The knowledge of which geometry aims is the knowledge of the eternal...... Plato

A french mathematician and a greatest philosopher named Rene Descartes, pioneered the use of algebra in Geometry. He suggested methods to study geometry by algebraic methods without making direct reference to the actual figures

This geometry was called co-ordinate geometry or analytical geometry and it is the branch of geometry in which algebraic equations are used to denote points, lines and curves.

Rectangular cartesian co-ordinate systems :

We shall right now focus on two-dimensional co-ordinate geometry in which two perpendicular lines called co-ordinate axes (x-axis and y-axis) are used to locate a point in the plane.



O is called origin. Any point P in this plane can be represented by a unique ordered pair (x, y), which are called co-ordinates of that point. x is called x co-ordinate or abscissa and y is called y co-ordinate or ordinate. The two perpendicular lines xox' and yoy' divide the plane in four regions which are called quadrants, numbered as shown in the figure.

Let us look at some of the formulae linked with points now.

Distance Formula :

In rectangular Cartesian coordinate system

The distance between the points A(x₁,y₁) and B(x₂,y₂) is = $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$.

Example #1: Find the value of x, if the distance between the points (x, 8) and (4, 3) is 13

Solution : Let P(x, 8) and Q(4, 3) be the given points. Then PQ = 13 (given)

$$(x-4)^2 + (8-3)^2 = 13 \implies (x-4)^2 + 25 = 169 \implies x = 16 \text{ or } x = -8$$

Self practice problems :

- (1) Show that four points (0, -1), (6, 7) (-2, 3) and (8, 3) are the vertices of a rectangle.
- (2) Find the co-ordinates of the circumcentre of the triangle whose vertices are (8, 6), (8, -2) and (2, -2). Also find its circumradius.

Ans. (2) (5, 2), 5

Section Formula :

If P(x, y) divides the line joining A(x_1, y_1) & B(x_2, y_2) in the ratio m : n, then;

$$x = \frac{mx_2 + nx_1}{m + n}; y = \frac{my_2 + ny_1}{m + n}$$

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalaw	war Road, Kota (Raj.)-324005
Resonance	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free: 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSE-1

- **Notes :** (i) If $\frac{m}{n}$ is positive, the division is internal, but if $\frac{m}{n}$ is negative, the division is external.
 - (ii) If P divides AB internally in the ratio m : n & Q divides AB externally in the ratio m : n then P & Q are said to be harmonic conjugate of each other w.r.t. AB.

Mathematically,
$$\frac{2}{AB} = \frac{1}{AP} + \frac{1}{AQ}$$
 i.e. AP, AB & AQ are in H.P.

Example # 2 : Find the co-ordinates of the point which divides the line segment joining the points (2, 5) and (-3, 7) in the ratio 2 : 3 (i) internally and (ii) externally.

Solution :

Let P(x, y) be the required point.



Example # 3 : Find the co-ordinates of points which trisect the line segment joining (2, -3) and (4, 5). **Solution :** Let A (2, -3) and B(4, 5) be the given points. Let the points of trisection be P and Q. Then $AP = PQ = QB = \lambda$ (say)

 $\begin{array}{ll} \therefore & PB = PQ + QB = 2\lambda \text{ and } AQ = AP + PQ = 2\lambda \\ \Rightarrow & AP : PB = \lambda : 2\lambda = 1 : 2 \text{ and } AQ : QB = 2\lambda : \lambda = 2 : 1 \\ & \text{So P divides AB internally in the ratio } 1 : 2 \text{ while Q divides internally in the ratio } 2 : 1 \end{array}$

So P divides AB internally in the ratio 1:2 while Q divides internally in the ratio 2:1

 $\therefore \qquad \text{the co-ordinates of P are } \left(\frac{4+4}{1+2}, \frac{5-6}{1+2}\right) = \left(\frac{8}{3}, -\frac{1}{3}\right)$ and the co-ordinates of Q are $\left(\frac{8+2}{1+2}, \frac{10-3}{1+2}\right) = \left(\frac{10}{3}, \frac{7}{3}\right)$ Hence, the points of trisection are $\left(\frac{8}{3}, -\frac{1}{3}\right)$ and $\left(\frac{10}{3}, \frac{7}{3}\right)$

Self practice problems :

- (3) In what ratio does the point (4, 1) divide the line segment joining the points (1, -2) and (5, 2).
- (4) The three vertices of a parallelogram taken in order are (-2, 0), (4, 2) and (5, 3) respectively. Find the co-ordinates of the fourth vertex.
- **Ans.** (3) 3:1 internally (4) (-1, 1)

The ratio in which a given line divides the line segment joining two points :

Let the given line ax + by + c = 0 divide the line segment joining $A(x_1, y_1) \& B(x_2, y_2)$ in the ratio m : n, $m = ax_1 + by_1 + c$

then $\frac{m}{n} = -\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c}$. If A & B are on the same side of the given line then m/n is negative but if A & B are on the same side of the given line then m/n is negative but if A &

B are on opposite sides of the given line, then m/n is positive

Example #4 : Find the ratio in which the line joining the points A (1, 2) and B(-3, 4) is divided by the line x + y - 5 = 0.

Solution : Let the line x + y = 5 divides AB in the ratio k : 1 at P

 $\begin{array}{rcl} \therefore & \text{co-ordinate of P are } \left(\frac{-3k+1}{k+1}, \frac{4k+2}{k+1} \right) \\ & \text{Since P lies on } x+y-5=0 \\ \therefore & \frac{-3k+1}{k+1} + \frac{4k+2}{k+1} - 5=0 \qquad \Rightarrow \qquad k=-\frac{1}{2} \\ \therefore & \text{Required ratio is 1 : 2 externally. .} \\ \hline \underline{Aliter:} \text{ Let the ratio is m : n} \\ \therefore & \frac{m}{n} = -\frac{(1\times 1+1\times 2-5)}{1\times (-3)+1\times 4-5} = -\frac{1}{2} \qquad \therefore \qquad \text{ratio is 1 : 2 externally.} \end{array}$

Self practice problem :

(5) If the line $2x-3y + \lambda = 0$ divides the line joining the points A (-1, 2) & B(-3, -3) internally in the ratio 2 : 3, find λ .

Ans. ¹⁸

5

Slope Formula :

If θ is the angle at which a straight line is inclined to the positive direction of x-axis, & $0^{\circ} \le \theta < 180^{\circ}, \theta \ne 90^{\circ}$, then the slope of the line, denoted by m, is defined by m = tan θ . If θ is 90°, m does not exist, but the line is parallel to the y-axis. If $\theta = 0$, then m = 0 & the line is parallel to the x-axis. If A (x₁, y₁) & B (x₂, y₂), x₁ \ne x₂, are points on a straight line, then the slope m of the line is given

$$by: m = \left(\frac{y_1 - y_2}{x_1 - x_2}\right).$$

Example # 5 : What is the slope of a line whose inclination with the positive direction of x-axis is :

30º 90^⁰ 135° (i) (iii) (ii) Here $\theta = 30^{\circ}$ Solution : (i) Slope = $\tan \theta = \tan 30^{\circ} = \frac{1}{\sqrt{2}}$. Here $\theta = 90^{\circ}$ (ii) The slope of line is not defined. *.*.. (iii) Here $\theta = 135^{\circ}$ Slope = $\tan \theta = \tan 135^{\circ} = \tan (180^{\circ} - 45^{\circ}) = -\tan 45^{\circ} = -1$. *.*.. Example # 6: Find the slope of the line passing through the points : (2, 7) and (-3, 4) (i) (ii) (6, 9) and (-2, 7)Solution : (i) Let A = (2, 7) and B = (-3, 4)Slope of AB = $\frac{4-7}{-3-2} = \frac{3}{5}$ (Using slope = $\frac{y_2 - y_1}{x_2 - x_1}$) ÷. A = (6, 9), B = (-2, 7) \therefore Slope of AB = $\frac{7-9}{-2-6} = \frac{1}{4}$ (ii) Let Self practice problems : Find the value of x, if the slope of the line joining (1, 5) and (x, -7) is 4. (6)What is the inclination of a line whose slope is (7)(iv) $-1/\sqrt{3}$ (iii) -1 (i) 0 (ii) 1 (i) 0º. (ii) 45º. (iii) 135º, (iv) 150º Ans. (6)- 2 (7)Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005 kesonanc Website: www.resonance.ac.in | E-mail : contact@resonance.ac.in ADVSL - 3 Educating for better tomorrow Toll Free : 1800 258 5555 | CIN: U80302RJ2007PLC024029

Condition of collinearity of three points :

Points A (x_1 , y_1), B (x_2 , y_2), C(x_3 , y_3) are collinear if

(i)
$$m_{AB} = m_{BC} = m_{CA} \text{ i.e. } \left(\frac{y_1 - y_2}{x_1 - x_2} \right) = \left(\frac{y_2 - y_3}{x_2 - x_3} \right)$$

(ii) $\Delta ABC = 0$ i.e. $\begin{vmatrix} x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$

- (iii) $AC = AB + BC \text{ or } AB \sim BC$
- (iv) A divides the line segment BC in some ratio.

Example #7: Show that the points (-2, -1), (2, 7) and (5, 13) are collinear.

Solution : Let (-2, -1) (2, 7) and (5, 13) be the co-ordinates of the points A, B and C respectively.

Slope of AB =
$$\frac{7+1}{2+2}$$
 = 2 and Slope of BC = $\frac{13-7}{5-2}$ = 2

- \therefore Slope of AB = slope of BC
- AB & BC are parallel
- A, B, C are collinear because B is on both lines AB and BC.

Self practice problem :

(8) Prove that the points (a, 0), (0, b) and (1, 1) are collinear if $\frac{1}{a} + \frac{1}{b} = 1$

Area of a Triangle :

If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of triangle ABC, then its area is equal to $\Delta ABC = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$, provided the vertices are considered in the counter clockwise sense. The

above formula will give a (-) ve area if the vertices (x_i, y_i) , i = 1, 2, 3 are placed in the clockwise sense.

Note: Area of n-sided polygon formed by points (x_1, y_1) ; (x_2, y_2) ;; (x_n, y_n) is given by

1	(x ₁	x ₂	x ₂	x ₃		x _{n-1}	x _n	-	x _n	X ₁	
2	<u>у</u> 1	y ₂	y ₂	y ₃	'	У _{п–1}	y _n	I	y _n	y ₁).

Here vertices are taken in order.

Example # 8 : If the co-ordinates of two points A and B are (2, 1) and (4, -3)respectively. Find the co-ordinates of any point P if PA = PB and Area of \triangle PAB = 6.

Solution : Let the co-ordinates of P be (x, y). Then $\begin{array}{l} PA = PB \qquad \Rightarrow \qquad PA^2 = PB^2 \\ \Rightarrow \qquad (x-2)^2 + (y-1)^2 = (x-4)^2 + (y+3)^2 \qquad \Rightarrow \qquad x-2y=5 \qquad \dots(i) \\ Now, \quad Area of \, \Delta PAB = 6 \Rightarrow \qquad \frac{1}{2} \begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ 4 & -3 & 1 \end{vmatrix} = \pm 6 \qquad \Rightarrow \qquad 4x + 2y - 10 = \pm 12 \\ \Rightarrow \qquad 4x + 2y = 22 \quad \text{or} \quad 4x + 2y = -2 \qquad \Rightarrow \qquad 2x + y = 11 \quad \text{or} \quad 2x + y = -1 \\ \text{Solving } 2x + y = 11 \text{ and } x - 2y = 5 \text{ we get } x = \frac{27}{5} \text{ , } y = \frac{1}{5} \text{ .} \\ \text{Solving } 2x + y = -1 \text{ and } x - 2y = 5, \text{ we get } x = \frac{3}{5} \text{ , } y = -\frac{11}{5} \text{ .} \\ \text{Thus, the co-ordinates of P are } \left(\frac{27}{5}, \frac{1}{5}\right) \text{or} \quad \left(\frac{3}{5}, -\frac{11}{5}\right) \end{array}$

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalav	war Road, Kota (Raj.)-324005
A Resonance	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVGL - 4

Self practice problems :

- (9)The area of a triangle is 5. Two of its vertices are (2, 1) and (3, -2). The third vertex lies on y = x + 3. Find the third vertex.
- (10)The coordinates of A, B, C are (6, 3), (-3, 5) & (4,-2) respectively and p is any point

(x, y).Show that the ratio of the areas of the triangles PBC and ABC is

Ans.	(9)	$\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$	
------	-----	--	--

Equation of a Straight Line in various forms :

Now let us understand, how a line can be represented with the help of an algebraic equation. A moving point (point with variable co-ordinates) is assumed on the line and a link is established between its co-ordinates with the help of some given parameters. There are various forms of lines depending on the specified parameter

Point - Slope form :

 $y - y_{x} = m (x - x_{x})$ is the equation of a straight line whose slope is m & which passes through the point $(X_1, Y_1).$

Example #9: Find the equation of a line passing through (3, -4) and inclined at an angle of 150° with the positive direction of x-axis.

Here, m = slope of the line = tan 150° = tan $(90^{\circ} + 60^{\circ}) = -\cot 60^{\circ} = -\frac{1}{\sqrt{3}}$, x₁ = 3, y₁ = -4 Solution :

So, the equation of the line is $y - y_1 = m (x - x_1)$

i.e.
$$y + 4 = -\frac{1}{\sqrt{3}} (x - 3)$$

 $x + \sqrt{3}y + 4\sqrt{3} - 3 = 0$

Self practice problem :

Find the equation of s a line passing through P(-3, 5) and whose slope is -2. (11)

Ans. 2x + y + 1 = 0

Slope-intercept form :

y = mx + c is the equation of a straight line whose slope is m & which makes an intercept c on the y-axis.

Example #10: Find the equation of a line with slope -3 and cutting off an intercept of 5 units on negative direction of y-axis.

Solution : Here m = -3 and c = -5. So, the equation of the line is y = mx + cy = -3x - 5i.e. or 3x + y + 5 = 0

Self practice problem :

Find the equation of a straight line which cuts off an intercept of length 3 on y- axis and whose (12)slope is - 3.

Ans. 3x + y - 3 = 0

 $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}$ (x - x₁) is the equation of a straight line which passes through the Two point form : points $(x_1, y_1) \& (x_2, y_2)$.

Example #11: Find the equation of the line joining the points (3, 4) and (-2, 5) Solution :

Here the two points are $(x_1, y_1) = (3, 4)$ and $(x_2, y_2) = (-2, 5)$.

So, the equation of the line in two-point form is

$$y - 4 = \frac{5 - 4}{-2 - 3} (x - 3) \implies x + 5y = 23$$



Self practice problem :(13)Find the equations of the sides of the triangle whose vertices are (-1, 8), (4, -2) and (-5, -3).
Also find the equation of the median through (-1, 8)Ans.2x + y - 6 = 0, x - 9y - 22 = 0, 11x - 4y + 43 = 0, 21x + y + 13 = 0Determinant form : Equation of line passing through (x_1, y_1) and (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$ Example # 12 : Find the equation of line passing through (2, 4) & (-1, 3).Solution : $\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 3 & 1 \end{vmatrix} = 0$ \Rightarrow x - 3y + 10 = 0

Self practice problem :

(14) Find the equation of the passing through (-2, 3) & (-1, -1). **Ans.** 4x + y + 5 = 0

Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$ is the equation of a straight line which makes intercepts a & b on OX & OY respectively.

- **Example # 13 :** Find the equation of the line which passes through the point (-3, 8) and the sum of its intercepts on the axes is 7.
- **Solution :** Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ (i) This passes through (-3, 8), therefore $-\frac{3}{a} + \frac{8}{b} = 1$ (ii) It is given that $a + b = 7 \Rightarrow b = 7 - a$. Putting b = 7 - a in (ii), we get $-\frac{3}{a} + \frac{8}{7-a} = 1$ $\Rightarrow a^2 + 4a - 21 = 0 \Rightarrow a = 3, -7$ For a = 3, b = 4 and for a = -7, b = 14Putting the values of a and b in (i), we get the equations of the lines $\frac{x}{3} + \frac{y}{4} = 1$ and $\frac{x}{-7} + \frac{y}{14} = 1$ or 4x + 3y = 12 and 2x - y + 14 = 0

Self practice problem :

(15) Find the equation of the line through (2, 3) so that the segment of the line intercepted between the axes is bisected at this point.

Ans. 3x + 2y = 12.

Perpendicular/Normal form :

 $x\cos \alpha + y\sin \alpha = p$ (where $p > 0, 0 \le \alpha < 2\pi$) is the equation of the straight line where the length of the perpendicular from the origin O on the line is p and this perpendicular makes an angle α with positive x-axis.

Example #14 : Find the equation of the line which is at a distance 3 from the origin and the perpendicular from the origin to the line makes an angle of 30° with the positive direction of the x-axis.

