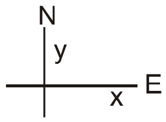
SOLUTIONS OF REALTIVE MOTION

EXERCISE # 1

PART - I

A-1. $\vec{V}_A = 15\text{m/sec}\,\hat{j}$ $\quad\quad$ $\vec{V}_B = 25\text{m/sec}(-\hat{j})$



(a) $\vec{V}_B - \vec{V}_A = -40\hat{j}$ $\quad$ i.e., 40 m/sec due south = 144 km/hr due south

(b) $0 - \vec{V}_B$ = 25 m/sec $\hat{j}$ $\quad$ i.e., 25 m/sec due north = 90 km/hr due north

(c) $\vec{V}_{M.G} = 15\hat{j} - \frac{18\times 5}{18}\hat{j} = 10\hat{j}$ $\quad$ i.e., 10 m/sec due north. = 36 km/hr due north

(d) $\vec{V}_{M.B} = 10\hat{j} - (-25)\hat{j} = 35\hat{j}$ $\quad$ i.e., 35 m/sec due north. = 126 km/hr due north

A-2. d = $\upsilon_r t$ where υ_r is the relative velocity

$\frac{3}{60\times 60} = \frac{0.075}{40+v}$ $\quad$ 120 + 3v = 270 $\quad$ 3v = 150 $\quad$ v = 50 $\frac{\text{km}}{\text{h}}$

A-3. Relative velocity of A with respect to B

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = 10 – 5 = 5 m/s

So time taken by A to meet B is –

t = $\frac{100}{v_{AB}}$ = $\frac{100}{5}$ = 20 sec.

A-4. v_r = 25 – 15 = 10m/s and a_r = –1m/s^2 so by $v^2 = u^2 + 2as$

S = $\frac{100}{2\times 1} = 50$m

B-1. $\vec{v}_A = 4\hat{i}$, $\vec{v}_B = -3\hat{j}$

$\vec{v}_{AB} = \vec{v}_A - \vec{V}_B$ = 4$\hat{i}$ – (–3$\hat{j}$) = 4$\hat{i}$ + 3$\hat{j}$

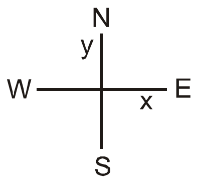
$\vec{v}_{BA} = \vec{v}_B - \vec{V}_A$ = – 3$\hat{j}$ – 4$\hat{i}$

$|\vec{v}_{AB}|$ = $\sqrt{4^2+3^2}$ = 5 unit

$|\vec{v}_{BA}| = \sqrt{3^2+4^2}$ = 5 unit.

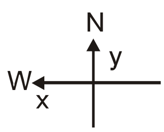
B-2. $\vec{V} = 12\hat{i} + 5\hat{j}$ $\quad\quad$ $|\vec{V}| = \sqrt{12^2+5^2}$ = 13 m/sec

tan θ = $\frac{5}{12}$ north of east

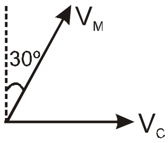


B-3. (a) $V_{G,B}$ = 0 – 20 = – 20 m/sec $\quad$ i.e., due east

(b) V_A = 15 m/sec $\quad$ V_B = 20 m/sec $\quad$ $|\vec{V}_A - \vec{V}_B| = \sqrt{20^2+15^2}$



B-4.

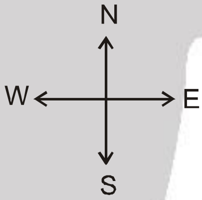


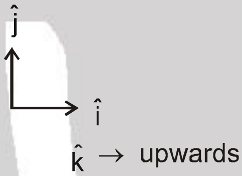
$\vec{V}_M - \vec{V}_C = 5 \sin 30\,\hat{i} + 5 \cos 30^o\,\hat{j} - 10\,\hat{i}$ $\quad \vec{V}_{MC} = \dfrac{5\sqrt{3}}{2}\hat{j} - 7.5\hat{i}$

Speed = $|\vec{V}_M - \vec{V}_C| = \sqrt{\dfrac{25\times 3}{4} + \dfrac{225}{4}} = 5\sqrt{3}$ km/hr **Ans**

$\tan\theta = \dfrac{\frac{5\sqrt{3}}{2}}{7.5} = \dfrac{1}{\sqrt{3}}$; direction θ = 30º North of west. **Ans**

B-5. $V_{ship} = \sqrt{2}\,\hat{j} + 1\hat{i} + 1\hat{k} = \hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$





C-1. $V_S + V_r = 16$ $\qquad V_S - V_r = 8$

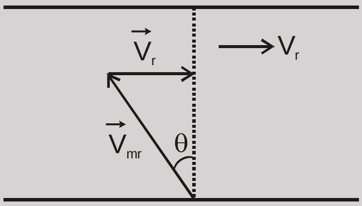
$\Rightarrow \quad V_S = 12$km/hr $\quad \Rightarrow \quad V_r = 4$ km/hr

C-2. (I) (a) $\dfrac{d}{\upsilon_{mR}} = \dfrac{1}{4}$hr

(b) Displacement along the river flow = $\upsilon_r t = \dfrac{3}{4}$ km

C-3. Given $\vec{V}_r$ = 5 m/min

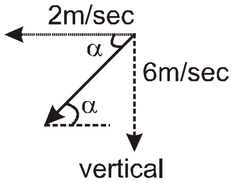
$\vec{V}_{mr}$ = 10 m/min



$\sin\theta = \dfrac{\vec{V}_r}{\vec{V}_{mr}} = \dfrac{5}{10} = \dfrac{1}{2}$

$\sin\theta = \dfrac{1}{2} \quad \Rightarrow \quad \theta = 30°$ (west of north)

D-1.

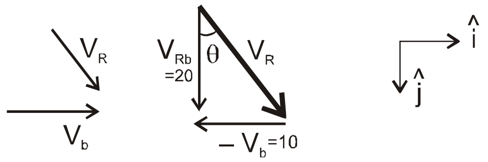


$\tan\alpha = 6/2 = 3$ $\qquad \alpha = \tan^{-1} 3$

D-2. $V_{RM} = V_R - V_M$

$V_R = V_{Rb} + V_b = 20\,\hat{j} + 10\,\hat{i}$

$\tan\theta = \dfrac{10}{20} = \dfrac{1}{2}$



$|V_r| = \sqrt{(20)^2 + (10)^2} = 10\sqrt{5}$ m/s

Making angle $\tan^{-1} 1/2$ with vertical.

D-3. $\vec{V}_m = 2\hat{i}$

$\vec{V}_r = \upsilon_x\hat{i} + \upsilon_y\hat{j}$

$\vec{\upsilon}_{r,m} = (\upsilon_x - 2)\hat{i} + \upsilon_y\hat{j} = \upsilon_y\hat{j}$

$\therefore \quad \upsilon_x = 2$ m/sec

$\vec{v}'_m = 4\hat{i}$

$\vec{\upsilon}'_{r,m} = \vec{\upsilon}_r - \vec{\upsilon}'_m$

$= (\upsilon_x - 4)\hat{i} + \upsilon_y\hat{j} = -2\hat{i} + \upsilon_y\hat{j}$

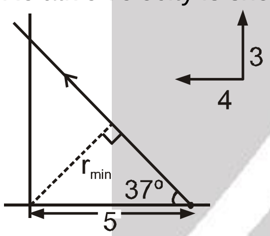
$\tan 45^\circ = \upsilon_y/2$

$\upsilon_y = 2$

so $\vec{\upsilon}_r = 2\hat{i} + 2\hat{j}$ so $\tan\theta = 1 \Rightarrow \theta = 45^\circ$

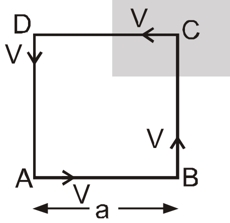
$\upsilon_r = 2\sqrt{2}$ m/sec.

E-1. Relative velocity is shown



$r_{min} = 5 \sin 37^\circ = 3$m.

E-2.



Approach velocity of A towards B = v

So, time taken = a/v.

PART - II

A-1. Acceleration of shell with respect to plane = g + a (downward)
and speed = 700 – 500 = 200 (upward)
To just escape from being hit

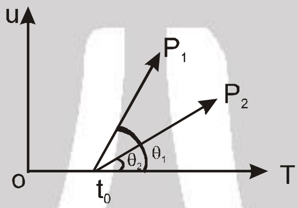
$$h > \frac{u_{rel}^2 - v_{rel}^2}{2(g+a)} \quad \Rightarrow \quad 1000 > \frac{(200)^2 - (0)^2}{2(g+a)}$$

$$g + a > 20 \quad \Rightarrow \quad a > 10 \text{ m/s}^2$$

A-2. $\upsilon_1 = 50 - gT$ $\quad\quad$ $\upsilon_2 = -50 - gT$ $\quad$ $v_r = \upsilon_1 - \upsilon_2 = 100$ m/sec

A-3. υ_1 = slope of C_1 line = constant
υ_2 = slope of C_2 line = constant
$\upsilon_1 - \upsilon_2 \neq 0$ but constant

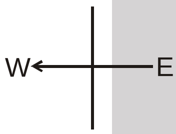
A-4. Slope of $\upsilon - t$ graph = acceleration



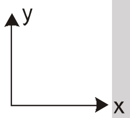
$\upsilon_1 = a_1 (t - t_0) = \tan\theta_1 t - \tan\theta_1 t_0$
$\upsilon_2 = a_2 (t - t_0) = \tan\theta_2 t - \tan\theta_2 t_0$
$\upsilon_r = \upsilon_1 - \upsilon_2 = (\tan\theta_1 - \tan\theta_2)\, t - t_0 (\tan\theta_1 - \tan\theta_2)$
So v_r contineously increases.

