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▸ STATISTICS :

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JEE (Main) Syllabus

Statistics : Calculation of mean, median, mode of grouped and ungrouped data. Calculation of standard deviation, variance and mean deviation for grouped and ungrouped data.

Statistics

1. Measures of central tendency :

An average value or central value of a distribution is the value of variable which is representative of the entire distribution, this representive value are called the measures of central tendency are of following type.

(A) Mathematical average
(i) Arithmetic mean or mean
(ii) Geometrical mean
(iii) Harmonic mean

(B) Positional average
(i) Median
(ii) Mode

2. Mean (Arithmetic mean)

If x_1, x_2, x_3 ,.......x_n are n values of variate x_i then their A.M. $\bar{x}$ is defined as

$$\bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

If x_1, x_2, x_3, ... x_n are values of veriate with frequencies f_1, f_2, f_3,.........f_n then their A.M. is given by

$$\bar{x} = \frac{f_1x_1 \times f_2x_2 + f_3x_3 + ...f_nf_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N}, \text{ where } N = \sum_{i=1} f_i$$

(i) Properties of arithmetic mean :

(a) Sum of deviation of variate from their A.M. is always zero that is $\Sigma\left(x_i - \bar{x}\right) = 0$.

(b) Sum of square of deviation of variate from their A.M. is minimum that is $\Sigma\left(x_i - \bar{x}\right)^2$ is minimum

(c) If $\bar{x}$ is mean of variate x_i then

A.M. of $(x_i + \lambda) = \bar{x} + \lambda$

A.M. of $\lambda_i . x_i = \lambda . \bar{x}$

A.M. of $(ax_i + b) = a\bar{x} + b$

(ii) Merits of arithmetic mean :

(a) It is rigidly defined.
(b) It is based on all the observation taken.
(c) It is calculated with reasonable ease.
(d) It is least affected by fluctuations in sampling.
(e) It is based on each observation and so it is a better representative of the data.
(f) It is relatively reliable
(g) Mathematical analysis of mean is possible.

(iii) Demerits of Arithmetic Mean :

(a) It is severely affected by the extreme values.

(b) It cannot be represented in the actual data since the mean does not coincide with any of the observed value.

(c) It cannot be computed unless all the items are known.

Example # 1 : Find mean of data 2, 4, 5, 6, 8, 17.

Solution : Mean = $\frac{2+4+5+6+8+17}{6}$ = 7

Example # 2 : Find the mean of the following distribution :

x :	4	6	9	10	15
f :	5	10	10	7	8

Solution : Calculation of Arithmetic Mean

x_i	f_i	$f_i x_i$
4	5	20
6	10	60
9	10	90
10	7	70
15	8	120
	$N = \sum f_i = 40$	$\sum f_i x_i = 360$

$\therefore$ Mean = $\bar{X} = \frac{\sum f_i x_i}{\sum f_i} = \frac{360}{40} = 9$

Example # 3 : Find the mean wage from the following data :

Wage (in Rs) :	800	820	860	900	920	980	1000
No. of workers :	7	14	19	25	20	10	5

Solution : Let the assumed mean be A = 900 and h = 20.

Calculation of Mean

Wage (in Rs) x_i	No. of workers f_i	$d_i = x_i - A = x_i - 900$	$u_i = \frac{x_i - 900}{20}$	$f_i u_i$
800	7	–100	–5	–35
820	14	–80	–4	–56
860	19	–40	–2	–38
900	25	0	0	0
920	20	20	1	20
980	10	80	4	40
1000	5	100	5	25
	$N = \sum f_i = 100$			$\sum f_i u_i = -44$

We have,

$N = 100, \sum f_i u_i = -44$, A = 900 and h = 20

$\therefore$ Mean = $\bar{X} = A + h\left(\frac{1}{N}\sum f_i u_i\right) \Rightarrow \bar{X} = 900 + 20 \times \frac{-44}{100} = 900 - 8.8 = 891.2$

Hence, mean wage = Rs. 891.2

3. <u>Geometric mean</u> :

If x_1, x_2, x_3 ,........x_3 are n positive values of variate then their geometric mean G is given by

$G = (x_1\, x_2\, x_3\, ...\, x_n)^{1/n}$

$\Rightarrow \quad G = \text{antilog}\left[\frac{1}{n}\sum_{i=1}^{n}\log\ x_i\right]$

4. Median :

The median of a series is values of middle term of series when the values are written is ascending order or descending order. Therefore median, divide on arranged series in two equal parts

(i) For ungrouped distribution :

If n be number of variates in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{ term, } (\text{when } n \text{ is odd}) \\ \text{Mean of } \left(\dfrac{n}{2}\right)^{th} \text{ and } \left(\dfrac{n}{2}+2\right)^{th} \text{ term } (\text{when } n \text{ is even}) \end{cases}$$

(ii) For ungrouped frequency distribution :

First we calculate cumulative frequency (sum of all frequencies). Let it be N then

$$\text{Median} \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{ term } (\text{when } n \text{ odd}) \\ \text{Mean of } \left(\dfrac{N}{2}\right) \& \left(\dfrac{N+2}{2}\right) (\text{when } n \text{ is even}) \end{cases}$$

5. Merits and demerits of median :

The following are some merits and demerits of median :

(i) Merits :

(a) It is easy to compute and understand.

(b) It is well defined an ideal average should be

(c) It can also be computed in case of frequency distribution with open ended classes.

(d) It is not affected by extreme values.

(e) It can be determined graphically.

(f) It is proper average for qualitative data where items are not measured but are scored.

(ii) Demerits :

(a) For computing median data needs to be arranged in ascending or descending order.

(b) It is not based on all the observations of the data.

(c) It cannot be given further algebraic treatment.

(d) It is affected by fluctuations of sampling.

(e) It is not accurate when the data is not large.

(f) In some cases median is determined approximately as the mid-point of two observations whereas for mean this does not happen.

Example # 4 : Find the median of observations 4, 6, 9, 4, 2, 8, 10

Solution : Values in ascending order are 2, 4, 4, 6, 8, 9, 10

here n = 7 so $\dfrac{n+1}{2}$ = 4

so median = 4^{th} observaiton = 6

Example # 5 : Obtain the median for the following frequency distribution :

x	11	13	15	18	21	23	30	40	50
f	8	10	11	16	20	25	15	9	6

Solution :

x	f	cf
11	8	8
13	10	18
15	11	29
18	16	45
21	20	65
23	25	90
30	15	105
40	9	114
50	6	120
	N = 120	

Here, N = 120 $\Rightarrow$ $\frac{N}{2} = 60$

We find that the cummulative frequency just greater than $\frac{N}{2}$ i.e., 60 is 65 and the value of x corresponding to 65 is 21. Therefore, Median = 21.

6. Harmonic Mean :

If $x_1, x_2, x_3, \ldots\ldots x_3$ are n non-zero values of variate then their harmonic mean H is defined as

$$H = \frac{n}{\frac{1}{x_i} + \frac{1}{x_2} + \ldots. + \frac{1}{x_n}} = \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

7. Mode :

If a frequency distribution the mode is the value of that variate which have the maximum frequency. Mode for

(i) For ungrouped distribution :

The value of variate which has maximum frequency.

(ii) For ungrouped frequency distribution :

The value of that variate which have maximum frequency.
Relationship between mean, median and mode.

- In symmetric distribution, mean = mode = median
- In skew (moderately asymmetrical) distribution,
  median divides mean and mode internally in 1 : 2 ratio.

$$\Rightarrow \quad \text{median} = \frac{2(\text{Mean}) + (\text{Mode})}{3}$$

8. Merits and demerits of mode :

The following are some merits and demerits of mode :

(i) Merits :

(a) It is readily comprehensible and easy to compute. In some case it can be computed merely by inspection.

(b) It is not affected by extreme values. It can be obtained even if the extreme values are not known.