Solution : Here p = 3, $\alpha = 30^{\circ}$

...

Equation of the line in the normal form is

$$x \cos 30^{\circ} + y \sin 30^{\circ} = 3 \text{ or } x \frac{\sqrt{3}}{2} + \frac{y}{2} = 3 \text{ or } \sqrt{3} x + y = 6$$



Self practice problem :

(16) The length of the perpendicular from the origin to a line is 7 and the line makes an angle of 150° with the positive direction (clock-wise) of y-axis. Find the equation of the line.

Ans. $\sqrt{3} x - y + 14 = 0$

General Form : ax + by + c = 0 is the equation of a straight line in the general form

In this case, slope of line =
$$-\frac{a}{b}$$

x - intercept = $-\frac{c}{a}$, y - intercept = $-\frac{c}{b}$

Example #15 : Find slope, x-intercept & y-intercept of the line 2x - 3y + 5 = 0.

Solution : Here,
$$a = 2, b = -3, c = 5$$

 \therefore slope $= -\frac{a}{b} = \frac{2}{3}$
x-intercept $= -\frac{c}{a} = -\frac{c}{3}$
y-intercept $= \frac{5}{3}$

Self practice problem :

(17) Find the slope, x-intercept & y-intercept of the line 3x - 5y - 8 = 0.

 $\frac{5}{2}$

Ans. (17) $\frac{3}{5}, \frac{8}{3}, -\frac{8}{5}$

Parametric form :

 $P(r) = (x, y) = (x_1 + r \cos \theta, y_1 + r \sin \theta) \text{ or } \frac{x - x_1}{\cos \theta} = \frac{y - y_1}{\sin \theta} = r \text{ is the equation of the line in}$

parametric form, where 'r' is the parameter whose absolute value is the distance of any point (x, y) on the line from the fixed point (x_1, y_1) on the line.

Remark : The above form is derived from point-slope form of line.

$$y - y_1 = m (x - x_1)$$
 where $m = \tan \theta$ \Rightarrow $y - y_1 = \frac{\sin \theta}{\cos \theta} (x - x_1)$

Example #16 : Find the equation of the line through the point A(1, 4) and making an angle of 45° with the positive direction of x-axis. Also determine the length of intercept on it between A and the line x + y - 10 = 0

Solution :

The equation of a line through A and making an angle of 45° with the x-axis is

$$\frac{x-1}{\cos 45^{\circ}} = \frac{y-4}{\sin 45^{\circ}} \text{ or } \frac{x-1}{\frac{1}{\sqrt{2}}} = \frac{y-4}{\frac{1}{\sqrt{2}}} \text{ or } x-y+3=0$$

Suppose this line meets the line x + y - 10 = 0 at P such that AP = r. Then the co-ordinates of P are given by

 $\frac{x-1}{\cos 45^{\circ}} = \frac{y-4}{\sin 45^{\circ}} = r \qquad \Rightarrow \qquad x = 1 + r \cos 45^{\circ}, y = 4 + r \sin 45^{\circ}$

$$\Rightarrow \qquad x = 1 + \frac{r}{\sqrt{2}} \ , \ y = 4 + \frac{r}{\sqrt{2}}$$

Thus, the co-ordinates of P are $\left(1+\frac{r}{\sqrt{2}}, \ 4+\frac{r}{\sqrt{2}}\right)$



Since P lies on x + y - 10 = 0, so 1 + $\frac{r}{\sqrt{2}}$ + 4 + $\frac{r}{\sqrt{2}}$ = 10 5 + $\sqrt{2}$ r = 10 \Rightarrow r = $\frac{5}{\sqrt{2}}$ \Rightarrow length AP = |r| = $\frac{5}{\sqrt{2}}$

Thus, the length of the intercept = $\frac{5}{\sqrt{2}}$.

Self practice problem :

- A straight line is drawn through the point A($\sqrt{3}$, 2) making an angle of $\pi/6$ with positive (18)direction of the x-axis. If it meets the straight line $\sqrt{3x} - 4y + 8 = 0$ in B, find the distance between A and B.
- Ans. 6 units

Angle between two straight lines in terms of their slopes:

If m₁ & m₂ are the slopes of two intersecting straight lines (m₁ m₂ \neq -1) & θ is the acute angle between them, then $\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$.

Notes : (i) Let m_1 , m_2 , m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$, then the tangent of interior angles of the Δ ABC formed by these lines are given by, tan A = $\frac{m_1 - m_2}{1 + m_1 m_2}$; tan

B =
$$\frac{m_2 - m_3}{1 + m_2 m_3}$$
 & tan C = $\frac{m_3 - m_1}{1 + m_3 m_1}$

(ii) The equation of lines passing through point (x_1, y_1) and making angle α with the line y = mx + c are given by :

 $(y - y_1) = \tan(\theta - \alpha) (x - x_1) \& (y - y_1) = \tan(\theta + \alpha) (x - x_1)$, where $\tan \theta = m$.



Example #17: The acute angle between two lines is $\pi/4$ and slope of one of them is -1/3. Find the slope of the other line.

If θ be the acute angle between the lines with slopes m₁ and m₂, then tan $\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$ Solution :

Let $\theta = \frac{\pi}{4}$ and $m_1 - 1/3$

$$\therefore \qquad \tan \frac{\pi}{4} = \left| \frac{-\frac{1}{3} - m_2}{1 - \frac{1}{3}m_2} \right| \qquad \Rightarrow \qquad 1 = \left| \frac{3m_2 + 1}{3 - m_2} \right| \qquad \Rightarrow \qquad \frac{3m_2 + 1}{3 - m_2} = 1 \text{ or } -1$$

Now
$$\frac{3m_2+1}{3-m_2} = 1$$
 \Rightarrow $m_2 = \frac{1}{2}$ and $\frac{3m_2+1}{3-m_2} = -1$ \Rightarrow $m_2 = -2$.

e of the other line is either 1/2 or -2

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhala	war Road, Kota (Raj.)-324005
	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
	Toll Free: 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L - 0

Example #18: Find the equation of the straight line which passes through the point (3,-2) and making angle 60° with the line $\sqrt{3} x + y = 1$.

Solution : Given line is $\sqrt{3} x + y = 1$..

 \Rightarrow

Slope of
$$(1) = -\sqrt{3}$$
.

Let slope of the required line be m. Also between these lines is given to be 60°.

$$\Rightarrow \qquad \tan 60^{\circ} = \left| \frac{m - \left(-\sqrt{3} \right)}{1 + m \left(-\sqrt{3} \right)} \right| \Rightarrow \qquad \sqrt{3} = \left| \frac{m + \sqrt{3}}{1 - \sqrt{3}m} \right| \Rightarrow \qquad \frac{m + \sqrt{3}}{1 - \sqrt{3}m} = \pm \sqrt{3}$$
$$\frac{m + \sqrt{3}}{1 - \sqrt{3}m} = \sqrt{3} \qquad \Rightarrow \qquad m + \sqrt{3} = \sqrt{3} - 3m \qquad \Rightarrow \qquad m = 0$$

the equation of the required line is y + 2 = 0

$$\frac{m+\sqrt{3}}{1-\sqrt{3}m} = -\sqrt{3} \qquad \qquad \Rightarrow \qquad m = \sqrt{3}$$

 \therefore The equation of the required line is $y + 2 = \sqrt{3} (x-3) \Rightarrow y \sqrt{3} = x - 2 - 3\sqrt{3}$

Self practice problem :

(19) A vertex of an equilateral triangle is (2, 3) and the equation of the opposite side is x + y = 2. Find the equation of the other sides of the triangle.

Ans. (19) $(2 - \sqrt{3})x - y + 2\sqrt{3} - 1 = 0$ and $(2 + \sqrt{3})x - y - 2\sqrt{3} - 1 = 0$.

Parallel Lines :

- (i) When two straight lines are parallel their slopes are equal. Thus any line parallel to y = mx + c is of the type y = mx + d, where 'd' is a parameter.
- (ii) Two lines ax + by + c = 0 and a'x + b'y + c' = 0 are parallel if $\frac{a}{a'} = \frac{b}{b'} \neq \frac{c}{c'}$.

Thus any line parallel to ax + by + c = 0 is of the type ax + by + k = 0, where k is a parameter.

(iii) The distance between two parallel lines with equations $ax + by + c_1 = 0$ &

ax + by + c₂ = 0 is =
$$\left| \frac{c_1 - c_2}{\sqrt{a^2 + b^2}} \right|$$

Note that coefficients of x & y in both the equations must be same.

(iv) The area of the parallelogram $=\frac{p_1p_2}{\sin\theta}$, where $p_1 \& p_2$ are distances between two pairs of

opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1 x + c_1$, $y = m_1 x + c_2$ and $y = m_2 x + d_1$, $y = m_2 x + d_2$ is

given by
$$\frac{(c_1 - c_2) (d_1 - d_2)}{m_1 - m_2}$$





...

...

Example #19 : Find the equation of the straight line that has y-intercept 5 and is parallel to the straight line 3x - 7y = 8.

- **Solution :** Given line is 3x 7y = 8
 - Slope of (1) is $\frac{3}{7}$

The required line is parallel to (1), so its slope is also 3/7, y-intercept of required line = 5 By using y = mx + c form, the equation of the required line is

$$y = \frac{3}{7} x + 5$$
 or $3x - 7y + 35 = 0$

Example #20 : Two sides of a square lie on the lines 5x - 12y + 6 = 0 and 5x - 12y = 20. What is its area ?

Solution : Clearly the length of the side of the square is equal to the distance between the parallel lines 5x - 12y + 6 = 0......(i) and 5x - 12y = 20(ii) Now, Distance between the parallel lines $= \frac{|6+20|}{\sqrt{5^2 + (-12)^2}} = 2$

Thus, the length of the side of the square is 2 and hence its area = 4

Example # 21 : Find the area of the parallelogram whose sides are x + 2y + 3 = 0, 3x + 4y - 5 = 0, 2x + 4y + 5 = 0 and 3x + 4y - 10 = 0

$$y = -\frac{1}{2}x - \frac{3}{2}$$

$$y = -\frac{3}{4}x + \frac{10}{4}$$

$$y = -\frac{1}{2}x - \frac{5}{4}$$

$$y = -\frac{3}{4}x + \frac{5}{4}$$

$$y = -\frac{1}{2}x - \frac{5}{4}$$

$$Here, \quad c_1 = -\frac{3}{2}, \qquad c_2 = -\frac{5}{4}, \qquad d_1 = \frac{10}{4}, d_2 = \frac{5}{4}, m_1 = -\frac{1}{2}, m_2 = -\frac{3}{4}$$

$$Area = \left| \frac{\left(-\frac{3}{2} + \frac{5}{4}\right)\left(\frac{10}{4} - \frac{5}{4}\right)}{\left(-\frac{1}{2} + \frac{3}{4}\right)} \right| = \frac{5}{4} \text{ sq. units}$$

Self practice problem :

...

(20) Find the area of parallelogram whose sides are given by 4x - 5y + 1 = 0, x - 3y - 6 = 0, 4x - 5y - 2 = 0 and 2x - 6y + 5 = 0

Ans. (20)
$$\frac{51}{14}$$
 sq. units

Perpendicular Lines:

(i) When two lines of slopes $m_1 \& m_2$ are at right angles, the product of their slopes is -1,

i.e. $m_1 m_2 = -1$. Thus any line perpendicular to y = mx + c is of the form

$$y = -\frac{1}{m} x + d$$
, where 'd' is any parameter.

(ii) Two lines ax + by + c = 0 and a'x + b'y + c' = 0 are perpendicular if aa' + bb' = 0. Thus any line perpendicular to ax + by + c = 0 is of the form bx - ay + k = 0, where 'k' is any parameter.



Example #22: Find the equation of the straight line that passes through the point (3, 4) and perpendicular to the line 3x + 2y + 5 = 0

Solution : The equation of a line perpendicular to 3x + 2y + 5 = 0 is $2x - 3y + \lambda = 0$ (i) This passes through the point (3, 4) \therefore $3 \times 2 - 3 \times 4 + \lambda = 0 \Rightarrow \lambda = 6$ Putting $\lambda = 6$ in (i), we get 2x - 3y + 6 = 0, which is the required equation.

<u>Aliter</u> The slope of the given line is -3/2. Since the required line is perpendicular to the given line. So, the slope of the required line is 2/3. As it passes through (3, 4). So, its equation is $y - 4 = \frac{2}{3}(x - 3)$ or 2x - 3y + 6 = 0

Self practice problem :

(21) The vertices of a triangle are A(10, 4), B (-4, 9) and C(-2, -1). Find the equation of its altitudes. Also find its orthocentre.

Ans. (21) x - 5y + 10 = 0, 12x + 5y + 3 = 0, 14x - 5y + 23 = 0, $\left(-1, \frac{9}{5}\right)$

Position of the point (x_1, y_1) relative of the line ax + by + c = 0:

If $ax_1 + by_1 + c$ is of the same sign as c, then the point (x_1, y_1) lie on the origin side of ax + by + c = 0. But if the sign of $ax_1 + by_1 + c$ is opposite to that of c, the point (x_1, y_1) will lie on the non-origin side of ax + by + c = 0.

In general two points (x_1, y_1) and (x_2, y_2) will lie on same side or opposite side of ax + by + c = 0 according as $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ are of same or opposite sign respectively.

Example #23: Show that (2, -1) and (-3, 3) lie on the opposite sides of the line 2x - 3y + 5 = 0.

- **Solution :** At (2, -1), the value of 2x 3y + 5 = 4 + 3 + 5 = 12 > 0.
 - At (-3, 3), the value of 2x 3y + 5 = -6 9 + 5 = -10 < 0

 \therefore The points (2, -1) and (-3, 3) are on the opposite sides of the given line.

Self practice problems :

- (22) Are the points (3, -4) and (2, 6) on the same or opposite side of the line 3x 4y = 8?
- (23) Which one of the points (1, 1), (-1, 2) and (2, 3) lies on the side of the line 4x + 3y 5 = 0 on which the origin lies?
- **Ans.** (22) Opposite sides (23) (-1, 2)

Length of perpendicular from a point on a line :

The length of perpendicular from P(x₁, y₁) on ax + by + c = 0 is $\left| \frac{a x_1 + b y_1 + c}{\sqrt{a^2 + b^2}} \right|$.





Example #24 : Find the distance between the line 4x - 3y + 8 = 0 and the point (-2, 3)

Solution : The required distance =
$$\left|\frac{(-2) \times 4 - 3 \times 3 + 8}{\sqrt{4^2 + (-3)^2}}\right| = \frac{9}{5}$$

Example #25 : Find all points on x - y + 2 = 0 that lie at a unit distance from the line 12x - 5y + 9 = 0.

Solution : Note that the co-ordinates of an arbitrary point on x - y + 2 = 0 can be obtained by putting x = t (or y = t) and then obtaining y (or x) from the equation of the line, where t is a parameter. Putting x = t in the equation x - y + 2 = 0 of the given line, we obtain y = 2 + t. So, co-ordinates of an arbitrary point on the given line are P(t, 2 + t). Let P(t, 2 + t) be the required point. Then, distance of P from the line 12x - 5y + 9 = 0 is unity i.e.

$$\Rightarrow \qquad \left| \frac{12t - 5(t + 2) + 9}{\sqrt{12^2 + 5^2}} \right| = 1 \Rightarrow \qquad |7t - 1| = 13$$
$$\Rightarrow \qquad 7t - 1 = \pm 13 \Rightarrow \qquad t = 2 \text{ or } t = -12/7$$

Hence, required points are (2,4) or (-12/7,2/7)

Self practice problem :

(24) Find the length of the altitudes from the vertices of the triangle with vertices : (-1, 1), (5, 2) and (3, -1).

Ans. (24)
$$\frac{16}{\sqrt{13}}$$
, $\frac{8}{\sqrt{5}}$, $\frac{16}{\sqrt{37}}$

Reflection of a point about a line :

(i) Foot of the perpendicular from a point (x_1, y_1) on the line ax + by + c = 0 is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

(ii) The image of a point (x_1, y_1) about the line ax + by + c = 0 is

$$\frac{x - x_1}{a} = \frac{y - y_1}{b} = -2\left(\frac{ax_1 + by_1 + c}{a^2 + b^2}\right)$$

Example #26 : Find the foot of perpendicular of the line drawn from P (2, -3) on the line x - 2y + 5 = 0. **Solution :** Slope of PM = -2

 $\therefore \quad \text{Equation of PM is} \\ 2x + y = 1 \qquad \dots \dots (i)$ solving equation (i) with x - 2y + 5 = 0, we get co-ordinates of M (-3/5, 11/5) $\int_{M}^{P (2,-3)} \frac{P^{(2,-3)}}{M \times -2y+5 = 0}$ **Aliter** Here, $\frac{x-2}{1} = \frac{y+3}{-2} = -\frac{2+6+5}{1+4} \Rightarrow \frac{x-2}{1} = \frac{y+3}{-2} = -\frac{13}{5} \Rightarrow x = -3/5$, y = 11/5

Example #27: Find the image of the point P(-1, 2) in the line mirror 2x - 3y + 4 = 0.

