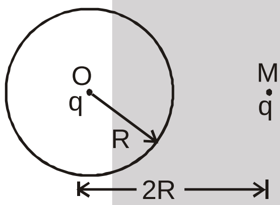


$\because$ Through the given volume : flux entering = flux coming out

i.e $\quad \phi_{in} = \phi_{out}$

$\therefore \quad \phi_{net} = \phi_{out} - \phi_{in} = 0$

I-3.



Electric flux due to charge q at O $\Rightarrow \dfrac{q}{\epsilon_0} = \dfrac{8.85 \times 10^{-8}}{8.85 \times 10^{-12}}$

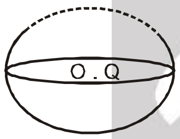
$= 10^4 \dfrac{N - m^2}{C}$

Electric flux due to charge q at M $\Rightarrow 0$

($\because$ flux centering = flux coming out)

(for spherical surface)

I-4.



Let us complete the sphere by drawing remaining hemisphere dotted as shown in Then total electric flux through complete sphere.

$= q/\epsilon_0 = \phi$

Hence flux through lower hemisphere $\Rightarrow \dfrac{q}{2\epsilon_0} = \dfrac{\phi}{2}$

I-5. Since : $\phi_{out} = 2\,\phi_{in}$

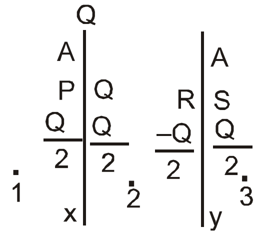
$\therefore \quad \phi_{net} = \phi_{out} = +$ ve

i.e net flux is coming out (+ve) from the surface.

$\therefore \quad$ There is net + ve charge inside closed surface

SECTION (J)

J-1. The charge distribution on the surface of two plates can be shown as



From the given diagram :

(a) surface charge density on inner part of plate x = $\frac{Q}{2A}$

(b) electric field at point 1 :

$Q/4A\in_0$ (P) ← 1 → $Q/4A\in_0$ (R) $\Rightarrow$ ← 1 $Q/2A\in_0$

← $Q/4A\in_0$ (Q) $\therefore$ $Q/2A\in_0$ (towards left)

← $Q/4A\in_0$ (S)

(c) electric field at point 2 : -

2 → $Q/4A\in_0$ (P) 2 → $Q/2A\in_0$

→ $Q/4A\in_0$ (Q)

→ $Q/4A\in_0$ (R) $\therefore$ $Q/2A\in_0$ (towards right)

← $Q/4A\in_0$ (S)

(d) electric field at point 3 : -

3 → $Q/4A\in_0$ (P) $\Rightarrow$ → $Q/2A\in_0$

→ $Q/4A\in_0$ (Q)

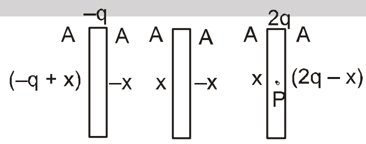
→ $Q/4A\in_0$ (R) $\therefore$ $Q/2A\in_0$ (towards right)

← $Q/4A\in_0$ (S)

J-2. Let the charge distribution on the plates is as shown in the figure
Equating electric field at point P inside the right most plate :

$$\frac{-q}{A\in_0} + 0 + \frac{x}{A\in_0} = \left(\frac{2q-x}{A\in_0}\right)$$

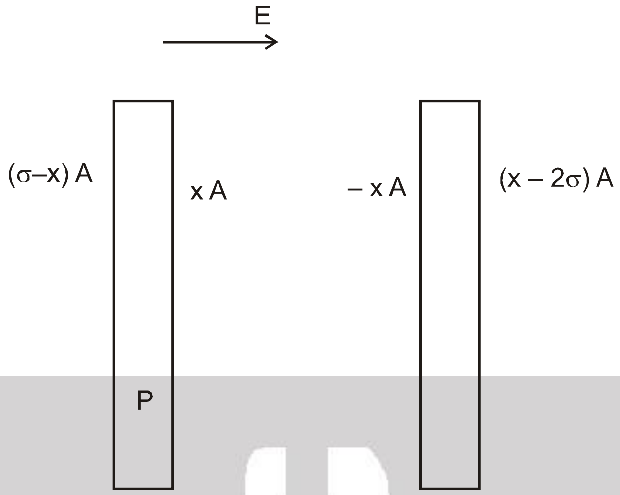
$\therefore$ $-q + x = 2q - x$ or $x = \frac{3q}{2}$



$\therefore$ Charge on outer surface of leftmost plate is

$$-q + \frac{3q}{2} = \frac{q}{2}$$

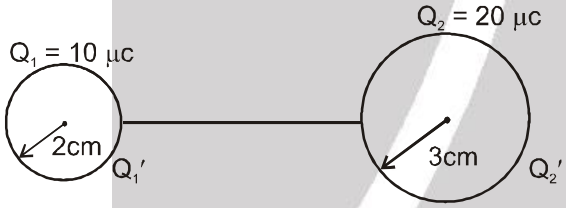
J-3. Let charge distribution on each surface at plates are shown in figure (after applying the electric field)



Now, at point P :

$$\frac{(\sigma - x)}{2 \epsilon_o} - \frac{(x - 2\sigma)}{2 \epsilon_o} = E \qquad \Rightarrow 2x + 3\sigma = E.\ 2\varepsilon_0 \qquad \therefore \qquad x = \frac{3\sigma + 2\varepsilon_0 E}{2}$$

J-4.



As shown, 10 μC and 20 μC are charge on two spheres before connecting with conducting wire.
Let after connecting with conducting wire, the charges on two spheres ⇒ Q_1' & Q_2'

(i) After connection

Potentials of both spheres are equal

$\therefore \quad V_1 = V_2$

or $\quad \frac{kQ_1'}{2cm} = \frac{kQ_2'}{3cm}\ 3\,Q_1' = 2Q_2'$

$\therefore \quad \frac{Q_1'}{Q_2'} = \frac{2}{3}$

(ii) $\quad \frac{Q_1'}{Q_2'} = \frac{2}{3}$ (i) & $Q_1' + Q_2'$ (ii) = 30 μC → charge conservation

$\therefore$ from (i) & (ii)

$Q_2' + \frac{2Q_2'}{3} = 30$

or $\quad \frac{5Q_2'}{3} = 30$

or $\quad Q_2' = \frac{90}{5}\ \mu c = 18\ \mu c$

& $\quad Q_1' = \frac{2}{3}\ Q_2' = \frac{2}{3} \times 18\ \mu c = 12\ \mu c$

(iii) $\frac{\sigma_1'}{\sigma_2'} = \frac{Q_1'/4\pi r_1^2}{\frac{Q_2'}{4\pi r_2^2}} = \frac{Q_1'}{Q_2'} \times \frac{r_2^2}{r_1^2}$

$= \frac{2}{3} \times \frac{9}{4} = \frac{3}{2}$

(iv) Heat produced during the process ⇒
initial energy of system – final energy of system

⇒ $U_i - U_f$

⇒ $\left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \left[\frac{1}{2}C_{eq}V^2\right]$

Where; $C_1 = 4\pi\epsilon_0 r_1$

$C_2 = 4\pi\epsilon_0 r_2$

$V_1 = \frac{kQ_1}{r_1}$; $V_2 = \frac{kQ_2}{r_2}$ & $V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$; $C_{eq} = (C_1 + C_2)$

∴ Heat ⇒ $\left[\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right] - \frac{1}{2}\left(\frac{C_1V_1 + C_2V_2}{C_1 + C_2}\right)^2 (C_1 + C_2)$

⇒ $\frac{C_1V_2 + C_2V_2^2}{2} - \frac{(C_1V_1 + C_2V_2)^2}{2(C_1 + C_2)}$

⇒ $\frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_2^2 + C_1C_2V_1^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{(C_1 + C_2)}\right]$

⇒ $\frac{C_1C_2(V_1 - V_2)^2}{2(C_1 + C_2)} = 2\pi\epsilon_0 \left(\frac{r_1 \ r_2}{r_1 + r_2}\right)(V_1 - V_2)^2$

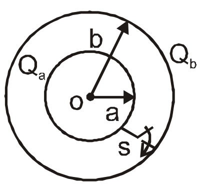
∴ Heat $= \frac{r_1 r_2}{(r_1 + r_2)} 2\pi\epsilon_0 (V_1 - V_2)^2$

⇒ $\frac{1}{2 \times 9 \times 10^9} \times \frac{2 \times 3 \times 10^{-4}}{5 \times 10^{-2}} \left[(9 \times 10^9)^2 \times \left[\frac{10 \times 10^{-6}}{2 \times 10^{-2}} - \frac{20 \times 10^{-6}}{3 \times 10^{-2}}\right]^2\right]$

$\frac{3}{5} \times 10^{-2} \times 9 \times 10^9 \left[5 \times 10^{-4} - \frac{20}{3} \times 10^{-4}\right]^2 = \frac{27}{5} \times 10^7 \left[\frac{-5}{3} \times 10^{-4}\right]^2 = \frac{27}{5} \times \frac{25}{9} \times 10^{-1}$

$= \frac{3}{2}$ Joules **Ans**

J-5.



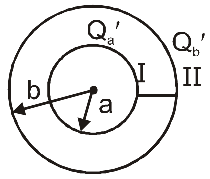
(initial)

Before connecting spheres charges are Q_a & Q_b on inner & outer sphere

After closing switch S :

Potentials of both becomes equal

Let charges on two spheres after connection are Q_a' & Q_b' as shown



(final)

$\therefore \quad Q_a' + Q_b' = Q_a + Q_b$ ....(1)

$V_I = V_{II}$

$\therefore \quad \dfrac{KQ_a'}{a} + \dfrac{KQ_b'}{b} = \dfrac{KQ_a'}{b} + \dfrac{KQ_b'}{b}$

$\therefore \quad Q_a' = 0$ ....(2)

Putting (2) in (1)

$0 + Q_b' = Q_a + Q_b$

$\therefore \quad Q_a' = Q_a + Q_b$ **Ans.**

i.e total charge is transferred to outer shell

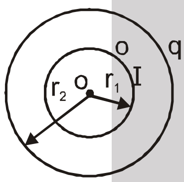
(ii) Heat produced during the process $\Rightarrow$ Initial energy of system – final energy of system

$\Rightarrow$ Heat $= V_i - V_f$

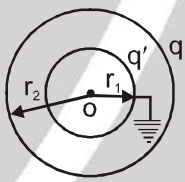
$$= \left[\frac{KQ_a^2}{2a} + \frac{KQ_a^2}{2b} + \frac{KQ_aQ_b}{b}\right] - \left[\frac{KQ_b'^2}{2b}\right] = \frac{KQ_a^2}{2a} + \frac{KQ_b^2}{2b} + \frac{KQ_aQ_b}{b} - \frac{K}{2b}(Q_a + Q_b)^2 n$$

$$= \frac{KQ_a^2}{2a} + \frac{KQ_b^2}{2b} + \frac{KQ_aQ_b}{b} - \frac{KQ_a^2}{2b} - \frac{KQ_b^2}{2b} - \frac{KQ_aQ_b}{b} = \frac{KQ_a^2}{2} \cdot \left[\frac{1}{a} - \frac{1}{b}\right]$$

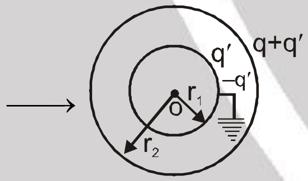
J-6.



(initially)



(finally)



Let charge q′ appears on the inner shell after grounding

(i) potential of inner shell = 0 (grounded)

$\therefore \quad \dfrac{kq'}{r_1} - \dfrac{kq'}{r_2} + \dfrac{k(q+q)}{r_2} = 0$

or $\quad \dfrac{q'}{r_1} = \dfrac{-q}{r_2}$

$\therefore \quad q' = \dfrac{-qr_1}{r_2}$

$\therefore$ charge on inner surface of outer shell $\Rightarrow -q'$

or $-q' = \dfrac{qr_1}{r_2}$ **Ans.**

(ii) Final charges on spheres :

inner sphere $\rightarrow q' = \dfrac{-qr_1}{r_2}$

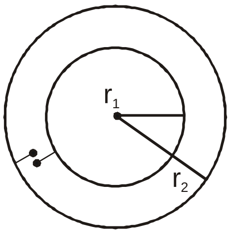
outer sphere $\rightarrow$ q

(iii) initial charges on inner sphere = 0

final charges on inner sphere = q′

or charge flown from inner sphere to earth $\Rightarrow -q' = \dfrac{qr_1}{r_2}$

J-7.



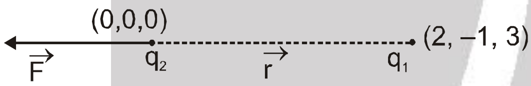
Initially charge on r_1 = q, then $\frac{kq}{r_1} = V_1$; $q = \frac{V_1 r_1}{k}$

Now, if both shells are connected to each other, total charge goes to outer shell so, how potential at 2nd shell : $V_2 = \frac{kq}{r_2} = V_1 \frac{r_1}{r_2}$

PART - II

SECTION (A)

A-1.

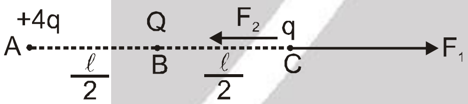


$\vec{F} = \frac{kq_1q_2}{r^3}(\vec{r})$; (By definition)

$\therefore \quad \vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2[(0-2)\hat{i} + \{0-(-1)\}\hat{j} + (0-3)\hat{k}]}{\left[\sqrt{(0-2)^2 + \{0-(-1)\}^2 + (0-3)^2}\right]^3}$

$= \frac{q_1q_2}{4\pi\varepsilon_0}\cdot\frac{(-2\hat{i}+\hat{j}-3\hat{k})}{(\sqrt{4+1+9})^3} = \frac{q_1q_2(-2\hat{i}+\hat{j}-3\hat{k})}{56\sqrt{14}\pi\varepsilon_0}$

A-2.



Charges are placed as shown on line AC.

For net force on q to be zero, Q must be of –ve sign. If F_1 is force on q due to 4q & F_2 due to Q

Then, $F_1 = F_2$ (magnitudewise)

or $\frac{k4q\cdot q}{\ell^2} = \frac{kQq}{\left(\frac{\ell}{2}\right)^2}$

$\therefore$ 4q = 4Q

or Q = q (in magnitude)

$\therefore$ Q = –q (with sign)

A-3. Final charge on both spheres = $\frac{40-20}{2}\mu C = 10\mu C$ (each) [Distibution by conducting]

$\therefore \frac{F_i}{F_f} = \frac{(q_1q_2)_i}{(q_1q_2)_f} = \frac{800}{100}$ = 8 : 1

A-4. Initially, $F = \frac{kq_1q_2}{r^2}$ ....(1)

Finally, $4F = \frac{kq_1q_2}{16R^2}$ ....(2)

$\Rightarrow \quad \frac{4kq_1q_2}{r^2} = \frac{4kq_1q_2}{16R^2}$ or $R = \frac{r}{8}$

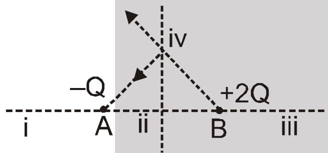
SECTION (B)

B-1. Time period of simple pendulum is given as

$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$; where = $g_{eff} = \frac{\sqrt{m^2g^2 + q^2E^2}}{m} = \sqrt{g + \left(\frac{qE}{m}\right)^2}$

$\therefore \quad T = 2\pi\sqrt{\frac{\ell}{\sqrt{g^2 + \left(\frac{qE}{m}\right)^2}}}$

B-2.



The electric field due to a point charge 'q' at distance 'r' from it is given as :

$E = \frac{kq}{r^2}$; more is q, more is r for E to have same magnitude

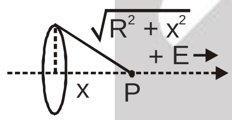
$\therefore$ By this mathematical analogy, electric field cannot be zero in the region iii

In region ii, electric field due to both charges is added & net electric field is towards left

Along ⊥ bisector line IV electric field due to both charges will be added vectorially & can't be zero

$\therefore$ E.F (net) can be zero in region I only (by mathematical analogy explained)

B-3. At point P on axis, $E = \frac{kqx}{(R^2 + x^2)^{3/2}}$



For max E, $\frac{dE}{dx} = 0 \quad \Rightarrow$ or $x = \frac{R}{\sqrt{2}}$

$\therefore$ Putting x in (i) $E_{max} = \frac{2kq}{3\sqrt{3}R^2}$

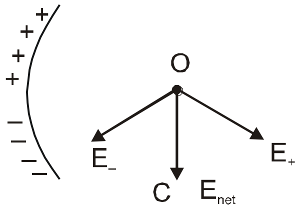
B-4. Force on charge q in electric field E $\Rightarrow$ F = qE $\Rightarrow a = \frac{qE}{m}$

$\therefore$ Distance travelled $\Rightarrow \quad x = \frac{1}{2} at^2 = \frac{1}{2}\left(\frac{qE}{m}\right)t^2$

Also, kinetic energy K.E = Work done by electric field E is $W_E = qE.\ \frac{1}{2}\left(\frac{qE}{m}\right)t^2 = \frac{E^2q^2t^2}{2m}$

B-5. By M.E. conservation between initial & final point :
$U_i + K_i = U_f + K_f$

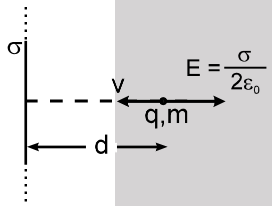
B-6. Given diagram shows :
The direction of E_{net} is along OC.



B-7. $T = 2\pi\sqrt{\dfrac{\ell}{g_{eff}}}$; where, $g_{eff} = \dfrac{mg - qE}{m}$

$= g - \dfrac{qE}{m}$ $\therefore$ Time period increases.

B-8.



$E = \dfrac{\sigma}{2\varepsilon_0}$, $F = \dfrac{\sigma q}{2\varepsilon_0}$

$a = \dfrac{\sigma q}{2\varepsilon_0 m}$; $v^2 = u^2 + 2as$; $0^2 = v^2 - 2\left(\dfrac{\sigma q}{2\varepsilon_0 m}\right)d$ $\Rightarrow$ or $v = \sqrt{\dfrac{\sigma q d}{\varepsilon_0 m}}$

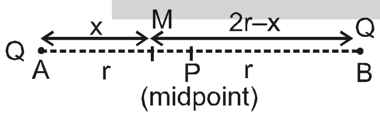
B-9. $W = Fr\cos\theta$ $\Rightarrow$ $\therefore$ $4 = (0.2)\,E\,(2)\cos 60^o$ $\Rightarrow$ $\therefore$ $E = 20$ N/C.

SECTION (C)

C-1. $\therefore$ $V = Er$, $\therefore$ $r = V/E = 6$m.

C-2 Apply the formula $V = \dfrac{kQ}{r}$

C-3.



Let the two charges at A & B are separated by distance 2r.
Let us consider a general point 'M' at distance
'x' from point 'A' in figure.

$\therefore$ V_m = Potential at M $= \dfrac{kQ}{x} + \dfrac{kQ}{(2r - x)}$

$\therefore$ $V_m = \left[\dfrac{1}{x} + \dfrac{1}{(2r-x)}\right]kQ = kQ\left[\dfrac{(2r)}{x(2r-x)}\right]$

For V_m to be max. or min :

$\frac{dV_m}{dx} = 0$

or $\frac{d}{dx}\left[kQ\frac{2r}{x(2r-x)}\right] = 0$

$\therefore$ $\frac{x(2r-x)(0) - kQ\ \ (2r)[2r-2x]}{[x(2r-x)]^2} = 0$

$\therefore$ x = r

& At x = r, $\frac{d^2V_m}{dx^2} > 0$ $\therefore$ x = r is min.

Hence potential continuously decreases from A to P and then increases

C-4. $\because$ $V_C = \frac{kQ}{r}$ $\therefore$ $V_C = \frac{9\times10^9 \times 1.5\times10^{-9}}{(0.5)}$ = 27 V.

C-5. $\because$ $E = \frac{E}{q}$ $E = \frac{3000}{3}$ = 1000 N/C.

& $\Delta V = Ed = \frac{1000\times1}{100}$ = 10V.

C-6. $\because$ $E = \frac{F}{q}$ $\therefore$ $E = \frac{2000}{5}$ = 400 N/C

Potential difference, $\Delta V = E.\ d = 400 \times \frac{2}{100}$ = 8V.

C-8. Since B and C are at same potential (lying on a line $\perp$ to electric field i.e. equipotential surface)

$\therefore$ $\Delta V_{AB} = \Delta V_{AC} = Eb$.

C-9. Property of equipotential surface

C-10. K.E. = VQ and momentum = $\sqrt{2m(KE)} = \sqrt{2mVQ}$

C-11. Potential at 5cm. $\Rightarrow$ 5cm = $V = \frac{kq}{(10cm)}$

($\because$ point lying inside the sphere)

Pontential at 15 cm V′ $\Rightarrow$ $\frac{kq}{15cm} = \frac{2}{3}$ V.

C-12. $\because$ $V = \frac{kq}{r} - \frac{kq}{3r}$ $V = \frac{2kq}{3r}$

$\therefore$ Field intensity at distance 3r from centre = $\frac{kq}{9r^2} = \frac{V}{6r}$

C-13. The whole volume of a uniformly charged spherical shell is equipotential.

C-14. Potential at origin will be given by

$$V = \frac{q}{4\pi\in_0}\left[\frac{1}{x_0} - \frac{1}{2x_0} + \frac{1}{3x_0} - \frac{1}{4}x_0 + ...\right] = \frac{q}{4\pi\in_0}.\frac{1}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...\right] = \frac{q}{4\pi\in_0\ x_0}\ \ln(2)$$

SECTION (D)

D-1. PE = q (V_{final} – $V_{initial}$)

PE = qΔV   PE decreases if q is +ve increases if q is –ve.

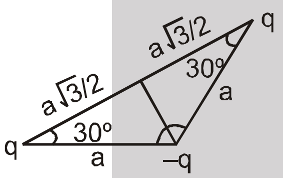
D-2. By conservation of machenical energy

$$\frac{1}{2}mv^2 = \frac{k\ q_1q_2}{r_1} - \frac{k\ q_1q_2}{r_2} \qquad \frac{1}{2}(2\times10^{-3})\ \ v^2 = 9\times10^9\times10^{-6}\times10^{-3}\left(\frac{1}{1}-\frac{1}{10}\right)$$

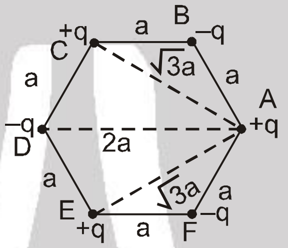
or $v^2 = 9\times10^3\times\frac{9}{10}$ or v = 90 m/sec

SECTION (E)

E-1.



$\Rightarrow$



(i) E.P.E. of charge +q at point A can be given as :

$E_A = \frac{-2kq^2}{a} + \frac{-2kq^2}{\sqrt{3}a} - \frac{kq^2}{2a}$ & E.P.E. of system

$\Rightarrow$ $E_S = \frac{E_A+E_B+E_C+E_D+E_E+E_F}{2}$ where $E_A = E_B = E_C = E_D = E_E = E_F$

$\therefore$ $E_S = 3\,E_A$

$\therefore$ $E_S = 6\left(-\frac{kq^2}{a}\right) + 6\left(\frac{kq^2}{a\sqrt{3}}\right) + 3\left(-\frac{kq^2}{2a}\right) = \frac{q^2}{\pi\in_0 a}\left[\frac{\sqrt{3}}{2}-\frac{15}{8}\right]$

E-2. P.E. of system = $\frac{2\ Kq^2}{a} + \frac{2xkq^2}{a} + \frac{xkq^2}{a} = 0$ where a is distance between charges.

or 2 + 3x = 0 $\therefore$ $x = -\frac{2}{3}$

SECTION (F)

F-1. E = Field near sphere = $\frac{V}{R} = \frac{8000}{1\times10^{-2}}$ = 8×10^5 V/m

$\therefore$ Energy density = $\frac{1}{2}\varepsilon_0E^2 = \frac{4\pi\varepsilon_0}{8\pi}$ $E^2 = \frac{8\times8\times10^{10}}{8\pi\times9\times10^9} = \frac{80}{9\pi}$ = 2.83 J/m^3.

F-2. Let q is charge and a is radius of single drop.

$\therefore$ $U_{single\ drop} = \frac{3kq^2}{5a}$

Now, charge on big drop = nq. & let Radius of big drop is R.

$\therefore$ By conservation of volume

$\Rightarrow$ $\frac{4}{3}\pi R^3 = n.\frac{4}{3}\pi a^3$ $\Rightarrow$ $R = an^{1/3}$.

$\therefore$ P.E. of big drop = $\frac{3}{5}\frac{k(qn)^2}{R} = \frac{3}{5}\frac{k.q^2n^2}{an^{1/3}} = Un^{\frac{5}{3}}$

SECTION (G)

G-1. (i) $E = -\frac{dV}{dr} = -(\text{slope of curve})$

$\therefore$ At r = 5 cm slope $= -\frac{5}{2}$ V/cm.

$= -2.5$ V/cm

$\therefore$ $E_{(at\ sin)} = 2.5$ V/cm

G-2. At origin, $E = -\frac{dV}{dr} = -2.5$ V/cm $= -250$ V/m

$\therefore$ F = force on 2C = q E = 2 × (– 250) N = – 500 N.