(c) Mode can be determined in distributions with open classes.

(d) Mode can be located on graph also.

(ii) Demerits :

(a) It is ill-defined. It is not always possible to find a clearly defined mode. In some cases, we may come across distributions with two modes. Such distributions are called bimodal. If a distribution has more than two modes, it is said to be multimodal.

(b) It is not based upon all the observation.

(c) Mode can be calculated by various formulae as such the value may differ from one to other. Therefore, it is not rigidly defined.

(d) It is affected to a greater extent by fluctuations of sampling.

Example # 6 : Find mode of data 2, 4, 6, 8, 8, 12, 17, 6, 8, 9.

Solution : 8 occurs maximum number of times so mode = 8

9. Measure of dispersion :

It is measure of deviation of its value about their central values. It gives an idea of scatterdness of different values from the central values.

Types :

(i) Range :

Difference between greatest values & least values of variates of a distribution are called the range of distribution.

Also coefficient of range = $\dfrac{\text{difference of extreme values}}{\text{sum of extreme values}} = \dfrac{L - S}{L + S}$

where L = largest value and S = smallest value

(ii) Mean deviation :

Mean deviation of a distribution is, the mean of absolute value of deviation of variate from their statistical average (median, mean or mode).

If A is any statistical average then mean deviation about A is defined as

Mean deviation = $\dfrac{\sum_{i=1}^{n} |x_i - A|}{n}$

Mean deviation = $\dfrac{\sum_{i=1}^{n} f_i |x_i - A|}{N}$ (for frequency distribution)

Example # 7 : Calculate mean deviation about median for the following data
3, 9, 5, 3, 12, 10, 18, 4, 7, 19, 21.

Solution : Data in ascending order is 3, 3, 4, 5, 7, 9, 10, 12, 18, 19, 21

Median = $\dfrac{n+1}{2}$ th value = 6^{th} value = 9

Mean deviation about median = $\dfrac{\sum_{i=1}^{11} |x_i - \text{median}|}{11} = \dfrac{58}{11}$

Example # 8 : Find mean deviation from mean

x	5	7	9	10	12	15
f	8	6	2	2	2	6

Solution :

x	f	fx	$x - \bar{x}$	$\lvert x - \bar{x}\rvert$	$f\lvert x - \bar{x}\rvert$
5	8	40	–4	4	32
7	6	42	–2	2	12
9	2	18	0	0	0
10	2	20	1	1	2
12	2	24	3	3	6
15	6	90	6	6	36
	N = 26	$\sum fx = 234$			$\sum f\lvert x - \bar{x}\rvert = 88$

$\bar{x} = \dfrac{\sum fx}{\sum f} = 9$ Now, M.D. $(\bar{x}) = \dfrac{\sum f|x - \bar{x}|}{\sum f} = \dfrac{88}{26} = 3.38$

(iii) Variance :

It is the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x). The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

so standard deviation = $+\sqrt{\text{variance}}$ formula

$$\sigma_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n} \Rightarrow \sigma_x^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2 = \frac{\sum_{i=1}^{n} x_i^2}{n} - (\bar{x})^2$$

$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma di}{n}\right)^2$, where $d_i = x_i - a$, where a = assumed mean

coefficient of S.D. $= \left(\frac{\sigma}{\bar{x}}\right) \Rightarrow$ coefficient of variation $= \left(\frac{\sigma}{\bar{x}}\right) \times 100$ (in percentage)

(a) Properties of variance :

• $\text{var}(x_i + \lambda) = \text{var}(x_i)$ • $\text{var}(\lambda.x_i) = \lambda^2(\text{var } x_i)$ • $\text{var}(a\,x_i + b) = a^2(\text{var } x_i)$

where λ, a, b are constant.

Example # 9 : Find the mean and variance of first n natural numbers.

Solution : $\bar{x} = \frac{\sum x}{n} = \frac{1+2+3+.\ .\ .\ .\ .+\ n}{n} = \frac{n+1}{2}$

Variance $= \frac{\sum x^2}{n} - (\bar{x})^2 = \frac{1^2+2^2+3^2+.\ .\ .\ .\ .+\ n^2}{n} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}$

Example # 10 : Find the variance and standard deviation of the following frequency distribution :

Variable (x_i)	2	4	6	8	10	12	14	16
Frequency (f_i)	4	4	5	15	8	5	4	5

Solution : Calculation of variance and standard deviation

Variable x_i	Frequency f_i	$f_i x_i$	$x_i = \bar{X}$ $= x_i - 9$	$(x_i - \bar{X})^2$	$f_i(x_i - \bar{X})^2$
2	4	8	–7	49	196
4	4	16	–5	25	100
6	5	30	–3	9	45
8	15	120	–1	1	15
10	8	80	1	1	8
12	5	60	3	9	45
14	4	56	5	25	100
16	5	80	7	49	245
	$N = \sum f_i = 50$	$\sum f_i x_i = 450$			$\sum f_i(x_i - \bar{X})^2 = 754$

Here N = 50, $\Sigma f_i x_i = 450$

$\therefore$ $\bar{X} = \frac{1}{N}(\Sigma f_i x_i) = \frac{450}{50} = 9$

We have $\Sigma f_i(x_i - \bar{X})^2 = 754$

$\therefore$ $\text{Var}(X) = \frac{1}{N}\left[\sum f_i (x_i - \bar{X})^2\right] = \frac{754}{50} = 15.08$

S.D. $= \sqrt{\text{Var}(X)} = \sqrt{15.08} = 3.88$

Example # 11 : Calculate the mean and standard deviation for the following data :

Wages upto (in Rs.)	15	30	45	60	75	90	105	120
No. of workers	12	30	65	107	157	202	222	230

Solution : We are given the cummulative frequency distribution. So first we will prepare the frequency distribution as given below :

Class Interval	Cummlative frequency	Mid–values	Frequency	$u_i = \frac{x_i - 67.5}{15}$	$f_i u_i$	$f_i u_i^2$
0 – 15	12	7.5	12	– 4	– 48	192
15 – 30	30	22.5	18	– 3	– 54	162
30 – 45	65	37.5	35	– 2	– 70	140
45 – 60	107	52.5	42	– 1	– 42	42
60 – 75	157	67.5	50	0	0	0
75 – 90	202	82.5	45	1	45	45
90 – 105	222	97.5	20	2	40	80
105 – 120	230	112.5	8	3	24	72
			$\sum f_i = 230$		$\sum f_i u_i = -105$	$\sum f_i u_i^2 = 733$

Here A = 67.5, h = 15, N = 230, $\Sigma f_i u_i = -105$ and $\Sigma f_i u_i^2 = 733$

$\therefore$ Mean = $A + h\left(\frac{1}{N}\sum f_i u_i\right) = 67.5 + 15\left(\frac{-105}{230}\right) = 67.5 - 6.85 = 60.65$

and $\text{Var}(X) = h^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]$

$\Rightarrow$ $\text{Var}(X) = 225\left[\frac{733}{230} - \left(\frac{-105}{230}\right)^2\right] = 225[3.18 - 0.2025] = 669.9375$

$\therefore$ $\text{S.D.} = \sqrt{\text{Var}(X)} = \sqrt{669.9375} = 25.883$

10. For NCERT and Board purpose :

(i) Arithmetic mean :

Arithmetic mean of continuous grouped data :

Take mid points of given classes as x_i and use formula as given for discrete grouped data.

Example # 12 : Find the mean of the following frequency distribution :

Class-interval :	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
No. of workers f :	7	10	15	8	10

Solution : Calculation of Mean

Class-interval	Mid-values (x_i)	Frequency f_i	$d_i = x_i - 25$	$u_i = \frac{x_i - 25}{10}$	$f_i u_i$
0 – 10	5	7	–20	–2	–14
10 – 20	15	10	–10	–1	–10
20 – 30	25	15	0	0	0
30 – 40	35	8	10	1	8
40 – 50	45	10	20	2	20
		$N = \sum f_i = 50$			$\sum f_i u_i = 4$

We have,

A = 25, h = 10, N = 50 and $\Sigma f_i u_i = 4$.