A-5. Initial relative velocity
$u_r = 50 - (-50) = 100$
$a_r = 20 - (20) = 0$
$s_r = \upsilon_r t + 1/2\, a_r t^2$
$100 = 100\, t$ $\quad\quad$ $t = 1$ hr
$s_A = 50\,(1) + 1/2\,(20)\,(1)^2 = 60$ km.

A-6.

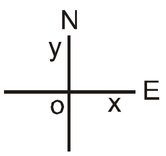


$\vec{V}_A = -500\,\hat{i} \quad \Rightarrow \quad \vec{V}_{GA} + \vec{V}_A = V_{GAS}$



$\vec{V}_{GA} = 1500\,\hat{i} \quad \Rightarrow \quad 1500\,\hat{i} - 500\,\hat{i} = 1000\,\hat{i}$

B-1.



$\vec{V}_r = 50(-\hat{j}) - 50\hat{i} = 50(-\hat{i} - \hat{j})$
i.e., in south west

B-2. $\vec{V}_{12} = \vec{V}_1 - \vec{V}_2$

$|\vec{V}_{12}| = \sqrt{V_1^2 + V_2^2 - 2V_1V_2\cos\theta}$

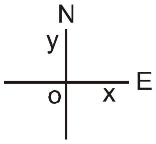
If $\cos\theta = -1$

$|\vec{V}_{12}|_{max} = \sqrt{V_1^2 + V_2^2 + 2V_1V_2}$

$|\vec{V}_{12}|_{max} = (V_1 + V_2)$

So $|\vec{V}_{12}|$ is maximum when $\cos\theta = -1$ and $\theta = \pi$

B-3.



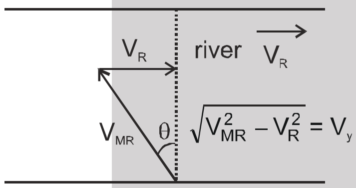
$\vec{V}_1 = 10\hat{i}$

$\vec{V}_2 = v\sin 30\hat{i} + v\cos 30\hat{j} = \dfrac{\upsilon}{2}\hat{i} + \dfrac{\upsilon\sqrt{3}}{2}\hat{j}$

$\vec{V}_2 - \vec{V}_1 = \left(\dfrac{\upsilon}{2} - 10\right)\hat{i} + \dfrac{\upsilon\sqrt{3}}{2}\hat{j} = \dfrac{\upsilon\sqrt{3}}{2}\hat{j}$

$\therefore \quad \upsilon/2 - 10 = 0 \quad$ or $\quad \upsilon = 20$

C-1. 15 min = 1/4 hr.



$t = \dfrac{d}{V_y} \Rightarrow \quad \dfrac{1}{4} = \dfrac{1}{\sqrt{V_{MR}^2 - V_R^2}} \quad = \dfrac{1}{4} = \dfrac{1}{\sqrt{5^2 - V_R^2}}$

$\Rightarrow \quad V_R = 3$ km/h

C-2. $V_{boat,\ river} = 9$ km/hr.

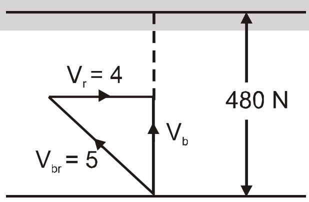
$V_{river,\ ground} = 12$ km/hr.

$V_{boat,\ ground} = (12\hat{i} + 9\hat{j})$ km/hr

$V_{boat,\ ground} = \sqrt{12^2 + 9^2} = 15$ km/hr.

C-3. $V_b = \sqrt{5^2 - 4^2} = 3$ m/s

$t = \dfrac{480}{3} = 160$ s



C-4. Let velocity of wind be

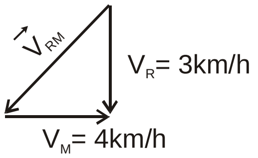
$(240 + v_1)\ 1/2 = 150 \quad \Rightarrow \quad v_1 = 60$

and $v_2 \times 1/2 = 40 \quad \Rightarrow \quad v_2 = 80$

so $v_{air} = \sqrt{v_1^2 + v_2^2} = 100$ km/hr

$\tan\theta = \dfrac{v_1}{v_2} \qquad \theta = 37^o$ west of south

D-1.



$V_{RH} = \sqrt{V_R^2 + V_M^2} = \sqrt{3^2 + 4^2} = 5$ km/h **Ans.**

D-2. $\vec{V}_r = \upsilon_y \hat{j}$

$\vec{\upsilon}_m = 5\hat{i}$

$\vec{V}_r - \vec{V}_m = (-5)\hat{i} + \upsilon_y \hat{j}$

tan θ = 1 = $\upsilon_y/5$

so υ_y = 5 km/hr

D-3. $\vec{V}_r = 10\hat{j}$

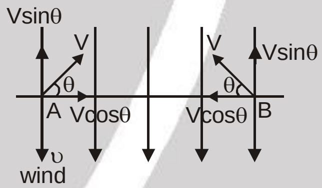
$\vec{V}_c = \upsilon \hat{i}$

$\vec{V}_r - \vec{V}_c = 10\hat{j} - \upsilon\hat{i}$

$|\vec{V}_r - \vec{V}_c| = \sqrt{10^2 + \upsilon^2} = 20$

$\upsilon = 10\sqrt{3}$

D-4. For no drift :



V sinθ = υ

$\sin\theta = \dfrac{\upsilon}{V}$ ∴ $t = t_{AB} + t_{BA}$

$t = \dfrac{2\ell}{V\cos\theta} = \dfrac{2\ell}{V\sqrt{1-\dfrac{\upsilon^2}{V^2}}}$ ⇒ $t = \dfrac{2\ell}{\sqrt{V^2 - \upsilon^2}}$

E-1. A and B collide if $\dfrac{\vec{r}_B - \vec{r}_A}{|\vec{r}_B - \vec{r}_A|} = \pm \dfrac{\vec{V}_B - \vec{V}_A}{|\vec{V}_B - \vec{V}_A|}$

$= \dfrac{4\hat{i} + 4\hat{j}}{4\sqrt{2}} = \pm \dfrac{(x-3)\hat{i} - 4\hat{j}}{|[(x-3)^2 + 4^2]^{1/2}|}$

By comparison

x – 3 = – 4 ⇒ x = –1.

ALTERNATE:

$\vec{r}_{AB} = -4\hat{i} - 4\hat{j}$

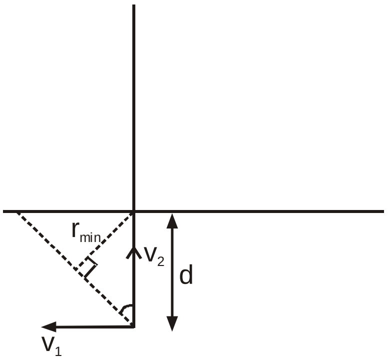
$\vec{v}_{AB} = (3-x)\hat{i} + 4\hat{j}$

to collide angle between $\vec{r}_{AB}$ & $\vec{v}_{AB}$ should be π i.e.

$= \dfrac{-4}{3-x} = \dfrac{-4}{4} - 1$ ⇒ x = – 1

E-2.

Velocity of B w.r.t. A



$\tan\theta = v_1/v_2$

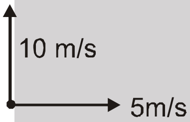
$r_{min} = d \sin\theta = d \cdot \frac{v_1}{\sqrt{v_1^2 + v_2^2}}$.

PART - III

1. In all cases, angle between velocity and net force (in the frame of observer) is in between 0° and 180° (excluding both values, in that path is straight line).

2. (A) $V_{BA} = 10 + 10 = 20$
so distance b/w B and A in 2sec. = 2 × 20 = 40 m

(B)

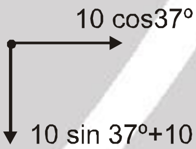


$\vec{V}_{BA} = 5\hat{i} - 10\hat{j}$

$\Rightarrow \quad |V_{BA}| = \sqrt{25+100} = 5\sqrt{5}$

Distance between A and B in 2 sec. = $10\sqrt{5}$ m

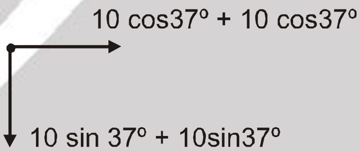
(C) $\vec{V}_{BA}$:-



so $|\vec{V}_{BA}| = 8\sqrt{5}$

so distance between A and B in 2 sec. = 2 × $8\sqrt{5}$ = $16\sqrt{5}$

(D) $\vec{V}_{BA}$: -



so $|\vec{V}_{BA}| = 20$

so distance between A and B in 2 sec. = 2 × 20 = 40 m.