Solution :

Let image of P is Q.

$$\therefore PM = MQ \& PQ \perp AB$$
Let Q is (h, k)

$$\therefore M is \left(\frac{h-1}{2}, \frac{k+2}{2}\right)$$
It lies on $2x - 3y + 4 = 0$.

$$A M B$$

$$P(-1,2)$$

$$2x - 3y + 4 = 0$$

$$\begin{array}{rcl} & \ddots & 2 \ \left(\frac{h-1}{2}\right) - 3 \ \left(\frac{k+2}{2}\right) + 4 = 0. \\ & \text{or} & 2h-3k=0 & \dots \\ & \text{slope of } PQ = \frac{k-2}{h+1} \\ & PQ \perp AB \\ & \ddots & \frac{k-2}{h+1} \times \frac{2}{3} = -1. \Rightarrow & 3h+2k-1=0. & \dots \\ & \text{soving (i) & (ii), we get } h = \frac{3}{13} \ , k = \frac{2}{13} \\ & & \ddots & \text{Image of } P(-1,2) \text{ is } Q\left(\frac{3}{13},\frac{2}{13}\right) \\ \hline \textbf{Aliter} & \text{The image of } P(-1,2) \text{ about the line} \\ & 2x-3y+4=0 \text{ is } \frac{x+1}{2} = \frac{y-2}{-3} = -2 \ \frac{[2(-1)-3(2)+4]}{2^2+(-3)^2} \\ & \frac{x+1}{2} = \frac{y-2}{-3} = \frac{8}{13} \\ \Rightarrow & 13x+13 = 16 \Rightarrow x = \frac{3}{13} \ \& & 13y-26 = -24 \ \Rightarrow y = \frac{2}{13} \\ & & \ddots & \text{image is } \left(\frac{3}{13},\frac{2}{13}\right) \end{array}$$

Self practice problems :

(25) Find the foot of perpendicular of the line drawn from (-2, -3) on the line 3x - 2y - 1 = 0.

(26) Find the image of the point (1, 2) in y-axis.

Ans.
$$(25)\left(\frac{-23}{13}, \frac{-41}{13}\right)$$
 (26) $(-1, 2)$

Centroid, Incentre & Excentre :

If A (x_1, y_1) , B (x_2, y_2) , C (x_3, y_3) are the vertices of triangle ABC, whose sides BC, CA, AB are of lengths a, b, c respectively, then the co-ordinates of the special points of triangle ABC are as follows :

Centroid G =
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Incentre I = $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$, and
Excentre (to A) I₁ = $\left(\frac{-ax_1 + bx_2 + cx_3}{-a + b + c}, \frac{-ay_1 + by_2 + cy_3}{-a + b + c}\right)$ and so on.

Notes : (i)

: (i) Incentre divides the angle bisectors in the ratio, (b+c) : a; (c+a) : b & (a+b) : c.

- (ii) Incentre and excentre are harmonic conjugate of each other w.r.t. the angle bisector on which they lie.
- (iii) Orthocentre, Centroid & Circumcentre are always collinear & centroid divides the line joining orthocentre & circumcentre in the ratio 2 : 1.
- (iv) In an isosceles triangle G, O, I & C lie on the same line and in an equilateral triangle, all these four points coincide.
- (v) In a right angled triangle orthocentre is at right angled vertex and circumcentre is mid point of hypotenuse
- (vi) In case of an obtuse angled triangle circumcentre and orthocentre both are out side the triangle.

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhala	war Road, Kota (Raj.)-324005
A Resonance	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSE - 15

(i)

Example # 28: Find the co-ordinates of (i) centroid (ii) in-centre of the triangle whose vertices are (0, 0), (5, 0) and (0, 12).

Solution :

We know that the co-ordinates of the centroid of a triangle whose angular points are
$$(x_1, y_1), (x_2, y_2) (x_3, y_3)$$
 are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$
So the co-ordinates of the centroid of a triangle whose vertices are (0, 0), (5, 0) and (0, 12) are $\left(\frac{0+5+0}{3}, \frac{0+0+12}{3}\right)$ or $\left(\frac{5}{3}, 4\right)$.
Let A (0, 0), B (5, 0) and C(0, 12) be the vertices of triangle ABC.

(ii) Let A (0, 0), B (5, 0) and C(0, 12) be the vertices of triangle ABC. Then c = AB = 5, b = CA = 12and a = BC = 13.

The co-ordinates of the in-centre are $\left(\frac{ax_1+bx_2+cx_3}{a+b+c}, \frac{ay_1+by_2+cy_3}{a+b+c}\right)$

or
$$\left(\frac{13 \times 0 + 12 \times 5 + 5 \times 0}{5 + 12 + 13}, \frac{13 \times 0 + 12 \times 0 + 5 \times 12}{5 + 12 + 13}\right)$$
 or (2, 2)

c′

Self practice problems :

- (27) Two vertices of a triangle are (3, -5) and (-7, 4). If the centroid is (2, -1), find the third vertex.
- (28) Find the co-ordinates of the centre of the circle inscribed in a triangle whose vertices are (-36, 7), (20, 7) and (0, -8)
- **Ans.** (27) (10, -2) (28) (-1, 0)

Bisectors of the angles between two lines:

Equations of the bisectors of angles between the lines ax + by + c = 0 &

$$a'x + b'y + c' = 0 (ab' \neq a'b) are : \frac{ax + by + c}{\sqrt{a^2 + b^2}} = \pm \frac{a'x + b'y + c}{\sqrt{a'^2 + b'^2}}$$

Note : Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y + c_1 = 0 & a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Example #29 : Find the equations of the bisectors of the angle between the straight lines 3x + y + 1 = 0 and x + 3y + 1 = 0.

Solution : The equations of the bisectors of the angles between 3x+y+1=0 and x+3y+1=0 are $\frac{3x+y+1}{\sqrt{3^2+1^2}}$

$$=\pm \frac{x+3y+1}{\sqrt{1^2+3^2}}$$
 or $3x + y + 1 = \pm (x + 3y + 1)$

Taking the positive sign, we get x = y as one bisector Taking the negative sign, we get 2x + 2y + 1 = 0 as the other bisector.

Self practice problem :

(29) Find the equations of the bisectors of the angles between the following pairs of straight lines 3x + 4y + 13 = 0 and 12x - 5y + 32 = 0

Ans. (29) 21x - 77y - 9 = 0 and 99x + 27y + 329 = 0

Methods to discriminate between the acute angle bisector & the obtuse angle bisector:

(i) If θ be the angle between one of the lines & one of the bisectors, find tan θ .

If $|\tan \theta| < 1$, then 2 $\theta < 90^\circ$ so that this bisector is the acute angle bisector.

If $|\tan \theta| > 1$, then we get the bisector to be the obtuse angle bisector.

(ii) Let $L_1 = 0 \& L_2 = 0$ are the given lines $\& u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0 \& L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0 \& u_2 = 0$ as shown in figure. If, $|p| < |q| \Rightarrow u_1$ is the acute angle bisector. $|p| > |q| \Rightarrow u_1$ is the obtuse angle bisector. $|p| = |q| \Rightarrow$ the lines $L_1 \& L_2$ are perpendicular. $L_1 = 0$

$$u_{1} = 0$$

(iii) If aa' + bb' < 0, then the equation of the bisector of this acute angle is

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

If, however, aa' + bb' > 0, the equation of the bisector of the obtuse angle is :

$$\frac{ax + by + c}{\sqrt{a^2 + b^2}} = + \frac{a'x + b'y + c'}{\sqrt{a'^2 + b'^2}}$$

Example #30 : For the straight lines 2x - y + 1 = 0 and x - 2y - 2 = 0, find the equation of the (i) bisector of the obtuse angle between them;

(ii) bisector of the acute angle between them;

Solution : 2x - y + 1 = 0(1) and x - 2y - 2 = 0(2) Here $a_1 = 2, a_2 = 1, b_1 = -1, b_2 = -2$ Now $a_1a_2 + b_1b_2 = 4 > 0$

 \therefore bisector of the obtuse angle between lines (1) and (2) will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = \frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

x + y + 3 = 0

and the equation of the bisector of the acute angle will be

$$\frac{2x - y + 1}{\sqrt{2^2 + (-1)^2}} = -\frac{x - 2y - 2}{\sqrt{1^2 + (-2)^2}}$$

or 3x - 3y = 1

Self practice problem :

or

(30) Find the equations of the bisectors of the angles between the lines x + y - 3 = 0 and 7x - y + 5 = 0 and state which of them bisects the acute angle between the lines.

Ans. (30) x - 3y + 10 = 0 (bisector of the obtuse angle); 6x + 2y - 5 = 0 (bisector of the acute angle)

Condition of Concurrency :

Three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ & $a_3x + b_3y + c_3 = 0$ are concurrent if $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Resonance [®]	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005		
	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in		
	Toll Free : 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L-15	

Alternatively : If three constants A, B & C (not all zero) can be found such that

 $A(a_1x + b_1y + c_1) + B(a_2x + b_2y + c_2) + C(a_3x + b_3y + c_3) \equiv 0$, then the three straight lines are concurrent.

Example #31 : If the straight lines x + 2y = 9, 3x + 5y = 5 and ax - by = 1 are concurrent. Then find the value of 35a + 22b

Solution :

Given lines are x + 2y = 9(1) 3x + 5y = 5(2) and ax - by = 1(3) Lines will be concurrent if $\Delta = 0$ $\Delta = \begin{vmatrix} 1 & 2 & -9 \\ 3 & 5 & -5 \\ a & -b & -1 \end{vmatrix} = 0 \Rightarrow 35a + 22b = -1$

Self practice problem :

(31) Find the value of m so that the lines 4x - 3y + 2 = 0, 3x + 4y - 4 = 0 and x + my + 6 = 0 may be concurrent. **Ans.** (31) -7

Family of Straight Lines :

The equation of a family of straight lines passing through the point of intersection of the lines,

 $L_1 \equiv a_1x + b_1y + c_1 = 0 \& L_2 \equiv a_2x + b_2y + c_2 = 0$ is given by $L_1 + k L_2 = 0$ i.e. $(a_1x + b_1y + c_1) + k(a_2x + b_2y + c_2) = 0$, where k is an arbitrary real number.

Note : (i) If $u_1 = ax + by + c$, $u_2 = a'x + b'y + d$, $u_3 = ax + by + c'$, $u_4 = a'x + b'y + d'$ then $u_1=0$; $u_2=0$; $u_3=0$; $u_4=0$ form a parallelogram. The diagonal BD can be given by $u_2u_3 - u_1u_4 = 0$.



(ii) The diagonal AC is also given by $u_1 + \lambda u_4 = 0$ and $u_2 + \mu u_3 = 0$, if the two equations are identical for some real λ and μ . [For getting the values of $\lambda \& \mu$ compare the coefficients of x, y & the constant terms].

Example #32 : If 3a + 2b + 5c = 0 and the set of lines ax + by + c = 0 passes through a fixed point . Find
co-ordinates of that point.Solution :3a + 2b + 5c = 0....... (i)ax + by + c = 0....... (ii)

ax + by + c = 0 Eliminating c, we get. ax + by $-\frac{1}{2}$ (3a + 2b) = 0

$$ax + by - \frac{1}{5} (3a + 2b) = 0 \Rightarrow a\left(x - \frac{3}{5}\right) + b\left(y - \frac{2}{5}\right) = 0 \Rightarrow \left(x - \frac{3}{5}\right) + \frac{b}{a}\left(y - \frac{2}{5}\right) = 0$$

It is of the form L₁ + λ L₂ = 0

Which passes through the point of intersection $\left(\frac{3}{5}, \frac{2}{5}\right)$ of L₁= 0 & L₂= 0 for all real values of a & b

Aliter: $3a + 2b + 5c = 0 \implies \frac{3}{5}a + \frac{2}{5}b + c = 0$ $\left(\frac{3}{5}, \frac{2}{5}\right)$ lies on the line ax + by + c = 0 Hence fixed point $\left(\frac{3}{5}, \frac{2}{5}\right)$

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhala	war Road, Kota (Raj.)-324005
	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
	Toll Free : 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-10

Example #33 : Obtain the equations of the lines passing through the intersection of lines 3x + 7y = 17 and x + 2y = 5 and is prependicular to the straight line 3x + 4y = 10.

Solution :	The equation of any line through the intersection of the	he given lines is
	$(x + 2y - 5) + \lambda (3x + 7y - 17) = 0$	
or	$x (3 \lambda + 1) + y (7\lambda + 2) - 17\lambda - 5 = 0$	(i)
	This is prependicular to the line $3x + 4y = 10$	
	$\therefore \left(-\frac{3\lambda+1}{7\lambda+2}\right)\left(-\frac{3}{4}\right) = -1 \Longrightarrow \lambda = -\frac{11}{37}$	

Putting this value of λ in (i), the equation of required line 4x - 3y + 2 = 0

Self practice problem :

- (32) Find the equation of the lines through the point of intersection of the lines x 3y + 1 = 0 and 2x + 5y 9 = 0 and whose distance from the origin is $\sqrt{5}$ Ans. (32) 2x + y - 5 = 0

A Pair of straight lines through origin:

- A homogeneous equation of degree two,
 "ax² + 2hxy + by² = 0" always represents a pair of straight lines passing through the origin if :
 - (a) $h^2 > ab$ \implies lines are real & distinct.
 - (b) $h^2 = ab$ \Rightarrow lines are coincident.

(c) $h^2 < ab$ \Rightarrow lines are imaginary with real point of intersection i.e. (0, 0)

$$y = m_x x$$

$$y = m_x x$$

$$y = m_x x$$

This equation is obtained by multiplying the two equations of lines $(m_1x - y) (m_2x - y) = 0$ $\Rightarrow m_1m_2x^2 - (m_1 + m_2) xy + y^2 = 0$

(ii) If $y = m_1 x \& y = m_2 x$ be the two equations represented by $ax^2 + 2hxy + by^2 = 0$, then;

$$m_1 + m_2 = -\frac{2h}{b} \& m_1 m_2 = \frac{a}{b}.$$

(iii) If θ is the acute angle between the pair of straight lines represented by,

$$ax^2 + 2hxy + by^2 = 0$$
, then $\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|$.

(iv) The condition that these lines are :

- (a) at right angles to each other is a + b = 0. i.e. co-efficient of $x^2 + co$ -efficient of $y^2 = 0$.
- (b) coincident is $h^2 = ab$.
- (c) equally inclined to the axis of x is h = 0.i.e. coeff. of xy = 0.

Note that a homogeneous equation of degree n represents n straight lines passing through origin.





0	
Example#34	: Show that the equation $18x^2 - 9xy + y^2 = 0$ represents a pair of distinct straight lines, each
	passing through the origin. Find the separate equations of these lines.
Solution :	The given equation is a homogeneous equation of second degree. So, it represents a pair of
	straight lines passing through the origin. Comparing the given equation with
	$ax^{2} + 2hxy + by^{2} = 0$, we obtain $a = 18$, $b = 1$ and $2h = -9$.
	$h^2 - ab = \frac{81}{4} - 18 = \frac{9}{4} > 0 \qquad \Rightarrow \qquad h^2 > ab$
	Hence, the given equation represents a pair of distinct lines passing through the origin.
	Now, $18x^2 - 9xy + y^2 = 0 \implies \left(\frac{y}{x}\right)^2 - 9 \left(\frac{y}{x}\right) + 18 = 0 \implies \left(\frac{y}{x} - 6\right) \left(\frac{y}{x} - 3\right) = 0$
	\Rightarrow -6=0 or -3=0 \Rightarrow y - 6x = 0 or y - 3x = 0
	So the given equation represents the straight lines $y - 3x = 0$ and $y - 6x = 0$.
Example#35	: Find the equations to the pair of lines through the origin which are perpendicular to the lines represented by $6x^2 - xy - 12y^2 = 0$.
Solution :	We have $6x^2 - xy - 12y^2 = 0$.
	$\Rightarrow 6x^2 - 9xy + 8xy - 12y^2 = 0 \Rightarrow (3x + 4y)(2x - 3y) = 0$
	$\Rightarrow 3x + 4y = 0 & 2x - 3y = 0$
Thus	the given equation represents the lines $3x + 4y = 0$ and $2x - 3y = 0$. The equations of the lines
passir	The grid equation of the model of the given lines are $4x - 3y = 0$ & $3x + 2y = 0$
there	combined equations $12x^2 - xy - 6y^2 = 0$
Example#36	: Find the angle between the pair of straight lines $x^2 - 3xy + 2y^2 = 0$

Solution : Given equation is $x^2 - 3xy + 2y^2 = 0$ Here a = coeff. of $x^2 = 1$, b = coeff. of $y^2 = 2$

and 2h = coeff. of xy = -3
$$\therefore$$
 h = $-\frac{3}{2}$

Now
$$\tan \theta = \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right| = \left| \frac{2\sqrt{\frac{9}{4} - 2}}{1 + 2} \right| = \frac{1}{3}$$

Where $\boldsymbol{\theta}$ is the acute angle between the lines.

$$\therefore$$
 acute angle between the lines is $\tan^{-1}\left(\frac{1}{3}\right)$ and obtuse angle between them is

$$\pi - \tan^{-1}\left(\frac{1}{3}\right)$$

Self practice problems :

(33) Find the area of the triangle formed by the lines $y^2 - 9xy + 18x^2 = 0$ and y = 9.