G-3. $E = -\frac{dV}{dx} = -10x - 10$

$\therefore$ $E_{(x=1m)} = -10(1) - 10 = -20$ V/m

G-4. $\Delta V = -E\Delta x \Rightarrow V_x - 0 = -E_0x.$ or $V_x = -E_0x.$

G-6. $E \alpha r \Rightarrow \int_0^v dV \alpha \int_0^r r dr \Rightarrow V \alpha (-\frac{r^2}{2}) \Rightarrow V \alpha r^2$

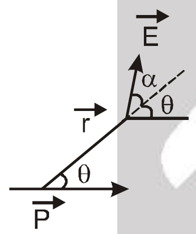
G-7. $-\int_{\ell\to\infty}^{\ell=0} \vec{E}\cdot d\vec{\ell}$ will be equal to potential at $\ell = 0$ i.e. (at centre) and potential at the centre of the ring is

$V_{centre} = \frac{Kq_{total}}{R} = \frac{(9\times10^9)\times(1.11\times10^{-10})}{(0.5)} = +2$ Volt. (Approx)

SECTION (H)

H-1. Since P & Q are axial & equatorial points, so electric fields are parallel to axis at both points.

H-2.



In shown diagram, $\vec{E}$ = Net electric field vector due to dipole. (by derivation) & $\tan\alpha = 1/2 \tan\theta$

$\therefore$ angle made by $\vec{E}$ with x-axis is $(\theta + \alpha)$

H-3. $\tau_{max} = pE \sin 90^o = 10^{-6} \times 2 \times 10^{-2} \times 1 \times 10^5$ N – m $= 2 \times 10^{-3}$ N-m

H-4. max PE $\Rightarrow$ position of unstable equilibrium $\Rightarrow$ $\theta = \pi$.

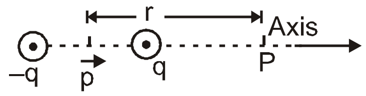
H-5. $\tau_{max} = PE = 4 \times 10^{-8} \times 2 \times 10^{-4} \times 4 \times 10^8$

$= 32 \times 10^{-4}$ N-m.

Work done $W = (P.E.)_f - (P.E.)_i = PE - (-PE)$

$= 2PE = 64 \times 10^{-4}$ N-m

H-6.



At a point 'P' on axis of dipole electric field $E = \frac{2kp}{r^3}$ and electric potential $V = \frac{kp}{r^2}$

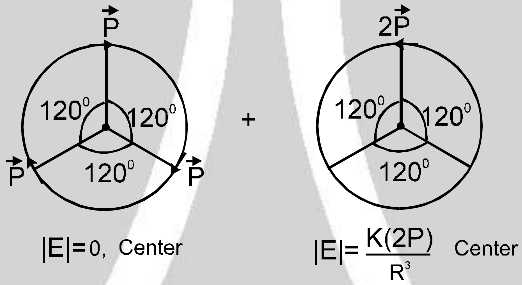
both nonzero and electric field along dipole on the axis.

H-7. Force on one dipole due to another

$= P\left(\frac{dE}{dr}\right)$ where E is field due to second dipole at first dipole.

$E \propto \frac{1}{r^3}$ $\quad\therefore\quad$ $\frac{dE}{dr} \propto \frac{1}{r^4}$ $\quad\therefore\quad$ Force $\propto \frac{1}{r^4}$

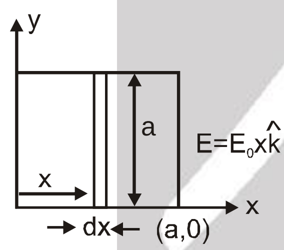
H-8. Given system is equivalent to :



$\therefore$ Ans : $\frac{2kp}{R^3}$

SECTION (I)

I-1.



flux through differential element $d\phi = E_0\, x\, a\, dx$.

$\therefore$ Net flux

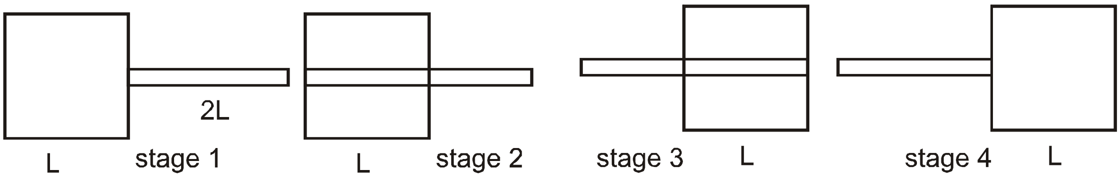
$\Rightarrow \quad \phi = E_0\, a \int_0^a x.dx = \frac{E_0 a^3}{2}$

I-3. Density of electric field lines at a point i.e. no. of lines per unit area shows magnitude of electric field at that point.

I-5. Net flux $= \phi_2 - \phi_1 = \frac{q_{in}}{\varepsilon_0}$ $\qquad q_{in} = \varepsilon_0 (\phi_2 - \phi_1)$

I-6. Since, dipole has net charge zero, so flux through sphere is zero with non-zero electric field at each point of sphere.

I-7.



From stage 1 to stage 2 $\Rightarrow$ enclosed charge is increasing means flux is increasing
From stage 2 to stage 3 $\Rightarrow$ enclosed charge is constant means flux is constant
From stage 3 to stage 4 $\Rightarrow$ enclosed charge is decreasing means flux is decreasing

I-8. Since same no of field lines are passing through both spherical surfaces, so flux has same value for both.

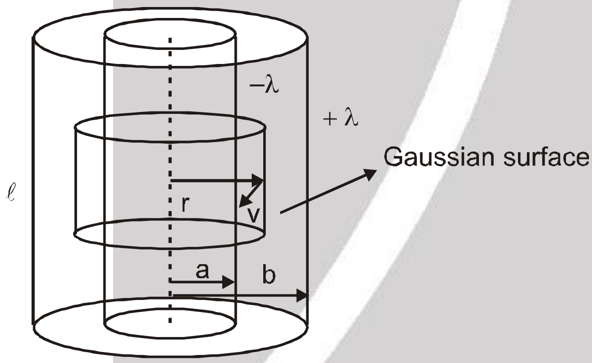
I-9. Each charge has its 1/8th part of electric field lines inside cube since there are 8 charges.

$\therefore$ Net enclosed charge = $\frac{q}{8} \times 8 = q$

$\therefore$ Net flux = $\frac{q_{in}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$.

$\therefore$ Fflux through one surface $= \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$

I-10.



Using Gauss's law for Gaussian surface shown in figure.

$\oint \vec{E}.d\vec{A} = \frac{q_{in}}{\varepsilon_0}$; E. $2\pi r\ell = \frac{\lambda\ell}{\varepsilon_0}$ $\therefore$ $E = \frac{\lambda}{2\pi\varepsilon_0 r}$

For circular motion.

$qE = \frac{mV^2}{r} = \frac{q\lambda}{2\pi\varepsilon_0 r}$ $\therefore$ $V = \sqrt{\frac{q\lambda}{2\pi\varepsilon_0 m}}$

I-11. For the closed surface made by disc and hemisphere

$q_{in} = 0$ $\therefore$ $\phi_{net} = 0$

$\phi_{disc} + \phi_{H.S} = 0$

$\therefore$ $\phi_{HS} = -\phi_{disc} = -\phi$

I-12. By definition

I-13. Electric lines of force never form a closed loop. Therefore, options (A), (C) and (D) are wrong.

I-14. Consider a Gaussian surface

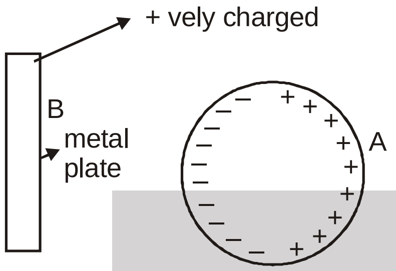
$\phi = (800 - 400)A = \frac{q_{in}}{\varepsilon_0}$

$q_{in} = 400\varepsilon_0 A$

SECTION (J) :

J-1. Electric field lines never enter a metallic conducter ($\because$ E = 0 inside a conductor) & they fall normally on the surface of a metallic conductor (Because whole surface is at same potential so lines are perpendicular to equipotenial surface).

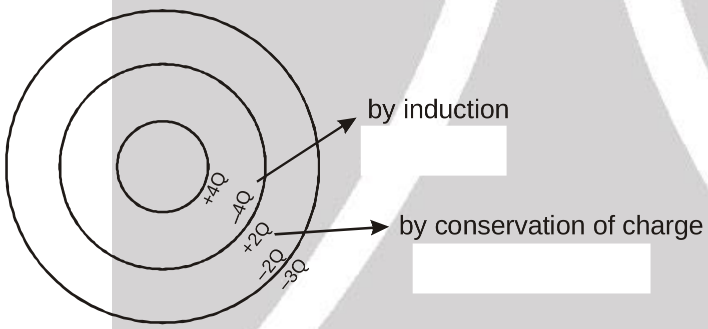
J-2.



The given diagram shows induction on sphere (metallic) due to metal plate.
Since distance between plate and –ve charge is less than that between plate and +ve charge. electric force acts on object towards plate.

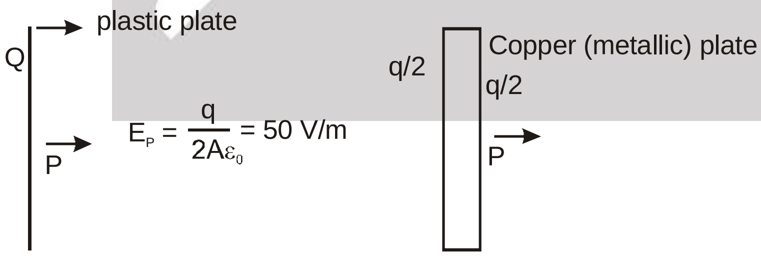
J-3. Induction takes place on outer surface of sphere producing non-uniform charge distribution & since external electric field can not enter the sphere, so interior remains charge free.

J-4.



Given diagram shows the charge distribution on shells due to induction & conservation of charge.

J-5.



$$E_P = \frac{q/2}{2A\varepsilon_0} + \frac{q/2}{2A\varepsilon_0} = \frac{q}{2A\varepsilon_0} = 50 V/m$$

J-6. Due to outer charge, since there is no charge induced inside the sphere, so no electric field is present inside the sphere.

J-7. Since field lines are always perpendicular to conductor surface field lines can't enter into conductor so only option C is correct.

J-8. Since A, B and C are at same potential electric field inside C must be zero. For this final charge on A and B must be zero and final charge on C = Q + q_1 + q_2. (By conservation of charge)

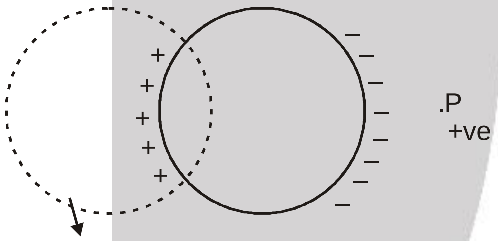
$\because$ All charge comes out to the surface of C.

J-9. Car (A conductor) behaves as electric field shield in which a person remains free from shock.

J-10. Potential of B = Potintial at the centre of B
= Potential due to induced charges + potential due to A.
= 0 + (+ ve) $\therefore$ Potential of B is +ve.

J-11. Since electric field produced by charge is conservative, so work done in closed zero path is zero.

J-12.



closed suface
cUh l rg
enclosed charge = +ve $\Rightarrow$ flux through closed suface = +ve.

J-13. Since distance between q and A is less then distance between q and B.

$\therefore$ $\sigma_A > \sigma_B$. & $E_A > E_B$.
but, $V_A = V_B$ because surface of conductor is equipotential.

PART - III

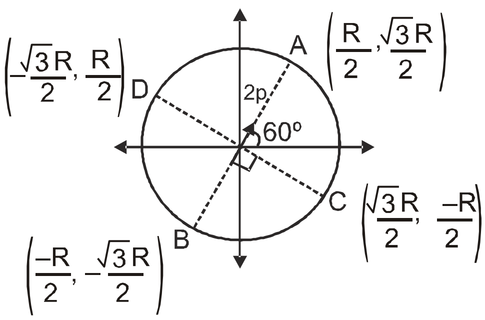
1. In situation A, B and C, shells I and II are not at same potential. Hence charge shall flow from sphere I to sphere II till both acquire same potential.
If charge flows, the potential energy of system decreases and heat is produced.
In situations A and B charges shall divide in some fixed ratio, but in situation C complete charge shall be transferred to shell II for potential of shell I and II to be same.

$\therefore$ (A) $\rightarrow$ p, q , (B) $\rightarrow$ p, q , (C) $\rightarrow$ p, q, s

In situation D, both the shells are at same potential, hence no charge flows through connecting wire.

$\therefore$ (D) $\rightarrow$ r, s

2. The resultant dipole moment has magnitude $\sqrt{(\sqrt{3}\ P)^2 + P^2}$ = 2P at an angle $\theta = \tan^{-1} \frac{\sqrt{3}\ P}{P}$ = 60° with positive x direction.



Diameter AB is along net dipole moment and diameter CD is normal to net dipole moment.

$\therefore$ Potential at A $\left(\frac{R}{2}, \frac{\sqrt{3}\,R}{2}\right)$ is maximum

Potential is zero at C $\left(\frac{\sqrt{3}\,R}{2}, -\frac{R}{2}\right)$ and D $\left(-\frac{\sqrt{3}\,R}{2}, \frac{R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{4p}{R^3}$ at A $\left(\frac{R}{2}, \frac{\sqrt{3}\,R}{2}\right)$ and B $\left(-\frac{R}{2}, -\frac{\sqrt{3}\,R}{2}\right)$

Magnitude of electric field is $\frac{1}{4\pi\varepsilon_0}\frac{2p}{R^3}$ at C $\left(\frac{\sqrt{3}\,R}{2}, -\frac{R}{2}\right)$ and D $\left(-\frac{\sqrt{3}\,R}{2}, \frac{R}{2}\right)$

EXERCISE-2

PART - I

1.

Let the two charges are q & (20 – q) μC

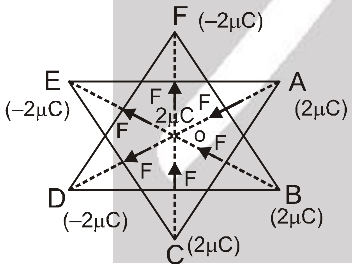
$\therefore F_e = \frac{K(q)(20-q)}{r^2}$

F_e will be max, when $\frac{dF_e}{dq} = 0$

or $\frac{dFe}{dq} = \frac{K}{r^2}(20 - 2q) = 0$

$\therefore$ q = 10 μC.

2.



The given figure shows force diagram for charge at O due to all other charges with r = $\frac{10}{\sqrt{3}}$ cm

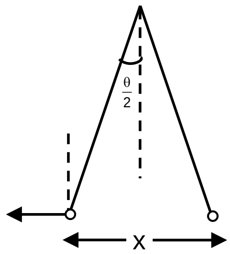
$\therefore$ $F_{net} = 2F + 4F\cos 60^o = 4F$

$= \frac{4k(2\mu c)(2\mu c)}{\left(\frac{10}{\sqrt{3}100}\right)^2} = \frac{4\times 9\times 10^9 \times 2\times 2\times 10^{-12}}{\left(\frac{1}{300}\right)} = 36 \times 4 \times 300 \times 10^{-3}$ N = 43.2 N. (Towards E)

3. Attraction is possible between a charged and a neutral object.

4. There is no point near electric dipole having E = 0.

5.



$$\frac{kq^2}{x^2} = T\sin\frac{\theta}{2}$$

$$mg = T\cos\frac{\theta}{2} \qquad q^2 = \frac{mgx^3}{2k\ell}$$

$$\frac{kq^2}{mgx^2} = \tan\frac{\theta}{2} \qquad q = \left(\frac{mg}{2k\ell}\right)^{\frac{1}{2}} x^{\frac{3}{2}} \qquad q^{\frac{-1}{3}} = \left(\frac{mg}{2k\ell}\right)^{-\frac{1}{6}} x^{-\frac{1}{2}}$$

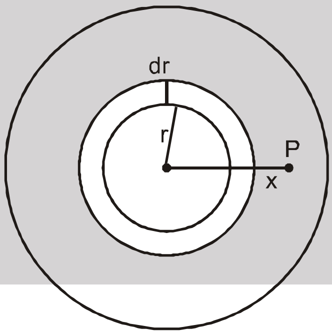
$$\frac{2kq^2}{mgx^2} = \frac{x}{\ell} \quad \Rightarrow \quad x^3 = \frac{2k\ell q^2}{mg}$$

$$2v = \frac{(\text{const})(c)2}{3} q^{-1/3} \quad \Rightarrow \quad x = (\text{const})q^{\frac{2}{3}}$$

$$2v = 2v = (\text{const})x^{-\frac{1}{2}} \quad \Rightarrow \quad \frac{dx}{dt} = (\text{const})\frac{2}{3}q^{-\frac{1}{3}}$$

6. a & b can't be both +ve or both – ve otherwise field would have been zero at their mid point.
b can't be positive even, otherwise the field would have been in –ve direction to the right of mid point
answer is (A)

7. Lets take a small spherical element of thickness dr.
Electric field at point P due to this element:



$$dE = \frac{K\,dq}{x^2}$$

$\therefore$ Total electric field :

$$E = \int \frac{K\,dq}{x^2}$$

$$E = \int \frac{K \text{ (density) (volume of the element)}}{x^2}$$

$$E = \int_{r=\infty}^{r=x} \frac{K(\rho_0 r^2)\ (4\pi r^2 dr)}{x^2}$$

$$E = \frac{K\rho_0}{x^2}\left(\frac{x^5}{5}\right) = (x^3) \quad \therefore \quad (E \propto x^3)$$

8. B & C are equipotential and field is conservative, therefore :

$\therefore \quad W_{CA} = W_{BA} = -\int_{2a}^{a} \frac{\lambda}{2\pi\varepsilon_0 r} q\, dr = \frac{q\lambda}{2\pi\varepsilon_0} \ell n\, 2.$

9. Comparison can be shown as :

$V \to 2V$

$\Rightarrow \quad k \to 4k$

$\Rightarrow \quad PE_{max} \to 4PE_{max}$

$\Rightarrow \quad r \to r/4$

10. Potential energy = $-\vec{p}_1 \cdot \vec{E}$; where, $\vec{E}$ = Electric field due to dipole p_2.

$\therefore \quad U_{12} = -(p_1)(E_2)$

$U_{12} = -(p_1)\left(\frac{2Kp_2\cos\theta}{r^3}\right)$

11. Total flux through closed cubical vessel = $\frac{q}{\varepsilon_0}$.

& Flux through one face = $\frac{1}{6}\left(\frac{q}{\varepsilon_0}\right)$

So, total flux passing through given cubical vessel is = $5\left(\frac{q}{6\varepsilon_0}\right)$; (as vessel has 5 faces)

12. $E = \frac{\sigma}{2\varepsilon_0}$ due to non-conducting sheet.

$\Delta E' = \frac{\sigma'}{\varepsilon_0}$ due to conducting sheet, but $\sigma' = \frac{\sigma}{2}$

$\therefore$ Result is same i.e. E' = E

13. Electric field at given location is only due to inner solid metalic sphere.

14. Inside the given sphere, there will not be any effect of external electric field. So net electric field will only be due to point charge 'q' at centre.

$\therefore$ Graph is (A)

15. In the above question, if Q' is removed then which option is correct:
On removing Q', no effect is there in previous situation as Q ' does not affect the electric field at inside point.

16. In a conductor, potential is same everywhere

$\therefore$ Potential at A = potential at centre = $V_{\text{due to p}} + V_{\text{due to induced charges}}$

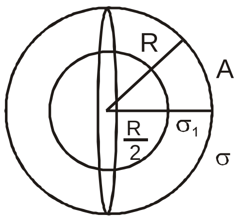
$= \frac{kp}{(r\sec\phi)^2} + 0 \qquad = \frac{kp\cos^2\phi}{r^2}$

17. By definition

18. Distribution of charge in the volume of sphere depends on uniformity of material of sphere.

19. Since, no external electric field can enter into a conductor so force experienced by Q = 0

20.



Let surface charge density on inner shell is σ_1

Due to inner sphere, field at A = $\frac{1}{4} \times \frac{\sigma_1}{\varepsilon_0} = \frac{\sigma_1}{4\varepsilon_0}$, and electrostatic pressure at point A. = $\frac{\sigma^2}{2\varepsilon_0} + \frac{\sigma_1\sigma}{4\varepsilon_0}$

Net force one hemisphere = $\left(\frac{\sigma^2}{2\varepsilon_0} + \frac{\sigma_1\sigma}{4\varepsilon_0}\right) \pi R^2 = 0$

$\Rightarrow \quad \sigma^2 + \frac{\sigma_1\sigma}{2} = 0$, or $\sigma_1 = -2\sigma$

21. $\frac{1}{4\pi\epsilon_0}\frac{q}{r} = v$

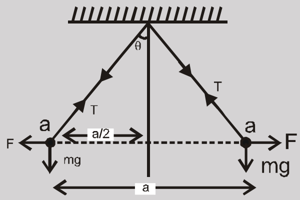
$n\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi a^3 \quad \Rightarrow \quad n = \frac{a^3}{2r^3}$

$v^1 = \frac{1}{4\pi\epsilon_0}\frac{n.q}{a} = v_r \frac{a^3}{2r^3}\frac{1}{a}$

$v^1 = \left(\frac{a^2}{2r^2}\right)v$

PART - II

1. As the charge on one of the balls is removed so electrostatic force between the balls is zero.
The balls will first go down and due to contact with each other, charge on one ball is equally distributed on both balls and then the balls get separated due to electrostatic repulsion.
At equilibrium :



Here F = Force (electrostatic) between two balls = $\frac{kq^2}{a^2}$

By force balance, T sinθ = F and T cosθ = mg

$\Rightarrow \quad mg \tan\theta = F$ ....(1) [for small angle $\tan\theta \approx \sin\theta = \frac{a}{2L}$]

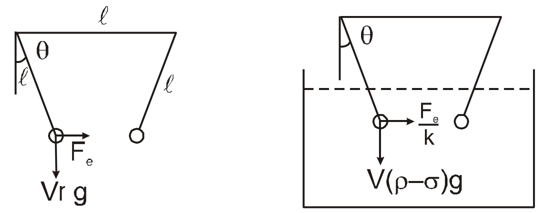
$\Rightarrow \quad mg \tan\theta = \frac{kq^2}{a^2} \Rightarrow$ or $kq^2 = \frac{mga^3}{2L}$

Now the balls are discharged and charge on each ball = $\frac{q}{2}$. & the distance between two ball = b.

By equation (1), $mg \tan\theta = F_1 = \frac{kq^2}{4b^2}$ & $mg\frac{b}{2L} = \frac{kq^2}{4b^2}$

By putting value of kq^2 , $\frac{a^3}{2L} = \frac{4b^3}{2L} \quad \Rightarrow \quad b = \frac{a}{(4)^{1/3}}$

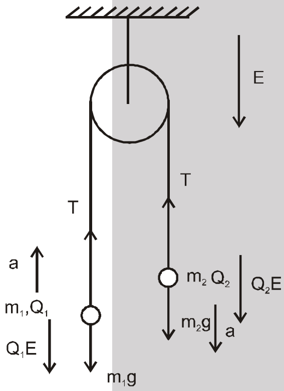
2.



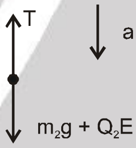
$\tan\theta = \frac{F_e}{V\rho g} = \frac{F_e/k}{V(\rho-\sigma)g} \quad \Rightarrow \quad k = \frac{\rho}{\rho-\sigma} = \frac{2.4}{2.4-0.8} = 1.5$

$\mathbf{k = \frac{\rho}{\rho-\sigma} = \frac{2.4}{2.4-0.8} = 1.5}$ **so 4k = 6**

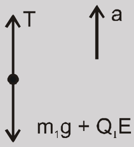
3.



Force on m_1 and m_2 due to electric field = Q_1E and Q_2E downward Let acceleration of both masses is 'a' as shown in figure:



∴ By F.B.D of m_2 : $m_2g + Q_2E - T = m_2a$ ....(1)



By F.B.D of m_1 : $T - m_1g - Q_1E = m_1a$ ....(2)

By both equations : $a = \frac{(m_2 - m_1)g + (Q_2 - Q_1)E}{m_1 + m_2}$

and by putting m_1 = 2m, m_2 = 3m and $Q_1 = Q_2$ we get a = 2m/sec^2

4.

