HINTS & SOLUTIONS

TOPIC : TRIGONOMETRY

EXERCISE # 1

PART - I

Section (A)

A-1. $180^o = \pi^c$

A-2. $\pi^c = 180^o$

A-3. **(i)** $2 \times \frac{1}{4} - 2 \times \frac{1}{4} = 0$

(ii) $3 + 2 + 3 \times \frac{1}{3} = 6$

A-4. **(i)** $\cos 210° = \cos(180 + 30) = \frac{-\sqrt{3}}{2}$

A-5. **(i)** $\frac{(-\cos\theta)\cos\theta}{\sin\theta \times (-\sin\theta)} = \cot^2\theta$

(ii) $\cos\theta - \cos\theta + \cos\theta - \cos\theta = 0$

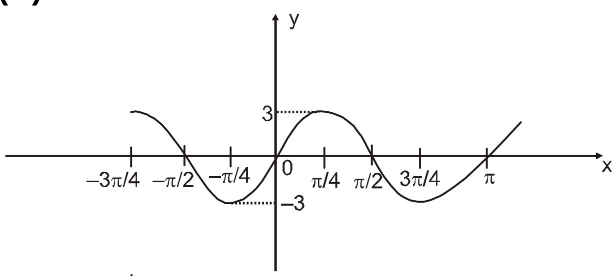
(iii) $\sin\theta \times \cos\theta\,[\tan\theta + \cot\theta] = 1$

A-6. $\tan\theta = -\frac{5}{12}$ $\because$ $\frac{3\pi}{2} < \theta < 2\pi$ $\Rightarrow$ $\sin\theta = -\frac{5}{13}$ and $\cot\theta = -\frac{12}{5}$

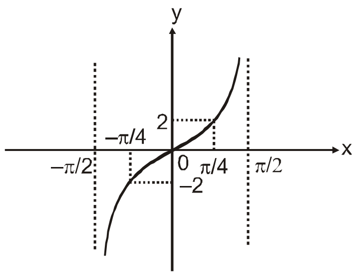
LHS $= \frac{-\ \sin\theta - \cot\theta}{-\operatorname{cosec}\theta - \operatorname{cosec}\theta} = \frac{\sin\theta + \cot\theta}{2\ \operatorname{cosec}\theta} = \frac{-\frac{5}{13} - \frac{12}{5}}{-2 \times \frac{13}{5}} = \frac{181}{338}$ = RHS

Section (B)

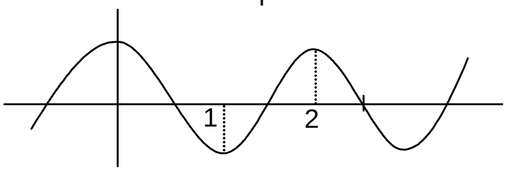
B-1. **(i)**



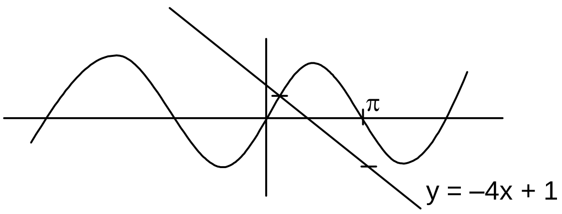
(ii)



(iii)



B-2.



B-3. $\tan\theta + \sec\theta = \frac{2}{3}$

$\sec\theta - \tan\theta = \frac{3}{2} \Rightarrow \quad 2\sec\theta = \frac{2}{3} + \frac{3}{2} = \frac{4+9}{6} \quad \sec\theta = \frac{13}{12}$

B-4. **(i)** $\sin(20^o + 40^o) = \sin 60^o = \frac{\sqrt{3}}{2}$

(ii) $\cos(100^o - 40^o) = \cos 60^o = \frac{1}{2}$

B-5. $\frac{1}{2}\left[\cos\frac{5\theta}{2} + \cos\frac{3\theta}{2} - \cos\frac{15\theta}{2} - \cos\frac{3\theta}{2}\right] = \frac{1}{2} \times 2\sin 5\theta \cdot \sin\frac{5\theta}{2}$

B-6 $A + B = 45° \quad \Rightarrow \quad \tan(A + B) = \tan(45^o) \quad \Rightarrow \quad \frac{\tan A + \tan B}{1 - \tan A \quad \tan B} = 1$

$\Rightarrow \quad \tan A + \tan B + \tan A \tan B = 1 \quad \Rightarrow \quad (1 + \tan A)(1 + \tan B) = 2$

put $\quad A = B = 22\frac{1^o}{2} \Rightarrow (1 + \tan 22\frac{1^o}{2})^2 = 2 \quad \Rightarrow \quad \tan 22\frac{1^o}{2} = \sqrt{2} - 1$

B-7. $a \sec\theta = 1 - b\tan\theta$ ... (1)

$a^2 \sec^2\theta = 5 + b^2 \tan^2\theta$ ... (2)

$(1)^2$ is $a^2 \sec^2\theta = 1 + b^2 \tan^2\theta - 2b\tan\theta$

From equation (2)

$5 + b^2\tan^2\theta = 1 + b^2\tan^2\theta - 2b\tan\theta \quad \Rightarrow \quad \tan\theta = -\frac{2}{b} \Rightarrow \quad \sec^2\theta = \frac{b^2+4}{b^2}$

so $\quad a^2 \frac{(b^2+4)}{b^2} = 5 + 4 \quad \Rightarrow \quad a^2b^2 + 4a^2 = 9b^2$

Section (C)

C-1. **(i)** $\sin(75^0 + 15^0) . \sin(75^0 - 15^0) = \sin 90^o . \sin 60^o = \sqrt{3}/2$

(ii) $\sin(45^0 + 15^0) . \sin(45^0 - 15^0) = \sin 60^o . \sin 30^o = \sqrt{3}/4$

C-2. **(i)** $4\sin 18° \cos 36° = 4\left(\frac{\sqrt{5}-1}{4} \times \frac{\sqrt{5}+1}{4}\right) = 4\left(\frac{4}{16}\right) = 1$

(ii) $\cos^2 72° - \sin^2 54° = \cos(126°)\cos(18°) = -\sin 36° \cos 18°$

$= -\frac{\sqrt{10-2\sqrt{5}}}{4} \times \frac{\sqrt{10+2\sqrt{5}}}{4} = -\frac{4\sqrt{5}}{16} = -\frac{\sqrt{5}}{4}$

(iii) We have, L.H.S. $= \cos^2 48^o - \sin^2 12^o$

$= \cos(48^o + 12^o)\cos(48^o - 12^o)[\cos^2 A - \sin^2 B = \cos(A + B)\cos(A - B)]$

$= \cos 60^o \cos 36^o = \frac{1}{2} \times \frac{\sqrt{5}+1}{4} = \frac{\sqrt{5}+1}{8} =$ R.H.S.

C-3. we have $a \cos\theta + b \sin\theta = c$ ...(i)

$\Rightarrow a\cos\theta = c - b\sin\theta \quad \Rightarrow a^2\cos^2\theta = (c - b\sin\theta)^2 \Rightarrow a^2(1 - \sin^2\theta) = c^2 - 2bc\sin\theta + b^2\sin^2\theta$

$\Rightarrow (a^2 + b^2)\sin^2\theta - 2bc\sin\theta + (c^2 - a^2) = 0$ ...(ii)

Since α, β are roots of equation (i). Therefore, $\sin\alpha$ and $\sin\beta$ are roots of equation

$\therefore \quad \sin\alpha\sin\beta = \dfrac{c^2 - a^2}{a^2 + b^2}$ ...(iii)

Again, $a\cos\theta + b\sin\theta = c$

$\Rightarrow \quad b\sin\theta = c - a\cos\theta$

$\Rightarrow \quad b^2\sin^2\theta = (c - a\cos\theta)^2$

$\Rightarrow \quad b^2(1 - \cos^2\theta) = (c - a\cos\theta)^2$

$\Rightarrow \quad (a^2 + b^2)\cos^2\theta - 2ac\cos\theta + c^2 - b^2 = 0$ ...(iv)

It is given that α, β are the the roots of equation (i), So, $\cos\alpha$, $\cos\beta$ are the roots of equation (iv).

$\therefore \quad \cos\alpha\cos\beta = \dfrac{c^2 - b^2}{a^2 + b^2}$ ...(v)

Now, $\quad \cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\Rightarrow \quad \cos(\alpha + \beta) = \dfrac{c^2 - b^2}{a^2 + b^2} - \dfrac{c^2 - a^2}{a^2 + b^2} = \dfrac{a^2 - b^2}{a^2 + b^2}$

C-4. LHS $= \sin\left\{\dfrac{\pi}{8} + \dfrac{A}{2} + \dfrac{\pi}{8} - \dfrac{A}{2}\right\} \sin\left\{\dfrac{\pi}{8} + \dfrac{A}{2} - \dfrac{\pi}{8} + \dfrac{A}{2}\right\} = \sin\dfrac{\pi}{4}.\sin A = \dfrac{1}{\sqrt{2}}\sin A$ = RHS

C-5. LHS $= \cos^2\alpha + \cos(\alpha + \beta)\{\cos\alpha\cos\beta - \sin\alpha\sin\beta - 2\cos\alpha\cos\beta\}$

$= \cos^2\alpha - \cos(\alpha + \beta).\cos(\alpha - \beta) = \cos^2\alpha - \cos^2\alpha + \sin^2\beta = \sin^2\beta$ = RHS

C-6. **(i)** $\dfrac{\sin^2 A - \sin^2 B}{\sin A\cos A - \sin B\cos B} = \dfrac{\sin(A + B)\sin(A - B)}{\frac{1}{2}\sin 2A - \frac{1}{2}\sin 2B} = \dfrac{2\sin(A + B)\sin(A - B)}{2\cos(A + B)\sin(A - B)} = \tan(A + B)$

(ii) $\cot(A + 15^o) - \tan(A - 15^o) = \dfrac{\cos(A + 15^o)}{\sin(A + 15^o)} - \dfrac{\sin(A - 15^o)}{\cos(A - 15^o)}$

$= \dfrac{\cos(A + 15^o)\cos(A - 15^o) - \sin(A + 15^o)\sin(A - 15^o)}{\sin(A + 15^o)\cos(A - 15^o)}$

$= \dfrac{(\cos^2 A - \sin^2 15^o) - (\sin^2 A - \sin^2 15^o)}{\frac{1}{2}(\sin 2A + \sin 30^o)} = \dfrac{2\cos 2A}{\sin 2A + \frac{1}{2}} = \dfrac{4\cos 2A}{2\sin 2A + 1}$

C-7. $0 < \theta < \dfrac{\pi}{4} \quad \Rightarrow \quad$ LHS $= \sqrt{2 + \sqrt{2 \times 2\cos^2 2\theta}} = \sqrt{2 + 2|\cos 2\theta|}$

$0 < \theta < \dfrac{\pi}{4} \quad \Rightarrow \quad 0 < 2\theta < \dfrac{\pi}{2}$

$\therefore \quad \cos 2\theta$ is + ve $\quad \Rightarrow \quad 2|\cos\theta| = 2\cos\theta$

$\because \quad \theta \in (0, \pi/4)$

C-8. $\dfrac{\cos^3 A - 4\cos^3 A + 3\cos A}{\cos A} + \dfrac{\sin^3 A + 3\sin A - 4\sin^3 A}{\sin A} = 3\cos A\,\dfrac{(1 - \cos^2 A)}{\cos A} + 3\sin A\,\dfrac{(1 - \sin^2 A)}{\sin A}$

$= 3(2 - 1) = 3$

C-9. **(i)** LHS $= \left\{-\left(\dfrac{1-\tan^2\left(\dfrac{\alpha-\pi}{4}\right)}{1+\tan^2\left(\dfrac{\alpha-\pi}{4}\right)}\right)+\cos\dfrac{\alpha}{2}\cot 4\alpha\right\}\sec\dfrac{9\alpha}{2}$

$= \left\{-\cos\left(\dfrac{\alpha-\pi}{2}\right)+\cos\dfrac{\alpha}{2}\cot 4\alpha\right\}\sec\dfrac{9\alpha}{2} = \left\{-\sin\dfrac{\alpha}{2}+\dfrac{\cos\frac{\alpha}{2}\cos 4\alpha}{\sin 4\alpha}\right\}\sec\dfrac{9\alpha}{2}$

$= \dfrac{1}{\sin 4\alpha}\left[\cos 4\alpha\cos\dfrac{\alpha}{2}-\sin 4\alpha\sin\dfrac{\alpha}{2}\right]\sec\dfrac{9\alpha}{2}$

$= \dfrac{1}{\sin 4\alpha}\times\cos\dfrac{9\alpha}{2}\,.\,\sec\dfrac{9\alpha}{2} = \text{cosec } 4\alpha = \text{RHS}$

(ii) $\dfrac{\cos\alpha\cos 3\alpha}{\sin 3\alpha\cos\alpha-\sin\alpha\cos 3\alpha}-\dfrac{\sin\alpha\sin 3\alpha}{\cos 3\alpha\sin\alpha-\cos\alpha\sin 3\alpha}$

