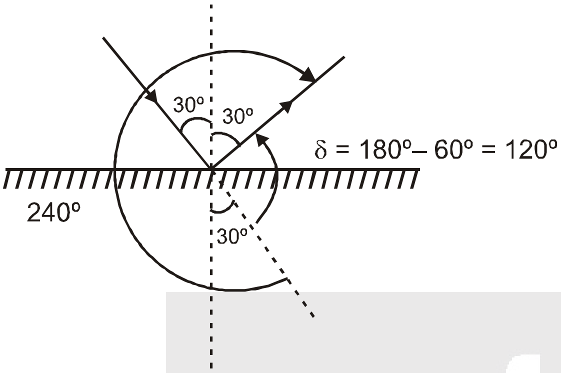
SOLUTIONS OF GEOMETRICAL OPTICS

EXERCISE–1

PART - I

SECTION (A)

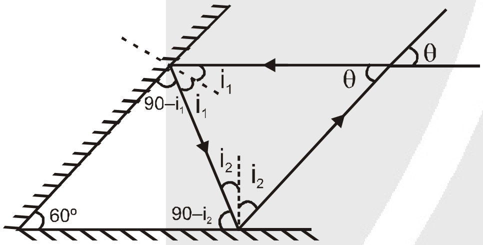
A-1.



δ = 120° Anticlockwise = (360° – 120°) clockwise.

A-2. Angle turned by the reflected ray = 2 (20°) – (10°) = 30° clockwise.

A-3.



$2i_1 + 2i_2 + \theta = 180°$ ....(i)

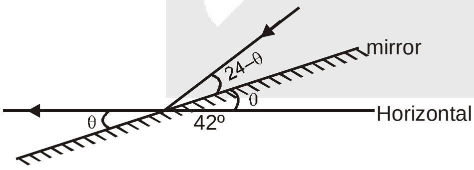
$90° – i_1 + 90° – i_2 + 60° = 180°$

$i_1 + i_2 = 60°$ ....(ii)

from eq. (i) and (ii)

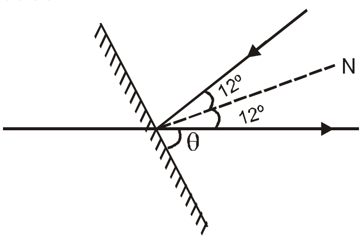
$120° + \theta = 180°$ $\theta = 60°$

A-4. case 1



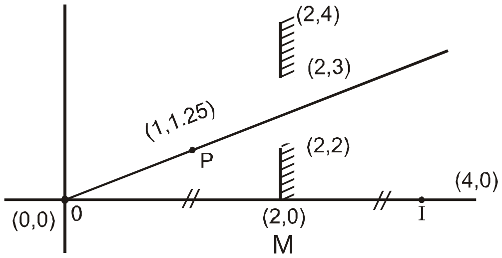
$24 – \theta = \theta$ $\theta = 12°$

case 2



$\theta + 12° = 90°$ $\Rightarrow$ $\theta = 78°$

A-5.



(a) Since both mirrors are in same plane images formed by both mirror concide
(b) MI = MO = 2 $\therefore$ I $\equiv$ (4, 0)
(c) Since ray passing through P is not falling on mirror.

A-6. (a) Position of image = $(1 \cos 60^\circ \hat{i}, -1 \sin 60^\circ \hat{j})$
(b) Velocity of image = $(1 \cos 60^\circ, +1 \sin 60^\circ)$ m/s.

SECTION (B)

B-1.

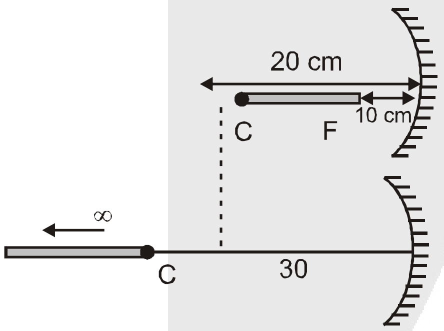
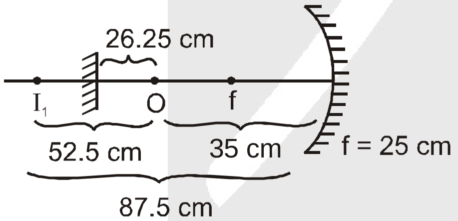


Image of end f and C is formed at infinity and C respectively
Hence image is of infinite length.

B-2.



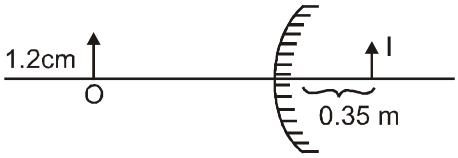
$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $\frac{1}{-25} = \frac{1}{v} + \frac{1}{-35}$ $v = \frac{-175}{2} = -87.5$ cm

OI_1 = 87.5 – 35 = 52.5 cm.
Distance between mirrors = 35 + 26.25 = 61.25 cm

B-3. Moon acts as object at infinity, so image is formed at focus.

$|m| = \left|\frac{d_i}{d_o}\right| = \left|\frac{-v}{u}\right|$ $\Rightarrow$ $d_i = \left|\frac{-v}{u}\right| d_o = \frac{11.4 \times 3450}{3.8 \times 10^8}$ km = 10.35 cm

B-4.



R = 1.2 m

$m = \frac{h_2}{h_1} = \frac{-v}{u}$ $\Rightarrow$ $h_2 = \frac{-v}{u} h_1 = \frac{-\ 0.35 \times 1.2\ \ cm}{-0.84} = 0.5$ cm.

B-5. u = + 40 cm f = – 40 cm

$\frac{1}{-\ 40} = \frac{1}{v} + \frac{1}{40}$ $\frac{1}{v} = -\frac{1}{20}$

$\Rightarrow$ v = – 20 cm So required position is 0.2 m form the mirror's pole.

B-6. (a) $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $\Rightarrow$ $\frac{1}{-40} = \frac{1}{v} + \frac{1}{-60}$ $\Rightarrow$ $\frac{1}{v} = \frac{1}{60} - \frac{1}{40} = \frac{2-3}{120}$ $\Rightarrow$ v = – 120 cm

$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt} = -\left(\frac{120}{60}\right)^2 \times 10 = -40$ cm/sec.

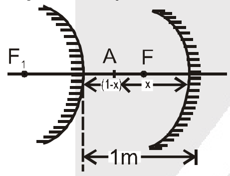
(b) $\frac{h_2}{h_1} = \frac{-v}{u}$ $\Rightarrow$ $\frac{dh_2}{dt} = -\left(\frac{v}{u}\right)\left(\frac{dh_1}{dt}\right) = -\left(\frac{-120}{-60}\right) \times 10$ cm/s = – 20 cm/s

B-7. $m = -\frac{v}{u} = 1.5$ v = – 1.5 u

v = – 1.5 × (– 20) = 30 cm

$\frac{1}{f} = \frac{1}{30} + \frac{1}{-20} = \frac{2-3}{60}$ f = – 60 cm.

B-8. Let the object be placed at a distance x from the concave mirror as shown.



for concave mirror, , u = – x and f = – F $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} = -\frac{1}{F} - \frac{1}{(-x)} = \frac{1}{x} - \frac{1}{F}$

$\therefore$ $v = \frac{Fx}{(F-x)}$

Also $m = \frac{-v}{u} = \frac{F}{(F-x)}$

For convex mirror, u = – (1 – x) and f = F

$\therefore$ $v = \frac{F(1-x)}{F+(1-x)}$ $m = -\frac{v}{u} = \frac{F}{(F+1-x)}$

But size of the images formed are same

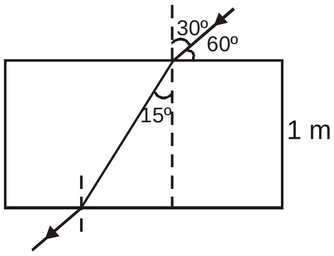
So, $\left|\frac{F}{(F-x)}\right| = \frac{F}{(F+1-x)}$ $\Rightarrow$ $\frac{-F}{(F-x)} = \frac{F}{F+1-x}$ (for F – x < 0)

for F – x > 0 (which is not possible).

$x = \frac{2F+1}{2} = \frac{2 \times 0.36 + 1}{2}$ m = 0.86 m. = 86 cm

SECTION (C)

C-1.

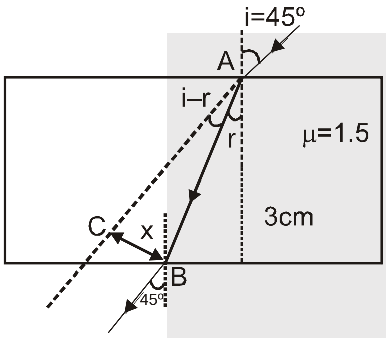


$$\mu = \frac{\sin 30^\circ}{\sin 15^\circ}$$

Distance travelled by light in slab = $\frac{1 \text{ m}}{\cos 15^\circ}$ speed of light in slab = $\frac{C}{\mu}$

$$t = \frac{1/\cos 15^\circ}{C/\mu} = \frac{\mu}{C \quad \cos 15^\circ} = \frac{2 \quad \sin 30^\circ}{2\sin 15^\circ \quad 3\times 10^8 \cos 15^\circ} = \frac{2\times 1/2}{1/2\times 3\times 10^8} = \frac{2}{3} \times 10^{-8} \text{ sec.}$$

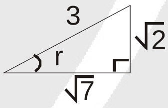
C-2.



Emergent ray is parallel to incident Ray, angle of emergence = 45º

$$\mu = \frac{\sin \quad 45}{\sin \quad r} = 1.5$$

$$\sin r = \frac{1}{\sqrt{2}\times 1.5} = \frac{\sqrt{2}}{3}$$



$AB = \frac{3}{\cos r}$ $\quad x = AB \sin (i - r)$

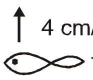
$$= \frac{3\left[\frac{1}{\sqrt{2}}\times\frac{\sqrt{7}}{3} - \frac{1}{\sqrt{2}}\times\frac{\sqrt{2}}{3}\right]}{\frac{\sqrt{7}}{3}} = 3\left[\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{7}}\right] \text{ cm}$$

C-3. Apparent depth = $\frac{t}{\mu} = \frac{40}{4} \times 3 = 30$ cm

C-4. Apparent depth = $\frac{n_1}{n_2} \times d = \frac{5}{2} \times 10 = 25$ cm

C-5. Bird ↓ $\qquad V_b$↓ → real velocity of bird.

velocity of bird as seen by fish



$= \mu V_b + V_f \qquad 16 = \frac{4}{3}\times V_b + 4. \qquad V_b = 9$ cm/sec

C-6. Apparent shift = $t\left(1-\frac{1}{\mu}\right) = 10\left(1-\frac{1}{2}\right) = 5$ cm towards slab.

Apparent distance = 10 + 10 + 20 – 5 = 35 cm.

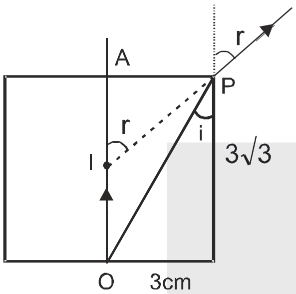
C-7. Apparent shift = $25\left(1-\frac{2}{3}\right)+15\left(1-\frac{2}{5}\right) = \frac{52}{3}$ cm. towards A

Apparent depth = $(25 + 15 - \frac{52}{3}) = \frac{68}{3}$ cm

C-8. Apparent shift = $1.4\left(1-\frac{1}{1.4}\right) + 2\left(1-\frac{1}{1}\right) + 1.3\left(1-\frac{1}{1.3}\right) + 2\left(1-\frac{1}{1}\right) + 1.2\left(1-\frac{1}{1.2}\right) + 2\left(1-\frac{1}{1}\right)$

= 0.4 + 0.3 + 0.2 = 0.9 cm towards the eye.
So image is formed 0.9 cm above P.

C-9.

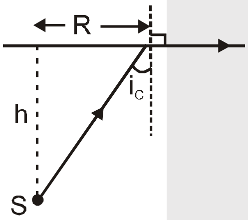


Applying snells law: $\sqrt{3}\ \sin 30^\circ = 1 \times \sin r$

$\Rightarrow \quad r = 60^\circ$

$\tan r = \frac{AP}{AI} \quad \Rightarrow \quad \sqrt{3} = \frac{3}{AI} \qquad AI = \sqrt{3}$ cm

C-10.

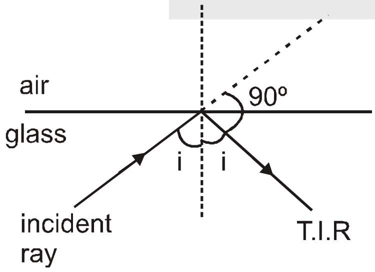


Ligth comes in air from water where refraction takes place.

$\sin i_C = \frac{R}{\sqrt{R^2+h^2}} = \frac{1}{\mu} \qquad \mu^2 R^2 = R^2 + h^2 \qquad R^2 = \frac{h^2}{\mu^2-1}$

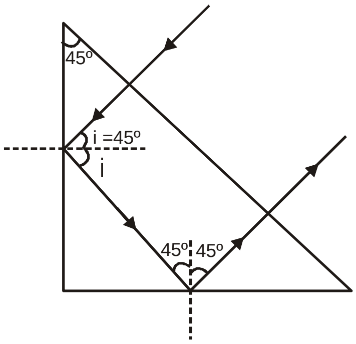
Area = $\pi R^2 = \frac{\pi h^2}{\mu^2-1}$

C-11.



$2i = 90 \qquad \Rightarrow \qquad i = 45^\circ$

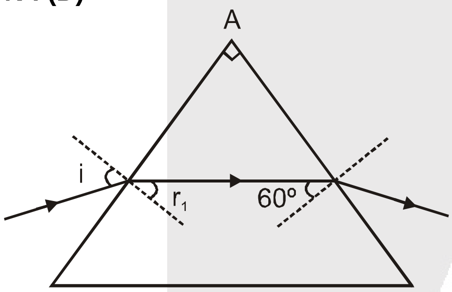
C-12.



$i > \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow \quad \sin\frac{\pi}{4} > \frac{1}{\mu} \quad \Rightarrow \mu > \frac{1}{\sin\frac{\pi}{4}} \quad \Rightarrow \quad \mu > \sqrt{2}$

SECTION : (D)

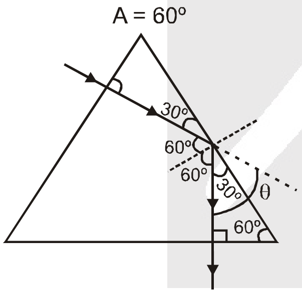


D-1.

$r_1 + 60^\circ = 90^\circ \quad \Rightarrow \quad r_1 = 30$

$\frac{\sin\ i}{\sin 30^\circ} = n = 2 \qquad \sin i = 2 \times \frac{1}{2} \qquad i = 90^\circ$

D-2.



Here $i_c = \sin^{-1}\frac{1}{1.5} = \sin^{-1}\frac{2}{3} < 60^\circ$

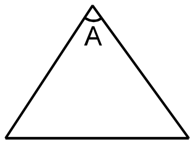
So, T.I.R. takes place at second surface

$\theta + 120^\circ = 180^\circ \qquad \theta = 60^\circ$

D-3. (i) $\delta = A\ (\mu - 1) = 3^\circ\left(\frac{3}{2} - 1\right) = 1.5^\circ$

(ii) $\delta = A\ (\mu - 1) = 3^\circ\ \left(\frac{3/2}{4/3} - 1\right) = 3^\circ\ \left(\frac{9}{8} - 1\right) = \frac{3^\circ}{8}$

D-4.



We have $A = r_1 + r_2$

$r_1 \le I_c$ ↔We have $A = r_1 + r_2$

$r_1 \le I_c$;

$r_2 \le I_c$ (for no T.I.R)

$r_1 + r_2 \le 2 I_c$

$A \le 2I_c \qquad \Rightarrow \qquad A \le 2\sin^{-1}(1/\mu) \qquad$ **[Ans** $2\sin^{-1}\frac{1}{\mu}$ **]**

SECTION (E)

E-1. For refraction at spherical surface

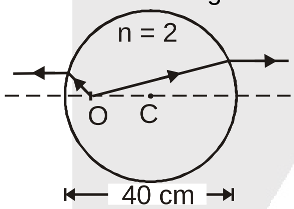
$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \qquad \Rightarrow \qquad \frac{2}{v} - \frac{1}{-10} = \frac{2-1}{+20}$$

$$\Rightarrow \qquad \frac{2}{v} + \frac{1}{10} = \frac{1}{20} \qquad \Rightarrow \qquad \frac{2}{v} = -\frac{1}{20} \qquad \Rightarrow \qquad v = -40 \text{ cm. (virtual)}$$

Using magnification formula

$$m = \frac{h_2}{h_1} = +\left(\frac{\mu_1}{\mu_2}\right)\left(\frac{v}{u}\right) \qquad \Rightarrow \qquad \frac{h_2}{2} = \frac{1\times(-40)}{2\times(-10)} \qquad \Rightarrow \qquad h_2 = +4 \text{ cm. (erect सीधा)}$$

E-2. When seen from air through nearest surface,



$$\frac{1}{-5} - \frac{2}{u} = \frac{1-2}{-20}$$

$$\frac{2}{u}\ \frac{2}{u} = \frac{-1}{20} - \frac{1}{5} = \frac{-1-4}{20}$$

$u = -8$ cm.

for second case,

$u = -(40 - 8) = -32$ cm

$$\frac{1}{v} - \frac{2}{-32} = \frac{1-2}{-20}$$

$$\frac{1}{v} = -\frac{1}{16} + \frac{1}{20} = \frac{-5+4}{80} \qquad v = -80 \text{ cm.}$$

E-3. For first refraction :

$$\frac{1.5}{v_1} - \frac{1}{-10} = \frac{1.5-1}{10} \qquad \Rightarrow \qquad v_1 = -30 \text{ cm.}$$

For second refractions

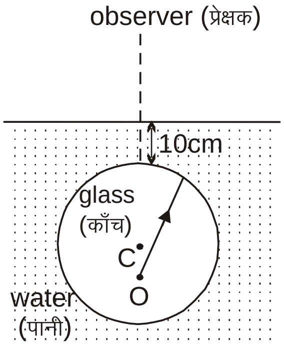
$u = -(30 + 20) = -50$ cm

$$\therefore \qquad \frac{1}{v_2} - \frac{1.5}{-50} = \frac{1-1.5}{-10}$$

$\Rightarrow \qquad v_2 = 50$ cm

Hence, final image is formed 50 cm right of B.

E-4.



For first refraction, (at the glass-water interface)

$$\frac{4/3}{v} - \frac{3/2}{-7.5} = \frac{4/3 - 3/2}{-5}$$

$$\frac{4}{3v} + \frac{3}{15} = \frac{1}{30}$$

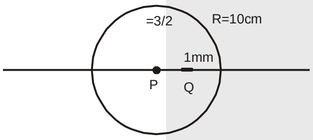
$$\frac{4}{3v} = \frac{-5}{30} \qquad \Rightarrow \qquad v = -8 \text{ cm}$$

For second refraction : (at air-water interface)

$$\text{Apparent depth} = \frac{(10+8)}{4/3}$$

$$= \frac{18 \times 3}{4} = \frac{54}{4} = \frac{27}{2} \text{ cm.}$$

E-5.



$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{3}{2 \times (-15)} = \frac{1 - 3/2}{-10}$$

$$\frac{1}{v} + \frac{1}{10} = \frac{-1}{10} \qquad \Rightarrow \qquad v = -20 \text{ cm (virtual)}$$

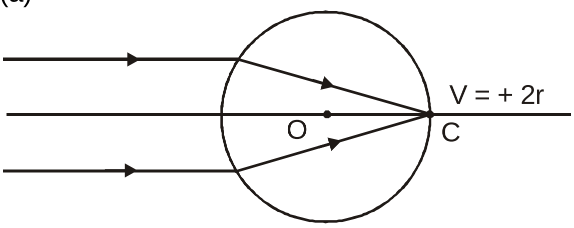
$$-\frac{n_2}{v^2} dv + \frac{n_1}{u^2} du = 0$$

$$\frac{1}{400} dv = \frac{3}{2 \times 225} \times 1\text{mm}$$

$$dv = \frac{400}{2 \times 75} \text{ mm} \qquad = \frac{8}{3} \text{ mm}$$

+ve dv $\Rightarrow$ no inversion

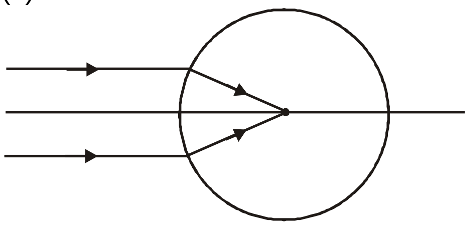
E-6. (a)



$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R} \qquad \frac{\mu}{2r} - \frac{1}{-\infty} = \frac{\mu - 1}{r}$$

$$\frac{\mu}{2r} = \frac{\mu - 1}{r} \quad \Rightarrow \quad \mu = 2\mu - 2 \quad \Rightarrow \quad \mu = 2$$

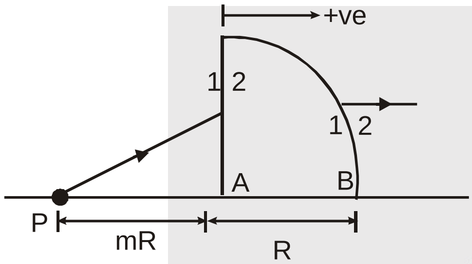
(b)



$$\frac{\mu}{r} - \frac{1}{-\infty} = \frac{\mu - 1}{r} \quad \Rightarrow \quad \frac{\mu}{r} = \frac{\mu - 1}{r} \qquad \mu = \mu - 1$$

$\Rightarrow$ $0 = -1$ not possible

E-7.



Applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

First on plane surface

$$\frac{1.5}{v_1} - \frac{1}{(-mR)} = \frac{1.5 - 1}{\infty} \qquad (R = \infty)$$

$\therefore$ $v_1 = (-1.5\ mR)$

Then on curved surface

$$= \frac{1}{\infty} - \frac{1.5}{-(1.5mR + R)} \qquad [\ v = \infty \text{ because final image is at infinity}]$$

$$\Rightarrow \frac{1.5}{(1.5m + 1)R} = \frac{0.5}{R} \quad \Rightarrow \quad 3 = 1.5\ m + 1$$

$$\Rightarrow \frac{3}{2} m = 2 \quad \text{or} \quad m = 4/3$$ **Ans.**

SECTION (F)

F-1. $\frac{1}{f} = (2 - 1)\left(\frac{1}{\pm 20} - \frac{1}{\pm 30}\right) \Rightarrow f = \pm\ 12\ \text{cm}, \pm\ 60\ \text{cm}.$

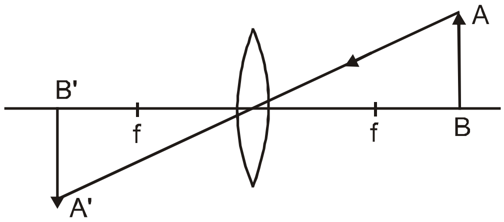
F-2. $\frac{1}{f_1} = \left(\frac{2}{1.5} - 1\right)\left(\frac{1}{-60} - \frac{1}{-40}\right)$

$\Rightarrow$ $f_1 = 360$ cm

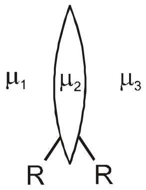
$$\frac{1}{f_2} = \left(\frac{2}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-40}\right) = 0 \Rightarrow \quad = f_2 = \infty$$

$$\frac{1}{f_3} = \left(\frac{2}{2.5} - 1\right)\left(\frac{1}{-60} - \frac{1}{-40}\right) \Rightarrow \quad f_3 = -\ 600\ \text{cm}$$

F-3.



F-4.



(a) For first refraction $\frac{\mu_2}{v_1} - \frac{\mu_1}{-\infty} = \frac{\mu_2 - \mu_1}{R}$ $\Rightarrow$ $v_1 = \frac{\mu_2 \; R}{\mu_2 - \mu_1}$

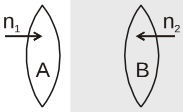
for second refraction, this image is object.

$$\frac{\mu_3}{v} - \frac{\mu_2}{v_1} = \frac{\mu_3 - \mu_2}{-R}$$

$$\Rightarrow \quad \frac{\mu_3}{v} - \frac{\mu_2 - \mu_1}{R} = \frac{\mu_2 - \mu_3}{R} \quad \Rightarrow \frac{\mu_3}{v} = \frac{2\mu_2 - \mu_1 - \mu_3}{R} \quad \Rightarrow \quad v = \frac{\mu_3 \; R}{2\mu_2 - \mu_1 - \mu_3}$$

(b) By interchanging μ_1 and μ_3 , $v = \frac{\mu_1 \; R}{2\mu_2 - \mu_3 - \mu_1}$

F-5. (i) $n_1 = 1.5$, $n_2 = 1.7$

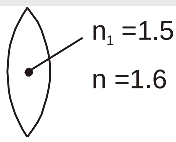


$$\frac{1}{f} = (\mu_{rel} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad \Rightarrow \quad \frac{1}{f} = (\mu_{\text{आपेक्षित}} - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{\frac{1}{f_A} = \left(\frac{n_1}{n} - 1\right)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]}{\frac{1}{f_B} = \left(\frac{n_2}{n} - 1\right)\left[\frac{1}{R_1} + \frac{1}{R_2}\right]} \quad \Rightarrow \quad \frac{f_B}{f_A} = \frac{n_1 - n}{n_2 - n} = \frac{1.5 - 1}{1.7 - 1} = \frac{0.5}{0.7}$$

So $\frac{f_A}{f_B} = \frac{0.7}{0.5} = 7/5$

(ii) For lens A



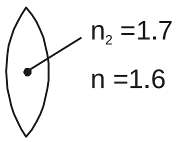
Since $\mu_{rel} < 1$

So $f = -$ ve

It will behave like cocave (diverging lens)

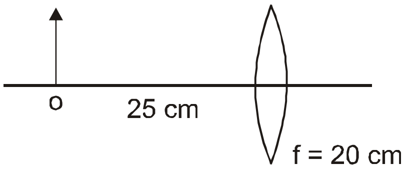
For lens B

$\mu_{rel} = \frac{n_2}{n} > 1$



So f = +ve, behaves like convex (converging lens)

F-6.



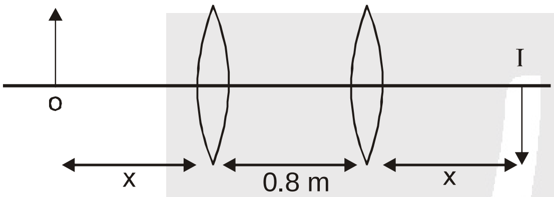
P = 5 D $\Rightarrow$ $f = \frac{1}{5}$ m = 20 cm

By lens formula

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\quad \frac{1}{20} = \frac{1}{v} - \frac{1}{-25}$ $\quad \Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{25}$

$\Rightarrow \quad \frac{1}{v} = \frac{5}{20 \times 25}$ $\quad \Rightarrow \quad$ v = 100 cm = 1 m

$m = \frac{h_2}{h_1} = +\frac{v}{u} = \frac{100}{-25} = -4$ $\quad \Rightarrow \quad$ $h_2 = -4$ cm.