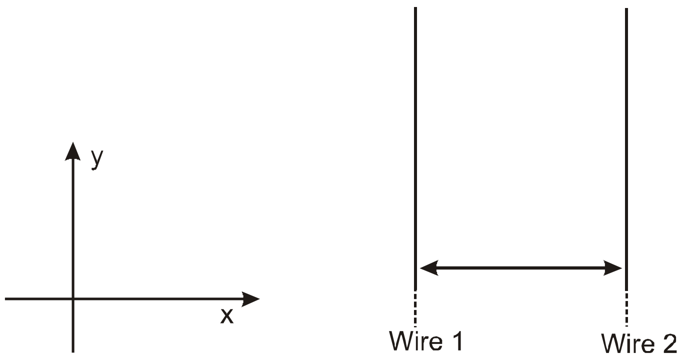


Figure shows two like charged infinitely long wires with $\lambda = 3 \times 10^{-6}$ cm

Now, electric field due to wire 1 on long wire 2 = $\frac{2k\lambda}{r}\hat{i}$

Now, force on unit length of wire (2) = Charge on unit length × $\frac{2k\lambda}{r}$

$= \lambda \times \frac{2k\lambda}{r} = \frac{2k\lambda^2}{r} = \frac{2\times9\times10^9\times9\times10^{-12}}{2\times10^{-2}}$ = 8.1 N/m

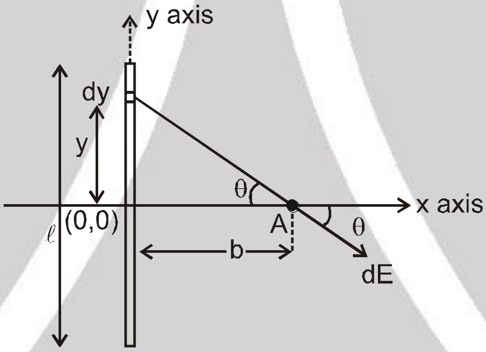
Now work done to bring them dr cm closer $\Rightarrow dW = \frac{2k\lambda^2}{r}dr$

$\Rightarrow \quad -\int dW = -\int_2^1 \frac{2k\lambda^2}{r}\,dr = 2k\lambda^2 \ln 2 = 0.1129$ J/m

$= \frac{11.29}{100}$ J/m

$\therefore$ x = 11

5. Let a rod of length ℓ and charge q and a point A are as shown in figure (at distance = b). Charge per unit length = $\frac{q}{\ell}$. Let a small component dy along the rod at distance y from centre of rod is considered.



Charge on the element dy = $\frac{q}{\ell}dy$

$\therefore$ Electric field at point A due to this charge $\Rightarrow$ $dE = \frac{K\left[\frac{q}{\ell}dy\right]}{y^2+b^2}$

Now, x component of dE = dE cos θ = $\frac{Kqdy}{\ell(y^2+b^2)}\cdot\frac{b}{\sqrt{y^2+b^2}}$

and y component of dE = – dE sin θ = $-\frac{Kqdy}{\ell(y^2+b^2)}\cdot\frac{y}{\sqrt{y^2+b^2}}$

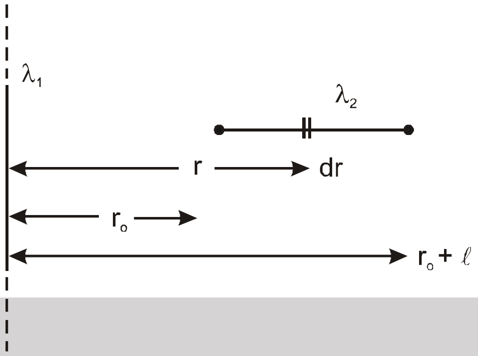
Now, electric field due to total length of rod = $\int dE_x = \int_{-\ell/2}^{\ell/2}\frac{Kqdy.b}{\ell(y^2+b^2)^{3/2}} = \frac{Kqb}{b^2}\left[\frac{1}{\sqrt{\frac{\ell^2}{4}+b^2}}\right]$

$= \frac{2Kq}{b\sqrt{\ell^2+4b^2}} = \frac{q}{2\pi\varepsilon_0 b\sqrt{\ell^2+4b^2}}$ (along x axis) and $\int dE_y = -\int_{-\ell/2}^{\ell/2}\frac{Kqydy}{\ell(y^2+b^2)^{3/2}} = 0$

So, net electric field is along x–axis and is equal to $\frac{q}{2\pi\varepsilon_0 b\sqrt{\ell^2+4b^2}}$

6. Electric field due to infinitely long charged wire at distance r from it = $\frac{2k\lambda}{r}$

Now, we take small element dr on wire (2) which is having distance r from wire (1)



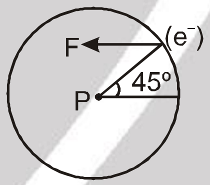
Now, force on this element = (λ_2 dr). $\frac{2k\lambda_1}{r}$

or $\quad dF = \frac{2k\lambda_1\lambda_2 dr}{r}$

Total force = $\int dF = \int_{r_o}^{r_o+\ell} \frac{2k\lambda_1\lambda_2}{r} dr = 2k\lambda_1\lambda_2 \, \ell n\left(\frac{r_o+\ell}{r_o}\right) = \frac{\lambda_1 \lambda_2}{4\pi\varepsilon_0} \ell n(4)$ (since $\ell = r_0$)

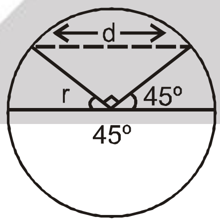
7. Electric field inside the cavity = $\frac{\rho\vec{a}}{3\varepsilon_0}$ $\left[\begin{array}{l}\text{Here } \vec{a} = \text{along line joining} \\ \text{centers of sphere and cavity}\end{array}\right]$

Force on the e^- inside cavity = $\frac{\rho\vec{a}}{3\varepsilon_0}$ (e)



Cavity ——→ $\qquad \therefore \qquad$ acceleration of electron $a_e = \frac{\rho ae}{3\varepsilon_o m}$.

Now for distance



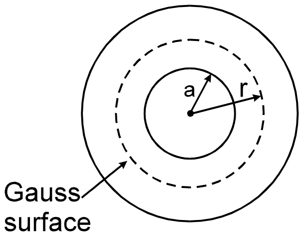
Cavity ——→ $\qquad d = \sqrt{r^2 + r^2} = \sqrt{2}r$

by S = ut + 1/2 at^2, $\quad \sqrt{2}r = \frac{1}{2} \times \frac{\rho ae}{3m\varepsilon_0} t^2$

$\Rightarrow t = \left(\frac{6\sqrt{2}\, m r \varepsilon_0}{e a \rho}\right)^{1/2}$

8. P.D.= $\int \vec{E}.\vec{dr}$ and E between spheres does not depend on charge on outer sphere.

9.



Applying Gauss's law on Gaussian surface shown in figure :

$$E\,4\pi r^2 = \frac{\int_a^r \rho 4\pi x^2 dx}{\epsilon_0} = \frac{\rho 4\pi (r^3 - a^3)}{3\epsilon_0} \qquad \therefore \qquad E = \frac{\rho (r^3 - a^3)}{3\epsilon_0 r^2}$$

Now using $\int dV = -\int \vec{E}.d\vec{r}$

$$\Rightarrow \qquad V_A - V_B = \int_A^B \vec{E}.d\vec{r} = \int_a^b \frac{\rho (r^3 - a^3)dr}{3\epsilon_0 r^2} = \int_a^b \frac{\rho}{3\epsilon_0} rdr - \int_a^b \frac{\rho}{3\epsilon_0}\frac{a^3}{r^2}dr$$

$$= \frac{\rho}{3\epsilon_0}\left[\frac{b^2 - a^2}{2}\right] - \frac{\rho a^3}{3\epsilon_0}\left[\frac{1}{a} - \frac{1}{b}\right]$$

$$= \frac{\rho}{3\epsilon_0}\left[\frac{b^2 - a^2}{2} - a^2 + \frac{a^3}{b}\right]$$

$$= \frac{\rho}{6\epsilon_0}\left[b^2 - a^2 - 2a^2 + \frac{2a^3}{b}\right]$$

$$= \frac{\rho}{6\epsilon_0}\left[b^2 - 3a^2 + \frac{2a^3}{b}\right]$$

Put b = 2a

$$V_A - V_B = \frac{\rho a^2}{3\varepsilon_0}$$

ALTERNATIVE:

$$V_B = \frac{kQ}{b} = \frac{k\rho}{b}\frac{4}{3}\pi (b^3 - a^3)$$

V_A can be found by taking a shell of radius r and calculate potential at 'A' due to that shell and integrating

$$V_A = \int_{r=a}^{b} \frac{k\rho 4\pi r^2 dr}{r} = \frac{K\rho 4\pi}{2}(b^2 - a^2)$$

$$\therefore \qquad V_B - V_A = \frac{4\pi k\rho}{6}\left[2b^2 - \frac{2a^3}{b} - 3b^2 + 3a^2\right]$$

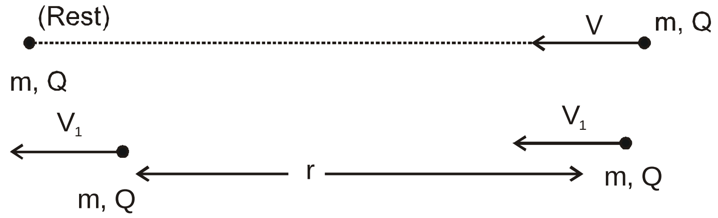
$$V_B - V_A = \frac{\rho}{6\epsilon_0}\left[3a^2 - b^2 - \frac{2a^3}{b}\right]$$

$$V_A - V_B = \frac{\rho}{6\epsilon_0}\left[-3a^2 + b^2 + \frac{2a^3}{b}\right]$$ **Ans....**

Put b = 2a

$$V_A - V_B = \frac{\rho a^2}{3\varepsilon_0}$$

10.



The particle is projected from infinity towards other particle. As the 2nd particle gets closer to 1st particle, force of repulsion is acting on both of them, which decreases one's speed and increases other's speed. At minimum separation, both particles have same velocity (v_1). Let closest distance of approach = r .;

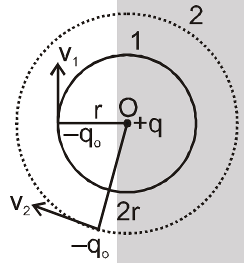
So, by energy conservation:

$$O + \frac{1}{2}mv^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_1^2 + \frac{kq^2}{r} \Rightarrow \frac{kq^2}{r} = \frac{1}{2}mv^2 - mv_1^2 \text{ ....(1)}$$

Also, by momentum conservation : $mv = mv_1 + mv_1 \Rightarrow v_1 = \frac{v}{2}$

So by eq (1) $\frac{kq^2}{r} = \frac{1}{2}mv^2 - \frac{1}{4}mv^2 = \frac{1}{4}mv^2 \Rightarrow r = \frac{4\,kq^2}{mv^2} = \frac{q^2}{\pi \epsilon_o mv^2}$

11.



Required centripetal force is acquired from electric force between two charges in situation (1)

or $\frac{mv_1^2}{r} = \frac{kqq_o}{r^2} \Rightarrow mv_1^2 = \frac{kqq_o}{r}$

In situation (2), $\frac{mv_2^2}{2r} = \frac{kqq_o}{(2r)^2} \Rightarrow mv_2^2 = \frac{kqq_o}{2r}$

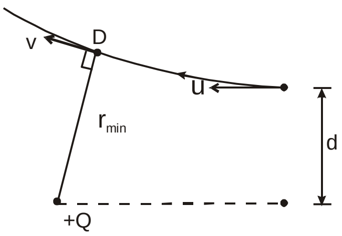
If additional energy E is added to situation (1) to change it to situation (2) then by energy conservation:

$$\frac{1}{2}mv_1^2 - \frac{kqq_o}{r} + E = \frac{1}{2}mv_2^2 - \frac{kqq_o}{2r}$$

or $E = \frac{1}{2}m(v_2^2 - v_1^2) + \frac{kqq_o}{r} - \frac{kqq_o}{2r}$ $\therefore$ $E = \frac{-kqq_o}{4r} + \frac{kqq_o}{2r} = \frac{kqq_o}{4r} = \frac{qq_o}{16\pi \epsilon_o r}$

12. The path of the particle will be as shown in the figure. At the point of minimum distance (D) the velocity of the particle will be $\perp$ to its position vector w.r.t. +Q.

Now by conservation of energy :



$\frac{1}{2}mu^2 + 0 = \frac{1}{2}mv^2 + \frac{KQq}{r_{min}}$ ......(1)

$\because$ Torque on q about Q is zero, hence angular momentum about Q will be conserved

$\Rightarrow$ $m\,v\,r_{min} = m\,ud$ ......(2)

$\therefore$ By putting (2) in (1) $\Rightarrow$ $\frac{1}{2}mu^2 = \frac{1}{2}m\left(\frac{ud}{r_{min}}\right)^2 + \frac{KQq}{r_{min}}$

$\Rightarrow$ $\frac{1}{2}mu^2\left(1-\frac{d^2}{r_{min}^2}\right) = \frac{mu^2d}{r_{min}}$ { $\because$ $KQq = mu^2d$ (given) }

$\Rightarrow$ $r^2_{min} - 2r_{min}\,d - d^2 = 0$ $\Rightarrow$ $r_{min} = \frac{2d \pm \sqrt{4d^2+4d^2}}{2} = d(1 \pm \sqrt{2})$

Distance cannot be negative

$\therefore$ **$r_{min} = d(1+\sqrt{2})$ = 1 m Ans.**

13. When 1st ball is released, its potential energy due to the rest of the system will be converted into kinetic energy :

$\therefore$ $K.E._1 = \sum_{i=2}^{i=2019}(P.E.)_{1,i}$

[Here $(P.E.)_{1,i}$ = Potential energy between 1st ball and ith ball]

$K.E_1 = (P.E)_{1,2} + \sum_{i=3}^{i=2019}(P.E.)_{1,i}$ ....(1)

Now, when 2nd ball is released, it also takes its self potential energy from system :

So, kinetic energy of 2nd ball : $K.E._2 = \sum_{i=3}^{i=2019}(P.E.)_{2,i}$

Now $(K.E)_1 - (K.E)_2 = (P.E)_{1,2} + \sum_{i=3}^{i=2019}(P.E)_{1,i} - \sum_{i=3}^{i=2019}(P.E)_{1,i}$

$(K.E)_1 - (K.E)_2 = (P.E)_{1,2}$

Given: $(K.E)_1 - (K.E)_2 = K$ and $(P.E)_{1,2}$

$= \frac{q^2}{4\pi\epsilon_o a}$; $K = \frac{q^2}{4\pi\epsilon_o a}$ $\Rightarrow q = \sqrt{4\pi\epsilon_o K a}$

14. Given, $V = 3x + 4y$

So, $\vec{E} = -\left[\frac{\partial v}{\partial x}\hat{i} + \frac{\partial v}{\partial y}\hat{j} + \frac{\partial v}{\partial z}\hat{k}\right]$ or $\vec{E} = -(3\hat{i} + 4\hat{j})$

So, force on particle $= q\vec{E} = -(3\hat{i} + 4\hat{j}) \times 1$ N

and acceleration of particle $= \frac{-(3\hat{i}+4\hat{j})}{10}$ m/s^2

Now, $a_x = -0.3$ m/s^2 and $a_y = -0.4$ m/s^2

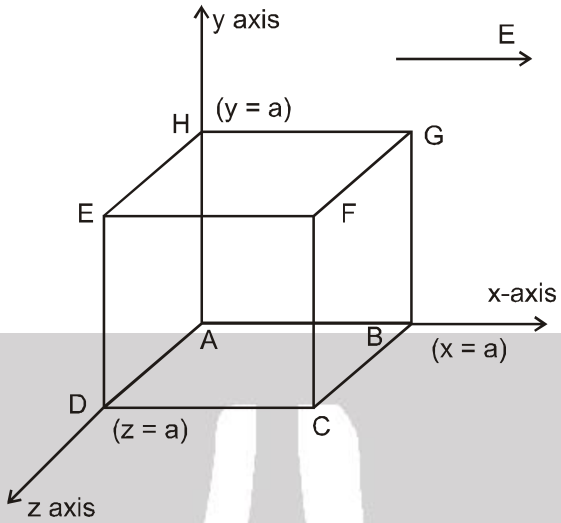
At x - axis y = 0 So by ($s = ut + \frac{1}{2}at^2$) in y direction :

$-3.2 = -\frac{1}{2} \times \frac{4}{10} \times t^2$ $\Rightarrow t = 4$ sec.

Now, velocity at 4 sec.$\Rightarrow v_x = \frac{-3}{10} \times 4 = -1.2$ m/sec $\Rightarrow$ $v_y = \frac{-4}{10} \times 4 = -1.6$ m/sec

So, velocity vector $= -1.2\hat{i} - 1.6\hat{j}$

15. Given $\vec{E} = E_0 x \hat{i}$ So flux = $\int E_0 x \hat{i} . d\vec{s}$



Now total flux = sum of flux through each face

$\therefore$ Total flux = $\int_{ABCD} \vec{E}.d\vec{s} + \int_{EFGH} \vec{E}.d\vec{s} + \int_{ABGH} \vec{E}.d\vec{s} + \int_{CDEF} \vec{E}.d\vec{s} + \int_{ADEH} \vec{E}.d\vec{s} + \int_{BCFG} \vec{E}.d\vec{s}$

first four terms are = 0, because angle between $\vec{E}$ and $d\vec{s}$ is 90º so $\vec{E}.d\vec{s} = 0$

Total flux = $\int_{ADEH} \vec{E}.d\vec{s} + \int_{BCFG} \vec{E}.d\vec{s}$

For ADEH: $\vec{E} = E_0 \hat{i} = E_0 \hat{i}\ (0) = 0$ [$\because$ x = 0]

for BCFG: $\vec{E} = E_0 x\ \hat{i} = E_0 a \hat{i}$

So, total flux = 0 + $\int E_0 a\ \hat{i} . ds \hat{i}$

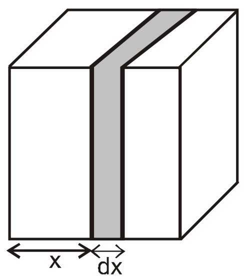
$= E_0 a \int ds = AE_0 a = E_0 a^3$

$\Rightarrow \therefore\ q_{in} = \epsilon_0 E_0 a^3 = \left[\dfrac{8.85 \times 10^{-12} \times 4 \times 10^3 \times 8 \times 10^{-6}}{2 \times 10^{-2}}\right] = 1.416 \times 10^{-11}$ C

16. Net flux through the cube, $\phi_{net} = \dfrac{q_{in}}{\varepsilon_0}$

To find q_{in}, let's divide the cube into small elements, and consider a small element of width dx as shown.

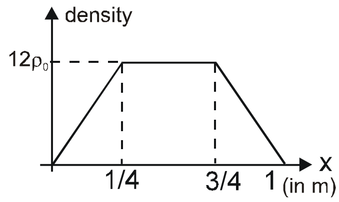
Charge on the small element = (ρ) (A.dx)



Total charge = $\int \rho A dx$

$= A\int \rho dx$

= (A) (Area of ρ–x graph)

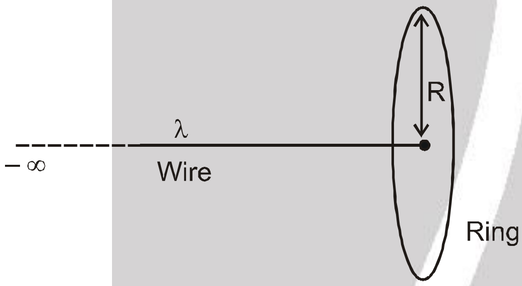


$= (1)^2 \left(\frac{1}{2}\left(1+\frac{1}{2}\right)(\rho_0)\right) 12$

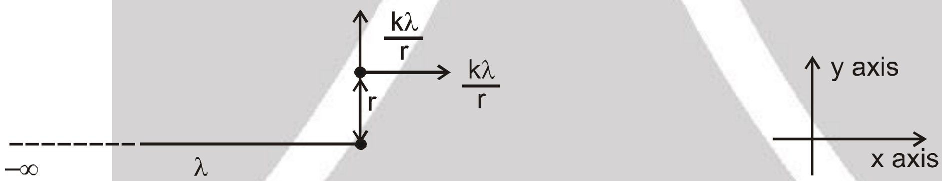
$= \frac{3}{4}\rho_0 \times 12.$

$\Rightarrow$ Net flux $\phi_{net} = \frac{q_{in}}{\varepsilon_0} = \frac{9\rho_0}{\varepsilon_0} = 9$ V.m (as $\rho_0 = 8.85 \times 10^{-12}$)

17.



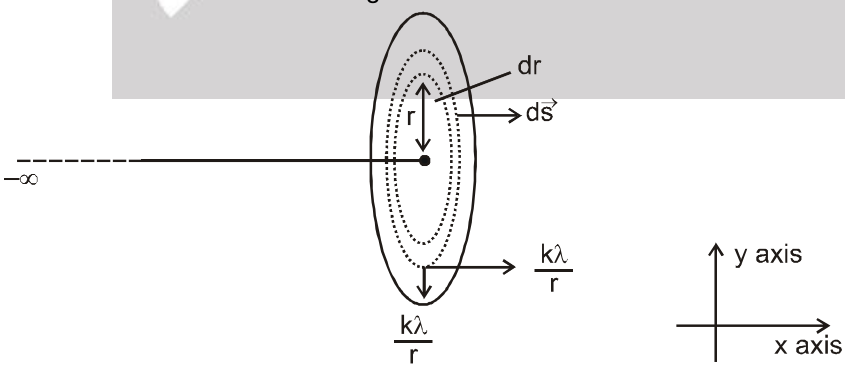
Let a wire and ring are placed as shown in figure. Due to semi-infinite wire, electric field at one of its end at distance r is as shown



So, the Electric field = $\frac{k\lambda}{r}\hat{i} + \frac{k\lambda}{r}\hat{j}$

Now we take a ring element of radius r and thickness dr.

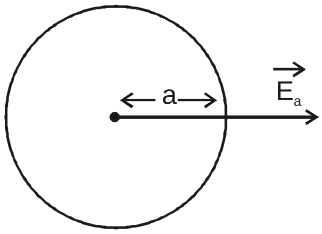
Let $\vec{ds}$ = Area vector of this ring element $\therefore$ $\vec{ds} = 2\pi r dr\ \hat{i}$



Now electric flux due to this element :

$= \int \frac{k\lambda}{r}\ \hat{i}\ d\vec{s} + \int \frac{k\lambda}{r}\ \hat{j}\ d\vec{s} = \int_o^R \frac{k\lambda}{r}.\, 2\pi r\, dr + 0 = 2\pi k \lambda .\, R = \frac{R\lambda}{2\epsilon_o}$

18. Given $\vec{E} = 2r\hat{r}$ So, at radius a $\Rightarrow \vec{E}_a = 2a\,\hat{r}$



$$\frac{q_{in}}{\epsilon_o} = \int \vec{E}_a.d\vec{s} \quad = \frac{q_{in}}{\epsilon_o} = E_a \int ds$$

$$\frac{q_{in}}{\epsilon_o} = E_a.\ 4\pi a^2 = 2a^3.4\pi \qquad \therefore q_{in} = 4\pi \epsilon_o\ 2a^3$$

19. Total charge $Q = \int \rho dV = \int_{r=0}^{r=R} (Kr^a)\ (4\pi r^2 dr) = \frac{4\pi k}{a+3}(R^{a+3})$

and $Q' = \int \rho d\ V = \int_{r=0}^{r=R/2} (Kr^a)\ (4\pi r^2 dr) = \frac{4\pi k}{a+3}\left(\frac{R}{2}\right)^{a+3}$

According to question

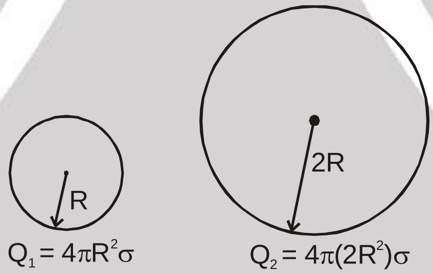
$$\frac{1}{4\pi\varepsilon_0}\ \frac{Q'}{(R/2)^2} = \frac{1}{8}\left(\frac{1}{4\pi\varepsilon_0}\ \frac{Q}{R^2}\right)$$

$(2)^{a+3} = 32$

Putting the value of Q and Q´ get

a = 2 **Ans. 2**

20. (A) Surface charge density = σ



Charge on bigger sphere $Q_2' = \left(\frac{Q_1 + Q_2}{C_1 + C_2}\right).\ C_2$

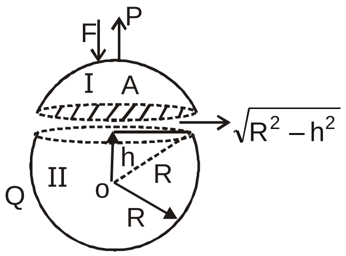
$$Q_2' = \left(\frac{4\pi R^2\sigma + 4\pi(2R)^2\sigma}{4\pi\epsilon_0 R + 4\pi\epsilon_0\ 2R}\right) \times 4\pi\epsilon_0 \times 2R = \frac{40\pi R^2\sigma}{3}$$

Surface charge density on bigger sphere $\sigma' = \frac{40\pi R^2\sigma}{3\times 16\pi R^2} = \frac{5}{6}\sigma$.

Put σ = 12 μC/m^2

Ans. 10

21.



Sphere is cut along the plane shown by dotted part.

P = Electrostatic pressure exerted on each point of sphere due to charge sphere due to charge on it

$\therefore$ force required to hold both pieces together F $\Rightarrow$ P × A (Where A = Area of shaded portion)

$\therefore$ $F = \frac{\sigma^2}{2\epsilon_0} \times A$ [Where $\frac{\sigma^2}{2\epsilon_0}$ = P = (electrosatic pressure on sphere)

$= \left[\frac{Q}{4\pi R^2}\right]^2 \left[\frac{1}{2\epsilon_0}\right]\left[\pi \left(\sqrt{R^2 - h^2}\right)^2\right]$

$= \frac{Q^2}{16\pi^2 R^4} \times \frac{1}{2\epsilon_0} \times \pi (R^2 - h^2)$ $F = \frac{Q^2(R^2 - h^2)}{32\pi\epsilon_0 R^4}$

Put h = R/2

$F = \frac{3kQ^2}{32R^2}$ **Ans.**

PART - III

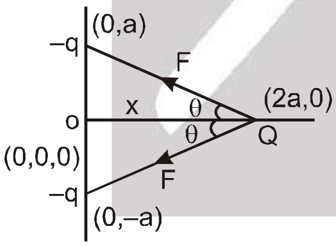
1. (A) Charging by conduction has charge distribution depending on size of bodies.