$\dfrac{\cos 3\alpha\cos\alpha}{\sin(2\alpha)}-\dfrac{\sin\alpha\sin 3\alpha}{-\sin 2\alpha}=\dfrac{\cos(3\alpha-\alpha)}{\sin 2\alpha}=\cot 2\alpha$

(iii) $\dfrac{\sec 8A-1}{\sec 4A-1}=\dfrac{1-\cos 8A}{1-\cos 4A}\times\dfrac{\cos 4A}{\cos 8A}=\dfrac{2\sin^2 4A}{2\sin^2 2A}\times\dfrac{\cos 4A}{\cos 8A}=\dfrac{\sin 8A\quad\sin 4A}{2\sin^2 2A\cos 8A}=\dfrac{\tan 8A}{\tan 2A}$

(iv) $\dfrac{\cos A+\sin A}{\cos A-\sin A}-\dfrac{\cos A-\sin A}{\cos A+\sin A}=\dfrac{4\sin A\cos A}{\cos^2 A-\sin^2 A}=\dfrac{2\sin 2A}{\cos 2A}=2\tan 2A$

C-10. $\dfrac{\sin 3\theta}{1+2\cos 2\theta}=\dfrac{3\sin\theta-4\sin^3\theta}{1+2(1-2\sin^2\theta)}=\dfrac{\sin\theta(3-4\sin^2\theta)}{3-4\sin^2\theta}=\sin\theta$. Now put $\theta = 15^o$

C-11. LHS = 4 (cos 20° + cos 40°) (cos² 20° + cos² 40° – cos 20° cos 40°)

$= 4(\cos 20° + \cos 40°)\left(\dfrac{1+\cos 40°}{2}+\dfrac{1+\cos 80°}{2}-\dfrac{1}{2}(\cos 60°+\cos 20°)\right)$

$= \dfrac{4}{2}(\cos 20° + \cos 40°)(2 + \cos 80° + \cos 40° - \dfrac{1}{2} - \cos 20°)$

$= 2(\cos 20° + \cos 40°)(2 + 2\cos 60°\cos 20° - \dfrac{1}{2} - \cos 20°)$

$= 2(\cos 20° + \cos 40°)(2 + \cos 20° - \dfrac{1}{2} - \cos 20°)$

$= 2\times\dfrac{3}{2}(\cos 20° + \cos 40°) = 3(\cos 20° + \cos 40°) = \text{RHS}$

C-12. **(i)** LHS $=\dfrac{\tan 3x}{\tan x}=\dfrac{3\tan x-\tan^3 x}{\tan(1-3\tan^2 x)}=\dfrac{3-\tan^2 x}{1-3\tan^2 x}=\dfrac{3-\left(\dfrac{1-\cos 2x}{1+\cos 2x}\right)}{1-3\left(\dfrac{1-\cos 2x}{1+\cos 2x}\right)}$ $(\because \cos 2x=\dfrac{1-\tan^2 x}{1+\tan^2 x})$

$=\dfrac{3+3\cos 2x-1+\cos 2x}{1+\cos 2x-3+3\cos 2x}=\dfrac{2\cos 2x+1}{2\cos 2x-1}$

(ii) $\dfrac{2\sin x}{\sin x(3-4\sin^2 x)}+\dfrac{\tan x(1-3\tan^2 x)}{\tan x(3-\tan^2 x)}$

$=\dfrac{2}{3-4\sin^2 x}+\dfrac{\cos^2 x-3\sin^2 x}{3\cos^2 x-\sin^2 x}=\dfrac{2}{3-4\sin^2 x}+\dfrac{1-\sin^2 x-3\sin^2 x}{3-3\sin^2 x-\sin^2 x}=\dfrac{2+1-4\sin^2 x}{3-4\sin^2 x}=1$

C-13. $\tan\theta \tan(60° + \theta) \tan(60° – \theta) = \tan 3\theta$

LHS = $\tan\theta \left(\frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\ \ \tan\theta}\right)\left(\frac{\sqrt{3}-\tan\theta}{1+\sqrt{3}\ \ \tan\theta}\right) = \tan\theta\left(\frac{3-\tan^2\theta}{1-3\tan^2\theta}\right) = \frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta} = \tan 3\theta$

Put $\theta = 20°$

$\therefore$ $\tan 20° \tan 80° \tan 40° = \tan 60° =$

C-14. **(i)** L.H.S = $(\text{cosec}\ \theta – \sin\theta)(\sec\theta – \cos\theta)(\tan\theta + \cot\theta)$

$= \left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\ \ \cos\theta}\right) = \frac{\sin^2\theta\ \cdot\ \cos^2\theta}{\sin^2\theta\ \cdot\ \cos^2\theta} = 1 = \text{RHS}$

(ii) LHS $= \frac{2\sin\theta\ \ \tan\theta\ \ (1-\tan\theta)+2\sin\theta\ \ \sec^2\theta}{(1+\tan\theta)^2}$

$= \frac{2\sin\theta\ \ \tan\theta+2\sin\theta\ \ (\sec^2\theta\ \ -\tan^2\theta)}{(1+\tan\theta)^2} = \frac{2\ \ \sin\theta\ \ (1+\tan\theta)}{(1+\tan\theta)^2} = \frac{2\ \ \sin\theta}{1+\tan\theta} = \text{RHS}$

(iii) LHS $= \sqrt{\frac{1-\sin A}{1+\sin A}} = \sqrt{\frac{1-\sin A}{1+\sin A}\times\frac{(1-\sin A)}{(1-\sin A)}} = \frac{1-\sin A}{|\cos A|}$

$= \pm(\sec A – \tan A) = \text{RHS}.$

(iv) $\frac{\cos A\,\text{cosec}A-\sin A\sec A}{\cos A+\sin A} = \frac{\cos^2 A-\sin^2 A}{\sin A\cos A(\cos A+\sin A)} = \frac{\cos A-\sin A}{\cos A\sin A}$

(v) $\frac{1}{\sec\alpha-\tan\alpha}-\frac{1}{\cos\alpha} = \frac{\cos\alpha}{1-\sin\alpha}-\frac{1}{\cos\alpha} = \frac{\cos^2\alpha-1+\sin\alpha}{(1-\sin\alpha)\cos\alpha} = \frac{\sin\alpha-\sin^2\alpha}{(1-\sin\alpha)\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}$

Now taking RHS $\frac{1}{\cos\alpha}-\frac{\cos\alpha}{\sin\alpha+1} = \frac{\sin\alpha+1-\cos^2\alpha}{(1+\sin\alpha)\cos\alpha} = \frac{\sin\alpha(1+\sin\alpha)}{(1+\sin\alpha)\cos\alpha} = \frac{\sin\alpha}{\cos\alpha}$

(vi) $\frac{\cos^3 A+\sin^3 A}{\cos A+\sin A}+\frac{\cos^3-\sin^3 A}{\cos A-\sin A}$ $\cos^2 A + \sin^2 A – \sin A\cos A + \cos^2 A + \sin^2 A + \sin A\cos A = 2$

Section (D)

D-1. LHS = $\cos\alpha + \cos\beta + 2\cos\left(\frac{2\ \ \gamma+\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\left[\cos\ \left(\frac{\alpha-\beta}{2}\right)+\cos\ \left(\frac{\alpha+\beta}{2}+\gamma\right)\right]$

$= 4\cos\left(\frac{\alpha+\beta}{2}\right)\ \cos\left\{\frac{\alpha-\beta+\alpha+\beta+2\ \ \gamma}{4}\right\}\cos\left\{\frac{\alpha-\beta-\alpha-\beta-2\ \ \gamma}{4}\right\}$

$= 4\cos\left(\frac{\alpha+\beta}{2}\right)\ \cos\left(\frac{\alpha+\gamma}{2}\right)\ \cos\left(\frac{\beta+\gamma}{2}\right) = \text{RHS}$

D-2. LHS = $2\sin\left(\frac{\pi}{2}-z\right)\cos(x – y) + 2\sin z\cos z$; $x + y = \left(\frac{\pi}{2}-z\right)$

$\Rightarrow$ $2\cos z\,\{\cos(x – y) + \cos(x + y)\}$ $(\because z = \frac{\pi}{2} – (x + y))$

$=$ $2\cos z \times 2\cos x\cos y = 4\cos x\cos y\cos z$

D-3. LHS = $\sin^2 x + \sin(y + z)\sin(y – z) = \sin^2 x + \sin(y+z)\sin(\pi – x) = \sin x\,[\sin(\pi – (y – z)) + \sin(y + z)]$
$= \sin x.\ 2\sin y\cos z = 2\sin x\sin y\cos z$

D-4. cos (S – A) + cos (S – B) + cos (S – C) + cos S

$= 2\cos\left(\frac{2S-A-B}{2}\right)\cos\left(\frac{B-A}{2}\right) + 2\cos\left(\frac{2S-C}{2}\right)\cos\left(\frac{-C}{2}\right)$

$= 2\cos\left(\frac{C}{2}\right)\cos\left(\frac{B-A}{2}\right) + 2\cos\left(\frac{A+B}{2}\right)\cos\frac{C}{2} = 2\cos\frac{C}{2}\left(2\cos\frac{A}{2}\cos\frac{B}{2}\right)$

D-5. LHS = 2sin(A + B) cos(A – B) + 2 sin C cos C ($\because$ A + B = – C)

= 2sinC {– cos(A – B) + cos(A + B)} = 2sinC { 2 sinA sin (– B)} = – 4 sinA sinB sinC

D-6. LHS = $\sin\theta + \sin(\theta+\phi) + \sin(\theta+2\phi) + \ldots\ldots\ldots\ldots + \sin(\theta+(n-1)\phi) = \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}}\cdot\sin\left(\frac{2\theta+(n-1)\phi}{2}\right)$

$\because$ $\phi = \frac{2\pi}{n}$ (External angle of regular polygon)

So LHS = $\frac{\sin\frac{n}{2}(2\pi/n)}{\sin(\pi/n)}\sin\left(\frac{2\theta+\frac{(n-1)2\pi}{n}}{2}\right)$ = 0 = RHS

D-7. LHS = $\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + \ldots\ldots + \sin^2 n\theta = \left(\frac{1-\cos2\theta}{2}\right)+\left(\frac{1-\cos4\theta}{2}\right)+\ldots\ldots\ldots\ldots+\left(\frac{1-\cos2n\theta}{2}\right)$

$= \frac{n}{2}-\frac{1}{2}$ [(cos2θ + cos4θ + cos6θ + ...+ cos 2nθ)]

$= \frac{n}{2}-\frac{1}{2}\left[\frac{\sin\frac{n(2\theta)}{2}}{\sin\frac{2\theta}{2}}\ .\ \cos\left(\frac{2\theta+2n\theta}{2}\right)\right] = \frac{n}{2}-\frac{1}{2}\left[\frac{\sin n\theta\ .\ \cos(n+1)\theta}{\sin\theta}\right]$ = RHS

D-8. **(i)** By using Series formulae $\cos A\cos 2A\cos 2^2A\cos 2^3A\ldots\ldots\ldots\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n\sin A}$

LHS = $\cos\frac{2\pi}{7}\cos\frac{4\pi}{7}\cos\frac{6\pi}{7} = -\cos\frac{\pi}{7}\cos\frac{2\pi}{7}\cos\frac{4\pi}{7} = -\frac{\sin\frac{8\pi}{7}}{2^3.\sin\frac{\pi}{7}} = \frac{1}{8}$ = RHS

(ii) LHS = $\cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{3\pi}{11}.\cos\frac{4\pi}{11}.\cos\frac{5\pi}{11}$

$= \cos\frac{\pi}{11}\cos\frac{2\pi}{11}\cos\frac{4\pi}{11}\cos\frac{8\pi}{11}\cos\frac{16\pi}{11} = \frac{\sin\frac{32\pi}{11}}{2^5.\sin\frac{\pi}{11}} = \frac{\sin\left(3\pi-\frac{\pi}{11}\right)}{32.\sin\frac{\pi}{11}} = \frac{1}{32}$ = RHS

Section (E)

E-1. Let y = $\cos x.\cos\left(\frac{2\pi}{3}+x\right)\cos\left(\frac{2\pi}{3}-x\right)$

$y = \frac{1}{2}\cos x\left[\cos\frac{4\pi}{3}+\cos2x\right] \Rightarrow y = \frac{1}{2}\cos x\left[\frac{-1+2\cos2x}{2}\right]$

$y = \frac{1}{4}\left[2\cos2x\cos x-\cos x\right] \Rightarrow y = \frac{1}{4}$[cos 3x + cos x – cosx]

$y = \frac{1}{4}\cos 3x$

$\because$ $-1 \le \cos 3x \le 1$

$y_{min} = -\frac{1}{4}$ and $y_{max} = \frac{1}{4}$

E-2. **(i)** $y = \cos 2x + \cos^2 x \Rightarrow y = 3\cos^2 x - 1 \Rightarrow 0 \le \cos^2 x \le 1$