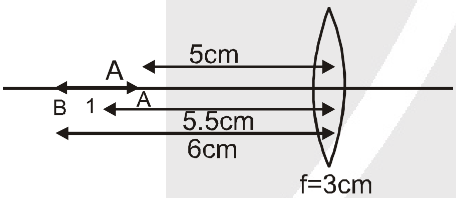


F-7. for first position

$m = \frac{v}{u} = \frac{0.8 + x}{-x} = -3$ $\quad \Rightarrow \quad$ 0.8 + x = 3x $\quad \Rightarrow \quad$ x = 0.4

$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\quad \Rightarrow \quad \frac{1}{f} = \frac{1}{1.2} - \frac{1}{-0.4}$ $\Rightarrow \frac{1}{f} = \frac{1}{1.2} + \frac{1}{0.4} = \frac{1+3}{1.2}$ $\quad \Rightarrow \quad$ 0.3 m

F-8.



For A

$\frac{1}{3} = \frac{1}{v_A} - \frac{1}{-5}$ $\quad \Rightarrow \quad \frac{1}{v_A} = \frac{1}{3} - \frac{1}{5} = \frac{2}{15}$ $\quad \Rightarrow \quad$ $v_A = 7.5$ cm.

For B

$\frac{1}{3} = \frac{1}{v_B} - \frac{1}{-6}$ $\quad \Rightarrow \quad \frac{1}{v_B} = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$ $\quad \Rightarrow \quad$ $v_B = 6$ cm.

size of image = $v_A - v_B$ = 1.5 cm.

F-9. Position of image of sun is at focus.

$|m| = \frac{d_i}{d_o} = \left|\frac{v}{u}\right|$ $\quad d_i = \frac{40 \times 1.5 \times 10^{11}}{1.5 \times 10^{13}}$ = 0.4 cm

F-10. $f = \frac{1}{P} = \frac{1}{2.5}$ m = 40 cm $\quad m = \frac{v}{u} = 4$ $\quad$ v = 4u.

Using lens formula

$\frac{1}{40} = \frac{1}{4u} - \frac{1}{u} = \frac{1-4}{4u}$ $\quad \frac{1}{40} = \frac{-3}{4u}$ $\quad \Rightarrow \quad$ u = – 30 cm

So, required distance = 30 cm.

F-11.

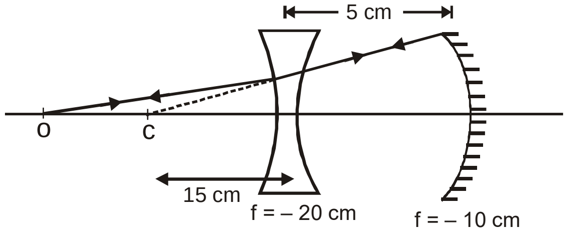
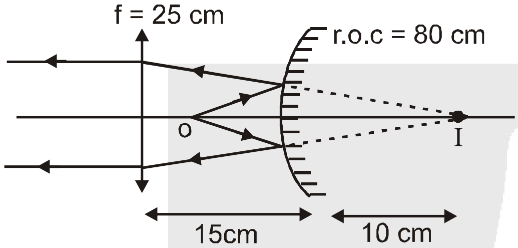


Image is formed at the object itself if the image formed due to lens is at centre of curvature of the mirror.

For refraction by lens,

$$\frac{1}{-20} = \frac{1}{-15} - \frac{1}{u} \quad \Rightarrow \quad u = -60 \text{ cm}$$

F-12.

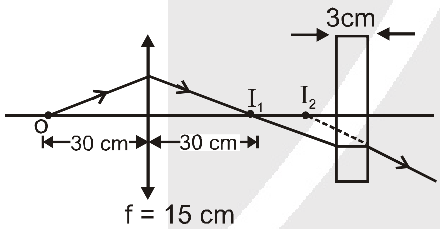


For final light ray to be parallel to the axis of the lens, the image formed by the mirror should be at the focus of the lens.

By mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ v = + 10 cm, f = 40 cm

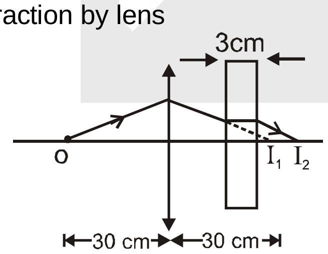
on solving $u = -\frac{40}{3}$ cm so distance of object from lens = $15 - \frac{40}{3} = \frac{5}{3}$ cm

F-13. Case- 1



Case- 2

For refraction by lens

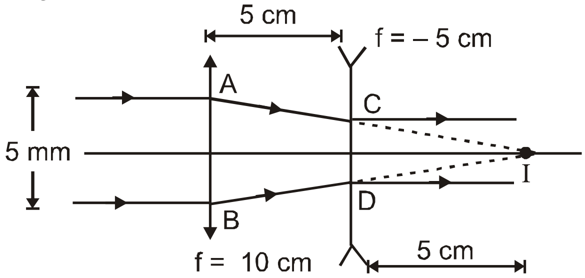


$$\frac{1}{15} = \frac{1}{v} - \frac{1}{-30} \quad \Rightarrow \quad \frac{1}{v} = \frac{1}{15} - \frac{1}{30} \quad \Rightarrow \quad v = 30 \text{ cm}$$

apparent shift by plate = $3\left(1 - \frac{1}{3/2}\right)$ = 1 cm.

Distance of final image from lens = (30 + 1) cm = 31 cm.

F-14. If light is incident from side of convex lens



by convex lens image is formed at its focus i.e. at a distance 10 cm from it, means at the focus of concave lens. therefore, the final beam will be || to incident ray.

$$\frac{AB}{CD} = \frac{10}{5}$$

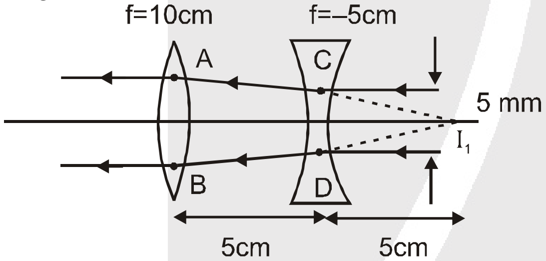
$\Rightarrow$ $CD = \frac{AB}{2} = \frac{5}{2}$ mm. = 2.5 mm

Power, P = constant

$\therefore$ $I_1A_1 = I_2A_2$

$\Rightarrow$ $\frac{I_2}{I_1} = \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2} = \left[\frac{5}{5/2}\right]^2 = 4$

If light is incident from side of concave lens,



first image is formed at the focus of concave lens means at the focus of convex lens. Therefore, final ray will be || to incident one.

$$\frac{AB}{CD} = \frac{10}{5}$$

$\Rightarrow$ AB = 2 × CD = 2 × 5 mm

$$\frac{I_2}{I_1} = \frac{A_1}{A_2} = \frac{D_1^2}{D_2^2} = \left(\frac{5}{10}\right)^2 = \frac{1}{4}.$$

SECTION (G)

G-1. **u** = – 12.5 cm, $m = \frac{v}{u} = -4$ $\Rightarrow$ v = + 50

Also, $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

$\Rightarrow$ $= \frac{1}{f} = \frac{1}{+50} - \frac{1}{-12.5} = \frac{+1+4.}{50} = \frac{1}{10}$

$f = 10 \text{ cm} = \frac{1}{10}$ m

$P = \frac{1}{f} = 10$ D.

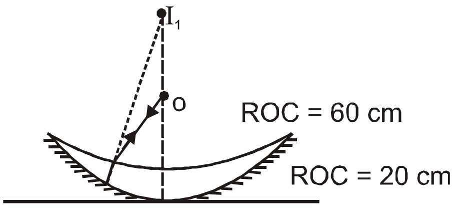
Power of each lens = $\frac{P}{2} = 5$ D.

G-2. $\frac{1}{f_1} = \frac{1}{30} - \frac{1}{-15} = \frac{1+2}{30} = \frac{1}{10}$ $\Rightarrow$ $f_1 = 10$ cm

$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{60} - \frac{1}{-15} = \frac{1+4}{60}$

$\Rightarrow$ $\frac{1}{10} + \frac{1}{f_2} = \frac{1}{12}$ $\frac{1}{f_2} = \frac{1}{12} - \frac{1}{10} = \frac{5-6}{60}$ $f_2 = -60$ cm

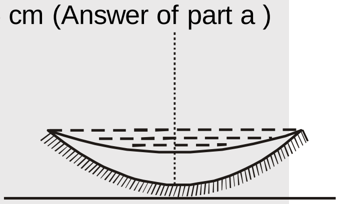
G-3. (a)



First image should form at centre of curvature of mirror after refraction by concave surface.

$\frac{1.5}{-20} - \frac{1}{u} = \frac{1.5-1}{-60}$ $\frac{1}{u} = \frac{1}{120} - \frac{1.5}{20} = \frac{1-9}{120}$ $u = -\frac{120}{8} = -15$ cm.

(b) After refraction by water lens image should form at 15 cm (Answer of part a)



$\frac{1}{f_{water}} = \frac{1}{-15} - \frac{1}{u} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-60}\right)$

$\Rightarrow$ $u = -13.86$ cm

$\Rightarrow$ So, distance through which the pin should be moved = (15 – 13.86) = 1.14 cm towards lens.

SECTION (H)

H-1. (a) $\omega = \frac{\mu_v - \mu_R}{\mu_y - 1} = \frac{1.68 - 1.53}{1.60 - 1} = \frac{0.15}{0.6} = \frac{1}{4}$

(b) $\theta = A(\mu_v - \mu_R) = 6^o (1.68 - 1.53) = 0.90^o$

H-2. Since deviation of mean ray is zero.

$A(\mu_{m1} - 1) = A_2(\mu_{m2} - 1)$

$6^o(1.6 - 1) = A_2(1.9 - 1)$

$A_2 = \frac{6^o \times 0.6}{0.9} = 4^o$

H-3. (a) For no net angular dispersion,

$2A(\mu_v - \mu_r) - A'(\mu_v' - \mu_r') = 0$

$\Rightarrow$ $\frac{A'}{A} = \frac{2(\mu_v - \mu_r)}{(\mu_v' - \mu_r')}$

(b) For no net deviation

$2A_C(\mu_C - 1) - A_f(\mu_f - 1) = 0$

$2A(\mu_y - 1) - A'(\mu_y' - 1) = 0$

$\frac{A'}{A} = \frac{2(\mu_y - 1)}{(\mu_y' - 1)}$

H-4. Dispersive power $= \frac{\delta_V - \delta_R}{\delta_Y} = \frac{(\mu_V - \mu_R)}{(\mu_Y - 1)} = \frac{(\mu_V - 1) - (\mu_R - 1)}{(\mu_y - 1)}$

We know that, $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (\mu - 1)K$

where $K = \frac{1}{R_1} - \frac{1}{R_2}$

So , $(\mu_V - 1)\, K = \frac{1}{98}$ ....(i)

$(\mu_R - 1)\, K = \frac{1}{100}$ ....(ii)

and $(\mu_Y - 1)\, K = \frac{1}{99}$ ....(iii)

$\therefore$ Dispersive power, $\omega = \frac{\frac{1}{98} - \frac{1}{100}}{\frac{1}{99}} = \frac{99}{4900}$

H-5. (a) For no deviation $(\mu_y - 1)\, A = (\mu'_y - 1)\, A'$

$A' = \frac{(\mu_y - 1)}{(\mu'_y - 1)}\, A = \frac{(1.3 - 1)}{(1.5 - 1)} \times 5^\circ = 3^\circ$

(b) Net angular dispersion produced

$= \theta_1 - \theta_2 = 0.08 \times (1.5 - 1) \times 3^o - 0.07\,(1.3 - 1) \times 5^o = 0.120^o - 0.105 = 0.015^o$

(c) Net deviation when prism are similarly directed

$\delta_1 + \delta_2 = (\mu_y - 1)\, A + (\mu'_y - 1)A' = 2 \times (1.5 - 1) \times 3^o = 3^o$

(d) Angular dispersion in above case

$= (\delta_v - \delta_r) = (\mu_y - 1)\, A\, (\omega + \omega') = (1.3 - 1) \times 5^\circ \times (0.07 + 0.08) = 0.225^\circ$

SECTION (I)

I-2. $v_e = \infty$

$u_e = f_e = 5$ cm

$M = -\frac{v_0}{u_0} \times \frac{D}{f_0} \Rightarrow 30 = -\frac{v_0}{u_0}\,\frac{25}{5}$

$v_0 = -6u_0$

$\Rightarrow u_0 = -1.45,\ v_0 = 8.75,\ L = v_0 + f_e = 13.75$

I-3. (a) $v_e = -2.5$ cm and $f_e = 6.25$ cm give $u_e = -5$ cm ; $v_0 = (15 - 5)$ cm = 10 cm.

$f_0 = u_0 = -2.5$ cm; Magnifying power $= \frac{10}{2.5} \times \frac{25}{5} = 20$

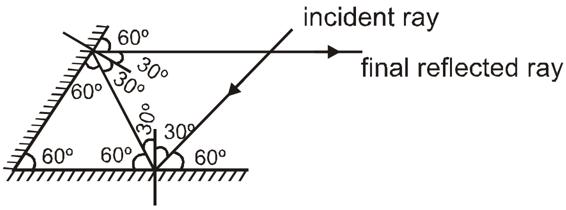
(b) $u_e = -6.25$ cm, $v_0 = (15 - 6.25)$ cm = 8.75, $f_0 = 2.0$ cm. Therefore,
$u_0 = -(70/27) = -2.59$ cm.

Magnifying power $= \frac{v_0}{|u_0|} \times (25/6.25) = \frac{27}{8} \times 4 = 13.5$

PART - II

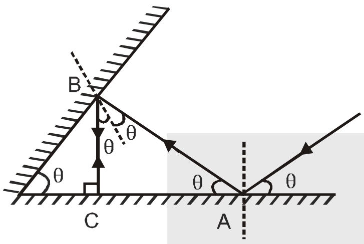
SECTION (A)

A-1.



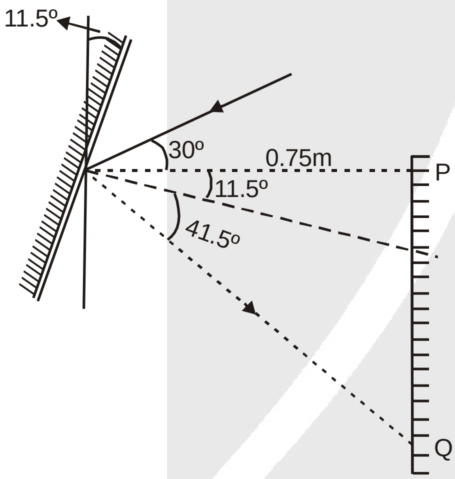
final ray is II to first mirror.

A-2.



In ΔABC में $\quad 90^o + 3\theta = 180^o \quad \Rightarrow \quad \theta = 30^o$

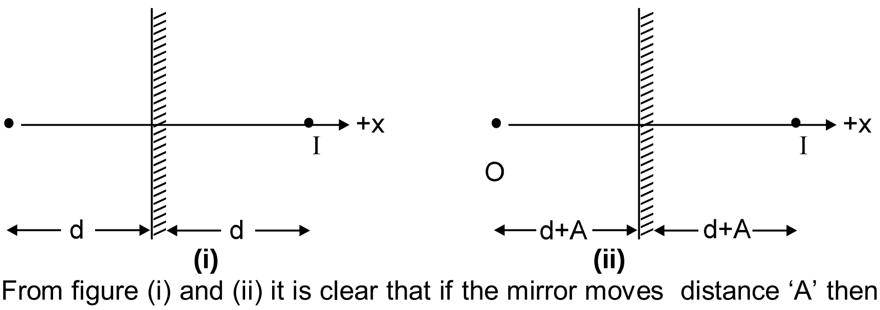
A-3.



$\tan (11.5 + 41.5)^o = \tan 53^o = \dfrac{PQ}{0.75}$

$\Rightarrow \quad PQ = \dfrac{4}{3} \times 0.75 = 1m.$

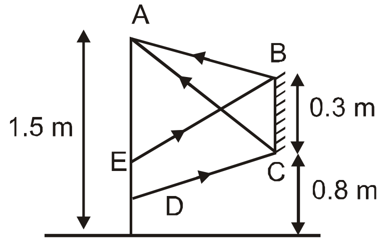
A-4.

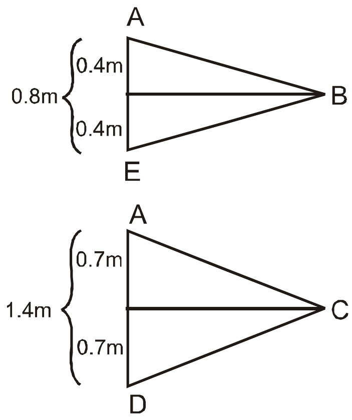


From figure (i) and (ii) it is clear that if the mirror moves distance ‘A’ then the image moves a distance ‘2A’.

Therefore Amplitude of SHM of image = 2A

A-5.





ED = AD – AE = 1.4 – 0.8 = 0.6 m

A-6. If time in the clock is T_1 & time in image clock is T_2 then.

$T_1 + T_2 = 12 : 00 : 00$

$4 : 25 : 37 + T_2 = 12 : 00 : 00$

$T_2 = 07 : 34 : 23$

A-7. $\vec{V}_{I,M} = -\vec{V}_{O,M}$ (normal to plane mirror)

$\Rightarrow \quad \vec{V}_I - \vec{V}_M = -(\vec{V}_0 - \vec{V}_M)$

$\Rightarrow \quad V_I - V\sin\theta = -(0 - V\sin\theta)$

$\Rightarrow \quad V_I = 2V\sin\theta$.

A-8. $\vec{V}_0 = 3\hat{i} + 4\hat{j} + 5\hat{k}$  $\vec{V}_m = 8\hat{i} + 5\hat{j} + 8\hat{k}$

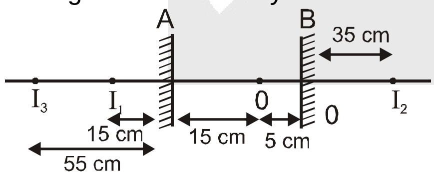
$V_{IZ} = 2V_{mz} - V_{oz} = 2 \times 8 - 5 = 11$

$V_{Ix} = V_{ox} = 3$

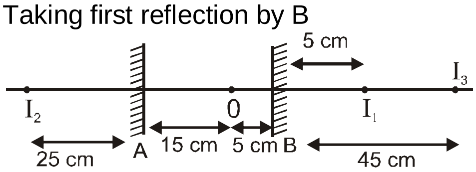
$V_{Iy} = V_{oy} = 4$

$\vec{V}_I = 3\hat{i} + 4\hat{j} + 11\hat{k}$

A-9. Taking first reflection by A.

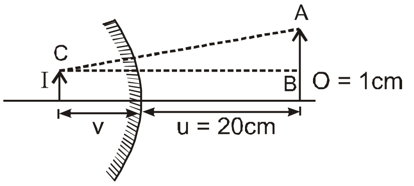


Taking first reflection by B



SECTION (B)

B-1.



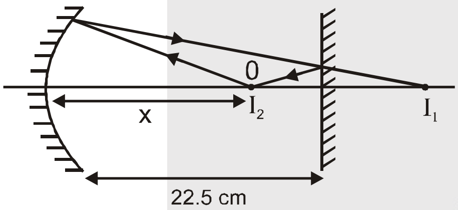
$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{10} - \frac{1}{(-20)} = +\frac{3}{20} \; ; \; v = +\frac{20}{3} \text{ cm}$$

$$I = -\frac{v}{u} \times O = -\frac{\frac{20}{3}}{(-20)} \times 1 = \frac{1}{3}$$

$\therefore$ The distance between tip of the object and image is = AC = $\sqrt{(BC)^2 + (AB)^2}$

$$S = \sqrt{\left(20 + \frac{20}{3}\right)^2 + \left(1 - \frac{1}{3}\right)^2} = \sqrt{\frac{6404}{9}} \text{ cm}$$

B-2.



I_1 is the image formed by concave mirror.
For reflection by concave mirror
u = – x, v = – (45 – x), f = – 10 cm,

$$\frac{1}{-10} = \frac{1}{-(45-x)} + \frac{1}{-x}$$

$$\frac{1}{10} = \frac{x+45-x}{x(45-x)} \Rightarrow x^2 - 45x + 450 = 0 \Rightarrow x = 15 \text{ cm}, 30 \text{ cm}$$

but x = 30 cm is not acceptable because x < 22.5 cm.

B -3. $v = \frac{uf}{u-f} = \frac{(-15)\times(-10)}{-15+10} = -30$ cm, $m = -\frac{v}{u} = -2$ $\therefore$ A′B′ = C′D′ = 2 × 1 = 2 mm

Now $\frac{B'C'}{BC} = \frac{A'D'}{AD} = \frac{v^2}{u^2} = 4 \Rightarrow$ B′C′ = A′D′ = 4 mm

$\therefore$ Perimeter length = 2 + 2 + 4 + 4 = **12 mm Ans.**

B-4. For M_1 : $v_1 = \frac{uf}{u-f} = \frac{(-30)\times(-20)}{(-30)-(-20)} = -60$

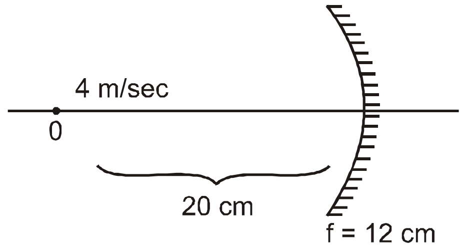
$\therefore$ $M = -\frac{v_1}{u} = -2.$

For M_2 : u = + 20. f = 10

$\therefore$ $\frac{1}{v} + \frac{1}{20} = \frac{1}{10} \Rightarrow$ v = 2 0

$m_2 = -\frac{20}{20} = -1$ $m = m_1 \times m_2 = +2$

B-5.



$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$ $\qquad \frac{1}{-12} = \frac{1}{v} + \frac{1}{-20}$ $\quad \Rightarrow \quad \frac{1}{v} = \frac{1}{20} - \frac{1}{12}$

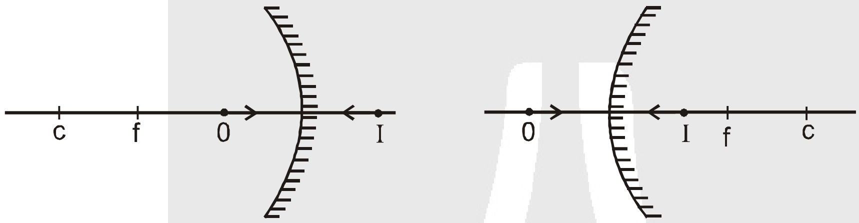
$\Rightarrow \qquad v = -30$ cm

velocity of image

$\frac{dV}{dt} = -\left(\frac{V^2}{u^2}\right)\frac{du}{dt} = -\left(\frac{-30}{-20}\right)^2 4. = -9$ cm/sec.

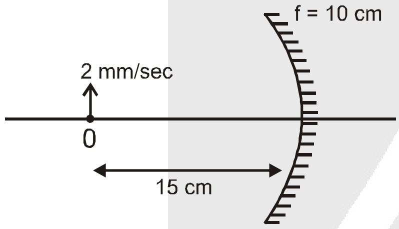
$\Rightarrow \qquad$ 9 cm/sec away from mirror.

B-6.



only in above two cases image moves towards mirror.

B-7.



$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-10} = \frac{1}{v} + \frac{1}{-15} \quad \Rightarrow \quad \frac{1}{v} = \frac{2-3}{30}$ $\qquad v = -30$ cm.

$m = \frac{I}{O} = -\frac{v}{u} = -\frac{-30}{-15} = -2$

$\frac{dI}{dt} = 2 \frac{d\theta}{dt} = 4$ mm/sec.

B-8. The image 'I' of object 'O' formed by plane mirror moves towards left. I acts a real object for concave mirror. As I moves towards left, its image formed by concave mirror (whether real or virtual) moves towards right.

B-9. Using mirror formula,

$\frac{1}{-10} = \frac{1}{v} + \frac{1}{-15} \quad \Rightarrow \quad v = -30$ cm.

$|\text{ Axial magnification }| = \frac{V^2}{u^2} = \left(\frac{30}{15}\right)^2 = 4$

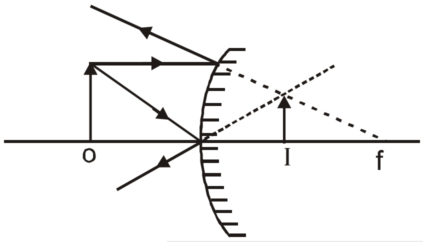
amplitude of image = 4 × 2 = 8 mm.

B-10. Using Newtons formula. x $\rightarrow$ distance of object from focus
$xy = f^2$ y $\rightarrow$ distance of object from focus
f $\rightarrow$ focal length.

$\Rightarrow by = (a/2)^2$, $y = \frac{a^2}{4b}$.

B-11. It is created at focus ie + 20 cm, when object is at infinity

B-12.



B-13. $\frac{I}{O} = -\frac{v}{u}$

If O and I are on same sides of PA . $\frac{I}{O}$ will be positive which implies v and u will be of opposite signs.

Similarly if O and I are on opp. sides, $\frac{I}{O}$ will be -ve which implies v and u will have same sign.

If O is on PA, I = $\left(-\frac{V}{u}\right)$ (O) = 0 $\Rightarrow$ I will also be on. P.A.

B-14. $\frac{1}{-f} = \frac{1}{-v} + \frac{1}{-u}$ $\Rightarrow$ $\frac{1}{v} = \frac{-1}{u} + \frac{1}{f}$

Slope = – 1 intercept = $\frac{1}{f}$ (positive)

B-15. For real inverted image formed by concave mirror.
v = – ve, u = – ve f = – ve

$\Rightarrow$ $\frac{u}{f}$ & $\frac{v}{f}$ are positive

$\Rightarrow$ A is right answer .

Alternative

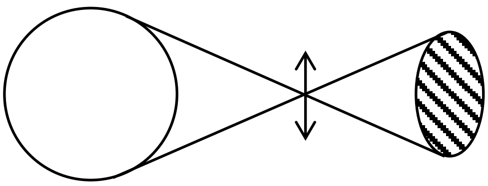
$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\Rightarrow$ $\frac{1}{v/f} + \frac{1}{u/f} = 1$ $\Rightarrow$ $\frac{1}{y} + \frac{1}{x} = 1$

$\Rightarrow$ xy = x + y

$\Rightarrow$ xy – x – y + 1 = 1 $\Rightarrow$ (x – 1) (y – 1) = 1 Hence (A)

B-16.