(B) Charge is invariant with velocity.

(C) Charge requires mass for existence

(D) Repulsion shows charge of both bodies because attraction can be there between charged and uncharged body.

2.



(i) From diagram, force on Q at general position x, is given by

$F_{net} = -2F\cos\theta = -\frac{kQqx}{(a^2 + x^2)^{3/2}}$ (Towards origin)

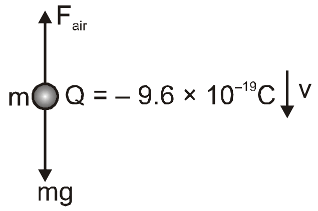
(ii) When charge moves from (2a, 0) to origin O, force keeps on acting on Q and becomes zero at O.

$\therefore$ Velocity of Q is max. at O.

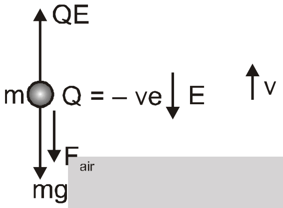
$\therefore$ Velocity of Q is max. at O.

(iii) Motion is SHM for very small displacements. & 2a is not very small os motion is periodic but not SHM.

3.



(initially) $\therefore$ $mg = f_{air}$



(ii) (finelly) $\therefore$ $QE = mg + f_{air} = 2mg$

$\therefore$ charge is –ve, so electric field 'E' is directed downwards.

& QE = 2 mg

& QE = 2 mg

$\therefore$ $E = \dfrac{2mg}{Q} = \dfrac{2 \times 1.6 \times 10^{-18} \times 10}{9.6 \times 10^{-19}} = \dfrac{1}{3} \times 10^2 \, NC^{-1}$

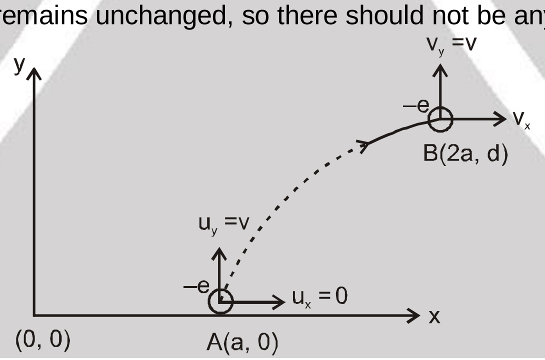
4. (i) At any point P inside the sphere, electric field $\Rightarrow E_P = \dfrac{kQr}{R^3}$.

$\therefore$ E_P increases as r increases.

(ii) At any point M outside the sphere, $E_M = \dfrac{kQ}{r^2}$

$\therefore$ E_M decreases as r increases.

5. As velocity along y-axis remains unchanged, so there should not be any electric field along y axis.



As velocity along x axis is increasing, so force on the electron must be along +x direction, so electric field must be towards –x direction.

So force on the electron is :

F = qE = eE

acceleration, $a = \dfrac{eE}{m}$ towards +x direction

From A $\rightarrow$ B

$S_y = u_y t$

or $d = vt$ $\Rightarrow$ $\therefore$ $t_{A \rightarrow B} = \dfrac{d}{V}$

From : A $\rightarrow$ B

$S_x = u_x t + \frac{1}{2} a_x t^2$

or $a = 0 + \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{d}{V}\right)^2$

$\Rightarrow$ $E = \frac{2amV^2}{ed^2}$ toward-x direction ....(1)

(A) Velocity along x axis at B :
From A $\rightarrow$ B
$V_x = u_x + a_x t$

or $V_x = 0 + \left(\frac{eE}{m}\right)\left(\frac{d}{V}\right)$ $\Rightarrow$ $V_x = \frac{eEd}{mV}$

where, $E = \frac{2amv^2}{ed^2}$ $\Rightarrow$ $\therefore V_x = \frac{2aV}{d}$

(D) Net velocity vector at B
$\vec{V}_B = V_x\hat{i} + V_y\hat{j}$

$\vec{V}_B = \frac{2aV}{d}\hat{i} + V\ \hat{j}$

(B) Rate of work done at B = Power = $\vec{F}\cdot\vec{V}_B$

$= \left(eE\hat{i}\right).\left(\frac{2aV}{d}\hat{i} + V\hat{j}\right)$

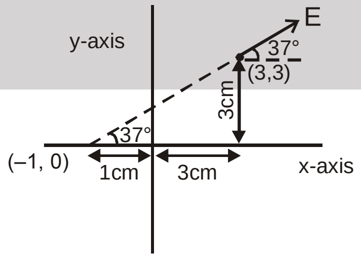
$= eE\left(\frac{2aV}{d}\right)$; where, $E = \frac{2amV^2}{ed^2}$

$\Rightarrow$ $\therefore P = \frac{4ma^2V^3}{d^3}$

(C) Rate of work done at A :
$P_A = \vec{F}\cdot\vec{V}_A$
$= \left(eE\hat{i}\right)\cdot\left(V\hat{j}\right) = 0$

6. By definition of electric field

7. At point (2cm, 0), field is along x-axis. It is possible only when the particle is situated on x-axis. Its position is located by extending electric field direction from point (3cm, 3cm). The point at which this extension intersects x-axis, is the location of the charge. That is (–1cm, 0)



For point (2cm, 0), r = 3 cm, E = 100 N/C

Using, $E = \frac{kQ}{r^2}$ $\Rightarrow Q = 10 \times 10^{-12}$ C

Potential at origin = $\frac{kQ}{r}$

where, r = 1cm, $Q = 10 \times 10^{-12}$ C
$\therefore$ V = 9V

8. $\dfrac{kQ}{(r+5cm)}$ = 100V & $\dfrac{kQ}{(r+10cm)}$ = 75 V

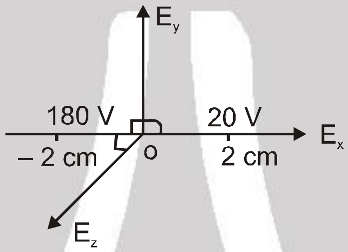
$\therefore$ Q = $\dfrac{5}{3}\times10^{-9}C$, r = 10 cm

$\therefore$ $V_{surface} = \dfrac{kQ}{r}$ = 150V $E_{surface} = \dfrac{kQ}{r^2}$ = 1500 V/m

$V_{centre} = \dfrac{3}{2}\ V_{surface} = \dfrac{3}{2}\times 150 = 225$ V

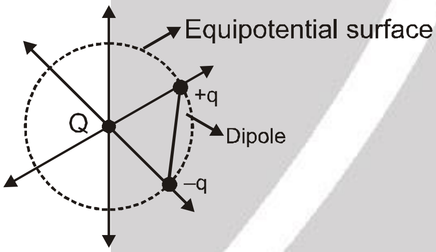
10. from given data $E_x = \dfrac{160}{4}$ V/cm = 40 V/cm

but E = $\sqrt{E_x^2+E_y^2+E_z^2}$ $\Rightarrow$ E may be equal or greater than 40 V/cm ie.



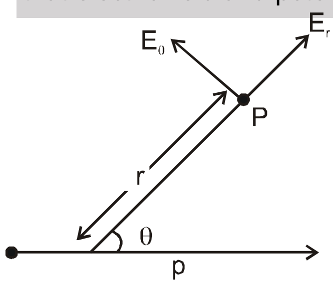
As shown, there can be electric fields ⊥ to x axis, which will not affect the electric potential difference but can increase net field.

11.



In all orientations, dipole experiences force, but does not experience torque if dipole has its dipole moment along or opposite to ELOF.
Dipole can never be in stable equilibrium & work done in moving dipole along an EPS of point charge Q will be zero.

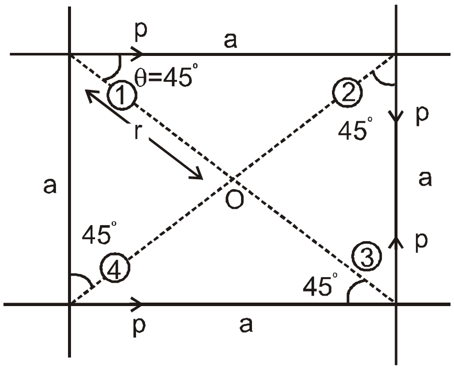
12. We know that electric field and potential due to dipole is



$E_r = \dfrac{2kp\cos\theta}{r^3}$ & $E_\theta = \dfrac{kp\sin\theta}{r^3}$ & $V_P = \dfrac{kp\cos\theta}{r^2}$

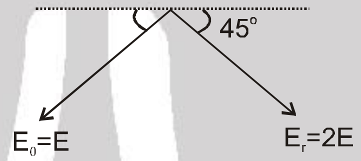
Now four dipoles are shown in figure.

Let $\left[\dfrac{2kp}{r^3}=E\right]$ & given, θ = 45º

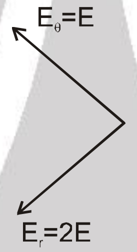


and $r = \frac{a}{\sqrt{2}}$; $E_r = \dfrac{2\,k\,p\cos 45^o}{\left(\frac{a}{\sqrt{2}}\right)^3} = \dfrac{4\,kp}{a^3} = 2E$; $E_\theta = \dfrac{k\,p\sin 45^o}{\left(\frac{a}{\sqrt{2}}\right)^3} = \dfrac{2\,kp}{a^3} = E$

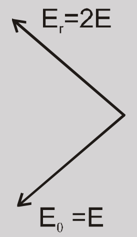
$\therefore$ Electric field due to 1st dipole



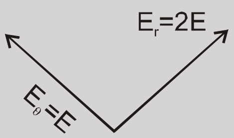
Electric field due to 2nd dipole



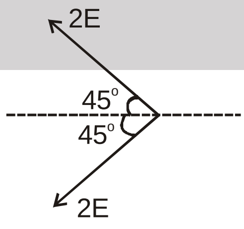
Electric field due to 3rd dipole



Electric field to 4th dipole



So, Net electric field at point O is :



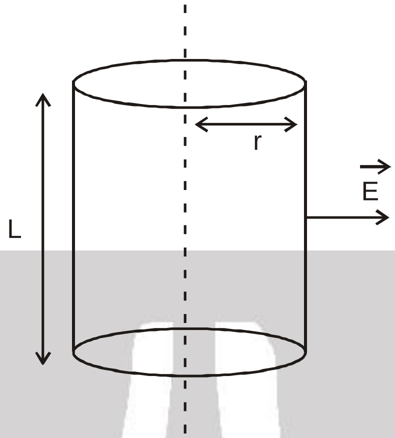
So, resultant = $2\sqrt{2}E = \dfrac{2\sqrt{2}.2kp}{a^3} = \dfrac{\sqrt{2}p}{\pi \in_o a^3}$

& Potential at point O = $4 \times \dfrac{k\,p\cos 45^o}{\left(\frac{a}{\sqrt{2}}\right)^2}$; $V = \dfrac{\sqrt{2}p}{\pi \in_o a^2}$

13. If charge is at A or D, its all field lines cut the given surface twice which means that net flux due to this charge remains zero and flux through given surface remains unchanged.

14. Case (i) $x < R$

Let a Gaussian surface is a cylinder of radius r and length equal to given cylinder



$\because$ $\vec{E}$ and $\vec{ds}$ are parallel to each other, so : $\dfrac{q_{in}}{\epsilon_o} = \int \vec{E}.\vec{ds} = \int E\, ds$

$\Rightarrow \dfrac{\rho(\pi r^2)L}{\epsilon_o} = E\int ds = E.\ 2\pi r\ L \qquad \Rightarrow E = \dfrac{\rho r}{2\epsilon_o}$

(ii) $x \geq R$: Again by $\dfrac{q_{in}}{\epsilon_o} = \int \vec{E}.\,d\vec{s} = E\int ds$

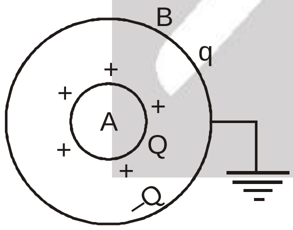
$\Rightarrow \dfrac{\rho(\pi R^2)L}{\epsilon_o} = E\,(2\pi\, r)L \Rightarrow E = \dfrac{\rho R^2}{2\epsilon_o\, x}$

15. Since, no. of electrons entering = no of electrons leaving.

$\Rightarrow$ Net enclosed charge is constant

$\Rightarrow$ Flux is constant

16.



(i) Due to earthing

Let total charge on B is q.

$V_B = 0 \qquad \therefore \qquad \dfrac{kq}{b} + \dfrac{kQ}{b} = 0 \qquad$ or $\qquad q = -Q$.

(ii) $\therefore$ All charge $q = -Q$

appears on inner surface of B due to induction

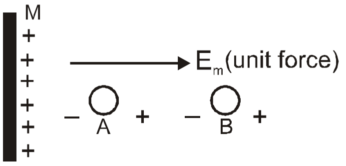
$\Rightarrow$ Charge on outer surface of B = 0

$\Rightarrow$ Field between A and B due to B = 0

Field between A and B due to A 0

Net field between A and B 0.

17. (i) Figure after induction is :

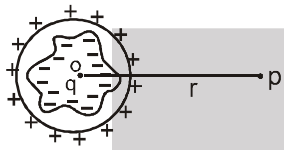


(ii) Field due to M is uniform $\therefore$ Force between M and A = 0.

Also, force between M and B = 0

& A and B attract each other due to induction.

18.



Due to induction, –q charge will induce at the inner surfac, and +q charge will appear on the outer surface. Due to the inner charges, there is no effect at outer points.

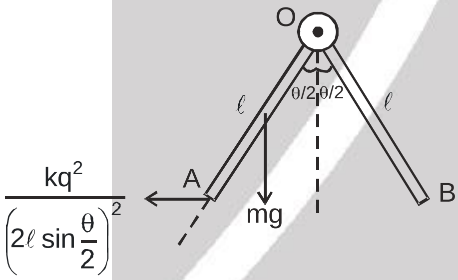
Internal disturbance is balance by internal charge and no effect found outside

PART - IV

1. For 30 C charge, angle $\in$ (5°, 9°) $\Rightarrow$ 7°

2. In (iii) most of the positive charge with run away to the metal knob. So due to less charge on the leaves, the leaves will come closer than before.

3.

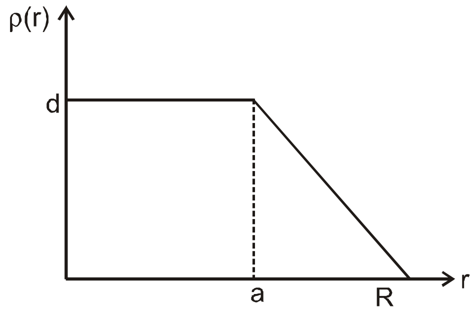


Applying torque balance about hinge point O.

$$\frac{kq^2}{\left(2\ell\sin\frac{\theta}{2}\right)^2}\left(\ell\cos\frac{\theta}{2}\right) = mg\left(\frac{\ell}{2}\right)\sin\frac{\theta}{2}$$

for small θ, $\sin\frac{\theta}{2} \to \frac{\theta}{2}$, $\cos\frac{\theta}{2} \to 1$ $\therefore$ $\theta = \sqrt{\frac{4kq^2}{mg\ell^2}}$

4. Electric field at r = R



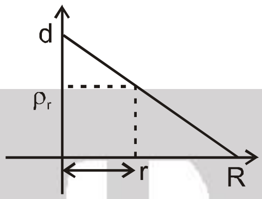
$E = \frac{KQ}{R^2}$ where Q = Total charge within the nucleus = Ze

So $E = \frac{KZe}{R^2}$

So electric field is independent of a

5. $Q = \int \rho_r 4\pi r^2 dr$

for a = 0 $\frac{d}{R} = \frac{\rho_r}{R - r}$



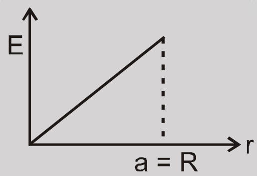
$\therefore \quad \rho_r = \frac{d}{R}(R - r)$ or, $Q = \int_0^R \frac{d}{R}(R - r)\, 4\pi r^2\, dr$

$= \frac{4\pi d}{R}\left[R\int_0^R r^2\, dr - \int_0^R r^3\, dr\right] = \frac{4\pi d}{R}\left[\frac{R^4}{3} - \frac{R^4}{4}\right] = \frac{\pi d R^3}{3}$

$\therefore \quad Q = Ze = \frac{\pi d R^3}{3}$ or $d = \frac{3Ze}{\pi R^3}$

6. From the formula of uniformly (volume) charged solid sphere

$E = \frac{\rho r}{3 \epsilon_0}$



For $E \propto r$, ρ should be constant throughout the volume of nucleus

This will be possible only when a = R.

7. $\because \phi = \frac{Q}{\epsilon_0}$

$\therefore = 2 \times 10^5 \times 8.85 \times 10^{-12}\ C = 1.77\ \mu C$

8. $\frac{(1.77 \times 10^{-6} + Q_A)}{\epsilon_0} = -4 \times 10^5 \Rightarrow Q_A = -5.31 \times 10^{-6}\ C$

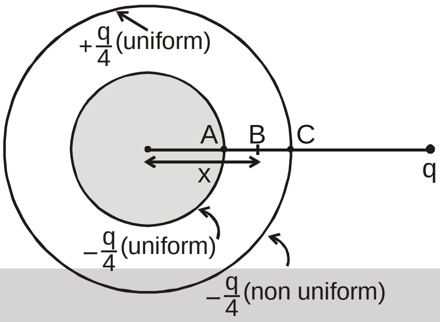
9. For all values of r, flux ϕ is non-zero i.e. no Gaussian sphere of radius r is possible in which net enclosed charge is zero.

10. The inner sphere is grounded, hence its potential is zero. The net charge on isolated outer sphere is zero. Let the charge on inner sphere be q'.

$\therefore$ Potential at centre of inner sphere is $= \frac{1}{4\pi\varepsilon_o}\frac{q'}{a} + 0 + \frac{1}{4\pi\varepsilon_o}\frac{q}{4a} = 0$

$\therefore \quad q' = -\frac{q}{4}$

11. The region in between conducting sphere and shell is shielded from charges on and outside the outer surface of shell. Hence, charge distribution on surface of sphere and inner surface of shell is uniform. The distribution of induced charge on outer surface of shell depends only on point charge q, hence is nonuniform. The charge distribution on all surfaces, is as shown.

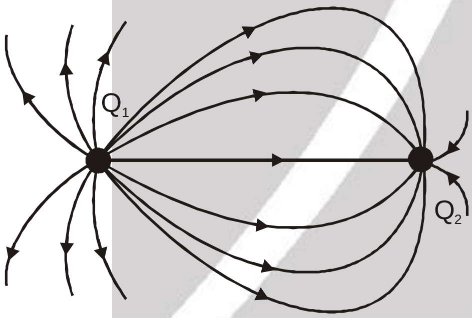


12. The electric field at B is = $\frac{1}{4\pi\varepsilon_o}.\frac{q}{4x^2}$ towards left.

$\therefore V_C = V_C - V_A = -\int_{2a}^{a} \frac{1}{4\pi\varepsilon_o}\frac{q}{4x^2}dx = \frac{1}{32\pi\varepsilon_o}.\frac{q}{a}$

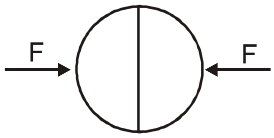
EXERCISE-3
PART - I

1.



From the diagram, it can be observed that Q_1 is positive and Q_2 is negative.
No. of lines on Q_1 is greater and number of lines is directly proportional to magnitude of charge.
So, $|Q_1| > |Q_2|$
Electric field will be zero to the right of Q_2 as it has small magnitude & opposite sign to that of Q_1.

2.



Electrostatics repulsive force ;

$F_{ele} = \left(\frac{\sigma^2}{2\varepsilon_0}\right)\pi R^2$;

$F = F_{ele} = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$

3. In equilibrium,

$mg = qE$

In absence of electric field,

$mg = 6\pi\eta rv$

$\Rightarrow \quad qE = 6\pi q rv$

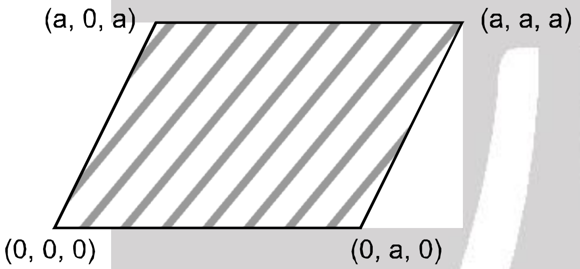
$m = \frac{4}{3}\pi r^3 d. = \frac{qE}{g}$

$\frac{4}{3}\pi\left(\frac{qE}{6\pi\eta v}\right)^3 d = \frac{qE}{g}$

After substituting value we get,

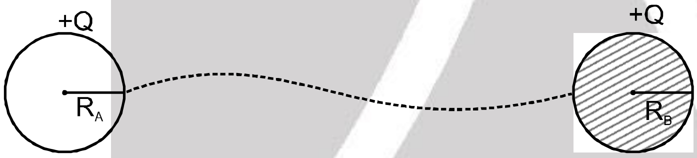
$q = 8 \times 10^{-19}$ C **Ans.**

4.



flux = (E_0 cos 45°) × area) $= \frac{E_0}{\sqrt{2}} \times a \times \sqrt{2}a = E_0a^2$

5.



$Q_A + Q_B = 2Q$ ...(i)

$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$ ...(ii)

(i) and (ii) $\Rightarrow Q_A = Q_B\left(\frac{R_A}{R_B}\right)$

& $\quad Q_B\left(1+\frac{R_A}{R_B}\right) = 2Q \quad \Rightarrow \quad Q_B = \frac{2Q}{\left(1+\frac{R_A}{R_B}\right)} = \frac{2QR_B}{R_A + R_B}$

& $\quad Q_A = \frac{2QR_A}{R_A + R_B} \quad \Rightarrow \quad Q_A > Q_B$

$\frac{\sigma_A}{\sigma_B} = \frac{Q_A / 4\pi R_A^2}{Q_B / 4\pi R_B^2} = \frac{R_B}{R_A}$ using (ii)

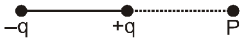
$E_A = \frac{\sigma_A}{\epsilon_0}$ & $E_B = \frac{\sigma_B}{\epsilon_0}$ $\because \sigma_A < \sigma_B$

$\Rightarrow E_A < E_B$ (at surface)

6. The frequency will be same $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$

but due to the constant qE force, the equilibrium position gets shifted by $\frac{qE}{K}$ in forward direction. So Ans. will be (A)

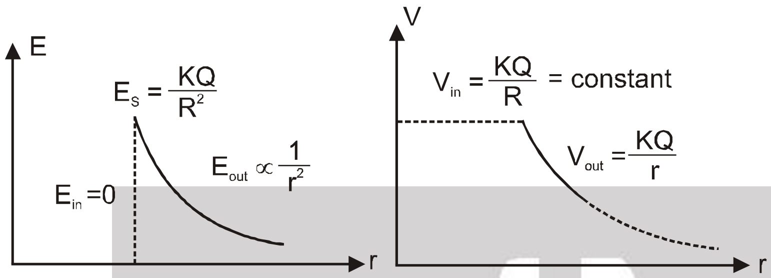
7. $\phi = \int E ds = \frac{Kq}{r^2} 4\pi r^2 = \frac{q}{\varepsilon_0}$



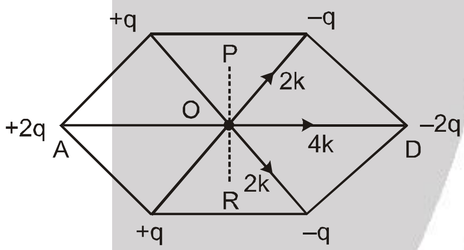
$W_{ext} = q(V_B – V_A)$

Comment : (D) is not crrect answer because it is not given that charge is moving slowly.

8.



9.

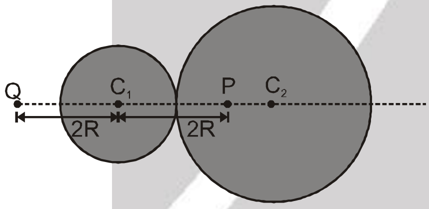


$E_0 = 6$ K (along OD)

$V_0 = 0$

Potential on line PR is zero

10.



At point P

If resultant electric field is zero

then $\frac{KQ_1}{4R^2} = \frac{KQ_2}{8R^3} R$

$\frac{\rho_1}{\rho_2} = 4$

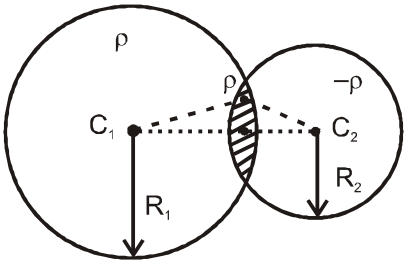
At point Q

If resultant electric field is zero

then $\frac{KQ_1}{4R^2} + \frac{KQ_2}{25R^2} = 0$

$\frac{\rho_1}{\rho_2} = -\frac{32}{25}$ (ρ_1 must be negative)

11.