EXERCISE # 2

PART - I

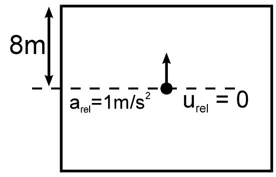
1. Relative velocity between either car (1st or 2nd) and 3rd car = u + 30
where u = velocity of 3rd car
Relative Displacement = 5 km
Time interval = 4 min.

$\therefore \quad u + 30 = \frac{5}{4}$ km/min $= \frac{5 \times 60}{4}$ km/h = 75 $\Rightarrow$ u = 45 km/h

2. $V_{rel} = \frac{S_{rel}}{t} = \frac{1000}{100} = 10$ m/s.

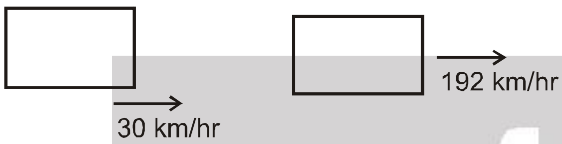
$\therefore \quad V_S - V_B = 10 \quad \Rightarrow \quad V_S = 10 + V_B = 10 + 10 = 20$ m/s. **Ans.**

3. Relative to lift initial velocity and acceleration of coin are 0 m/s and 1 m/s^2 upwards



$\therefore \quad 8 = \frac{1}{2}(1)\, t^2 \quad$ or $\quad t = 4$ second

4.



30 km/h = $\frac{25}{3}$ m/s, 192 km/hr = $\frac{160}{3}$ m/s

Muzzle speed = velocity of bullet w.r.t. revolver

= velocity of bullet w.r.t. van

$150 = V_b - V_v$

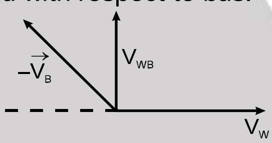
$150 = V_b - \frac{25}{3} \quad \Rightarrow \quad V_b = \frac{475}{3}$ m/s w.r.t. ground

Now speed with which bullet hit thief's car

= velocity of bullet w.r.t car = $V_{bc} = V_b - V_c$

$= \frac{475}{3} - \frac{160}{3} = \frac{315}{3} = 105$ m/s **Ans.**

5. Flag will flutter in the direction of wind with respect to bus.



and $\vec{V}_{WB} = \vec{V}_W - \vec{V}_B = \vec{V}_W + (-\vec{V}_B)$ (Addition of two vector always lies between them)

$(-\vec{V}_B)$ must lie in any direction between north & west. So bus will be moving in any direction between south east. **(C)**

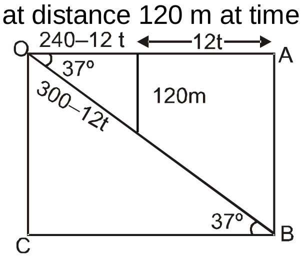
6. Let $\hat{i}$ and $\hat{j}$ be unit vectors in direction of east and north respectively.

$\therefore \quad \vec{V}_{DC} = 20\hat{j}$, $\vec{V}_{BC} = 20\hat{i}$ and $\vec{V}_{BA} = -20\hat{j}$

$\therefore \quad \vec{V}_{DA} = \vec{V}_{DC} + \vec{V}_{CB} + \vec{V}_{BA} = 20\hat{j} - 20\hat{i} - 20\hat{j} = -20\hat{i}$

$\therefore \quad \vec{V}_{DA} = -20\hat{i}$

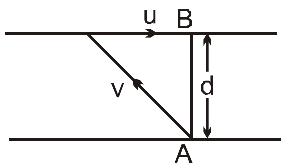
7. Position of P and Q when they are at distance 120 m at time t after motion start



Velocity of Q along y-direction is initially 12cos37º.

Later on it increases it to 15cos37º = 12 m/s
Earlier Q was travelling with less velocity along y direction. So, it will reach point O later.
So P reaches first at point O

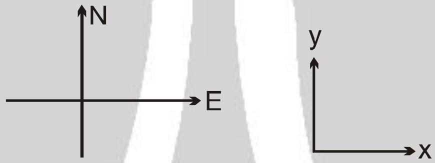
8. Let v = man's speed in still water and
u = speed of river water



$\Rightarrow \quad t = \dfrac{d}{\sqrt{v^2 - u^2}} \qquad \Rightarrow \quad v^2 - u^2 = \dfrac{d^2}{t^2}$

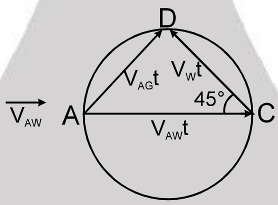
$\Rightarrow \quad T = \dfrac{d}{v+u} \qquad \Rightarrow \quad (v + u)^2 = \dfrac{d^2}{T^2} \quad \Rightarrow \quad \dfrac{(v+u)^2}{v^2 - u^2} = \dfrac{t^2}{T^2}$

$\Rightarrow \quad \dfrac{v+u}{v-u} = \dfrac{t^2}{T^2}$



$\Rightarrow \quad \dfrac{(v+u)+(v-u)}{(v+u)-(v-u)} = \dfrac{t^2+T^2}{t^2-T^2} \qquad \Rightarrow \quad \dfrac{v}{u} = \dfrac{t^2+T^2}{t^2-T^2}$

9. In absence of wind A reaches to C and in presence of wind it reaches to D in same time so wind must deflect from C to D so wind blow in the direction of CD

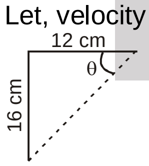


$\vec{V}_{AG} = \vec{V}_{AW} + \vec{V}_{WG} \qquad \Rightarrow \qquad \vec{V}_{AG}\ t = \vec{V}_{AW}t + \vec{V}_{WG}\ t$

AC = $\vec{V}_{AW}t$

CD = $\vec{V}_{WG}t$

10. $V_{R/G(x)} = 0$, $V_{R/G(y)} = 10$ m/s
Let, velocity of man = v



$\tan\theta = \dfrac{16}{12} = \dfrac{4}{3}$

Then, $v_{R/man} = v$ (opposite to man)
For the required condition :

$\tan\theta = \dfrac{V_{R/M(y)}}{V_{R/M(x)}} = \dfrac{10}{v} = \dfrac{4}{3} \qquad \Rightarrow \qquad V = \dfrac{10 \times 3}{4} = 7.5$ **Ans.**

11. $T = \frac{2(5)}{10} = 1\text{sec}$

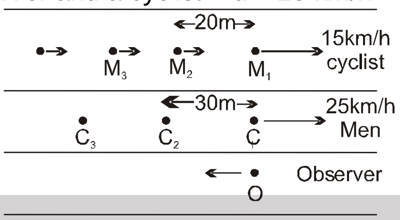
h = 50 + 3(1) = 53m

PART - II

1. Let u = speed of observer.

Relative velocity between observer and a man = u + 15 km/h.

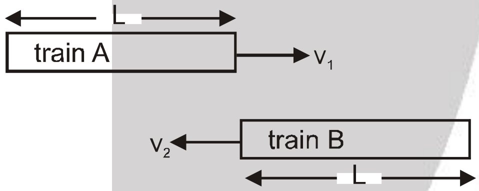
Relative velocity between observer and a cyclist = u + 25 km/h.



Hence, to for a man and a cyclist to meet simultaneously

$$\frac{20\text{m}}{(u+15)\text{km/h}} = \frac{30\text{m}}{(u+25)\text{km/h}} \Rightarrow u = 5 \text{ km/h}$$

2.



$t_1 = 3 = \frac{2L}{v_1+v_2} \Rightarrow v_1 + v_2 = \frac{2L}{3}$ ....(i)

$t_2 = 2.5 = \frac{2L}{1.5v_1+v_2}$ $1.5\,v_1 + v_2 = \frac{4L}{5}$ ....(ii)

by (i) and (ii)

$$v_1 = \frac{4L}{15}\ ;\ v_2 = \frac{2L}{5}$$

Now, $t_3 = \frac{2L}{|v_1 - v_2|} = \frac{2L}{2L/15} = 15$ sec.