$D_I = \frac{5}{10} \times 0.5 \times \frac{\pi}{180} = \frac{\pi}{180 \times 4} = 4.36mm \approx 4.4mm$

SECTION (C)

C-1. $\mu = \frac{\lambda_V}{\lambda_m} = \frac{6000}{4000} = 1.5$

C-2. $i = 2r$

$1 \sin i = n \sin r$

$\Rightarrow \quad 2 \sin i/2 \cos i/2 = n \sin i/2$

$\Rightarrow \quad \cos i/2 = (n/2)$

$\Rightarrow \quad i = 2 \cos^{-1} (n/2)$

C-3. Displacement = $\frac{t \sin(i-r)}{\cos r}$ and $1 \sin i = n \times \sin r$

Since i and r are small angles. and $i = nr$

Displacement = $t (i - r)$

$\therefore \quad$ Displacement = $t\, i \left(1 - \frac{r}{i}\right) = t\, \theta \left(1 - \frac{1}{n}\right) = \frac{t\theta \ (n-1)}{n}$

C-4. As n varies 'y', parallel slabs can be taken, and we know in parallel slabs

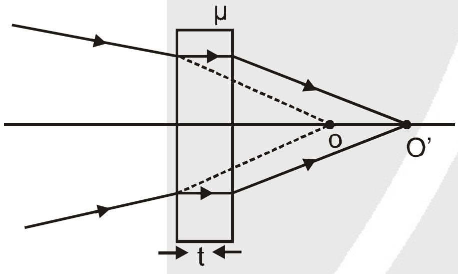
$n_r \sin i_r$ = constant. as $n_1 \sin i_1 = 1 \times \sin 90° = 1$ = constant

$n_{final} = n_{air} = 1$

$\Rightarrow \quad 1 = 1 \times \sin r_{final} \Rightarrow r_{final} = 90°$

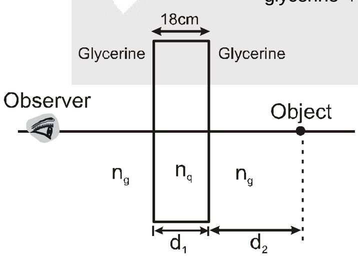
$\therefore \quad$ Deviation is zero.

C-5.



$oo' = t \left(1 - \frac{1}{\mu}\right)$

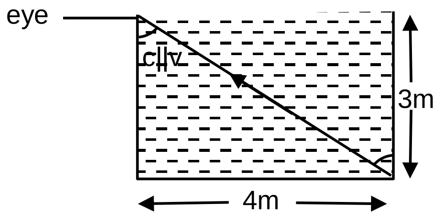
C-6. $n_{quartz} = 2$; $n_{glycerine} = \frac{4}{3} \Rightarrow \frac{n_{quartz\ \text{क्वार्टज}}}{n_{glycerine\ \text{ग्लिसरीन}}} = \frac{2}{4/3} = \frac{3}{2} = \mu_{rel}$



shift $= t \left(1 - \frac{1}{\mu_{rel}}\right) = 18 \left(1 - \frac{1}{3/2}\right) = 6$ cm

C-7. $\sin \theta = \frac{1}{\mu} = \frac{C_A}{C_B} \quad \Rightarrow \quad C_B = \frac{V}{\sin\theta}$

C-8.



$\tan\theta = \frac{4}{3} \Rightarrow \theta = 53° = \text{critical angle}$

$\sin c = \frac{n_r}{n_d}$

$\frac{4}{5} = \frac{1}{\mu} \Rightarrow \qquad \mu = \frac{5}{4} = 1.25$

C-9. $\Delta y = t\left(1 - \frac{1}{\mu}\right) = t\left(1 - \frac{2}{3}\right) = \frac{t}{3}$ closer.

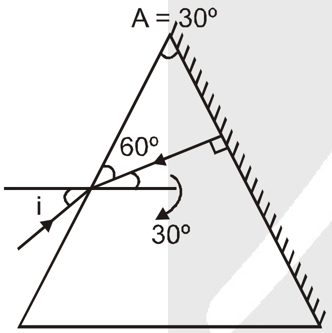
SECTION (D)

D-1 . $r_2 = \sin^{-1}\left(\frac{1}{\mu}\right) = 45^o$

$r_1 = A - r_2 = 75^o - 45^o = 30^o$

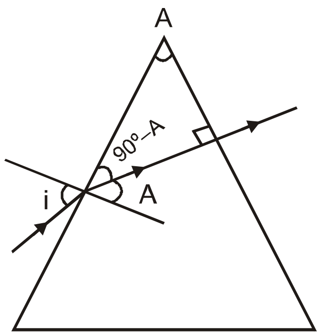
$\frac{\sin\ i}{\sin\ r_1} = \sqrt{2} \quad \Rightarrow \quad \sin i = \sqrt{2} \sin 30^o = \sqrt{2} \times \frac{1}{2} \quad \Rightarrow \quad i = 45^o.$

D-2.



$\frac{\sin i}{\sin 30^o} = \sqrt{2} \quad \Rightarrow \quad \sin i = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad i = 45^o.$

D-3.



$\frac{\sin i}{\sin\ A} = \mu$ since i and A are small angle. $\frac{i}{A} = \mu$

D-4. **(a)** and **(b)**

$$\mu = \frac{\sin\left(\frac{A+\delta_m}{2}\right)}{\sin\frac{A}{2}} \Rightarrow \sqrt{2} = \frac{\sin\left(\frac{60^o+\delta_m}{2}\right)}{\sin 30^o} \Rightarrow \frac{60^o+\delta_m}{2} = 45^o$$

$\therefore$ $\delta_{min} = 30^o$ Also $i + e = A + \delta$.

for $\delta = \delta_{min}$ $2i = 60^o + 30^o$ $\Rightarrow$ $i = 45^o$

(c) for $\delta = \delta_{max}$

$e = 90^o$ $\Rightarrow$ $r_2 = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\Rightarrow$ $r_2 = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^o$ $\Rightarrow$ $r_1 = A - r_2 = 15^o$

$$\frac{\sin i}{\sin 15^o} = \mu = \sqrt{2}$$

$\sin i = \sqrt{2}\ \sin 15^o$

$i = \sin^{-1}(\sqrt{2}\ \sin 15^o)$

$\delta_{max} = i + e - A = 30^o + \sin^{-1}(\sqrt{2}\ \sin 15^o)$

D-5. For transmission

$r_2 \le \sin^{-1}(1/\mu)$ & $r_1 \le \sin^{-1}(1/\mu)$

$r_1 + r_2 \le 2\sin^{-1}(1/\mu)$ $A \le 2\sin^{-1}(1/\mu)$

$\sin^{-1}(1/\mu) \ge 45^o$ $\Rightarrow$ $\frac{1}{\mu} \ge \frac{1}{\sqrt{2}}$ $\Rightarrow$ $\mu \le \sqrt{2}$.

D-6. Deviation by prism.

$\delta_1 = A(\mu - 1) = 4^o(1.5 - 1)$ $\Rightarrow$ $\delta_1 = 2^o$

for plane mirror

$i = 2^o$

$\delta_2 = 180^o - 2i = 176^o$ $\Rightarrow$ $\delta = \delta_1 + \delta_2 = 178^o$

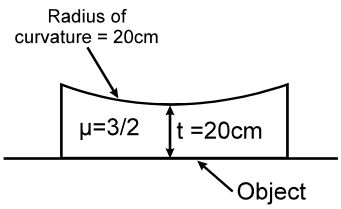
SECTION (E)

E-1. $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ $\frac{\mu_2}{v} - \frac{\mu_1}{-R} = \frac{\mu_2 - \mu_1}{-R}$

$v = -R$ for all values of μ.

E-2. $\frac{1}{v} - \frac{3}{2\times 30} = \frac{1-\frac{3}{2}}{+20}$ $\frac{1}{v} = -\frac{1}{40} + \frac{1}{20} = +\frac{1}{40}$ $v = 40$ cm.

E-3.



Considering refraction at the curved surface,

$u = -20$; $\mu_2 = 1$

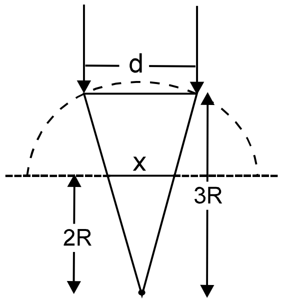
$\mu_1 = 3/2$; $R = +20$

applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ $\Rightarrow$ $\frac{1}{v} - \frac{3/2}{-20} = \frac{1-3/2}{20}$ $\Rightarrow$ $v = -10$

i.e. 10 cm below the curved surface or 10 cm above the actual position of flower.

E-4. Using refraction formula at curved surface,

$$\frac{3}{2v} - \frac{1}{\infty} = \frac{\frac{3}{2}-1}{R} \; ; \qquad \frac{3}{2V} = \frac{1}{2R} \; ; \; V = 3R \; ;$$

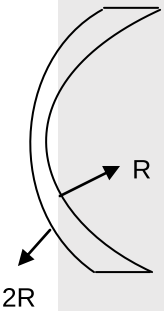


From figure $\frac{x}{2R} = \frac{d}{3R}$; $\qquad x = \frac{2}{3}$ d.

SECTION (F)

F-1. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow \frac{1}{-24} = (1.5-1)\left(\frac{1}{2R} - \frac{1}{R}\right) \Rightarrow \frac{1}{-24} = \frac{1}{2}\left(-\frac{1}{2R}\right)$

R = 6 cm $\qquad \Rightarrow \qquad$ 2R = 12 cm



F-2. $\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{f} = (1.63-1)\left(\frac{2}{R_A}\right) = (n_B - 1)\left(\frac{2}{R_B}\right)$

n_B –1 = 0.63 × $\frac{R_B}{R_1} = \frac{0.63}{0.9}$ = 0.7 n_B = 1.7

F-3. P = $(\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ ....(i)

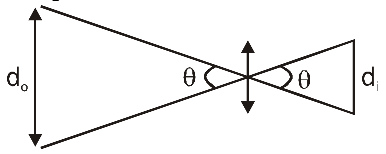
$P_0 = \left(\frac{\mu}{\mu_o} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$ .....(ii)

$\frac{P}{P_0} = \frac{(\mu-1)\ \mu_0}{(\mu - \mu_0)} \qquad P_0 = \frac{P\ (\mu - \mu_0)}{\mu_0(\mu-1)}$

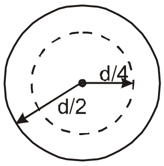
F-4. Lens changes its behaviour if R.I. of surrounding becomes greater than R.I. of lens.

$\mu_{lens} < 1.33$

F-5. Image of sun is formed in the focal plane. So,



Diameter of image = fθ = $\frac{100 \times 0.5^o}{180^o} \times \pi \times 10$ mm = 9.

F-6. 

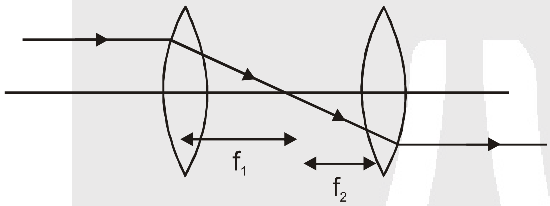
Initial area $= \frac{\pi d^2}{4}$

after blockening, area that allows light $= \frac{\pi d^2}{4} - \frac{\pi d^2}{16} = \frac{3}{4} . \frac{\pi d^2}{4}$

It is $\frac{3}{4}$th of the total area of the lens that would allow the light, hence

Intensity is now $\frac{3I}{4}$. There will be no change in focal length

F-7. $f_A = f_B = f_C = f_{net} \quad \Rightarrow \quad P_A = P_B = P_C = P_{net} = P$

F-8. 

Distance between lens is

F-9. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \quad \Rightarrow \quad \frac{1}{+f} = \frac{1}{+v} - \frac{1}{-u} \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \quad \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$

slope ढाल = – 1 intercept = $\frac{1}{f}$

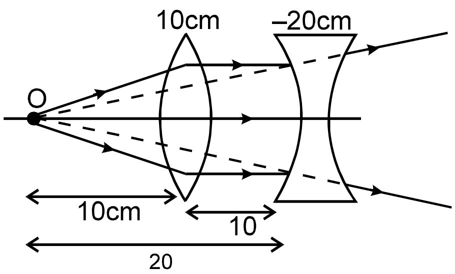
F-10. For vertical erect image by diverging lens.
u, v and f are negative

$\therefore \quad \frac{u}{f} = +ve$ and $\frac{v}{f} = +ve$

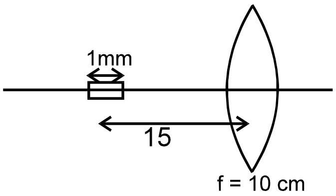
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \qquad 1 = \frac{f}{v} - \frac{f}{u} \qquad \frac{1}{y} = \frac{1}{x} + 1$

$y = \frac{x}{x+1}$ since x & y are +ve graph lies in first quadrant.

Also, at x = 0, y = 0 and at x = ∞ ,, y = 1 Hence, (D)

F-11. 

F-12.



$$\frac{1}{10} = \frac{1}{v} - \frac{1}{(-15)} \Rightarrow v = +30 \text{ cm}$$

for small object $|dv| = \frac{v^2}{u^2}\ |du| = \left(\frac{30}{15}\right)^2 \times 1 = 4$ mm

F-13. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ...(1)

$m = \frac{v}{u}$ ...(2)

from (1) and (2) $m = \frac{f}{f+u}$

here $m = -\frac{18}{2} = -9$ {only real images can be formed on the screen, which is inverted}

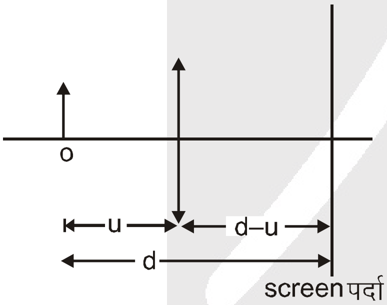
$\therefore \quad -9 = \frac{f}{f+(-10)}$

$\therefore \quad -9f + 90 = f$

$10f = 90$

$f = 9$ cm

F-14.

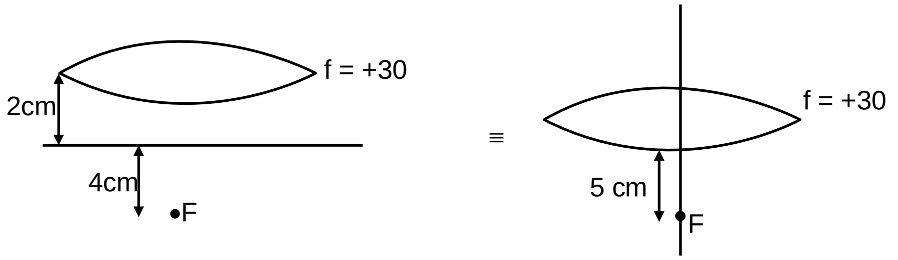


$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{(d-u)} - \frac{1}{(-u)} = \frac{1}{f}$$

$\Rightarrow u^2 - du + df = 0 \quad \Rightarrow d = \frac{u^2}{(f-u)}$

for minimum d, $u = -2f$ Hence, $d_{min.} = 4f$

F-15.



water $m = \frac{4}{3}$

$\frac{1}{v} - \frac{1}{-5} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{6}{30}$

$\frac{1}{v} = -\frac{5}{30}$

$v = -6cm$

SECTION (G)

G-1. Focal length of plano–convex lens is 10cm.

$$\frac{1}{10} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} - \frac{1}{-R}\right)$$

$$\frac{1}{10} = \frac{1}{2} \times \frac{1}{R} \quad \Rightarrow \quad R = 5 \text{ cm}$$

Let focal length of water lens is f_w

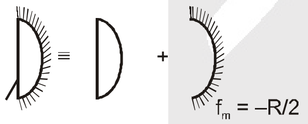
$$\frac{1}{f_w} = \left(\frac{4}{3} - 1\right)\left(\frac{1}{-5} - \frac{1}{5}\right)$$

$$\frac{1}{f_w} = \frac{1}{3} \times \left(-\frac{2}{5}\right) = \frac{-2}{15}$$

Optical power of system

$$P = \frac{1}{f} + \frac{1}{f} + \frac{1}{f_w} = \frac{1}{0.1} + \frac{1}{0.1} + \left(-\frac{40}{3}\right) = 6.67 \text{ D.}$$

G-2.



$\frac{1}{-10} = \frac{2}{-R} - \frac{2}{f_\ell}$

$\frac{2}{R} = \frac{1}{10} - \frac{2}{56} = \frac{56-20}{560} = \frac{36}{560}$

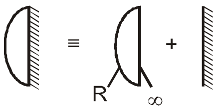
$\frac{1}{R} = \frac{18}{560}$

$(\mu - 1)\ \frac{18}{560} = \frac{1}{56}$

$\mu - 1 = \frac{10}{18}$

$\mu = 1 + \frac{10}{18} = \frac{28}{18} = \frac{14}{9}$.

G-3.

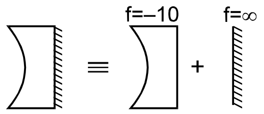


$$\frac{1}{f} = \frac{1}{\infty} - \frac{2}{f_\ell} = -\frac{2}{f_\ell} = \frac{1}{-28}$$

$f_\ell = 56$ cm $\Rightarrow$ $(\mu - 1)\left(\frac{1}{R}\right) = \frac{1}{56}$ ....(i)

$$\left(\frac{14}{9} - 1\right)\frac{1}{R} = \frac{1}{56} = \frac{280}{9} \text{ cm}$$

G-4.



$$\frac{1}{F} = \frac{1}{f_m} - \frac{2}{f_L} = 0 - \frac{2}{-10} \Rightarrow F = 5$$

G-5. $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} = 0$

$= \frac{1}{25} + \frac{1}{-20} - \frac{d}{-500} = 0$

$= \frac{20-25}{500} = -\frac{d}{500}$

d = 5 cm.

SECTION (H)

H-1. Obvious from theory

H-2. $C = \sin^{-1}\left(\frac{1}{\mu}\right)$ μ is greatest for voilet $\Rightarrow$ C is minimum for voilet.

H-3. Apparent shift $t\left(1 - \frac{1}{\mu}\right)$ μ is least for red $\Rightarrow$ shift in least for red.

H-4. $\omega = \dfrac{n_v - n_r}{\left(\dfrac{n_v + n_r}{2}\right) - 1} = \dfrac{6}{25}$.

H-5. ω is property of material.

H-6. Dispersion will not occur for a monochromatic light.

SECTION (I)

I-1. $MP = \left(1 + \frac{D}{f}\right) = \left(1 + \frac{25}{5}\right) = 6$

I-3. In normal adjustment

$m = -\frac{f_0}{f_e}$

so $50 = -\frac{100}{f_e}$ $\Rightarrow$ $f_e = -2$ cm

($\because$ eyepiece is concave lens)

and $L = f_0 + f_e = 100 - 2 = 98$ cm

I-5. $m = 1 + \frac{D}{f}$

I-7. For normal adjustment

$m = -\frac{f_0}{f_e}$

When final image is at least distance of distinct vision from eyepiece,

$m' = -\frac{f_0}{f_e}\left(1+\frac{f_e}{d}\right) = 10\left(1+\frac{5}{25}\right) = 12$

I-8. $f = \frac{1}{p} = \frac{1}{2}$ metre

f = 0.5 m this is positive so lense is convex lense.

I-9. By using $m_\infty = \frac{(L_\infty - f_0 - f_e).D}{f_0 f_e}$

$\Rightarrow 45 = \frac{(L_\infty - 1 - 5)\times 25}{1 \times 5} \Rightarrow L_\infty = 15$ cm

I-10. $m_\infty = \frac{v_0}{u_0} \times \frac{D}{f_e}$

From $\frac{1}{f_0} = \frac{1}{v_0} - \frac{1}{u_0} \quad \Rightarrow \quad \frac{1}{(+1.2)} = \frac{1}{v_0} - \frac{1}{(-1.25)} \Rightarrow v_0 = 30$ cm

$\therefore \quad |m_\infty| = \frac{30}{1.25} \times \frac{25}{3} = 200$

PART - III

1. (A) For converging lens (convex lens)

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

u = – x, v = y, f = d (+ ve constant)

$\frac{1}{v} + \frac{1}{x} = \frac{1}{d}$

$\frac{1}{y} = \frac{1}{d} - \frac{1}{x}$

at x = 0 y = 0

For x = 0 to x = d, y = – ve

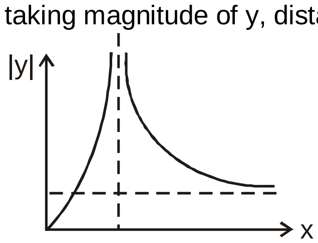
so, if x ↑ y ↓ and |y| ↑

At x = d, y = ∞

when x > d , y + ve, and

at x = ∞ , y = d

taking magnitude of y, distance graph is shown.



(B) For converging mirror (concave mirror)

$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$u = -x$, $f = -\dfrac{R}{2}$, $v = y$

$$\frac{1}{y} - \frac{1}{x} = -\frac{2}{R}$$

$$\frac{1}{y} = \frac{1}{x} - \frac{2}{R}$$

At $x = 0$, $y = 0$

for $0 < x < \dfrac{R}{2}$, $y = +ve$

and as x increases $\dfrac{1}{y}$ decrease so $y \uparrow$ upto $x = \dfrac{R}{2}$

At $x = \dfrac{R}{2}$, $y = \infty$

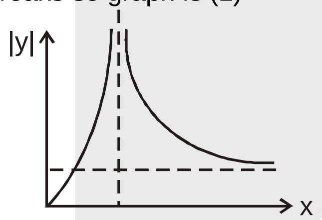
So, graph is (1)

when $x > \dfrac{R}{2}$ y (–ve)

and as $x \uparrow$, $1/y \downarrow$, $y \uparrow$ so, $|y| \downarrow$

At $x = \infty$, $y = -\dfrac{R}{2}$

graph breaks so graph is (1)



(C) For diverging Lens (concave lens)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$u = -x$, $f = -d$ $v = y$

$$\frac{1}{y} + \frac{1}{x} = -\frac{1}{d}$$

$$\frac{1}{y} = -\frac{1}{x} - \frac{1}{d}$$

$\Rightarrow$ y is always –ve

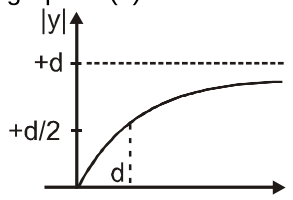
At $x = 0$, $y = 0$

As $x \uparrow$, $y \downarrow$ so, $|y| \uparrow$

At $x = d$, $y = \dfrac{-d}{2}$

or $x = \infty$, $y = -d$

graph is (2)



(D) For diverging Mirror (convex mirror)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$u = -x, \qquad f = +\frac{R}{2}, \qquad v = y$

$\frac{1}{y} - \frac{1}{x} = \frac{2}{R} \quad \Rightarrow \quad \frac{1}{y} = \frac{1}{x} + \frac{2}{R} \quad \Rightarrow \quad y = +ve$

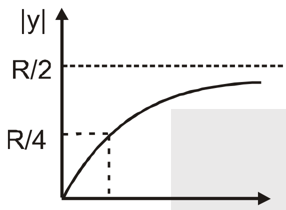
At $x = 0$, $\qquad y = 0$

$\frac{dy}{dx} = \frac{y^2}{x^2}$

$x \uparrow, \; y \uparrow$

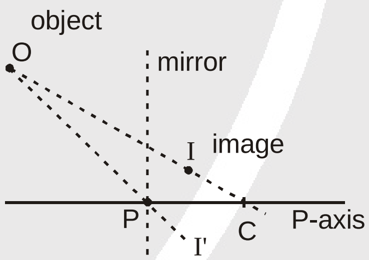
At $x = \frac{R}{2}$, $\qquad y = \frac{R}{4}$, $\qquad$ At $x = \infty$, $\qquad y = \frac{R}{2}$

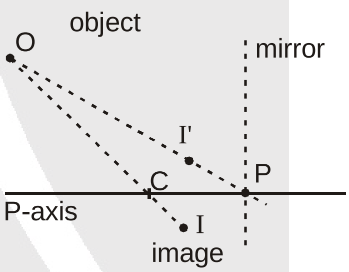
taking magnitude of y distance graph is



graph is (2)

2. For spherical mirror, line joining object and its image crosses principal axis at centre of curvature. The line joining object and image inverted about principal axis cuts the principal axis at the pole. The from figure below.





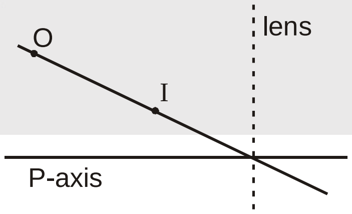
We can conclude

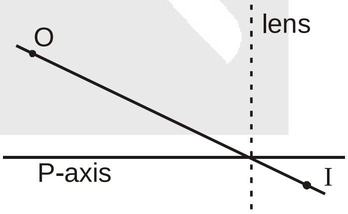
(A) If object and image are on same side of principal axis, they are on opposite side of mirror.

(B) If object and image are on opposite side of principal axis, they are on same side of mirror.

For a lens, the line joining object and image cuts the principal axis at optical centre.

Then from figures below.





We can conclude

(C) If object and image are on same side of principal axis, they are also on same side of lens.

(D) If object and image are on opposite side of principal axis. They are also on opposite side of lens.

3. $V_{Im} = -m^2 V_{om}$ (for all types of mirrors)

for A; $\qquad m = 1$

for B; $\qquad |m| > 1$

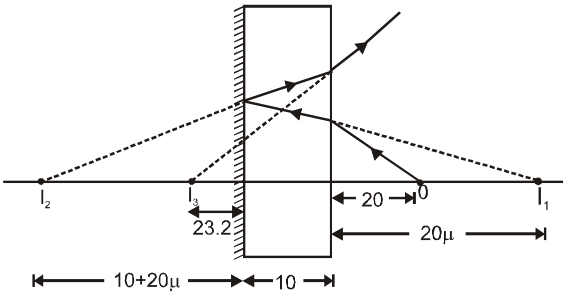
for C & D; $\qquad |m| < 1$

In case of A & C image is virtual (behind the mirror) and in case of B & D image is real (in front of mirror). Since image and object moves opposite to each other with respect to mirror so when a real object moves closer, virtual image also moves closer while real image moves away from mirror.

EXERCISE–2

PART - I

1.



Distance of I_1 from refracting surface = 20 μ
Distance of I_2 from reflecting surface = $20\mu + 10$
Distance of I_1 from reflecting surface = $10 + 20\mu$
Distance of I_2 from refracting surface = $20 + 20\mu$
Distance of I_3 from refracting surface

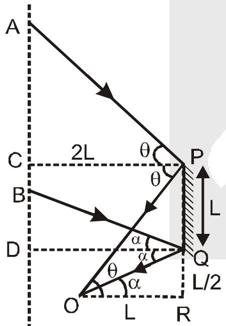
$$= \frac{20 + 20\mu}{\mu} = 10 + 23.2 = \frac{20}{\mu} + 20 = 13.2$$

$$\mu = \frac{20}{13.2} = \frac{200}{132} \text{ cm.}$$

2. Deviation by prism = $A(\mu - 1) = 4^o (1.5 - 1) = 2^o$
For 90^o total deviation, deviation by mirror
$= 90^o - 2^o = 88^o$
$180^o - 2i = 88^o$
$2i = 92^o$
$i = 46^o$
Mirror should be rotated 1^o anticlockwise.