$y_{max} = 3 - 1 = 2 \Rightarrow y_{min} = 0 - 1 = -1$

(ii) $y = \cos^2\left(\frac{\pi}{4}+x\right) + (\sin x - \cos x)^2 = \cos^2\left(\frac{\pi}{4}+x\right) + 2\left(\cos^2\left(\frac{\pi}{4}+x\right)\right)$

$y = 3\cos^2\left(\frac{\pi}{4}+x\right) \quad \because 0 \le \cos^2\theta \le 1 \Rightarrow y_{max} = 3.1 = 3 \Rightarrow y_{min} = 0$

E-3. **(i)** $y = 10\cos^2 x - 6\sin x\cos x + 2\sin^2 x = 5(1 + \cos 2x) - 3\sin 2x + 1 - \cos 2x$

$= 4\cos 2x - 3\sin 2x + 6 \quad \because -\sqrt{a^2+b^2} \le a\cos\theta + b\sin\theta \le \sqrt{a^2+b^2}$

$y_{max} = 5 + 6 = 11 \Rightarrow y_{min} = -5 + 6 = 1$

(ii) $y = 3\cos\left(\theta+\frac{\pi}{3}\right) + 5\cos\theta + 3 \Rightarrow y = 3\cos\theta \cdot \frac{1}{2} - 3\frac{\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3$

$y = \frac{3}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 5\cos\theta + 3 \Rightarrow y = \frac{13}{2}\cos\theta - \frac{3\sqrt{3}}{2}\sin\theta + 3$

$y_{max} = \sqrt{\frac{169}{4}+\frac{27}{4}} + 3 = 7 + 3 = 10 \Rightarrow y_{min} = -\sqrt{\frac{169}{4}+\frac{27}{4}} + 3 = -7 + 3 = -4$

Section (F)

F-1. Obvious

F-2. **(i)** $\sin 9\theta = \sin\theta \Rightarrow \sin 9\theta - \sin\theta = 0 \Rightarrow 2\cos 5\theta \sin 4\theta = 0 \Rightarrow \cos 5\theta = 0$ or

$\sin 4\theta = 0 \Rightarrow 5\theta = (2n+1)\frac{\pi}{2}$ or $4\theta = m\pi \Rightarrow \theta = (2n+1)\frac{\pi}{10}$ or $\theta = \frac{m\pi}{4}$.

(ii) $\cot\theta + \tan\theta = 2\operatorname{cosec}\theta \Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{2}{\sin\theta}$

$\Rightarrow \frac{\cos^2\theta+\sin^2\theta}{\sin\theta\cos\theta} = \frac{2}{\sin\theta} \Rightarrow 1 = 2\cos\theta \Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}$

(iii) $\sin 2\theta = \cos 3\theta \Rightarrow \cos\left(\frac{\pi}{2}-2\theta\right) = \cos 3\theta \Rightarrow \frac{\pi}{2} - 2\theta = 2n\pi \pm 3\theta \Rightarrow \frac{\pi}{2} - 2\theta \pm 3\theta = 2n\pi$

$\Rightarrow \theta = 2n\pi - \frac{\pi}{2}, \frac{\frac{\pi}{2}-2n\pi}{5} \Rightarrow \theta = 2n\pi - \frac{\pi}{2}, \frac{\pi}{5}\left(\frac{1}{2}-2n\right)$

(iv) $\cot\theta = \tan 8\theta \Rightarrow \tan 8\theta = \tan\left(\frac{\pi}{2}-\theta\right) \Rightarrow 8\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = (2n+1)\frac{\pi}{18}$

(v) $\cot\theta - \tan\theta = 2 \quad \frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta} = 2 \Rightarrow 2\cot 2\theta = 2 \Rightarrow 2\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = \frac{n\pi}{2} + \frac{\pi}{8} = \frac{\pi}{2}\left(n+\frac{1}{4}\right)$

(vi) $\operatorname{cosec}\theta = \cot\theta + \sqrt{3}$

$\Rightarrow \frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta} = \sqrt{3} \Rightarrow 1 - \cos\theta = \sqrt{3}\sin\theta \Rightarrow 2\sin^2\frac{\theta}{2} = \sqrt{3}\left(2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\right)$

$\Rightarrow \sin\frac{\theta}{2} = 0$ or $\tan\frac{\theta}{2} = \sqrt{3} \Rightarrow \frac{\theta}{2} = n\pi$ or $\frac{\theta}{2} = n\pi + \frac{\pi}{3}$

$\Rightarrow \theta = 2n\pi$ or $\theta = 2n\pi + \frac{2\pi}{3}$

But for $\theta = 2n\pi$, cosecθ is not defined $\theta = 2n\pi$ cosecθ

$\therefore \quad \theta = 2n\pi + \frac{2\pi}{3}$

(vii) $\tan 2\theta \tan\theta = 1 \Rightarrow \sin 2\theta \sin\theta = \cos 2\theta \cos\theta$

$\Rightarrow 0 = \cos 3\theta \Rightarrow 3\theta = (2n + 1)\ \frac{\pi}{2} \Rightarrow \theta = (2n + 1)\ \frac{\pi}{6}$.

(viii) $\tan\theta + \tan 2\theta + \sqrt{3}\ \tan\theta \tan 2\theta = \sqrt{3}$

$\Rightarrow \quad \tan\theta + \tan 2\theta = \sqrt{3}\ (1 - \tan\theta \tan 2\theta)$

$\Rightarrow \quad \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta \tan 2\theta} = \sqrt{3}$

$\Rightarrow \quad \tan 3\theta = \tan\frac{\pi}{3} \Rightarrow 3\theta = n\pi + \frac{\pi}{3} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{9}$

F-3. (i) $\sin\theta + \sin 3\theta + \sin 5\theta = 0 \quad \Rightarrow \quad \sin\theta + \sin 5\theta + \sin 3\theta = 0$

$2 \sin 3\theta \cos 2\theta + \sin 3\theta = 0 \quad \Rightarrow \quad \sin 3\theta = 0$ and $\cos 2\theta = -\frac{1}{2}$

$\theta = \frac{n\pi}{3}$, $n \in I$, $\quad \theta = \left(n \pm \frac{1}{3}\right)\pi, n \in I$

(ii) $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta \quad \Rightarrow \quad \cos\theta - \cos 2\theta = \sin 2\theta - \sin\theta$

$\Rightarrow 2 \sin\frac{3\theta}{2} \sin\frac{\theta}{2} = 2 \cos\frac{3\theta}{2} \sin\frac{\theta}{2} \quad \Rightarrow \quad \sin\frac{\theta}{2} = 0$ or $\tan\frac{3\theta}{2} = 1$

$\Rightarrow \frac{\theta}{2} = n\pi$ or $\frac{3\theta}{2} = n\pi + \frac{\pi}{4} \quad \Rightarrow \quad \theta = 2n\pi$ or $\theta = \frac{2n\pi}{3} + \frac{\pi}{6}$

(iii) $\cos^2 x + \cos^2 2x + \cos^2 3x = 1 \quad \Rightarrow \quad \frac{1+\cos 2x}{2} + \frac{1+\cos 4x}{2} + \frac{1+\cos 6x}{2} = 1$

$\Rightarrow \cos2x + \cos4x + \cos6x = -1 \quad \Rightarrow \quad 2\cos4x \cos2x = -2\cos^2 2x$

$\Rightarrow \cos2x = 0$ or $\cos4x + \cos2x = 0$

$\Rightarrow 2x = (2n + 1)\ \frac{\pi}{2}$ or $2\cos3x \cos x = 0$

$\Rightarrow x = (2n + 1)\frac{\pi}{4}, (2n + 1)\frac{\pi}{6}, (2n + 1)\ \frac{\pi}{2}$

Now $x = (2n + 1)\frac{\pi}{6} = \frac{n\pi}{3} + \frac{\pi}{6}$ may also be written as

$x = (3k + 1)\ \frac{\pi}{3} + \frac{\pi}{6}, (3k + 2)\ \frac{\pi}{3} + \frac{\pi}{6}, (3k)\ \frac{\pi}{3} + \frac{\pi}{6}$

$= k\pi + \frac{\pi}{2}, k\pi + \frac{5\pi}{6}, k\pi + \frac{\pi}{6} = (k + 1)\ \pi - \frac{\pi}{6}, k\pi + \frac{\pi}{6}$

$(k\pi + \frac{\pi}{2}$ is same as $(2n + 1)\ \frac{\pi}{2}) = m\pi \pm \frac{\pi}{6}$

(iv) $\sin^2 n\theta - \sin^2(n - 1)\theta = \sin^2\theta$

$\Rightarrow \sin(2n - 1)\theta \sin\theta = \sin^2\theta$

$\Rightarrow \sin\theta = 0$ or $\sin (2n - 1)\theta = \sin\theta$

$\Rightarrow \theta = m\pi, \sin(2n - 1)\theta - \sin\theta = 0$

$\Rightarrow 2\cos n\theta \sin\frac{(2n-2)}{2}\ \theta = 0$

$\Rightarrow n\theta = (2p + 1)\ \frac{\pi}{2}, (n - 1)\ \theta = \lambda\pi$

$\Rightarrow \theta = m\pi\ , \frac{\lambda\pi}{n-1}\ , \left(p + \frac{1}{2}\right)\frac{\pi}{n}$

F-4. **(i)** $\tan^2\theta - (1+\sqrt{3})\tan\theta + \sqrt{3} = 0$

After factorization we get $\Rightarrow \tan\theta = 1, \sqrt{3} \Rightarrow \theta = n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}$.

(ii) $4\cos\theta - 3\sec\theta = 2\tan\theta \Rightarrow 4\cos\theta - \frac{3}{\cos\theta} = \frac{2\sin\theta}{\cos\theta} \Rightarrow 4\cos^2\theta - 3 = 2\sin\theta$

$\Rightarrow 4 - 4\sin^2\theta - 3 = 2\sin\theta \Rightarrow 4\sin^2\theta + 2\sin\theta - 1 = 0$

$\Rightarrow \sin\theta = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-2\pm2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4} \Rightarrow \sin\theta = \frac{-(\sqrt{5}+1)}{4}, \frac{\sqrt{5}-1}{4} = -\cos 36^o, \sin 18^o$

$= -\sin 54^o, \sin 18^o = \sin\left(\frac{-3\pi}{10}\right), \sin\frac{\pi}{10} \Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{10}$ or $n\pi - (-1)^n\frac{3\pi}{10}$.

(iii) $\tan x \cdot \tan\left(x+\frac{\pi}{3}\right) \cdot \tan\left(x+\frac{2\pi}{3}\right) = \sqrt{3} \Rightarrow \tan x\left[\frac{\tan x+\sqrt{3}}{1-\sqrt{3}\tan x}\right]\left[\frac{\tan x-\sqrt{3}}{1+\sqrt{3}\tan x}\right] = \sqrt{3}$

$\Rightarrow \frac{\tan^3 x - 3\tan x}{1-3\tan^2 x} = \sqrt{3} \Rightarrow -\tan 3x = \tan\frac{\pi}{3} \Rightarrow \tan 3x = \tan\left(-\frac{\pi}{3}\right)$

$\Rightarrow 3x = n\pi - \frac{\pi}{3} \Rightarrow x = \frac{n\pi}{3} - \frac{\pi}{9}$

F-5. **(i)** $\sqrt{3}\sin\theta - \cos\theta = \sqrt{2} \Rightarrow 2\left(\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta\right) = \sqrt{2}$

$\Rightarrow 2\sin\left(\theta-\frac{\pi}{6}\right) = \sqrt{2} \Rightarrow \sin\left(\theta-\frac{\pi}{6}\right) = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4} \Rightarrow \theta - \frac{\pi}{6} = n\pi + (-1)^n\frac{\pi}{4}$.