$\therefore$ Mean = $A + h\left\{\frac{1}{N}\Sigma f_i u_i\right\}$ $\Rightarrow$ mean = $25 + 10 \times \frac{4}{50} = 25.8$

Example # 13 : Find the mean marks of students from the following cumulative frequency distribution :

Marks	Number of students	Marks	Number of students
0 and above	80	60 and above	28
10 and above	77	70 and above	16
20 and above	72	80 and above	10
30 and above	65	90 and above	8
40 and above	55	100 and above	0
50 and above	43		

Solution : Here we have, the cumulative frequency distribution. So, first we convert it into an ordinary frequency distribution. We observe that there are 80 students getting marks greater than or equal to 0 and 77 students have secured 10 and more marks. Therefore, the number of students getting marks between 0 and 10 is 80 – 77 = 3.
Similarly, the number of students getting marks between 10 and 20 is 77 – 72 = 5 and so on. Thus, we obtain the following frequency distribution.

Marks	Number of students	Marks	Number of students
0 – 10	3	50 – 60	15
10 – 20	5	60 – 70	12
20 – 30	7	70 – 80	6
30 – 40	10	80 – 90	2
40 – 50	12	90 – 100	8

Now, we compute arithmetic mean by taking 55 as the assumed mean.
Computation of Mean

Marks	Mid-value (x_i)	Frequency (f_i)	$u_i = \frac{x_i - 55}{10}$	f_iu_i
0 – 10	5	3	–5	–15
10 – 20	15	5	–4	–20
20 – 30	25	7	–3	–21
30 – 40	35	10	–2	–20
40 – 50	45	12	–1	–12
50 – 60	55	15	0	0
60 – 70	65	12	1	12
70 – 80	75	6	2	12
80 – 90	85	2	3	6
90 – 100	95	8	4	32
Total		$\Sigma f_i = 80$		$\Sigma f_iu_i = -26$

We have,
$N = \Sigma f_i = 80$, $\Sigma f_iu_i = -26$, $A = 55$ and $h = 10$

$$\therefore \quad \bar{X} = A + h\left\{\frac{1}{N}\Sigma f_iu_i\right\} \Rightarrow \quad \bar{X} = 55 + 10 \times \frac{-26}{80} = 55 - 3.25 = 51.75 \text{ Marks}$$

(ii) Median :

(a) Median of continuous frequency distribution :

Let the number of observation be N. Prepare the cumulative frequency table. Find the median class i.e. the class in which the observation whose cumulative frequency is equal to or just greater than $\frac{N}{2}$ lies.

The median value is given by the formula : Median

$$(M) = \ell + \left[\frac{\left(\frac{N}{2}\right) - c}{f}\right] \times h \text{ where}$$

N = total frequency = $\sum f_i$

ℓ = lower limit of median class

f = frequency of the median class

c = cumulative frequency of the class preceding the median class

h = class interval (width) of the median class

Example # 14 : Calculate the median from the following distribution :

Class	5 – 10	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35	35 – 40	40 – 45
Frequency	5	6	15	10	5	4	2	2

Solution :

Class	Frequency	Cumulative Frequency
5 – 10	5	5
10 – 15	6	11
15 – 20	15	26
20 – 25	10	36
25 – 30	5	41
30 – 35	4	45
35 – 40	2	47
40 – 45	2	49
	N = 49	

We have, N = 49 $\therefore \quad \frac{N}{2} = \frac{49}{2} = 24.5$

The cummulative frequency just greater than $\frac{N}{2}$ is 26 and the corresponding class is 15-20.

Thus 15-20 is the median class such that ℓ = 15, f = 15, F = 11 and h = 5

$$\therefore \text{ Median} = \ell + \frac{\frac{N}{2} - F}{f} \times h = 15 + \frac{24.5 - 11}{15} \times 5 = 15 + \frac{13.5}{3} = 19.5$$

(iii) Mode :

(a) **Mode for continuous frequency distribution :**

First find the modal class i.e. the class which has maximum frequency. The modal class can be determined either by inspecting or with the help of grouping.

The mode is given by the formula :

$$\text{Mode} = \ell + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times h$$

where ℓ = lower limit of the modal class

h = width of the modal class

f_m = frequency of the modal class

f_{m-1} = frequency of the class preceding modal class

f_{m+1} = frequency of the class succeeding modal class

Example # 15 : Compute the mode for the following frequency distribution :

Size of items	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20	20 – 24	24 – 28	28 – 32	32 – 36	36 – 40
Frequency	5	7	9	17	12	10	6	3	1	0

Solution : Here, the maximum frequency is 17 and the corresponding class is 12-16 So 12-16 is the modal class.

We have, $\ell = 12$, h = 4, $f = 17$, $f_1 = 9$ and $f_2 = 12$

$$\therefore \text{ Mode} = \ell + \frac{f - f_1}{2f - f_1 - f_2} \times h \Rightarrow \qquad \text{Mode} = 12 + \frac{17 - 9}{34 - 9 - 12} \times 4$$

$$\Rightarrow \text{Mode} = 12 + \frac{8}{13} \times 4 = 12 + \frac{32}{13} = 12 + 10.66 = 32.66$$

(iv) Mean deviation :

(a) Mean deviation of continuous frequency distribution : For calculating mean deviation of a continuous frequency distribution, the procedure is same as for a discrete frequency distribution. The only difference is that here we have to obtain the midpoints of the various classes and take the deviations of these mid point from the given average A.

Example # 16 : Find the mean deviation about the median of the following frequency distribution :

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequency	8	10	12	9	5

Solution : Calculation of mean deviation about the median

Class	Mid values (x_i)	Frequency (f_i)	Cumulative Frequency (c.f.)	$\lvert x_i - 14\rvert$	$f_i\lvert x_i - 14\rvert$
0 – 6	3	8	8	11	88
6 – 12	9	10	18	5	50
12 – 18	15	12	30	1	12
18 – 24	21	9	39	7	63
24 – 30	27	5	44	13	65
			$N = \sum f_i = 44$		$\sum f_i\lvert x_i - 14\rvert = 278$

Here N = 44, so $\frac{N}{2}$ = 22 and the cumulative frequency just greater than $\frac{N}{2}$ is 30. Thus 12-18 is the median class.

Now Median = $\ell + \frac{N/2 - F}{f} \times h$, where $\ell = 12$, h = 6, $f = 12$, F = 18

or Median = $12 + \frac{22 - 18}{12} \times 6 = 12 + \frac{4 \times 6}{12} = 14$

Mean deviation about median = $\frac{1}{N} \sum f_i \lvert x_i - 14\rvert = \frac{278}{44} = 6.318$

Example # 17 : Find the mean deviation from the mean for the following data :

Classes	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequencies	2	3	8	14	8	3	2

Solution : We prepare the table as follows :

Computation of mean deviation from mean

Classes	Mid–values (x_i)	frequencies f_i	$f_i\ x_i$	$\lvert x_i - \overline{X}\rvert = \lvert x_i - 45\rvert$	$f_i\lvert x_i - \overline{X}\rvert$
10 – 20	15	2	30	30	60
20 – 30	25	3	75	20	60
30 – 40	35	8	280	10	80
40 – 50	45	14	630	0	0
50 – 60	55	8	440	10	80
60 – 70	65	3	195	20	60
70 – 80	75	2	150	30	60
		$n = \sum f_i = 40$	$\sum f_i x_i = 1800$		$\sum f_i \lvert x_i - \overline{X}\rvert = 400$

We have, N = 40 and $\sum f_i\ x_i = 1800$ $\therefore\ \overline{X} = \dfrac{\sum f_i\ x_i}{N} = \dfrac{1800}{40} = 45$

Now $\sum f_i\ \lvert x_i - \overline{X}\rvert = 400$ and $N = \sum f_i = 40$

$\therefore$ $\quad$ $\text{M.D.} = \dfrac{1}{N}\sum f_i\ \lvert x_i - \overline{X}\rvert = \dfrac{400}{40} = 10$

(v) Variance and standard deviation :

(a) Variance of a grouped or continuous frequency distribution : In a grouped or continuous frequency distribution, any of the formulae discussed in discrete frequency distribution can be used.