3. Answer is A because A net angle of dispersion by each surface slope is equal to zero.

4.



$$\tan\theta = \frac{PR}{OR} = \frac{AC}{PC}$$

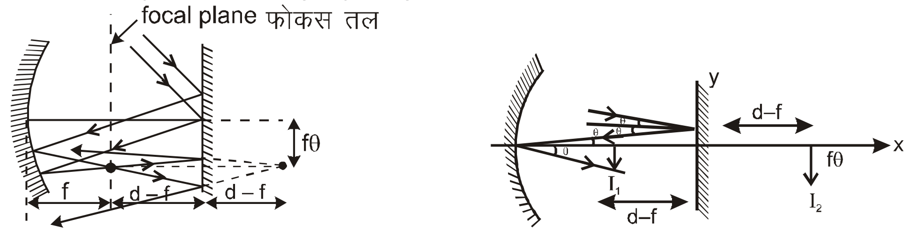
$$\Rightarrow \quad \frac{3L/2}{L} = \frac{AC}{2L} \quad \Rightarrow \quad AC = 3L$$

$$\tan\alpha = \frac{QR}{OR} = \frac{BD}{DQ} \quad \Rightarrow \quad BD = \frac{L/2}{L} \times 2L = L$$

$\therefore$ AB = AD – BD = (3L + L) – L = 3L.
From A to B observer can observe M_1
Making similar diagram for lower mirror
Total visible distance = 3L + 3L = 6L

Total time = $\frac{6L}{u}$.

5. Can be understood by the following ray diagrams :



Since, rays are almost perpendicular to y-axis
Image will form at focus of size = fθ .

6. For m = 2

$m = -\frac{v}{u} = 2$

$V = -2u$ ....(i)

$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{-2u} + \frac{1}{u}$

$\Rightarrow \quad \frac{1}{f} = \frac{1}{2u} \quad \Rightarrow \quad u = \frac{f}{2}$ & $v = -f$

Distance between object & image = f + f/2 = 3f/2

For m = –2

$m = -\frac{v}{u} = -2$

$v = 2u$

$\Rightarrow \quad \frac{1}{f} = \frac{1}{2u} + \frac{1}{u} \quad \Rightarrow \quad u = \frac{3f}{2}$ & $v = 3f$

Distance between object & image = $3f - \frac{3f}{2}$.

7. In the question, the image is inverted and magnified. so, it is formed due to concave mirror with image and object on the same side of mirror and the object closer to the mirror Hence, (B)

8. For M_1

$v = \frac{uf}{u-f} = \frac{-15 \times (-10)}{-15-(-10)} = -30$ cm

For M_2 u = 10 cm

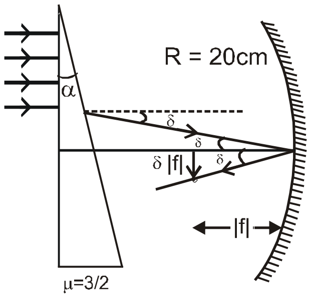
$\therefore \quad v = \frac{10 \times (-10)}{10-(-10)} = -5$ cm

magnification $m = \frac{-v}{u} = -\left(\frac{-5}{10}\right) = \frac{1}{2}$

so, distance of image from CD = $\frac{1}{2} \times 3 = \frac{3}{2}$ cm

$\therefore$ distance of image from AB = $3 - \frac{3}{2} = \frac{3}{2}$ cm

9. Deviation by prism $= 1.8^o \left(\frac{3}{2}-1\right) = 0.9^o$



R = 20 cm |f| = 10 cm

Image will form on focal plane

Distance of image from y–axis = |f| δ

$= \quad 100 \times \frac{0.9\pi}{180}$ mm $= 1.57$ mm

10. $i = \frac{\pi}{2},\ e = \frac{\pi}{4},\ A = \frac{\pi}{4}$

$\frac{\sin i}{\sin r_1} = \frac{\sin e}{\sin r_2} = \mu$

$\Rightarrow \quad \sin r_1 = \frac{1}{\mu} \quad$ and $\quad \sin r_2 = \frac{1}{\sqrt{2}\,\mu}$

Since $r_1 + r_2 = A = \frac{\pi}{4} \quad \Rightarrow \quad r_1 = \frac{\pi}{4} - r_2$

$\Rightarrow \quad \sin r_1 = \frac{1}{\sqrt{2}}\cos r_2 - \frac{1}{\sqrt{2}}\sin r_2$

$= \quad \sqrt{2}\sin r_1 + \sin r_2 = \cos r_2$

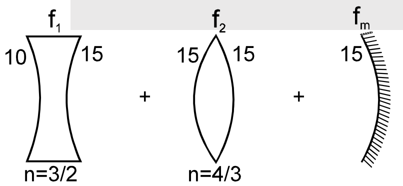
$= \quad \frac{\sqrt{2}}{\mu} + \frac{1}{\sqrt{2}\mu} = \sqrt{1 - \frac{1}{2\mu^2}}$

$= \quad \frac{1}{\mu^2}\left(2 + \frac{1}{2} + 2\right) = 1 - \frac{1}{2\mu^2}$

$= \quad \frac{1}{\mu^2}\left(\frac{9}{2} + \frac{1}{2}\right) = 1$

$\mu^2 = 5 \quad \Rightarrow \quad \mu = \sqrt{5}$.

11.



$\frac{1}{f_1} = \left(\frac{3}{2}-1\right)\left(\frac{-1}{10}-\frac{1}{15}\right) = -\frac{1}{12}$; $\frac{1}{f_2} = \left(\frac{4}{3}-1\right)\left(\frac{2}{15}\right) = \left(\frac{2}{45}\right)$; $\frac{1}{f_m} = -\frac{2}{15}$

$\Rightarrow \quad \frac{1}{f_\ell} = \frac{1}{f_1} + \frac{1}{f_2} \qquad \frac{1}{f_{eq}} = \frac{1}{f_m} - \frac{2}{f_\ell} = -\frac{1}{18} \Rightarrow f_{eq} = -18$ cm

So, the combination behaves as a concave mirror

12. If the angle of incidence is greater than critical angle for all colours, the beam will be totally internally reflected. The beam may not always suffer refraction and hence dispersion. Hence both statements are true. Since statement-2 has no connection with TIR, it is not an explanation of statement-1

13. $\frac{1}{f} = \frac{2(\mu - 1)}{R}$

$\frac{1}{12} = \frac{2(1.5 - 1)}{R} = \frac{1}{R}$

R = 12 cm

$\frac{1}{f_w} = \frac{2[\mu_r - 1]}{R}$

$\frac{1}{f_w} = \frac{2\left[\frac{1.5}{1.35} - 1\right]}{12} = \frac{2 \times 0.15}{12 \times 1.35}$

$f_w = 54$

$\frac{1}{f_{eq}} = \frac{2}{54} \quad \Rightarrow \quad f_{eq} = 27$ cm

14. $m = \frac{v}{u} \Rightarrow \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow 1 - \frac{v}{u} = \frac{v}{f} \Rightarrow 1 - m = \frac{v}{f}$

$m = 1 - \frac{v}{f} = \frac{f - v}{f}$

and $\frac{u}{v} - 1 = \frac{u}{f} \Rightarrow \frac{u}{v} = \frac{1 + u}{f} = \frac{f + u}{f}$

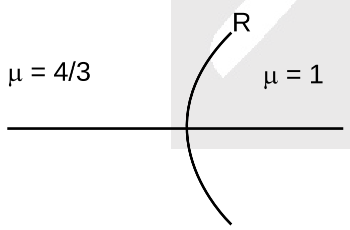
$m = \frac{v}{u} = \frac{f}{f + u}$

$u = -(f + x) \quad \Rightarrow \quad m = \frac{f}{f - f - x} = -\frac{f}{x}$

$v = f + y \Rightarrow m = \frac{f - f - y}{f} = -\frac{y}{f}$

$m \propto \frac{1}{x}$, $m \propto \frac{1}{y}$

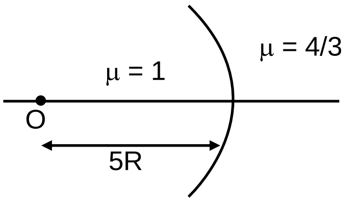
15. e-1



$\frac{1}{v} - \frac{\frac{4}{3}}{-\infty} = \frac{1 - \frac{4}{3}}{R} = -\frac{1}{3R} \Rightarrow v = -3R$

$m_1 = \frac{-\frac{3R}{1}}{\frac{\infty}{-\frac{4}{3}}} = +0$

e-2 :



$$\frac{4}{3v} - \frac{1}{-5R} = \frac{\frac{4}{3}-1}{-R}$$

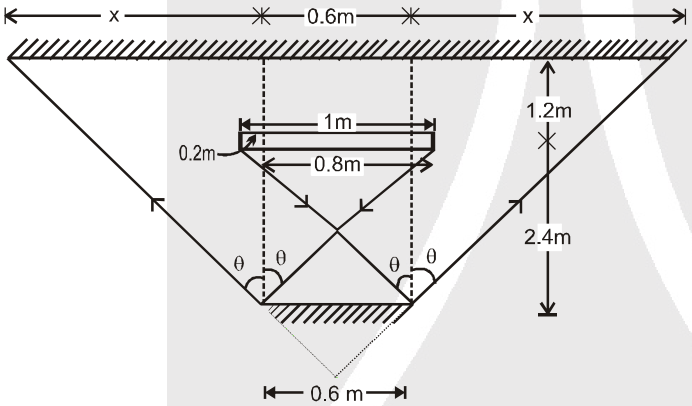
$$\frac{4}{3v} + \frac{1}{5R} = -\frac{1}{3R} \Rightarrow \frac{4}{3v} = -\frac{1}{3R} - \frac{1}{5R} = -\frac{8}{15R}$$

$$v = -\frac{5R}{2}$$

$$m_2 = \frac{-\frac{5R}{2(4/3)}}{-\frac{5R}{1}} = +\frac{3}{8}$$

PART - II

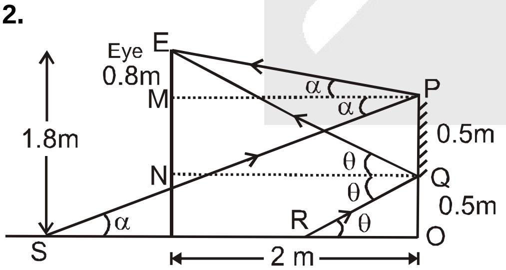
1. The ray diagram is as shown in the fig.



Now $\tan\theta = \frac{x}{(1.2+2.4)} = \frac{0.8}{2.4}$ $x = 1.2$ m.

Hence, required distance = 2x + 0.6 = 3

2.



From figure, ΔEMP and और ΔPOS are similar.

So , $\frac{EM}{PO} = \frac{MP}{OS} \Rightarrow OS = 1 \times \frac{2}{0.8}$ m = 2.5 m.

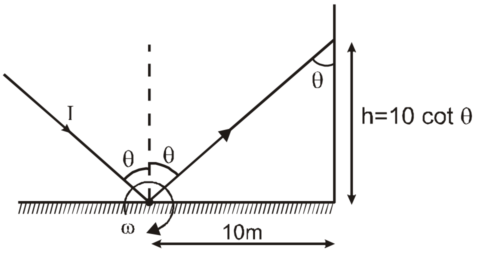
Similarly , ΔENQ and ΔQOR are similar.

So, अत: $\frac{EN}{QO} = \frac{QN}{RO} \Rightarrow RO = \frac{0.5\times 2}{(1.8-0.5)}$ m. $= \frac{10}{13}$ m.

Hence, length of wall visible in mirror.

$= OS - OR = (\frac{5}{2} - \frac{10}{13})$ m $= \frac{45}{26}$; x = 45

3.

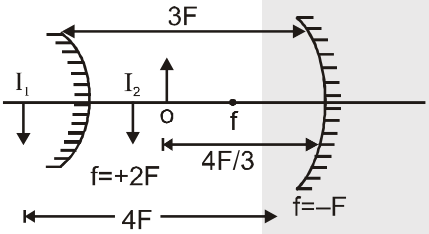


When mirror is rotated with angular speed ω, the reflected ray rotates with angular speed 2ω (= 36 rad/s)

speed of the spot = $\left|\frac{dh}{dt}\right| = \left|\frac{d}{dt}(10\cot\theta)\right|$

$= \left|-10\,\mathrm{cosec}^2\theta \quad \frac{d\theta}{dt}\right| = \left| -\frac{10}{(\;0.\;\;6)^2}\times 36 \right|$ = 1000 m/s.

4.



For reflection by concave mirror.

$\frac{1}{-F} = \frac{1}{v} + \frac{1}{-4F/3}$ ⇒ v = – 4F Hence $m_1 = \frac{-v}{u} = -3$

For reflection by concave mirror,

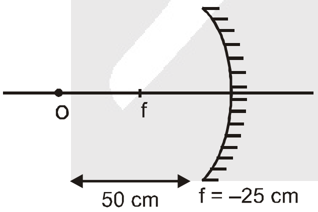
u = F , f = + 2F $\frac{1}{2F} = \frac{1}{v} + \frac{1}{F}$ $\frac{1}{v} = \frac{1}{2F} - \frac{1}{F} = -\frac{1}{2F}$ v = – 2F

magnification by concave mirror.

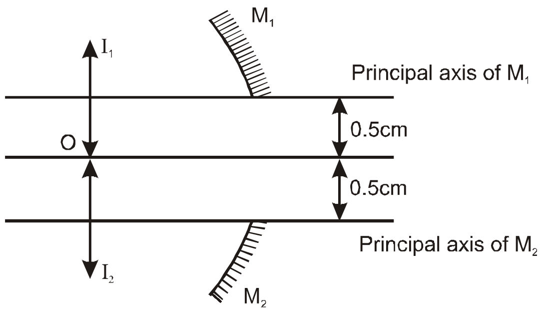
$m_2 = -\frac{v}{u} = -\left(-\frac{2F}{F}\right) = 2$

Net magnification = $m_1 m_2$ = (– 3) (2) = – 6

5.



$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$, $\frac{1}{-25} = \frac{1}{v} + \frac{1}{-50}$ v = – 50 cm



For M_1 $\quad m = \frac{I_1}{o} = -\frac{v}{u} \Rightarrow \frac{I_1}{-0.5} = -\left(\frac{-50}{-50}\right) \Rightarrow I_1 = 0.5$ cm

For M_2 $\quad m = \frac{I_2}{o} = \frac{-v}{u} \Rightarrow \frac{I_2}{0.5} = -\left(\frac{-50}{-50}\right) \Rightarrow I_2 = -0.5$ cm

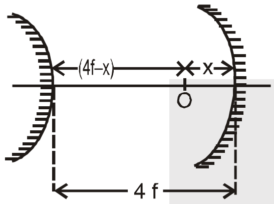
$\therefore$ Distance between I_1 and I_2
= 0.5 + 0.5 + 1 = 2 cm.Ans

6. Let the situation be as shown in figure with the object and image shown at A. For image formation by concave mirror,

$u = -x$

$f = -f$

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{f} + \frac{1}{x}$$



$$\Rightarrow v = \frac{fx}{f - x}$$

$\therefore$ Distance of the image from the convex mirror = $4f + \frac{fx}{f - x} = \frac{4f^2 - 3fx}{(f - x)}$

Now for image formation by convex mirror,

$$u = -\frac{(4f^2 - 3fx)}{(f - x)}$$

$f = f$

By the question,

$v = -(4f - x)$

$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ gives ,

$$\frac{1}{-(4f - x)} = \frac{1}{f} + \frac{(f - x)}{(4f^2 - 3fx)}$$

on solving, we get,

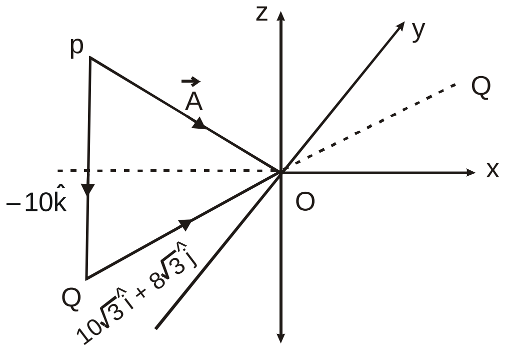
$x^2 - 6fx + 6f^2 = 0$

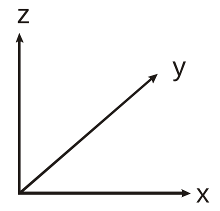
which gives,, $x = \left(3 \pm \sqrt{3}\right) f$ But $x < 4f$

Hence अत: , $x = \left(3 - \sqrt{3}\right) 10$ cm = 12.68 cm

so integer next to x is 13 Ans.

7.





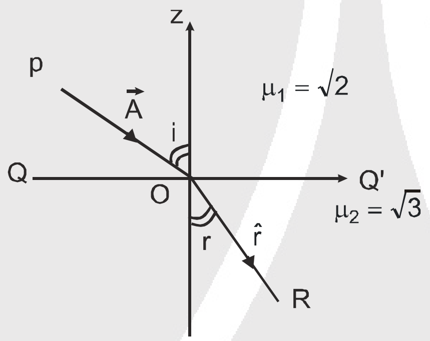
Incident ray $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\,\hat{j} - 10\hat{k}$

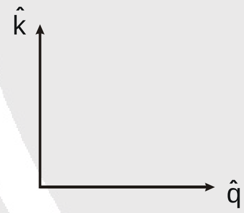
$= (6\sqrt{3}\,\hat{i} + 8\sqrt{3}\,\hat{j}) + (10\hat{k})$

$= \overrightarrow{QO} + \overrightarrow{PQ}$ (As shown in figure)

Note that $\overrightarrow{QO}$ is lying on x– y plane

Now QQ' and Z axis are mutually perpendicular. Hence we can show them in two-dimensional figure as below.





Vector $\vec{A}$ makes an angle i with z-axis, given by

$$i = \cos^{-1}\left\{\frac{10}{\sqrt{(10)^2 + (6\sqrt{3})^2 + (8\sqrt{3})^2}}\right\} = \cos^{-1}\left\{\frac{1}{2}\right\} \quad i = 60^o$$

Unit vector in the direction of QOQ' will be -

$$\hat{q} = \frac{6\sqrt{3}\ \ \hat{i} + 8\sqrt{3}\ \ \hat{j}}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2}} = \frac{1}{5}(3\ \ \hat{i} + 4\ \ \hat{j})$$

Snell's law gives

$$\frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin\ \ i}{\sin\ \ r} = \frac{\sin\ \ (60^o)}{\sin\ \ r} \qquad \therefore \qquad \sin r = \frac{\sqrt{3}/2}{\sqrt{3}/\sqrt{2}} = \frac{1}{\sqrt{2}} \qquad \therefore \qquad r = 45^o$$

Now we have to find a unit vector in refracted ray's direction OR. Say it is $\hat{r}$ whose magnitude is 1. Thus

$$\hat{r} = (\ 1\sin r\ )\ \hat{q} - (\ 1\cos r\ )\ \hat{k} = \frac{1}{\sqrt{2}}\ [\hat{q} - \hat{k}\,]$$

$$= \frac{1}{\sqrt{2}}\left[\frac{1}{5}(3\ \ \hat{i} + 4\hat{j}) - \hat{k}\right]$$

$$\hat{r} = \frac{1}{5\sqrt{2}}(3\ \ \hat{i} + 4\hat{j} - 5\hat{k}) = \frac{1}{5}\left(\frac{3}{\sqrt{2}}\hat{i} + 2\sqrt{2}\hat{j} - \frac{5}{\sqrt{2}}\hat{k}\right)$$

$a = \frac{3}{\sqrt{2}}$, $b = 2\sqrt{2}$ so $a \times b = 6$ **Ans**

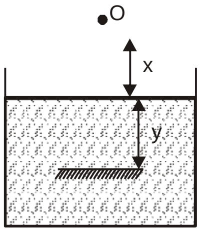
8. **(a)** The final image formed by slab has a fixed separation from 'O'.

Shift produced is $t\left(1-\frac{1}{3/2}\right) = \frac{t}{3}$ where 't' is thickness which is constant. The shift produced in the position of the image w.r.t. to 'O' is fixed

$\therefore$ velocity of final image is zero w.r.t. to 'O'

(b) Velocity of the final image = velocity of object = **6 cm/s Ans.**

9.



Apparent distance of mirror from O

$= x + \frac{y}{\mu}$

Distance of final image from O

$= 2\left(x + \frac{y}{\mu}\right)$

velocity of image

$= 2\left(\frac{dx}{dt} + \frac{1}{\mu}\frac{dy}{dt}\right) = \frac{2}{\mu} \times 4 = 6$

Ans 6 cm/s

10. Apparent shift in position of the object due to refraction through the slab $= d\left(1-\frac{1}{\mu_{rel}}\right)$

$= 3\left(1-\frac{1}{\frac{3}{2}}\right)$ = 1 cm towards the mirror

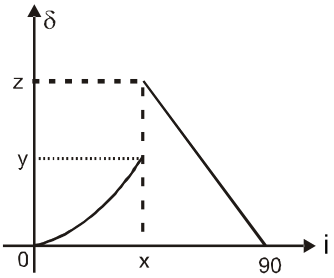
Now, for reflection from the mirror,

u = – 30 cm. $f = -\frac{R}{2} = -10$ cm

$\therefore$ $\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$ gives $\frac{1}{v} = -\frac{1}{10} - \frac{1}{(-30)}$ $\therefore$ v = – 15 cm

Thus, image is formed 15 cm, right of the mirror but because of second refraction through the slab the image is shifted 1 cm away from the mirror. Hence final image is formed at a distance 16 cm away from mirror.

11. (i)



x is the minimum angle of incidence for total internal reflection. $x = C = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

for refraction

if $i = C \Rightarrow r = \frac{\pi}{2} \Rightarrow \delta = (r - i) = \frac{\pi}{2} - C. \Rightarrow y = \frac{\pi}{2} - \frac{\pi}{3}. = \frac{\pi}{6}$

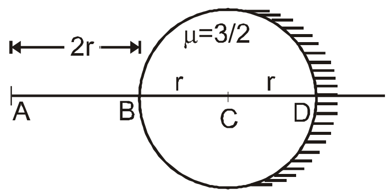
for total internal reflection,

if $i = C \Rightarrow \delta = 180^o - 2\,i = 180 - 2\,C. \Rightarrow z = \pi - 2\left(\frac{\pi}{3}\right) = \left(\frac{\pi}{3}\right)$

$x+y+z = \frac{\pi}{3} + \frac{\pi}{6} + \frac{\pi}{3} = \frac{5\pi}{6}$

Ans n = 5

12.



Let the object be placed at A

Then for refraction at the unsilvered part $u = -2r$, $R = r$.

$\therefore \quad \frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$= \frac{3/2}{v} - \frac{1}{(-2r)} = \frac{\frac{3}{2} - 1}{r} \Rightarrow v = \infty$

so, image formed at infinity will act as an object for the silvered part and hence the image will be formed at the focus of the concave mirror i.e., at $\frac{r}{2}$ distance left ward from D.

Again for refraction at the unsilvered surface $u = \left(r + \frac{r}{2}\right) = \frac{3r}{2}$

$\therefore \quad \frac{1}{v} - \frac{\frac{3}{2}}{\frac{3r}{2}} = \frac{1 - \frac{3}{2}}{r}$ or, $v = 2r$

Hence the final image is formed at D.

13. For refraction by upper surface

$\frac{1.6}{v_1} - \frac{1}{-2} = \frac{1.6 - 1}{1}$

$\Rightarrow \frac{1.6}{v_1} = 0.6 - 0.5 = 0.1 \Rightarrow v_1 = 16$ m

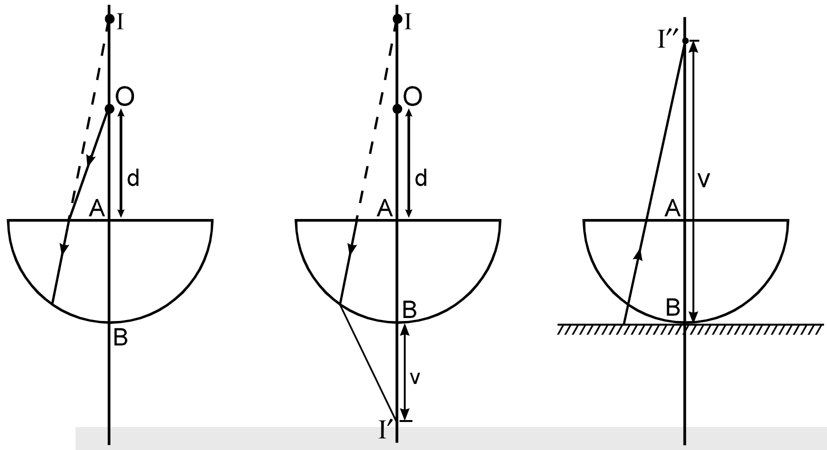
For refraction by lower surface

$\frac{2}{v_2} - \frac{1}{-2} = \frac{2-1}{1} \Rightarrow \frac{2}{v_2} = 1 - 0.5 = 0.5 \Rightarrow v_2 = \frac{2}{0.5} = 4$m

Distance between images = (16 – 4) = 12m.

14. The image of object O by refraction at plane surface is formed at I such that

AI = $\frac{4}{3}$ d



(a) (b) (c)

I acts as object for curved surface. The curved surface makes image of I at I′

$$\frac{1}{v} - \frac{\frac{4}{3}}{-\left(R + \frac{4}{3}d\right)} = \frac{1 - \frac{4}{3}}{-R} \quad \text{or} \quad \frac{1}{v} = \frac{1}{3R} - \frac{4}{3R + 4d}$$

I′ acts as object for mirror. Mirror makes its image at I″ distant v above B

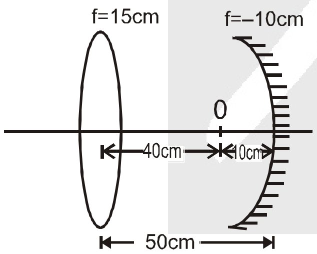
I″ acts as virtual object for the curved surface which makes its image at infinity

$$\frac{\frac{4}{3}}{\infty} - \left[\frac{1}{3R} - \frac{4}{3R + 4d}\right] = \frac{\frac{1}{3}}{R}$$

Solving we get d = $\frac{3}{4}$ R = $\frac{3}{4}$ × 4 = 3 cm

Ans. 30

15.



For image formation by lens :

u = – 40 cm f = + 15 cm

$$\frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{15} - \frac{1}{40} \qquad \therefore \qquad v = 24 \text{ cm}$$

So, image is formed at a distance 24 cm left of the lens.

For image formation by the mirror and the lens,

u = – 10 cm f = – 10 cm $\frac{1}{v} = \frac{1}{f} - \frac{1}{u} = -\frac{1}{10} + \frac{1}{10} = 0.$ v = – ∞

for lens, u = – ∞ f = 15 cm v = – 15 cm

Thus, image is formed at a point 15 cm towards left from the mirror.