(ii) $5\sin\theta + 2\cos\theta = 5 \Rightarrow \frac{5}{\sqrt{29}}\sin\theta + \frac{2}{\sqrt{29}}\cos\theta = \frac{5}{\sqrt{29}}$

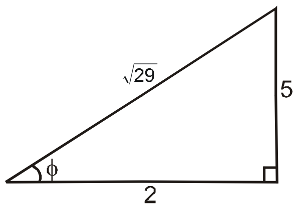
$\Rightarrow \sin\phi\sin\theta + \cos\phi\cos\theta = \frac{5}{\sqrt{29}} \Rightarrow \cos(\theta-\phi) = \sin\phi = \cos\left(\frac{\pi}{2}-\phi\right)$

$\Rightarrow \theta - \phi = 2n\pi \pm \left(\frac{\pi}{2}-\phi\right) \Rightarrow \theta = 2n\pi \pm \frac{\pi}{2} \mp \phi + \phi \Rightarrow \theta = 2n\pi \pm \frac{\pi}{2}, 2n\pi \pm \frac{\pi}{2} + 2\phi$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}, 2n\pi - \frac{\pi}{2} + 2\phi$ For $\theta = 2n\pi - \frac{\pi}{2} + 2\phi$,

We have $\theta = 2n\pi + 2\left(\phi - \frac{\pi}{4}\right) = 2n\pi + 2\left(\tan^{-1}\frac{5}{2} - \tan^{-1}1\right)$

$= 2n\pi + 2\tan^{-1}\left(\frac{\frac{5}{2}-1}{1+\frac{5}{2}}\right) = 2n\pi + 2\tan^{-1}\left(\frac{3}{7}\right)$



$\therefore$ $\theta = 2n\pi + \frac{\pi}{2}$ or $2n\pi + 2\alpha$ where $\tan^{-1}\frac{3}{7} = \alpha$

Section (G)

G-1. tanx $\in$[–1, 1] $\Rightarrow$ $x \in \left[n\pi - \frac{\pi}{4}, n\pi + \frac{\pi}{4}\right]$: n $\in$ I

G-2. (2sinx + 1) (sinx – 1) > 0

$\Rightarrow$ $\sin x < -\frac{1}{2}$

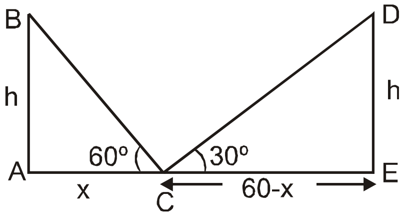
$\therefore$ $x \in \left(\frac{7\pi}{6}, \frac{11\pi}{6}\right)$ $\left(2n\pi + \frac{7\pi}{6},\ 2n\pi + \frac{11\pi}{6}\right)$

G-3. $\sqrt{\sqrt{3}\cot\theta} < 1$

$\Rightarrow 0 \le \cot\theta < \frac{1}{\sqrt{3}}$

$\Rightarrow \theta \in (n\pi + \pi/3,\ n\pi + \pi/2]$

G-4.



In ΔABC $\frac{h}{x} = \tan 60º = \sqrt{3}$

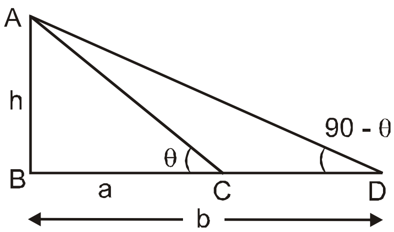
$h = \sqrt{3}\,x$

In ΔCDE $\frac{h}{60 - x} = \tan 30º = \frac{1}{\sqrt{3}}$

$\sqrt{3}\,h = 60 - x \Rightarrow 3x = 60 - x \Rightarrow x = 15$ m

$\therefore$ $h = 15\sqrt{3}$ meter

G-5.



In ΔABC $\tan\theta = \frac{h}{a}$ .... (i)

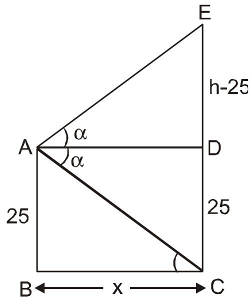
In ΔABD $\tan(90 - \theta) = \frac{h}{b}$ ....(ii)

By equation (i) and (ii) $\tan\theta . \cot\theta = \frac{h}{a} \cdot \frac{h}{b}$

$\Rightarrow h^2 = ab$

$\Rightarrow h = \sqrt{ab}$

G-6.



In Δ ABC $\tan\alpha = \frac{25}{x}$

In Δ ADE $\tan\alpha = \frac{h-25}{x}$ $\frac{25}{x} = \frac{h-25}{x}$

$\Rightarrow$ h = 50 meter

PART - II

Section (A)

A-1. $\cos(540^\circ - \theta) - \sin(630^\circ - \theta) = -\cos\theta + \cos\theta = 0$

A-2. tan 1° tan2° tan3° .... tan45° .... tan 87° tan88° tan 89°
= tan 1° tan2° tan3° .... tan45° ...... cot3° cot2° cot1° =1

A-3. $x = y\left(-\frac{1}{2}\right) = z\left(-\frac{1}{2}\right) \Rightarrow \frac{x}{-1} = \frac{y}{2} = \frac{z}{2} = \lambda$ (say)

$\therefore$ $xy + yz + zx = -2\lambda^2 + 4\lambda^2 - 2\lambda^2 = 0$

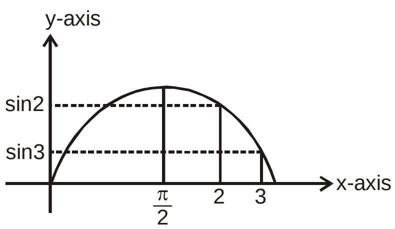
A-4. $0^\circ < x < 90^\circ$ & $\cos x = \frac{3}{\sqrt{10}} \Rightarrow \log_{10}\sin x + \log_{10}\cos x + \log_{10}\tan x$

$= \log_{10}(\sin x \cos x \tan x) = \log_{10}(1 - \cos^2 x) = \log_{10}(1 - 9/10) = \log_{10}\left(\frac{1}{10}\right) = -1$ Ans.

A-5. $\tan\alpha + \cot\alpha = a \Rightarrow \tan^2\alpha + \cot^2\alpha + 2 = a^2 \Rightarrow \tan^4\alpha + \cot^4\alpha = (a^2 - 2)^2 - 2 = a^4 - 4a^2 + 2$

Section (B)

B-1.



B-2. $\text{cosec}\theta - \cot\theta = \alpha$

$\text{cosec}\theta + \cot\theta = \frac{1}{\theta}$ $\Rightarrow$ $\cot\theta = \frac{1}{2}\left(\frac{1}{\alpha} - \alpha\right)$

B-3. square & add $a^2 + b^2 = 9 + 16 = 25$

B-4. $\frac{(-\cot x)\sin x + \cos^3 x}{\sin x(-\cot x)} = \frac{-\cos x}{-\cos x}(1 - \cos^2 x) = \sin^2 x$

B-5. $3\{\cos^4\alpha + \sin^4\alpha\} - 2\{\cos^6\alpha + \sin^6\alpha\}$

$= 3\{1 - 2\sin^2\alpha\cos^2\alpha\} - 2\{1 \times (\cos^4\alpha + \sin^4\alpha - \sin^2\alpha\cos^2\alpha)\}$

$= 3 - 6\sin^2\alpha\cos^2\alpha - 2\{1 - 3\sin^2\alpha\cos^2\alpha\} = 3 - 6\sin^2\alpha\cos^2\alpha - 2 + 6\sin^2\alpha\cos^2\alpha = 1$

B-6. $\left(1+\cos\frac{\pi}{10}\right)\left(1+\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{3\pi}{10}\right)\left(1-\cos\frac{\pi}{10}\right)=\left(1-\cos^2\frac{\pi}{10}\right)\left(1-\cos^2\frac{3\pi}{10}\right)$

$\sin^2\frac{\pi}{10}\,.\,\sin^2\frac{3\pi}{10}=\left(\frac{\sqrt{5}-1}{4}\;.\;\frac{\sqrt{5}+1}{4}\right)^2=\left(\frac{4}{16}\right)^2=\frac{1}{16}$

B-7. $\frac{\sin 24^\circ\cos 6^\circ-\sin 6^\circ\cos 24^\circ}{\sin 21^\circ\cos 39^\circ-\sin 39^\circ\cos 21^\circ}=\frac{\sin(24^\circ-6^\circ)}{\sin(21^\circ-39^\circ)}=\frac{\sin 18^\circ}{\sin(-18^\circ)}=-1$

B-8. $\tan A+\tan B=a \quad\Rightarrow\quad \tan A\tan B=b \Rightarrow \tan(A+B)=\frac{a}{1-b}$

$\therefore\quad \sin^2(A+B)=\left[\frac{|a|}{\sqrt{a^2+(1-b)^2}}\right]^2=\frac{a^2}{a^2+(1-b)^2}$

B-9. $\tan A-\tan B=x \Rightarrow \cot B-\cot A=y \Rightarrow \frac{\tan A-\tan B}{\tan A\;\tan B}=y$

$\Rightarrow\quad \tan A\tan B=\frac{x}{y}$ Now $\cot(A-B)=\frac{1}{\tan(A-B)}=\frac{1+\tan A\tan B}{\tan A-\tan B}=\frac{1+\frac{x}{y}}{x}=\frac{1}{x}+\frac{1}{y}$

B-10. $\frac{\tan(180^\circ-25^\circ)-\tan(90^\circ+25^\circ)}{1+(\tan(180^\circ-25^\circ)\;\tan(90^\circ+25^\circ)}=\frac{-\tan 25^\circ+\frac{1}{\tan 25^\circ}}{2}=\frac{1-x^2}{2\;x}$

B-11. $\because\quad \cot(A+B)=\cot 225^\circ=1 \quad\Rightarrow\quad \frac{\cot A\;\cot B-1}{\cot A+\cot B}=1$

$\Rightarrow \cot A\cot B=1+\cot A+\cot B$

Now $\frac{\cot A\;.\;\cot B}{1+\cot A+\cot B+\cot A\;\cot B}=\frac{1+\cot A+\cot B}{2(1+\cot A+\cot B)}=\frac{1}{2}$

B-12. $203^\circ+22^\circ=225^\circ \quad\Rightarrow\quad \tan(203^\circ+22^\circ)=\tan 225^\circ=1$

$\Rightarrow \frac{\tan 203^\circ+\tan 22^\circ}{1-\tan 203^\circ\;.\;\tan 22^\circ}=1 \quad\Rightarrow\quad \tan 203^\circ+\tan 22^\circ+\tan 203^\circ.\tan 22^\circ=1$

Section (C)

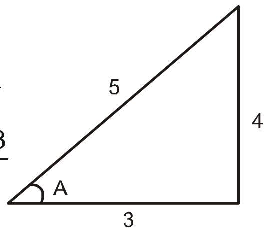
C-1. $\frac{1-\tan^2 15^\circ}{1+\tan^2 15^\circ}=\cos(2\times 15^\circ)=\cos 30^\circ$

C-2. $\tan A=\frac{4}{3} \quad\Rightarrow\quad A\to$ III[rd] quadrant

$5\sin 2A+3\sin A+4\cos A=10\sin A\cos A+3\sin A+4\cos A=10\sin A\cos A+3\sin A+4\cos A=0$

$\sin A=\frac{-4}{5}$

$\cos A=\frac{-3}{5}$



C-3. $\cos A = \frac{3}{4} \Rightarrow 16\cos^2\frac{A}{2} - 32\sin\frac{A}{2}\sin\frac{5A}{2} = \frac{16(1+\cos A)}{2} - 16\,(\cos 2A - \cos 3A)$

$= \frac{16(1+\cos A)}{2} - 16\{(2\cos^2 A - 1) - (4\cos^3 A - 3\cos A)\} = 8\left(1+\frac{3}{4}\right) - 16\left\{2\times\frac{9}{16} - 1 - 4\times\frac{27}{64} + 3\times\frac{3}{4}\right\} = 3$

C-4. $\tan^2\theta = 2\tan^2\phi + 1$ ... (i)

$\cos 2\theta + \sin^2\phi = \frac{1-\tan^2\theta}{1+\tan^2\theta} + \sin^2\phi = \frac{1-2\tan^2\phi - 1}{1+2\tan^2\phi+1} + \sin^2\phi = \frac{-2\tan^2\phi}{2\;(1+\tan^2\phi)} + \sin^2\phi = -\sin^2\phi + \sin^2\phi = 0.$

Which is independent of ϕ

C-5. $\alpha \in \left[\frac{\pi}{2},\ \pi\right] \Rightarrow \frac{\alpha}{2} \in \left[\frac{\pi}{4},\ \frac{\pi}{2}\right]$

$\sqrt{1+\sin\alpha} - \sqrt{1-\sin\alpha} = \sqrt{\left(\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right)^2} - \sqrt{\left(\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right)^2} = \left|\sin\frac{\alpha}{2}+\cos\frac{\alpha}{2}\right| - \left|\sin\frac{\alpha}{2}-\cos\frac{\alpha}{2}\right|$

$= \sin\frac{\alpha}{2}+\cos\frac{\alpha}{2} - \sin\frac{\alpha}{2}+\cos\frac{\alpha}{2} = 2\cos\frac{\alpha}{2}\ \left(\because \text{ for } \frac{\alpha}{2}\in\left[\frac{\pi}{4},\ \frac{\pi}{2}\right],\ \sin\frac{\alpha}{2} > \cos\frac{\alpha}{2}\right)$

C-6. $\frac{1}{\cos\ (270^o+20^o)} + \frac{1}{\sqrt{3}\sin\ (270^o-20^o)} = \frac{1}{\sin 20^o} - \frac{1}{\sqrt{3}\cos 20^o}$

$= \frac{2\left(\frac{\sqrt{3}}{2}\cos 20^o - \frac{1}{2}\sin 20^o\right)}{2\frac{\sqrt{3}}{2}\sin 20^o\ \cos 20^o} = \frac{4\sin(60^o-20^o)}{\sqrt{3}\sin 40^o} = \frac{4}{\sqrt{3}}\ \frac{\sin 40^o}{\sin 40^o} = \frac{4\sqrt{3}}{3}$.