For electrostatic field,

$\vec{E}_P = \vec{E}_1 + \vec{E}_2$

$= \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1P} + \frac{(-\rho)}{3\varepsilon_0}\overrightarrow{C_2P}$

$= \frac{\rho}{3\varepsilon_0}(\overrightarrow{C_1P} + \overrightarrow{PC_2})$

$\vec{E}_P = \frac{\rho}{3\varepsilon_0}\overrightarrow{C_1C_2}$

For electrostatic potential, since electric field is non zero so it is not equipotential.

12. $E_1 = \frac{KQ}{R^2}$

$E_2 = \frac{k(2Q)}{R^2} \Rightarrow E_2 = \frac{2kQ}{R^2}$

$E_3 = \frac{k(4Q)\ R}{(2R)^3} \Rightarrow E_3 = \frac{kQ}{2R^2}$

$E_3 < E_1 < E_2$

13. $\frac{Q}{4\pi \in_0 r_0^2} = \frac{\lambda}{2\pi \in_0 r_0} = \frac{\sigma}{2 \in_0}$

$Q = 2\pi\sigma r_0^2$ A incorrect

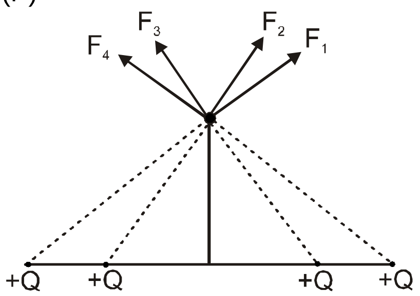
$r_0 = \frac{\lambda}{\pi\sigma}$ B incorrect

$E_1\left(\frac{r_0}{2}\right) = \frac{4E_1(r_0)}{1}$

$E_2\left(\frac{r_0}{2}\right) = 2E_2(r_0) \Rightarrow$ C correct

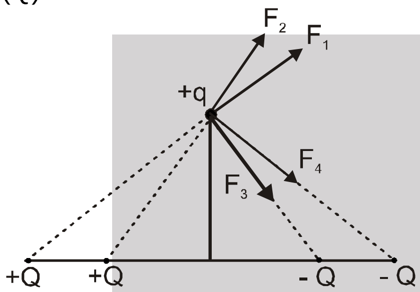
$E_3\left(\frac{r_0}{2}\right) = E_3(r_0) = E_2(r_0)$ D incorrect

14. (P)



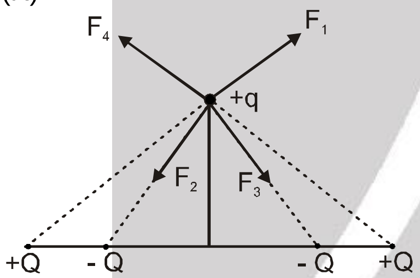
Component of forces along x-axis will vanish. Net force along +ve y-axis

(Q)



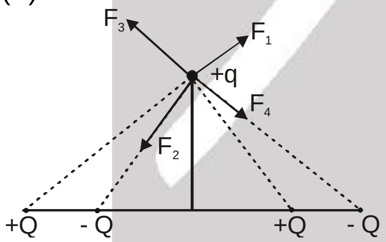
Component of forces along y-axis will vanish. Net force along +ve x-axis

(R)



Component of forces along x-axis will vanish. Net force along -ve y-axis.

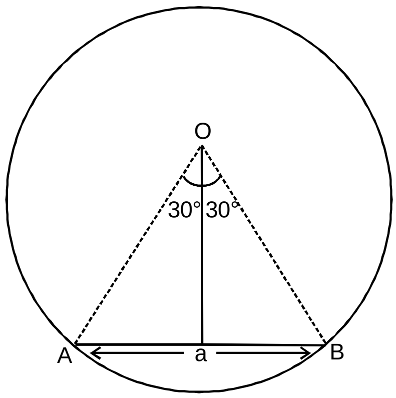
(S)



Component of forces along y-axis will vanish. Net force along -ve x-axis.

Ans. **(A)** P—3, Q—1, R—4, S—2

15. Flux from total cylindrical surface (angle = 2π)



$$= \frac{Q_{in}}{\varepsilon_0}$$

Flux from cylindrical surface AB = flux from the given surface

$$= \frac{Q_{in}}{6\varepsilon_0} = \frac{\lambda\ell}{6\varepsilon_0} = n = 6$$

16. As +q is displaced towards right, the repulsion of right side wire will dominate and the net force on +q will be towards left, and vice versa

$$F_{restoring} = q\left(\frac{2k\lambda}{d-x} - \frac{2k\lambda}{d+x}\right)$$

$$F_{restoring} = \frac{2k\lambda(2x)q}{d^2 - x^2} \approx \left(\frac{4k\lambda q}{d^2}\right)x$$

Hence SHM

For –q, as it is displaced towards right the attraction of right side wire will dominate, which forces the –q charge to move in the same direction of displacement similarity for other side

Hence it is not SHM.

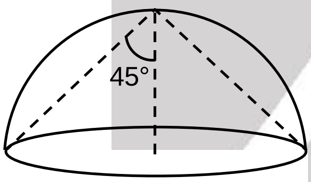
17. Electric field in cavity

$$\vec{E} = \frac{\rho\overrightarrow{OP}}{3\epsilon_0}$$

$OP = R_1 - R_2$

$$= \frac{\rho\vec{a}}{3\epsilon_0}$$

18.



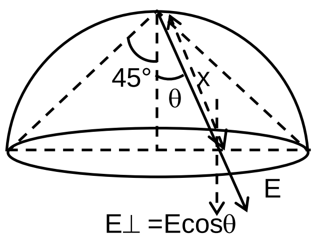
(A) ϕ total due to charge Q is $= Q/\varepsilon_0$

so ϕ through the curved and flat surface will be less than Q/ε_0

(B) The component of the electric field perpendicular to the flat surface will decrease so we move away from the centre as the distance increases (magnitude of electric field decreases) as well as the angle between the normal and electric field will increase.

Hence the component of the electric field normal to the flat surface is not constant.

Aliter :



$$x = \frac{R}{\cos\theta}$$

$$E = \frac{KQ}{x^2} = \frac{KQ\cos^2\theta}{R^2}$$

$$E_\perp = \frac{KQ\cos^3\theta}{R^2}$$

As we move away from centre $\theta\uparrow$ $\cos\theta\downarrow$ so $E_\perp\downarrow$

(C) Since the circumference is equidistant from 'Q' it will be equipotential $V = \frac{KQ}{\sqrt{2}R}$

(D) $\Omega = 2\pi(1 - \cos\theta)$; $\theta = 45°$

$$\phi = -\frac{\Omega}{4\pi} \times \frac{Q}{\varepsilon_0} = -\frac{2\pi(1-\cos\theta)}{4\pi}\frac{Q}{\varepsilon_0} = -\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

19. $Q_{enc} = \lambda\sqrt{3}\,R$

$$\phi = \frac{Q_{enc}}{\epsilon_0} = \frac{\sqrt{3}\lambda R}{\epsilon_0}$$

20. $E = qE_0 \sin\omega t = m\frac{dv}{dt}$

$$\int_0^v dv = \frac{qE_0}{m}\int_0^{\pi/\omega}\sin\omega t\,dt$$

$$v = \frac{qE_0}{\omega m}\left(-\cos\omega t\right)_0^{\pi/\omega}$$

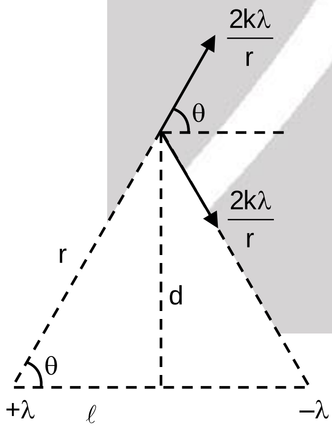
$$= -\frac{qE_0}{m\omega}((-\cos\pi) - (-\cos 0)) = \frac{2qE_0}{m\omega} = 2\text{m/s}$$

21. (1) in case of point charge $E = \frac{KQ}{d^2}$

(2) In case of dipole $E = \frac{Kp}{d^3}$

(3) For an infinite long line charge $E = \frac{2K\lambda}{d}$

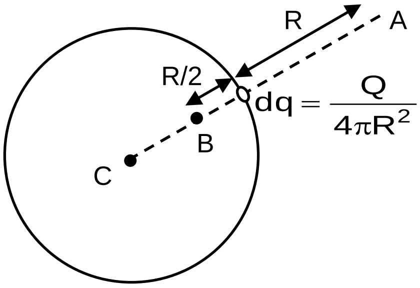
(4)



$$E = \frac{K\lambda}{r}\cos\theta = \frac{k\lambda}{\sqrt{d^2+\ell^2}} = \frac{\ell}{\sqrt{d^2+\ell^2}} = \frac{4k\lambda\ell}{(d^2+\ell^2)} \sim \frac{2k\lambda\ell}{d^2}$$

(5) $E = \frac{\sigma}{2\varepsilon_0}$

22.



$dq = \frac{Q}{4\pi R^2} dA = Q\alpha$

Given V at surface

$V_0 = \frac{KQ}{R}$

V at C

$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0\ (1 - \alpha)$

V at B

$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0\ (1 - 2\alpha)$

$\therefore\ \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$

E at A

$E_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$

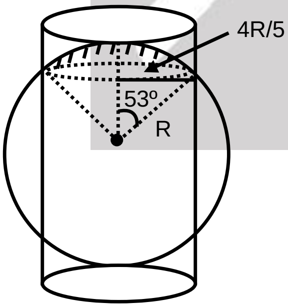
So reduced by $\frac{\alpha V_0}{R}$

E at C

$E_C = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$

So increased by $\frac{\alpha V_0}{R}$ **Ans. (1)**

23. (1) for h = 2R $r = \frac{4R}{5}$



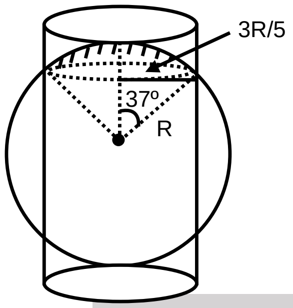
Shaded charge = $2\pi\ (1 - \cos 53^o) \times \frac{Q}{4\pi} = \frac{Q}{5}$

$\therefore\quad q_{enclosed} = \frac{2Q}{5}$

$\therefore\quad \phi = \frac{2Q}{5\varepsilon_0}$

$\therefore\quad$ for h > 2R $r = \frac{4R}{5}$ $\quad\therefore\quad \phi = \frac{2Q}{5\varepsilon_0}$

(2) for h = 2R $\quad r = \frac{3R}{5}$



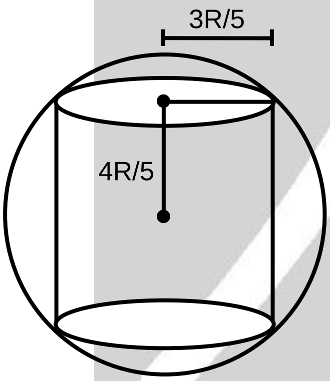
Shaded charge = $2\pi(1 - \cos 37^\circ) \times \frac{Q}{4\pi} = \frac{Q}{10}$

$\therefore \quad q_{enclosed} = \frac{Q}{5}$

$\therefore \quad \phi = \frac{Q}{5\varepsilon_0}$

$\therefore \quad$ for h > 2R $r = \frac{3R}{5} \qquad \therefore \phi = \frac{Q}{5\varepsilon_0}$

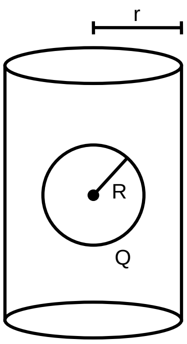
(3) suppose $h = \frac{8R}{5} \qquad r = \frac{3R}{5}$



$\phi = 0$

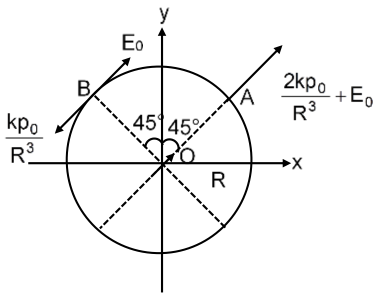
so for $h < \frac{8R}{5} \quad \phi = 0$

(4) h > 2R $\qquad$ r > R



$\phi = \frac{Q}{\varepsilon_0}$ Clearly from Gauss' Law

24. R ≫ dipole size
circle is equipotential



So, E_{net} Should be ⊥ to surface so $\frac{kp_0}{r^3} = E_0 \Rightarrow r = \left(\frac{kp_0}{E_0}\right)^{1/3}$

At point B net electric field will be zero.
$E_B = 0$

$(E_A)_{Net} = \frac{2kp_0}{r^3} + E_0 = 3E_0$

Electric field at point A s
$(E_B)_{Net} = 0$

PART - II

1. $\vec{E} = \left(\frac{2k\lambda}{r}\right)(-\hat{j}) \Rightarrow \vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{j})$

$\lambda = \frac{q}{\pi r} \Rightarrow \vec{E} = \frac{q}{2\pi^2\varepsilon_0 r^2}(-\hat{j})$

2. Consider a spherical shell of radius x and thickness dx.
Charge on it dq
$dq = \rho \times 4\pi x^2 . dx$

$dq = \rho_0 \left(\frac{5}{4} - \frac{x}{R}\right) \times 4\pi x^2\, dx$

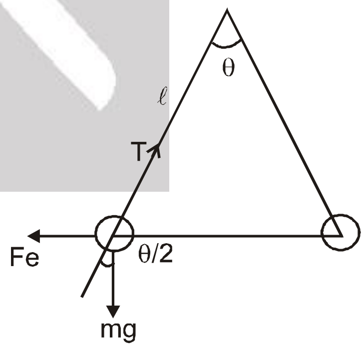
$q = 4\pi\rho_0 \int_0^r \left(\frac{5x^2}{4} - \frac{x^3}{R}\right) dx$

$q = 4\rho_0 \left(\frac{5r^3}{3\times4} - \frac{r^4}{4R}\right)$

$E = \frac{kq}{r^2} = \frac{1}{4\pi r^2} \times 4\pi\rho_0 \left(\frac{5r^3}{3\times4} - \frac{r^4}{4R}\right)$

$E = \frac{\rho_0 r}{4\rho_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

3. At equilibrium



$\tan\theta/2 = \frac{F_e}{mg} = \frac{1}{4\pi\epsilon_0}\frac{q^2}{[\ell \ \sin(\theta/2)]^2} . \frac{1}{mg}$

When suspended in liquid

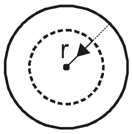
$\tan\frac{\theta}{2} = \frac{1}{4\pi\epsilon_0 \ K}\frac{q^2}{[\ell \ \sin(\theta/2)]^2}\ \frac{1}{(mg - F_B)}$

$= \frac{1}{4\pi\epsilon_0 \ K}\frac{q^2}{[\ell \ \sin(\theta/2)]^2} . \frac{1}{(mg - \frac{m}{1.6}\times 0.8 \ \ g)}$

on comparing the two equation we get

$K\left(1 - \frac{0.8}{1.6}\right) = 1 \Rightarrow K = 2.$

4.



$\phi = ar^2 + b$

$E = -\dfrac{d\phi}{dt} = -2ar$

$\oint \vec{E}.\overrightarrow{dS} = \dfrac{q}{\varepsilon_0}$

$-2ar \cdot 4\pi r^2 = \dfrac{q}{\varepsilon_0}$

$q = -8\,\varepsilon_0 a\pi r^3$

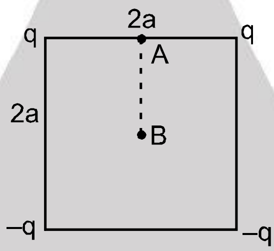
$\rho = \dfrac{q}{\frac{4}{3}\pi r^3}$

$\rho = -6a\varepsilon_0$ **Ans.**

5. Potential at point A,

$V_A = \dfrac{2Kq}{a} - \dfrac{2Kq}{a\sqrt{5}}$

Potential at point B,



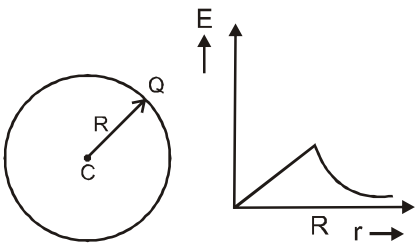
$V_B = 0$

∴ Using work energy theroem,

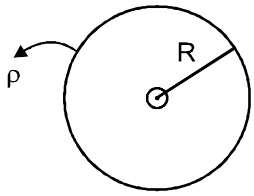
$W_{AB})_{electric} = Q(V_A - V_B)$

$= \dfrac{2KqQ}{a}\left[1 - \dfrac{1}{\sqrt{5}}\right] = \left(\dfrac{1}{4\pi \in_0}\right)\dfrac{2Qq}{a}\left[1 - \dfrac{1}{\sqrt{5}}\right]$

6.



7. $U_c = \frac{3}{2}\frac{KQ}{R}q$

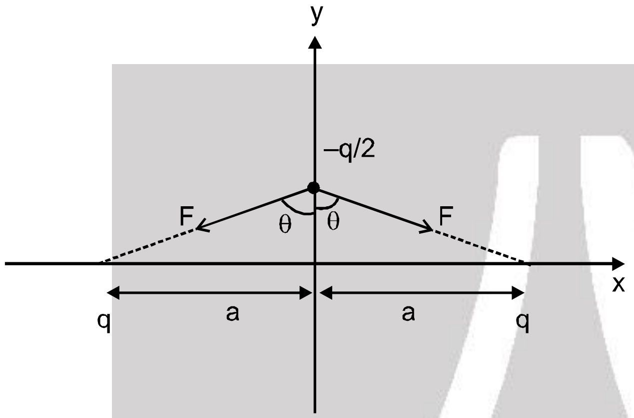


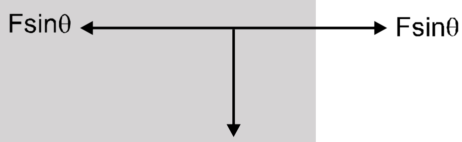
$U_S = \frac{KQ}{R}q$

$\therefore \quad \Delta U = \frac{KQ}{2R}q$

$= \frac{1}{4\pi\varepsilon_0}.\frac{1}{2R}\rho\frac{4\pi R^3}{3}q = \frac{\rho R^2 q}{6\varepsilon_0}$

8.





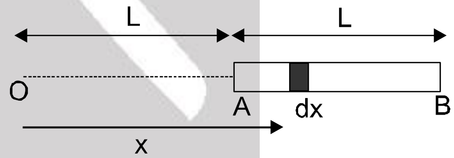
$\Rightarrow$

$\Rightarrow \quad F_{net} = 2F\cos\theta$

$F_{net} = \frac{2kq\left(\frac{q}{2}\right)}{\left(\sqrt{y^2+a^2}\right)^2} \cdot \frac{y}{\sqrt{y^2+a^2}}$

$F_{net} = \frac{2kq\left(\frac{q}{2}\right)y}{(y^2+a^2)^{3/2}} \quad \Rightarrow \frac{kq^2y}{a^3} \quad \propto y$ **Ans. (1)**

9. $V = \int_L^{2L}\frac{kdq}{x}$



$= \int_L^{2L}\frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{L}\right)dx}{x} = \frac{q}{4\pi\varepsilon_0 L}\ell n(2)$ **Ans. (4)**

10. $V_A - V_0 = -\int_O^A E_x dx$

$V_A - V_0 = \int_0^2 30x^2 dx$

$= -30\ \frac{2^3}{3} = -80V$

11. (2) and (3) is not possible since field lines should originate from positive and terminate to negative charge.
(4) is not possible since field lines must be smooth.
(1) satisfies all required condition.

12. $V_0 = \frac{KQ}{R}$

$V_{(r > R)} = \frac{KQ}{r}$

$V_{(r < R)} = \frac{KQ}{2R^3}(3R^2 - r^2)$

$V_{centre_{केन्द्र}} = \frac{3}{2}\frac{KQ}{R} = \frac{3V_0}{2}$

V at R_2 (R_2 पर V) = $\frac{5V_0}{4} = \frac{kQ}{2R^3}(3R^2 - R_2^2)$

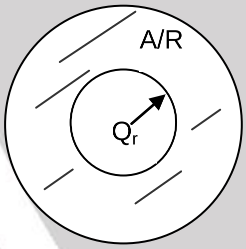
$\Rightarrow \quad \frac{5}{2} = 3 - \frac{R_2^2}{R^2} \Rightarrow R_2 = \frac{R}{\sqrt{2}}$

V at R_3 (R_3 V)= $\frac{3V_0}{4} = \frac{kQ}{R_3} \Rightarrow R_3 = \frac{4}{3}R$

V at R_4 (R_4 V) = $\frac{V_0}{4} = \frac{kQ}{R_4} \Rightarrow R_4 = 4R$

$\therefore \quad R_4 - R_3 = 4R - \frac{4}{3}R = \frac{8R}{3} > R_2$

13. $(E)(4\pi r^2) = \dfrac{Q + \int_a^r \frac{A}{r} 4\pi r^2 dr}{\varepsilon_0}$



$\Rightarrow \quad (E)\, 4\pi r^2 = \dfrac{Q + \frac{4\pi A}{2}(r^2 - a^2)}{\varepsilon_0}$

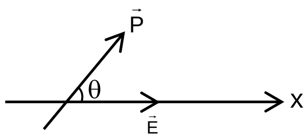
$\Rightarrow \quad E = \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{A}{\varepsilon_0 2r^2}(r^2 - a^2)$

$= \frac{Q}{4\pi\varepsilon_0 r^2} + \frac{A}{2\varepsilon_0} - \frac{Aa^2}{2\varepsilon_0 r^2}$

$\frac{Q}{4\pi\varepsilon_0} = \frac{Aa^2}{2\varepsilon_0}$

$A = \frac{Q}{2\pi a^2}$

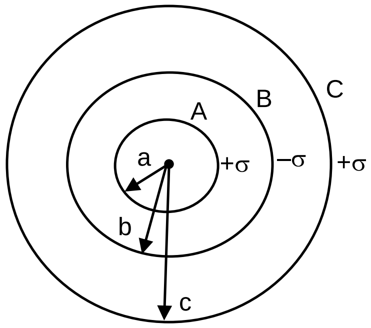
14.