One at 15 cm and the other at 24 cm from the lens away from the mirror so distance between images 9 cm

16. $\frac{h_i}{h_0} = \frac{f-v}{f} \Rightarrow h_i = -\frac{v}{f}h_0 + h_0$

$\Rightarrow \quad |h_i| = -\frac{v}{f}h_0 + h_0 \quad -\infty \le v \le f$

$|h_i| = \frac{v}{f}h_0 - h_0 \qquad f \le v \le -\infty$

So $\quad h_2 = h_0 = 1$ cm

From second eq. $v_2 = 2f$

Or When $v \to 0$, $u \to 0$ & $h_i \to h_0$ so $h_2 = h_0 = 1$ cm Image of same height is obtained when $v = 2f$ so $v_2 = 2f$

17. For first refraction

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{3}{2\times\infty} - \frac{1}{(-x)} = \frac{3/2-1}{+10}$$

$$\frac{1}{x} = \frac{1}{20} \quad \Rightarrow \quad x = 20 \text{ cm.}$$

18. For refraction through water surface,

$u = -12$ cm, $\qquad$ using $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\Rightarrow \frac{4}{3v} + \frac{1}{12} = 0 \Rightarrow \quad v = -16$ cm

Now, for lens,

$\frac{1}{f_a} = (\mu_g - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \qquad \frac{1}{f_w} = \left(\frac{\mu_g}{\mu_w} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$f_w = \frac{(\mu_g - 1)}{(\frac{\mu_g}{\mu_w} - 1)} \qquad f_a = \frac{\frac{3}{2}-1}{\left(\frac{3/2}{4/3} - 1\right)} \times 10 = 40$ cm

For refraction through the lens,

$u = -(16 + 44)\text{cm} = -\ 60$ cm

$f = +40$ cm

$\therefore \quad \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{40} - \frac{1}{60} = \frac{1}{120}$

$v = 120$ cm

For refraction from water to glass slab,

$u = 120$ cm $\quad \mu_2 = \frac{3}{2}$, $\quad \mu_1 = \frac{4}{3} \qquad \frac{\frac{3}{2}}{v} - \frac{\frac{4}{3}}{120} = 0. \quad (\therefore \quad R = \infty)$

$v = 135$ cm

Again for refraction from glass to air,

$u = 135$ cm $\quad \mu_2 = 1$, $\quad \mu_1 = \frac{3}{2}$

$\therefore \quad \frac{1}{v} - \frac{\frac{3}{2}}{135} = 0 \qquad \therefore \quad v = 90$ cm

19. $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \Rightarrow -\frac{df}{f^2} = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)dn$

(differentiating both sides)

$\Rightarrow -\frac{df}{f}\ (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{1}{R_1} - \frac{1}{R_2}\right)\ dn \ \Rightarrow - df = f\ \frac{dn}{n-1}$

but $dn = n_v - n_R$ ∴ $-d\,f = \frac{n_v - n_R}{n-1} f \Rightarrow df = -\omega f \Rightarrow f_v - f_R = -\omega f$

$= -0.04 \times 10 = -0.4$ cm

Ans.: $f_R - f_v = 4$ **mm**

PART - III

1. $f = -20$ m & $m = -\frac{v}{u} = 2$

using v = 2u.

$\frac{1}{-20} = \frac{1}{2u} + \frac{1}{u} \Rightarrow \frac{3}{2u} = \frac{1}{-20}$ $u = -30$ cm

using v = – 2u

$\frac{1}{-20} = \frac{1}{-2u} + \frac{1}{u} \Rightarrow \frac{1}{2u} = \frac{1}{-20}$ $u = -10$ cm

2. (A) No, when object is between infinite and focus ,image is real.
(C) when object is between pole and focus, image is magnified.
(D) when object is between pole and focus image formed by convex mirror is real.

3. $\frac{C_y}{C_x} = \frac{\sin r}{\sin\ i} = \tan 30^o = \frac{1}{\sqrt{3}}$

$C_y = \frac{1}{\sqrt{3}} C_x$.

since y is denser, total internal reflection can take place when ray is incident from y.

4. (A) is not true for minimum deviation.
(B) is true only if refracting side are equal.
(C) Two angles for maximum deviation are 90º and $i_{min.}$
(D) $\delta_{min.} = (\mu - 1)$ A.

5. A = 60º
δ = 40º
i + e = A + δ
⇒ i + e = 100º
and i – e = 20º
⇒ i = 60º or या 40º

6. $\frac{\mu_1}{v} - \frac{\mu_2}{u} = \frac{\mu_1 - \mu_2}{R}$

$\Rightarrow \frac{\mu_1}{v} - \frac{\mu_2}{u} < 0$

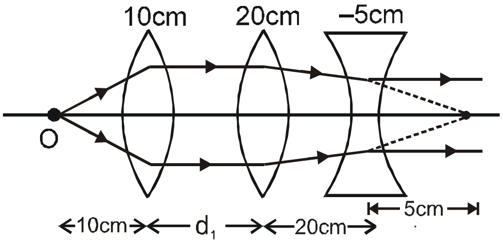
$\Rightarrow \frac{\mu_1}{v} < \frac{\mu_2}{u}$

$\Rightarrow \frac{v}{u} > \frac{\mu_1}{\mu_2}$

$\Rightarrow \frac{v}{u} > 0$

⇒ v and u must have same sign.

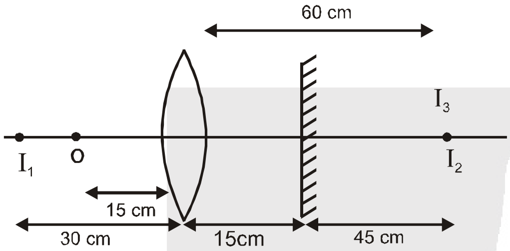
7.



Clearly, final rays are parallel to principal axis for any value of d_1 and $d_2 = (20 - 5) = 15$ cm.

8. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$ $\qquad \frac{1}{+30} = \frac{1}{v} - \frac{1}{-15}$

$\frac{1}{v} = \frac{1}{30} - \frac{1}{15} = -\frac{1}{30}$ $\qquad v = -30$



For plane mirror

$u = -30 - 15 = -45$ cm $\qquad \Rightarrow \qquad v = +45$ cm

For second refraction

$u = -60$, $\qquad f = 30$ cm

$\frac{1}{30} = \frac{1}{v} - \frac{1}{-60}$ $\qquad \frac{1}{v} = \frac{1}{30} - \frac{1}{60} = -\frac{1}{60}$ $\qquad v = +60$ cm

final image is real and 60 cm left from lens.

9. In fig (i)

image formed by each half will have same x-coordinate but different y-coordinates. Since their principal axis are not same.

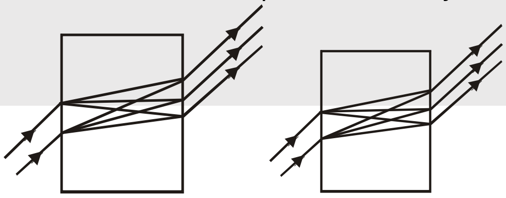
And image formed by the combination of the two halves will be have different x-coordinate. Hence, three images are formed.

In fig (ii) and fig. (iii), combinations have same focal length.

10. The light splits in different colours inside the slab due to dispersion.

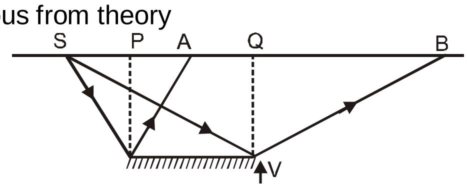
But the emergent rays will be parallel and will overlap with others hence giving white emergent beam.

Inside the slab rays of different colours are not parallel and they intersect each other.



11. Obvious from theory

12.



Here, sp = PA and SQ = QB

so, position of A and B doesn't depend on separation of mirror from the wall so, the patch AB will not move on the wall.

$\therefore$ SA and SB are constant

So, AB = constant.

13. The image will look like white donkey because a small part of lines can form complete image. The image will be less intense because some light will stopped by streakes.

14. Refer to Q.no. F-4. Ex.1, Part–I

$f = \dfrac{n_1R}{2n_2 - n_1 - n_3}$ or $\dfrac{n_3R}{2n_2 - n_1 - n_3}$

If $n_2 < \dfrac{n_1 + n_3}{2}$ $\Rightarrow$ f is –ve $\Rightarrow$ lens is diverging

If $n_2 > \dfrac{n_1 + n_3}{2}$ $\Rightarrow$ f is +ve $\Rightarrow$ lens is converging.

If $n_2 = n_1 + n_2$ $\Rightarrow$ $f = \infty$ neither converging nor diverging.

15. For d_1 = 120 m $\dfrac{3/2}{v} - \dfrac{1}{(-120)} = \dfrac{3/2 - 1}{60}$

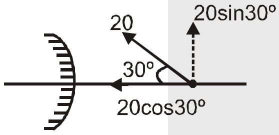
$\Rightarrow$ $v = \infty$

so, the ray is incident normally on the mirror. so for any value of d_2, ray retraces its path. so I_f is at O. for d_1

I_f, O d_1 = 240 cm $\dfrac{3/2}{v} - \dfrac{1}{(-240)} = \dfrac{3/2 - 1}{60}$ $\Rightarrow$ v = 360 cm.

If first image is formed at mirror ray retraced its path to form image at O.

16. We have $v = \dfrac{uf}{u - f} = \dfrac{(-10)(10)}{-10 - 10} = +5$



$\therefore$ $v_{ix} = -\dfrac{v^2}{u^2} v_{ox}$

$= -\left(\dfrac{5}{-10}\right)^2 \times 20.\dfrac{\sqrt{3}}{2} = -\dfrac{5\sqrt{3}}{2}$ mm/sec

and $v_{iy} = -\left(\dfrac{v}{u}\right) v_{oy} = -\left(\dfrac{5}{-10}\right) \times 20 \times \dfrac{1}{2}$ = 5 mm/s.

Hence $\tan\theta = \left|\dfrac{v_{iy}}{u_{ix}}\right| = \dfrac{5}{5\sqrt{3}/2} = \dfrac{2}{\sqrt{3}}$

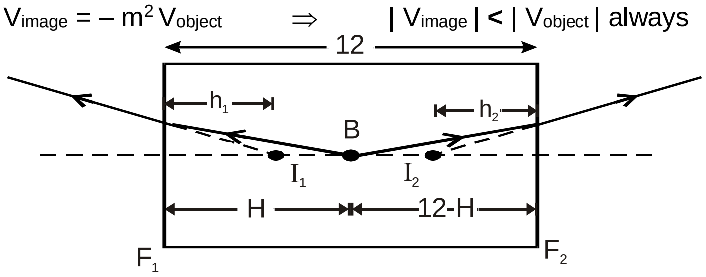
and $v_i = \sqrt{\left(\dfrac{5\sqrt{3}}{2}\right)^2 + (5)^2} = \dfrac{5\sqrt{7}}{2}$ mm/s

17. For convex mirror

|m| < 1 for any real object

Now, $V_{image} = -m^2 V_{object}$ $\Rightarrow$ $|V_{image}| < |V_{object}|$ always

18.



Let the bubble B is at distance H from the face F_1 of the cube.

$$h_1 = \frac{n_a}{n_C} H = 5 \text{ cm}$$

Similarly when looking from opposite face F_2,

$$h_2 = \frac{n_a}{n_C} (12 - H) = 3 \text{ cm}$$

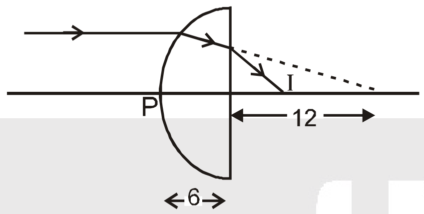
Solving H = 7.5 cm and $n_C = 1.5$

19. As in figure (i), rays are parallel incident on curved part.

Therefore $\frac{1}{v} - \frac{3/2}{\infty} = \frac{1-3/2}{-6}$ ⇒ v = 12 cm.

For figure (ii), image created by curved part acts as object for the flat part.

For curved part



$\frac{1.5}{v} - \frac{1}{\infty} = \frac{0.5}{6}$ ⇒ v = 18 cm

from flat surface, object is at 12 cm to the right.

for flat part

$\frac{1}{v} - \frac{3/2}{12} = 0$ ⇒ $\frac{1}{v} - \frac{1}{8}$ ⇒ v = 8.

So, distance of final image from P = 6 + 8 = 14 cm. (right)

20. $\delta = i - r$ when i ↑, r ↑, δ ↑

$\delta_{max} = 90 - \sin^{-1}\frac{1}{u} = \cos^{-1}\frac{1}{u}$

21. Combination may behave converging

In that case (a), (b) & (d) are possible.

If combination behaves like diverging c will be correct. So all the options are correct.

PART - IV

1. From passage, (D) is correct.

2. From passage, (C) is correct.

3. From points (2) and (3) of passage :

f and f' must be of opposite sign.

Also $\omega_C < \omega_D$ and $f_C < f_D$

which is satisfied only by (D).

4. $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0$

⇒ $\frac{\omega_1}{\omega_2} = -\frac{f_1}{f_2} = \frac{1}{2}$ .... (1)

⇒ $\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{40}$ ....(2)

After solving (1) & (2)

$f_1 = 20$ cm

$f_2 = -40$ cm.

5. Chromatic aberration doesn't occur in case of spherical mirrors.

6. $\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$

Here v = 2.5 (Distance of Retina as position of image is fixed)

u = – x

$\frac{1}{f} = \frac{1}{2.5} + \frac{1}{x}$ For f_{min} : x is minimum $\frac{1}{f_{min}} = \frac{1}{2.5} + \frac{1}{25}$

7. For f_{max} : x is maximum f_{max} $\frac{1}{f_{max}} = \frac{1}{2.5} + \frac{1}{\infty}$

8. For near sighted man lens should make the image of the object with in 100 cm range

For lens u = – ∞ v = – 100

$\frac{1}{f_{lens}} = \frac{1}{-100} - \frac{1}{-\infty}$

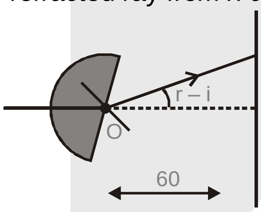
9. For far sighted man lens should make image of the nearby object at distance beyond 100 cm

For grown up person least distance is 25 cm for lens u = – 25, v = – 100

$\frac{1}{f} = \frac{1}{-100} - \frac{1}{(-25)}$ $\Rightarrow \frac{1}{f} = \frac{3}{100}$

P = + 3 so no. of spectacle is = +3.

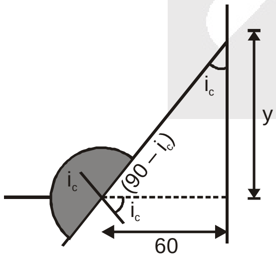
10. Angle of refracted ray from x-axis θ = r – i



For small angles

r = μi

$\theta = (\mu - 1)\, i \Rightarrow \frac{d\theta}{dt} = (\mu - 1)\frac{di}{dt} = \left(\frac{5}{3} - 1\right)6 = 4$ rad/s



11. Brigth spot will disappear due to TIR

$\tan(90 - i_c) = \frac{y}{60}$

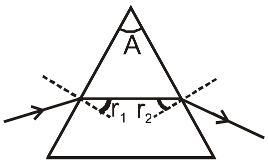
$\frac{4}{3} = \frac{y}{60}$

y = 80 cm

EXERCISE–3

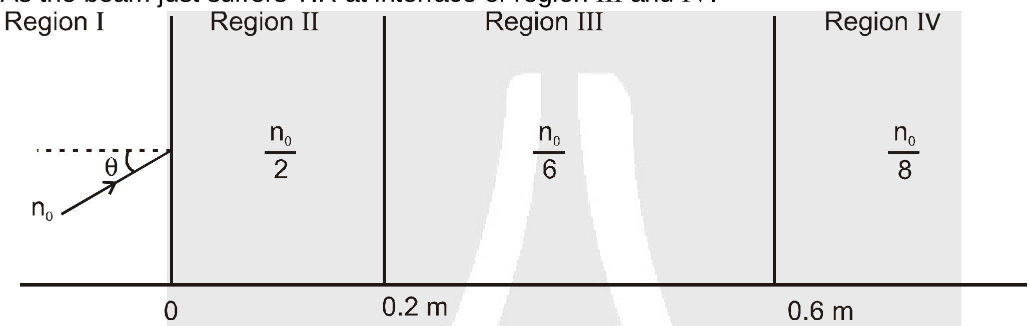
PART - I

1.



For minimum deviation
i = e and $r_1 = r_2 = A/2 = 30^o$
which is independent of nature of light.
Both the lights are set for minimum deviation so angle of refraction will be 30° for both colours.
Ans. (A)

2. As the beam just suffers TIR at interface of region III and IV.



$$n_0 \sin\theta = \frac{n_0}{2}\sin\theta_1 = \frac{n_0}{6}\sin\theta_2 = \frac{n_0}{8}\sin 90^\circ$$

$$\sin\theta = \frac{1}{8}$$

$$\theta = \sin^{-1}\frac{1}{8}$$ **Ans. (B)**

3. (A) $f < 0$,

$$v = \frac{uf}{u-f} = \frac{f}{1-f/u} = \frac{u}{u/f-1} \text{ and } m = -\frac{v}{u}$$

values of v may be positive, negative or infinity, also it can have values less than or greater than u.

(A) → (p,q,r,s)

(B) In this case f is positive.
So v will be positive and less than u.
(B) → (q)

(C) $v = \dfrac{f}{1+f/u} = \dfrac{u}{u/f+1}$

Here $u < 0$ $f > 0$
v may be positive, negative or infinity
v may be greater than or less than u
So (C) → (p,q,r,s)

(D) $\dfrac{1}{f} = (\mu-1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$

for diagram $R_2 > R_1$
⇒ f is +ve
This is same as in case (C) for the nature of image.
So (D) → (p,q,r,s)

4. $x' = \frac{x}{n_{rel}}$, $v' = \frac{v}{n_{rel}} = \frac{\sqrt{2\times10\times(20-12.8)}}{1}\times\frac{4}{3}$ = 16 m/s.

5. $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ or $\frac{1}{-|v|}+\frac{1}{-|u|}=\frac{1}{-|f|}$ $\Rightarrow$ $|v| = \frac{|u|\ |f|}{|u|-|f|}$

For |u| = 42, |f| = 24 ; $|v| = \frac{(42)\ (24)}{42-24}$ = 56 cm so (42, 56) is correct observation

For |u| = 48 or |u| = 2f or |v| = 2f so (48, 48) is correct observation

For |u| = 66 cm ; |f| = 24 cm

$|v| = \frac{(66)\ (24)}{66-24} \approx$ 36 cm which is not in the permissible limit

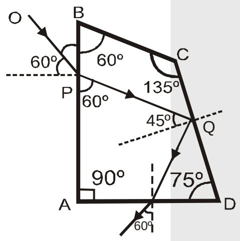
so (66, 33), is incorrect recorded

For |u| = 78, |f| = 24 cm

$|v| = \frac{(78)\ (24)}{78-24} \approx$ 32 cm which is also not in the permissible limit.

so (78, 39), is incorrect recorded.

6.



By refraction at face AB :

$1.\sin 60^\circ = \sqrt{3}\ .\ \sin r_1$

So $r_1 = 30^\circ$

This shows that the refracted ray is parallel to side BC of prism. For side 'CD' angle of incidence will be 45º, which can be calculated from quadrilateral PBCQ.

By refraction at face CD :

$\sqrt{3}\ \sin 45^\circ = 1\ \sin r_2$

So $\sin r_2 = \frac{\sqrt{3}}{\sqrt{2}}$

which is impossible. So, there will be T.I.R. at face CD.

Now, by geometry angle of incidence at AD will be 30º. So, angle of emergence will be 60º.

Hence, angle between incident and emergent beams is 90º

7. When object distance is 25.

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\frac{1}{v} - \frac{1}{(-25)} = \frac{1}{20}$ $\Rightarrow$ v = 100 cm.

$m_{25} = \frac{v}{u} = \frac{100}{-25} = -4.$

When object distance is 50.

$\frac{1}{v} - \frac{1}{(-50)} = \frac{1}{20}$ $\Rightarrow$ $u = \frac{100}{3}$ cm

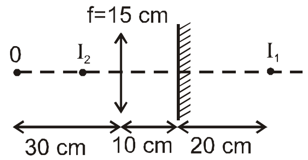
$m_{50} = \frac{\frac{100}{3}}{-50} = -\frac{2}{3}$

$$\frac{m_{25}}{m_{50}} = \frac{-4}{-\frac{2}{3}} = 6.$$

Alternate :

$$\frac{m_{25}}{m_{50}} = \frac{\frac{f}{20-25}}{\frac{f}{20-50}} = \frac{-30}{-5} = 6$$

8.



First image,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-30} = \frac{1}{15}$$

v = 30, image in formed 20 cm behind the mirror.
Second image, by plane mirror will be at 20 cm infront of plane mirror.

For third image, $\frac{1}{v} - \frac{1}{10} = \frac{1}{15}$

$$\frac{1}{v} = \frac{1}{10} + \frac{1}{15} = \frac{3+2}{30} = \frac{5}{30}$$

v = 6 cm

Ans. Final image is real & formed at a distance of 16 cm from mirror.

9. R = 20 m, f = 10 m
For mirror,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{25/3} + \frac{1}{u_1} = \frac{1}{10}$$

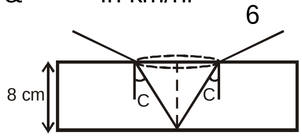
$\frac{1}{u_1} = \frac{1}{10} - \frac{3}{25} = -\frac{1}{50}$ $\Rightarrow$ $u_1 = -50$ cm

& $\frac{1}{50/7} + \frac{1}{u_2} = \frac{1}{10}$ $\Rightarrow$ $\frac{1}{u_2} = -\frac{1}{25}$ $\Rightarrow$ $u_2 = -25$ cm

So, speed = $\left|\frac{\Delta u}{\Delta t}\right| = \frac{25}{30}$ m/sec. = $\frac{5}{6}$ m/sec.

& in km/hr = $\frac{5}{6} \times \frac{18}{5}$ = 3 km/hr.

10.



$\tan C = \frac{R}{8}$ ....(i)

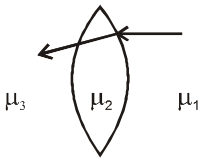
$\frac{5}{3} \sin C = 1.\sin 90^o$

$\sin C = \frac{3}{5} \Rightarrow$ C = 37º

$$\frac{3}{4} = \frac{R}{8}$$

R = 6 cm. **Ans.**

11. (A)

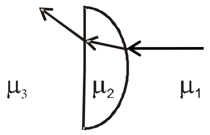


$\mu_2 = \mu_3$

As there is no deviation. As the light bends towards normal in denser medium $\mu_2 > \mu_1$

p – A & C

(B)



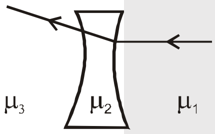
As light bends away from normal

$\mu_2 < \mu_1$

& $\mu_3 < \mu_2$

q – B & D

(C)

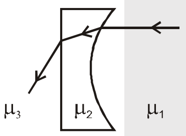


$\mu_2 = \mu_3$ (As no deviation)

$\mu_2 > \mu_1$ (As light bends towards normal)

r – C & A

(D)



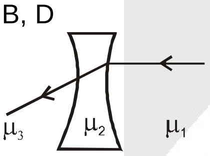
$\mu_2 < \mu_1$

$\mu_3 < \mu_2$

As light bends away from normal

s – B, D

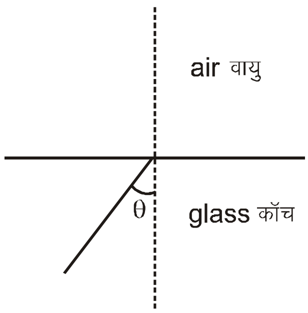
(E)



$\mu_2 = \mu_3$ As no deviation of light

$\mu_2 < \mu_1$ As light bend away from normal

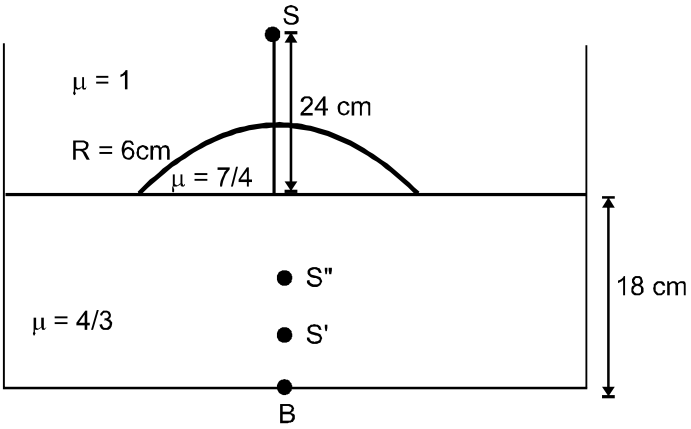
12.



Initially most of part will be transmitted. When $\theta > i_C$, all the light rays will be total internal reflected. So transmitted intensity = 0

So correct answer is (C)

13.



$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$$\frac{7}{4v}-\frac{1}{-24}=\frac{\frac{7}{4}-1}{6}$$

$$\frac{7}{4v}=\frac{3}{24}-\frac{1}{24}=\frac{2}{24}=\frac{1}{12}$$

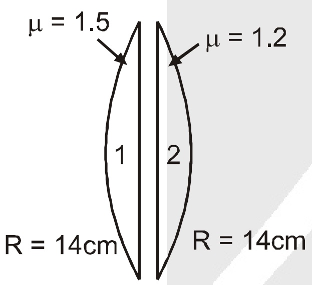
$$\frac{7\times12}{4}=V \;= 21 \text{ cm}$$

$$\frac{21}{OS"}=\frac{7/4}{4/3}$$

OS" = 16

∴ BS" = 2cm

14.



$$\frac{1}{f_1}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{f_1}=(1.5-1)\left[\frac{1}{14}-\frac{1}{\infty}\right]$$

$$\frac{1}{f_1}=\frac{0.5}{14}$$

$$\frac{1}{f_2}=(1.2-1)\left[\frac{1}{\infty}-\frac{1}{-14}\right]$$

$$\frac{1}{f_2}=\frac{0.2}{14}$$

$$\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}\;=\;\frac{0.5}{14}+\frac{0.2}{14}$$

$\frac{1}{f} = \frac{0.7}{14}$

$\frac{1}{v} = \frac{7}{140} - \frac{1}{40} = \frac{1}{20} - \frac{1}{40}$

$\frac{1}{v} = \frac{2-1}{40}$

v = 40 cm

15. $n = \frac{c}{v}$

for metamaterials

$v = \frac{c}{|n|}$

16. Meta material has a negative refractive index

$\therefore$ $\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$ $\Rightarrow$ n_2 is negative

$\therefore$ θ_2 negative

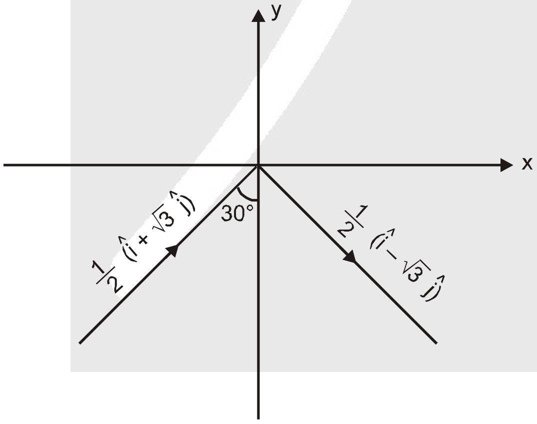
17. v = 8 m (magnification = $-\frac{1}{3} = \frac{v}{u}$)

u = –24m

$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{\infty} + \frac{1}{R}\right)$

R = 3m

18. Angle between given rays is 120°
so angle of incidence is 30°



19. $\frac{1}{f_{film}} = (n_1 - 1)\left(\frac{1}{R} - \frac{1}{R}\right)$ $\Rightarrow$ $f_{film} = \infty$ (infinite)

No effect of presence of film.