Ans. (33)
$$\frac{27}{4}$$
 sq. units

General equation of second degree representing a pair of Straight lines :

(i) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if :

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0$$
, i.e. if $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$.

Such an equation is obtained again by multiplying the two equation of lines $(a_1x + b_1y + c_1) (a_2x + b_2y + c_2) = 0$

(ii) The angle θ between the two lines representing by a general equation is the same as that between the two lines represented by its homogeneous part only.

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalaw	war Road, Kota (Raj.)-324005
	Website: www.resonance.ac.in E-mail : contact@resonance.ac.in	
	Toll Free : 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L - 18

Example #38 : Prove that the equation
$$6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$$
 represents a pair of straight lines.
Find the co-ordinates of their point of intersection.
Solution : Given equation is $6x^2 + 13xy + 6y^2 + 8x + 7y + 2 = 0$
Writing the equation (1) as a quadratic equation in x we have
 $6x^2 + (13y + 8) x + 6y^2 + 7y + 2 = 0$
 $\therefore x = \frac{-(13y + 8) \pm \sqrt{(13y + 8)^2 - 4.6(6y^2 + 7y + 2)}}{12}$
 $= \frac{-(13y + 8) \pm \sqrt{(169y^2 + 208y + 64 - 144y^2 - 168y - 48)}}{12}$
 $= \frac{-(13y + 8) \pm \sqrt{25y^2 + 40y + 16}}{12} = \frac{-(13y + 8) \pm (5y + 4)}{12}$
 $\Rightarrow 12x = -13y - 8 + 5y + 4$ or $12x = -13y - 8 - 5y - 4$
 $\Rightarrow 3x + 2y + 1 = 0$ or $2x + 3y + 2 = 0$
Hence equation (1) represents a pair of straight lines whose equation are
 $3x + 2y + 1 = 0$ (1) and $2x + 3y + 2 = 0$ (2)
Solving these two equations, the required point of intersection is

Self practice problem :

(35) Find the combined equation of the straight lines passing through the point (1, 1) and parallel to the lines represented by the equation $x^2 - 5xy+4y^2+x+2y-2 = 0$ and find the angle between them.

Ans. (35) $x^2 - 5xy + 4y^2 + 3x - 3y = 0$, $\tan^{-1}\left(\frac{3}{5}\right)$

Homogenization :

This method is used to write the joint equation of two lines connecting origin to the points of intersection of a given line and a given second degree curve. The equation of a pair of straight lines joining origin to the points of intersection of the line $L \equiv \ell x + my + n = 0$ and a second degree curve

$$S \equiv ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$

is
$$ax^2 + 2hxy + by^2 + 2gx\left(\frac{\ell x + my}{-n}\right) + 2fy\left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right)^2 = 0.$$

The equation is obtained by homogenizing the equation of curve with the help of equation of line.



- **Notes :** (i) Here we have written 1 as $\frac{\ell x + my}{-n}$ and converted all terms of the curve to second degree expressions
 - (ii) Equation of any curve passing through the points of intersection of two curves $C_1 = 0$ and $C_2 = 0$ is given by $\lambda C_1 + \mu C_2 = 0$, where $\lambda \& \mu$ are parameters.

Example #39 : All chords of the curve $2x^2 - 3y^2 + 4x - 2y = 0$ which subtend a right angle at the origin passes through a fixed point. Find that point

Solution : Let Ax + By = 1 be a chord of the curve $2x^2 - 3y^2 + 4x - 2y = 0$

Equation of the given curve is $2x^2 - 3y^2 + 4x - 2y = 0$

and equation of the chord is Ax + By = 1

....(i)



Making equation (1) homogeneous equation of the second degree in x any y with the help of

(1), we have
$$2x^2 - 3y^2 + 4x (Ax + By) - 2y(Ax + By) = 0$$

 $x^2(2 + 4A) + y^2 (-3 - 2B) + xy (4B - 2A) = 0$ (ii)
since line represented by (ii) are at right angle
 \Rightarrow coefficient of x^2 + coefficient of $y^2 = 0$
 $\Rightarrow 2 + 4A + (-3 - 2B) = 0$
 $4A - 2B = 1$
chord $Ax + By = 1$
 $x = 4$ $y = -2$
fixed point $(4, -2)$

Self practice problems :

- (36) Find the equation of the straight lines joining the origin to the points of intersection of the line 3x + 4y - 5 = 0 and the curve $2x^2 + 3y^2 = 5$.
- (37) Find the equation of the straight lines joining the origin to the points of intersection of the line lx + my + n = 0 and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.

Ans. (36) $x^2 - y^2 - 24xy = 0$ (37) $4alx^2 + 4amxy + ny^2 = 0$; 4al + n = 0



Exercise-1

A Marked questions are recommended for Revision.

PART - I : SUBJECTIVE QUESTIONS

Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope

- **A-1.** (i) Prove that the points (2a, 4a), (2a, 6a) and (2a + $\sqrt{3}a$, 5a) are the vertices of an equilateral triangle whose side is 2a.
 - (ii) Find the points which trisect the line segment joining the points (0, 0) and (9, 12).
- A-2. (i) In what ratio does the point $\left(\frac{1}{2}, 6\right)$ divide the line segment joining the points (3, 5) and (-7, 9)?
 - (ii) In which ratio P(2a 2, 4a 6)) divides Q(2a 3, 3a 7) and R(2a, 6a 4).
- A-3. (i) Find the value of λ such that points P(1,2), Q(-2, 3) and R(λ + 1, λ) are not forming a triangle ?
 - (ii) Find the ratio in which the line segment joining of the points (1, 2) and (-2, 3) is divided by the line 3x + 4y = 7
 - (iii) Find the harmonic conjugate of the point R (5, 1) with respect to points P (2, 10) and Q (6, -2).
- **A-4.** A and B are the points (3, 4) and (5, -2) respectively. Find the co-ordinates of a point P such that PA = PB and the area of the triangle PAB = 10.
- **A-5.** Find the area of the quadrilateral with vertices as the points given in each of the following : (i) (0, 0), (4, 3), (6, 0), (0, 3) (ii) (0, 0), (a, 0), (a, b), (0, b)

Section (B) : Different forms of straight lines and Angle between lines

- **B-1.** Reduce $x + \sqrt{3} y + 4 = 0$ to the :
 - (i) Slope intercepts form and find its slope and y-intercept.
 - (ii) Intercepts form and find its intercepts on the axes.
 - (iii) Normal form and find values of P and α .
- **B-2.** Find number of straight line passing through (2, 4) and forming a triangle of 16 sq. cm with the coordinate axis.
- **B-3.** Find the equation of the straight line that passes through the point A(-5, -4) and is such that the portion intercepted between the axes is divided by the point A in the ratio 1 : 2 (internally).
- **B-4.** The co-ordinates of the mid-points of the sides of a triangle are (2, 1), (5, 3) and (3, 7). Find the length and equation of its sides.
- **B-5.** Find the straight line cutting an intercept of one unit on negative x-axis and inclined at 45° (in anticlockwise direction) with positive direction of x-axis
- **B-6.** Through the point P(4, 1) a line is drawn to meet the line 3x y = 0 at Q where PQ = $\frac{11}{2\sqrt{2}}$. Determine the equation of line.
- **B-7.** A line having slope '1' is drawn from a point A(-3, 0) cuts a curve $y = x^2 + x + 1$ at P & Q. Find $\ell(AP) = \ell(AQ)$.

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	j.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-21

- **B-8.** Find the direction in which a straight line must be drawn through the point (1, 2), so that its point of intersection with the line x + y = 4 may be at a distance $\frac{\sqrt{6}}{3}$ from this point.
- **B-9.** Through the point (3, 4) are drawn two straight lines each inclined at 45° to the straight line x y = 2. Find their equations and also find the area of triangle bounded by the three lines.
- **B-10.** Find the equation of a straight lines which passes through the point (2, 1) and makes an angle of $\pi/4$ with the straight line 2x + 3y + 4 = 0
- **B-11.** From (1, 4) you travel $5\sqrt{2}$ units by making 135° angle with positive x-axis (anticlockwise) and then 4 units by making 120° angle with positive x-axis (clockwise) to reach Q. Find co-ordinates of point Q.
- **B-12.** The ends of the hypotenuse of a right angled triangle are (6, 0) and (0, 6), then find the locus of third vertex of triangle.
- **B-13.** A point moves in the x-y plane such that the sum of its distances from two mutually perpendicular lines is always equal to 3, then find the area enclosed by the locus of the point.
- **B-14.** One side of a rectangle lies along the line 4x + 7y + 5 = 0. Two of its vertices are (-3, 1) and (1, 1). Then find the equations of other sides.

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot, Area of Parallelogram

- C-1.2Plot the region
(i) $6x + 2y \ge 31$ (ii) $2x + 5y \le 10$ (iii) 8x + 3y + 6 > 0(iv) x > 2
- **C-2.** Find coordinates of the foot of perpendicular, image and equation of perpendicular drawn from the point (2, 3) to the line y = 3x 4.
- **C-3.** Starting at the origin, a beam of light hits a mirror (in the form of a line) at the point A(4, 8) and reflected line passes through the point B (8, 12). Compute the slope of the mirror.
- **C-4.** Find the nearest point on the line 3x + 4y 1 = 0 from the origin.
- **C-5.** Find the position of the origin with respect to the triangle whose sides are x + 1 = 0, 3x 4y 5 = 0, 5x + 12y 27 = 0.
- **C-6.** Find the area of parallelogram whose two sides are y = x + 3, 2x y + 1 = 0 and remaining two sides are passing through (0, 0).
- **C-7.** Is there a real value of λ for which the image of the point $(\lambda, \lambda 1)$ by the line mirror $3x + y = 6\lambda$ is the point $(\lambda^2 + 1, \lambda)$? If so find λ .
- **C-8.** Find the equations of two straight lines which are parallel to x + 7y + 2 = 0 and at $\sqrt{2}$ distance away from it.
- **C-9.** Prove that the area of the parallelogram contained by the lines 4y 3x a = 0, 3y 4x + a = 0,

4y - 3x - 3a = 0 and 3y - 4x + 2a = 0 is $\frac{2}{7}a^{2}$.

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

D-1. For triangle whose vertices are (0, 0), (5, 12) and (16, 12). Find coordinates of (i) Centroid (ii) Circumcentre

	leg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005										
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in										
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-22									

Excentre opposite to vertex (5, 12) (iv)

- (iii) Incentre D-2. Find the sum of coordinates of the orthocentre of the triangle whose sides are x = 3, y = 4 and 3x + 4y = 6.
- D-3. Find equations of altitudes and the co-ordinates of the othocentre of the triangle whose sides are 3x - 2y = 6, 3x + 4y + 12 = 0 and 3x - 8y + 12 = 0.
- D-4. A triangle has the lines $y = m_x$ and $y = m_x$ for two of its sides, where m_y , m_y are the roots of the equation $x^2 + ax - 1 = 0$, then find the orthocentre of triangle.
- D-5. Prove that the circumcentre, orthocentre, incentre & centroid of the triangle formed by the points A(-1, 11); B(-9, -8); C(15, -2) are collinear, without actually finding any of them.
- D-6. Find locus of centroid of $\triangle AOB$ if line AB passes through (3, 2), A and B are on coordinate axes.
- D-7. Find the locus of the centroid of a triangle whose vertices are (a cos t, a sin t), (b sin t, -b cos t) and (1, 0), where 't' is the parameter.
- Show that equation of the locus of a point which moves so that difference of its distance from two given D-8. points (ae, 0) and (-ae, 0) is equal to 2a is $\frac{x^2}{a^2} - \frac{y^2}{a^2(e^2 - 1)} = 1$.
- D-9. Find the locus of point of intersection of the lines x cos α + y sin α = a and x sin α - y cos α = b, where α is a parameter.
- D-10. The area of the triangle formed by the intersection of a line parallel to x-axis and passing through P(h, k) with the lines y = x and x + y = 2 is $4h^2$. Find the locus of the point P.

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

- E-1. Find equations of acute and obtuse angle bisectors of the angle between the lines 4x + 3y 7 = 0 and 24x + 7y - 31 = 0.
- E-2. Find the equation of a straight line passing through the point (4, 5) and equally inclined to the lines 3x = 4y + 7 and 5y = 12x + 6.
- E-3. The line x + 3y - 2 = 0 bisects the angle between a pair of straight lines of which one has equation x - 7y + 5 = 0, then find equation of other line
- Find the value of λ such that lines x + 2y = 3, 3x y = 1 and $\lambda x + y = 2$ can not form a triangle. E-4.æ
- Find values of λ for which line y = x + 1, $y = \lambda x + 2$ and $y = (\lambda^2 + \lambda 1) x + 3$ are con-current. E-5.
- E-6. Find the equation to the straight line passing through
 - The point (3, 2) and the point of intersection of the lines 2x + 3y = 1 and 3x 4y = 6. (i)
 - The intersection of the lines x + 2y + 3 = 0 and 3x + 4y + 7 = 0 and perpendicular to the straight (ii) line y - x = 8.
- E-7. Find the locus of the circumcentre of a triangle whose two sides are along the co-ordinate axes and third side passes through the point of intersection of the lines ax + by + c = 0 and $\ell x + my + n = 0$.

Section (F) : Pair of straight lines, Homogenization

F-1. If the slope of one of the lines represented by $ax^2 + 2hxy + by^2 = 0$ be the nth power of the other, then prove that $(ab^{n})^{n+1} + (a^{n}b)^{n+1} + 2h = 0$

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005										
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in										
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-23									

- **F-2.** For what value of λ does the equation $12x^2 10xy + 2y^2 + 11x 5y + \lambda = 0$ represent a pair of straight lines ? Find their
 - (i) equations, (ii) point of intersection, (iii) acute angle between them.
- **F-3.** (i) Find the integral values of 'h' for which $hx^2 5xy + 4hy^2 + x + 2y 2 = 0$ represents two real straight lines.
 - (ii) If the pair of lines represented by equation $k(k 3) x^2 + 16xy + (k + 1)y^2 = 0$ are perpendicular to each other, then find k.
- **F-4.** Find the equation of the straight lines joining the origin to the points of intersection of the line lx + my + n = 0 and the curve $y^2 = 4ax$. Also, find the condition of their perpendicularity.
- **F-5.** Find the condition that the diagonals of the parallelogram formed by the lines ax + by + c = 0; ax + by + c' = 0; a'x + b'y + c = 0, a'x + b'y + c' = 0 are at right angles. Also find the equation to the diagonals of the parallelogram.

PART - II : ONLY ONE OPTION CORRECT TYPE

- Section (A): Distance formula, section formula, Area of Triangle & polygon, Collinearity, slope
- A-1. Mid point of A(0, 0) and B(1024, 2048) is A₁. mid point of A₁ and B is A₂ and so on. Coordinates of A₁₀ are.
 (A) (1022, 2044)
 (B) (1025, 2050)
 (C) (1023, 2046)
 (D) (1, 2)
- A-2. If the points (k, 2 2k), (1 k, 2k) and (-k 4, 6 2k) be collinear, the number of possible values of k are (A) 4 (B) 2 (C) 1 (D) 3
- A-3. Given a $\triangle ABC$ with unequal sides. P is the set of all points which is equidistant from B & C and Q is the set of all point which is equidistant from sides AB and AC. Then $n(P \cap Q)$ equals : (A) 1 (B) 2 (C) 3 (D) Infinite
- A-4. A line segment AB is divided internally and externally in the same ratio (> 1) at P and Q respectively and M is mid point of AB.

Statement-1: MP, MB, MQ are in G.P.

Statement-2 AP, AB and AQ are in HP.