$PE\sin\theta = P(\sqrt{3}E)\sin(90° - \theta)$

$\tan\theta = \sqrt{3}$; $\theta = 60°$

15.



$$V_B = \frac{1}{4\pi\varepsilon_0}\frac{4\pi a^2\sigma}{b} - \frac{1}{4\pi\varepsilon_0}\frac{4\pi b^2\sigma}{b} + \frac{1}{4\pi\varepsilon_0}\frac{4\pi c^2\sigma}{C} = \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - b^2}{b} + c\right)$$

16. $\Delta KE + \Delta U = 0$

$$\left(\frac{1}{2}mv^2 - 0\right) + q(v_f - v_i) = 0$$

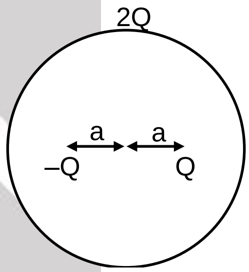
$$\frac{1}{2}mv^2 + q\left[-2k\lambda \ell n\left(\frac{R}{R_0}\right)\right] = 0$$

$$\frac{1}{2}mv^2 = 2k\lambda q \ell n\left(\frac{R}{R_0}\right)$$

$$v = \left[\frac{4k\lambda q}{m}\ell n\left(\frac{R}{R_0}\right)\right]^{1/2}$$

17. (i) Electric field outside sphere does not depends on inside charge, it depends on only outer charge.
(ii) Surface charge density on inner surface is non-uniform.
(iii) Surface charge density on inner surface is non-uniform.
(iv) Surface charge density on outer surface does not depend on $|\vec{P}|$

18. $\int_0^R kr\, 4\pi r^2 dr = 2Q$



$k\pi R^4 = 2Q$ …….. (1)

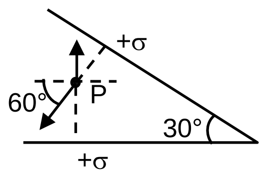
$$\frac{KQ^2}{4a^2} = KQ\frac{\int_0^a kr 4\pi r^2 dr}{a^2}$$

$$\frac{KQ^2}{4a^2} = Q\,Kk4\pi\frac{a^2}{4}$$

$$\frac{KQ^2}{4a^2} = QK\left(\frac{2Q}{R^4}\right)a^2$$

$R = a8^{1/4}$

19. $\vec{E} = \frac{\sigma}{2\varepsilon_0}\cos 60°(-\hat{x}) + \left[\frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0}\sin 60°\right](\hat{y})$



$$\vec{E} = \frac{\sigma}{2\varepsilon_0}\left[\left(1 - \frac{\sqrt{3}}{2}\right)\hat{y} - \frac{1}{2}\hat{x}\right]$$

20. Magnitude of electric field is constant & the surface is equipotential

21. **Since** $\vec{p}.\vec{r} = 0$

$\vec{E}$ must be antiparallel to $\vec{p}$

So, $\vec{E} = -\lambda(\vec{p})$

where λ is a arbitrary positive constant

Now $\vec{A} = a\hat{i} + b\hat{j} + c\hat{k}$

$\vec{A} \parallel \vec{E}$

$$\frac{a}{\lambda} = \frac{b}{3\lambda} = \frac{c}{-2\lambda} = k$$

so $\vec{A} = \lambda k(\hat{i} + 3\hat{j} - 2\hat{k})$

22. For a solid sphere

$E = \frac{\rho r}{3\varepsilon_0}$

$E_A = \frac{-\rho R}{2(3\varepsilon_0)}$

$|E_A| = \frac{\rho R}{6\varepsilon_0}$

Electric field at point B = $E_B = E_{1A} + E_{2A}$

E_{1A} = Electric Field Due to solid sphere of radius R at point B = $\frac{\rho R}{3\varepsilon_0}$

E_{2A} = Electric Field Due to solid sphere of radius R/2 (which having charge density $-\rho$)

$= -\frac{KQ' \times 4}{9R^2} = -\frac{\rho R}{54\varepsilon_0}$

$E_B = E_{1A} + E_{2A} = \frac{\rho R}{3\varepsilon_0} - \frac{\rho R}{54\varepsilon_0} = \frac{17\rho R}{54\varepsilon_0}$

$$\frac{|E_A|}{|E_B|} = \frac{9}{17}$$

23. Flux via ABCD

$\phi_1 = \int \vec{E}.d\vec{A} = 0$

Flux via BCEF

$\phi_2 = \int \vec{E}.d\vec{A}$

$\phi_2 = \vec{E}.\vec{A} = (4x\hat{i} - (y^2 + 1)\hat{j}) \cdot 4\hat{i}$

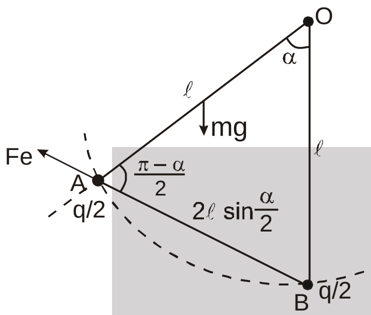
$= 16x$, $x = 3$

$\phi_2 = 48 \frac{N-m^2}{C}$; $\phi_1 - \phi_2 = -48 \frac{N-m^2}{C}$

HIGH LEVEL PROBLEMS (HLP)

1. The electrostatic force exerted by charge at B on charge at A is

$$F_e = \frac{1}{4\pi\in_0}\frac{\left(\frac{q}{2}\right)^2}{(AB)^2} = \frac{1}{4\pi\varepsilon_0}\frac{\left(\frac{q}{2}\right)^2}{\left(2\ell\sin\frac{\alpha}{2}\right)^2} = \frac{1}{64\pi\in_0}\frac{q^2}{\ell^2\sin^2\frac{\alpha}{2}}$$



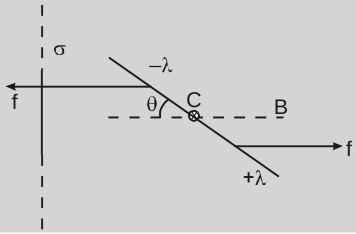
The rod AO is in equilibrium, hence net torque on rod about point O is

$$F_e\,\ell\sin\left(\frac{\pi}{2}-\frac{\alpha}{2}\right) - mg(\sin\alpha)\,\frac{\ell}{2} = 0$$

$\Rightarrow \quad \frac{2}{64\pi\in_0}\frac{q^2}{\ell^2\sin^2\frac{\alpha}{2}}\cos\alpha/2 = mg\sin\alpha \quad \Rightarrow \quad$ solving, we get

$$q = 4\ell\sqrt{4\pi\in_0 mg\sin\left(\frac{\alpha}{2}\right)}\,\sin\frac{\alpha}{2}$$ **Ans.**

2. The sheet produces a uniform electric field $E = \frac{\sigma}{2\in_0}$ towards right. The part AC and CB will experience electric force F as shown. They can be considered to be acting at the mid points of those parts respectively. The rod will experience torque about the point 'c' in the anticlockwise direction



Whose magnitude is $\tau = F\,\frac{\ell}{2}\sin\theta \simeq \frac{F\ell}{2}\,\theta$; But $F = \lambda.\,\frac{\ell}{2}\,.\,\frac{\sigma}{2\in_0} = \frac{\lambda\ell\sigma}{4\in_0}$

$\therefore \quad \tau = \left(\frac{\lambda\ell^2\sigma}{8\in_0}\right)\theta$

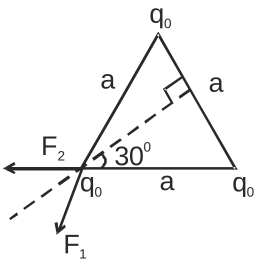
Now, since τ is towards the mean position & $\tau \propto \theta$

$\therefore$ it will perform SHM → Hence proved

& $\quad \tau = I\,\alpha = \frac{\lambda\ell^2\sigma}{8\in_0}\theta \Rightarrow \quad \frac{m\ell^2}{12}\alpha = \frac{\lambda\ell^2\sigma}{8\in_0}$ or $\alpha = \left(\frac{3\lambda\sigma}{2m\in_0}\right)\theta$

$\therefore \quad \omega^2 = \frac{3\lambda\sigma}{2m\in_0} = \left(\frac{2\pi}{T}\right)^2 \Rightarrow$ $\mathbf{T = 2\pi\sqrt{\frac{2m\in_0}{3\lambda\sigma}}}$ **Ans.**

3. (i)



$$F_1 = F_2 = \frac{kq_0^2}{a^2}$$

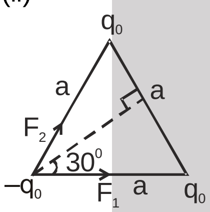
$F = F_1\cos30^0 + F_2\cos30^0$

$= 2F_1\cos30^0$

$\therefore \quad F = F_1\sqrt{3}$

or $\quad F = \frac{\sqrt{3}kq_0^{\ 2}}{a^2}$, away from the charges along perpendicular bisector of line joining remaining two charges.

(ii)

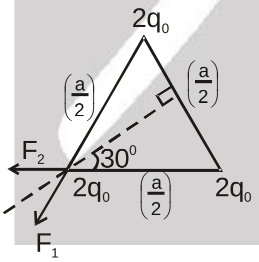


$$F_1 = F_2 = \frac{kq_0^2}{a^2}$$

$F = F_1\cos30^0 + F_2\cos30^0 = 2F_1\cos30^0 = F_1\sqrt{3}$

$= \frac{\sqrt{3}kq_0^{\ 2}}{a^2}$ towards the charges along perpendicular bisector of line joining remaining two charges.

(iii)



$$F_1 = F_2 = \frac{k(2q_0)^2}{\left(\frac{a}{2}\right)^2} = \frac{16kq_0^2}{a^2}$$

$F = F_1\cos30^0 + F_2\cos30^0$

$= 2F_1\cos30^0$

$= F_1\sqrt{3}$

$$= \frac{\sqrt{3}k\left(2q_0\right)^2}{\left(\frac{a}{2}\right)^2}$$

$= \frac{16\sqrt{3}kq_0^{\ 2}}{a^2}$ away from the charges along angle bisector.

4. Assume 'ρ' and '–ρ' in the cavity then

$$V_\rho = \frac{3}{2}\frac{K}{R}\left(\rho \cdot \frac{4}{3}\pi R^3\right)$$

$$V_{-\rho} = \frac{K\left[-\rho \cdot \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]}{\frac{R}{2}}$$

$$\therefore \quad V_C = V_\rho + V_{-\rho} = 2\pi K \rho R^2 - \frac{\pi K \rho R^2}{3} = \frac{5\pi K \rho R^2}{3}$$

$$\therefore \quad V = \frac{5\rho R^2}{12 \in_0}$$ **Ans.**

5. In the remaining three quadrants, put three more quarter sheets to convert this given arrangement to that of infinite sheet. Now contribution from all the four quarters to the z - component will be same. Hence due to a quarter component of E.F. along z axis at point (0, 0, z) will be,

$$\vec{E}_z = \frac{1}{4}\left(\frac{\sigma}{2 \in_0}\right)\frac{\vec{z}}{z} = \frac{\sigma}{8 \in_0}\hat{k}$$

Hence potential difference between points (0, 0, d) and (0, 0, 2d) will be,

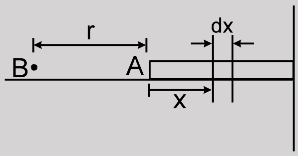
$$V_{2d} - V_d = -\int_d^{2d} \vec{E}.d\vec{\ell} \;; \qquad \text{where, } d\vec{\ell} = dz\hat{k} \;;\; \vec{E}_z = \frac{\sigma}{8 \in_0}\frac{|\vec{z}|}{z} = \frac{\sigma}{8 \in_0}\hat{k}$$

$$V_{2d} - V_d = -\int_d^{2d} \frac{\sigma}{8 \in_0}\hat{k}\,.\,dz\hat{k} = -\frac{\sigma}{8 \in_0}\int_d^{2d} dz \;;\quad V_d - V_{2d} = \frac{\sigma}{8 \in_0}|d|$$

Ans. $\frac{\sigma}{8 \in_0}$, $\frac{\sigma}{8 \in_0}|d|$

6. Let the closest distance of approach be r

Consider an element of length 'dx' on rod at a distance x from end 'A' of rod.



Potential at point B due to the element = $\frac{k\lambda dx}{r+x}$.

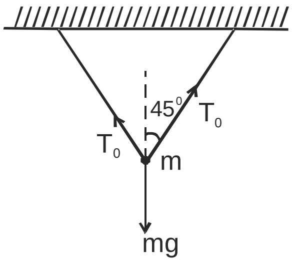
$$\therefore \quad \text{Potential at B, due to the rod} = \int_0^L \frac{k\lambda dx}{r+x} = k\lambda\,\ell n\left(\frac{r+L}{r}\right)$$

Now applying conservation of energy

$$0 + \frac{1}{2}mv^2 = q\left[k\lambda\ell n\left(\frac{r+L}{r}\right)\right] + 0 \Rightarrow \qquad \frac{r+L}{r} = e^{\frac{2\pi\in_0 mv^2}{\lambda q}}$$

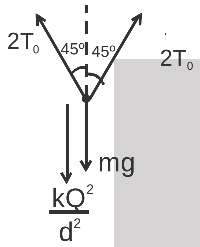
$$\Rightarrow \quad r = \frac{L}{\left(e^{\frac{2\pi\in_0 mv^2}{\lambda q}} - 1\right)}$$ **.....Ans.**

7.



Initial situation

$2T_0 \cos 45^0 = mg \qquad T_0 = \dfrac{mg}{\sqrt{2}}$



Final FBD

$4T_0 \cos 45^0 = mg + \dfrac{kQ^2}{d^2} \quad \Rightarrow \quad 2\sqrt{2}\, T_0 = mg + \dfrac{kQ^2}{d^2}$

$\therefore \quad mg = \dfrac{kQ^2}{d^2} \quad$ or $\quad Q = d\sqrt{\dfrac{mg}{k}}$

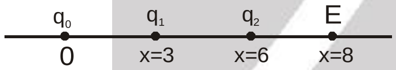
$= 4.2 \times 10^{-2} \sqrt{\dfrac{5.88 \times 10^{-4} \times 9.8}{9 \times 10^{9}}}$

$= 3.36 \times 10^{-8}$ C

Now, $\quad T_0 = \dfrac{mg}{\sqrt{2}} = \dfrac{5.88 \times 10^{-4} \times 9.8}{\sqrt{2}} = 4.075 \times 10^{-3}$ N

$\therefore \quad 2T_0 = 8.15 \times 10^{-3}$ N

8.



Electric field, $E = 20.25 = k\left(\dfrac{q_0}{8^2} + \dfrac{q_1}{5^2} + \dfrac{q_2}{2^2}\right)$

$\therefore \quad 20.25 = 9 \times 10^9 \left(\dfrac{16 \times 10^{-9}}{64} + \dfrac{q_1}{25} + \dfrac{12 \times 10^{-9}}{4}\right)$

$20.25 = 2.25 + \dfrac{9 \times 10^9}{25} q_1 + 27$

$\therefore \quad q_1 = -25 \times 10^{-9}$ C

9. $\quad \therefore \quad d = ut + \dfrac{1}{2}at^2$

$\therefore \quad d = \dfrac{1}{2}at^2$

or $\quad d = \dfrac{1}{2}\dfrac{eE}{m}t^2 \qquad \& \qquad E = \dfrac{\sigma}{\varepsilon_0}$

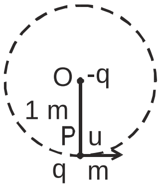
$\therefore \quad d = \dfrac{1}{2}\dfrac{e}{m}\dfrac{\sigma}{\varepsilon_0}t^2 \qquad \sigma = \dfrac{2m\varepsilon_0 d}{et^2}$

$= \dfrac{2 \times 9.1 \times 10^{-31} \times 8.85 \times 10^{-12} \times 2 \times 10^{-2}}{1.6 \times 10^{-19} \times 4 \times 10^{-12}} = \dfrac{2 \times 9.1 \times 8.85 \times 2 \times 10^{-14}}{1.6 \times 4}$

$= 0.503 \times 10^{-12}\ \frac{c}{m^2}$

charge density on outer surfaces of plates is equal in magnitude as well as in sign. So there is no contribution in electric field between the plates by charges on outer surfaces. So we cannot find out charge density on the outer surfaces.

10.



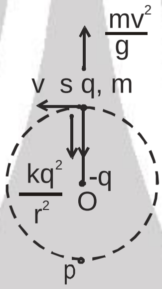
Here, $q = 3 \times 10^{-6}$ C

$r = 1$ m

u = initial horizontal velocity of ball.

At highest point S tension in the string becomes zero.

& let velocity at this point = v



$\therefore \quad mg + \frac{kq^2}{r^2} = \frac{mv^2}{r}$ ....(i)

From conservation of energy between P & S

$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + 2mgr$ ....(ii)

$\therefore \quad u^2 = v^2 + 4gr$

From (i)

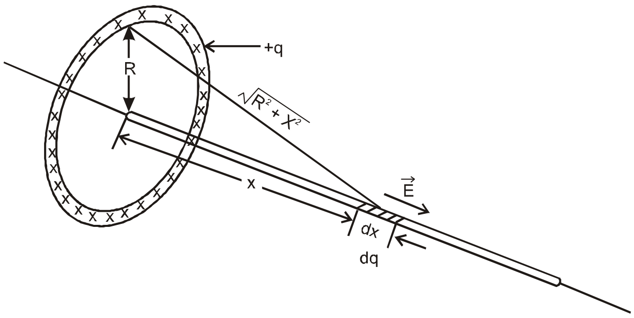
$$u^2 = gr + \frac{kq^2}{mr} + 4gr = 5gr + \frac{kq^2}{rm} = 5 \times 10 \times 1 + \frac{9\times10^9 \times 9\times10^{-12}}{1\times10^{-2}}$$

or $\quad u^2 = 50 + 8.1$

$\therefore \quad u = 7.62$ m/sec.

11. The system of the ring charge and line charge may be represented as shown in the figure. Here, the electric field intensity due to the ring charge +q at a point distant x on the axis is given by :

$E = \frac{1}{4\pi\varepsilon_0}\frac{qx}{(R^2+x^2)^{3/2}}$ .... (along the axis of ring i.e. along wire)



The force due to electric field of ring charge on a small charge element dq concentrated in small length dx of the line charge is given by

dF = E dq

or, $dF = \frac{1}{4\pi\varepsilon_0} \times \frac{qx}{(R^2 + x^2)^{3/2}} \lambda\ dx$

Here, λ = linear charge density of the thread.

so, $dF = \frac{1}{4\pi\varepsilon_0} \times \frac{q\lambda x\,dx}{(R^2 + x^2)^{3/2}}$

so, $F = \frac{q\lambda}{4\pi\varepsilon_0} \int_0^{\infty} \frac{xdx}{(R^2 + x^2)^{3/2}}$

let $(R^2 + x^{2)} = t$

so, $2x\ dx = dt$

so, $x\ dx = \frac{dt}{2}$

so, $F = \frac{q\lambda}{8\pi\varepsilon_0} \int_{t=R}^{t=\infty} \frac{dt}{t^{3/2}}$

or, $= \frac{q\lambda}{8\pi\varepsilon_0} \left[\frac{t^{-3/2+1}}{-3/2+1} \right]_{t=R}^{t=\infty}$

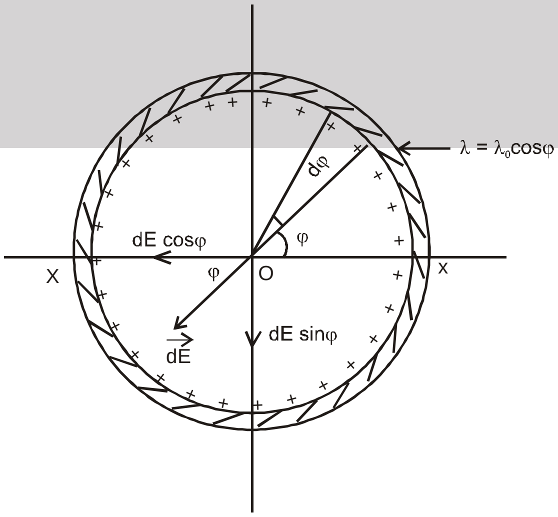
or, $= -\frac{q\lambda}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{t}} \right]_{t=R}^{t=\infty}$

[because, $t = R^2 + x^2$]

so, $F = -\frac{q\lambda}{4\pi\varepsilon_0} \left[\frac{1}{\sqrt{\infty^2}} - \frac{1}{\sqrt{R^2}} \right]$

or $F = \frac{q\lambda}{4\pi\varepsilon_0 R}$

12. **(a)** Lets take a small element at an angle φ subtending angle $d\varphi$ at the center. Charge on this element will be $dq = \lambda\ (Rd\varphi) = \lambda_0 \cos\varphi (Rd\varphi)$



Due to this element, electric field at center will be

$dE = \frac{kdq}{R^2}$

The y component dE sin φ will be cancelled by the opposite element of lower half and the x component dE cos φ will be added up

So $\quad E_{net} = \int dE\cos\varphi$

$$E_{net} = \int_{\varphi=0}^{\varphi=2\pi} \frac{K(\lambda_0\cos\varphi)\ \ Rd\varphi}{R^2}\cos\varphi = \frac{\lambda}{4\varepsilon_0 R}$$

(b) Let the ring plane coincides with y-z plane shown in fig. We consider a small element AB (of length dl) on ring.

Here dl = R$d\theta$ where R is the radius of ring.

Also, from fig. $y = R\sin\theta$ and $z = R\cos\theta$

The electric charge on the considered element is $dq = \lambda dl$

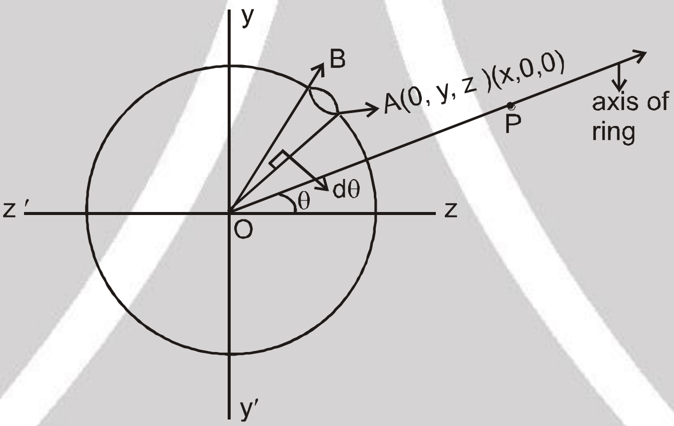
$= \lambda_0\cos\theta\,(Rd\theta) = \lambda_0 R\cos\theta d\theta$

The axis of the ring is X-axis.

The electric field at point P due to considered element is

$$\overrightarrow{dE} = \frac{dq(\vec{r}_p - \vec{r}_A)}{4\pi\varepsilon_0|\vec{r}_p - \vec{r}_A|^3} \text{ or } \overrightarrow{dE} = \frac{(\lambda_0 R\cos\theta d\theta)(x\hat{i} - y\hat{j} - z\hat{k})}{4\pi\varepsilon_0|x\hat{i} - y\hat{j} - z\hat{k}|^3}$$

or $\quad \overrightarrow{dE} = \dfrac{\lambda_0 R\cos\theta d\theta}{4\pi\varepsilon_0}\ \dfrac{(x\hat{i} - R\sin\theta\hat{j} - R\cos\theta\hat{k})}{(x^2+y^2+z^2)^{3/2}}$



$$= \frac{\lambda_0 R\cos\theta d\theta}{4\pi\varepsilon_0}\ \frac{(x\hat{i} - R\sin\theta\hat{j} - R\cos\theta\hat{k})}{(x^2+R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$= \frac{\lambda_0 R\cos\theta d\theta}{4\pi\varepsilon_0}\ \frac{(x\hat{i} - R\sin\theta\hat{j} - R\cos\theta\hat{k})}{(x^2+R^2)^{3/2}}$$

$\therefore \quad \overrightarrow{dE}\ \dfrac{\lambda_0 R}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}\ (x\cos\theta d\theta\,\hat{i} - R\sin\theta\cos\theta\,\hat{j} - R\cos^2\theta d\theta\,\hat{k})$

$\therefore \quad dE_x = \dfrac{\lambda_0 Rx\cos\theta d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$

$dE_y = \dfrac{-\lambda_0 R^2\sin\theta\cos\theta d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$

and $\quad dE_2 = \dfrac{-\lambda_0 R^2\cos^2\theta d\theta}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}$

$\therefore \quad E_x = \int dE_x = \dfrac{\lambda_0 Rx}{4\pi\varepsilon_0(R^2+x^2)^{3/2}}\int_0^{2\pi} \cos\theta d\theta$

After integrating, $E_x = 0$ and

$$E_y = \int dE_y = \frac{-\lambda_0 R^2}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi} \frac{\sin 2\theta}{2} d\theta$$

$$= \frac{-\lambda_0 R^2}{8\pi\varepsilon_0 (R^2+x^2)^{3/2}} \left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi} = 0$$

$\therefore$ $E_y = 0$

similarly,

$$E_z = \int dE_z = \frac{-\lambda_0 R^2}{4\pi\varepsilon_0 (R^2+x^2)^{3/2}} \int_0^{2\pi} \cos^2\theta d\theta = \frac{-\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

$\therefore$ $\vec{E} = E_x \hat{i} + E_y \hat{j} + E_z \hat{k}$

$\therefore$ $|\vec{E}| = E = |E_z \hat{k}|$ $(\because E_x = 0, E_y = 0)$

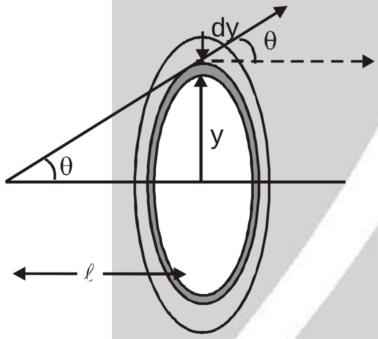
$$= \frac{\lambda_0 R^2}{4\varepsilon_0 (R^2+x^2)^{3/2}}$$

For x > > R, $R^2 + x^2 = x^2$

$\therefore$ $E = \dfrac{\lambda_0 R^2}{4\varepsilon_0 x^3} = \dfrac{P}{4\pi\varepsilon_0 x^3}$

Where $P = \lambda_0 \pi R^2$

13.



Electric field at distance y on the circle due to both charges is

$$E = 2 \times \frac{kq}{(\ell^2+y^2)} \times \cos\theta = \frac{2kq}{\ell^2+y^2} \times \frac{\ell}{\sqrt{\ell^2+y^2}}$$

$E = \dfrac{2kq\ell}{[\ell^2+y^2]^{3/2}}$ (Along the dotted line)

flax through the width dy of circle $d\phi = E\,(2\pi y.\,dy)$ (Angle = 0º)

$$\int d\phi = 2\pi\,(2\,kq\ell) \int_0^R \frac{ydy}{[\ell^2+y^2]^{3/2}}$$

Let $\ell^2 + y^2 = x$

$\therefore$ $2ydy = dx$

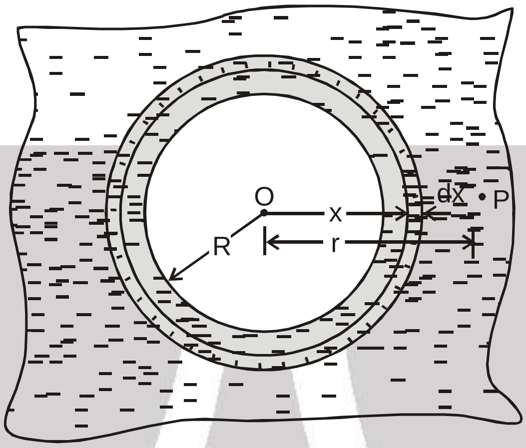
$$= \frac{q\ell}{\epsilon_0} \int_{\ell^2}^{\ell^2+R^2} \frac{dx}{2(x)^{3/2}} \qquad = \frac{q\ell}{2\,\epsilon_0} \cdot \left[\frac{x^{\frac{-3}{2}+1}}{\frac{-3}{2}+1}\right]_{\ell^2}^{\ell^2+R^2}$$

$$= -\frac{q\ell}{\epsilon_0}\left[\frac{1}{\sqrt{R^2+\ell^2}} - \frac{1}{\ell}\right] \quad = \frac{q}{\varepsilon_0}\left[1 - \frac{1}{\sqrt{1 + \left(\frac{R}{\ell}\right)^2}}\right]$$

14. To calculate the electric field due to the charged sphere and the space surrounding the sphere, a shell of radius x and thickness dx whose centre is the centre of the sphere is taken and electric field due to this shell and charged sphere at a distance r from O is obtained as given below

$E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} + \int_R^r \frac{1}{4\pi\varepsilon_0} \frac{(4\pi x^2 dx\rho)}{r^2}$ [where, q = charge considered on the ball]

Where, first term is the field strength of spherical charge q and second integral term is the field strength of space surrounding the charged sphere.