C-7. $3A = 2A + A \quad \Rightarrow \quad \tan 3A = \tan(2A + A)$

$\Rightarrow \quad \tan 3A = \frac{\tan\ 2A + \tan A}{1-\tan 2A\ \tan A}$

$\Rightarrow \quad \tan 3A - \tan A\tan 2A\tan 3A = \tan 2A + \tan A$

$\Rightarrow \quad \tan 3A - \tan 2A - \tan A = \tan A\tan 2A\tan 3A$

C-8. $\frac{\cos 20^\circ + 8\sin 70^\circ\sin 50^\circ\sin 10^\circ}{\sin^2 80^\circ} = \frac{\cos 20^\circ + 4\ \{(\cos 60^\circ - \cos 80^\circ)\sin 50^\circ\}}{\sin^2 80^\circ}$

$= \frac{\cos 20^\circ + 2\ (1-2\ \cos 80^\circ)\sin 50^\circ}{\sin^2 80^\circ} = \frac{\cos 20^\circ + 2\ \sin 50^\circ - 2\ (\sin 130^\circ - \sin 30^\circ)}{\sin^2 80^\circ}$

$= \frac{\cos 20^\circ + 2\ \sin 50^\circ - 2\sin\ (180^\circ - 50^\circ) + 2\sin 30^\circ}{\sin^2 80^\circ} = \frac{1+\cos 20^\circ}{1+\cos 20^\circ} \times 2 \because \sin^2(80^\circ) = \cos^2 10^\circ = 2$

C-9. $= \frac{1}{2}\ (2\sin 12^\circ\,.\sin 48^\circ\,.\sin 54^\circ) = \frac{1}{2}\ [(\cos 36^\circ - \cos 60^\circ)\sin 54^\circ] = \frac{1}{2}\ [\sin^2 54^\circ - \frac{1}{2}\sin 54^\circ]$

$= \frac{1}{4}\left[2\ \frac{(\sqrt{5}+1)^2}{16} - \frac{(\sqrt{5}+1)}{4}\right] = \frac{1}{4}\left[\frac{5+1+2\sqrt{5}}{8} - \frac{(\sqrt{5}+1)}{4}\right] = \frac{1}{32}\ [6 + 2\sqrt{5} - 2\sqrt{5} - 2] = \frac{1}{8}$

C-10. A = tan 6° tan 42° ; B = cot 66° cot 78° ⇒ $\frac{A}{B}$ = tan 6° tan 42° tan 66° tan 78°

$$\Rightarrow \quad \frac{A}{B} = \frac{\tan 6^\circ \tan (60^\circ - 6^\circ) \tan (60^\circ + 6^\circ)}{\tan 54^\circ} . \tan 78^\circ \tan 42^\circ$$

$$\Rightarrow \quad \frac{A}{B} = \frac{\tan 18^\circ . \tan (60^\circ - 18^\circ) \tan (60^\circ + 18^\circ)}{\tan 54^\circ} = \frac{\tan 54^\circ}{\tan 54^\circ} \Rightarrow \frac{A}{B} = 1 \Rightarrow A = B$$

Section (D)

D-1. tan A + tan B + tan C = 6, tan A tan B = 2

In any Δ ABC,

tan A + tan B + tan C = tan A tan B tan C

⇒ 6 = 2 tan C ⇒ tan C = 3

∴ tan A + tan B + 3 = 6

⇒ tan A + tan B = 3 & tan A tan B = 2

Now $(\tan A - \tan B)^2 = (\tan A + \tan B)^2 - 4\tan A \tan B = 9 - 8 = 1$

⇒ tan A – tan B = ± 1

∴ tan A – tan B = 1 or tan A – tan B = – 1

tan A – tan B = 1	tan A – tan B = – 1
tan A + tan B = 3	tan A + tan B = 3
on solving	on solving
tan A = 2	tan A = 1
tan B =1	tan B = 2

D-2. Add & subtract cot α.

(tan α – cotα) + 2 tan 2α + 4 tan 4α + 8 cot 8α + cotα

= – 2 cot2 α + 2 tan 2α + 4 tan4α + 8 cot 8α + cotα

= – 4 cot4 α + 4 tan 4α + 8 cot 8α + cotα

= – 8 cot 8α + 8 cot 8α + cot α = cotα

D-3. $1 + \cos\frac{\pi}{7} + \cos\frac{2\pi}{7} + \cos\frac{3\pi}{7} - \cos\frac{3\pi}{7} - \cos\frac{2\pi}{7} - \cos\frac{\pi}{7} = 1$

D-4. $\cos\frac{\pi}{10}\cos\frac{2\pi}{10}\cos\frac{4\pi}{10}\cos\frac{8\pi}{10}\cos\frac{16\pi}{10}$

$$= \frac{\sin 2^5 \frac{\pi}{10}}{2^5 \sin\frac{\pi}{10}} = \frac{1}{32}\,\frac{\sin\frac{32\pi}{10}}{\sin\frac{\pi}{10}} = \frac{1}{32}\,\frac{\sin\left(3\pi + \frac{2\pi}{10}\right)}{\sin\left(\frac{\pi}{10}\right)} = -\frac{1}{32}$$

$$\frac{2 \sin\frac{\pi}{10}\cos\frac{\pi}{10}}{\sin\frac{\pi}{10}} = -\frac{1}{16}\cos\frac{\pi}{10} = -\frac{1}{64}\sqrt{10+2\sqrt{5}}$$

D-5. $\cos\frac{\pi}{19} + \cos\frac{3\pi}{19} + \cos\frac{5\pi}{19} + ... + \cos\frac{17\pi}{19}$

here $A = \frac{\pi}{19}$, $D = \frac{2\pi}{19}$, $n = 9$

$\because$ cos A + cos (A + D) + cos(A + 2D) + ... + cos (A + (n – 1)D)

$$= \frac{\sin\left(\frac{nD}{2}\right)}{\sin\frac{D}{2}} . \cos\left(\frac{2A+(n-1)D}{2}\right) = \frac{\sin 9\times\frac{\pi}{19}}{\sin\frac{\pi}{19}} \times \cos\left(\frac{\frac{\pi}{19}+\frac{17\pi}{19}}{2}\right) = \frac{\sin\frac{9\pi}{19}}{\sin\frac{\pi}{19}} \times \cos\frac{9\pi}{19}$$

$$= \frac{1}{2} . \frac{\sin\left(\frac{18\pi}{19}\right)}{\sin\frac{\pi}{19}} = \frac{1}{2} . \frac{\sin\frac{\pi}{19}}{\sin\frac{\pi}{19}} = \frac{1}{2}$$

Section (E)

E-1. $f(\theta) = \sin^4\theta + \cos^2\theta = \sin^2\theta (1 - \cos^2\theta) + \cos^2\theta = \sin^2\theta + \cos^2\theta - \sin^2\theta \cos^2\theta$

$f(\theta) = 1 - \frac{1}{4} \sin^2 2\theta$

$\because \; 0 \le \sin^2 2\theta \le 1$

$f(\theta)_{max} = 1 \Rightarrow f(\theta)_{min} = 1 - \frac{1}{4} = 3/4$

$\therefore$ Range is $\left[\frac{3}{4}, \; 1\right]$

E-2. $\cos^2x + \sec^2x + 3\sec^2x \ge 2 + 3 \ge 5$

E-3. $y = 1 + 2 \sin x + 3 \cos^2x \quad \Rightarrow \quad y = 1 + 2\sin x + 3 - 3 \sin^2 x$

$y = 1 - (3 \sin^2x - 2 \sin x - 3) \quad \Rightarrow \quad y = 1 - 3 (\sin^2x - \frac{2}{3} \sin x + \frac{1}{9} - \frac{1}{9} - 1)$

$y = 1 - 3 \left[\left(\sin x - \frac{1}{3}\right)^2 - \frac{10}{9}\right] = -3 \left(\sin x - \frac{1}{3}\right)^2 + \frac{13}{3}$

$y_{max} = \frac{13}{3}$, $\quad y_{min} = -3 \left(\frac{16}{9}\right) + \frac{13}{3} = -1$

E-4. $12 \sin\theta - 9 \sin^2\theta = 4 - (3 \sin\theta - 2)^2$ whose maximum value is 4 when $\sin\theta = \frac{2}{3}$

E-5. $y = 10 \cos^2x - 6 \sin \cos x + 2 \sin^2x = 5 (1 + \cos 2x) - 3 \sin 2x + 1 - \cos 2x = 4 \cos 2x - 3 \sin 2x + 6$

$\because \; \sqrt{a^2+b^2} - \le a \cos\theta + b \sin\theta \le \sqrt{a^2+b^2}$

$y_{max} = 5 + 6 = 11$

$y_{min} = -5 + 6 = 1$

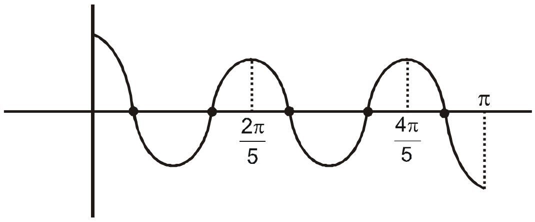
Section (F)

F-1. $4\sin\theta.\cos\theta - 2\cos\theta - 2\sqrt{3} \sin\theta + \sqrt{3} = 0 \;\Rightarrow\; 2\cos\theta (2\sin\theta - 1) - \sqrt{3} (2\sin\theta - 1) = 0$

$\Rightarrow (2\sin\theta - 1) (2\cos\theta - \sqrt{3}) = 0 \Rightarrow \sin\theta = \frac{1}{2}, \cos\theta = \frac{\sqrt{3}}{2} \quad \Rightarrow \theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{11\pi}{6}$

F-2. $2 \sin\theta + \tan\theta = 0 \Rightarrow \sin\theta = 0$ or $2 + \frac{1}{\cos\theta} = 0 \Rightarrow \theta = n\pi$ or $2 = -\frac{1}{\cos\theta} \Rightarrow \cos\theta = -\frac{1}{2} \Rightarrow \theta = 2n\pi \pm \frac{2\pi}{3}$.

F-3. sin x . tan 4x = cos x $\Rightarrow$ sin x sin 4x = cos x cos 4x $\Rightarrow$ cos 5x = 0 $\Rightarrow$ five solutions.



F-4. $\sin 7x + \sin 4x + \sin x = 0 \Rightarrow 2\sin 4x \cos 3x + \sin 4x = 0$

$\Rightarrow \sin 4x = 0$ or $\cos 3x = -\frac{1}{2} \Rightarrow 4x = n\pi$ or $3x = 2n\pi \pm \frac{2\pi}{3}$

$\Rightarrow x = \frac{n\pi}{4}, \frac{2n\pi}{3} \pm \frac{2\pi}{9} = 0, \frac{\pi}{4}, \frac{\pi}{2}, \frac{2\pi}{9}, \frac{4\pi}{9}$.

F-5. sinx + sin5x = sin2x + sin4x $\Rightarrow$ 2sin 3x cos 2x = 2 sin 3x cos x

sin 3x = 0 or cos2x = cosx $\Rightarrow$ 3x = nπ or 2x = 2n$\pi\pm$x

$\Rightarrow x = \frac{n\pi}{3}, 2n\pi, \frac{2n\pi}{3} \Rightarrow x = \frac{n\pi}{3}$ (It includes all three possible)

F-6. $2\cos 2x = 6\cos^2 x - 4 \Rightarrow 2(2\cos^2 x - 1) = 6\cos^2 x - 4 \Rightarrow 2\cos^2 x = 2 \Rightarrow \cos^2 x = 1 \Rightarrow x = n\pi$.

F-7. $2\cos^2(\pi + x) + 3\sin(\pi + x) = 0 \Rightarrow 2\cos^2 x - 3\sin x = 0 \Rightarrow 2 - 2\sin^2 x - 3\sin x = 0$

$\Rightarrow 2\sin^2 x + 3\sin x - 2 = 0 \Rightarrow \sin x = -2, \frac{1}{2} \Rightarrow \sin x = \frac{1}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}$

F-8. $\frac{\cos 3\theta}{2\cos 2\theta - 1} = \frac{1}{2} \Rightarrow 2(4\cos^3\theta - 3\cos\theta) = 2(2\cos^2\theta - 1) - 1 \Rightarrow 8\cos^3\theta - 4\cos^2\theta - 6\cos\theta + 3 = 0$

$\Rightarrow (4\cos^2\theta - 3)(2\cos\theta - 1) = 0 \Rightarrow \cos\theta = \frac{1}{2}, \pm\frac{\sqrt{3}}{2}$

But when $\cos\theta = \pm\frac{\sqrt{3}}{2}$ then $2\cos 2\theta - 1 = 0$

$\therefore$ rejecting this value, $\cos\theta = \frac{1}{2}$ is valid only $\Rightarrow \theta = 2n\pi \pm \frac{\pi}{3}, n \in I$

F-9. $\cos 2\theta + 3\cos\theta = 0 \Rightarrow 2\cos^2\theta + 3\cos\theta - 1 = 0 \Rightarrow \cos\theta = \frac{3 \pm \sqrt{9+8}}{4} = \frac{-3 \pm \sqrt{17}}{4}$

As $-1 \le \cos\theta \le 1$

$\therefore \quad \cos\theta = \frac{-3 + \sqrt{17}}{4}$ only $\Rightarrow \theta = 2n\pi \pm \alpha$ where $\cos\alpha = \frac{\sqrt{17} - 3}{4}$

F-10. $\sin\theta + 7\cos\theta = 5 \Rightarrow \frac{2t}{1+t^2} + \frac{7(1-t^2)}{1+t^2} = 5$

where $t = \tan\left(\frac{\theta}{2}\right) \Rightarrow 2t + 7 - 7t^2 = 5 + 5t^2 \Rightarrow \tan\frac{\theta}{2}$ is root of $12t^2 - 2t - 2 = 0$ or $6t^2 - t - 1 = 0$.