<u>From Air to Glass :</u>

Using single spherical Refraction :-

$\frac{n_2}{v} - \frac{1}{u} = \frac{n_2 - 1}{R}$

$\frac{1.5}{v} - \frac{1}{\infty} = \frac{1.5-1}{R}$ $\Rightarrow$ v = 3R

$f_1 = 3R$

From Glass to Air :-

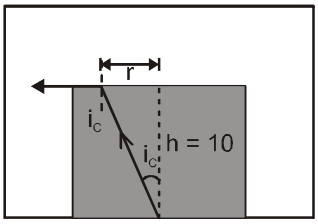
$$\frac{1}{v} - \frac{n_2}{u} = \frac{1-n_2}{-R}$$

$$\Rightarrow \quad \frac{1}{v} - \frac{1.5}{\infty} = \frac{1-1.5}{-R}$$

$\Rightarrow \quad v = 2R$

$\therefore \quad f_2 = 2R$

20.



$$\text{Sin } i_C = \frac{r}{\sqrt{r^2+h^2}}$$

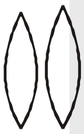
$$\Rightarrow \quad \frac{n_\ell}{n_B} = \frac{r}{\sqrt{r^2+h^2}}$$

$$\Rightarrow \quad n_\ell = \frac{r}{\sqrt{r^2+h^2}} \times 2.72$$

$$= \quad \frac{5.77}{11.54} \times 2.72 = 1.36$$

21. (P)

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}+\frac{1}{r}\right) = \frac{1}{r} \quad \Rightarrow \quad f = rs$$



$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = \frac{2}{r} \quad \Rightarrow \quad f_{eq} = \frac{r}{2}$$

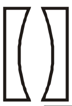
(Q)

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(\frac{1}{r}\right) \quad \Rightarrow \quad f = 2r$$

$$\Rightarrow \frac{1}{f} + \frac{1}{f} = \frac{2}{f} = \frac{1}{r} \quad \Rightarrow \quad f_{eq} = r$$

(R)

$$\frac{1}{f} = \left(\frac{3}{2}-1\right)\left(-\frac{1}{r}\right) = -\frac{1}{2r} \quad f = -2r$$



$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{f} + \frac{1}{f} = -\frac{2}{2r} \quad \Rightarrow \quad f_{eq} = -r$$

(S)

$$\Rightarrow \frac{1}{f_{eq}} = \frac{1}{r} + \frac{1}{-2r} = \frac{1}{2r} \quad \Rightarrow \quad f_{eq} = 2r$$

Ans. (B) P–2 Q –4 R–3 S – 1

22. For mirror

$$M = \frac{f}{f-u} = -\frac{V}{u}$$

$$M = \frac{-10}{-10+15} = -\frac{V}{-15}$$

M = – 2,

v = – 30 cm

For lens

$$M' = \frac{f}{f+u} = \frac{10}{10-20} = -1$$

$M_1 = 2$

In liquid

$$\frac{f'}{f} = \frac{\mu-1}{\left(\frac{\mu}{\mu_0}-1\right)} = \frac{7}{4}$$ {f' is the focal length of lens in medium of refractive index $\mu_0 = \frac{7}{6}$}

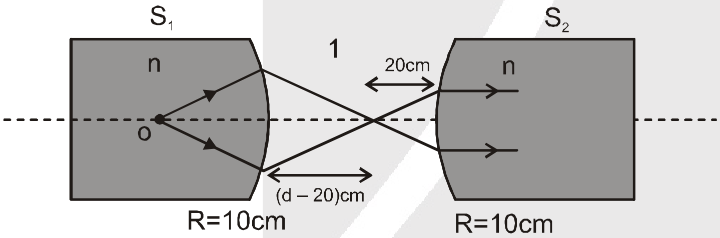
$$f' = \frac{70}{4} \text{ cm}$$

$$M' = \frac{f'}{f'+u} = \frac{\frac{70}{4}}{\frac{70}{4}-20} = -7$$

$M_2 = 14$

$$\left|\frac{M_2}{M_1}\right| = 7$$

23.



at glass rod S_2

$1 \rightarrow n$ refraction

$$\frac{n}{\infty} - \frac{1}{u_2} = \frac{n-1}{+10}$$

$\Rightarrow$ $u_2 = -20$

at glass rod S_1

for $n \rightarrow 1$ refraction

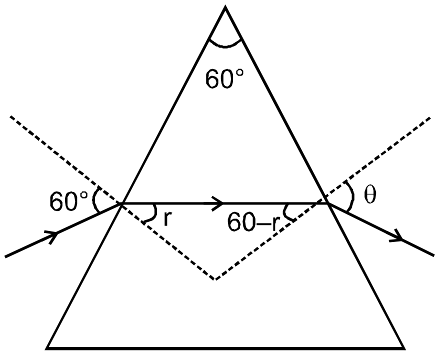
$V_1 = d - 20$

$$\frac{1}{d-20} - \frac{n}{(-50)} = \frac{1-n}{-10}$$

$$\frac{1}{d-20} + \frac{n}{50} = +\frac{1}{20}$$

d = 70 cm

24.



sin 60 = n sin r ....(1)

sinθ = n sin (60–r)...............(2)

Differentiating eq...(2)

$\cos\theta \dfrac{d\theta}{dn} = -n\cos(60-r)\dfrac{dr}{dn} + \sin(60-r)$

Differentiating eq...(1)

$n\cos r\dfrac{dr}{dn} + \sin r = 0$

$\cos\theta \dfrac{d\theta}{dn} = -n\cos(60-r)\left(\dfrac{-\tan r}{n}\right) + \sin(60-r)$

$\dfrac{d\theta}{dn} = \dfrac{1}{\cos\theta}\left(+\cos(60-r)\tan r + \sin(60-r)\right)$

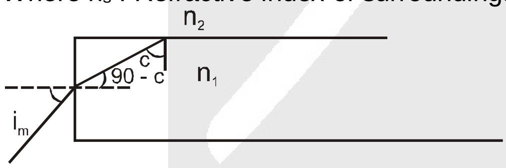
$\dfrac{d\theta}{dn} = \dfrac{1}{\cos 60}\left(\cos 30 \times \tan 30 + \sin 30\right)$

$= 2\left(\dfrac{1}{2}+\dfrac{1}{2}\right) = 2$

25. $n_s \sin i_m = n_1 \sin(90 - c)$

$\sin c = \dfrac{n_2}{n_1}$

Where n_s : Refractive index of surroundings



$NA = \sin i_m = \sqrt{1-\dfrac{n_2^2}{n_1^2}} = \dfrac{1}{n_s}\sqrt{n_1^2 - n_2^2}$

For S_1 in Air

$NA = \dfrac{1}{1}\sqrt{\dfrac{45}{16}-\dfrac{9}{4}} = \dfrac{3}{4}$

For S_1 in $n_s = \dfrac{6}{\sqrt{15}}$

$NA = \dfrac{\sqrt{15}}{6}\sqrt{\dfrac{45}{16}-\dfrac{9}{4}} = \dfrac{3\sqrt{15}}{24}$

For S_1 in water

$NA = \dfrac{1}{(4/3)}\sqrt{\dfrac{45}{16}-\dfrac{9}{4}} = \dfrac{3}{4}\left(\dfrac{3}{4}\right) = \dfrac{9}{16}$

For S_2 in Air

$$NA = \frac{1}{1}\sqrt{\frac{64}{25}-\frac{49}{25}} = \frac{\sqrt{15}}{5}$$

For S_2 in water

$$NA = \frac{1}{(4/3)}\sqrt{\frac{64}{25}-\frac{49}{25}} = \frac{3}{4}\frac{\sqrt{15}}{5}$$

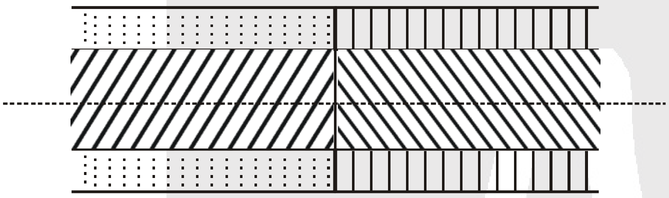
For S_2 in $n_s = \frac{16}{3\sqrt{15}}$

$$NA = \frac{3\sqrt{15}}{16}\frac{\sqrt{15}}{5} = \frac{9}{16}$$

For S_2 sin $n_s = \frac{4}{\sqrt{15}}$

$$NA = \frac{\sqrt{15}}{4}\sqrt{\frac{64}{25}-\frac{49}{25}} = \frac{\sqrt{15}}{4}\frac{\sqrt{15}}{5} = \frac{3}{4}$$

26.



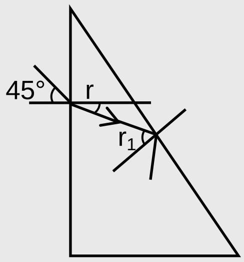
$$NA = \frac{1}{n_s}\sqrt{n_1^2 - n_2^2}$$

$NA_2 < NA_1$

Therefore the numerical aperture of combined structure is equal to the lesser of the two numerical aperture, which is NA_2

27. $\frac{\sin 45°}{\sin r} = \sqrt{2} \Rightarrow r = 30°$

Also $r_1 \geq 45°$ for internal reflection.

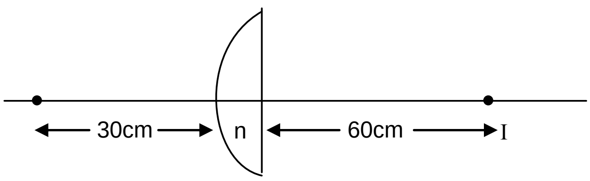


$90 - \theta = r + 90 - r_1$

$r_1 > 30 + \theta \geq 45°$

$\theta \geq 15°$ **Ans. (A)**

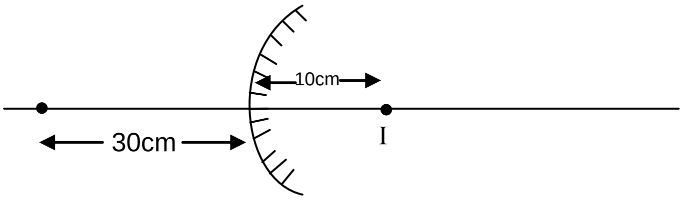
28. **Case–I**



$$\frac{1}{60}+\frac{1}{30}=\frac{1}{f_1} \Rightarrow \frac{1}{f_1}=\frac{1}{60}+\frac{2}{60}$$

$$f_1 = \frac{R}{n-1} = +20cm$$

Case –2



$$\frac{1}{10} - \frac{1}{30} = \frac{1}{f_2}$$

$$\frac{3}{30} - \frac{1}{30} = \frac{1}{f_2} = \frac{2}{30}$$

$f_2 = 15 = \frac{R}{2} \Rightarrow R = 30$

R = 30 cm

$$\frac{R}{n-1} = +20\text{cm} = \frac{30}{n-1}$$

$\Rightarrow$ = 2n – 2 = 3

$\Rightarrow$ f_1 = +20 cm

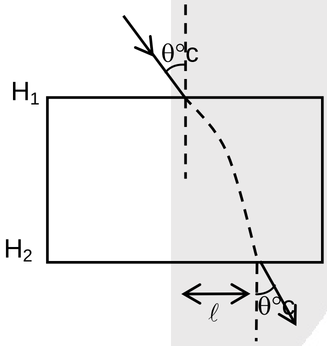
RI of lens is 2.5.

ROC of convex surface is 30 cm

Faint image is erect and virtual

focal length of lens is 20 cm

29.

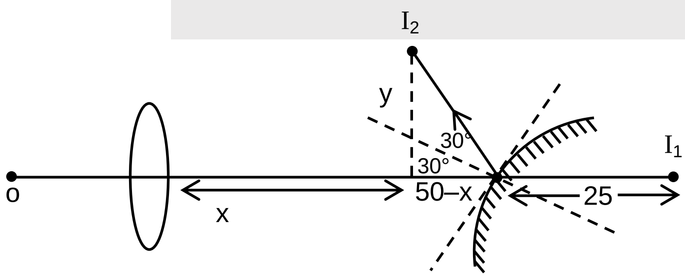


(A*) $n_1\sin\theta_1 = n_2\sin\theta_2$

(C*) ℓ is independent of n_2

(D) ℓ is dependent on n(z)

30.



For I_1 $\Rightarrow$ u = –50 f = +30 v = +75cm

final image is I_2

$\frac{y}{50-x}$ = tan 60 = $\sqrt{3}$ $\quad$ $y = 50\sqrt{3} - \sqrt{3}\,x$

$y + \sqrt{3}\,x = 50\sqrt{3}$

A and B both option satisfy this,

size of image > size of object

$\sqrt{y^2+(50-x)^2}$ sin30° > 25 sin30°

$y^2 + (50 - x)^2 > 625$

for option (A) $\frac{625}{3}+\frac{625}{9}$ < 625

for option (B) 625(3) + 625 > 625

so B is correct.

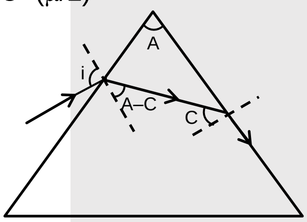
31.

$$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\frac{A}{2}} = \frac{\text{Sin}A}{\sin\frac{A}{2}} = 2\cos\frac{A}{2}$$

$\sin i = \mu \sin\frac{A}{2}$ $\Rightarrow$ $\sin i = 2\sin\frac{A}{2}\cos\frac{A}{2} = \sin A$

i = A $\quad\quad$ $r_1 = \frac{A}{2}$

$A = 2\cos^{-1}(\mu/2)$



$\sin C = \frac{1}{\mu}$, $\cos C = \sqrt{1-\frac{1}{\mu_2}}$

$\sin i = \mu \sin(A - C) = \mu(\sin A \cos C - \cos A \sin C)$

$= \mu\left(\sin A\sqrt{1-\frac{1}{\mu^2}} - \frac{\cos A}{\mu}\right) = \left(\sin A\sqrt{\mu^2-1}-\cos A\right) = \left(\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right)$

$i = \sin^{-1}\left(\sin A\sqrt{4\cos^2\frac{A}{2}-1}-\cos A\right)$

(C) In this option, it has been assumed that the refracting sides (Incident and emergent side) of isosceles triangle are equal. Therefore, the ray inside the prism is parallel to the prism.

32. $1.6\sin\theta = (n - m\Delta n)\sin 90°$

$1.6\sin\theta = n - m\Delta n$

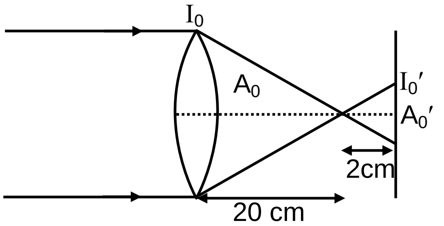
$1.6 \times \frac{1}{2} = 1.6 - m(0.1)$

0.8 = 1.6 – m(0.1)

m × 0.1 = 0.8

m = 8

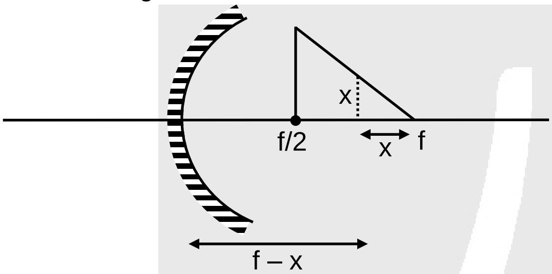
33.



$$\frac{A_0'}{A_0} = \left(\frac{2}{20}\right)^2 = \frac{1}{100} \quad \Rightarrow \quad A_0' = \frac{A_0}{100}$$

$$I_0' = \frac{I_0 A_0}{\frac{A_0}{100}} = 100 I_0 = 130\,kW/m^2$$

34. For small angle the answer would be 'D'. So all the points which are near to focus fill have magnification

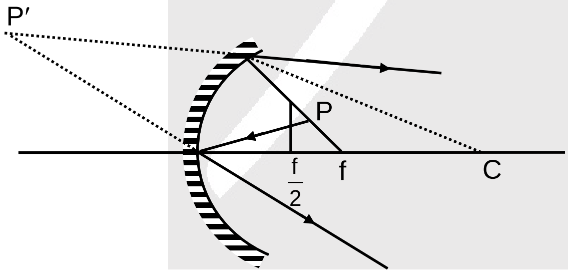


$$m = \frac{f}{f-u}$$

$\Rightarrow$ $m = \dfrac{-f}{-f-(-f+x)} = \dfrac{f}{x}$

Size of image = $x \times \dfrac{f}{x} = f$

So, the height of image for all such objects will be constant.



The angle of the object is 45º the focus will not be defined for all points.

35. $\dfrac{1}{f_0} = \dfrac{2(n-1)}{R}$ .....(1)

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_2} = (n+\Delta n-1)\left(\frac{1}{R} - \frac{1}{\infty}\right)$$

$$\frac{1}{f_0+\Delta f_0} = \frac{(n-1)}{R} + (n+\Delta n-1)\left(\frac{1}{R}\right)$$

$$\frac{1}{f_0+\Delta f_0} = \frac{2n+\Delta n-2}{R} \quad .....(2)$$

$$(1)/(2) \Rightarrow \frac{f_0+\Delta f_0}{f_0} = \frac{\frac{2(n-1)}{R}}{\frac{2n+\Delta n-2}{R}}$$

$$1+\frac{\Delta f_0}{f_0} = \frac{2(n-1)}{2n+\Delta n-2}$$

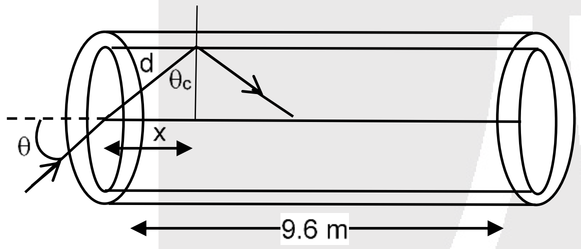
$$\frac{\Delta f_0}{f_0} = \frac{-\Delta n}{(2n+\Delta n-2)}$$

$$\frac{\Delta f_0}{20} = -\frac{10^{-3}}{3+10^{-3}-2} \quad \Rightarrow \quad \Delta f_0 = -2 \times 10^{-2}$$

$|\Delta f_0| = 0.02$ cm

Ans. 1, 2, 4

36.



$1.8 \sin\theta_c = 1.44 \sin\theta$

$$\sin\theta_C = \frac{1.44}{1.50} = \frac{24}{25}$$

$$\therefore \quad \sin\theta_C = \frac{x}{d} = \frac{24}{25}$$

$$d = \frac{25x}{24}$$

$\therefore$ Total length travel by light = $\frac{25}{24}\times 9.6 = 10m$

$$\therefore \quad t = \frac{S}{\left(\frac{C}{n_1}\right)} = \frac{10}{\frac{3\times 10^8}{1.5}} = \frac{1}{2}\times 10^{-7} = 5 \times 10^{-8}$$

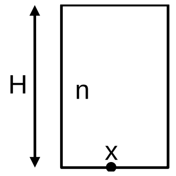
t = 50ns

$t = 50 \times 10^{-9}$

$\therefore$ Ans = 50

37.

Case-I :



H = 30 cm

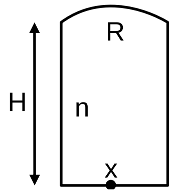
n = 3/2

$H_1 = H/n \Rightarrow \frac{30\times 2}{3} = 20$ cm

Case-II :

R = 300 cm

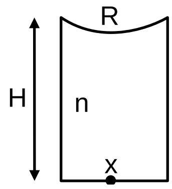
$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}$$



$$\frac{1}{-H_2}-\frac{3}{-2\times 30}=\frac{1-\frac{3}{2}}{-300}$$

$$H_2=\frac{600}{29}=20.684\text{cm}$$

Case-III :



$$\frac{n_2}{v}-\frac{n_1}{u}=\frac{n_2-n_1}{R}\ ;\ \frac{1}{-H_3}-\frac{3}{-2\times 30}=\frac{1-\frac{3}{2}}{300}$$

$$;\ H_3=\frac{600}{31}=19.354\text{cm}$$

38.

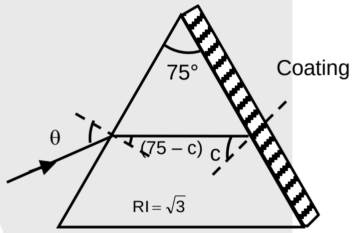
For T.I.R at coating

$\sin c = \dfrac{n}{\sqrt{3}}$

Appling snell's law at first surface

$\sin\theta=\sqrt{3}\sin(75-c)$

for limiting condition, at θ = 60°



$\sin 60 = \sqrt{3}\sin(75-c)$

$\dfrac{\sqrt{3}}{2}=\sqrt{3}\sin(75-c)$

$\dfrac{1}{2}=\sin(75-c) \quad\Rightarrow\quad \sin 30=\sin(75-c)$

$30 = 75 - c \quad\Rightarrow\quad c = 45°$

$\dfrac{n}{\sqrt{3}}=\dfrac{1}{\sqrt{2}} \quad\Rightarrow\quad n^2=\dfrac{3}{2}$ = 1.50

PART - II

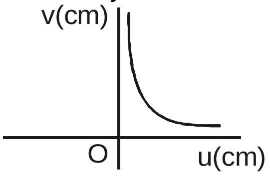
1. For a convex lens

u = –ve f = +ve

If v = ∞, u = f and if u = –∞ , v = f.

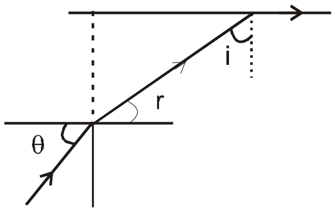
We have v = +ve and u = –ve

and u and v are symmetrical. Hence graph is shown,



Since graph is for distance so it should be lie in first quardent.

2.



$\sin\theta = \frac{2}{\sqrt{3}} \quad \sin r = \frac{2}{\sqrt{3}} \cos i$ ....(i)

and $\frac{2}{\sqrt{3}} \sin i = \sin 90^\circ \Rightarrow \quad i = 60^\circ$ ....(ii)

From (i) and (ii)

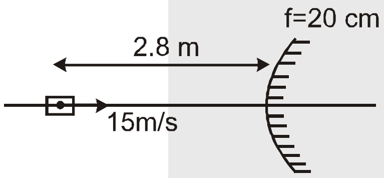
$\sin\theta = \frac{1}{\sqrt{3}}$

3. v = u and $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$\frac{2}{v} = \frac{1}{f}$

$\Rightarrow \quad v = 2f, \; u = 2f$

4. Mirror formula :



$\frac{1}{v} + \frac{1}{-280} = \frac{1}{20}$

$\frac{1}{v} + \frac{1}{20} + \frac{1}{280}$

$\frac{1}{v} + \frac{14+1}{280}$

$v = \frac{280}{15}$

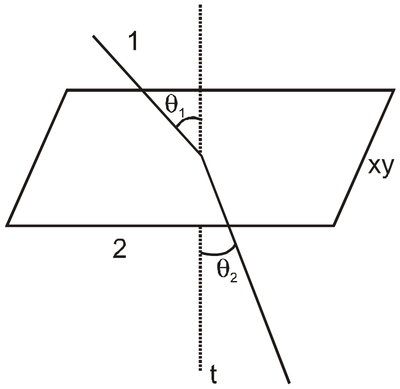
$v_I = -\left(\frac{v}{u}\right)^2 . v_{om}$

$\therefore \; v_I = -\left(\frac{280}{15 \times 280}\right)^2 .15$

$\therefore \; v_I = \frac{-15}{15 \times 15}$

$v_I = -\frac{1}{15}$ m/s **Ans.**

5. X–Y axis



$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$

$$\cos\theta_1 = \frac{10}{\sqrt{(6\sqrt{3})^2 + (8\sqrt{3})^2 + 100}} = \frac{10}{\sqrt{400}} = \frac{10}{20}$$

$$\cos\theta_1 = \frac{1}{2}$$

$$\theta_1 = 60°$$

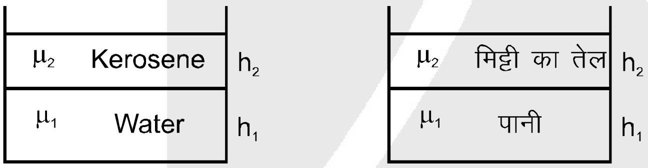
$$\sqrt{2}\ \sin 60° = \sqrt{3}\ \sin\theta_2$$

$$\sqrt{2} \times \frac{\sqrt{3}}{2} = \sqrt{3}\ \sin\theta_2$$

$$\sin\theta_2 = \frac{1}{\sqrt{2}}$$

$$\theta_2 = 45°$$

6.



Apparent shift :

$$= h_1\left(1 - \frac{1}{\mu_1}\right) + h_2\left(1 - \frac{1}{\mu_2}\right).$$

7. $\mu_R < \mu_B$

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_B} > \frac{1}{f_R}$$

$f_R > f_B$.

8.
$$\frac{1}{f} = \frac{1}{12} + \frac{1}{240} = \frac{20+1}{240}$$

$$f = \frac{240}{21} m$$

$$\text{shift} = 1(1 - \frac{2}{3}) = \frac{1}{3}$$

$$\text{Now } v' = 12 - \frac{1}{3} = \frac{35}{3} \text{ cm}$$

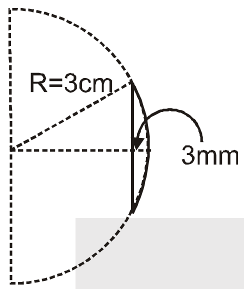
$\therefore \ \frac{21}{240} = \frac{3}{35} - \frac{1}{u}$

$\frac{1}{u} = \frac{3}{35} - \frac{21}{240} = \frac{1}{5}\left(\frac{3}{7} - \frac{21}{48}\right)$

$\frac{5}{u} = \left|\frac{144-147}{48\times 7}\right|$

u = 560 cm = 5.6 m

9.