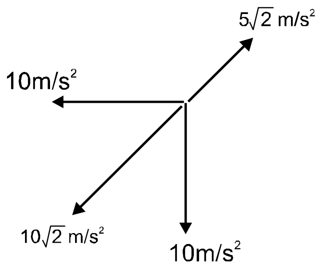
3. B catches C in time t then $t = \frac{d}{u-10}$

Seperation by this time has increased by 'd' between A and C hence

$(10 - 5) \times \frac{d}{(u-10)} = d$

u = 15 m/s

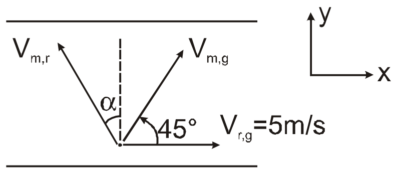
4.



w.r.t. man

5. $\vec{V}_{m,g} = \vec{V}_{m,r} + \vec{V}_{r,g}$

As resulting velocity $\vec{V}_{m,g}$ is at 45° with river flow



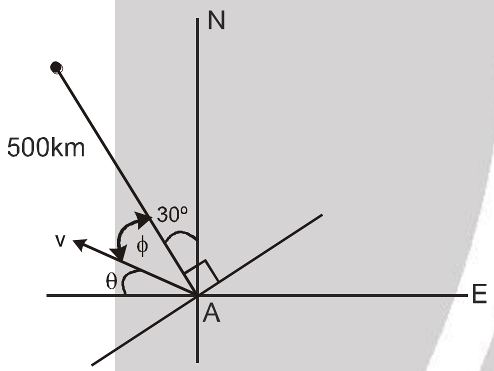
i.e. $V_{r,g} - V_{m,r} \sin\alpha = V_{m,g} \cos 45°$ ....(1)

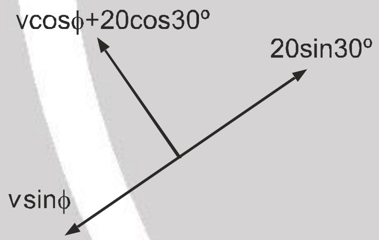
and $\dfrac{60m}{V_{mr}\cos\alpha} = 6$ sec. ....(2)

Solving (1) & (2)

$V_{m,r} = 5\sqrt{5}$ m/s

6.





Velocity of plane w.r.t. ground

Velocity of plane w.r.t ground is along AB so perpendicular component (to line AB) of velocity is zero.

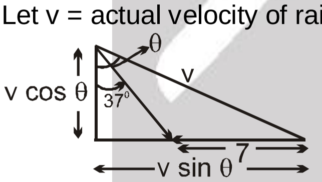
$v \sin\phi = 20 \sin 30°$

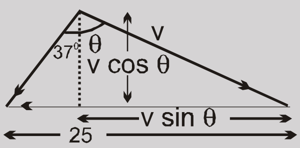
$v \sin\phi = 20 \sin 30°$

$\sin\phi = \dfrac{10}{150} = \dfrac{1}{15}$

$\phi = \sin^{-1}\left(\dfrac{1}{15}\right)$

7. Let v = actual velocity of rain and θ = its angle with vertical :





In fig. (A)

$v \sin\theta = 7 + v \cos\theta . \tan 37° = 7 + 3/4\, v \cos\theta$

$\Rightarrow$ $4 v \sin\theta - 3 v \cos\theta = 28$ ....(1)

In fig. (B)

$25 = v \sin\theta + v \cos\theta . \tan 37° = v \sin\theta + 3/4\, v \cos\theta$

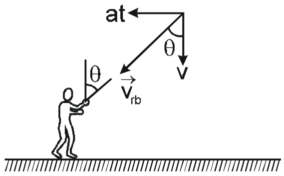
$\Rightarrow$ $4 v \sin\theta + 3 v \cos\theta = 100$ ....(2)

Solving (1) and (2)

$v = 20$m/s and $\theta = 53°$

8. At any time t, rain will appear to the boy as shown in picture.

$\tan\theta = at/v$



$\vec{v}_{rb}$ = velocity of rain w.r.t. boy

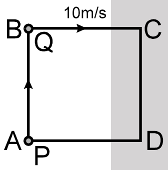
Boy should hold his umbrella at an angle θ from the vertical

$\therefore \quad \tan\theta = \dfrac{at}{v} \qquad \sec^2\theta \dfrac{d\theta}{dt} = \dfrac{a}{v}$

$$\Rightarrow \quad \frac{d\theta}{dt} = \frac{a}{v\sec^2\theta} = \frac{a}{v[1+\tan^2\theta]} = \frac{a}{v\left[1+\dfrac{a^2t^2}{v^2}\right]} = \frac{av}{v^2+a^2t^2} = \frac{2\times 2}{4+4t^2} = \frac{1}{1+t^2}$$

$\dfrac{d\theta}{dt} = \dfrac{1}{1+t^2}$ **Ans.** $\dfrac{d\theta}{dt} = \dfrac{1}{1+t^2}$

9.



a = side of square = 8m

They meet when Q displace 8 × 3 m more than P

$\Rightarrow$ Relative displacement = Relative velocity × time.

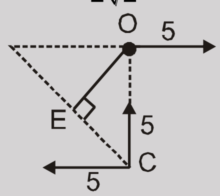
8 × 3 = (10 – 2) t $\Rightarrow$ t = 3 sec **Ans. 3 sec**

10. The positions of the persons at 12:00 PM will be as shown in figure. Such that OC = 5 × $\dfrac{3}{2}$ km = $\dfrac{15}{2}$ km

Velocity of man at C with respect to man at O will be along CE such that $\tan\theta = 5/5 = 1$

$\therefore \quad \theta = 45°$

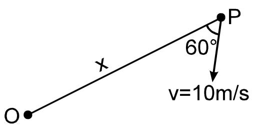
$\therefore$ Least distance = OE = OC sin 45° = $\dfrac{15}{2\sqrt{2}}$ km **Ans.**



Time taken = $\dfrac{CE}{5\sqrt{2}} = \dfrac{15}{2\sqrt{2}\times 5\sqrt{2}} = \dfrac{3}{4}$ hr.

So, the person will be closest at 12:45 PM **Ans.**

11.



Velocity of approach of P and O is $\rightarrow \dfrac{dx}{dt} = v\cos 60° = 5$ m/s

It can be seen that velocity of approach is always constant.

$\therefore$ P reaches O after = $\dfrac{100}{5}$ = 20 sec.

PART - III

1. Relative Intial velocities

$u_r = 20 - (0) = 20$ m/s

Relative acceleration

$a_r = 0$

Relative velocity between them after time

$v_r = u_r + a_r .t$

= 20m/s

= constant

$\Rightarrow$ (A) is correct

$\Rightarrow$ Since they are thrown from same height

$\Rightarrow$ Speed is same after reaching ground

$\Rightarrow$ Same KE when they hit the ground

$\Rightarrow$ (B) is correct

The time taken by the first stone to come to same height from where it was thrown.

$$\frac{2u}{g} = \frac{2\times 20}{10}$$

$\therefore$ Time interval between two stone when both are at A and going downwards = 4 – 2 = 2s.

Since, relative velocity is Constant between them. So time interval between their hitting the ground = 2 s.

$\Rightarrow$ (C) is correct

Option (D) is obvious from conservation of energy.

2. For first case (when lift is ascending with an acceleration a)

$$t_1 = \frac{2v}{g+a} \qquad \text{...........(i)}$$

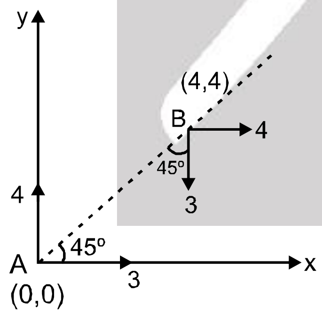
for second case (when lift is descending with an acceleration a)

$$t_2 = \frac{2v}{g-a} \qquad \text{...........(ii)}$$

on solving equation (i) and (ii) we get

$$V = \frac{gt_1t_2}{t_1+t_2} \quad \& \quad a = g\left(\frac{t_2-t_1}{t_1+t_2}\right)$$

3.