F-11. $\tan\theta = -1 \Rightarrow \theta = \frac{3\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{7\pi}{4}$ in $[0, 2\pi]$

common value is $x = \frac{7\pi}{4}$ general solution is $2n\pi + \frac{7\pi}{4}$, $n \in I$.

F-12. $\sin(2A + B) = \frac{1}{2} \Rightarrow 2A + B = 30^o$ or 150^o

A,B,C are in AP $\Rightarrow$ B = 60º

$\therefore 2A = -30^o$ or $90^o \Rightarrow 2A = 90^o \Rightarrow A = 45^o$

$\therefore C = 180^o - A - B = 75^o$

Section (G)

G-1. $\tan^2 3x < 1$

$-1 < \tan 3x < 1$

$n\pi - \frac{\pi}{4} < 3x < \frac{\pi}{4} + n\pi$

$x \in \left(\frac{n\pi}{3} - \frac{\pi}{12}, \frac{n\pi}{3} + \frac{\pi}{12}\right)$

G-2. $(\cos x - 3)(2\cos - 1) < 0$

$2\cos x - 1 > 6$

$\cos x > \frac{1}{2}$

$2n\pi - \frac{\pi}{3} < x < \frac{\pi}{3} + 2n\pi$

G-3. $\cos 2x \le \cos x \Rightarrow 2\cos^2 x - \cos x - 1 \le 0 \Rightarrow 2\cos^2 x - 2\cos x + \cos x - 1 \le 0$

$\Rightarrow 2\cos x(\cos x - 1) + 1(\cos x - 1) \le 0 \Rightarrow (\cos x - 1)(2\cos x + 1) \le 0$

$\Rightarrow \cos x \in \left[-\frac{1}{2}, 1\right]$

$\therefore x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$ General soltution is $x \in \left[2n\pi - \frac{2\pi}{3}, 2n\pi + \frac{2\pi}{3}\right]$.

G-4. $\tan^2 x - \tan x - \sqrt{3}\tan x + \sqrt{3} < 0$

$\tan x(\tan x - 1) - \sqrt{3}(\tan x - 1) < 0 \Rightarrow (\tan x - 1)(\tan x - \sqrt{3}) < 0$

$\Rightarrow \quad \tan x \in (1, \sqrt{3})$

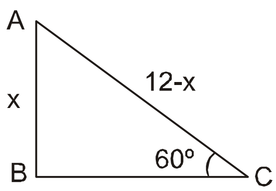
$\Rightarrow \quad x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\therefore$ general solution is $x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right)$, $n \in I$

G-5. $\sin 60^o = \frac{x}{12 - x} \Rightarrow \frac{\sqrt{3}}{2} = \frac{x}{12 - x} \Rightarrow 12\sqrt{3} - \sqrt{3}\ x = 2x$

$x(2 + \sqrt{3}) = 12\sqrt{3} \Rightarrow x = \frac{12\sqrt{3}}{2 + \sqrt{3}} x \frac{2 - \sqrt{3}}{2 - \sqrt{3}} \Rightarrow x = 12(2\sqrt{3} - 3)$

$x = 12(2 \times 1.732 - 3) \Rightarrow x = 12 \times 0.464 \Rightarrow x = 5.568$ m



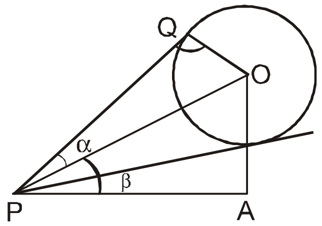
G-6. $\frac{AP}{AB} = \frac{2}{1} \Rightarrow AP = 2K, AB = K, AC = \frac{K}{2}$

$\tan(\alpha + \beta) = \frac{AB}{AP} = \frac{1}{2}$, $\tan\alpha = \frac{AC}{AP} = \frac{1}{4}$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} \Rightarrow \frac{1}{2} = \frac{\frac{1}{4} + \tan\beta}{1 - \frac{1}{4}\tan\beta} \Rightarrow \frac{1}{2} = \frac{1 + 4\tan\beta}{4 - \tan\beta}$

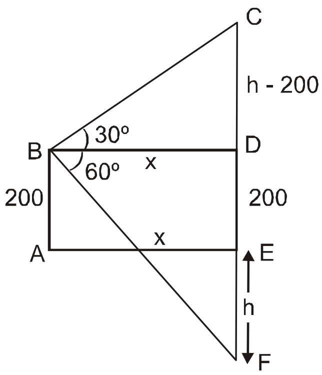
$\Rightarrow 4 - \tan\beta = 2 + 8\tan\beta \Rightarrow \tan\beta = \frac{2}{9} \Rightarrow \beta = \tan^{-1}\left(\frac{2}{9}\right)$

G-7. Let height of centre of balloon is OA = h and P is eye of observer men h = OP sin β OP = r cosec $\frac{\alpha}{2}$



$\therefore$ h = r sin β cosec $\frac{\alpha}{2}$

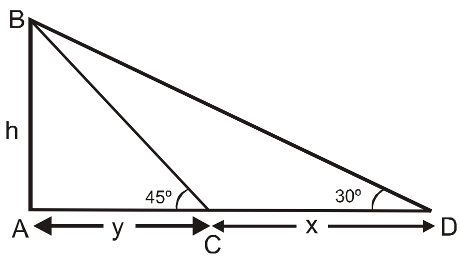
G-8.



In ΔBCD $\quad \frac{h - 200}{x} = \tan 30°$

In ΔBDF $\quad \frac{h + 200}{x} = \tan 60° \quad \Rightarrow \quad \frac{h - 200}{h + 200} = \frac{1}{3} \quad \Rightarrow \quad$ h = 400 meter

G-9.



$\tan 45° = \frac{h}{y}$, $\tan 30° = \frac{h}{y + x} \Rightarrow \sqrt{3} = \frac{y + x}{y} \Rightarrow \frac{x}{y} = \sqrt{3} - 1$

speed = $\frac{x}{12}$ m/min y = $\left(\frac{\sqrt{3} + 1}{2}\right)$x. Now taken to cover distance CA = $\frac{y}{\left(\frac{x}{12}\right)}$ = 12 $\frac{y}{x}$ = 6 $(\sqrt{3} + 1)$ min.

PART - III

1. **(i)** tan9° – tan27° – tan 63° + tan81° = (tan 9º + tan81º) – (tan 27º + tan 63º)

$$= \frac{\sin 90^o}{\cos 9^o \quad \cos 81^o} - \frac{\sin 90^o}{\cos 27^o \cos 63^o} = \frac{2}{2\sin 9^o \cos 9^o} - \frac{2}{2\sin 27^o \cos 27^o}$$

$$= \frac{2}{\sin 18^o} - \frac{2}{\sin 54^o} = \frac{2}{\frac{\sqrt{5}-1}{4}} - \frac{2}{\frac{\sqrt{5}+1}{4}} = \frac{8 \ (\sqrt{5}+1-\sqrt{5}+1)}{4} = 4$$

(ii) $\text{cosec } 10° - \sqrt{3} \ \sec 10° = 2\left(\frac{1}{2}\cos 10° - \frac{\sqrt{3}}{2}\sin 10°\right) \times \frac{1}{\sin 10^o \quad \cos 10^o} \times \frac{2}{2} = 4$

(iii) $2\sqrt{2}\sin 10° \left(\frac{\sec 5°}{2} + \frac{\cos 40°}{\sin 5°} - 2\sin 35°\right)$

$$= 2\sqrt{2}\left(\frac{2\sin 5^o \cos 5^o \sec 5^o}{2} + \frac{2\sin 5^o \cos 5^o \cos 40^o}{\sin 5^o} - 2\sin 35^o \sin 10^o\right)$$

$= 2\sqrt{2}$ (sin5º + 2cos45º + cos 35º – cos 25º) = $2\sqrt{2}$ (sin5º + 2cos45º + 2sin 30º sin (– 5º))

$= 2\sqrt{2}\ (\sqrt{2}) = 4$

(iv) $\cot 70° + 4\cos 70° = \frac{\cos 70°}{\sin 70°} + 4\cos 70^o = \frac{\cos 70^o + 4\cos 70^o \sin 70^o}{\sin 70^o} = \frac{\cos 70^o + 2\sin 140^o}{\sin 70^o}$

$$= \frac{(\cos 70^o + \sin 140^o) + \sin 140^o}{\sin 70^o} = \frac{(\sin 20^o + \sin 140^o) + \sin 140^o}{\sin 70^o}$$

$$= \frac{2\sin 80^o \times \cos 60^o + \sin 140^o}{\sin 70^o} = \frac{2\sin 120^o \times \cos 20^o}{\sin 70^o} = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

2. **(A)** By using A.M. ≥ G.M. ⇒ $x + \frac{1}{x} = 2\cos\theta \geq 2$ or ≤ -2 ⇒ cos θ = 1 or –1

(B) sin θ + cosec θ = 2 ⇒ By using A.M. ≥ G.M.

∵ $\sin\theta + \frac{1}{\sin\theta} \geq 2$ or ≤ -2 but given that sin θ + cosec θ = 2 ⇒ $\sin\theta + \frac{1}{\sin\theta} = 2$

which is possible only when sin θ = 1

∴ $\sin^{2008}\theta + \text{cosec}^{2008}\theta = \sin^{2008}\theta + \frac{1}{\sin^{2008}\theta} = 1 + 1 = 2$

(C) $\sin^4\theta + \cos^4\theta = (\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta = 1 - \frac{1}{2}\sin^2 2\theta$

∵ $0 \leq \sin^2 2\theta \leq 1$

∴ $\frac{1}{2} \leq 1 - \frac{1}{2}\sin^2 2\theta \leq 1$

∴ maximum value = 1

(D) $2\sin^2\theta + 3\cos^2\theta = 2\sin^2\theta + 3 - 3\sin^2\theta = 3 - \sin^2\theta$

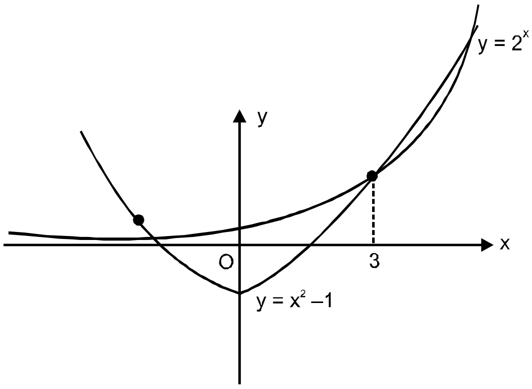
∵ $0 \leq \sin^2\theta \leq 1$

∴ $2 \leq 3 - \sin^2\theta \leq 3$

∴ least value = 2

3. **(A)** $\sin^2\theta + 3\cos\theta = 3 \Rightarrow 1 - \cos^2\theta + 3\cos\theta = 3$
$\Rightarrow \cos^2\theta - 3\cos\theta + 2 = 0 \Rightarrow \cos\theta = 1, 2$
$\Rightarrow \cos\theta = 1 (\cos\theta \neq 2) \quad \Rightarrow \theta = 0$ in $[-\pi, \pi]$
No. of solution = 1

(B) sin x. tan 4x = cos x
$\Rightarrow \sin x . \frac{\sin 4x}{\cos 4x} = \cos x$
$\Rightarrow$ sin4x sinx – cos4x cosx = 0
$\Rightarrow$ cos5x = 0
$\Rightarrow 5x = (2n + 1)\pi/2$
$\Rightarrow x = (2n + 1)\pi/10$
$\Rightarrow x = \frac{\pi}{10}, \frac{3\pi}{10}, \frac{5\pi}{10}, \frac{7\pi}{10}, \frac{9\pi}{10}$ in $(0, \pi)$
So there are five solutions.