Example # 18 : Calculate the mean and standard deviation for the following distribution :

Marks	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of Students	3	6	13	15	14	5	4

Solution : Calculation of Standard deviation

Class interval	Frequency f_i	Mid– values x_i	$u_i = \dfrac{x_i - 55}{10}$	$f_i u_i$	u_i^2	$f_i u_i^2$
20 – 30	3	25	– 3	– 9	9	27
30 – 40	6	35	– 2	–12	4	24
40 – 50	13	45	–1	– 13	1	13
50 – 60	15	55	0	0	0	0
60 – 70	14	65	1	14	1	14
70 – 80	5	75	2	10	4	20
80 – 90	4	85	3	12	9	36
	$N = \sum f_i = 60$			$\sum f_i u_i = 2$		$\sum f_i u_i^2 = 134$

Here N = 60, $\sum f_i u_i = 2$, $\sum f_i u_i^2 = 134$ and h = 10

$\therefore$ $\quad$ Mean $= \overline{X} = A + h\left(\dfrac{1}{N}\sum f_i\ u_i\right) \Rightarrow \overline{X} = 500 + 10\left(\dfrac{2}{60}\right) = 55.333$

$\text{Var}(X) = h^2\left[\dfrac{1}{N}\sum f_i\ u_i^2 - \left(\dfrac{1}{N}\sum f_i\ u_i\right)^2\right] = 100\left[\dfrac{134}{60} - \left(\dfrac{2}{60}\right)^2\right] = 222.9$

$\therefore$ $\quad$ $\text{S.D.} = \sqrt{\text{Var}\ (X)} = \sqrt{222.9} = 14.94$

Example # 19 : Suppose that samples of polythene bags from two manufactures, A and B are tested by a prospective buyer for bursting pressure, with the following results :

Bursting presure in kg	Number of bags manufactured by manufacturer A	B
5 – 10	2	9
10 – 15	9	11
15 – 20	29	18
20 – 25	54	32
25 – 30	11	27
30 – 35	5	13

Which set of the bag has the highest average bursting pressure ? Which has more uniform pressure ?

Solution: For determining the set of bags having higher average bursting pressure, we compute mean and for finding out set of bags having more uniform pressure we compute coefficient of variation.

Manufacturer A :

Computation of mean and standard deviation

Bursting pressure	Mid– values x_i	f_i	$u_i = \frac{x_i - 17.5}{5}$	$f_i u_i$	$f_i u_i^2$
5 – 10	7.5	2	– 2	– 4	8
10 – 15	12.5	9	– 1	– 9	9
15 – 20	17.5	29	0	0	0
20 – 25	22.5	54	1	54	54
25 – 30	27.5	11	2	22	44
30 – 35	32.5	5	3	15	45
		$N = \sum f_i = 110$	$\sum u_i = 3$	$\sum f_i u_i = 78$	$\sum f_i u_i^2 = 160$

$$\bar{X}_A = a + h\left(\frac{\sum f_i \ u_i}{N}\right)$$

$\Rightarrow$ $\bar{X}_A = 17.5 + 5 \times \frac{78}{110}$ [$\because$ h = 5, a = 17.5]

$\Rightarrow$ $\bar{X}_A = 17.5 + 3.5 = 21$ $\Rightarrow$ $\sigma_A^2 = h^2\left[\frac{1}{N}\sum f_i \ u_i^2 - \left(\frac{1}{N}\sum f_i \ u_i\right)^2\right]$

$\Rightarrow$ $\sigma_A^2 = 25\left[\frac{160}{110} - \left(\frac{78}{110}\right)^2\right]$ $\Rightarrow$ $\sigma_A^2 = 25\left(\frac{17600 - 6084}{110 \times 110}\right) = 23.79$

$\Rightarrow$ $\sigma_A = \sqrt{23.79} = 4.87$

$\therefore$ Coefficient of variation = $\frac{\sigma_A}{\bar{X}_A} \times 100 = \frac{4.87}{21} \times 100 = 23.19$

Manufacurer B :

Bursting pressure	Mid–value x_i	f_i	$u_i = \frac{x_i - 17.5}{5}$	$f_i u_i$	$f_i u_i^2$
5 – 10	7.5	9	– 2	– 18	36
10 – 15	12.5	11	– 1	– 11	11
15 – 20	17.5	18	0	0	0
20 – 25	22.5	32	1	32	32
25 – 30	27.5	27	2	54	108
30 – 35	32.5	13	3	39	117
		$N = \sum f_i = 110$	$\sum u_i = 3$	$\sum f_i u_i = 96$	$\sum f_i u_i^2 = 304$

$$\overline{X}_B = a + h\left(\frac{\sum f_i \; u_i}{N}\right) \Rightarrow \overline{X}_B = 17.5 + 5 \times \frac{96}{110} = 17.5 + 4.36 = 21.81$$

$$\sigma_B^2 = h^2\left[\frac{1}{N}\left(\sum f_i \; u_i^2\right) - \left(\frac{1}{N}\sum f_i \; u_i\right)^2\right]$$

$$\Rightarrow \quad \sigma_B^2 = 25\left[\frac{304}{110} - \left(\frac{96}{110}\right)^2\right] \Rightarrow \sigma_B^2 = 25\left(\frac{33440 - 9216}{110 \times 110}\right) = 50.04$$

$\sigma_B = 7.07$

$\therefore$ Coefficient of variation $= \frac{\sigma_B}{\overline{X}_B} \times 100 = \frac{7.07}{21.81} \times 100 = 32.41$

We observe that the average brusting pressure is higher for manufacturer B. So, bags manufactured by B have higher bursting pressure.

The coefficient of variation is less for manufacturer A. So bags manufactured by A have more uniform pressure.

Exercise-1

Marked questions are recommended for Revision

OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)

Section (A) : Mean, median & mode

A-1. Find the A.M. of the series 1, 2, 4, 8, 16, ..., 2^n

(1) $\frac{2^{n+1}-1}{n+1}$ (2) $\frac{2^{n+2}-1}{n}$ (3) $\frac{2^n-1}{n+1}$ (4) $\frac{2^n-1}{n}$

A-2. The average weight of 9 men is x kg. After another men joins the group, the average increases by 5%. Still another man joins and average returns to old level of x kg. Which one of the following true?

(1) the 10th & 11th men weight same

(2) the 10th man weight half as much as the 11th man

(3) the 10th man weight as much as the 11th man

(4) None of these

A-3. N observations on a variable x are x_i = A + iB for i = 1, 2, 3, ..., n where A, B are real constants. The mean of the observation is

(1) $A + B\frac{(n+1)}{2}$ (2) $nA + B\frac{(n+1)}{2}$ (3) $A + Bn\frac{(n+1)}{2}$ (4) $A + B\left(\frac{n}{2}\right)$

A-4. Find the median of values 10, 14, 11, 9, 8, 12, 6.

(1) 8 (2) 10 (3) 9 (4) 11

A-5. If a variable takes the discrete values $\alpha + 4$, $\alpha - \frac{7}{2}$, $\alpha - \frac{5}{2}$, $\alpha - 3$, $\alpha - 2$, $\alpha +$, $\frac{1}{2}$ $\alpha - \frac{1}{2}$, $\alpha + 5$ ($\alpha > 0$), then the median is

(1) $\alpha - \frac{5}{4}$ (2) $\alpha - \frac{1}{2}$ (3) $\alpha - 2$ (4) $\alpha + \frac{5}{4}$

A-6. Find the mode of data 1, 2, 5, 3, 2, 3, 0, 5, 2.