$v_{BA} = v_B - v_A$

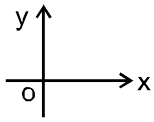
$= [4\hat{i} - 3\hat{j}] - [3\hat{i} + 4\hat{j}] = \hat{i} - 7\hat{j}$

v_{app} = 4 cos45º + 3 cos45º + 3 cos45º – 4 cos45º

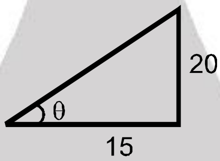
= 6 cos45º

= $3\sqrt{2}$ m/s

4. $u_T = 15$ m/s (velocity of truck)

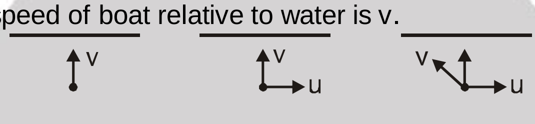


Range = 60 m
Range = distance travelled by truck $u_T \times T$
$60 = 15 \times T \Rightarrow T = 4s$ = Time of flight (of ball)
$T = \frac{2u_y}{g}$ where u_y = Vertical component of ball's vel. {wrt ground}
$\therefore \frac{T \times g}{2} = u_y$ i.e, $u_y = \frac{4 \times 10}{2} = 20$ m/s
Now vel. of truck = u_x = horizontal component of ball's vel. (wrt ground)
$\Rightarrow u_x = 15$ m/s (wrt ground)
This is because both cover same horizontal distance in same time with constant velocity along horizontal.
Now, velocity, of vall wrt track = V_{BT} Then $V_{BTx} = V_{Bx} - V_{Tx}$
i.e., velocity ball wrt truck (along x axis) = velocity of ball (wrt earth, along x axis) – velocity of truck (along x axis)
$\therefore V_{BT\,x} = 15 - 15 = 0$
Similarly, $V_{BT\,y} = V_{By} - V_{Ty} = 20 - 0$
$V_{BTy} = 20$ m/s.
$\Rightarrow \vec{V}_{BT} = \vec{V}_{BTx} + \vec{V}_{BTy} = 0 + 20 \text{ m/s } \hat{j}$
$\Rightarrow$ velocity of ball wrt truck = 20 m/s upwards
velocity of ball, $\vec{V} = \vec{V}_x + \vec{V}_y$ $\vec{V} = 15\hat{i} + 20\hat{j}$



$\tan\theta = 4/3$ $\theta = 53^o$
$\therefore$ Speed = $|\vec{V}| = \sqrt{15^2 + 20^2} = 5\sqrt{3^2 + 4^2} = 25$ m/s
i.e., vel. of ball (wrt ground) = 25 m/s at an angle of 53º with the horizontal (as shown)

5. Speed of river is u and speed of boat relative to water is v.



Boat A river frame Boat A ground frame Boat B ground frame

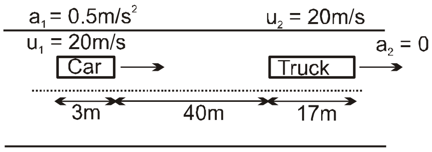
Speed of boat A observed from ground = $\sqrt{u^2 + v^2}$
Speed of boat B observed from ground = $\sqrt{v^2 - u^2}$
From river frame, speed of boat A and B will be same.

6. (A) Absolute velocity of ball = 40 m/s (upwards)
$h_{max} = h_i = f_f = 10 + \frac{(40)^2}{2 \times 10}$
h = 90 m
(B) Maximum height from lift = $\frac{(30)^2}{2 \times 10} = 45$ m
(C) The ball unless meet the elevator again when displacement of ball = displacement of lift
$40\,t - 1/2 \times 10 \times t^2 = 10 \times t \Rightarrow t = 6s.$
(D) with respect to elevator V_{ball} = 30 m/s downward $\therefore V_{ball}$ with respect to ground = 30 – 10 = 20 m/s

PART - IV

1. Displacement of car relative to truck

x_r = 40 + 17 + 3 + 40 = 100 m.

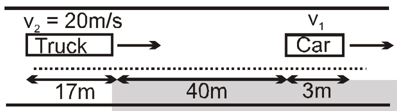


t = 0

Initilly

Relative initial velocity between car and truck

u_r = 20 – 20 = 0



t = t

Finally

Relative acceleration between car and truck

a_r = 0.5 – 0 = 0.5 m/s^2

Let required time = t .

$\therefore$ II equation of motion

$x_r = u_r.t + 1/2\ a_r.t^2$

$\Rightarrow$ 100 = 0 + 1/2 × 0.5 × t^2

$\Rightarrow$ t = 20 sec.

2. Distance travelled by car

$x_c = ut + 1/2\ at^2$

$= 20 \times 20 + 1/2 \times 0.5 \times 20^2 = 500\text{m}$

3. Final speed of the car

= u + at

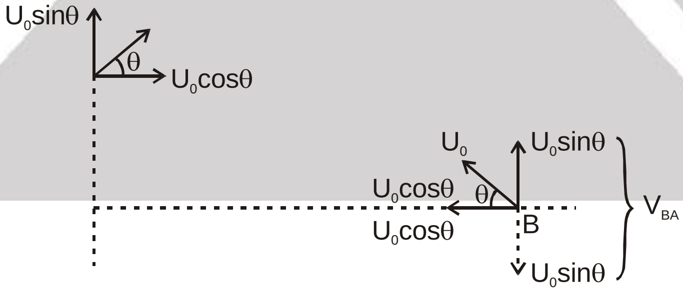
= 20 + 0.5 × 20 = 30m/s.

4 & 5. The path of a projectile as observed by other projectile is a straight line.

$V_A = u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$. $V_{AB} = (2u\cos\theta)\,\hat{i}$

$V_B = -u\cos\theta\,\hat{i} + (u\sin\theta - gt)\,\hat{j}$.

$a_{BA} = g - g = 0$



The vertical component $u_0 \sin\theta$ will get cancelled. The relative velocity will only be horizontal which is equal to $2u_0 \cos\theta$.

Hence, B will travel horizontally towards left with respect to A with constant speed $2u_0 \cos\theta$ and minimum distance will be h.

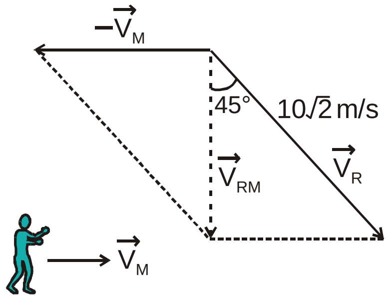
6. Time to attain this separation will be $\dfrac{S_{rel}}{V_{rel}} = \dfrac{\ell}{2u_0\cos\theta}$.

7 to 9. In the first case :

From the figure it is clear that

$\vec{V}_{RM}$ is 10 m/s downwards and

$\vec{V}_{M}$ is 10 m/s towards right.



In the second case :

Velocity of rain as observed by man becomes times in magnitude.

$\therefore$ New velocity of rain

$\vec{V}_{R'} = \vec{V}_{R'M} + \vec{V}_M$

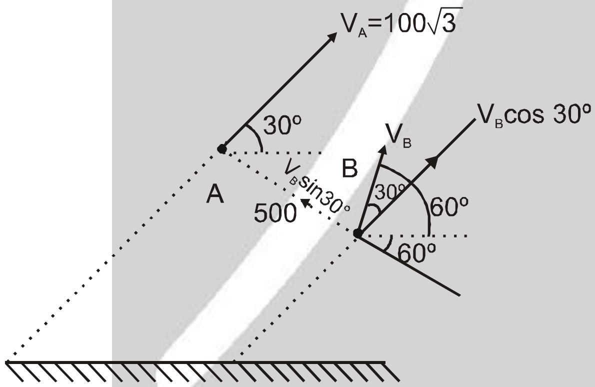
$\therefore$ The angle rain makes with vertical is $\tan\theta = \dfrac{10}{10\sqrt{3}}$

or $\theta = 30°$

$\therefore$ Change in angle of rain = 45 – 30 = 15°.

EXERCISE # 3
PART - I

1.



For relative motion perpendicular to line of motion of A

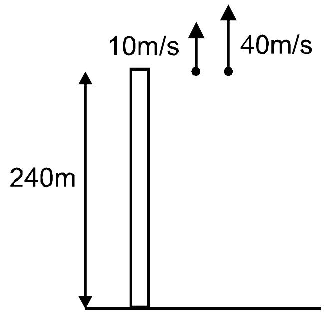
$V_A = 100\sqrt{3} = V_B \cos 30°$

$\Rightarrow$ $V_B = 100$ m/s

$t_0 = \dfrac{50}{V_B \sin 30°} = \dfrac{500}{200 \times \frac{1}{2}} = 5\,\text{sec}$ **Ans.**

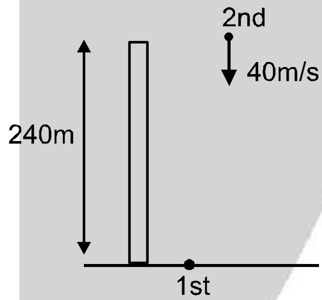
PART - II

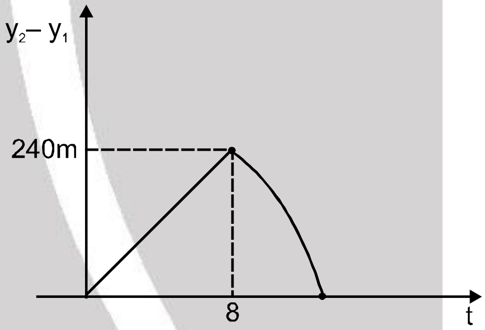
1.



$-240 = 10\,t - 1/2 \times 10t^2$
$5t^2 - 10t - 240 = 0$
$t^2 - 2t - 48 = 0$
$t^2 - 8t + 6t - 48 = 0$
$t = 8, -6$

The first particle will strike ground at 8 seconds upto 8 second, relative velocity is 30 m/s and relative acceleration is zero. After 8 second magnitude of relative velocity will increase upto 12 seconds when second particle strikes the ground.