(C) $(1 - \tan^2\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$
$\Rightarrow (1 - \tan^4\theta) + 2^{\tan^2\theta} = 0$
$\Rightarrow (1 - x^2) + 2^x = 0$ where $x = \tan^2\theta$
$\Rightarrow 2^x = x^2 - 1 \Rightarrow x = 3$
from graph number of solutions = 4

(D) $[\sin x] + [\sqrt{2}\cos x] = -3$
$\Rightarrow [\sin x] = -1$ and $[\cos x] = -2$
$\Rightarrow \pi < x < 2\pi$ and $-2 \leq \sqrt{2}\cos x < -1$
$\Rightarrow -2 \leq \cos x < -\frac{1}{\sqrt{2}}$
$\Rightarrow -1 \leq \cos x < -\frac{1}{\sqrt{2}}$
$\Rightarrow \pi \leq x < \frac{5\pi}{4}$ for $x \in [0, \; 2\pi]$; $\pi \leq x < \frac{5\pi}{4}, \; x \in [0, 2\pi]$
$\therefore \pi < x < \frac{5\pi}{4}$
$\Rightarrow 2\pi < 2x < \frac{5\pi}{2}$
$\Rightarrow 0 < \sin 2x < 1$

$\Rightarrow [\sin 2x] = 0$

EXERCISE # 2

PART - I

1. $\because$ tan A < 0 and A + B + C = 180°

$\Rightarrow$ A > 90° $\Rightarrow$ B + C < 90° $\Rightarrow$ tan (B + C) > 0

$\Rightarrow \dfrac{\tan B + \tan C}{1 - \tan B \tan C} > 0$ $\Rightarrow$ 1 – tan B tan C > 0 $\Rightarrow$ tan B tan C < 1

2. Clearly α = 30º and $\theta \in$ (60º, 90º).
Hence $\theta + \alpha$ lies in (90º, 120º).

3. $\dfrac{\sin A}{\sin B} = \dfrac{\sqrt{3}}{2}, \dfrac{\cos A}{\cos B} = \dfrac{\sqrt{5}}{2}$, 0 < A, B < $\pi/2$

$\Rightarrow \tan A = \dfrac{\sqrt{3}}{\sqrt{5}} \quad \dfrac{\sin B}{\cos B}$

tan A = $\dfrac{\sqrt{3}}{\sqrt{5}}$ tan B ..... (1)

$\dfrac{\sin A \cos A}{\sin B \cos B} = \dfrac{\sqrt{15}}{4} \Rightarrow \dfrac{\tan A \ .\sec^2 B}{\tan B \ .\sec^2 A} = \dfrac{\sqrt{15}}{4}$

from (1)

$\Rightarrow \dfrac{\sqrt{3}}{\sqrt{5}} \dfrac{(1+\tan^2 B)}{(1+\tan^2 A)} = \dfrac{\sqrt{15}}{4}$ $\Rightarrow$ 4 + 4 $\tan^2$ B = 5 + 5 $\tan^2$ A

$\Rightarrow$ –1 + 4 $\tan^2$ B = 5 × $\dfrac{3}{5}$ $\tan^2$ B $\Rightarrow$ tan B = ±1

$\Rightarrow$ tan B = +1 ($\because$ 0 < B < $\dfrac{\pi}{2}$)

Now tanA + tanB = $\dfrac{\sqrt{3}}{\sqrt{5}} + 1 = \dfrac{\sqrt{3}+\sqrt{5}}{\sqrt{5}}$ Ans.

4. AC = $2\sqrt{2}$ P $\Rightarrow \dfrac{P}{AD} = \dfrac{DC}{P}$ = tanθ

$\because$ AD + DC = $2\sqrt{2}$ P $\Rightarrow \dfrac{P}{\tan\theta}$ + P tanθ = $2\sqrt{2}$ P

$\Rightarrow \dfrac{\cos^2\theta + \sin^2\theta}{2\sin\theta \ \cos\theta} = \sqrt{2}$ $\Rightarrow$ sin 2θ = $\dfrac{1}{\sqrt{2}}$ $\Rightarrow$ $\theta = \dfrac{\pi}{8}$

So $\phi = \pi - \left(\dfrac{\pi}{2} + \dfrac{\pi}{8}\right)$ $\Rightarrow$ $\phi = \dfrac{3\pi}{8}$

5. 3 cos x + 2 cos 3x = cos y .....(i)

3 sin x + 2 sin 3x = sin y .....(ii)

$(i)^2 + (ii)^2$ gives

9 + 4 + 12 cos x cos 3x + 12 sin x sin 3x = 1

$\Rightarrow$ 13 + 12 (sin x sin 3x + cos x cos 3x) = 1

$\Rightarrow$ 13 + 12 cos 2x = 1

$\Rightarrow$ 12 cos 2x = – 12

$\Rightarrow$ cos 2x = – 1

6. $\dfrac{\cos 3\theta}{\cos\theta}$ = 4 cos² θ – 3 = 2 (1 + cos 2θ) – 3 = 2 cos2θ – 1 = 2 cos (α – β) – 1

$(\cos^2\alpha + \sin^2\alpha) + (\cos^2\beta + \sin^2\beta) + 2\cos(\alpha - \beta) = a^2 + b^2$

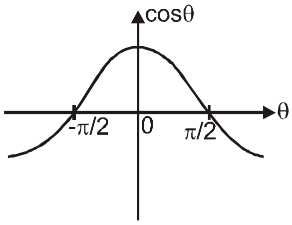
$2\cos(\alpha - \beta) = a^2 + b^2 - 2 \Rightarrow \dfrac{\cos 3\theta}{\cos\theta} = a^2 + b^2 - 3$

7. $\dfrac{3\pi}{4} < \alpha < \pi$, then $\sqrt{2\cot\alpha + \dfrac{1}{\sin^2\alpha}} = \sqrt{\text{cosec}^2\alpha + 2\cot\alpha} = \sqrt{\cot^2\alpha + 1 + 2\cot\alpha} = \sqrt{(1+\cot\alpha)^2}$

$\because \quad \dfrac{3\pi}{4} < \alpha < \pi = -(1 + \cot\alpha)$

8. $\dfrac{-\pi}{2} < \theta < \dfrac{\pi}{2}$ $\dfrac{\sin\theta + \sin 2\theta}{1 + \cos\theta + \cos 2\theta} = \dfrac{\sin\theta\,(1 + 2\cos\theta)}{2\cos^2\theta + \cos\theta} = \dfrac{\sin\theta\,(1 + 2\cos\theta)}{\cos\theta\,(1 + 2\cos\theta)} = \tan\theta$

∴ Range ∈ $(-\infty, \infty)$



$\because \cos\theta \neq \dfrac{-1}{2}$ for $\theta \in \left(\dfrac{-\pi}{2}, \dfrac{\pi}{2}\right)$.

9. $a_1 + a_2 \cos 2x + a_3 \sin^2 x = 0 \Rightarrow a_1 + a_2 (1 - 2\sin^2 x) + a_3 \sin^2 x = 0$

$\Rightarrow (a_1 + a_2) + \sin^2 x\,(a_3 - 2a_2) = 0$ is an identity $\Rightarrow a_1 + a_2 = 0$ & $a_3 - 2a_2 = 0$

$\Rightarrow \dfrac{a_1}{-1} = \dfrac{a_2}{+1} = \dfrac{a_3}{+2}$ ∴ infinite triplets are possible

10. $A + B + C = \dfrac{3\pi}{2} \Rightarrow \cos 2A + \cos 2B + \cos 2C = 2\cos(A+B).\cos(A-B) + 1 - 2\sin^2 C$

$= 2\cos\left(\dfrac{3\pi}{2} - C\right).\cos(A - B) - 2\sin^2 C + 1 \qquad \left(\because A + B + C = \dfrac{3\pi}{2}\right)$

$= -2\sin C\,\{\cos(A - B) + \sin C\} + 1 = -2\sin C\,\{\cos(A - B) + \sin\left(\dfrac{3\pi}{2} - (A + B)\right)\} + 1$

$= -2\sin C\,\{\cos(A - B) - \cos(A + B)\} + 1 = 1 - 4\sin A \sin B \sin C.$

11. As $\cos A = \cos B \cos C$, so $\Rightarrow \tan B \tan C = \dfrac{\sin B \quad \sin C}{\cos B \quad \cos C} = \dfrac{2}{2\cos A} \times \sin B \sin C$

$= \dfrac{2}{2\cos A}\,(\cos(B - C) - \cos(B + C)) = \dfrac{2}{2\cos A}(\cos(B - C) + \cos(B + C) - 2\cos(B + C))$

$= \dfrac{2}{2\cos A}\,\{(2\cos A - 2\cos(B + C)\} = \dfrac{4\cos A}{2\cos A} = 2$

12. $\tan^2\alpha + 2\sqrt{3}\tan\alpha - 1 = 0 \qquad \Rightarrow \tan\alpha = \dfrac{-2\sqrt{3} \pm \sqrt{12+4}}{2} = \dfrac{-2\sqrt{3} \pm 4}{2} = -\sqrt{3} \pm 2$

$= 2 - \sqrt{3}, -\left(2 + \sqrt{3}\right) = \tan 15^\circ, -\cot 15^\circ \qquad = \tan\dfrac{\pi}{12}, \tan\left(\dfrac{-5\pi}{12}\right) \Rightarrow \alpha = n\pi + \dfrac{\pi}{12}, n\pi - \dfrac{5\pi}{12}$

or $n\pi + \dfrac{\pi}{12}, (2n - 1)\dfrac{\pi}{2} + \dfrac{\pi}{12} \Rightarrow \alpha = \dfrac{2n\pi}{2} + \dfrac{\pi}{12}, (2n - 1)\dfrac{\pi}{2} + \dfrac{\pi}{12} \Rightarrow \alpha = \dfrac{n\pi}{2} + \dfrac{\pi}{12}.$

13. $\tan x + \tan\left(x + \frac{\pi}{3}\right) + \tan\left(x + \frac{2\pi}{3}\right) = 3 \Rightarrow 3\tan 3x = 3 \Rightarrow \tan 3x = 1 \Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}$, $n \in I$

14. $1 + 2\,\text{cosec}\, x = \dfrac{-\sec^2\left(\frac{x}{2}\right)}{2} \Rightarrow 1 + \dfrac{2}{\sin x} = \dfrac{-1}{1+\cos x}$

$\Rightarrow (2 + \sin x)(1 + \cos x) = -\sin x \Rightarrow 2 + 2\cos x + \sin x + \sin x \cos x = -\sin x$

$\Rightarrow 2(\sin x + \cos x) + \sin x \cos x + 2 = 0$

Put $\sin x + \cos x = t \quad \Rightarrow 1 + 2 \sin x \cos x = t^2$

$\therefore \quad 2t + \dfrac{t^2 - 1}{2} + 2 = 0 \Rightarrow t^2 + 4t + 3 = 0$

$\Rightarrow t = -1, -3 \Rightarrow \sin x + \cos x = -1 \Rightarrow \cos\left(x - \frac{\pi}{4}\right) = \frac{-1}{\sqrt{2}} = \cos\frac{3\pi}{4}$

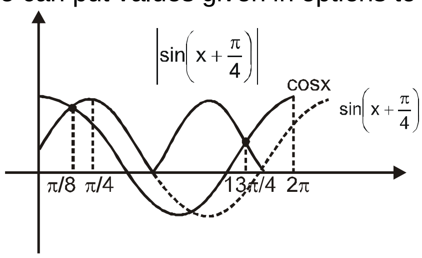
$\Rightarrow x - \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4} \Rightarrow x = 2n\pi + \pi,\ 2n\pi - \frac{\pi}{2} \Rightarrow x = 2n\pi + \pi$ at which cosec x is not defined

$\therefore \quad x = 2n\pi - \frac{\pi}{2}$.

15. $2\cos x = \sqrt{2 + 2\sin 2x} \Rightarrow \sqrt{2}\cos x = \sqrt{1 + \sin 2x} = |\sin x + \cos x|$

$\Rightarrow \cos x = \left|\frac{1}{\sqrt{2}}(\sin x + \cos x)\right| \Rightarrow \cos x = \left|\sin\left(x + \frac{\pi}{4}\right)\right|$

$\Rightarrow$ see from graph or we can put values given in options to verify.



16. $|\cos x| = \cos x - 2\sin x$

Case-I : $\cos x \geq 0 \Rightarrow \cos x = \cos x - 2\sin x \Rightarrow \sin x = 0 \Rightarrow x = n\pi = 2m\pi$ or $(2m + 1)\pi$

As $\cos x \geq 0$

$\therefore x = 2m\pi$

Case-II : $\cos x < 0 \Rightarrow -\cos x = \cos x - 2\sin x \Rightarrow \tan x = 1 \Rightarrow x = n\pi + \frac{\pi}{4}$

$\Rightarrow x = 2m\pi + \frac{\pi}{4}$ or $(2m + 1)\pi + \frac{\pi}{4}$ As $\cos x < 0$

$\therefore x = (2m + 1)\pi + \frac{\pi}{4}$.