$\therefore \quad E = \frac{1}{4\pi\varepsilon_0} \times \frac{q}{r^2} + \int_R^r \frac{1}{4\pi\varepsilon_0} \frac{4\pi x^2 dx \frac{\alpha}{x}}{r^2}$

$\left[\text{since}, \rho\left(=\frac{\alpha}{r}\right) \text{at } r = x \text{ is given by } \rho = \frac{\alpha}{x}\right]$

so, $E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{\alpha}{\varepsilon_0 r^2} \int_R^r x dx$

or, $= \frac{q}{4\pi\varepsilon_0 r^2} + \frac{\alpha}{\varepsilon_0 r^2} \left[\frac{x^2}{2}\right]_R^r$

or, $= \frac{q}{4\pi\varepsilon_0 r^2} + \frac{\alpha}{\varepsilon_0 r^2} \left[\frac{r^2}{2} - \frac{R^2}{2}\right]$

or, $= \frac{q}{4\pi\varepsilon_0 r^2} + \frac{\alpha r^2}{2\varepsilon_0 r^2} + \frac{\alpha R^2}{2\varepsilon_0 r^2}$

so, $E = \frac{q}{4\pi\varepsilon_0 r^2} + \frac{\alpha}{2\varepsilon_0} - \frac{\alpha R^2}{2\varepsilon_0 r^2}$

Now, for E to be independent of r, sum of the first and third terms must be zero.

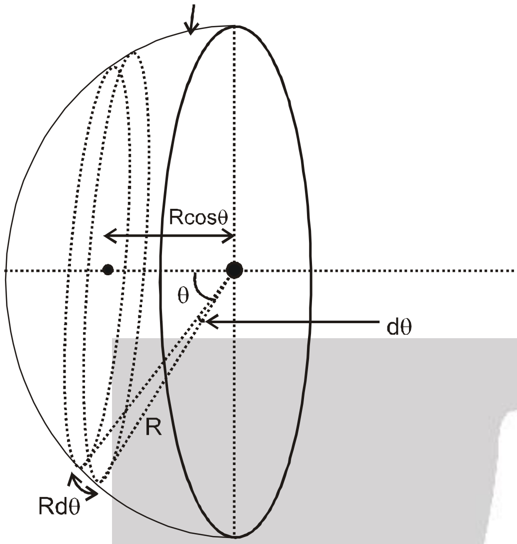
so, $\frac{q}{4\pi\varepsilon_0 r^2} - \frac{\alpha R^2}{2\varepsilon_0 r^2} = 0$

or, $q = 2\pi\alpha R^2$

So, resultant field, independent of r, is given as

$E = \frac{1}{2} \frac{\alpha}{\varepsilon_0}$

15.



Electric potential at centre :

If we assume an imaginary identical hemisphere of same charge distribution to complete the sphere, then potential at centre = $\frac{kQ_{total}}{R}$. So, due to symmetry, potential at centre due to left half is equal to Right half. So, potential due to a hemisphere at centre = $\frac{kQ_{total}}{2R} = \frac{k(2Q_{hemisphere})}{2R}$

$$= \frac{kQ}{R} = \frac{k(2\pi R^2.\sigma)}{R} \quad \text{or} \quad V = \frac{\sigma R}{2\in_0}$$

Alternative : -

∵ Each charge is at same distance of R from centre

So, Potential $= \frac{k\Sigma Q}{R} = \frac{k(2\pi R^2.\sigma)}{R}$; $V = \frac{\sigma R}{2\in_0}$

Electric Field

To calculate the electric field strength at centre, we take a ring element which makes angle θ on the centre and having width of R dθ. Due to this ring, electric field strength at centre :

$$\Rightarrow \quad d\vec{E} = \frac{k.dq.x}{(x^2+r^2)^{3/2}}\,\hat{i}$$

{here dq = charge on Ring ; r = radius of ring, x = distance b/w centre of ring and hemisphere}

By figure, x = R cosθ and dq = σ.2π R sinθ (Rdθ)

$$\Rightarrow \quad d\vec{E} = \frac{k.\,[\sigma.2\pi R\sin\theta]\;R\,d\theta\;.\,R\cos\theta}{R^3}$$

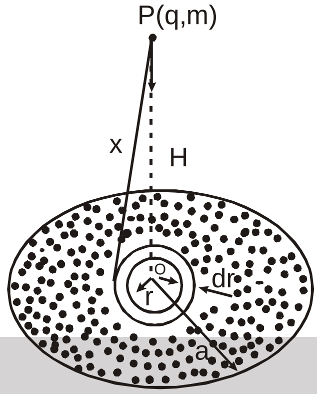
$$d\vec{E} = \pi\, k\, \sigma\, [\sin 2\theta\, d\theta]$$

$$\int d\vec{E} = \frac{\sigma}{4\in_o}\int_o^{\frac{\pi}{2}} \sin 2\theta d\theta\;\hat{i} = \frac{\sigma}{4\in_o}\left[\frac{-\cos 2\theta}{2}\right]_0^{\pi/2}\hat{i} = \frac{\sigma}{4\in_o\times 2}\;[-\cos\pi+\cos 0]\;\hat{i}$$

$$\vec{E} = \frac{\sigma}{4\in_o}\cdot\frac{2\hat{i}}{2} = \frac{\sigma}{4\in_o}\,\hat{i}$$

16. **Potential at a height H on the axis of the disc ie. V(P) : →**

The charge dq contained in the ring shown in figure



$dq = (2\pi r dr)\sigma$

Potential at P due to this ring,

$$dV = \frac{1}{4\pi \in_0}\frac{dq}{x}\text{; where, } x = \sqrt{H^2 + r^2}$$

$$dV = \frac{1}{4\pi \in_0}\frac{(2\pi r dr)\sigma}{\sqrt{H^2 + r^2}} = \frac{\sigma}{2 \in_0}\frac{r dr}{\sqrt{H^2 + r^2}}$$

∴ Potential due to the complete disc,

$$V_p = \int_{r=0}^{r=a} dV = \frac{\sigma}{2 \in_0}\int_{r=0}^{r=a}\frac{r dr}{\sqrt{H^2 + r^2}}$$

or, $V_p = \frac{\sigma}{2 \in_0}[\sqrt{H^2 + a^2} - H]$

Potential at centre, (O) will be

$V_O = \frac{\sigma a}{2 \in_0}$; (H = 0)

(i) Particle is released from P and it just reaches point O. Therefore, from conservation of mechanical energy :

Decrease in gravitational potential energy = Increase in electrostatic potential energy

($\Delta KE = 0$ because $K_i = K_f = 0$)

∴ $mgH = q[V_o - V_p]$

or $gH = \left(\frac{q}{m}\right)\left(\frac{\sigma}{2 \in_0}\right)[a - \sqrt{a^2 + H^2} + H]$ ......(1)

Also, $\frac{q}{m} = \frac{4 \in_0 g}{\sigma}$ ∴ $\frac{q\sigma}{2 \in_0 m} = 2g$

Substituting in (1), we get

$gH = 2g[a + H - \sqrt{a^2 + H^2}]$

or $= (a + H) - \sqrt{a^2 + H^2}$ or $= a + \frac{H}{2}$

or $a^2 + H^2 = a^2 + \frac{H^2}{4} + aH$ or $\frac{3}{4}H^2 = aH$

or $a = \frac{3H}{4}$

∴ $H = (4/3)a$ **Ans.**

(ii) Potential energy of the particle at height H = Electrostatic potential energy + gravitational potential energy

$\therefore \quad U = qV + mgH$

Here V = Potential at height H

$\therefore \quad U = \frac{\sigma q}{2\epsilon_0}[\sqrt{a^2+H^2} - H] + mgH \quad ...(2)$

At equilibrium position,

$F = \frac{-dU}{dH} = 0$

Differentiating (2) w.r.t. H :

or $\quad mg + \frac{\sigma q}{2\epsilon_0}\left[\left(\frac{1}{2}\right)(2H)\frac{1}{\sqrt{a^2+H^2}} - 1\right] = 0$

$\therefore \quad mg + 2mg\left[\frac{H}{\sqrt{a^2+H^2}} - 1\right] = 0$

or $\quad 1 + \frac{2H}{\sqrt{a^2+H^2}} - 2 = 0$ or $\quad \frac{2H}{\sqrt{a^2+H^2}} = 1$

or $\quad \frac{H^2}{a^2+H^2} = \frac{1}{4}$ or $\quad 3H^2 = a^2$ or $\quad H = \frac{a}{\sqrt{3}}$ **Ans.**

From equation (2), we can write :

U–H equation as

$U = mg\,(2\sqrt{a^2+H^2} - H)$

$\therefore \quad U = 2mga$ at $H = 0$ and

$U = U_{min} = \sqrt{3}\ mga$ at $H = \frac{a}{\sqrt{3}}$

Therefore U–H graph will be as shown.

Note that at $H = \frac{a}{\sqrt{3}}$, U is minimum.

Therefore, $H = \frac{a}{\sqrt{3}}$ is stable equilibrium position.

17.

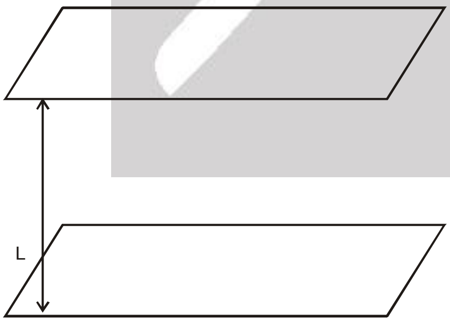


Figure shows two parallel plates electric field between plates E = V/d $\Rightarrow \quad E = \frac{at}{L}$

force on electron = $qE = \frac{eat}{L}$,

So acceleration of $e^- = \frac{eat}{mL}$

By acceleration = $\frac{dv}{dt}$; (v = velocity)

$$\frac{dv}{dt} = \frac{eat}{mL} \Rightarrow dv = \frac{eat}{mL}\,dt \quad \Rightarrow \quad \int_0^V dv = \int_0^t \frac{eat\,dt}{mL} \quad \Rightarrow \quad v = \frac{eat^2}{2mL}$$

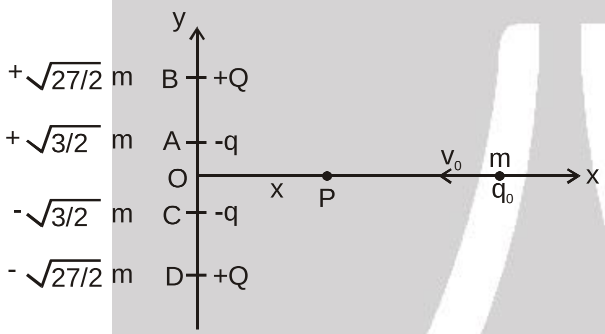
Again, $v = \frac{dx}{dt} = \frac{eat^2}{2mL}$ ...(1) $\Rightarrow \int_0^L dx = \int_0^t \frac{eat^2dt}{2mL} \Rightarrow \quad L = \frac{eat^3}{6mL}$

$\therefore \quad t = \left(\frac{6mL^2}{ea}\right)^{\frac{1}{3}}$

putting in eqn. (1)

$$v = \frac{ea}{2mL}\left(\frac{6mL^2}{ea}\right)^{\frac{2}{3}} = \frac{ea}{2mL}\left(\frac{36m^2L^4}{e^2a^2}\right)^{\frac{1}{3}}; \quad v = \left(\frac{9eaL}{2m}\right)^{\frac{1}{3}}$$

18.



In the figure $q = 1\ \mu C = 10^{-6}$ C $\quad q_0 = 0.1\ \mu C = 10^{-7}$C and $m = 6 \times 10^{-4}$ Kg
and $\quad Q = 8\mu C = 8 \times 10^{-6}$ C
Let P be any point at a distance x from origin O.Then

$$AP = CP = \sqrt{\frac{3}{2} + x^2}$$

$$\text{and } BP = DP = \sqrt{\frac{27}{2} + x^2}$$

Electric potential at point P will be–

$$V_P = \frac{2KQ}{BP} - \frac{2Kq}{AP}; \text{ where, } K = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\ Nm^2/C^2$$

$$\therefore \quad V_P = 2 \times 9 \times 10^9 \left[\frac{8 \times 10^{-6}}{\sqrt{\frac{27}{2} + x^2}} - \frac{10^{-6}}{\sqrt{\frac{3}{2} + x^2}}\right]$$

$$\text{or} \quad V = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2} + x^2}} - \frac{1}{\sqrt{\frac{3}{2} + x^2}}\right] \quad ....(1)$$

$\therefore$ Electric field at P is–

$$E_P = -\frac{dV}{dX} = 1.8 \times 10^4 \left[(8)\left(-\frac{1}{2}\right)\left(\frac{27}{2} + x^2\right)^{-3/2} - (1)\left(-\frac{1}{2}\right)\left(\frac{3}{2} + x^2\right)^{-3/2}\right](2x)$$

E = 0 on axis where $\rightarrow$ x = 0 or

$$\frac{8}{\left(\frac{27}{2} + x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2} + x^2\right)^{3/2}} \Rightarrow \frac{(4)^{3/2}}{\left(\frac{27}{2} + x^2\right)^{3/2}} = \frac{1}{\left(\frac{3}{2} + x^2\right)^{3/2}}$$

$\therefore \quad \left(\frac{27}{2} + x^2\right) = 4\left(\frac{3}{2} + x^2\right)$

This equation gives, $x = \pm\sqrt{\frac{5}{2}}$ m

The least value of kinetic energy of the particle at infinity should be enough to take the particle upto $x = +\sqrt{\frac{5}{2}}$ m because

at $x = \sqrt{\frac{5}{2}}$ + m, E = 0 ⇒ Electrostatic force on charge q is zero or Fe = 0.

For $x > \sqrt{\frac{5}{2}}$ m , E is repulsive (towards positive x–axis)

and For $x < \sqrt{\frac{5}{2}}$ m , E is attractive (towards negative x–axis)

Now, from equation (1), potential at $x = \sqrt{\frac{5}{2}}$ m

$$V_P = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2}+\frac{5}{2}}} - \frac{1}{\sqrt{\frac{3}{2}+\frac{5}{2}}}\right]$$

Applying energy conservation at x = ∞ and $x = \sqrt{\frac{5}{2}}$ m

$$\frac{1}{2}mv_0^2 = q_0V \quad ....(2)$$

$\therefore \quad v_0 = \sqrt{\frac{2q_0V}{m}}$

Substituting the values

$$v_0 = \sqrt{\frac{2\times10^{-7}\times2.7\times10^4}{6\times10^{-4}}}$$

or $\quad v_0 = 3$ m/s

$\therefore$ Minimum value of v_0 is 3 m/s. **Ans. (i)**

From equation (1), potential at origin (x = 0) is

$$V_0 = 1.8 \times 10^4 \left[\frac{8}{\sqrt{\frac{27}{2}}} - \frac{1}{\sqrt{\frac{3}{2}}}\right]$$

$$\approx 2.45 \times 10^4 \text{ V}$$

Let K be the kinetic energy of the particle at origin.

Applying energy conservation at x = 0 and at x = ∞

$K + q_0V_0 = 1/2\, mv_0^2$

But, $\quad 1/2\, mv_0^2 = q_0 V$ from equation (2)

$\therefore \quad K = q_0 (V - V_0)$

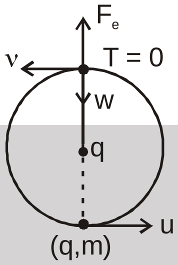
or $\quad K = (10^{-7}) (2.7 \times 10^4 - 2.45 \times 10^4) \sim 2.5 \times 10^{-4}$ J **Ans (ii)**

→ **Note :** E = 0 or Fe on q_0 is zero at x = 0 and $x = \pm\sqrt{\frac{5}{2}}$ m of these, x = 0 is stable equilibrium position and $x = \pm\sqrt{\frac{5}{2}}$ is unstable equilibrium position.

19. Given, $q = 1\mu c = 10^{-6}$ C
& $m = 2 \times 10^{-3}$ Kg and
$\ell = 0.8$ m

Let u be the speed of the particle at its lowest point and v its speed at highest point.
At highest point, three forces are acting on the particle.
(i) Electrostatic repulsion

$$F_e = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{\ell^2} \qquad \text{(outwards)}$$



(ii) Weight W = mg (inwards), and
(iii) Tension T (inwards)
T = 0, if the particle has just to complete the circle and the necessary centripetal force is provided by W – F_e i.e., $\frac{mv^2}{\ell} = W - F_e$

or $v^2 = \frac{\ell}{m}\left(mg - \frac{1}{4\pi\varepsilon_0}.\frac{q^2}{\ell^2}\right)$

$v^2 = \frac{0.8}{2\times10^{-3}}\left(2\times10^{-3}\times10 - \frac{9.0\times10^9\times(10^{-6})^2}{(0.8)^2}\right)$ m^2 / s^2

or $v^2 = 2.4$ m^2 / s^2 .....(1)

Now the electrostatic potential energy at the lowest and highest points are equal. Hence from conservation of mechanical energy
Increase in gravitational potential energy = Decrease in kinetic energy

or $mg(2l) = \frac{1}{2} m (u^2 - v^2)$

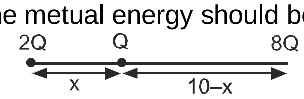
or $u^2 = v^2 + 4\,gl$

Substituting the values of v^2 from equation (1) we get
$u^2 = 2.4 + 4\,(10)\,(0.8) = 34.4$ m^2/ s^2.

$\therefore$ u = 5.86 m/s **Ans(2)**

Therefore, minimum horizontal velocity imparted to the lower ball, so that it can make compete revolution,is 5.86 m/s.

20. For potential energy of this system to be minimum, point charges 2Q & 8Q must be placed at the end positions of straight line. So that the metual energy should be minizied



$$PE = \frac{2KQ^2}{x} + \frac{8KQ^2}{10-x} + \frac{16KQ^2}{10}$$

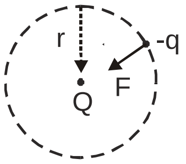
For minimum PE of system

$$\frac{d(PE)}{dx} = 0$$

$\Rightarrow -\frac{2KQ^2}{x^2} + \frac{8KQ^2}{(10-x)^2} = 0 \quad \Rightarrow \quad (10 - x)^2 = 4x^2$

$\Rightarrow 10 - x = \pm 2x \quad \Rightarrow \quad x = \frac{10}{3}$ cm (from 2q charge)

21.



Centripetal force $F = \frac{mv^2}{r}$

$\therefore \quad \frac{kQq}{r^2} = \frac{mv^2}{r}$

$\Rightarrow \quad \frac{kQq}{mr} = v^2$

& $\quad T = \frac{2\pi r}{v}$

$\therefore \quad T^2 = \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^2 (mr)}{kQq}$

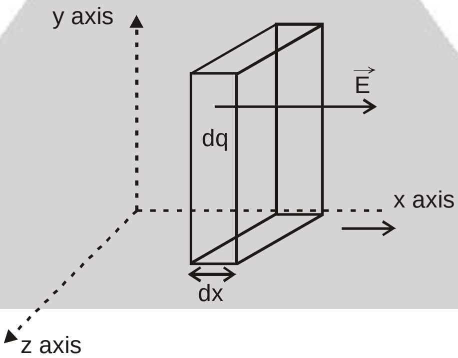
$\therefore \quad T^2 \alpha r^3$. Hence proved

22. Given that field potential is variable only in x direction. so for Given value of x (also we can conclude that electric field is parallel to x-axis) Potential is constant in Y and Z direction. Now, by taking a small volume in space at distance x. Cross section area of this element is A and width is dx [Given $\phi = ax^3 + b$]

Electric field $\quad \vec{E} = -\frac{d\phi}{dx}\hat{i} = -3ax^2\hat{i}$

Now electric field is constant for a particular value of x and it is parallel to area vector of this small elemental volume.

By Guess theorem :



$\int \vec{E}.\overrightarrow{dA} = \frac{q_{in}}{\varepsilon_0}$

$E\int dA = \frac{q_{in}}{\varepsilon_0}$

$EA = \frac{q_{in}}{\varepsilon_0} \quad \Rightarrow \quad A\,dE = \frac{dq_{in}}{\varepsilon_0}$

$A.\,6axdx = \frac{\rho.dx.A}{\varepsilon_0} \quad \Rightarrow \rho = 6a\varepsilon_0 x$

23. Capacities of conducting spheres are in the ratio of their radii. Let C_1 and C_2 be the capacities of S_1 and S_2, then

$$\frac{C_2}{C_1} = \frac{R}{r}$$

(a) Charges are distributed in the ratio of their capacities. Let in the first contact, charge acquired by S_2 is q_1. Therefore, charge on S_1 will be $Q - q_1$. Say it is q_1'

$$\therefore \quad \frac{q_1}{q_1'} = \frac{q_1}{Q - q_1} = \frac{C_2}{C_1} = \frac{R}{r}.$$

It implies that Q charge is to be distributed in S_2 and S_1 in the ratio of R/r.

$$\therefore \quad q_1 = Q\left(\frac{R}{R+r}\right) \qquad \text{......(1)}$$

In the second contact, S_1 again acquires the same charge Q.

Therefore, total charge in S_1 and S_2 will be $Q + q_1 = Q\left(1 + \frac{R}{R+r}\right)$

This charge is again distributed in the same ratio. Therefore, charge on S_2 in second contact,

$$\therefore \quad q_2 = Q\left(1 + \frac{R}{R+r}\right)\left(\frac{R}{R+r}\right) = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2\right]$$

Similarly, $$q_3 = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \left(\frac{R}{R+r}\right)^3\right]$$

and $$q_n = Q\left[\frac{R}{R+r} + \left(\frac{R}{R+r}\right)^2 + \ldots\ldots + \left(\frac{R}{R+r}\right)^n\right]$$

or $$q_n = Q\,\frac{R}{r}\left[1 - \left(\frac{R}{R+r}\right)^n\right] \qquad \text{..........(i)}$$

$$\left[S_n = \frac{a(1-r^n)}{(1-r)}\right]$$

Therefore, electrostatic energy of S_2 after n such contacts

$$U_n = \frac{q_n^{\,2}}{2C} = \frac{q_n^{\,2}}{2(4\pi \in_0 R)} \text{ or } \qquad U_n = \frac{q_n^{\,2}}{8\pi \in_0 R} \qquad \textbf{Ans.}$$

where q_n can be written from equation (1).

(b) $$q_n = \frac{QR}{(R+r)}\left[1 + \frac{R}{R+r} \ldots\ldots \left(\frac{R}{R+r}\right)^{n-1}\right]$$

as $n \to \infty$

$$\therefore \quad q_\infty = \frac{QR}{R+r}\,\frac{1}{1 - \frac{R}{R+r}} = \left(\frac{QR}{r}\right)$$

$$\therefore \quad U_\infty = \frac{q_\infty^2}{2c} = \frac{Q^2R^2/r^2}{8\pi \in_0 R} \quad \text{or} \quad U_\infty = \frac{Q^2R}{8\pi \in_0 r^2}$$

24. Given, $\vec{E} = \frac{a(x\hat{i} + y\hat{j})}{x^2 \times y^2}$

Let $d\vec{s} = \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{R}\,ds$; where ds = magnitude of small area considered on surface of sphere.

and $\frac{x\hat{i} + y\hat{j} + z\hat{k}}{R}$ = unit vector along the radius

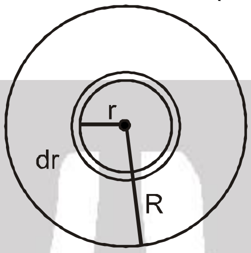
So $\sqrt{x^2+y^2+z^2} = R$ Now flux of $\vec{E} = \int \vec{E}.\,ds$

$$\phi = \int\left(\frac{a(x\hat{i} + y\hat{j})}{x^2+y^2}\right) \bullet \frac{(x\hat{i} + y\hat{j} + z\hat{k})}{R}\,ds \qquad \phi = \int \frac{a(x^2 + y^2)}{R(x^2+y^2)}\,ds = \frac{a}{R}\int ds$$

$$\phi = 4\,\pi R^2.\ \frac{a}{R} = 4\pi Ra \quad \text{and } \phi = \frac{Q}{\epsilon_0} \qquad \therefore \quad Q = \phi\,\epsilon_0 = 4\pi\,Ra\,\epsilon_0$$

25. Given, $\rho = \rho_0 r$

(a) Let a spherical element of thickness dr and radius r (r < R) is considered



Charge inside this sphere $q_{in} = \int_0^r \rho_0\ r.4\pi r^2 dr = \pi\rho_0 r^4$

Electric field at r = $\frac{Kq_{in}}{r^2}\hat{r} = \frac{K.\rho_0\pi r^4}{r^2}\hat{r} = \frac{\rho_0 r^2}{4\varepsilon_0}\hat{r}$

For potential : ⟶

V due to inner shell (or charge) = $\frac{Kq_{in}}{r}$

and V due to outer charge = $\int_r^R \frac{kdq}{r}$

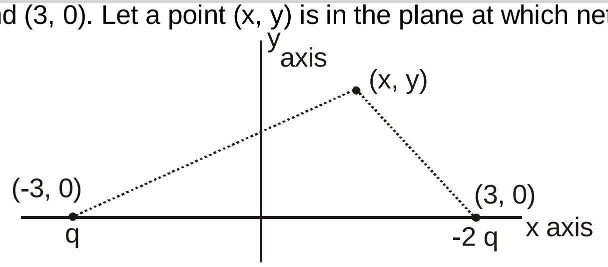
$$\therefore \text{ Total potential} = \frac{kq_{in}}{r} + \int_r^R \frac{kdq}{r} = \frac{1}{4\pi\varepsilon_0}\frac{(\pi\rho_0 r^4)}{r} + \int_r^R \frac{(4\pi r^2 dr)\rho_0}{4\pi\varepsilon_0 r}$$

$$= \frac{\rho_0 r^3}{4\varepsilon_0} + \frac{\rho_0}{\varepsilon_0}\left[\frac{R^3}{3} - \frac{r^3}{3}\right] = \frac{\rho_0\left[4R^3 - r^3\right]}{12\varepsilon_0}$$

(b) When r > R, Electric field = $\frac{kq_{in}}{r^2}\hat{r} = \frac{\rho_0\pi R^4}{4\pi\varepsilon_0 r^2}\hat{r} = \frac{\rho_0 R^4}{4\varepsilon_0 r^2}\hat{r}$

and Potential = $\frac{kq_{in}}{r} = \frac{1}{4\pi\varepsilon_0}.\frac{\rho_0\pi R^4}{r} = \frac{\rho_0 R^4}{4\varepsilon_0 r}$

26. Let charge q and –2q are placed in x – y plane and at x – axis ; Co-ordinate of q and –2q are respectively (–3, 0) and (3, 0). Let a point (x, y) is in the plane at which net potential is equal to zero



$$\therefore \quad \frac{kq}{\sqrt{(x+3)^2+y^2}} + \frac{k(-2q)}{\sqrt{(x-3)^2+y^2}} = 0 \quad \Rightarrow \quad 4[(x+3)^2 + y^2] = [(x-3)^2 + y^2]$$

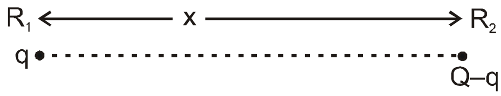
$\Rightarrow$ $4x^2 + 24x + 36 + 4y^2 = x^2 + 9 - 6x + y^2$

$\therefore$ $3x^2 + 3y^2 + 30x + 27 = 0$ $\therefore$ $x^2 + y^2 + 10x + 9 = 0$

or $(x + 5)^2 + (y - 0)^2 = 16$

This is eq^n of circle of radius 4m and centre at (–5, 0)

27.