(1) 3 (2) 1 (3) 5 (4) 2

A-7. Consider the following statements related to measure of central tendency of 50 positive numbers

1. Median is not influenced by extreme values in set of numbers
2. The harmonic mean is unreliable if one or more of the numbers is non-zero

which of the above statement is/are correct

(1) 1 only (2) 2 only (3) both 1 and 2 (4) neither 1 nor 2

A-8. M(x_1, x_2, x_3 .... x_n) defines a measure of central tendency based on n values x_1, x_2, x_3, ...x_n consider the following measured of central tendency

(1) Arithmetic mean (2) Median (3) Geometric mean

which of the above measure satisfie/ satisfies the property

$\frac{M(x_1, x_2, x_3 .... x_n)}{M(y_1, y_2, y_3 .... y_n)} = M.\left(\frac{x_1}{y_1}, \frac{x_2}{y_2}, \frac{x_3}{y_3}, ..., \frac{x_n}{y_n}\right)$?

select the correct answer using the code below

(1) 1 only (2) 2 only (3) 3 only (4) 1 and 3

A-9. If values a, b, c,...... j, p occurs with frequencies $^{10}C_0$, $^{10}C_1$, $^{10}C_2$,..., $^{10}C_{10}$ then mode is
(1) a (2) e (3) f (4) k

A-10. The mean of 21 observations (all different) is 40. If each observations greater than the median are increased 21, then mean of observations will become
(1) 50 (2) 50.5 (3) 30 (4) 45

A-11. The mean of series $x_1, x_2, x_3, \ldots, x_n$ is $\bar{x}$, then mean of the series $x_i + 2i$, i = 1, 2, 3, . . . , n will be
(1) $\bar{x} + n$ (2) $\bar{x} + n + 1$ (3) $\bar{x} + 2$ (4) $\bar{x} + 2n$

A-12. If the difference between mean and mode is 63, the difference between mean and median is
(1) 189 (2) 21 (3) 31.5 (4) 48.5

A-13. Mean of variates 1.2.3, 2.3.4, ..., n(n + 1)(n + 2)
(1) $\frac{n(n+1)(n+2)}{4}$ (2) $\frac{(n+1)(n+2)(n+3)}{2}$ (3) $\frac{(n+1)(n+2)(n+3)}{4}$ (4) $\frac{n(n+1)(n+3)}{2}$

Section (B) : Range, coefficient of range, mean deviation and coefficient of mean deviaiton.

B-1. Range of data 7, 8, 2, 1, 3, 13, 18 is
(1) 10 (2) 15 (3) 17 (4) 11

B-2. Coefficient of range 5, 2, 3, 4, 6, 8, 10 is
(1) $\frac{2}{3}$ (2) $\frac{1}{3}$ (3) $\frac{3}{5}$ (4) $\frac{1}{2}$

B-3 The scores of a batsman in ten innings are : 38, 70, 48, 34, 42, 55, 63, 46, 54, 44. Find the mean deviation about median.
(1) $\frac{43}{5}$ (2) $\frac{44}{5}$ (3) $\frac{41}{5}$ (4) $\frac{42}{5}$

B-4. The mean deviation about median of variates 13, 14, 15, ... , 99, 100 is
(1) 1936 (2) 21.5 (3) 23.5 (4) 22

B-5. The mean deviation of an ungrouped data is 50. If each observation is increased by 2%, then the new mean deviation is
(1) 50 (2) 51 (3) 49 (4) 50.5

B-6. The mean deviation of an ungrouped data is 80. If each observation is decreased by 5%, then the new mean deviation is
(1) 76 (2) 77 (3) 78 (4) 79

B-7. The mean deviation from mean of the observations a, a + d, a + 2d, ... a + 2nd is
(1) $\frac{n(n+1)d^2}{3}$ (2) $\frac{n(n+1)}{2}d^2$ (3) $a + \frac{n(n+1)d^2}{2}$ (4) $\frac{n(n+1)|d|}{(2n+1)}$

B-8. Mean deviation from the mean for the observations – 1, 0, 4 is
(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{2}{3}$ (3) 2 (4) 3

B-9. If $\bar{X}$ is the mean of $x_1, x_2, x_3 \ldots\ldots x_n$. Then the algebraic sum of the deviations about mean $\bar{X}$ is
(1) 0 (2) $\frac{\bar{X}}{n}$ (3) $n\bar{X}$ (4) $(n–1)\bar{X}$

B-10. If the algebraic sum of deviations of 20 observation from 30 is 20, then the mean of the observation is
(1) 30 (2) 30.1 (3) 29 (4) 31.

Section (C) : Variance, Standard deviation and coefficient of variation.

C-1. Variance of first 20 natural number is

(1) $\frac{133}{4}$ (2) $\frac{379}{12}$ (3) $\frac{133}{2}$ (4) $\frac{399}{4}$

C-2. The mean & variance of 7 observations are 8, 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, then the LCM of remaining two observations is

(1) 16 (2) 24 (3) 20 (4) 14

C-3. If n = 10, $\bar{x} = \sqrt{12}$, $\sum x^2 = 1560$, then standard deviation σ is

(1) 12 (2) 13 (3) $\sqrt{166}$ (4) $\sqrt{12}$

C-4. The mean of distribution is 4 if coefficient of variation is 58%. Then standard deviation of distribution is

(1) 2.23 (2) 3.23 (3) 2.32 (4) 2.75

C-5. The sum of squares of deviations for 10 observations taken from mean 50 is 250. The co-efficient of variation is

(1) 50% (2) 10% (3) 40% (4) 30%

C-6. Standard deviation is independent of

(1) change of scale & origin (2) change of scale but not origin

(3) change of origin but not scale (4) neither change of scale non origin

C-7 What is standard deviation of the set of observations 32, 28, 29, 30, 31?

(1) 1.6 (2) $\sqrt{2}$ (3) 2 (4) None of these

C-8. If the standard deviation of $x_1, x_2, ... x_n$ is 3.5, then the standard deviation of

$-2x_1 - 3, -2x_2 - 3, ...-2x_n - 3$ is

(1) – 7 (2) – 4 (3) 7 (4) 1.75

C-9. The marks of some students were listed out of a maximum 100. The standard deviation of marks was found to be 9. Subsequently the marks raised to a maximum of 150 and standard deviation of new marks was calculeted. The new standard deviation

(1) 9 (2) 13.5 (3) –13.5 (4) –9

Exercise-2

Marked questions are recommended for Revision

PART - I : OBJECTIVE QUESTIONS (SINGLE CHOICE CORRECT)

1. Find the arithmetic mean of ${}^{2n+1}C_0$, ${}^{2n+1}C_1$, ...${}^{2n+1}C_n$.

(1) $\frac{2^{2n+2}}{n}$ (2) $\frac{2^{2n}}{n+1}$ (3) $\frac{2^{n+1}}{n+1}$ (4) None of these

2. A variable takes the values of 0, 1, 2, ... , n with frequencies proportional to the binomial coefficients ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ... , ${}^nC_n$, then mean of the distribution is

(1) $\frac{n(n+1)}{4}$ (2) $\frac{n}{2}$ (3) $\frac{n(n-1)}{2}$ (4) $\frac{n(n+1)}{2}$

3. Following is the record of goals scored by team A in football session

Numbers of goals scored	0	1	2	3	4
Numbers of match is	1	9	7	5	3

for team 'B' mean number of goals scored per match was 2 goals with standard deviation 1.25 . The team which is more consistant

(1) A (2) B
(3) A & B both are equal (4) neither A nor B

4. The mean of two samples of sizes 200 and 300 were found to be 25, 10 respectively. Their standarddeviations were 3 and 4 respectively. Find the variance of combined sample of size 500

(1) 70 (2) 60 (3) 67.2 (4) 80

5. The first of the two samples has 100 items with mean 15 and S.D. 3. If the whole group has 250 items with mean 15.6 and S.D. = $\sqrt{13.44}$ then S.D. of the second group is

(1) 5 (2) 4 (3) 6 (4) 3.52

6. The average marks of 10 students in a class was 60 with standard deviation 4. While the average marks of other 10 students was 40 with a standard deviation 6. If all the 20 students are taken together, their standard deviation will be

(1) 5.0 (2) 7.5 (3) 9.8 (4) 11.2

7. The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. From these observations three are 1, 2 and 6 and $\lambda = |x_1 - x_2| + 8$ where x_1 & x_2 are remaining observations. Then number of solution of equation $10 - x^2 - 2x = \lambda$ are

(1) 1 (2) 2 (3) 3 (4) 4

8. The mean and variance of 10 numbers were calculated as 11.3 and 3.3 respectively. It was subsequently found that one of the a number was misread as 10 instead of 12. How does the variance change.