2. $\vec{V}_r = 40\hat{i} + 50\hat{j}$

$\vec{r}_r = -80\hat{i} - 150\hat{j}$

$t_{min} = \dfrac{|\vec{V}_r \cdot \vec{r}_r|}{|\vec{V}_r|^2} = \dfrac{10700}{4100} = \dfrac{107}{41} = 2.6$ sec.

3. $X_1 = -3t^2 + 8t + 10$

$\vec{v}_1 = (-6t + 8)\hat{i} = 2\hat{i}$

$Y_2 = 5 - 8t^3$

$\vec{v}_2 = -24t^2\hat{j}$

$\sqrt{v} = |\vec{v}_2 - \vec{v}_1| = |-24\hat{j} - 2\hat{i}|$

$\sqrt{v} = \sqrt{24^2 + 2^2}$

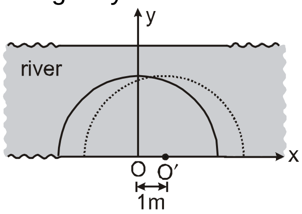
$v = 580$

HIGH LEVEL PROBLEMS (HLP)

1. **Method - 1**

If the river is still, the man will be at a distance 3 meters from origin O after 1 second. The locus of all the point where man can reach at t = 1 second is a semicircle of radius 3 and centre at O (dotted semicircle shown in figure).

The river flows to right with a speed 1 m/s. Hence there shall be additional shift in position by 1 m/s × 1 sec = 1m towards right. Hence the locus of all points giving possible position after one second will be the dotted semicircle shifted to right by 1 m as shown in figure.



Hence locus all the points where the man can be at t = 1 sec. is a semicircle of radius 3 and centre at O′ (1 m, 0 m)

$\therefore$ Equation of locus of all the points is

$(x - 1)^2 + (y - 0)^2 = 3^2$

or $(x - 1)^2 + y^2 = 9$

Method - 2

Let the relative velocity of the man make angle 'θ' with the x-axis.

Then at time 't' :

$x = (3\cos\theta + 1)\, t$

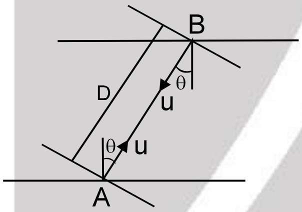
and $y = 3\sin\theta\, t$

$\Rightarrow$ $(x - t)^2 + y^2 = (3\cos\theta)^2 t^2 + (3\sin\theta)^2 t^2$

$(x - t)^2 + y^2 = 9t^2$

at $t = 1$ sec. the required equation is $(x - 1)^2 + y^2 = 9$.

2. (a)

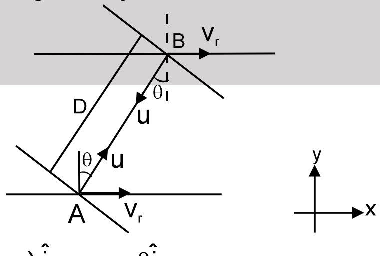


Velocity of approach along line joing them

$= u - V_r \sin\theta + u + V_r \sin\theta = 2u$

So time $t = \dfrac{D}{2u}$ **Ans.**

(b) For path to be at right angle to each other, their velocity vector with respect to ground must be right angle. Taking axis system as shown



$\vec{V}_A = (u\sin\theta + v_r)\,\hat{i} + u\cos\theta\,\hat{j}$

$\vec{V}_B = (v_r - u\sin\theta)\,\hat{i} - u\cos\theta\,\hat{j}$

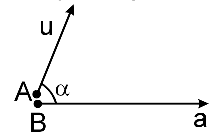
For $\vec{V}_A$ & $\vec{V}_B$ to perpendicular $\vec{V}_A \cdot \vec{V}_B = 0$

$(u\sin\theta + v_r)(v_r - u\sin\theta) - u^2\cos^2\theta = 0$

$\Rightarrow$ $v_r = u$

Speed of river should be equal to the speed of the swimmer relative to river. Ans.

3. Let particle A is moving with uniform velocity and particle B is moving with constant acceleration.



At any time, velocity of B with respect to A is

$\vec{V}_{BA} = \vec{V}_{BG} - \vec{V}_{AG} = (\vec{a})t - \vec{u}$

$V_{BA} = \left| \vec{V}_{BA} \right| = \sqrt{u^2 + a^2t^2 - 2uat\cos\alpha}$

For relative velocity to be least

$$\frac{dv_{BA}}{dt} = 0 \quad \Rightarrow \quad 2a^2t - 2ua\cos\alpha = 0 \quad \Rightarrow \quad t = \frac{u\cos\alpha}{a}$$

$(V_{BA})_{least} = \sqrt{u^2 + u^2\cos^2\alpha - 2u^2\cos^2\alpha}$

$(V_{BA})_{least} = u \sin\alpha$

At time $t = \dfrac{u\cos\alpha}{a}$

distance between the two particles

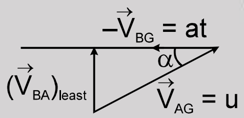
$$s_{BA} = \sqrt{x^2 + y^2} = \sqrt{\left(-u\cos\alpha t + \frac{1}{2}at^2\right)^2 + (u\sin\alpha t)^2}$$

$$= \sqrt{\left(-\frac{u^2\cos^2\alpha}{a} + \frac{u^2\cos^2\alpha}{2a}\right)^2 + \left(\frac{u^2\sin\alpha\cos\alpha}{a}\right)^2}$$

$$= \sqrt{\frac{u^4\cos^4\alpha}{4a^2} + \frac{u^4\sin^2\alpha \quad \cos^2\alpha}{a^2}} = \frac{u^2\cos\alpha}{2a}\sqrt{\cos^2\alpha + 4\sin^2\alpha}$$

$$s_{BA} = \frac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha}$$

Alternative Method :



$(V_{BA})_{least} = u \sin\alpha$

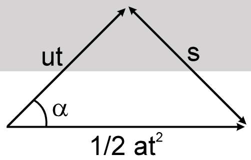
relative velocity of B w.r.t. A is least at time t which is given by

$u\cos\alpha = at$

$t = \dfrac{u\cos\alpha}{a}$

for distance between A and B at the time t

By cosine formula

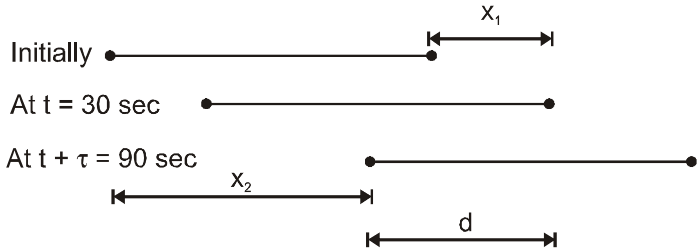


$$s = \sqrt{u^2t^2 + \left(\frac{1}{2}at^2\right)^2 - 2(ut)\left(\frac{1}{2}at^2\right)\cos\alpha}$$

$$= \frac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha} \quad \text{using } t = \frac{u\cos\alpha}{a}$$

Ans. (a) $(v_{rel})_{least} = u \sin\alpha$ (b) separation = $\dfrac{u^2\cos\alpha}{2a}\sqrt{1 + 3\sin^2\alpha}$

4. In the frame of train, the distance between A and B remains constant which is equal to ℓ = 350. Hence, in the frame of train the distance between two events is equal to AB = ℓ = 350 m.



Distance between these points with respect to ground

$$d = \ell + x_1 - x_2$$

$$= \ell + \frac{1}{2}\omega t^2 - \frac{1}{2}\omega(t+\tau)^2$$

$$= \ell - \omega\tau\left(t+\frac{\tau}{2}\right) \simeq 240 \text{ m}$$

Time between these two events = τ = 60 sec.

Velocity of frame V = $\frac{240}{60}$ = 4 m/s.