- (A) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is correct explanation for STATEMENT-1
- (B) STATEMENT-1 is true, STATEMENT-2 is true and STATEMENT-2 is not correct explanation for
 - STATEMENT-1Pro
- (C) STATEMENT-1 is true, STATEMENT-2 is false
- (D) STATEMENT-1 is false, STATEMENT-2 is true
- (E) Both STATEMENTS are false
- **A-5.** Find the area of the triangle formed by the mid points of sides of the triangle whose vertices are (2, 1), (-2, 3), (4, -3)
 - (A) 1.5 sq. units (B) 3 sq. units (C) 6 sq. units (D) 12 sq. units

Section (B) : Different forms of straight lines and Angle between lines

- **B-1.** A straight line through P (1, 2) is such that its intercept between the axes is bisected at P. Its equation is :
 - (A) x + 2y = 5 (B) x y + 1 = 0 (C) x + y 3 = 0 (D) 2x + y 4 = 0
- **B-2.** The number of integral points (integral point means both the coordinates should be integer) exactly in the interior of the triangle with vertices (0, 0), (0, 21) and (21, 0), is (A) 133 (B) 190 (C) 233 (D) 105

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	j.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV5L - 24



B-3. The line joining two points A (2, 0) and B (3, 1) is rotated about A in the anticlock wise direction through an angle of 15°. The equation of the line in the new position is :

(A) $x - \sqrt{3} y - 2 = 0$ (B) x - 2y - 2 = 0 (C) $\sqrt{3} x - y - 2\sqrt{3} = 0$ (D) $\sqrt{2} x - y - 2\sqrt{2} = 0$

- **B-4.** In a $\triangle ABC$, side AB has the equation 2x + 3y = 29 and the side AC has the equation x + 2y = 16. If the mid point of BC is (5, 6), then the equation of BC is (A) 2x + y = 16 (B) x + y = 11 (C) 2x - y = 4 (D) x + y = 10
- **B-5.** A square of side 'a' lies above the x-axis and has one vertex at the origin. The side passing through the origin makes an angle $\alpha \left(0 < \alpha < \frac{\pi}{4} \right)$ with the positive direction of x-axis. The equation of its diagonal not passing through the origin is : (A) y (cos $\alpha - \sin \alpha$) - x (sin $\alpha - \cos \alpha$) = a (B) y (cos $\alpha + \sin \alpha$) + x (sin $\alpha - \cos \alpha$) = a (C) y (cos $\alpha + \sin \alpha$) + x (sin $\alpha + \cos \alpha$) = a (D) y (cos $\alpha + \sin \alpha$) + x (cos $\alpha - \sin \alpha$) = a
- **B-6.** Find equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis. (A) $\sqrt{3} \times -y = 0$ (B) $\sqrt{3} \times +y = 8$ (C) $\times +\sqrt{3} \times y = 8$ (D) $\times -\sqrt{3} \times y = 8$
- **B-7.** The distance of the point (2, 3) from the line 2x 3y + 9 = 0 measured along a line x y + 1 = 0 is : (A) $5\sqrt{3}$ (B) $4\sqrt{2}$ (C) $3\sqrt{2}$ (D) $2\sqrt{2}$
- **B-8.** If a point P(x, y) from where line drawn cuts coordinate axes at A and B (with A on x-axis and B on yaxis) satisfies $\alpha \cdot \frac{x^2}{PB^2} + \beta \frac{y^2}{PA^2} = 1$, then $\alpha + \beta$ is (A) 1 (B) 2 (C) 3 (D) 4
- **B-9.** Two particles start from the point (2, -1), one moving 2 units along the line x + y = 1 and the other 5 units along the line x 2y = 4. If the particles move towards increasing y, then their new positions are (A) $(2-\sqrt{2},\sqrt{2}-1)$, $(2\sqrt{5}+2,\sqrt{5}-1)$ (B) $(2\sqrt{5}+2,\sqrt{5}-1)$, $(2\sqrt{2},\sqrt{2}+1)$

(C)
$$(2+\sqrt{2}, \sqrt{2}+1), (2\sqrt{5}+2, \sqrt{5}+1)$$
 (D) none of these

- **B-10.** Equation of a straight line passing through the origin and making with x axis an acute angle twice the size of the angle made by the line y = (0.2) x with the x axis, is : (A) y = (0.4) x (B) y = (5/12) x (C) 6y - 5x = 0 (D) 6y + 5x = 0
- **B-11.** The points A (1, 3) and C (5,1) are the oppositive vertices of rectangle. The equation of line passing through other two vertices and of gradient 2, is (A) 2x+y-8 = 0 (B) 2x-y-4 = 0 (C) 2x-y+4 = 0 (D) 2x+y = 0
- **B-12.** The point (-4,5) is the vertex of a square and one of its digonals is 7x-y+8 = 0. The equation of the other diagonal is (A) 7x-y+23 = 0 (B) 7y + x = 30 (C) 7y + x = 31 (D) x - 7y = 30
- B-13. ➤ Two straight lines x + 2y = 2 and x + 2y = 6 are given, then find the equation of the line parallel to given lines and divided distance between lines in the ratio 2 : 1 internally
 (A) 3x + 6y + 8 = 0
 (B) 3x + 6y = 14
 (C) 3x + 6y + 14 = 0
 (D) 3x + 2y = 10

Section (C) : Position of point, linear inequation, perpendicular distance, image & foot

C-1. The set of values of 'b' for which the origin and the point (1, 1) lie on the same side of the straight line,

 $\begin{array}{ll} a^2x + a \ by + 1 = 0 & \forall & a \in \mathsf{R}, \ b > 0 \ are: \\ (\mathsf{A}) \ b \in (2, \, 4) & (\mathsf{B}) \ b \in (0, \, 2) \\ \end{array} \ (\mathsf{C}) \ b \in [0, \, 2] \\ \end{array}$

(D) (2, ∞)

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005										
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in										
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV5L - 25									

- **C-2.** The point (a², a + 1) is a point in the angle between the lines 3x y + 1 = 0 and x + 2y 5 = 0containing the origin, then (A) $a \ge 1$ or $a \le -3$ (B) $a \in (-3, 0) \cup (1/3, 1)$
 - (C) $a \in (0, 1)$ (D) $a \in (-\infty, 0)$
- **C-3.** Find area of region represented by 3x + 4y > 12, 4x + 3y > 12 and x + y < 4

(A)
$$\frac{8}{7}$$
 (B) $\frac{4}{7}$ (C) $\frac{7}{8}$ (D) $\frac{8}{7}$

- **C-4.** The image of the point A (1, 2) by the line mirror y = x is the point B and the image of B by the line mirror y = 0 is the point (α , β), then : (A) $\alpha = 1, \beta = -2$ (B) $\alpha = 0, \beta = 0$ (C) $\alpha = 2, \beta = -1$ (D) $\alpha = 1, \beta = -1$
- **C-5.** The equations of the perpendicular bisector of the sides AB and AC of a \triangle ABC are x y + 5 = 0 and x + 2y = 0 respectively. If the point A is (1, -2), then the equation of the line BC is : (A) 14x + 23y = 40 (B) 14x - 23y = 40 (C) 23x + 14y = 40 (D) 23x - 14y = 40
- **C-6.** A light beam emanating from the point A(3, 10) reflects from the straight line 2x + y 6 = 0 and then passes through the point B(4, 3). The equation of the reflected beam is $x + 3y \lambda = 0$, then the value of λ is (A) 11 (B) 12 (C) 13 (D) 14

Section (D) : Centroid, orthocentre, circumcentre, incentre, excentre, Locus

(B) $\left(\frac{a}{2}, \frac{b}{2}\right)$

- **D-1.** The orthocentre of the triangle ABC is 'B' and the circumcentre is 'S' (a, b). If A is the origin, then the co–ordinates of C are :
 - (A) (2a, 2b)

C)	$\left(\sqrt{a^2 + b^2}\right)$, 0)	(D)

none

- **D-2.** A triangle ABC with vertices A (-1, 0), B (-2, 3/4) & C (-3, -7/6) has its orthocentre H. Then the orthocentre of triangle BCH will be : (A) (-3, -2) (B) (1, 3) (C) (-1, 2) (D) none of these
- **D-3.** Find locus of centroid of $\triangle ABC$, if B(1, 1), C(4, 2) and A lies on the line y = x + 3. (A) 3x + 3y + 1 = 0 (B) x + y = 3 (C) 3x - 3y + 1 = 0 (D) x - y = 3
- **D-4.** The locus of the mid-point of the distance between the axes of the variable line x cos α + y sin α = p, where p is constant, is

(A) $x^2 + y^2 = 4p^2$ (B) $\frac{1}{x^2} + \frac{1}{y^2} = \frac{4}{p^2}$ (C) $x^2 + y^2 = \frac{4}{p^2}$ (D) $\frac{1}{x^2} - \frac{1}{y^2} = \frac{2}{p^2}$

D-5. Find the locus of a point which moves so that sum of the squares of its distance from the axes is equal to 3.

(A)
$$x^2 + y^2 = 9$$
 (B) $x^2 + y^2 = 3$ (C) $|x| + |y| = 3$ (D) $x^2 - y^2 = 3$

D-6. A variable straight line passes through a fixed point (a, b) intersecting the co-ordinates axes at A & B. If 'O' is the origin, then the locus of the centroid of the triangle OAB is :

(A)
$$bx + ay - 3xy = 0$$
 (B) $bx + ay - 2xy = 0$ (C) $ax + by - 3xy = 0$ (D) $ax + by - 2xy = 0$

D-7. Consider a triangle ABC, whose vertices are A(-2, 1), B(1, 3) and C(x, y). If C is a moving point such that area of ∆ABC is constant, then locus of C is :
 (A) Straight line
 (B) Circle
 (C) Ray
 (D) Parabola



- D-8. If the equation of the locus of a point equidistant from the points (a_1, b_2) and (a_2, b_3) is $(a_1 - a_2) x + (b_1 - b_2) y + c = 0$, then the value of 'c' is :
 - (A) $\frac{1}{2} (a_2^2 + b_2^2 a_1^2 b_1^2)$ (B) $a_1^2 - a_2^2 + b_1^2 - b_2^2$ (C) $\frac{1}{2} (a_1^2 + a_2^2 + b_1^2 + b_2^2)$ (D) $\sqrt{a_1^2 + b_1^2 - a_2^2 - b_2^2}$

Section (E) : Angle Bisector, condition of concurrency, family of straight lines

- The equation of bisectors of two lines L, & L₂ are $2 \times -16 \times y 5 = 0$ and $64 \times +8 \times y + 35 = 0$. If the line E-1. L, passes through (-11, 4), the equation of acute angle bisector of L, & L_a is : (A) 2x - 16y - 5 = 0 (B) 64x + 8y + 35 = 0 (C) 2x + 16y + 5 = 0(D) 2x + 16y - 5 = 0
- E-2. The equation of the internal bisector of $\angle BAC$ of $\triangle ABC$ with vertices A(5, 2), B(2, 3) and C(6, 5) is (A) 2x + y + 12 = 0 (B) x + 2y - 12 = 0(C) 2x + y - 12 = 0 (D) 2x - y - 12 = 0
- E-3. The equation of the bisector of the angle between two lines 3x - 4y + 12 = 0 and 12x - 5y + 7 = 0which contains the point (-1, 4) is : (A) 21x + 27y - 121 = 0(C) 21x + 27y + 191 = 0

B)
$$21x - 27y + 121 = 0$$

D) $\frac{-3x + 4y - 12}{5} = \frac{12x - 5y + 7}{13}$

- E-4. The least positive value of t so that the lines x = t + a, y + 16 = 0 and y = ax (where a is real variable) are concurrent is (A) 2 (B) (C) 16 (D) 8 4
- E-5. Consider the family of lines $5x + 3y - 2 + \lambda_1 (3x - y - 4) = 0$ and $x - y + 1 + \lambda_2 (2x - y - 2) = 0$. Equation of a straight line that belong to both families is -(A) 25x - 62y + 86 = 0 (B) 62x - 25y + 86 = 0 (C) 25x - 62y = 86 (D) 5x - 2y - 7 = 0
- The equation of a line of the system $2x + y + 4 + \lambda (x 2y 3) = 0$ which is at a distance $\sqrt{10}$ units from E-6. point A(2, -3) is (B) 3x - y + 1 = 0 (C) y - 3x + 1 = 0(D) y + 3x - 2 = 0(A) 3x + y + 1 = 0
- **E-7.** The lines ax + by + c = 0, where 3a + 2b + 4c = 0, are concurrent at the point (D) $\left(\frac{3}{4}, \frac{1}{2}\right)$ (A) $\left(\frac{1}{2}, \frac{3}{4}\right)$ (C) (3, 1) (B) (1, 3)
- Find the equation of a straight line which passes through the point of intersection of the straight lines E-8. x + y - 5 = 0 and x - y + 3 = 0 and perpendicular to the straight line intersecting x-axis at the point (-2, 0) and the y-axis at the point (0, -3), (B) 2x - 3y + 10 = 0 (C) 2x - 5y + 10 = 0 (D) 2x + 5y + 10 = 0(A) 2x + 3y + 10 = 0
- **E-9.** The line parallel to the x-axis and passing through the intersection of the lines ax + 2by + 3b = 0and bx - 2ay - 3a = 0, where $(a, b) \neq (0, 0)$ is (A) Above the x-axis at a distance of 3/2 from it (B) Above the x-axis at a distance of 2/3 from it
 - (C) Below the x-axis at a distance of 3/2 from it (D) Below the x-axis at a distance of 2/3 from it

Section (F) : Pair of straight lines, Homogenization

- **F-1.** If the slope of one line of the pair of lines represented by $ax^2 + 10xy + y^2 = 0$ is four times the slope of the other line, then a =
 - (A) 1 (B) 2 (C) 4 (D) 16



The equation of second degree $x^2 + 2\sqrt{2}xy + 2y^2 + 4x + 4\sqrt{2}y + 1 = 0$ represents a pair of straight F-2. lines. The distance between them is (B) $\frac{4}{\sqrt{3}}$ (D) 2√3 (A) 4 (C) 2 F-3. The straight lines joining the origin to the points of intersection of the line 2x + y = 1 and curve $3x^{2} + 4xy - 4x + 1 = 0$ include an angle : (B) $\frac{\pi}{3}$ (A) $\frac{\pi}{2}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{6}$ **PART - III : MATCH THE COLUMN** Column -I Column – II 1.2 The points A(-4, -1), B (-2, -4), C (4, 0) Square (A) 🔈 (p) and D(2, 3) are the vertices of (B) The figure formed by the lines 2x + 5y + 4 = 0, (q) Rectangle 5x + 2y + 7 = 0, 2x + 5y + 3 = 0 and 5x + 2y + 6 = 0 is A quadrilateral whose diagonal are Rhombus (C) (r) perpendicular bisectors of each other must be (D) A quadrilateral whose diagonals are angle Parallelogram (s) bisector of sides and bisect each other must be Match The Followings : 2. Column –I Column – II (A) 🔊 P lies on the line y = x and Q lies on y = 2x. The equation for 36 (p) the locus of the mid point of PQ, if |PQ| = 4, is $25x^2 - \lambda xy + 13y^2 = 4$, then λ equals (B) The line $(K + 1)^2 x + ky - 2K^2 - 2 = 0$ passes through a point (α, β) regardless of value K. Then $(\alpha - \beta)$ equals : (q) 6 (C) Let two lines be $C_1 : 3x - 4y + 1 = 0$ and $C_2 : 8x + 6y + 1 = 0$ suppose (α, β) is a point which is at unit distance from each of the given lines. then sum of all possible value of α is -4/5 (r) If distance between the pair of parallel lines (D) 🕰 - 1 (s) $x^{2} + 2xy + y^{2} - 8ax - 8ay - 9a^{2} = 0$ is $25\sqrt{2}$, then 'a/5' is equal to

Exercise-2

A Marked questions are recommended for Revision.