(1) variance decreases (2) variance increases
(3) nothing can be said about variance (4) variance remains unchanged.

Comprehension # 1_ (Q. No. 9 to 11)

As median divides an arranged series into two equal parts, in similar way quartile divides an arranged series in 4 equal part. For ungrouped frequency distribution formula of finding i^{th} quartile =

$Q_i = \left\{ i . \left(\frac{N+1}{4} \right) \right\}^{th}$ term, i = 1, 2, 3.

Quartile deviation : half of difference between upper quartile & lower quartile.

$\Rightarrow$ Quartile deviation (Q.D.) = $\frac{1}{2}$ $(Q_3 - Q_1)$ $\Rightarrow$ Coefficient of quartile deviation = $\frac{Q_3 - Q_1}{Q_3 + Q_1}$.

9. If 1, 2, 3, 4, 5, 6, 7, are numbers then Q_1 & Q_3 are respectively

(1) 2, 4 (2) 2, 6 (3) 4, 6 (4) 3, 5

10. Quartile deviation of the following numbers 10, 8, 12, 11, 14, 9, 6 is

(1) 2 (2) 10 (3) 1/2 (4) 1

11. Coefficient of quartile deviation of numbers 6, 8, 9, 10, 11, 12, 14 is

(1) 1/5 (2) 1/10 (3) 2/5 (4) 1/4

Comprehension # 2 (Q. No. 12 to 14)

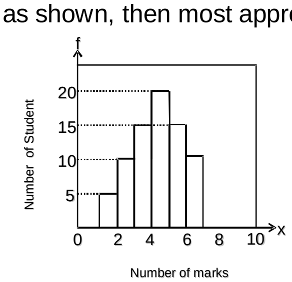
To analyse data using mean, median and mode, we need to use the most appropriate measure of central tendency. The mean is useful for predicting future results when there are no extreme values in the data set. The median may be more useful than the mean when there are extreme values in the data set as it is not affected by the extreme values. The median is most commonly quoted figure used to measure property prices as mean property price is affected by a few expensive properties that are not representative of the general property market. The mode is useful when the most common item or characteristic of a data set is required. The mode has applications in printing. It is important to print more of the most popular books.

12. For the data shown, the value of appropriate measure of central tendency is

No.of staff	1	2	4	5	3	2
Salary (In rupees)	15000	10000	7000	12000	90000	95000

(1) 95000 (2) 18350 (3) 90000 (4) 12000

13. For a normally distributed sample as shown, then most appropriate representative of data is



(1) Mean (2) Mode

(3) Median (4) any one of mean or median

14. Based upon collection of data of numbers of days it snows, rains or it is sunny in a month for three month December, January and February of last year, the weather forecast points that snow is likely to be in January. Which measure is used for this forecast ?

(1) Mean (2) Mode (3) Median (4) Range

PART - II : MISCELLANEOUS QUESTIONS

Section (A) : ASSERTION/REASONING

DIRECTIONS :

Each question has 4 choices (1), (2), (3) and (4) out of which ONLY ONE is correct.

(1) Both the statements are true.

(2) Statement-I is true, but Statement-II is false.

(3) Statement-I is false, but Statement-II is true.

(4) Both the statements are false.

A-1. **Statement-1 :** If $\sum_{i=1}^{9}(x_i - 8) = 9$ and $\sum_{i=1}^{9}(x_i - 8)^2 = 45$ then S.D. of $x_1, x_2, ..., x_9$ is 2.

Statement-2 : S.D. is independent of change of origin.

A-2. **Statement-1 :** Mean cannot be represented graphically.

Statement-2 : Mean may not coincide with anyone of the actual values.

Section (B) : MATCH THE COLUMN

B-1

	Column - I		Column - II
(1)	Better measure of central tendency for data 1, 7, 8, 9, 9 is	(p)	Mean
(2)	Which is not independent of change of scale ?	(q)	Median
(3)	Which is not dependent on change of origin ?	(r)	Mode
(4)	The value of range of data is always greater than or equal to	(s)	S.D.

B-2 Let $n \in N$.

	Column I		Column II
(1)	S.D. of 2, 4, 6, ... 2n ; $n \in N$	(p)	n
(2)	S.D. of 1, 3, 5, ..., 2n – 1 ; $n \in N$	(q)	$\frac{n+1}{2}$
(3)	Mean of 1,3,5, ...2n–1 ; $n \in N$	(r)	$\sqrt{\frac{n^2-1}{3}}$
(4)	Median of 1,3,5,.....,2n – 1	(s)	$\sqrt{\frac{n^2-1}{12}}$

Section (C) : ONE OR MORE THAN ONE OPTIONS CORRECT

C-1. If the daily earnings (in rupees) of 12 workers in a factory are 16, 11, 3, 7, 5, 28, 9, 31, 28, 43, 15 ,17, then which of the following is(are) true

(1) mean = 17.75 (2) median = 15.5

(3) mode = 28 (4) mean deviation about mean = $9.8\overline{3}$

C-2 If first sample of 25 variates has the mean 40 and standard deviation 5 and a second sample of 35 variates has the mean 45 and standard deviation 2, then which of the following is/are true

(1) mean of combined sample space = 42.917

(2) mean of combined sample space = 32.9

(3) standard deviation of combined sample space = 3.34

(4) standard deviation of combined sample space = 4.34

C-3. Which of the following is false :

(1) Algebraic sum of deviations of observation from their mean is zero

(2) If S.D. of x_i(i = 1, 2, 3, ... n) is σ then S.D. of hx_i is $h\sigma$

(3) Median is severely affected by fluctuations in extreme values

(4) Mean deviations of a given set of observations is least when taken about their median.

Exercise-3

✍ **Marked questions are recommended for Revision**

*** Marked Questions may have more than one correct option.**

PART - I : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

1. The mean of the number a, b, 8, 5, 10 is 6 and the variance is 6.80. Then which one of the folowing gives possible values of a and b ? **[AIEEE 2008, (3, –1), 105]**

(1) a = 3, b = 4 (2) a = 0, b = 7 (3) a = 5, b = 2 (4) a = 1, b = 6

2. **Statement-I** The variance of first n even natural numbers is $\frac{n^2-1}{4}$ **[AIEEE 2009, (4, –1), 144]**

Statement-II The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$ **.**

(1) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1.