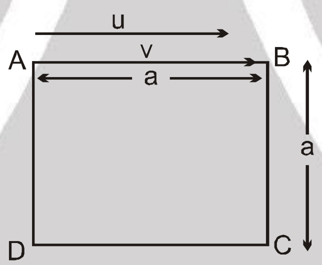
5. (a) Relative velocity along AB $\rightarrow$ u + v

BC $\rightarrow \sqrt{v^2 - u^2}$

CD $\rightarrow$ v – u

DA $\rightarrow \sqrt{v^2 - u^2}$

$\therefore$ time taken



$$t = \frac{a}{v+u} + \frac{a}{\sqrt{v^2-u^2}} + \frac{a}{v-u} + \frac{a}{\sqrt{v^2-u^2}}$$

$$= \frac{2a}{\sqrt{v^2-u^2}} + a\left(\frac{1}{v+u}+\frac{1}{v-u}\right)$$

$$= \frac{2a}{\sqrt{v^2-u^2}} + a\left(\frac{v+u+v-u}{v^2-u^2}\right)$$

$$= \frac{2a}{\sqrt{v^2-u^2}} + \frac{2av}{v^2-u^2}$$

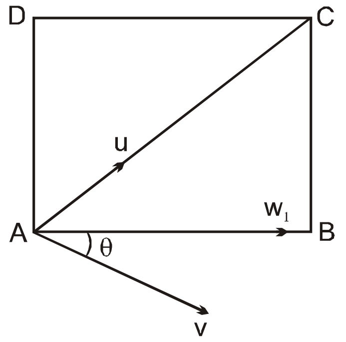
$$= 2a\left[\frac{\sqrt{v^2-u^2}+v}{v^2-u^2}\right]$$

$$t = 2a\left[\frac{v+\sqrt{v^2-u^2}}{v^2-u^2}\right]$$

$$t = \frac{2a}{v^2-u^2}\left[v+\sqrt{v^2-u^2}\right]$$

(b) **Along AB**

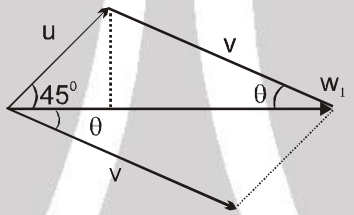
Let w_1 is the resultant velocity



$$w_1 = \frac{u}{\sqrt{2}} + v\cos\theta$$

$$= \frac{u}{\sqrt{2}} + v\sqrt{1-\sin^2\theta} = \frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}$$

Similarly, we can show that



Along BC Resultant velocity

$$w_2 = v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}$$

Along CD Resultant velocity

$$w_3 = v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}$$

Along DA Resultant velocity

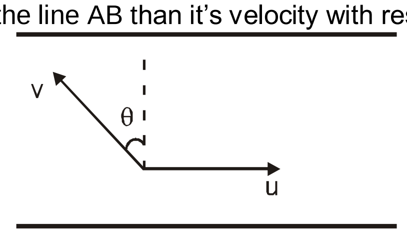
$$w_4 = \frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}$$

∴ Total time taken :-

$$t = \frac{a}{w_1} + \frac{a}{w_2} + \frac{a}{w_3} + \frac{a}{w_4}$$

$$= \frac{2a}{\frac{u}{\sqrt{2}} + v\sqrt{1-\frac{u^2}{2v^2}}} + \frac{2a}{v\sqrt{1-\frac{u^2}{2v^2}} - \frac{u}{\sqrt{2}}} = \frac{2\sqrt{2}a\sqrt{2v^2-u^2}}{v^2-u^2}$$

6. Let man swim at angle θ with the line AB than it's velocity with respect to ground any time t is as shown.



along y-axis

$V_y = V\cos\theta = 5\cos\theta$

So time taken to cross the river $t = \dfrac{d}{5\cos\theta}$ ....(i)

and velocity along x-axis

$$V_x = u - V\sin\theta \quad \Rightarrow \quad \frac{dx}{dt} = \frac{t}{2} - 5\sin\theta$$

$$\int_0^x dx = \int_0^t \left(\frac{t}{2} - 5\sin\theta\right)dt$$

$$x = \frac{t^2}{4} - 5t\sin\theta$$

For complete motion x = 0

$$\Rightarrow \quad \frac{t^2}{4} - 5t\sin\theta = 0 \quad \Rightarrow \quad t = 20\sin\theta \quad ....(ii)$$

by eq (i) and (ii)

$$100\cos\theta\sin\theta = d = 48 \text{ m}$$

$$\sin 2\theta = \frac{24}{25} \quad \Rightarrow \quad 2\theta = \sin^{-1}\left(\frac{24}{25}\right)$$

$$2\theta = 74^\circ \quad \Rightarrow \quad \theta = 37^\circ$$

$$2\theta = 106^\circ \quad \Rightarrow \quad \theta = 53^\circ$$

from eq (ii) (ii)

for $\theta = 37^\circ \Rightarrow$ t = 12 sec

for $\theta = 53^\circ \Rightarrow$ t = 16 sec

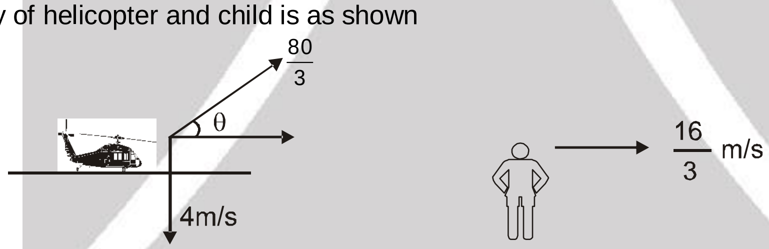
also y-coordinate of man

$$y = 5t\cos\theta \quad ....(iii)$$

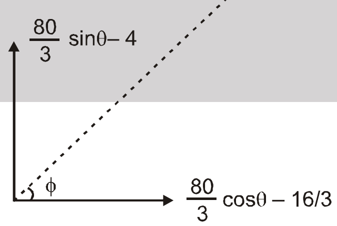
$$x = \frac{y^2}{100\cos^2\theta} - y\tan\theta$$

by eq (ii) and (iii) $\quad x = \dfrac{y^2}{64} - \dfrac{3y}{4}$

7. Velocity of helicopter and child is as shown



Now velocity of helicopter with respect to child

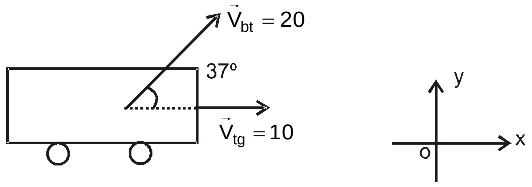


$$\tan\phi = \frac{\frac{80}{3}\sin\theta - 4}{\frac{80}{3}\cos\theta - \frac{16}{3}} = \frac{600}{800} \quad \Rightarrow \quad \tan\theta = \frac{3}{4} \quad \Rightarrow \quad \theta = 37^\circ$$

Now horizontal velocity = $\dfrac{80}{3}\cos\theta - \dfrac{16}{3} = \dfrac{80}{3}\cos 37^\circ - \dfrac{16}{3}$ = 16 m/s

So time taken = $\dfrac{800}{16}$ = 50 sec.

8. t = 0



$\vec{v}_{bg} = \vec{v}_{bt} + \vec{v}_{tg}$

$\vec{v}_{bt} = 20 \cos 37^o \hat{i} + 20 \sin 37^o \hat{j}$

$= 20 \times \frac{4}{5} \hat{i} + 20 \times \frac{3}{5} \hat{j} = 16\hat{i} + 12\hat{j}$

$\vec{v}_{tg} = 10\hat{i}$

$\therefore \quad \vec{v}_{bg} = 26\hat{i} + 12\hat{j}$

At t = 0 $\quad x_b = 0 \quad x_t = 0$

Let they meet after time t

$X_{b(t)} - X_{b(0)} = 26\,t$

$X_{train(t)} - X_{train(o)} = 10t + \frac{1}{2} at^2$

Let The ball return to train after time t $\quad \therefore \quad t = \frac{2.v_y}{g}$

$t = \frac{2 \times 12}{10} = \frac{12}{5}$ s

$X_{b(t)} = X_{train\,(t)} \quad \therefore \quad 26.t = 10\,t + \frac{1}{2} at^2$

$16 = \frac{1}{2} at$

$\frac{32}{t} = a = \frac{40}{3}$ m/s^2

$V_{train(t)} = 10 + at.$

$V_{train\,(t)} = 10 + \frac{40}{3} \times \frac{12}{5} = 10 + 32 = 42$ m/s

9. Let after time t, A is at P and B is at Q. Let T = Total time. Their velocities after time t

$V_A = at$ ....(1)

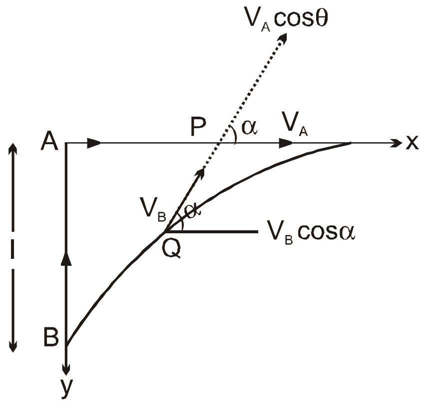
$V_B = bt$ ....(2)

Let distance PQ = x.