PART - I : ONLY ONE OPTION CORRECT TYPE

1.	On the portion of the straight line $x + 2y = 4$ intercepted between the axes, a square is constructed on the side of the line away from the origin. Then the point of intersection of its diagonals has co-ordinates													
	(A) (2, 3)	(B) (3, 2)	(C) (3, 3)	(D) (2, 2)										
2.	If the straight line ax +	by + p = 0 and $x\cos\alpha$ +	ysin α = p enclosed an a	angle of $\frac{\pi}{4}$ and the line xsin α –										
	$y\cos\alpha = 0$ meets them a (A) 4	at the same point, then a (B) 3	² + b ² is (C) 2	(D) 1										

	teg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005									
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in									
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-20								

- **3.** A \triangle ABC is formed by the lines 2x 3y 6 = 0, 3x y + 3 = 0 and 3x + 4y 12 = 0. If the points P(α , 0) and Q(0, β) always lie on or inside the \triangle ABC, then ; (A) $\alpha \in [-1, 2] \& \beta \in [-2, 3]$ (B) $\alpha \in [-1, 3] \& \beta \in [-2, 4]$ (C) $\alpha \in [-2, 4] \& \beta \in [-3, 4]$ (D) $\alpha \in [-1, 3] \& \beta \in [-2, 3]$
- 4.2. If $P\left(1+\frac{t}{\sqrt{2}},2+\frac{t}{\sqrt{2}}\right)$ be any point on a line, then the range of values of t for which the point P lies

between the parallel lines x + 2y = 1 and 2x + 4y = 15 is

$$(A) - \frac{4\sqrt{2}}{3} < t < \frac{5\sqrt{2}}{6} \qquad (B) \ 0 < t < \frac{5\sqrt{2}}{6} \qquad (C) - \frac{4\sqrt{2}}{5} < t < 0 \qquad (D) - \frac{4\sqrt{2}}{3} < t < \frac{\sqrt{2}}{6}$$

5. The point A(4, 1) undergoes following transformations successively :
(i) reflection about line y = x
(ii) translation through a distance of 3 units in the positive direction of x-axis
(iii) rotation through an angle 105° in anti-clockwise direction about origin O. Then the final position of point A is

(A)
$$\left(\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$$
 (B) $\left(-2, 7\sqrt{2}\right)$ (C) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (D) $\left(-2\sqrt{6}, 2\sqrt{2}\right)$

6. Given two points A = (-2, 0) and B = (0, 4), then find coordinate of a point P lying on the line 2x - 3y = 9 so that perimeter of \triangle APB is least.

(A)
$$\left(\frac{42}{13}, -\frac{11}{3}\right)$$
 (B) $\left(\frac{84}{13}, -\frac{74}{13}\right)$ (C) $\left(\frac{21}{17}, -\frac{37}{17}\right)$ (D) $(0, -3)$

- **7.** A ray of light is sent from the point (1, 4). Upon reaching the x-axis, the ray is reflected from the point (3, 0). This reflected ray is again reflected by the line x + y = 5 and intersect y-axis at P. Find the co-ordinate of P.
 - $(A) \left(\frac{1}{2}, 0\right) \qquad (B) \left(0, \frac{-1}{2}\right) \qquad (C) \left(0, \frac{1}{3}\right) \qquad (D) \left(2, \frac{1}{-2}\right)$
- 8. AB is a variable line sliding between the co-ordinate axes in such a way that A lies on X-axis and B lies on Y-axis. If P is a variable point on AB such that PA = b, PB = a and AB = a + b, then equation of locus of P is

(A)
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$
 (B) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (C) $x^2 + y^2 = a^2 + b^2$ (D) $x^2 - y^2 = a^2 + b^2$

9.2 If the distance of any point (x, y) from the origin is defined as d(x, y) = max { |x|, |y|}, d(x, y) = a (where 'a' is non-zero constant), then the locus is
(A) A circle
(B) Straight line
(C) A square
(D) A triangle

10. ➤ Two ends A & B of a straight line segment of constant length 'c' slide upon the fixed rectangular axes OX & OY respectively. If the rectangle OAPB is completed. Then find locus of the foot of the perpendicular drawn from P to AB.
(A) x^{2/3} + y^{2/3} = c^{2/3}
(B) x^{2/3} + y^{2/3} = c^{1/3}
(C) x^{1/3} + y^{1/3} = c^{2/3}
(D) x^{1/3} + y^{1/3} = c^{1/3}

11. Let the line $\frac{x}{a} + \frac{y}{b} = 1$ cuts the x and y axes at A and B respectively. Now a line parallel to the given line cuts the coordinate axis at P and Q and points P and Q are joined to B and A respectively. The locus of intersection of the joining lines is $(A) \frac{x}{b} - \frac{y}{b} = 0$ $(B) \frac{x}{b} + \frac{y}{b} = 0$ $(C) \frac{x}{b} - \frac{y}{b} = 0$ $(D) \frac{x}{b} + \frac{y}{b} = 0$

(A)
$$\frac{x}{a} - \frac{y}{b} = 0$$
 (B) $\frac{x}{a} + \frac{y}{b} = 0$ (C) $\frac{x}{b} - \frac{y}{a} = 0$ (D) $\frac{x}{b} + \frac{y}{a} = 0$



- 12. A variable line whose slope is -2 cuts the x and y axes respectively at points A and C. A rhombus ABCD is completed such that vertex B lies on the line y = x. Then the locus of vertex D is (A) 2x + y = 1(B) x - y = 0(C) x + y = 0(D) x + 2y = 0
- 13. ABCD is a square away from origin of side length 'a'. Its side AB slides between x and y-axes in first quadrant with A on x-axis and B on y-axis. The locus of the foot of perpendicular dropped from the point E on the diagonal AC (where E is the midpoint of the side AD), is

(B) $(y - x)^2 + (x - 3y)^2 = \frac{a^2}{2}$ (A) $(y - x)^2 + (x - 3y)^2 = a^2$ (C) $(y - x)^2 + (x - 3y)^2 = \frac{a^2}{4}$ (D) None of these

- Let orthocentre of $\triangle ABC$ is (4, 6). If A = (4, 7) and B = (-2, 4), the find coordinates of vertex C 14_. (C) (-5, -4)(A) (5, 4) (B) (4, 5) (D) (-4, -5)
- Let Incentre of $\triangle ABC$ is (2, 5). If A = (1, 13) and B = (-4, 1), then find coordinates of C 15_. (A) (1, 10) (B) (10, 1) (C) (8, 2) (D) (9, 3)

The locus of circumcentre of the triangle formed by vertices A((-pq - p - q), -(1 + p)(1 + q)), 16.2 B(pq + p - q, (1 + p)(1 + q)), C(pq + q - p, (1 + p)(1 + q)) is (B) y - x = 0(C) $x^2 + y^2 = 1$ (D) xy = 1 (A) y + x = 0

- Let two sides of rectangle of area 20 units are along lines x y = 0 and x + y = 2, then the locus of point 17. of intersection of diagnals is
 - (A) $(x-1)^2 + (y-1)^2 = 10$ or $(y-1)^2 + (x-1)^2 = 10$ (B) $(x-1)^2 - (y-1)^2 = 10$ or $(y-1)^2 - (x-1)^2 = 10$ (C) $(x + 1)^2 - (y + 1)^2 = 10$ or $(y + 1)^2 - (x + 1)^2 = 10$ (D) $(x-1)^2 + (y-1)^2 = 10$ or $(y+1)^2 - (x+1)^2 = 10$
- 18. Area of the triangle formed by the line x + y = 3 and angle bisectors of the pair of straight lines $x^2 - y^2 + 2y = 1$ is (A) 2 sq units (B) 4 sq. units (C) 6 sq. units (D) 8 sq. units
- 19. Equation of the line pair through the origin and perpendicular to the line pair $xy - 3y^2 + y - 2x + 10 = 0$ is (A) $xy - 3y^2 = 0$ (B) $xy + 3x^2 = 0$ (C) $xy + 3y^2 = 0$ (D) $x^2 - y^2 = 0$

20.2 Find the equation of the two straight lines which together with those given by the equation $6x^2 - xy - y^2 + x + 12y - 35 = 0$ will make a parallelogram whose diagonals intersect in the origin. (A) $6x^2 - xy - y^2 - x - 12y - 35 = 0$ (B) $6x^2 - xy - y^2 - x - 12y + 35 = 0$ (C) $6x^2 - xy - y^2 - x + 12y - 35 = 0$ (D) $6x^2 - xy - y^2 + x - 12y - 35 = 0$

The curve passing through the points of intersection of $S_1 \equiv \frac{x^2}{x^2} + \frac{y^2}{x^2} - 1 = 0$ and 21.

 $S_2 \equiv x^2 + y^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines which are

- (A) equally inclined to the x axis
- (B) perpendicular to each other

(C) parallel to each other

(D) Not equally inclined to y-axis



PART-II: NUMERICAL VALUE QUESTIONS

INSTRUCTION:

- The answer to each question is NUMERICAL VALUE with two digit integer and decimal upto two digit.
- If the numerical value has more than two decimal places truncate/round-off the value to TWO decimal placed.

1. If the points $\left(\frac{a^3}{a-1}, \frac{a^2-3}{a-1}\right)$, $\left(\frac{b^3}{b-1}, \frac{b^2-3}{b-1}\right)$ and $\left(\frac{c^3}{c-1}, \frac{c^2-3}{c-1}\right)$ are collinear for three distinct values a,

b, c and $a \neq 1$, $b \neq 1$ and $c \neq 1$, then find the value of abc-(ab + bc + ac) + 3 (a + b + c).

- **2.** The complete set of values of λ for which point $(\lambda, \lambda + 1)$ is an interior points of ΔABC , where $A \equiv (0, 3)$, $B \equiv (-2, 0)$ and $C \equiv (6, 1)$ is (p,q) then value of $p^2 + q^2$ is
- **3.** Let ABC be a triangle such that the coordinates of the vertex A are (-3, 1). Equation of the median through B is 2x + y 3 = 0 and equation of the angular bisector of C is 7x 4y 1 = 0. Find the slope of line BC.
- 4. A(3, 4), B(0, 0) and C(3, 0) are vertices of ∆ABC. If 'P' is the point inside the ∆ABC, such that d(P, BC) ≤ min. {d(P, AB), d (P, AC)}. Then the maximum of d (P, BC) is. (where d(P, BC) represent distance between P and BC).
- **5.** Drawn from the origin are two mutually perpendicular straight lines forming an isosceles triangle together with the straight line 2x + y = 13. Then find the area of the triangle.
- 6. On the straight line y = x + 2, a point (a, b) is such that the sum of the square of distances from the straight lines 3x 4y + 8 = 0 and 3x y 1 = 0 is least, then find value of (a + b).
- 7. Parallelogram ABCD is cut by (2n –1) number of parallel lines in which one is diagnal AC. Distance between any two nearest lines is same which is also equal to distance of B, D from respective nearest

line anong these. Ratio of area of smallest triangle so formed to area of parallelogram is $\frac{1}{32}$. Find n.

- 8. A is a variable point on x-axis and B(0,b) is a fixed point. A equilateral triangle ABC is completed with vertex C away from origin. If the locus of the point C is $\alpha x + \beta y = b$, then $\alpha^3 + \beta^3$ is
- **9.** The portion of the line ax + by 1 = 0, intercepted between the lines ax + y + 1 = 0 and x + by = 0 subtends a right angle at the origin and the condition in a and b is $\lambda a + b + b^2 = 0$, then find value of λ .
- 10. If the straight lines joining the origin and the points of intersection of the curve $5x^2 24xy 6y^2 + 4x 2y + 3 = 0$ and x + ky 1 = 0 are equally inclined to the x-axis, then find the value of |k|.
- 11. If the points of intersection of curves $C_1 = 4y^2 2\lambda x^2 2xy 9x + 3$ and $C_2 = 2x^2 + 3y^2 - 4xy + 3x - 1$ subtends a right angle at origin, then find the value of λ .
- 12. Let the integral points inside or on the boundary of region bounded by straight lines as shown in figure is equal to k, then \sqrt{k} is equal to

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005								
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in								
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L-31							



PART - III : ONE OR MORE THAN ONE OPTIONS CORRECT TYPE

- 1. Point P(2, 3) lies on the line 4x + 3y = 17. Then find the co-ordinates of points farthest from the line which are at 5 units distance from the P. (A) (6, 6) (B) (6, -6) (C) (2, 0) (D) (-2, 0)
- 2. Find the equation of the line passing through the point (2, 3) & making intercept of length 2 units between the lines y + 2x = 3 & y + 2x = 5. (A) 3x - 4y = 18 (B) x = 2 (C) 3x + 4y = 18 (D) x + 2 = 0
- **3.** In a triangle ABC, co-ordinates of A are (1, 2) and the equations to the medians through B and C are x + y = 5 and x = 4 respectively. Then the co-ordinates of B and C will be (A) (-2, 7), (4, 3) (B) (7, -2), (4, 3) (C) (2, 7), (-4, 3) (D) (2, -7), (3, -4)
- **4.** A is a point on either of two rays $y + \sqrt{3} |x| = 2$ at a distance of $\frac{4}{\sqrt{3}}$ units from their point of intersection. The co-ordinates of the foot of perpendicular from A on the bisector of the angle between

them is/are

- (A) $\left(-\frac{2}{\sqrt{3}}, 2\right)$ (B) (0, 0) (C) $\left(\frac{2}{\sqrt{3}}, 2\right)$ (D) (0, 4)
- 5. If one side of a square is parallel to 3x 4y = 0 & its area being 16 while centre being (1, 1), then find equation of sides of square.

(A) 3x - 4y + 11 = 0 (B) 3x - 4y - 9 = 0 (C) 4x + 3y + 3 = 0 (D) 4x + 3y - 17 = 0

- 6. Find the equations of the sides of a triangle having (4, -1) as a vertex, if the lines x - 1 = 0 and x - y - 1 = 0 are the equations of two internal bisectors of its angles. (A) 2x - y + 3 = 0 (B) x + 2y - 6 = 0 (C) 2x + y - 7 = 0 (D) x - 2y - 6 = 0
- 7. A straight line L with negative slope passes through the point (8, 2) and cuts the positive coordinate axes at points P and Q, then the correct statement(s) among the following is/are (O is origin)
 - (A) The absolute minimum value of OP + OQ, where O is origin is $18\sqrt{2}$
 - (B) Minimum area of $\triangle OPQ$ is 32
 - (C) The absolute minimum value of OP + OQ, where O is origin is 18

(D) Area of $\triangle OPQ$ is minimum for slope $\left(\frac{-1}{4}\right)$.

- 8. The equation of the diagonals of a rectangle are y + 8x 17 = 0 and y 8x + 7 = 0. If the area of the rectangle is 8 sq. units, find the equation of the sides of the rectangle. (A) y = 1 (B) y = 9 (C) x = 1 (D) x = 2.
- 9. Two adjacent sides of a rhombus are 2x + 3y = a 5 and 3x + 2y = 4 2a and its diagonals intersect at the point (1, 2), then a can be –

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005								
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in								
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L-32							

- (A) -16 (B) 16 (C) $-\frac{10}{3}$ (D) $\frac{10}{3}$
- **10.** A line $L_1 = 3y 2x 6 = 0$ is rotated about its point of intersection with y-axis in clockwise direction to make it L_2 such that the area formed by L_1 , L_2 , x-axis and line x = 5 is $\frac{49}{3}$ sq units if its point of intersection with x = 5 lies below x-axis then points lying on the equation of L_2 are (A) (3, -1) (B) (4,2) (C) (1,1) (D) (3,3)
- **11.** Let $D(x_4, y_4)$ be a point such that ABCD is a square & M & P are the midpoints of the sides BC & CD respectively, then
 - (A) Ratio of the areas of $\triangle AMP$ and the square is 3 : 8
 - (B) Ratio of the areas of \triangle MCP & \triangle AMD is 1 : 1
 - (C) Ratio of the areas of $\triangle ABM \& \triangle ADP$ is 1 : 1
 - (D) Ratio of the areas of the quandrilateral AMCP and the square is 1:3

12. The equations of perpendicular of the sides AB & AC of \triangle ABC are x - y - 4 = 0 and 2x - y - 5 = 0 respectively. If the vertex A is (-2, 3) and circumcenter is $\left(\frac{3}{2}, \frac{5}{2}\right)$, then which of the

following is true. (A) equation of median of side AB is x - y + 1 = 0(B) centroid of triangle ABC is (3, 1) (C) vertex C is (4, 0) (D) Area of triangle ABC is 12.