(2) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1

(3) Statement-1 is True, Statement-2 is False

(4) Statement-1 is False, Statement-2 is True

3. If the mean deviation of numbers 1, 1 + d, 1 + 2d, ...,1 + 100d from their mean is 255, then the value of d is equal to- **[AIEEE 2009, (4, –1), 144]**

(1) 10.0 (2) 20.0 (3) 10.1 (4) 20.2

4.✍ For two data sets, each of size 5, the variance are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is **[AIEEE 2010, (4, –1), 144**

(1) $\frac{11}{2}$ (2) 6 (3) $\frac{13}{2}$ (4) $\frac{5}{2}$

5. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals :
[AIEEE 2011, (4, –1), 120]
(1) 2 (2) 3 (3) 4 (4) 5

6. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **[AIEEE 2011, (4, –1), 120]**
(1) 32, 2 (2) 32, 4 (3) 28, 2 (4) 28, 4

7. Let x_1 , x_2 , ..., x_n be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be the variance
Statement-1 : Variance of $2x_1$, $2x_2$, ..., $2x_n$ is $4\sigma^2$. **[AIEEE 2012, (4, –1), 120]**
Statement-2 : Arithmetic mean $2x_1$, $2x_2$, ..., $2x_n$ is $4\bar{x}$.
(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, statement-2 is true; statement-2 is a correct explanation for Statement-1.
(3) Statement-1 is true, statement-2 is true; statement-2 is **not** a correct explanation for Statement-1.
(4) Statement-1 is true, statement-2 is false.

8. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given ? **[AIEEE - 2013, (4, –1),360]**
(1) mean (2) median (3) mode (4) variance

9. The variance of first 50 even natural number is **[AIEEE - 2014, (4, –1),360]**
(1) $\frac{833}{4}$ (2) 833 (3) 437 (4) $\frac{437}{4}$

10. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is **[JEE(Main) 2015, (4, – 1), 120]**
(1) 16.8 (2) 16.0 (3) 15.8 (4) 14.0

11. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true?
[JEE(Main) 2016, (4, – 1), 120]
(1) $3a^2 - 32a + 84 = 0$ (2) $3a^2 - 34a + 91 = 0$ (3) $3a^2 - 23a + 44 = 0$ (4) $3a^2 - 26a + 55 = 0$

12. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items x_1, x_2, ... , x_9 is :
[JEE(Main) 2018, (4, – 1), 120]
(1) 2 (2) 3 (3) 9 (4) 4

13. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. the variance (in cm^2) of the height of these six students is :
[JEE(Main) 09-01-19 P-1, (4, – 1), 120]
(1) 20 (2) 22 (3) 16 (4) 18

14. A data consists of n observations : **[JEE(Main) 09-01-19 P-2, (4, – 1), 120]**

x_1, x_2, ..., x_n. If $\sum_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5n$, then the standard deviation of this data is :

(1) 5 (2) 2 (3) $\sqrt{5}$ (4) $\sqrt{7}$

15. The mean of five observations is 5 and their variance is 9.20. If three of the given five observations are 1, 3 and 8, then a ratio of other two observations is : **[JEE(Main) 10-01-19 P-1, (4, – 1), 120]**

(1) 4 : 9 (2) 6 : 7 (3) 5 : 8 (4) 10 : 3

16. If mean and standard deviation of 5 observations x_1, x_2, x_3, x_4, x_5, are 10 and 3, respectively, then the variance of 6 observations x_1, x_2, ….x_5 and –50 is equal to : **[JEE(Main) 10-01-19 P-2, (4, – 1), 120]**

(1) 586.5 (2) 507.5 (3) 582.5 (4) 509.5

17. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2}$ – d each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2}$ + d each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals : **[JEE(Main) 11-01-19 P-1, (4, – 1), 120]**

(1) $\sqrt{2}$ (2) $\frac{2}{3}$ (3) 2 (4) $\frac{\sqrt{5}}{2}$

18. If the sum of the deviations of 50 observations from 30 is 50, then the mean of these observations is :
[JEE(Main) 12-01-19 P-1, (4, – 1), 120]

(1) 50 (2) 51 (3) 31 (4) 30

19. The mean and the variance of five observation are 4 and 5.20, respectively. If three of the observations are 3, 4 and 4 ; then the absolute value of the difference of the other two observations, is
[JEE(Main) 12-01-19 P-2, (4, – 1), 120]

(1) 1 (2) 3 (3) 5 (4) 7

Answers

EXERCISE - 1

PART- I

Section (A)

A-1.	(1)	**A-2.**	(4)	**A-3.**	(1)	**A-4.**	(2)	**A-5.**	(1)	**A-6.**	(4)	**A-7.**	(1)
A-8.	(3)	**A-9.**	(3)	**A-10.**	(1)	**A-11.**	(2)	**A-12.**	(2)	**A-13.**	(3)		

Section (B)

B-1.	(3)	**B-2.**	(1)	**B-3**	(1)	**B-4.**	(4)	**B-5**	(2)	**B-6.**	(1)	**B-7.**	(4)
B-8.	(3)	**B-9.**	(1)	**B-10.**	(4)								

Section (C)

C-1.	(1)	**C-2.**	(2)	**C-3.**	(1)	**C-4.**	(3)	**C-5.**	(2)	**C-6.**	(3)	**C-7**	(2)
C-8.	(3)	**C-9.**	(2)										

EXERCISE - 2

PART- I

1.	(2)	**2.**	(2)	**3.**	(1)	**4.**	(3)	**5.**	(2)	**6.**	(4)	**7.**	(1)
8.	(1)	**9.**	(2)	**10.**	(1)	**11.**	(1)	**12.**	(4)	**13.**	(1)	**14.**	(2)

PART- II

Section (A)

A-1. **(1)** **A-2.** **(1)**

Section (B)

B-1 (A) $\rightarrow$ (q, r) ; (B) $\rightarrow$ (p, q, r, s) ; (C) $\rightarrow$ (s) ; (D) $\rightarrow$ (s)

B-2 A $\rightarrow$ r ; B $\rightarrow$ r ; C $\rightarrow$ p ; D $\rightarrow$ p

Section (C)

C-1. (1,2,3,4) **C-2** (1,4) **C-3.** (2,3)

EXERCISE - 3

PART- I

1.	(1)	**2.**	(4)	**3.**	(3)	**4.**	(1)	**5.**	(3)	**6.**	(1)	**7.**	(4)
8.	(4)	**9.**	(2)	**10.**	(4)	**11.**	(1)	**12.**	(1)	**13.**	(1)	**14.**	(3)
15.	(1)	**16.**	(2)	**17.**	(1)	**18.**	(3)	**19.**	(4)				

Additional Problems For Self Practice (APSP)

PART - I : PRACTICE TEST PAPER

This Section is not meant for classroom discussion. It is being given to promote self-study and self testing amongst the Resonance students.

Max. Marks : 120 **Max. Time : 1 Hr.**

Important Instructions :

1. The test is of **1 hour** duration and max. marks 120.

2. The test consists **30** questions, **4 marks** each.

3. Only one choice is correct **1 mark** will be deducted for incorrect response. No deduction from the total score will be made if no response is indicated for an item in the answer sheet.

4. There is only one correct response for each question. Filling up more than one response in any question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instructions 3 above.

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1. The mean weight of 150 person in a group is 60 kg. The mean weight of men in the group is 70 kg and that of women is 55kg. Find the number of men.