Velocity of approch along PQ

$= V_B - V_A \cos\alpha$

$\Rightarrow \quad -\frac{dx}{dt} = V_B - V_A \cos\alpha = bt - at \cos\alpha$



$\Rightarrow \quad -\int_{\ell}^{0} dx = \int_0^T (bt - at\cos\alpha)dt$

$\Rightarrow \quad \ell = \frac{bT^2}{2} - a\int_0^T t\cos\alpha\, dt$ .... (3)

For motion along x-axis :

$$\int_0^T V_B \cos\alpha\, dt = \frac{1}{2}aT^2$$

$$\int_0^T bt\cos\alpha\, dt = \frac{1}{2}aT^2$$

$\Rightarrow \quad \int_0^T t\cos\alpha\, dt = \frac{1}{2}\frac{aT^2}{b}$ .....(4)

Put into (3) :

$$\ell = \frac{bT^2}{2} - a \times \frac{1}{2}\frac{aT^2}{b}$$

$\Rightarrow \quad T = \sqrt{\frac{\ell b}{b^2 - a^2}}$

Now, distance travelled by B

$$s = \int_0^T V_B\, dt = \int_0^T bt\, dt = \frac{1}{2}bT^2$$

$\Rightarrow \quad s = \frac{1}{2}b.\frac{2\ell b}{b^2 - a^2} = \frac{\ell b}{b^2 - a^2}$ **Ans.**

10. v = velocity of swimmer in still mater = 2.5 km/hr.
u = velocity of stream = 2 km/hr.
For Swimmer 1
Time taken to reach point B

$t = \frac{d}{\sqrt{v^2 - u^2}}$ .... (1)

For Swimmer 2
at v_0 = velocity of walking along shore
Time to reach C

$$t_1 = \frac{d}{v}$$

Time taken in coming from C to B

$$t_2 = \frac{d\frac{u}{v}}{v_0} = \frac{ud}{v\, v_0}$$

$\therefore$ Total time

$t = t_1 + t_2$

$= \frac{d}{v} + \frac{ud}{v\,v_0}$ .... (2)

From (1) and (2)

$$\frac{d}{\sqrt{v^2 - u^2}} = \frac{d}{v} + \frac{ud}{v\,v_0}$$

$\Rightarrow$ $v_0 = \left(\frac{\sqrt{v^2-u^2}}{v - \sqrt{v^2-u^2}}\right) \times u$

Putting u = 2 and v = 2.5 km/hr ; v = 2.52.5 km/hr

$\Rightarrow$ v_0 = 3 km/hr.

11. We have

$\vec{v} = \vec{v}_0 + \vec{v}\,'$ ...(1)

From the vector diagram [of equation (1)] and using properties of triangle

$v'^2 = v_0^2 + v^2 - 2v_0 v \cos(\pi - \phi)$

or, $v' = \sqrt{v_0^2 + v^2 + 2v_0 v\cos\phi}$ = 40 km/hr ...(2)

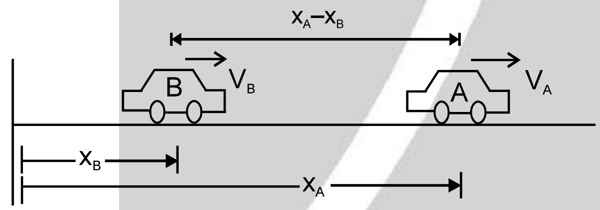
and $\frac{v'}{\sin(\pi-\phi)} = \frac{v}{\sin\theta}$ or, $\sin\theta = \frac{v\sin\phi}{v'}$

or $\theta = \sin^{-1}\left(\frac{v\sin\phi}{v'}\right)$

Using (2) and putting the values of v and d

$\theta = 19°$

12.



As given

$(V_A - V_B) \propto x_A - x_B$

$(V_A - V_B) = K(x_A - x_B)$

when $x_A - x_B = 10$ We have $V_A - V_B = 10$

We get

10 = K10 $\Rightarrow$ K = 1

$\Rightarrow$ $V_A - V_B = (x_A - x_B)$ ....(1)

Now Let

$x_A - x_B = y$ ....(2)

On differentiating with respect to 't' on both side.

$\Rightarrow$ $\frac{dx_A}{dt} - \frac{dx_B}{dt} = \frac{dy}{dt}$

$\Rightarrow$ Using (1) and (2)

$\frac{d(x_A - x_B)}{dt} = x_A - x_B$

$\frac{d(x_A - x_B)}{x_A - x_B} = dt$

$\Rightarrow$ $\left[\ell n(x_A - x_B)\right]_{10}^{20} = t$

t = (log$_e$2) sec ***Required Answer.***

13. (a)

Accelerations of particle and block are shown in figure.

Acceleration of particle with respect to block

= acceleration of particle – acceleration of block

$= (g \sin\theta\,\hat{i} + g\cos\theta\,\hat{j}) - (g\sin\theta\,\hat{i}) = g\cos\theta\,\hat{j}$

Now motion of particle with respect to block will be a projectile as shown.

The only difference is, g will be replaced by g cos θ

$\therefore \quad PQ = \text{Range (R)} = \dfrac{u^2 \sin 2\alpha}{g\cos\theta}$

$PQ = \dfrac{u^2 \sin 2\alpha}{g\cos\theta}$ **Ans.**

(b) Horizontal displacement of particle with respect to ground is zero. This implies that initial velocity of particle with respect to ground is only vertical, or there is no horizontal component of the absolute velocity of the particle.

Let v be the velocity of the block down the plane.

Velocity of particle with respect to block $= u\cos(\alpha+\theta)\,\hat{i} + u\sin(\alpha+\theta)\,\hat{j}$

Velocity of block $= -v\cos\theta\,\hat{i} - v\sin\theta\,\hat{j}$

$\therefore$ Velocity of particle with respect to ground $= \{u\cos(\alpha+\theta) - v\cos\theta\} + \{u\sin(\alpha+\theta) - v\sin\theta\}\,\hat{j}$

Now as we said earlier with that horizontal component of absolute velocity should be zero.

Therefore,

$u\cos(\alpha+\theta) - v\cos\theta = 0$

or $v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ (down the plane)

$v = \dfrac{u\cos(\alpha+\theta)}{\cos\theta}$ **Ans.**

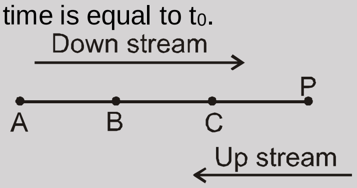
14. At t = 0, raft (a float of timber) and motor boat are at point A. The velocity of raft is equal to velocity of stream.

At $t = \tau = 60$ min, the motor boat is at point P and raft is at point B.

$\therefore$ The time taken by raft to reach from A to B = the time taken by motor boat to reach at P from A.

At $t = \tau + t_0$, both meet at point C,

So, the time taken by raft to reach at C from B is equal to the time taken by the motor boat to reach at C from P in upstream motion. This time is equal to t_0.



Let

v_A = actual velocity of motor boat,

v_B = actual velocity of stream = velocity of raft

$\therefore$ During down stream,

v_c = relative velocity of motor with respect to stream.

$\therefore \quad v_0 = v_A - v_B$

$\therefore \quad v_A = v_0 + v_B$

$\therefore \quad \tau = \dfrac{AP}{v_A} = \dfrac{AP}{v_0 + v_B}$

But AB = distance travelled by raft in time $\tau = v_B\tau$

During upstream,

$v_0 = v_A + v_B$

$\therefore \quad v_A = v_0 - v_B$

$\therefore \quad PC$ = distance travelled by motor boat in upstream in time $t_0 = (v_0 - v_B)\,t_0$

BC = distance travelled by raft in time $t_0 = v_B t_0$
According to fig.

$\because$ $AP - PC = AC = \ell$

or $(v_0 + v_B)\,\tau - (v_0 - v_B)\,t_0 = \ell$

or $v_0 t + v_B\, t - v_0 t_0 + v_B t_0 = \ell$ ....(i)

Also, $\because$ $AB + BC = \ell$

or $v_B t + v_B t_0 = \ell$

or $v_B = \dfrac{\ell}{\tau + t_0}$ ....(ii)

From equation (i) and (ii) we get

$v_0 t + v_B t - v_0 t_0 + v_B t_0 = \ell$

or $v_0\tau + \dfrac{\ell}{\tau + t_0}\tau - v_0 t_0 + \dfrac{\ell}{t_0 + \tau} t_0 = \ell$

or $v_0\tau^2 + \tau\ell - v_0 t_0^2 + \ell t_0 = \ell\,(\tau + t_0)$

or $v_0\tau^2 - v_0 t_0^2 = \ell\,(\tau + t_0) - \tau\ell - t_0\ell$

or $\tau = t_0$

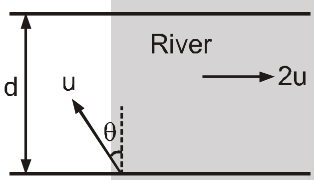
From (i) we have

$v_0\tau + v_B\tau - v_0 t_0 + v_B t_0 = \ell$

putting $\tau = t_0$

we get, $v_B = \dfrac{\ell}{2\tau}$.

15.



Let river velocity is 2u

time to cross river $t = \dfrac{d}{u\cos\theta}$

Drift $x = (2u - u\sin\theta)t = (2u - u\sin\theta)\dfrac{d}{u\cos\theta}$

Drift $x = (2\sec\theta - \tan\theta)d$

$\dfrac{dx}{d\theta} = (2\sec\theta\tan\theta - \sec^2\theta)d = 0 \quad \Rightarrow \quad 2\tan\theta = \sec\theta$

$\theta = 30^\circ$ with the river flow current

angle with stream $30^\circ + 90^\circ = 120^\circ$.