- **13.** Triangle ABC lies in the cartesian plane and has an area of 70 sq. units. The coordinates of B and C are (12, 19) and (23, 20) respecitvely and the coordinates of A are (p, q). The median to the side BC has slope -5, then which can be corrected. (A) p + q = 47 (B) p + q = 27 (C) p - q = 17 (D) p - q = 13
- 14. All the points lying on or inside the triangle formed by the points (1, 3), (5, 6) and (-1, 2) satisfy

$$(A) \ 3x + 2y \ge 0 \qquad \qquad (B) \ 2x + y + 1 \ge 0 \qquad \qquad (C) \ 2x + 3y - 12 \ge 0 \qquad (D) \ 2x + 11 \ge 0$$

- **15.** A = (4, 2) and B = (2, 4) are two given points and a point P on the line 3x + 2y + 10 = 0 is given then which of the following is/are true.
 - (A) (PA + PB) is minimum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$ (B) (PA + PB) is maximum when $P\left(\frac{-14}{5}, \frac{-4}{5}\right)$ (C) |PA - PB | is maximum when P(- 22, 28) (D) (PA - PB) is minimum when P(- 22, 28).
- **16.** A line passing through P = $(\sqrt{3}, 0)$ and making an angle of 60° with positive direction of x-axis cuts the parabola $y^2 = x + 2$ at A and B, then :

(A)
$$PA + PB = \frac{2}{3}$$

(B) $|PA - PB| = \frac{2}{3}$
(C) $(PA) (PB) = \frac{4(2 + \sqrt{3})}{3}$
(D) $\frac{1}{PA} + \frac{1}{PB} = \frac{2 - \sqrt{3}}{2}$

17. Let $u = ax + by + a \sqrt[3]{b} = 0$, $v = bx - ay + b \sqrt[3]{b} = 0$, where $a, b \in R$ be two straight lines, then find the equations of the bisectors of the angles formed by $k_1u - k_2v = 0$ & $k_1u + k_2v = 0$ for non zero real $k_1 \& k_2$ are :

(A)
$$u = 0$$
 (B) $k_2 u + k_1 v = 0$ (C) $k_2 u - k_1 v = 0$ (D) $v = 0$



- **18.** The sides of a triangle are the straight line x + y = 1, 7y = x and $\sqrt{3} y + x = 0$. Then which of the following is an interior points of triangle ? (A) circumcentre (B) centroid (C) incentre (D) orthocentre
- **19.** The line ℓ_1 passing through the point (1, 1) and the ℓ_2 passes through the point (- 1, 1). If the difference of the slope of lines is 2. Find the locus of the point of intersection of the ℓ_1 and ℓ_2 .

(A)
$$x^2 = y$$
 (B) $y = 2 - x^2$ (C) $y^2 = x$ (D) $x = 2 - y^2$

20. The two lines pairs $y^2 - 4y + 3 = 0$ and $x^2 + 4xy + 4y^2 - 5x - 10y + 4 = 0$ enclose a 4 sided convex polygon, then the correct statement among the following is/are (A) Area of polygon is 6 (B) Length of its diagonals are $\sqrt{5} \& \sqrt{53}$ (C) Point of intersection of diagonals is (-2, 2) (D) Polygon is parallelogram.

21. If the distance between the lines represented $9x^2 - 24xy + 16y^2 + k(6x - 8y) = 0$ is 4, then k may be (A) 3 (B) 10 (C) -10 (D) 7

PART - IV : COMPREHENSION

Comprehenssion #1 (Q. NO. 1 to 3)

Let ABC be an acute angled triangle and AD, BE and CF are its medians, where E and F are the points (3, 4) and (1, 2) respectively and centroid of \triangle ABC is G(3, 2), then answer the following questions :

- 1.The equation of side AB is
(A) 2x + y = 4(B) x + y 3 = 0(C) 4x 2y = 0(D) none of these2.Co-ordinates of D are
(A) (7, -4)(B) (5, 0)(C) (7, 4)(D) (-3, 0)3.Height of altitude drawn from point A is (in units)
 - (A) $4\sqrt{2}$ (B) $3\sqrt{2}$ (C) $6\sqrt{2}$ (D) $2\sqrt{3}$

Comprehension # 2 (Q.No. 4 to 6)

If vertices of triangle are P(p₁, p₂), Q(q₁, q₂), R(r₁, r₂), then area of $\triangle PQR = \frac{1}{2} \begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix}$ and if P, Q, R

are collinear, then $\begin{vmatrix} p_1 & p_2 & 1 \\ q_1 & q_2 & 1 \\ r_1 & r_1 & 1 \end{vmatrix} = 0.$

On the basis of above answer the following question.

4. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of median through A.

(A)	x x ₁ x ₂	y 1 / ₁ 1 / ₂ 1	-	x x ₁ x ₃	y 1 y ₁ 1 y ₃ 1	= 0.	(B)	x x ₁ x ₂	у У ₁ У ₂	1 1 1	+	x x ₁ x ₃	у У ₁ У ₃	1 1 1	= 0
(C)	x y x ₁ y x ₂ y	/ 1 / ₁ 1 ₂ 1	+	x y x ₃ y x ₁ y	y 1 / ₃ 1 / ₁ 1	= 0	(D)	None	of tl	nese	Э				



5. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find equation of line through A and parallel to BC

(A)	x x ₁ x ₂	y y ₁ y ₂	1 1 1	_	x x ₁ x ₃	y y ₁ y ₃	1 1 1	= 0.	(B)	x x ₁ x ₂	y y ₁ y ₂	1 1 1	+	x x ₁ x ₃	y y ₁ y ₃	1 1 1	= 0
(C)	x x ₂ x ₁	y y ₂ y ₁	1 1 1	+	x x ₃ x ₁	y y ₃ y ₁	1 1 1	= 0	(D)	x x ₁ x ₂	у У ₁ У2	1 1 1	=	x x ₃ x ₁	y y ₃ y ₁	1 1 1	

6. If $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices of the triangle then find the equation of internal angle bisector through A

(A) b	x x ₁ x ₂	у У ₁ У ₂	1 1 1	- C	x x ₁ x ₃	y y ₁ y ₃	1 1 1	= 0	(B) c	x x ₁ x ₂	y y ₁ y ₂	1 1 1	+	b	x x ₁ x ₃	y y ₁ y ₃	1 1 1	= 0
(C) b	x x ₁ x ₂	y y ₁ y ₂	1 1 + 1	- c > x	x (₁) (₃)	y 1 y ₁ 1 / ₃ 1	=	0	(D) c	x x ₁ x ₂	y y ₁ y ₂	1 1 1	-	b	x x ₁ x ₃	y y ₁ y ₃	1 1 1	= 0

COMPREHENSION # 3 (7-9)

The equation of pair of straight lines bisecting the angles between the straight lines $ax^{2} + 2hxy + by^{2} = 0$ is $\frac{x^{2} - y^{2}}{a - b} = \frac{xy}{h}$ The equation of pair of straight lines bisecting the angles between the straight lines $ax^{2} + 2hxy + by^{2} + 2gx + 2fy + c = 0$ is $\frac{(x - \alpha)^{2} - (y - \beta)^{2}}{a - b} = \frac{(x - \alpha)(y - \beta)}{h}$ where (α, β) is point of intersecting of lines $ax^{2} + 2hxy + by^{2} + 2gx + 2fy + c = 0$

- 7. Find the combined equation of the bisectors of the angle between the lines represented by $3x^2 - 5xy + 4y^2 = 0$ (A) $5x^2 - 2xy - 5y^2 = 0$ (B) $5x^2 - 5y^2 + 4xy = 0$ (C) $5x^2 - 5y^2 + 2xy = 0$ (D) $5x^2 - 5y^2 - xy = 0$
- 8. Find the combined equation of the bisectors of the angle between the lines represented by $2x^2 - 3y^2 + xy - x + 11y - 10 = 0$ (A) $x^2 - y^2 + xy + 4x + 2 = 0$ (B) $x^2 - y^2 - 10xy + 8x + 12y - 10 = 0$ (C) $x^2 - y^2 - 5xy + 3x + 7y - 5 = 0$ (D) None of these

9. Let combined equation of lines ℓ_1 and ℓ_2 is $x^2 - 2pxy - y^2 = 0$ combined equation of lines ℓ_3 and ℓ_4 is $x^2 - 2qxy - y^2 + 2gx + 2fy + c = 0$. If angle between lines (ℓ_1, ℓ_3) , (ℓ_1, ℓ_4) , (ℓ_2, ℓ_3) and (ℓ_2, ℓ_4) is equal to 45°, then find the value of pq. (A) -4 (B) 4 (C) -1 (D) 1

八

Comprehension (10 & 11)

Origin of coordinate system xy is shifted to (h, k) to make new coordinate system XY. X and Y are parallel to x and y. New co-ordinates of point P(x, y) are P(X, Y). x, y, X, Y are related as given below. X = x - h



- **10.** Co-ordinates of (-7, 9) if origin is shifted to (2, 4) without changing direction of axes, are (A) (-5,13) (B) (-7,4) (C) (-9,5) (D) (-9,13)
- **11.** If co-ordinate axes are so translated such that ordinate of (4, 12) becomes zero while abscissa remains same. Then new coordinates of point (-8, -2) are (A) (-8, 14) (B) (-8, 10) (C) (-8, -14) (D) (-8, -10)

Exercise-3

PART - I : JEE (ADVANCED) / IIT-JEE PROBLEMS (PREVIOUS YEARS)

* Marked Questions may have more than one correct option.

Let O(0, 0), P(3, 4), Q(6, 0) be the vertices of the triangle OPQ. The point R inside the triangle OPQ is such that the triangles OPR, PQR, OQR are of equal area. The co-ordinates of R are
 [IIT-JEE - 2007, P-II, (3, - 1), 81]

			[III-JEE - 2007, P-II, (3, –
$(A)\left(\frac{4}{3},3\right)$	$(B)\left(3,\frac{2}{3}\right)$	$(C)\left(3,\frac{4}{3}\right)$	$(D)\left(\frac{4}{3},\frac{2}{3}\right)$

Lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q, respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R. [IIT-JEE - 2007, P-II, (3, -1), 81] STATEMENT - 1 : The ratio PR : RQ equals $2\sqrt{2} : \sqrt{5}$.

because

2.

STATEMENT – 2: In any triangle, bisector of an angle divides the triangle into two similar triangles.

- (A) Statement 1 is True, Statement 2 is True; Statement-2 is a correct explanation for Statement 1
- (B) Statement -1 is True, Statement 2 is True; Statement-2 is **NOT** a correct explanation for Statement 1
- (C) Statement 1 is True, Statement 2 is False
- (D) Statement 1 is False, Statement 2 is True



3.

Consider three points [IIT-JEE - 2008, P-II, (3, -1), 81] $P = (-\sin(\beta - \alpha), -\cos\beta), Q = (\cos(\beta - \alpha), \sin\beta)$ and $R = (\cos(\beta - \alpha + \theta), \sin(\beta - \theta))$, where $0 < \alpha, \beta, \theta < \frac{\pi}{4}$. Then,

(A) P lies on the line segment RQ (C) R lies on the line segment QP

(B) Q lies on the line segment PR (D) P, Q, R are non-collinear

- 4.2 The locus of the orthocentre of the triangle formed by the lines [IIT-JEE - 2009, Paper-2, (3, -1), 80] (1 + p) x - py + p (1 + p) = 0, (1 + q) x - qy + q(1 + q) = 0 and y = 0, where $p \neq q$, is (D) a straight line (A) a hyperbola (B) a parabola (C) an ellipse
- A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also 5. intersects the x-axis, then the equation of L is [IIT-JEE 2011, Paper-1, (3, -1), 80] (B) $y - \sqrt{3} x + 2 + 3\sqrt{3} = 0$ (A) $y + \sqrt{3} x + 2 - 3\sqrt{3} = 0$ (C) $\sqrt{3} v - x + 3 + 2 \sqrt{3} = 0$ (D) $\sqrt{3} y + x - 3 + 2\sqrt{3} = 0$
- 6. For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0and bx + ay + c = 0 is less than $2\sqrt{2}$. Then [JEE (Advanced) 2013, Paper-1, (2, 0)/60] (A) a + b - c > 0(B) a - b + c < 0(C) a - b + c > 0(D) a + b - c < 0
- For a point P in the plane, let $d_{1}(P)$ and $d_{2}(P)$ be the distance of the point P from the lines x y = 0 and 7.2 x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is [JEE (Advanced) 2014, Paper-1, (3, 0)/60]

PART - II : JEE (MAIN) / AIEEE PROBLEMS (PREVIOUS YEARS)

The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the 1.

equation	$\frac{x}{c} + \frac{y}{3} = 1$. Then the	ne distance between	L and K is	[AIEEE - 201	0 (8, –2), 144]
(1) $\sqrt{17}$	(2)	$\frac{17}{\sqrt{15}}$	(3) $\frac{23}{\sqrt{17}}$	(4) $\frac{23}{\sqrt{15}}$	

- The line $L_1 : y x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The 2. bisector of the acute angle between L₁ and L₂ intersects L₃ at R. [AIEEE - 2011, I(4, -1), 120] Statement-1 : The ratio PR : RQ equals $2\sqrt{2}$: $\sqrt{5}$ Statement-2: In any triangle, bisector of an angle divides the triangle into two similar triangles.
 - (1) Statement-1 is true, Statement-2 is true; Statement-2 is correct explanation for Statement-1
 - (2) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
 - (3) Statement-1 is true. Statement-2 is false
 - (4) Statement-1 is false, Statement-2 is true
- The lines x + y = |a| and ax y = 1 intersect each other in the first quadrant. Then the set of all 3.2 possible values of a is the interval : [AIEEE - 2011, II(4, -1), 120] (1) (0, ∞) (3) (−1, ∞) (4) (-1, 1](2) [1, ∞)
- If A(2, -3) and B(-2, 1) are two vertices of a triangle and third vertex moves on the line 2x + 3y = 9, 4. then the locus of the centroid of the triangle is : [AIEEE - 2011, II(4, -1), 120] (2) 2x + 3y = 1(3) 2x + 3y = 3(1) x - y = 1(4) 2x - 3y = 1

5. If the line 2x + y = k passes through the point which divides the line segment joining the points (1, 1) and (2, 4) in the ratio 3 : 2, then k equals : [AIEEE-2012, (4, -1)/120] (1) 29 (4) $\frac{11}{5}$

	(2) 5	(3) 6	
5			

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	ij.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-3/

6.	A line is drawn through the point $(1, 2)$ to meet the coordinate axes at P and Q such that it forms a triangle OPQ, where O is the origin. if the area of the triangle OPQ is least, then the slope of the line PQ is : [AIEEE-2012, (4, -1)/120]							
	$(1) - \frac{1}{4}$	(2) – 4	(3) – 2	$(4) - \frac{1}{2}$				
7.	A ray of light along x +	$\sqrt{3} y = \sqrt{3}$ gets reflected	d upon reaching x-axis, t	the equation of the reflected ray				
	is			[AIEEE - 2013, (4, -1),360]				
	(1) $y = x + \sqrt{3}$	(2) $\sqrt{3} y = x - \sqrt{3}$	(3) $y = \sqrt{3} x - \sqrt{3}$	(4) $\sqrt{3} y = x - 1$				
8.	The x-coordinate of the (0, 1) (1, 1) and (1, 0) is	e incentre of the triangle s :	e that has the coordinate	es of mid points of its sides as [AIEEE - 201 <mark>3, (4, –1),360]</mark>				
	(1) 2 + √2	(2) 2 - $\sqrt{2}$	(3) 1 + √2	(4) $1 - \sqrt{2}$ [AIEEE - 2013, (4, -1),360]				
9.	Let PS be the median of line passing through (1) $(1) 4x + 7y + 3 = 0$	of the triangle with vertic , – 1) and parallel to PS i (2) 2x – 9y – 11 = 0	es P(2, 2), Q (6, - 1), a s : [JEE (3) 4x - 7y - 11 = 0	nd R (7, 3). The equation of the E(Main) 2014, (4, - 1), 120] (4) 2x + 9y + 7 = 0				
10.	Let a, b, c and d be nor 5bx + 2by + d = 0 lies in	n-zero numbers. If the po n the fourth quadrant and	int of intersection of the I is equidistant from the t [JEE	lines 4ax + 2ay + c = 0 and wo axes then : E(Main) 2014, (4, – 1), 120]				
11.24	(1) 3bc – 2ad = 0 The number of points, vertices (0, 0), (0, 41) a (1) 901	(2) 3bc + 2ad = 0 having both co-ordinate and (41, 0) is (2) 861	(3) 2bc - 3ad = 0 s as integers, that lie in [JE (3) 820	 (4) 2bc + 3ad = 0 the interior of the triangle with E(Main) 2015, (4, -1), 120] (4) 780 				
12.	Two sides of a rhombu (–1, –2), then which on	s are along the lines, x – e of the following is a ver	y + 1 = 0 and $7x - y - 5$ tex of this rhombus ? [J]	= 0. if its diagonals intersect at EE(Main) 2016, (4, – 1), 120]				
	(1) (-3, -8)	$(2)\left(\frac{1}{3},-\frac{8}{3}\right)$	$(3)\left(-\frac{10}{3},-\frac{7}{3}\right)$	(4) (-3, -9)				
13.	Let k be an integer suc Then the orthocentre o	th that the triangle with v f this triangle is at the po	ertices (k, –3k), (5, k) ar int :	nd (–k, 2) has area 28 sq. units. JEE(Main) 2017, (4, – 1), 120]				
	$(1)\left(2,-\frac{1}{2}\right)$	$(2)\left(1,\frac{3}{4}\right)$	$(3)\left(1,-\frac{3}{4}\right)$	$(4)\left(2,\frac{1}{2}\right)$				
14.	A straight line through a straight line through a is the origin and the red	a fixed point (2,3) interse ctangle OPRQ is complet	ects the coordinate axes red, then the locus of R is	at distinct points P and Q. If O s Main) 2018 (4 - 1) 1201				
	(1) $3x + 2y = xy$	(2) $3x + 2y = 6x$	(3) $3x + 2y = 6$	(4) $2x + 3y = xy$				
15.	Consider the set all lin statements is true ? (1) The lines are not co	nes px + qy + r = 0 su [J oncurrent	ch that 3p + 2q + 4r = EE(Main) 2019, Online (2) The lines are all par	0. Which one of the following (09-01-19),P-1 (4, - 1), 120] callel				
	(3) The lines are concu	rrent at the point $\left(\frac{3}{4}, \frac{1}{2}\right)$	(4) Each the line passe	s through the origin.				