(1) 50 (2) 75 (3) 100 (4) 25

2. Then mean of 11 observations is 25. If each observation is decreased by 5, the new mean will be

(1) 25 (2) 30 (3) 20 (4) 15

3. The marks of some students were listed out of a maximum 60. The standard deviation of marks was found to be 5. Subsequently the marks raised to a maximum of 100 and variance of new marks was calculeted .The new variance

(1) $\frac{25}{3}$ (2) $\frac{625}{3}$ (3) $\frac{625}{9}$ (4) $\frac{15}{9}$

4. Range of data 13, 14, 19, 21, 17, 14, 14, 12 is

(1) 7 (2) 14 (3) 9 (4) 21

5. Variance of first 10 natural numbers is

(1) $\frac{133}{4}$ (2) $\frac{33}{4}$ (3) 33 (4) $\frac{33}{2}$

6. Find the median of values 12, 17, 19, 8, 4, 23, 27

(1) 27 (2) 23 (3) 17 (4) 18

7. Find the mode of the data 3, 1, 1, 2, 3, 0, –3, 4, 1 2, 3, 3, 5

(1) 1 (2) 2 (3) 0 (4) 3

8. Coefficient of range 5, 2, 4, 3, 8, 11, is

(1) $\frac{9}{13}$ (2) $\frac{1}{11}$ (3) $\frac{6}{10}$ (4) $\frac{6}{16}$

9. If difference between mean and mode is 3, the difference between mean and median is

(1) 3 (2) 1 (3) 4 (4) 2

10 Mode of the data:

xi (variate)	frequency
${}^4C_1$	${}^8C_0$
${}^4C_2$	${}^8C_2$
${}^4C_3$	${}^8C_4$
${}^4C_4$	${}^8C_6$

(1) ${}^4C_2$ (2) ${}^8C_2$ (3) ${}^8C_4$ (4) ${}^4C_3$

11. If $\sum_{i=1}^{11}(x_i-4)=11$ and $\sum_{i=1}^{11}(x_i-4)^2=44$ then find variance of $x_1, x_2, x_3 .....x_{11}$.

(1) 4 (2) 3 (3) 7 (4) 11

12 Mean of 1, 4, 7, 10, 13..........n terms is

(1) (3n–1)n (2) (3n–1) $\frac{n}{2}$ (3) $\frac{3n-1}{2}$ (4) (3n–1)

13. If var(x_i) = λ then var($2x_i$ + 3) is

(1) 2λ + 3 (2) $2\lambda^2$ (3) 4λ (4) 4λ + 9

14. If S. D. of $x_1, x_2, x_3.....x_4$ is 3 then S. D of $-4x_1, -4x_2..........-4x_n$ is

(1) –12 (2) 12 (3) –6 (4) 6

15. Consider the following statement and choose correct option

(I) variance can not be negative

(ii) S.D can not be negative

(III) Median is influenced by extreme value in set of numbers.

(1) TTT (2) FTT (3) FTF (4) TTF

16. The mean and variance of 7 observation are 7 and $\frac{100}{7}$. If 5 of the observation are 2, 4, 7, 11, 10, find the remaining 2 observations.

(1) 3, 6 (2) 3, 12 (3) 4, 11 (4) 5, 10

17. The mean of distribution is 6, If coefficient of variation is 50%, then standard deviation of distribution is

(1) 9 (2) 3 (3) 300 (4) 4

18. The mean deviation about median of variation 53, 54, 55......100 is

(1) 11.5 (2) 12 (3) 12.5 (4) 13

19. The mean of two samples of sizes 20 and 10 were found to be 11, 8 respectively. Their variance were 4 and 34 respectively. Find the variance of combined sample of size 30.

(1) 19 (2) 19.5 (3) 18.5 (4) 16

20. If standard deviation of 1, 2, 3, 4, 5 is $\sqrt{2}$ then which of the following is correct.

(1) standard deviation of 1, 4, 9, 16, 25, is 2

(2) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2000}$

(3) standard deviation of 1001, 1002, 1003, 1004, 1005 is $\sqrt{2}$

(4) standard deviation of 1, 8, 27, 16, 25 is $\sqrt{2}$

21. The mean and variance of 100 numbers were calculated as 11 and 2 respectively. Later it was found that one of the number was misread 5 instead of 9. How does the variance change.

(1) Variance doesn't change (2) Variance Increases

(3) Variance decreases (4) Can't comment

22 The variance of first 5 even natural numbers is

(1) 6 (2) 7 (3) 8 (4) 9

23 If variance of x_1,x_2,x_3,x_4,x_5 is σ^2, find variance of $3x_1+4$, $3x_2+4$, $3x_3+4$, $3x_4+4$, $3x_5+4$

(1) $4\sigma^2+3$ (2) $4\sigma^2+9$ (3) $9\sigma^2$ (4) $4\sigma^2-3$

24. Find the mean deviation about median of 34, 38, 42, 55, 63, 46, 54, 44, 70,48

(1) 8.2 (2) 8.4 (3) 8.6 (4) 8.8

25. Variance of first n natural numbers.

(1) $\frac{n^2-1}{24}$ (2) $\frac{n^2-1}{12}$ (3) $\frac{n^2-1}{6}$ (4) $\frac{n^2-1}{3}$

26. In a batch of 20 students 8 have failed. The marks of the successful candidates are 23, 27, 29, 18, 17, 19, 21, 27, 20, 24, 26, 28 the median marks are

(1) 22 (2) 18 (3) 18.5 (4) can't determine

27. The coefficient of variation of two series are 60% and 70% if their standard deviation are 21 and 14, then find ratio of their AMs

(1) $\frac{6}{7}$ (2) $\frac{2}{3}$ (3) $\frac{4}{7}$ (4) $\frac{7}{4}$

28. The mean of 2 samples of sizes 50 & 40 were found to be 63 and 54. Their variance were 81 & 36. Find the variance of combined sample of size 90

(1) 9 (2) 81 (3) 3 (4) 243

29. The mean and median of some data is 14 and 12 . Later it was discovered that every data element should be increased by 2 units then new mean and median will be

(1) 16, 12 (2) 16, 14 (3) 14, 12 (4) 10, 8

30. Rohan worked for a firm as given below

No. of weeks	Days each week he worked
2 weeks	1 day each week
14 weeks	2 day each week
8 weeks	5 day each week
32 weeks	7 day each week

What is the mean number of days rohan works per week

(1) 5 (2) 6 (3) 5.5 (4) 5.25

Practice Test (JEE-Main Pattern)

OBJECTIVE RESPONSE SHEET (ORS)

Que.	1	2	3	4	5	6	7	8	9	10
Ans.										
Que.	11	12	13	14	15	16	17	18	19	20
Ans.										
Que.	21	22	23	24	25	26	27	28	29	30
Ans.										

PART - II : PRACTICE QUESTIONS

1. If a variate X is expressed as a linear function of two variates U and V in the form X = a U + b V, then mean $\overline{X}$ of X is

(1) $a\overline{U} + b\overline{V}$ (2) $\overline{U} + \overline{V}$ (3) $a\overline{U} + a\overline{U}$ (4) None of these

2. The AM of n numbers of a series is $\overline{X}$. If the sum of first (n – 1) terms is k, then the nth number is

(1) $\overline{X} - k$ (2) $n\overline{X} - k$ (3) $\overline{X} - nk$ (4) $\frac{\overline{X}}{3}$

3. If $\overline{X}_1$ and $\overline{X}_2$ are the means of two distributions such that $\overline{X}_1 < \overline{X}_2$ and $\overline{X}$ is the mean of the combined distribution, then

(1) $\overline{X} < \overline{X}_1$ (2) $\overline{X} > \overline{X}_2$ (3) $\overline{X} = \frac{\overline{X}_1 + \overline{X}_2}{2}$ (4) $\overline{X}_1 < \overline{X} < \overline{X}_2$

4. Coefficient of variation of two distribution are 50% and 60% and their arithmetic means are 30 and 25 respectively. Difference of their standard deviations is

(1) 0 (2) 1 (3) 1.5 (4) 2.5

APSP Answers

PART- I

1.	(1)	**2.**	(3)	**3..**	(3)	**4.**	(3)	**5.**	(2)	**6.**	(3)	**7.**	(4)
8.	(1)	**9.**	(2)	**10**	(4)	**11.**	(2)	**12**	(3)	**13.**	(3)	**14.**	(2)
15.	(4)	**16.**	(2)	**17.**	(2)	**18.**	(2)	**19.**	(4)	**20.**	(3)	**21.**	(3)
22	(3)	**23**	(3)	**24.**	(3)	**25.**	(2)	**26.**	(3)	**27.**	(4)	**28.**	(2)
29.	(2)	**30.**	(4)										

PART- II

1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(1)