Let total charge is Q and charge on ball (1) is q so charge on 2nd ball = Q – q

Now x >>> R_1 and R_2 so we can neglect potential energy of interaction,

So, total energy = $\dfrac{q^2}{8\pi \in_o R_1} + \dfrac{(Q-q)^2}{8\pi \in_o R_2} = E$

For E to be minimum : $\dfrac{dE}{dq} = 0 \Rightarrow \dfrac{1}{8\pi \in_o}\left[\dfrac{2q}{R_1} + \dfrac{2q-2Q}{R_2}\right] = 0 \Rightarrow \dfrac{q}{R_1} = \dfrac{Q-q}{R_2}$

$\Rightarrow \quad q = \dfrac{R_1 Q}{R_1 + R_2}$ and $Q - q = \dfrac{Q R_2}{R_1 + R_2}$

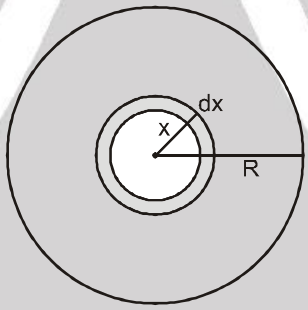
So ratio of charge = $\dfrac{Q_1}{Q_2} = \dfrac{q}{Q-q} = \dfrac{R_1}{R_2}$

28. **(i) E at a point inside the ball (r < R) :**

Consider an elemental shell of radius x and thickness dx. Electric field due to this small element at a distance r from centre:

$dE = \dfrac{Kdq}{r^2}$, where dq = ρ dV [dV = Volume of elemental shell]

$\therefore \quad dq = \rho_0 \left(1 - \dfrac{x}{R}\right)(4\pi x^2\, dx)$



$\therefore \quad E_{net} = \displaystyle\int_{x=0}^{x=r} \dfrac{K\rho_0 (1-\frac{x}{R})\,(4\pi x^2 dx)}{r^2} = \dfrac{\rho_0}{\varepsilon_0 r^2}\int_0^r \left(x^2 - \dfrac{x^3}{R}\right)dx = \dfrac{\rho_0}{\varepsilon_0 r^2}\left[\dfrac{r^3}{3} - \dfrac{r^4}{4R}\right]$

Solving we get, $E_{in} = \dfrac{\rho_0 r}{3\varepsilon_0}\left(1 - \dfrac{3r}{4R}\right)$

E outside the ball (r > R) →

$E_{net} = \displaystyle\int_{x=0}^{x=R} \dfrac{Kdq}{r^2}$

$E_{net} = \displaystyle\int_{x=0}^{x=R} \dfrac{K\rho_0 (1-\frac{x}{R})(4\pi x^2 dx)}{r^2}$

$E_{net} = \dfrac{\rho R^3}{12\varepsilon_0 r^2}$

(ii) E will be maximum at inside point when $\dfrac{dE}{dr} = 0$

$\therefore \quad \frac{\rho}{3\varepsilon_0}\left(1-\frac{6r}{4R}\right)=0 \Rightarrow \quad r = \frac{2R}{3}$

$\therefore \quad E_{max} = (E_{in})_{r=\frac{2R}{3}}$

$$= \frac{\rho_0\left(\frac{2R}{3}\right)}{3\varepsilon_0}\left(1-\frac{3\left(\frac{2R}{3}\right)}{4R}\right) = \frac{\rho_0 R}{9\varepsilon_0}$$

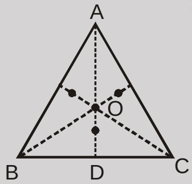
29. (a) $\vec{F} = \frac{2KQq\left(\frac{a}{\sqrt{3}}-\delta\right)}{\left(a^2+\left(\frac{a}{\sqrt{3}}-\delta\right)^2\right)^{3/2}} - \frac{KQq}{\left(\frac{2a}{\sqrt{3}}+\delta\right)^2}$

Here $K = 1/4\pi\varepsilon_0$ and direction is upward (towards A)

(b) Using binomial approximation, $\vec{F} = KQq\frac{9\sqrt{3}}{16}\frac{\delta}{a^3}$ (upward) which is linear in δ. Hence charge will oscillate simple harmonically about O when released.

(c) $\vec{F}_D = \frac{KQq}{3a^2}$ (downward)

(d) For small δ force on the test charge is upwards while for large δ (eg. at D) force is downwards. So there is a neutral point between O and D. By symmetry there will be neutral points on other medians also. In figure x. Below all possible (4) neutral points are shown by •.

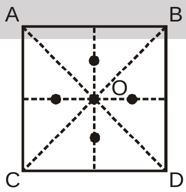


(e) Let the distance along P be x and O to be at (0, 0). Electric potential of a test charge along OP can be written as

$$V(x) = \frac{KQ}{\sqrt{x^2+(4/3)}} + \frac{KQ}{\sqrt{(x+1)^2+(1/3)}} + \frac{KQ}{\sqrt{(x-1)^2+(1/3)}} \approx KQ\sqrt{\frac{3}{4}}\left(3+\frac{9}{16}x^2\right)$$

We can see that $V(x) \propto x^2$, hence it is a stable equilibrium.

(f) Equilibrium points are indicated by •.



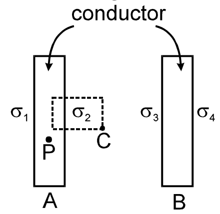
(g) N + 1

30. flux leaving q_1 is equal to flux entering $-q_2$.

$(1-\cos\alpha)\frac{q_1}{\varepsilon_0} = (1-\cos\beta)\frac{q_2}{\varepsilon_0}$

$q_1 \sin^2\left(\frac{\alpha}{2}\right) = q_2 \sin^2\left(\frac{\beta}{2}\right)$

31. By considering Gaussian surface as shown in figure, and applying Gauss law



We have $EA = \frac{\sigma_2 A}{\epsilon_0}$

$\therefore \quad E = \frac{\sigma_2}{\epsilon_0}$

$\therefore \quad \sigma_2 = \epsilon_0 E$

Also, $\sigma_3 = -\sigma_2 = -\epsilon_0 E$.

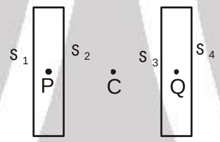
We can not find σ_1 and σ_4.

E at P = 0

$\Rightarrow \quad \sigma_1 = \sigma_4$

$\sigma_1 = \sigma_4$ ......Ans.

ALTERNATIVE :



In conductor $E_P = 0$

$\therefore \quad \sigma_1 A = \sigma_2 A + \sigma_3 A + \sigma_4 A$

$\Rightarrow \quad \sigma_1 = \sigma_2 + \sigma_3 + \sigma_4$ ....(1)

and $E_Q = 0$

$\Rightarrow \quad \sigma_4 A = \sigma_3 A + \sigma_2 A + \sigma_1 A$ ....(2)

By (1) and (2)

$\sigma_4 = \sigma_3 + \sigma_2 + \sigma_2 + \sigma_3 + \sigma_4 \quad \Rightarrow \quad \sigma_2 = -\sigma_3$ ....(3)

Using (3) in (1)

$\sigma_1 = \sigma_4$ **....Ans.**

Now : $E_c = \frac{\sigma_2}{2\epsilon_0} - \frac{\sigma_3}{2\epsilon_0} = \frac{2\sigma_2}{2\epsilon_0} = \frac{\sigma_2}{\epsilon_0} = E$

$\therefore \quad \sigma_2 = \epsilon_0 E$ **....Ans.**

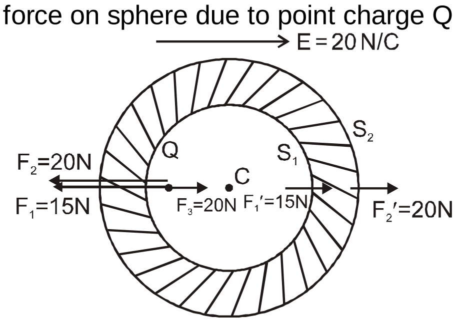
$\sigma_3 = -\epsilon_0 E$ **....Ans.**

$\sigma_1 = \sigma_4$, $\sigma_2 = \epsilon_0 E$, $\sigma_3 = -\epsilon_0 E$ ....Ans.

32. The net force on point charge Q at A is zero in the cavity due to external electric field and external induced chages on body of conductor.

Hence force on point Q due to induced chages is 35 N towards left.

By action reaction principle, force on sphere due to point charge Q is 35 N rightward.



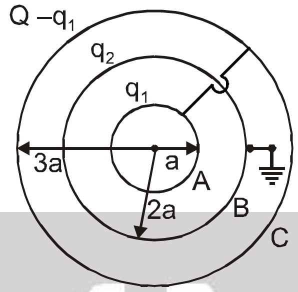
F_1 = Force on Q due to induced charge on S_1 = 15 N,

F_1' = Force on S_1 due to Q = 15 N

F_2 = Force on Q due to induced charge on S_2 = 20 N

F_2' = Force on S_2 due to Q = 20 N
F_3 = Force on Q due to electric field E
F_1, F_1' and F_2, F_2' are action reaction force.

33. Let q_1 and q_2 are charges on A and B respectively
From given conditions: Charge on A & C after connection with wire are q_1 and $Q - q_1$ on B, charge is q_2



$V_A = V_C$ and $V_B = 0$

$\Rightarrow \quad V_B = \dfrac{K(Q-q_1)}{3a} + \dfrac{Kq_2}{2a} + \dfrac{Kq_1}{2a} = 0$

$\Rightarrow \quad 2Q + q_1 + 3q_2 = 0$ ....(1)

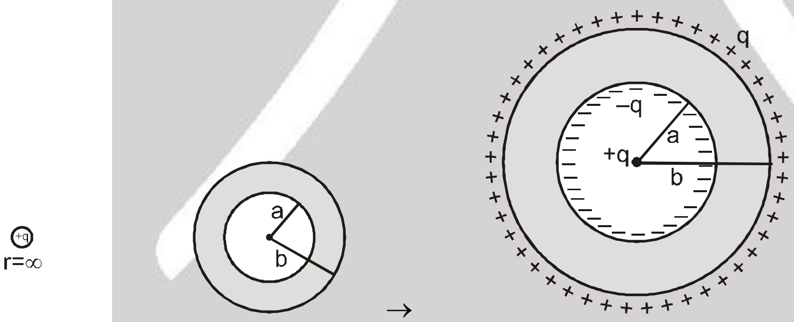
Using $V_C = V_A$

$\dfrac{K(Q-q_1)}{3a} + \dfrac{Kq_2}{3a} + \dfrac{Kq_1}{3a} = \dfrac{Kq_1}{a} + \dfrac{K(Q-q_1)}{3a} + \dfrac{Kq_2}{2a}$

$\Rightarrow q_1 = -\dfrac{q_2}{4}$ ....(2)

Using it in (1), $q_2 = -\dfrac{8}{11}Q$

34.



Work done by external agent :
$W_{ext} = U_F - U_i$

$\therefore \quad W_{ext} = \left(\dfrac{Kq^2}{2a} + \dfrac{Kq^2}{2b} + \dfrac{K(+q)\ (-q)}{(a)} + \dfrac{K(-q)\ (+q)}{b} + \dfrac{K(+q)\ (+q)}{b}\right) - 0 = \dfrac{Kq^2}{2b} - \dfrac{Kq^2}{2a}$

35. The charged sphere will polarize the neutral one, which acquires a dipole moment p proportional to the electric field created by the charged sphere

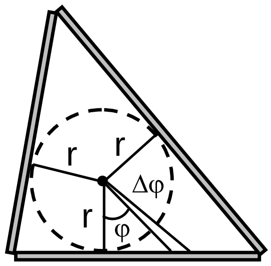
$p \propto E \propto \dfrac{q}{R^2}$

The force between the dipole and the charged sphere is given by the product of the dipole moment and the gradient of the electric field at the dipole.

$F \propto \dfrac{pq}{R^3} \propto \dfrac{q^2}{R^5}$

$q' = 4\sqrt{2}q$

36. We are going to prove that the electric field strength is zero at the socalled incentre, the centre of the triangle's inscribed circle (which has radius r in the figure)



Let us consider a small length of rod at position P on one of the sides of the triangle; let it subtend an angle $\Delta\varphi$ at the incentre (see figure). Its distance from the incentre is r/cos φ. Its small length Δx can be found by noting that P is a distance $x = r \tan \varphi$ along the rod from the fixed point Q and so $\Delta x = (r\,\Delta\varphi) / (\cos^2 \varphi)$. Consequently the charge it carries is

$$\Delta q = \frac{\lambda r \Delta\varphi}{\cos^2 \varphi}$$

where λ is the linear charge density on the rods. The magnitude of the elementary contribution of this small piece to the electric field at the incentre is

$$\Delta E = \frac{1}{4\pi\varepsilon_0}\frac{\Delta q \cos^2\varphi}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda r \Delta\varphi}{r^2}$$

It can be seen from this result that the same electric field (in both magnitude and direction) would be produced by an arc of the inscribed circle that subtends $\Delta\varphi$ at the circle's centre and carries the same linear charge density λ as the rod.

Summing up the contributions of the small arc pieces correspondingto all threesides of the triangle, we will, because of the circular symmetry, obtain zero net field. It follows that the electric field strength produced by the charged sides of the triangle is also zero at the incentre.

37. According to Newton's third law, the insulating plate acts on the point charge with a force of the same magnitude (but opposite direction) as the point charge does on the plate. We calculate the magnitude of this latter force.

Divide the plate (notionally) into small pieces, and denote the area of the i^{th} piece by ΔA_i. Because of the uniform charge distribution, the charge on this small piece is

$$\Delta Q_i = \frac{Q}{d^2}\Delta A_i$$

and so the electric force acting on it is Fi = Ei_Qi, where Ei is the magnitude of the electric field produced by the point charge q at the position of the small piece.

The force acting on the insulating plate, as a whole, can be calculated as the vector sum of the forces acting on the individual pieces of the plate. Because of the axial symmetry, the net force is erpendicular to the plate, and so it is sufficient to sum the perpendicular components of the forces :

$$F = \sum_i F_i \cos\theta_i = \sum_i E_i \frac{Q}{d^2}\Delta A_i \cos\theta_i = \frac{Q}{d^2}\sum_i E_i \Delta A_i \cos\theta_i$$

where θ_i is the angle between the normal to the plate and the line that connects the point charge to the ith piece of it.

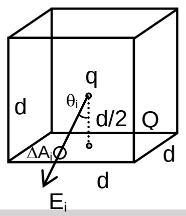
The sum in the given expression is nothing other than the electric flux through the square sheet produced by the point charge q :

$$\psi = \sum_i E_i \Delta A_i \cos\theta_i$$

and can be evaluated as follows.

Let us imagine that a cube of edge d is constructed symmetrically around the point charge (see figure). Then, the distance of the point charge from each side of the cube is just d/2. According to Gauss's law, the total electric flux passing through the six sides of the cube is q/ε_0 and so the flux through a single side is one-sixth of this :

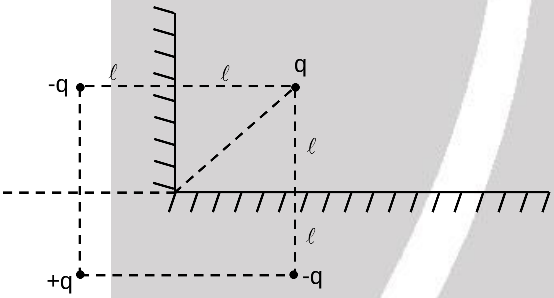
$$\psi = \frac{q}{6\varepsilon_0}$$



Using this and our previous observations, we calculate the magnitude of the force acting on the point charge due to the presence of the charged insulating plate as

$$F = \frac{Qq}{6e_0d^2}$$

38.



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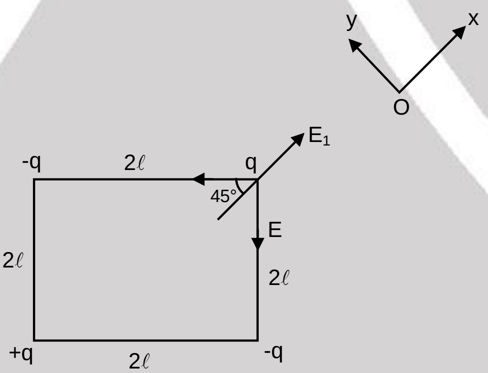


$$E_1 = \frac{q}{4\pi\varepsilon_0(2\sqrt{2}\ell)^2} = \frac{q}{32\pi\varepsilon_0\ell^2}$$

$$E = \frac{q}{4\pi\varepsilon_0(2\ell)^2} = \frac{q}{16\pi\varepsilon_0\ell^2}$$

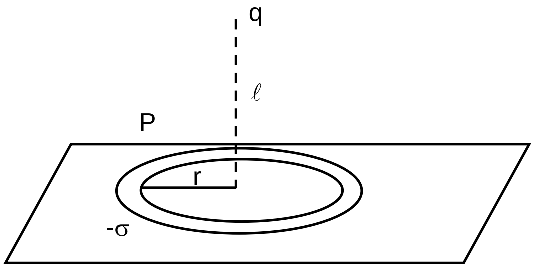
net field at charge q : $E_{net} = E_1 - 2E\cos 45°$

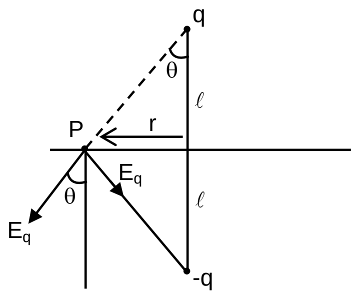
$$= \frac{q}{32\pi\varepsilon_0\ell^2} - \frac{2q}{16\pi\varepsilon_0\ell^2}\frac{1}{\sqrt{2}} = \frac{q}{32\pi\varepsilon_0\ell^2}\left(1 - \frac{4}{\sqrt{2}}\right)$$

$$F_{net} = \frac{q^2}{32\pi\varepsilon_0\ell^2}\left(1 - 2\sqrt{2}\right)$$

(Attractive nature)

39.





E_{net} at point P :

$$E_{net} = 2E_q\cos\theta$$

$$= 2\left(\frac{q}{4\pi\varepsilon_0(r^2+\ell^2)}\right)\frac{\ell}{\sqrt{r^2+\ell^2}}$$

$$E_{net} = \frac{2q\ell}{4\pi\varepsilon_0(r^2+\ell^2)^{3/2}}$$

At point P field due to conducting sheet charge will he half of above calculated E_{net} :

$$\frac{-\sigma}{2\varepsilon_0} = \frac{E_{not}}{2} = \frac{2\ell}{4\pi\varepsilon_0(r^2+\ell^2)^{3/2}}$$

$$-\sigma = \frac{2q\ell}{4\pi(r^2+\ell^2)^{3/2}}$$

$$\sigma = \frac{-q\ell}{2\pi(r^2+\ell^2)^{3/2}}$$

Calculation of charge induced on sheet :

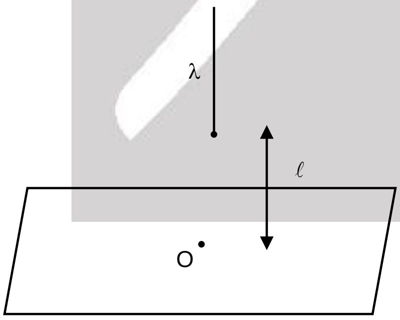
$dq_{in} = 62\pi rdr$

$$q_{in} = -\int_0^{\infty}\frac{2\ell}{2\pi(r^2+\ell^2)^{3/2}}2\pi r.dr$$

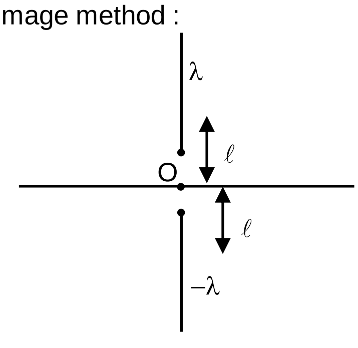
$q_{in} = -q$

40.

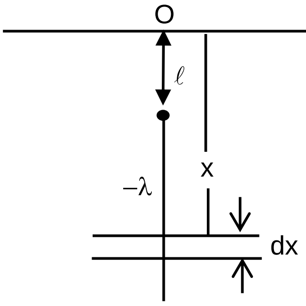
(a)



Using image method :



Calculation of field at point O due to charge of image.



$$dE = \frac{-(\lambda dx)}{4\pi\varepsilon_0 x^2} = \frac{-\lambda}{4\pi\varepsilon_0}\frac{dx}{x^2}$$

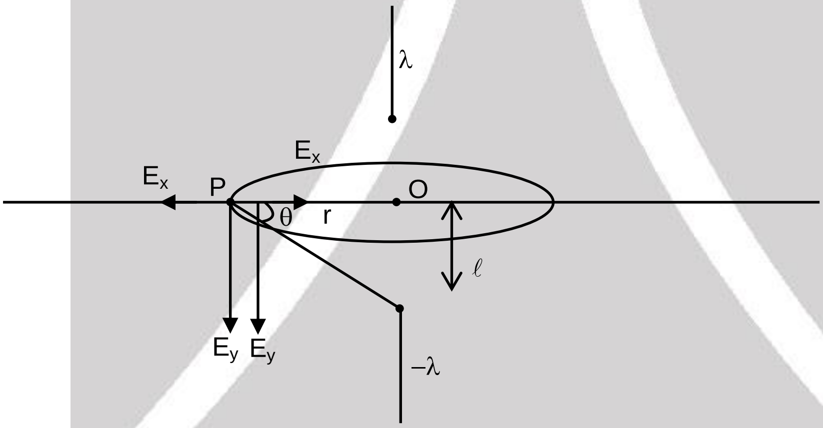
$$\int dE = \frac{-\lambda}{4\pi\varepsilon_0}\int_{\ell}^{\infty}\frac{dx}{x^2}$$

$$E = \frac{-\lambda}{4\pi\varepsilon_0 \ell}$$

Since charge density at O is σ then.

$$\frac{\sigma}{2\varepsilon_0} = \frac{-\lambda}{4\pi\varepsilon_0 \ell} \quad \Rightarrow \quad \sigma = \frac{-\lambda}{2\pi\ell}$$

(b)



Field at point P is only along y-axis because field in the x-driection will be cancelled.
Hence field due to plane = field dut to (–λ) image charge in y direction

$$E_y = \frac{-\lambda}{4\pi\varepsilon_0 r}\left[\cos 90^\circ - \cos\theta\right] = \frac{\lambda\cos\theta}{4\pi\varepsilon_0 r} = \frac{\lambda r}{4\pi\varepsilon_0 r\sqrt{r^2+\ell^2}}$$

$$E_y = \frac{\lambda}{4\pi\varepsilon_0\sqrt{r^2+\ell^2}} = \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = \frac{\lambda}{2\pi\varepsilon_0\sqrt{r^2+\ell^2}}$$