八





(1) 26x - 122y - 1675 = 0(3) 122y - 26x - 1675 = 0 [JEE(Main) 2019, Online (09-01-19),P-2 (4, - 1), 120] (2) 26x + 61y + 1675 = 0

- (4) 122y + 26x + 1675 = 0
- 17.2
 Two vertices of a triangle are (0, 2) and (4, 3). If its orthocentre is at the origin, then its third vertex lies in which quadrant ?

 (1) third
 (2) second

 (3) first
 (4) fourth

 18.
 Let O(0,0) and A (0,1) be two fixed points. Then the locus of a point P such that the perimeter of △AOP is 4, is :

 (1) $9x^2 - 8y^2 + 8y = 16$ (2) $8x^2 - 9y^2 + 9y = 18$

 (3) $9x^2 + 8y^2 - 8y = 16$ (4) $8x^2 + 9y^2 - 9y = 18$

 [JEE(Main) 2019, Online (08-04-19), P-1 (4, -1), 120]

- **19.** A straight line L at a distance of 4 units from the origin makes positive intercepts on the coordinate axes and the perpendicular form the origin to this line makes an angle of 60° with the line x + y = 0. Then an equation of the line L is -
 - (1) $(\sqrt{3} + 1)x + (\sqrt{3} 1)y = 8\sqrt{2}$
 - (3) $\sqrt{3}x + y = 8$

(2)
$$(\sqrt{3}-1)x + (\sqrt{3}+1)y = 8\sqrt{2}$$

 $(4) \quad x + \sqrt{3}y = 8$

[JEE(Main) 2019, Online (12-04-19), P-2 (4, -1), 120]

20. The locus of the mid-point of the perpendiculars drawn from points on the line, x = 2y to the line x = y is: (1) 3x - 2y = 0 (2) 3x - 3y = 0 (3) 5x - 7y = 0 (4) 7x - 5y = 0[JEE(Main) 2020, Online (07, 01, 20) B 2 (4, -1), 120]

[JEE(Main) 2020, Online (07-01-20), P-2 (4, -1), 120]



Answers



	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	aj.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L-40

<u>Strai</u>	ght Line	e											$\backslash \setminus$
E-5. E-7.	0 2xy(ma	E-6. ⊩bℓ) + x((i) an – ℓc)	43x – 2 + y (mc	9y = 71 – bn) =	0.	(ii)	x + y +	2 = 0				
Secu	,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,,								(3)	5)	(1		
F-2.১	$\lambda = 2,$	(i) 3x –	y + 2 =0	, 4x – 2y	/ + 1 = 0	, (ii) poir	nt of inte	rsection	$\left(-\frac{3}{2},-\frac{3}{2}\right)$	$\left[\frac{5}{2}\right]$, (iii)	$\tan^{-1}\left(\frac{1}{7}\right)$	<u>,</u>),	
F-3. F-5.	(i) h = 1 a² + b²	= (a') ² +	(ii) k = 1 (b')²⇒ (a	a + a')x -	F-4. + (b + b')	$4alx^2 + c + c$ PAR^2	4amxy + c' = 0 ⇒ Γ - II	- ny² = 0 (a – a') :	; 4al + n x + (b – l	= 0 c')y = 0	× ×	,	
Sectio	on (A):												
A-1.	(C)	A-2.	(B)	A-3.	(B)	A-4.	(A)	A-5.	(A)				
Sectio	on (B) :	:									_		
B-1.	(D)	B-2.	(B)	B-3.	(C)	B-4.	(B)	B-5.	(D)	B-6.	(B)	B-7.	(B)
B-0. Sactic	(B)	в-9.	(A)	B-10.	(В)	в-11.	(B)	В-12.	(C)	Б-13.	(B)		
C-1.	(B)	C-2.	(B)	C-3.	(A)	C-4.	(C)	C-5.	(A)	C-6.	(C)		
Sectio	on (D) :	:	()				(-)		()		(-)		
D-1.	(A)	D-2.	(D)	D-3.	(C)	D-4.	(B)	D-5.	(B)	D-6.	(A)	D-7.	(A)
D-8. Sootic	(A) >p (E)												
Secuc	лі (с) і (д)	F-2	(\mathbf{C})	F-3	(Δ)	F-4	(D)	E-5	(D)	F-6	(B)	F-7	(D)
E-8.	(A) (B)	E-9.	(C)	L-J.	(~)	L - T .	(D)	L-J.	(D)	L-0.	(D)	L-/.	(D)
Sectio	on (F) :	:	. ,										
F-1.	(D)	F-2.	(C)	F-3.	(A)	6							
						PAR1	r — III						
1.	$(A) \rightarrow$	(q, s) (B)	\rightarrow (r, s)	$(C) \rightarrow ($	(r, s) (D)	\rightarrow (r, s)		2. (A)	\rightarrow (p), ($B) \rightarrow (q)$	$(C) \rightarrow$	(r), (C	$(\mathfrak{s}) \rightarrow (\mathfrak{s})$
						EXERC	ISE - 2	2					
				1.1		PAR	т_і						
1.	(C)	2.	(C)	3.	(D)	4.	(A)	5.	(D)	6.	(C)	7.	(B)
8.	(A)	9.	(C)	10.	(A)	11.	(A)	12.	(C)	13.	(C)	14.	(A)
15.	(B)	16.	(A)	17.	(B)	18.	(A)	19.	(B)	20.	(A)	21.	(A)
						PAR	Г — II						
1.	00.00	2.	02.98	3.	18.00	4.	01.00	5.	33.80	6.	04.72 o	r 04.7	'3
7.	04.00	8. 🗸	04.19 o	r 04.20		9.	02.00	10.	02.60	11.	09.50	12.	18.19
						PAR1	- III – T						
1.	(AD)	2.	(BC)	3.	(B)	4.	(B)	5.	(ABCD)	6.	(ACD)	7.	(BCD)
8.	(ABCD)) 9.	(AD)	10.	(AC)	11.	(AC)	12.	(C)	13.	(ABD)	14.	(ABD)
15.	(AC)	16.	(BC)	17.	(AD)	18.	(BC)	19.	(AB)	20.	(ABD)	21.	(BC)
						PART	- IV						
1.	(A)	2.	(B)	3.	(C)	4.	(B)	5.	(A)	6.	(C)	4.	(B)
5.	(A)	6.	(C)	7.	(A)	8.	(D)	9.	(C)	10.	(C)	11.	(C)
					E	XERC	ISE - 3						
						PAR	T – I						
1.	(C)	2.	(C)	3.	(D)	4.	(D)	5.	(B)	6.	(A) or (C) or I	Bonus
7.	(6)		. /		、 /		、 /		、 /		., (,	
						PAR	Г — II						
1.	(3)	2.	(3)	3.	(2)	4.	(2)	5.	(3)	6.	(3)	7.	(2)
8.	(2)	9.	(4)	10.	(1)	11.	(4)	12.	(2)	13.	(4)	14.	(1)

八

 Resonance Educating for better tomorrow
 Reg. & Corp. Office : CG Tower, A-40 & 52, IPIA, Near City Mail, JnaiaWar Hoad, Kota (Raj.)-324005

 Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029
 ADVSL - 41

Stra	ight Lir	ıe /										——八
15.	(3)	16.	(1)	17.	(2)	18.	(3)	19.	(2)	20.	(3)	





Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005 Website : www.resonance.ac.in | E-mail : contact@resonance.ac.in Toll Free : 1800 200 2244 | 1800 258 5555 | CIN: U80302RJ2007PLC024029

ADVSL - 42

High Level Problems (HLP)

> Marked Questions may have for Revision Questions.

- **1.** The vertices of a triangle OBC are O(0,0) B(-3,-1) and C(-1,-3). Find the equation of line parallel to BC and intersecting the sides OB and OC, whose perpendicular distance from the point (0,0) is $\frac{1}{2}$.
- 2. A variable line, drawn through the point of intersection of the straight lines

 $\frac{x}{a} + \frac{y}{b} = 1$ and $\frac{x}{b} + \frac{y}{a} = 1$, meets the coordinate axes in A & B. Show that the locus of the mid point of AB is the curve 2xy(a + b) = ab(x + y).

- **3.** From the vertices A, B, C of a triangle ABC, perpendiculars AD, BE, CF are drawn to any straight line. Show that the perpendiculars from D, E, F to BC, CA, AB respectively are concurrent.
- 4. A triangle is formed by the lines whose equations are AB : x + y 5 = 0, BC : x + 7y 7 = 0 and CA : 7x + y + 14 = 0. Find the bisector of the interior angle at B and the exterior angle at C. Determine the nature of the interior angle at A and find the equation of the bisector.
- **5.** The coordinates of the feet of \perp from the vertices of a Δ on the opposite sides are (20, 25), (8, 16) and (8, 9). Then find the coordinates of a vertices of the Δ
- 6. Let P is any point inside the triangle ABC of side lengths 6, 5, 5 units and p₁, p₂, p₃ be the lengths of perpendiculars drawn from P to the sides of triangle. Find the maximum value of p₁.p₂.p₃.
- **7.** Let in $\triangle PAB$, A is (0, 0), B is (a, 0) and P is variable such that $\angle PBA$ is equal to three times $\angle PAB$ the, find the locus of P.
- 8. Through a fixed point any straight line is drawn meeting two given parallel straight lines in P and Q, through P and Q straight lines are drawn in fixed directions, meeting in R. Prove that the locus of R is straight line.
- **9.** Through the origin O a straight line is drawn to cut the lines $y = m_1 x + C_1$ and $y = m_2 x + C_2$ at Q and R. respectively. Find the locus of the point P on this variable line, such that OP is the geometric mean of OQ and OR.
- **10.** The sides of a triangle are $L_r = x\cos \alpha_r + y \sin \alpha_r p_r = 0$ for r = 1, 2, 3. Show that its orthocentre is given by $L_1 \cos (\alpha_2 - \alpha_3) = L_2 \cos (\alpha_3 - \alpha_1) = L_3 \cos (\alpha_1 - \alpha_2)$.
- **11.** A line passes through a fixed point R intersecting a fixed line at P. A point Q on RP such that $\frac{RP}{RQ}$ is constant. Then show that locus of Q is a straight line.
- **12.** A triangle ABC with a = 8, b = 6 and c = 10 slides on the coordinate axes with vertices A and B on the x-axis and the y-axis respectively. Find the locus of the vertex C.
- **13.** The line $L_1 = 4x + 3y 12 = 0$ intersects the x and the y-axis at A and B respectively. A variable line perpendicular to L_1 intersects the x and the y-axis at P and Q respectively. Find the locus of the circumcentre of triangle ABQ.
- 14. Show that the orthocentre of Δ formed by the straight lines, $ax^2 + 2hxy + by^2 = 0$ and $\ell x + my = 1$ is a

point (x', y') such that $\frac{x'}{\ell} = \frac{y'}{m} = \frac{a+b}{am^2 - 2h\ell m + b\ell^2}$.

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	aj.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV5L - 45

- **15.** The distance of a point (x_1, y_1) from each of two straight lines which passes through the origin of co–ordinates is δ , find the combined equation of these straight lines.
- **16.** A point moves so that the distance between the feet of the perpendiculars from it on the lines $ax^2 + 2h xy + by^2 = 0$ is a constant 2 d. Show that the equation to its locus is, $(x^2 + y^2) (h^2 - ab) = d^2 \{(a - b)^2 + 4h^2\}.$
- 17. Show that the pair of lines given by $a^2 x^2 + 2h(a + b) xy + b^2 y^2 = 0$ is equally inclined to the pair given by $ax^2 + 2hxy + by^2 = 0$.
- **18.** All the chords of the curve $3x^2 y^2 2x + 4y = 0$ which subtend a right angle at the origin are concurrent. Does this result also hold for the curve, $3x^2 + 3y^2 2x + 4y = 0$? If yes, what is the point of concurrence.
- **19.** The straight lines $(A^2 3B^2)x^2 + 8AB xy + (B^2 3A^2) y^2 = 0$ form a Δ with the line Ax + By + C = 0, then prove that

(i) Area of
$$\Delta$$
 is $\frac{C^2}{\sqrt{3}(A^2 + B^2)}$

- (ii) Δ is equilateral
- (iii) The orthocentre of Δ does not lie on one of its vertexs

Comprehension (Q. 20 & 21)

If coordinate system xy is being rotated through an angle θ in anti clock wise direction about the origin as shown in the diagram, Coordinates of P(x, y) has been change to P(X, Y) in new coordinate system XY, then x, y, X, Y are related as given below.



- **20.** If the axes are rotated through 60° in anticlockwise direction about origin. Find co-ordinats of point (2, 6) in new co-ordinate axes.
- **21.** If axes are rotated through an acute angle in clockwise direction about origin so that equation $x^2 + 2xy + y^2 2x + 2y = 0$ becomes free from xy in its new position, then find equation in new position
- **22.** Find the acute angle between two straight lines passing through the point M(-6, -8) and the points in which the line segment 2x + y + 10 = 0 enclosed between the co-ordinate axes is divided in the ratio 1 : 2 : 2 in the direction from the point of its intersection with the x axis to the point of intersection with the y axis.

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Ra	aj.)-324005
	Website : www.resonance.ac.in E-mail : contact@resonance.ac.in	
Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADVSL-44

23_. Let A lies on 3x - 4y + 1 = 0. B lies on 4x + 3y - 7 = 0 and C is (-2, 5). If ABCD is rhombus, then find locus of D.



DA = DC

$$(h + 2)^{2} + (A - 5)^{2} = \left(\frac{3h - 4k + 1}{5}\right)^{2}$$

$$25((x + 2)^{2} + (y - 5)^{2}) = (3x - 4y + 1)^{2}$$

- 24_. Let D is point on line $\ell_1 : x + y 2 = 0$ and S(3, 3) is fixed point. ℓ_2 is the line perpendicular to DS and passing through S. If M is another point on line ℓ_1 (other than D), then find locus of point of intersection of ℓ_2 and angle bisector of \angle MDS.
- **25_.** A variable line cuts the line 2y = x 2 and 2y = -x + 2 in points A and B respectively. If A lies in first quadrant, B lies in 4th quadrant and area of $\triangle AOB$ is 4, then find locus of (i) mid point of AB (ii) centroid of $\triangle OAB$
- **26.** An equilateral triangle PQR is formed where P (1, 3) is fixed point and Q is moving point on line x = 3. Find the locus of R.

Answers

1.
$$x + y + \frac{1}{\sqrt{2}} = 0.$$
 4. $3x + 6y - 16 = 0; 8x + 8y + 7 = 0; 12x + 6y - 11 = 0$

- **5.** (5, 10). (50, -5), (15, 30) **6.** $p_1p_2p_3 \le \frac{256}{75}$ **7.** $4x^3 4xy^2 3ax^2 + ay^2 = 0$.
- **9.** $(y m_1 x) (y m_2 x) = c_1 c_2$ **12.** $\frac{y}{x} = \frac{3}{4}$ or 3x 4y = 0 **13.** 6x 8y + 7 = 0
- **15.** $(y_1^2 \delta^2) x^2 2 x_1 y_1 xy + (x_1^2 \delta^2) y^2 = 0$ **18.** (1, -2), yes (1/3, -2/3)
- **20.** $(1 + 3\sqrt{3}, -\sqrt{3} + 3)$ **21.** $x^2 + \sqrt{2}y = 0$ **22.** $\pi/4$
- **23.** $25((x+2)^2 + (y-5)^2) = (3x-4y+1)^2$ **24.** $(x-3)^2 + (y-3)^2 = \left(\frac{x+y-2}{\sqrt{2}}\right)^2$
- **25.** (i) $(x-1)^2 4y^2 = 9$ (ii) $(x-2/3)^2 + 4y^2 = 4$ **26.** $(x-2) = \pm \sqrt{3} (y-3 \pm \sqrt{3})$

	Reg. & Corp. Office : CG Tower, A-46 & 52, IPIA, Near City Mall, Jhalawar Road, Kota (Raj.)-324005					
		Website : www.resonance.ac.in E-mail : contact@resonance.ac.in				
	Educating for better tomorrow	Toll Free : 1800 200 2244 1800 258 5555 CIN: U80302RJ2007PLC024029	ADV3L - 45			