$n = \frac{3}{2}$

$3^2 + (R - 3mm)^2 = R^2$

$\Rightarrow 3^2 + R^2 - 2R(3mm) + (3mm)^2 = R^2$

$\Rightarrow R \approx 15$ cm

$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{15}\right) \Rightarrow f = 30cm$

Ans (3)

11. $\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f} = \frac{1}{2x} \Rightarrow f = 2x$ here $\left(\frac{1}{x} = \frac{1}{R_1} - \frac{1}{R_2}\right)$

$\frac{1}{f_1} = \left(\frac{3/2}{4/3} - 1\right)\frac{1}{x}$

$\frac{1}{f_2} = \left(\frac{3/2}{5/3} - 1\right)\left(\frac{1}{x}\right)$; $\Rightarrow$ f_2 is negative

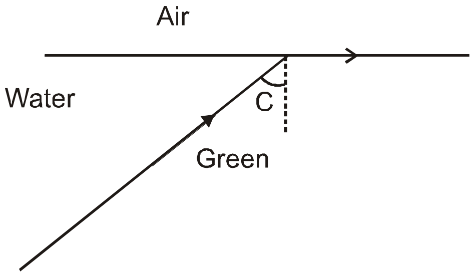
$\frac{1}{f_1} = \frac{1}{8x} = \frac{1}{4(2x)} = \frac{1}{4f}$

$\Rightarrow f_1 = 4f$

Analytically, If a lense is inserted in a denser sourrounding the sign of focal length changes and if lens is inserted in a rarer sourrounding , the sign of focal length remain same.

If lense is inserted in rarer medium the focal length increases.

12.

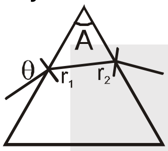


here $\sin C = \dfrac{1}{n_{water}}$

and $n_{water} = a + \dfrac{b}{\lambda^2}$

If frequency is less $\Rightarrow \lambda$ is greater and hence R.I. (n_{water}) is less and therefore, critical angle increases.

13.



For transmission $r_2 < i_C$

$A - r_1 < i_C$

$\sin (A - r_1) < \sin i_C$

$\sin (A - r_1) < \dfrac{1}{\mu}$

$A - r_1 < \sin^{-1}\left(\dfrac{1}{\mu}\right)$

$r_1 > A - \sin^{-1}\left(\dfrac{1}{\mu}\right)$

$\sin r_1 > \sin\left[A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right]$

$\therefore \quad \sin r_1 = \dfrac{\sin\theta}{\mu}$

$\dfrac{\sin\theta}{\mu} > \sin\left[A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right]$

$\theta > \sin^{-1}\left[\mu \sin\left\{A - \sin^{-1}\left(\dfrac{1}{\mu}\right)\right\}\right]$

14. $\theta = \dfrac{10}{x}$

$\theta_1 = \dfrac{10}{x}(20)$

Now 20 times nearer

15. When i = 35° and e = 79° then δ = 40°

$\delta = i + e - A$

40° = 35 + 79 – A

A = 74°

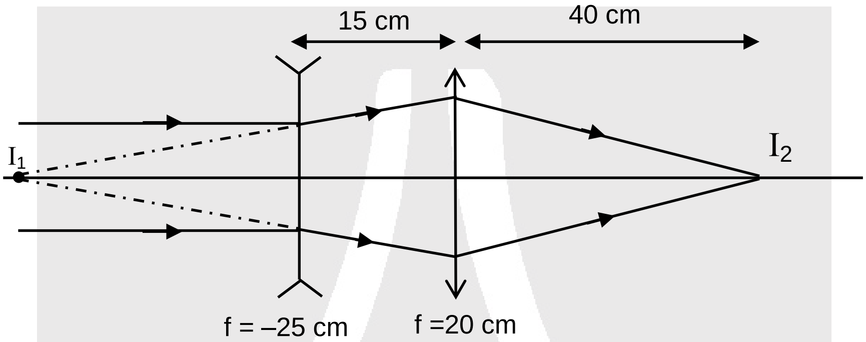
Since $i \neq e$ so δ_{min} will less than 40°

$$n = \frac{\sin\left(\frac{\delta_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

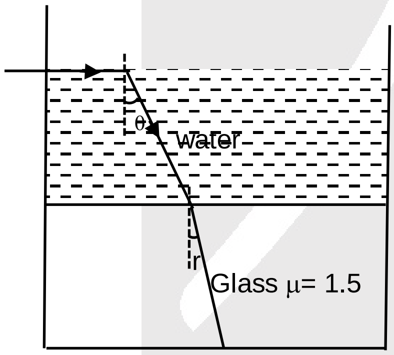
$$n = \frac{\sin\left(\frac{40° + 74}{2}\right)}{\sin\left(\frac{74}{2}\right)} = \frac{\sin(57°)}{\sin(37°)} = \frac{0.84}{0.60} = 1.4$$

Since δ_{min} will be less than 40° so
n will be less than 1.4
so the closest answer will be 1.5

16. Image formed by first lens is I_1 which is 25 cm left of diverging lens.
For second lens u = 40 cm (i.e. at 2F) so final image will be 40 cm right of converging lens.
Image will be real.



17.



1.sin90° = μsin$\theta \Rightarrow$ sinθ = $\frac{1}{\mu}$

μsinθ = 1.5 sinr

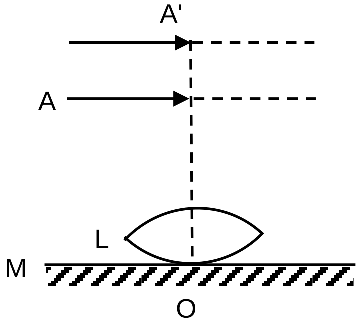
μtanθ = 1.5 $\Rightarrow$ tanθ = $\frac{1.5}{\mu}$

sinθ = $\frac{3}{\sqrt{9 + 4\mu^2}} = \frac{1}{\mu}$

$9\mu^2 = 9 + 4\mu^2 \Rightarrow \mu = \frac{3}{\sqrt{5}}$

18. $\delta = A(\mu - 1)$ for thin prism, then more is the refractive index, more will be the deviation

19.



Initially image is formed at A itself.

$\therefore$ After refraction from lens the rays must be incident normally on the plane mirror

$$\frac{1}{v} - \frac{1}{-OA} = \frac{1}{f}$$

f = OA = 18 cm

$$\frac{1}{f} = \left(\frac{3}{2} - 1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$$

R = f = 18 cm

After filling the liquid between lens and mirror and placing the object at A' same thing occurs

$$\frac{\mu_\ell}{v} = -\frac{1}{-OA'} = \frac{\frac{3}{2}-1}{R} + \left(\frac{\mu_\ell + \frac{3}{2}}{-R}\right)$$

$\Rightarrow$ $\frac{\mu_\ell}{\infty} + \frac{1}{27} = \frac{1}{36} - \frac{\mu_\ell + \frac{3}{2}}{18}$ $\Rightarrow$ $\frac{\mu_\ell - \frac{3}{2}}{18} = \frac{1}{36} - \frac{1}{27}$

$\Rightarrow$ $\frac{\mu_\ell - \frac{3}{2}}{18} = \frac{-9}{36 \times 27}$ $\Rightarrow$ $\mu_\ell = \frac{3}{2} - \frac{1}{6} = \frac{4}{3}$

20. **Case-I**

If final image is at least distance of clear vision

M.P. = $\frac{L}{f_0}\left(1 + \frac{D}{f_e}\right)$; 375 = $\frac{150}{5}\left[1 + \frac{25}{f_e}\right]$

$$\frac{375}{30} = 1 + \frac{25}{f_e}$$

$$\frac{345}{30} = \frac{25}{f_e}$$

f_e = $\frac{750}{345}$ = 2.17 cm; $f_e \approx$ 22 mm

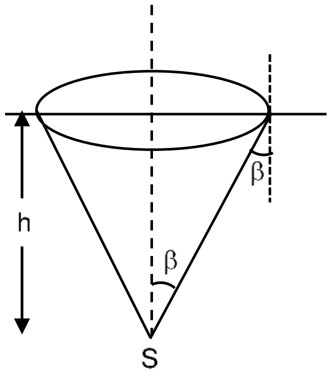
Case-II

If final image is at infinity

M.P. = $\frac{L}{f_0}\left(\frac{D}{f_e}\right)$ = 375

f_e = 22 mm

21.



$\sin\beta = \frac{3}{4}$, $\cos\beta = \frac{\sqrt{7}}{4}$

Solid angle $d\Omega = 2\pi R^2 (1 - \cos \beta)$

Percentage of light = $\frac{2\pi R^2(1-\cos\beta)}{4\pi R^2} \times 100$

$= \frac{1-\cos\beta}{2} \times 100 = \left(\frac{4-\sqrt{7}}{8}\right) \times 100 \approx 17\%$

HIGH LEVEL PROBLEMS

SUBJECTIVE QUESTIONS :

1. For AB

$\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$

$\frac{1}{v_1} = \frac{2}{-20} + \frac{1}{15} = -\frac{1}{10} + \frac{1}{15} = \frac{-3+2}{30} = -\frac{1}{30}$

$v_1 = -30$

$m_1 = -\frac{v}{u} = -\frac{(-30)}{-15} = -2$

For CD

$\frac{1}{v} = \frac{1}{-10} + \frac{1}{20} = \frac{-2+1}{20} = -\frac{1}{20}$

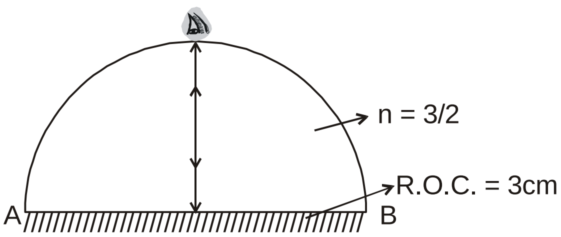
$v = -20$

$m_2 = -\frac{v}{u} = -\frac{(-20)}{-20} = -1$

$m_2 = -1$

Total = 2 + 10 + 4 = 16 cm

2. (i)



For AB face AB $\frac{1}{v} + \frac{1}{u} = \frac{2}{R}$ $R_{AB} = \infty$

$\frac{1}{v} = -\frac{1}{u}$ $|v| = |u|$

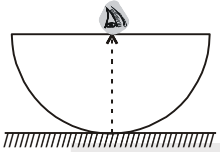
For spherical surface

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{\frac{3}{2}}{-3} = \frac{1 - \frac{3}{2}}{-3}$$

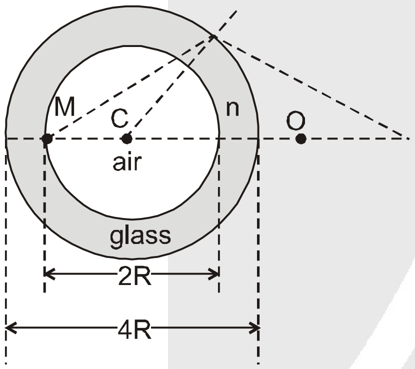
v = – 3

So no shift

(ii) 

$D_{app} = d_{actual} / n_{relative} = \frac{3}{3/2} = 2$ cm

so shift = 3 – 2 = 1 cm upward

3. 

For Ist refraction

$$\frac{n}{v_1} - \frac{1}{-2R} = \frac{n-1}{-R} \quad \text{........... (i)}$$

For IInd refraction

$$\frac{1}{v_2} - \frac{n}{v_1 - R} = \frac{1-n}{-2R} \quad \text{..........(ii)}$$

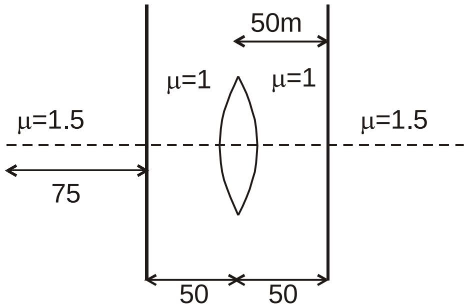
From (i) and (ii)

$$v_2 = \frac{2R(4n-1)}{1-3n}$$

so shift = $|v_2 - (-3R)|$

$$= 3R + 2\frac{(4n-1)R}{1-3n} = \frac{(n-1)\ R}{(3n-1)}$$

4.



I[st] refraction

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1}{v} - \frac{1.5}{(-75)} = \frac{1-1.5}{\infty} = 0$$

$$\frac{1}{v} = -\frac{1.5}{75} \qquad v = -50 \text{ cm}$$

u´ = –50 – 50 = –100 cm   Object for second refraction.

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{-100} = \frac{1}{60} \qquad \frac{1}{v} = \frac{1}{60} - \frac{1}{100}$$

v = 6 × 25 = 150 cm

v = 150 cm

III[rd] Refraction

u = 150 – 50 = 100

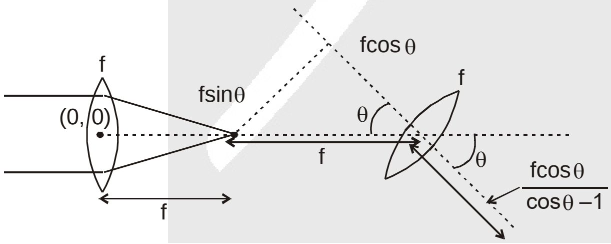
$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{100} = 0 \quad \Rightarrow \quad \frac{1.5}{v} = \frac{1}{100}$$

v = 150 cm from last surface   v = 150 cm

so from lens = 50 + 150 = 200 cm

5.



$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

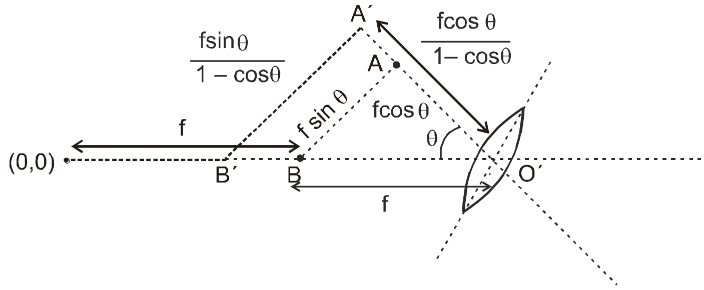
$$\frac{1}{v} - \frac{1}{-f\cos\theta} = \frac{1}{f}$$

$$v = \frac{f\cos\theta}{\cos - 1} = -\frac{f\cos\theta}{(1-\cos\theta)}$$

$$m = \frac{v}{u} = \frac{f\cos\theta \ / \ [\cos\theta - 1]}{-f\cos\theta} \quad \Rightarrow \quad m = \left(\frac{1}{1-\cos\theta}\right)$$

$$\therefore \frac{A'B'}{AB} = \left(\frac{f\cos\theta}{1-\cos\theta}\right) / \ f\cos\theta$$

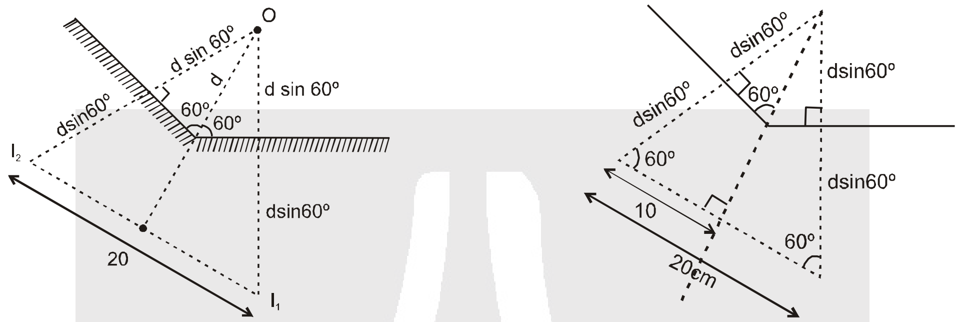
$$\Rightarrow \qquad A'B' = \frac{f\sin\theta}{1-\cos\theta}$$



From similar triangles, B´ lies on the principle axis of first lens.

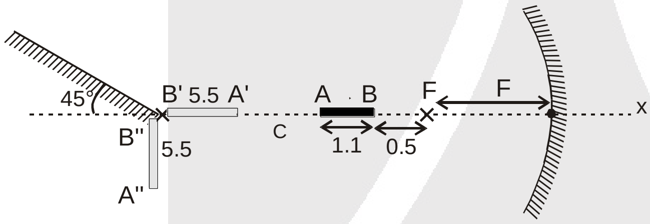
$\therefore$ y = 0 and $x = 2f - O'B' = 2f - \frac{f}{(1-\cos\theta)} = \frac{f(1-2\cos\theta)}{(1-\cos\theta)}$

6.



OI_1I_2 is equilateral triangle $\Rightarrow$ 2d sin 60º = 20 $\Rightarrow$ $d = \frac{20}{\sqrt{3}}$ cm.

7.



Let A and B represent the feet and head of the kid.
Now, for image formation of A and B by concave mirror,

$u_A = -(F + 1.6)$, $f = -F$

$u_B = -(F + 0.5)$, $f = -F$

$v_A = \frac{u_A f}{(u-f)} = \frac{-(F+1.6)\ \ (-F)}{(-F-1.6+F)} = \frac{-F(F+1.6)}{1.6}$

$v_B = \frac{-F(F+0.5)}{0.5}$

size of the image = $|v_B| - |v_A|$ $\Rightarrow$ $5.5 = \frac{F(F+0.5)}{0.5} - \frac{F(F+1.6)}{1.6}$

F = 2 ft

The plane mirror should be placed at angle of 45° with -ve x-axis (as shown) to get the required vertical image A'' B'' .

Altier :

From Newton's formula,

$xy = f^2$

so for A, $(1.1 + 0.5)y = f^2$ ...(i)

and for B,

$(0.5) \times (y + 5.5) + f^2$ ....(ii)

from (i) and (ii)

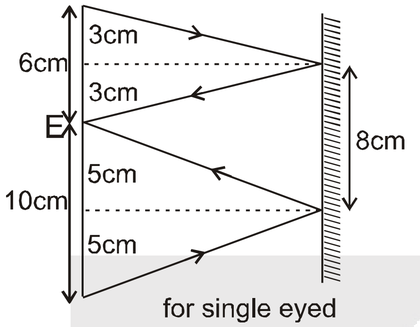
f = 2ft.

8. For height $\frac{h}{2} = \frac{24}{2} = 12$

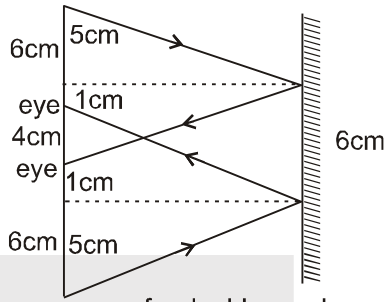
for width

(1) Single eyed person $\frac{h}{2} = \frac{16}{2} = 8$

(2) Double eyed person $\frac{12}{2} = 6$



for single eyed

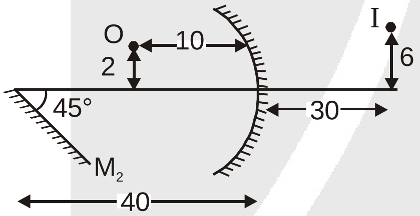


for double eyed

9. For M_1, u = – 10, f = – 15, h = 2.

Using mirror formula $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ $\Rightarrow \frac{1}{v} + \frac{1}{-10} = \frac{1}{-15}$

$\Rightarrow \frac{1}{v} = \frac{1}{10} - \frac{1}{15} = \frac{3-2}{30}$ $\Rightarrow$ v = 30 cm & $\frac{h_2}{h_1} = -\frac{v}{u}$ $\Rightarrow$ h_2 = 6 cm



The image formed by the plane mirror is at 70 below the principal axis & 70 + 6 – 30 = 46 of the concave mirror. $\therefore$ **coordinates of I_2 w.r.t. P = (– 46, – 70) Ans.**

10. u = – 6m, f = – 2m $\Rightarrow$ v = – 3m

let the height of object be Y_o,

then height of image is $Y_i = -\left(\frac{v}{u}\right)Y_o$.

$\therefore$ y-component of velocity of image $V_{iy} = \frac{dY_i}{dt}$

or $V_{iy} = \frac{v.Y_o}{u^2}\left(\frac{du}{dt}\right) - \frac{Y_0}{u}\left(\frac{dv}{dt}\right) - \frac{v}{u}\left(\frac{dY_o}{dt}\right)$ (differentiating both sides) .....(A)

$\frac{du}{dt}$ = x – component of velocity of object = 10 cos37° = 8 m/s

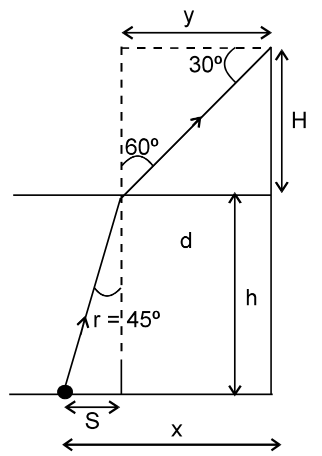
$\frac{dv}{dt}$ = x – component of velocity of image = $V_{ix} = -\left(\frac{v}{u}\right)^2\frac{du}{dt}$ = – 2m/s

$\frac{dY_o}{dt}$ = Y – component of velocity of object = 10 sin 37° = 6 m/s

(from equation A); $V_{iY} = \frac{dY_i}{dt} = \frac{(-3)(1)}{(-6)^2}.8 - \frac{1}{(-6)}(-2) \frac{(-3)}{(-6)}6$ – = – 4 m/s

$\therefore$ $\vec{V_i} = V_{ix}\,\hat{i} + V_{iy}\,\hat{j} = -2\,\hat{i} - 4\,\hat{j}$ **Ans.**

11.



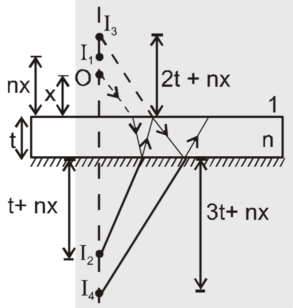
$\sin 60^\circ = \sqrt{\frac{3}{2}}\ \sin r \Rightarrow r = 45^\circ$

S = h = 1 m

$y = H \tan 60^0 = 3m$

$\therefore$ x= S + y = 4m = 4000 mm

12.

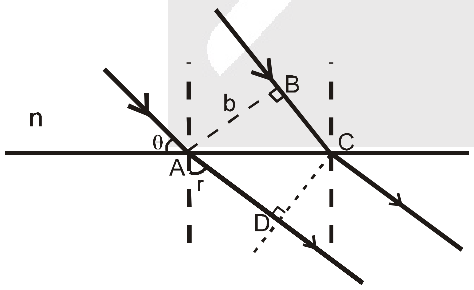


The figure shows the image formation in different steps.

Given 3t + nx – (t + nx) = 2t = 4

$\Rightarrow$ **t = 2 cm** **Ans.**

13. CD = beam diameter outside



$CD = AC \cos r = \frac{AB}{\sin\theta} \cos r$;

by Snell's law , $n \times \sin(90 - \theta) = 1 \times \sin r \Rightarrow \sin r = n\cos\theta$

$\therefore \cos r = \sqrt{1 - n^2 \cos^2\theta}$

Ans.: **CD =** $\frac{b\,.\sqrt{1 - n^2\cos^2\theta}}{\sin\theta}$

14. (a) $y = x \tan\left(1 - \frac{x}{R}\right) \longrightarrow$ actual trajectory of projectile.

$\therefore$ apparent trajectory or image of the trajectory will have 'x' unchanged but 'y' multiplied by a factor of R.I. = 4/3

$\therefore$ Equation is, $y' = \frac{4}{3}$ $y = \frac{4}{3} x \tan 37^o \left(1 - \frac{x}{100}\right)$ or $y = \left(x - \frac{x^2}{100}\right)$

$\frac{u^2 \sin(2 \times 37^\circ)}{g} = 100 \Rightarrow u = 25\sqrt{\frac{5}{3}}$

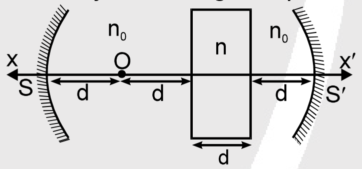
(b) Similarly 'x' component of velocity will remain unchanged, but the 'y' component of velocity will be multiplied by a factor of R.I. = 4/3 or at any time 't'.

$\vec{v}\ (t)_{image} = u \cos 37^o \hat{i} + \frac{4}{3}(u \sin 37^o - g\,t)\ \hat{j} = 25\sqrt{\frac{5}{3}} \times \frac{4}{5}\ \hat{i} + \frac{4}{3}\left[25\sqrt{\frac{5}{3}} \times \frac{3}{5} - 10\,t\right]\hat{j}$

$\vec{v}\ (t)_{image} = 20\sqrt{\frac{5}{3}}\ \hat{i} + 20\left[\sqrt{\frac{5}{3}} - \frac{2}{3}\,t\right]\hat{j}$

15. $u_1 = -d,\ \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow v = \frac{uf}{u-f} \Rightarrow v_1 = \frac{(-d)(-f)}{-d + f} = \frac{df}{f - d}$

{u_1, v_1 coordinates of object & image resp. w.r.t. pole S and positive axis as x}



and $v_2 = v_1 - d\left(1 - \frac{n_0}{n}\right) = \frac{df}{f - d} - d\left(1 - \frac{n}{n_0}\right)$

[v_2 is coordinate of image after refraction by the slab considering origin at S and positive direction as x axis]

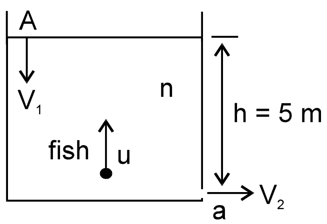
and $u_3 = -(v_2 + 4d)$

$\Rightarrow \quad v_3 = \left|\frac{u_3\ (-f)}{u_3 - (-\ f)}\right|$

[u_3 = coordinate of v_2 considering origin at S′ and positive direction as x′. v_3 = coordinates of image of u_3, origin at S′ and positive direction as x′]

$\left|\frac{(v_2 + 4d)f}{-v_2 - 4d + f}\right| \Rightarrow$ distance $D = \left|\frac{\left(\frac{df}{f-d} - d\left(1 - \frac{n_0}{n}\right) + 4d\right) f}{\frac{df}{d-f} + d\left(1 - \frac{n_0}{n}\right) - 4d + f}\right|$

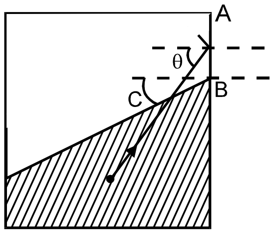
16.



$V_2 = \sqrt{2gh}$

$A V_1 = aV_2 \Rightarrow V_1 = 10^{-3} \times 10 = 10^{-2}$ m/s

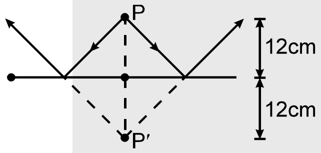
Velocity of fish observed $= \frac{(u + V_1)}{n} - V_1 = \frac{6 + 10^{-2}}{\frac{4}{3}} - 0.01 = 4.4975$ m/s

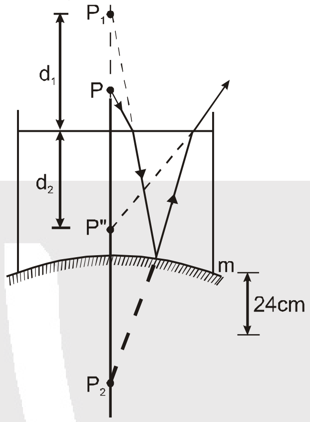
17. 

$\sin C = \frac{1}{2}$; $C = 30°$

If S is anywhere in the shaded region, the light rays from S will strike AB making an angle more than critical angle.

18. Due to reflection at plane water surface and other image formation is shown in the figure.





Due to refraction at water $d_1 = \frac{12}{\frac{1}{4/3}}$ = 16 cm. For M_1, P_1 is an object.

for this $\frac{1}{v} + \frac{1}{-40} = \frac{1}{+60} \Rightarrow$ v = 24 cm this is at P_2

It will act as object for water surface which makes image at P″. $d_2 = \frac{24+24}{4/3}$ = 36 cm

final images are P′ and P″

distance P′P″ = 36 – 12 = 24 cm. Ans.

19. $\vec{A} = 2\hat{i} - 2\hat{j} + \hat{k}$, $\vec{N} = \hat{j} - 2\hat{k}$ $\vec{R} = a\hat{i} + b\hat{j} + c\hat{k}$; $\because$ $\vec{R}$ is a unit vector

$\therefore$ $a^2 + b^2 + c^2 = 1$ .... (1)

For all these vectors to be in a plane.

$(\vec{A} \times \vec{N}).\vec{R} = 0 \Rightarrow \left[3\hat{i} + 4\hat{j} + 2\hat{k}\right].\left[a\hat{i} + b\hat{j} + c\hat{k}\right] = 0$

$\Rightarrow$ 3a + 4b + 2c = 0 .... (2)

Now अब

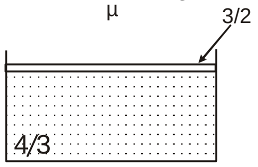
$\vec{A}.\vec{N} = |\vec{A}|\ |\vec{N}|\ \cos(\pi - i)$

and $\vec{R}.\vec{N} = |\vec{R}|\ |\vec{N}|\ \cos i$

$\therefore \frac{\vec{A}\ .\ \vec{N}}{|\vec{A}||\vec{N}|} = \frac{-\vec{R}.\ \vec{N}}{|\vec{R}||\vec{N}|} \Rightarrow \frac{-2-2}{3} = \frac{-\ (b-2c)}{1} \Rightarrow b - 2c = \frac{4}{3}$ ....(3)

The equation (1), (2) and (3) are the required relations.

20. Since width of glass sheet is negligible so neglecting effect of glass sheet.

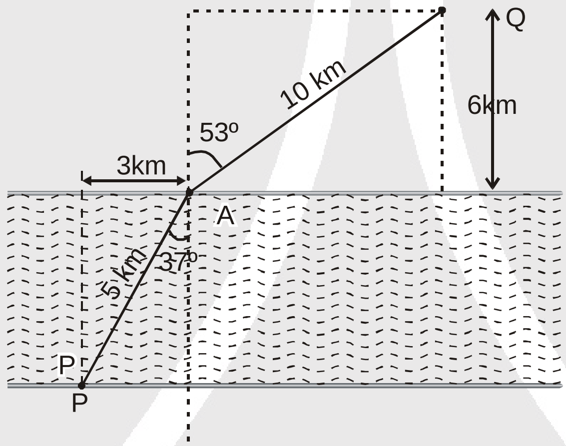


$d_{app} = \frac{d_{act}}{\mu_{rel}}$

$8 = \frac{10}{\left(\frac{4}{3\mu}\right)}$

$\Rightarrow \quad 8\,\frac{4}{3\mu} = 10 \quad \Rightarrow \quad \mu = \frac{32}{30} = \mathbf{\frac{16}{15}}$ **Ans.**

21. As we know that light travels in a path such as to reach from one point to another in shortest possible time.
Therefore, the man must travel along that path on which light would have travelled in moving from P to Q.
By Snell's law;



$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \quad \Rightarrow \quad \sin r = \frac{\mu_1}{\mu_2}.\sin i$

$\sin r = \frac{3}{5}.\frac{4}{3} = \frac{4}{5}$

$\Rightarrow \quad r = 53^o$

$\therefore \quad AQ = 10$ Km.

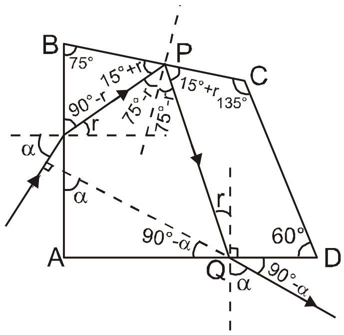
From P to A :

$t_1 = \frac{5}{3}$

From A to Q : $t_2 = \frac{10}{4} = \frac{5}{2}$

$T = t_1 + t_2 = \frac{5}{3} + \frac{5}{2} = \frac{25}{6}$ hr. = 250 minutes

22.



From the figure since incidence angle at face AB is α and refraction angle is r ; condition that emergent ray is perpendicular to incidence ray $\Rightarrow$ angle of emergence at face QD is $\alpha \Rightarrow$ incidence angle at face QD is r.

In quadrilateral PQDC 15° + r + 135° + 60° + r + 90° = 180° $\Rightarrow$ r = 30°

Now, $75° - r = 45° > c \;\Rightarrow \dfrac{1}{\sqrt{2}} > \dfrac{1}{n} \;\Rightarrow n > \sqrt{2}$

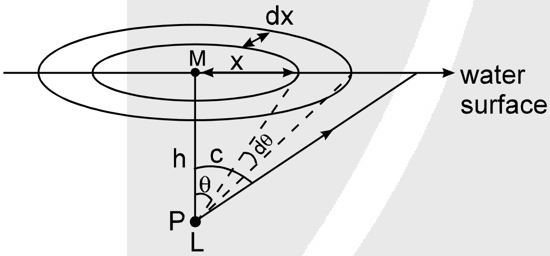
Also, $\sin\alpha = n \sin r = n \sin 30° =$

$\Rightarrow \quad n = 2\sin\alpha \;\Rightarrow \dfrac{n}{2} \le 1 \;\Rightarrow n \le 2$

$\therefore \quad \sqrt{2} < n \le 2$ and $\dfrac{\sin\alpha}{\sin r} = n \Rightarrow \sqrt{2} < 2\sin\alpha \le 2$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} < \sin\alpha \le 1 \;\Rightarrow 45° < \alpha \le 90°.$

23. **(a)**



Let the point source of power (P watts) be situated at L .

All the rays (from source) entering the cone of half angle C (critical angle) about axis LM escape out of water.

The power crossing the ring of radius x and width dx on surface of water is

$dP = (I\cos\theta)\, 2\pi x\, dx = \dfrac{P}{4\pi(h\sec\theta)^2} \cos\theta\, 2\pi x\, dx$ (I = intensity of light at point on annular ring.)

$x = h\tan\theta \qquad dx = h\sec^2\theta\, d\theta$

$\therefore \quad dP = \dfrac{P}{2}\sin\theta\, d\theta \qquad \therefore$ power escaping out of water is

$$P = \int dP = \int_0^C \frac{P}{2}\sin\theta\, d\theta = \frac{P}{2}(1 - \cos C)$$

where C is critical angle

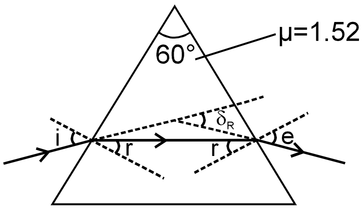
$\therefore \quad$ fraction of power escaped $= \dfrac{1}{2}(1 - \cos C) = \dfrac{1}{2}\left[1 - \dfrac{\sqrt{n^2 - 1}}{n}\right]$

(b) for n = 4/3 ; $f = \dfrac{1}{2}\left[1 - \dfrac{\sqrt{\left(\frac{4}{3}\right)^2 - 1}}{\frac{4}{3}}\right]$

24. (a) $\mu_R = 1.52$

$\mu_v = 1.6$

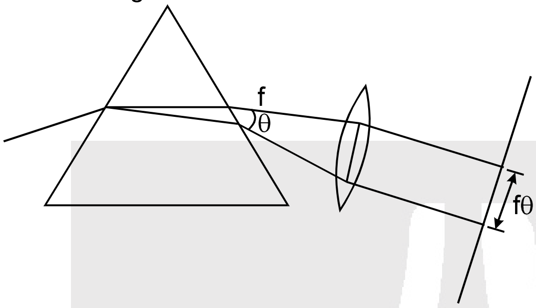
Minimum deviation condition for red is r = 30°



⇒ (1) sin i = (1.52) sin30°

i = 49.7°, δ_R = (49.7) 2 – 60° = 39.4°

(b) For violet light



(1) sin 49.7° = (1.6) sin r

∴ r = 28.4°

r′ = 31.6° (r + r′ = A)

(1) sin e = (1.6) sin 31.6°

∴ e = 56°,

⇒ δ_v = i + e – A = 49.7° + 56° – 60°

= 45.7°

∴ angular width = $\delta_v - \delta_R$ = 6.3°

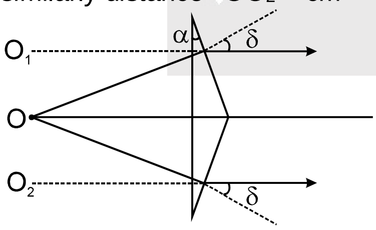
(c) $f\,\theta = 100 \times 6.3° \times \frac{\pi}{180°}$ cm = 11 cm

[Ans. (a) 49.7º, (b) 56º – 49.7º = 6.3º (c) fθ = 11cm]

25. For thin prism $\delta = (\mu - 1)\,\alpha = (1.5 - 1)\,1.8° = (0.5)\,1.8° \times \frac{\pi}{180}$ rad $= \frac{\pi}{200}$ rad

∴ Distance $OO_1 = 10 \times \frac{\pi}{200} = \frac{\pi}{20}$ cm

similarly distance OO_2 = cm



∴ $O_1O_2 = \frac{\pi}{10}$ cm.

For mirror O_1, O_2 are two point objects, at a distance of 30 cm.

Now applying mirror formula

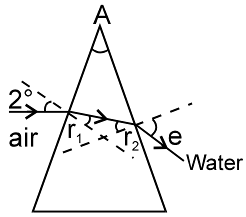
$$\frac{1}{v} + \frac{1}{-30} = \frac{1}{-10} \qquad \therefore \qquad v = -15 \text{ cm}$$

Lateral magnification

$$= -\frac{v}{u} = \frac{-(-15)}{-30} = -\frac{1}{2}$$ (– ve sign indicates inverted image)

$\therefore$ If O_1' and O_2' are the images formed, then distance between them

$O_1'O_2' = \frac{1}{2}O_1O_2 = \frac{1}{2} . \frac{\pi}{10} = \frac{\pi}{20}$ cm.

26. 

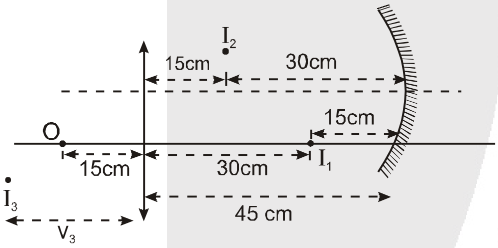
we can use sin x = x (where x is in radians)

$\sin i = \frac{3}{2}\sin r_1 \Rightarrow i = \frac{3}{2} r_1 \Rightarrow 2^o \times \frac{\pi}{180^o} = \frac{3}{2}\times\frac{\pi}{180^o}\times r_1^o$

$\Rightarrow r_1 = \frac{4}{3}^{\circ}; \Rightarrow r_2 = A - r_1 = \frac{8}{3}^{\circ}$

$\therefore \frac{3}{2}\sin r_2 = \frac{4}{3}\text{ sine} \Rightarrow e = \frac{9}{8} r_2 = 3°$

$\therefore \delta = i + e - A = 2° + 3° - 4° = 1°$

27. 

I_1 is the image of object O formed by the lens.

$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f}$ $u_1 = -15$ $f_1 = 10$

Solving we get

$v_1 = 30$ cm

I_1 acts as source for mirror

$\therefore u_2 = -(45 - v_1) = -15$ cm

I_2 is the image formed by the mirror

$\therefore \frac{1}{v_2} = \frac{1}{f_m} - \frac{1}{u_2} = -\frac{1}{10} + \frac{1}{15}$ $\therefore v_2 = -30$ cm

The height of I_2 above principal axis of lens is $= \frac{v_2}{u_2} \times 1 + 1 = 3$cm

I_2 acts a source for lens

$\therefore$ $u_3 = -(45 - v_2) = -15$ cm

$\therefore u_3 = -(45 - v_2) = -15$ cm

Hence the lens forms an image I_3 at a distance $v_3 = 30$ cm to the left of lens and at a distance

The height of I_3 above principal axis of lens is $= \left|\frac{v_3}{u_3}\right| \times 3$ cm $= 6$ cm

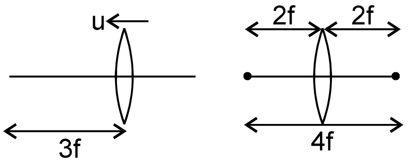
$\therefore$ required distance $= \sqrt{30^2 + 6^2} = 6\sqrt{26}$ cm

28. In the time interval described, the image is always real at any time t.

u (x-co-ordinate of object) = – [3 f – u t]

by lens formula $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\Rightarrow \frac{1}{v} = \frac{1}{f} + \frac{1}{u} = \frac{1}{f} + \frac{1}{-(3f - ut)} = \frac{-3f + ut + f}{f(3f - ut)}$ $v = \frac{f(3f - ut)}{ut - 2f}$



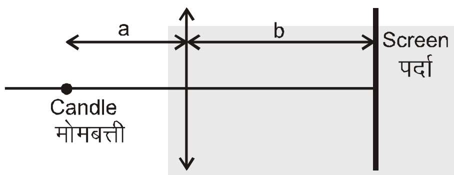
Now by differentiating the lens formula once we get

$v_{iL} = \frac{v^2}{u^2}\ v_{OL} \quad \Rightarrow \quad v_i - v_L = \frac{f^2\,(3f-ut)^2}{(ut-2f)^2} \times \frac{1}{(3f-ut)^2}\,.\,(v_O - v_L)$

$= u + \frac{f^2}{(ut-2f)^2}\ (0-u) = u\left[1 - \frac{f^2}{(ut-2f)^2}\right]$

Ans.: $\mathbf{v_i = u}\left[1-\left\{\frac{f}{ut-2f}\right\}^2\right]$

29.



a + b = D b – a = x 2 b = D + x

$b = (D + x)/2$ $a = (D - x)/2$

$\frac{1}{b} - \frac{1}{-a} = \frac{1}{f} \quad \Rightarrow \quad \frac{2}{D+x} + \frac{2}{D-x} = \frac{1}{f}$

$2.\ \frac{D-x+D+x}{D^2-x^2} = \frac{1}{f} \Rightarrow \frac{4\ D}{D^2-x^2} = \frac{1}{f} \quad \Rightarrow$ $\mathbf{f = (D^2 - x^2)/4D}$ **Ans.**]

30. From lens Maker's formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

we have $\frac{1}{0.3} = \left(\frac{3}{2}-1\right)\left(\frac{1}{R} - \frac{1}{-R}\right)$

(Here $R_1 = R$ and $R_2 = -R$)

∴ R = 0.3 m

Now applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ at air glass surface, we get,

$\frac{\mu_2-\mu_1}{R} = \frac{\frac{3}{2}-1}{0.3}$

∴ $v_1 = 2.7$ m

i.e., first image I_1 will be formed at 2.7 m from the lens. This will act as the virtual object for glass water surface. Therefore, applying $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2-\mu_1}{R}$ at glass water surface,

We have $\frac{4/3}{v_2} - \frac{3/2}{2.7} = \frac{4/3 - 3/2}{-0.3}$

∴ $v_2 = 1.2$ m

i.e., second image I_2 is formed at 1.2 m from the lens or 0.4 m from the plane mirror. This will act as a virtual object for mirror. Therefore, third real image I_3 will be formed at a distance of 0.4 m in front of the mirror after reflection from it. Now this image will work as a real object for water-glass interface. Hence, applying

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ We get $\frac{\frac{3}{2}}{v_4} - \frac{\frac{4}{3}}{-(0.8 - 0.4)} = \frac{\frac{3}{2} - \frac{4}{3}}{0.3}$

$\therefore$ $v_4 = -0.54$ m

i.e., fourth image is formed to the right of the lens at a distance of 0.54 m from it. Now finally applying the same formula for glass-air surface,

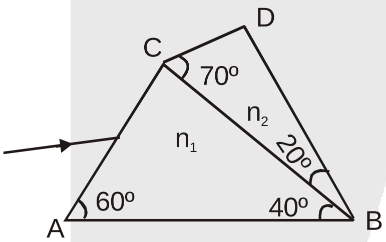
$\frac{1}{v_5} - \frac{\frac{3}{2}}{-0.54} = \frac{1 - \frac{3}{2}}{-0.3}$ $\therefore$ $v_5 = -0.9$ m

i.e., position of final image is 0.9 m relative to the lens (rightward) or the image is formed 0.1 m behind the mirror.

31. $n_1 = 1.20 + \frac{1.80 \times 10^4}{\lambda_0^{\,2}}$ and

$n_2 = 1.45 + \frac{1.80 \times 10^4}{\lambda_0^{\,2}}$

Here λ is in nm.



(i) The incident ray will not deviate at BC if

$n_1 = n_2$

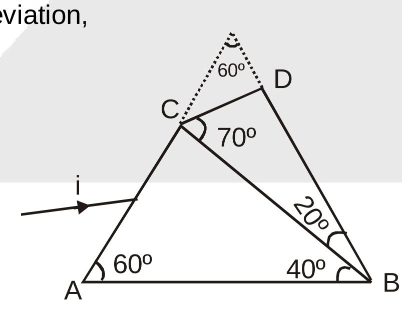
$\Rightarrow$ $1.20 + \frac{10.8 \times 10^4}{\lambda_0^{\,2}} = 1.45 + \frac{1.80 \times 10^4}{\lambda_0^{\,2}}$ $(\lambda = \lambda_0)$

$\Rightarrow$ $\frac{9 \times 10^4}{\lambda_0^{\,2}} = 0.25$

or $\lambda_0 =$

or $\lambda_0 = 600$ nm **Ans.**

(ii) The given system is a part of an equilateral prism of prism angle 60º as shown in figure. At minimum deviation,



$r_1 = r_2 = \frac{60^\circ}{2} = 30^\circ = r$ (say)

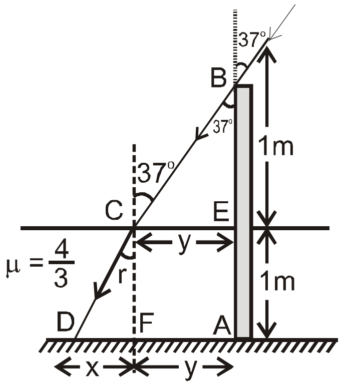
$\therefore$ $n_1 = \frac{\sin\ i}{\sin\ r}$

$\therefore$ $\sin i = n_1 . \sin 30^\circ$

$\sin i = \left\{1.20 + \frac{10.8 \times 10^4}{(600)^2}\right\}\left(\frac{1}{2}\right) = \frac{1.5}{2} = \frac{3}{4}$ ($\lambda = \lambda_0$ 600 nm)

or $i = \sin^{-1}(3/4)$ **Ans.**

32.



Let in the adjoining figure AB be the pole. So, AD represents its shadow on the bed.
In ΔBCE में,

$$\tan 37^o = \frac{CE}{BE} = \frac{y}{1}$$

$$\Rightarrow y = \frac{3}{4} \text{ m.}$$

Also due to refraction of sunray at the water surface

$$\sin 37° = \frac{4}{3} \sin r$$

$$\Rightarrow \sin r = \frac{9}{20}$$

$$\therefore \tan r \approx 0.5$$

So, In Δ CDF,

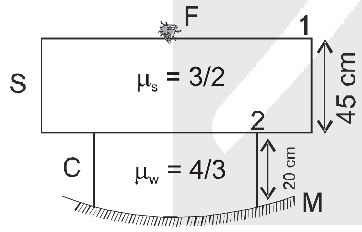
$$\text{we have } \tan r = \frac{DF}{CF}$$

$$\Rightarrow 0.5 = \frac{x}{1}$$

$$\therefore x = 0.5$$

∴ Length of the shadow AD = x + y = 1.25 m.

33.



For refraction at surface - 2,

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{4}{3v} - \frac{\frac{3}{2}}{(-45)} = 0 \quad (\because R = \infty)$$

∴ v = – 40 cm

For reflection from mirror,

u = – (20 + 40) cm = – 60 cm. f = – 20 cm

$$\therefore \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} \quad \text{gives} \quad v = -30 \text{ cm}$$

Again, for refraction from surface - 2.

u = 30 – 20 = 10 cm.

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ gives $\qquad \frac{\mu_s}{v} - \frac{\mu_w}{u} = \frac{\mu_s - \mu_w}{R}$

$\frac{3}{2v} - \frac{4}{3 \times (+10)} = 0 \qquad v = \frac{45}{4}$ cm

For refraction from surface - 1

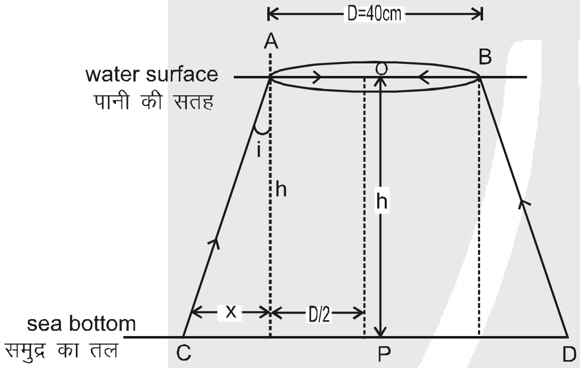
$u = \left(45 - \frac{45}{4}\right)$ cm $= \frac{3 \times 45}{4}$

$\frac{\mu_a}{v} - \frac{\mu_s}{u} = 0. \qquad (\because R = \infty)$

$\therefore \qquad v = \frac{-45}{2} = -22.5$ cm

Hence, image is formed 22.5 cm below surface 1

34. For TIR $\sin i = \frac{1}{\mu_w}$



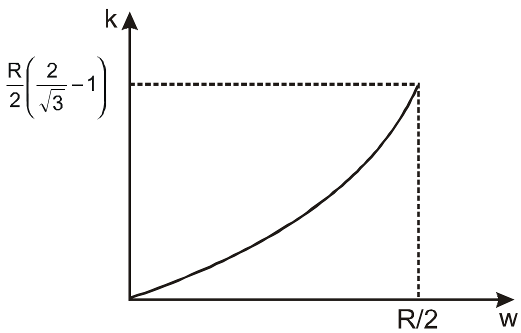
$\Rightarrow \tan i = \frac{1}{\sqrt{\mu_w^2 - 1}} = \frac{x}{h}$

$\Rightarrow x = \frac{h}{\sqrt{\mu_w^2 - 1}}$

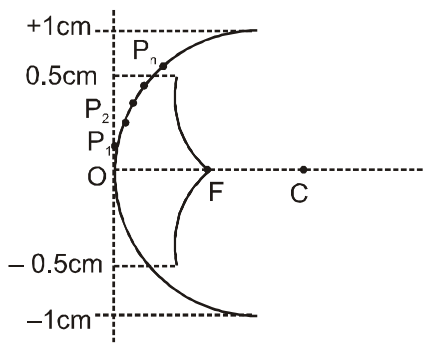
Hence required area $s = \pi\left(x + \frac{D}{2}\right)^2 = \pi\left[\frac{h}{\sqrt{\mu_w^2 - 1}} + \frac{D}{2}\right]^2$

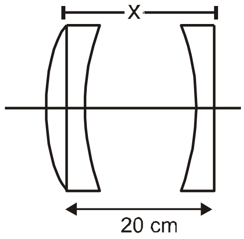
35. (a) $k = \frac{R}{2}\left[\frac{R}{(R^2 - \omega^2)^{1/2}} - 1\right]$

(b)



(c)



36. 

$f_1 = \frac{(+20)(-40)}{(20-40)}$ $\quad\quad f_2 = -40 = 40$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{x}{f_1 f_2}$

$= \frac{1}{40} + \frac{1}{-40} - \frac{20}{40 \times (-40)}$

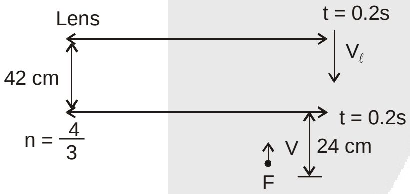
$\frac{1}{f} = \frac{1}{80}$

f = 80 cm.

37. At t = 0.2 sec , velocity of lens

V_ℓ = gt = 2m/s (downwards)

For lens the fish appears to approach with a speed of $2 + \left(1 \times \frac{3}{4}\right) = \frac{11}{4}$ m/s



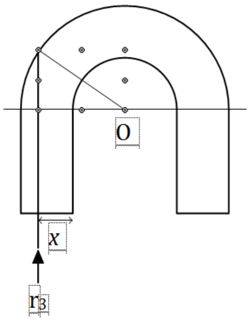
at a distance of $\left(42 + \frac{24}{\left(\frac{4}{3}\right)}\right)$ = 60 cm.

distance of image of fish from lens , V = $\frac{-60 \times 90}{-60 + 90}$ = –180 cm.

Velocity of image w.r.t. lens $\quad V_i = \left(\frac{v^2}{u^2}\right)\frac{du}{dt} = \left(\frac{-180}{-60}\right)^2 \times \frac{11}{4} = \frac{99}{4}$ m/s

velocity of image w.r.t. observer = $V_i - 2 = \frac{99}{4} - 2 = \frac{91}{4}$ m/s = 22.75 m/s (upwards)

38.



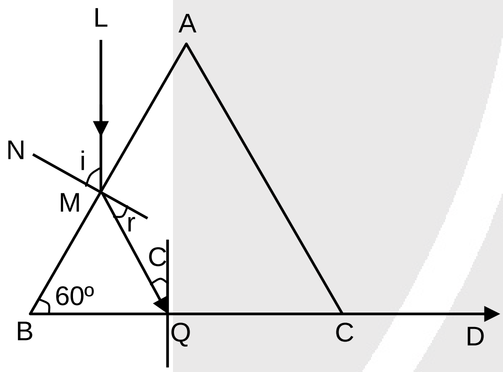
Let the intensity of beam be I_0. Flux entering through glass slab will be dII_0. Assume that any light ray up to distance *x* from the edge BC undergoes at least one total internal reflection. Then the flux going through at least one total internal reflection will be $(d - x)\, II_0$. Also

$$\frac{R+x}{R+d} = \frac{n_{water}}{n_{glass}}$$

For R/ d = 2 and n_{water} = 1.33, x = 2d/ 3

Fraction of light = 0.33

39.



1 sin i = 1.6 sin r

= 1.6 sin (60 - C)

where sin C = 1/1.6

on solving sin i = .58212

i = 35.6º