17. $4^{\tan x} - 3.2^{\tan x} + 2 \leq 0 \quad \Rightarrow \quad t^2 - 3t + 2 \leq 0, \quad t = 2^{\tan x}$

$(t - 1)(t - 2) \leq 0 \quad \Rightarrow \quad t \in [1, 2] \Rightarrow 1 \leq 2^{\tan x} \leq 2$

$\Rightarrow \quad 2^0 \leq 2^{\tan x} \leq 2^1 \quad \Rightarrow \quad 0 \leq \tan x \leq 1 \quad \Rightarrow \quad x \in \left[0, \frac{\pi}{4}\right]$

$\because \quad x \in \left[n\pi, \ n\pi + \frac{\pi}{4}\right]$, $n \in I$

18. Clearly $5 - 2\sin x > 0$

case-I : If $6\sin x - 1 \le 0 \Rightarrow -1 \le \sin x \le \frac{1}{6}$ then inequality is true

case-II : If $\frac{1}{6} < \sin x < 1$ then we have $5 - 2\sin x \ge 36\sin^2 x - 12\sin x + 1$

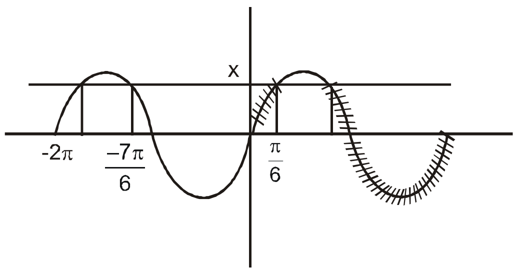
$36\sin^2 x - 10\sin x - 4 \le 0 \Rightarrow 18\sin^2 x - 5\sin x - 2 \le 0$

$18\sin^2 x - 9\sin x + 4\sin x - 2 \le 0 \Rightarrow 9\sin x(2\sin x - 1) + 2(2\sin x - 1) \le 0$

$\Rightarrow (2\sin x - 1)(9\sin x + 2) \le 0$

$\Rightarrow \sin x \in \left[-\frac{2}{9}, \frac{1}{2}\right] \quad \therefore \ \sin x \in \left(\frac{1}{6}, \frac{1}{2}\right]$

$\therefore \ \sin x \in \left[-1, \frac{1}{2}\right]$



PART - II

1. $\because$ $19\sin\alpha = 29\sin\beta \Rightarrow \frac{\sin\alpha}{\sin\beta} = \frac{29}{19} \Rightarrow \frac{\sin\alpha + \sin\beta}{\sin\alpha - \sin\beta} = \frac{48}{10} \Rightarrow \frac{\tan\left(\frac{\alpha+\beta}{2}\right)}{\tan\left(\frac{\alpha-\beta}{2}\right)} = 04.80$

2. $5 - 12\tan\theta = 11\sec\theta \Rightarrow 25 + 144\tan^2\theta - 120\tan\theta = 121 + 121\tan^2\theta$

$23\tan^2\theta - 120\tan\theta - 96 = 0 \Rightarrow \tan\alpha + \tan\beta = \frac{120}{23} \Rightarrow \tan\alpha\tan\beta = -\frac{96}{23}$

$\tan(\alpha+\beta) = \frac{\frac{120}{23}}{1 + \frac{96}{23}} = \frac{120}{119} \Rightarrow \sin(\alpha+\beta) = -\frac{120}{169}$.

3. $4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + \sqrt{4\sin^4 x + 4\sin^2 x\cos^2 x} = 4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) + |2\sin x| = 4\cos^2\left(\frac{\pi}{4} - \frac{x}{2}\right) - 2\sin x$

$= 2\left(1 + \cos\left(\frac{\pi}{2} - x\right)\right) - 2\sin x = 2$

4. $\cos A\cos B\cos C = \lambda(\cos 3A + \cos 3B + \cos 3C)$

$\Rightarrow$ $\cos A\cos B\cos C = \lambda(4\cos^3 A - 3\cos A + 4\cos^3 B - 3\cos B + 4\cos^3 C - 3\cos C)$

$\Rightarrow$ $\cos A\cos B\cos C = \lambda(4(\cos^3 A + \cos^3 B + \cos^3 C) - 3 \times 0)$

$\because$ $\cos A + \cos B + \cos C = 0 \Rightarrow \cos^3 A + \cos^3 B + \cos^3 C = 3\cos A\cos B\cos C$

$\Rightarrow$ $\cos A\cos B\cos C = 12\lambda\cos A\cos B\cos C \Rightarrow \lambda = \frac{1}{12}$

5. $-5 \le 2\lambda + 1 \le 5 \Rightarrow -6 \le 2\lambda \le 4 \Rightarrow -3 \le \lambda \le 2$

$\lambda = -3, -2, -1, 0, 1, 2$

$\lambda^2 = 9, 4, 1, 0, 1, 4$

6. $a\cos^3\alpha + 3a\cos\alpha\sin^2\alpha = m$ ... (i)

$a\sin^3\alpha + 3a\cos^2\alpha\sin\alpha = n$ ... (ii)

then $(m + n)^{2/3} + (m - n)^{2/3}$

$= (a(\cos^3\alpha + \sin^3\alpha) + 3a\cos\alpha\sin\alpha(\cos\alpha + \sin\alpha))^{2/3}$

$+ (a(\cos^3\alpha - \sin^3\alpha) + 3a\cos\alpha\sin\alpha(\sin\alpha - \cos\alpha))^{2/3}$

$= a^{2/3}[\{\cos^3\alpha + \sin^3\alpha + 3\cos\alpha\sin\alpha(\cos\alpha + \sin\alpha)\}^{2/3} + \{\cos^3\alpha - \sin^3\alpha - 3\cos\alpha\sin\alpha(\cos\alpha - \sin\alpha)\}^{2/3}]$

$= a^{2/3}[\{(\cos\alpha + \sin\alpha)^3\}^{2/3} + \{(\cos\alpha - \sin\alpha)^3\}^{2/3}]$

$= a^{2/3}[(\cos\alpha + \sin\alpha)^2 + (\cos\alpha - \sin\alpha)^2] = a^{2/3}[1 + 1] = 2a^{2/3}$

7. $2\cos x + \sin x = 1$ ..... (1)

$4\cos^2 x = (1 - \sin x)^2 \Rightarrow 4 - 4\sin^2 x = 1 + \sin^2 x - 2\sin x$

$5\sin^2 x - 2\sin x - 3 = 0 \Rightarrow (\sin x - 1)(5\sin x + 3) = 0 \Rightarrow \sin x = 1, \sin x = -\frac{3}{5}$

$\because \quad \cos x = \frac{1 - \sin x}{2}$ (from equation (1))

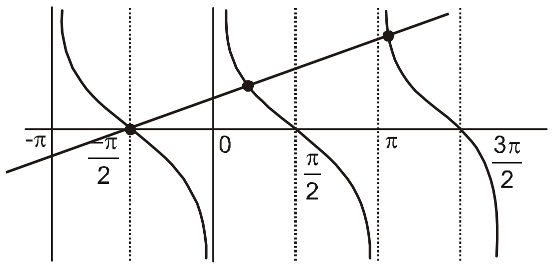
$\therefore$ when $\sin x = 1$

$7\cos x + 6\sin x = 7\left(\frac{1-\sin x}{2}\right) + 6\sin x = 7\left(\frac{1-1}{2}\right) + 6 \times 1 = 6$ Ans.

and when $\sin x = \frac{-3}{5}$ then $7\cos x + 6\sin x = 7\left(\frac{1+\frac{3}{5}}{2}\right) - \frac{6 \times 3}{5} = \frac{28 - 18}{5} = 2$ Ans.

8. $\cot x = \frac{\pi}{2} + x$; $x \in \left[-\pi, \frac{3\pi}{2}\right]$

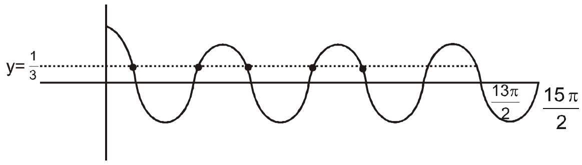
let $y = \cot x$ & $y = \frac{\pi}{2} + x$



3 solutions.

9. $2\tan^2 x - 5\sec x - 1 = 0 \Rightarrow 2(\sec^2 x - 1) - 5\sec x - 1 = 0 \Rightarrow 2\sec^2 x - 5\sec x - 3 = 0$

$\Rightarrow \sec x = \frac{6}{2}, \frac{-1}{2} = 3, \frac{-1}{2} \quad \Rightarrow \sec x = 3 \left(\sec x \neq \frac{-1}{2}\right)$



$\Rightarrow \cos x = \frac{1}{3} \quad \Rightarrow$ 7 solutions in $\left[0, \frac{15\pi}{2}\right]$

$\therefore$ n = 15.

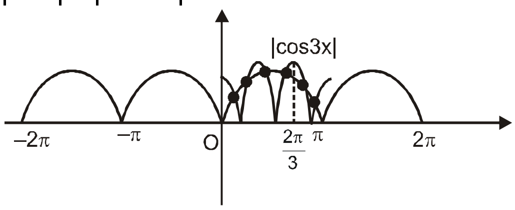
10. $\cos 2x + a\sin x = 2a - 7 \Rightarrow 1 - 2\sin^2 x + a\sin x = 2a - 7$

$\Rightarrow 2\sin^2 x - a\sin x + 2(a - 4) = 0$ .... (1)

$\Rightarrow \sin x = \frac{a \pm \sqrt{a^2 - 16(a-4)}}{4} = \frac{a \pm \sqrt{(a-8)^2}}{4} = \frac{a \pm (a-8)}{4} = \frac{2a-8}{4}, \frac{8}{4}$

$\Rightarrow \sin x = \frac{a-4}{2} \Rightarrow -1 \leq \frac{a-4}{2} \leq 1 \Rightarrow 2 \leq a \leq 6 \Rightarrow a = 2, 3, 4, 5, 6$

11. $|\sin x| = |\cos 3x|$



Number of solutions in $[0, \pi] = 6$

Number of solutions in $[-2\pi, 2\pi] = 24$

12. $\sum \cos A \operatorname{cosec} B \operatorname{cosec} C = \frac{\cos A}{\sin B \ \sin C} + \frac{\cos B}{\sin A \ \sin C} + \frac{\cos C}{\sin A \ \sin B}$

$= \frac{\cos A \sin A + \cos B \sin B + \cos C \sin C}{\sin A \sin B \sin C} = \frac{\sin 2A + \sin 2B + \sin 2C}{2\sin A \sin B \sin C} = \frac{4\sin A \sin B \sin C}{2\sin A \sin B \sin C}$

(using conditional identity) = 2

13. $A + B + C = \pi$

LHS = $\tan C(\tan A + \tan B) + \tan A . \tan B = \tan C \ \frac{\sin(A+B)}{\cos A \cos B} + \tan A \tan B$

$= \frac{\sin C \sin(\pi - C)}{\cos A \cos B \cos C} + \frac{\sin A \sin B}{\cos A \cos B} = \frac{\sin^2 C + \sin A \sin B \cos C}{\cos A \cos B \cos C} = \frac{1 - \cos^2 C + \sin A \sin B \cos C}{\cos A \cos B \cos C}$

$= \frac{1 - \cos C(\cos C - \sin A \sin B)}{\cos A \cos B \cos C} = \frac{1 - \cos C(-\ \cos(A+B) - \sin A \sin B)}{\cos A \cos B \cos C}$

$= \frac{1 + \cos A \cos B \cos C}{\cos A \cos B \cos C}$ = 1 + secA secB secC = RHS.

14. $4\cos^3 x - 4\cos^2 x + \cos x - 1 = 0 \Rightarrow (4\cos^2 x + 1)(\cos x - 1) = 0 \Rightarrow \cos x = 1$

$x = 2n\pi$

solutions in the interval [0, 315] are $0, 2\pi, 4\pi, \ldots, 100\pi$

$\therefore$ arithmatic mean = $\frac{0 + 2\pi + 4\pi + \ldots + 100\pi}{51} = 50\pi$

15. $\alpha - \beta = 0, -2\pi$ or 2π

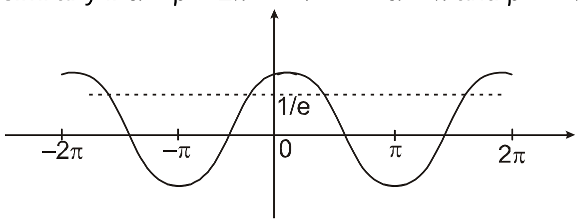
$\alpha - \beta = 0 \Rightarrow \alpha = \beta \Rightarrow \cos 2\beta = \frac{1}{e}$

This is true for '4' value of 'α', 'β'

If $\alpha - \beta = -2\pi \Rightarrow \alpha = -\pi$ and $\beta = \pi$

and $\cos(\alpha + \beta) = 1 \Rightarrow$ (No solution)

similarly if $\alpha - \beta = 2\pi \Rightarrow \alpha = \pi$ and $\beta = -\pi$



again no solution results

16. $\sec^2\theta . \text{cosec}^2\theta + 2\,\text{cosec}^2\theta = 8 \Rightarrow \frac{1}{\sin^2\theta\cos^2\theta} + \frac{2}{\sin^2\theta} = 8 \Rightarrow 1 + 2\cos^2\theta = 8\sin^2\theta\cos^2\theta$

$\Rightarrow 8\sin^2\theta\cos^2\theta - 2\cos^2\theta - 1 = 0$

$\Rightarrow 8(1 - \cos^2\theta)\cos^2\theta - 2\cos^2\theta - 1 = 0 \Rightarrow 8\cos^4\theta - 6\cos^2\theta + 1 = 0$

$\Rightarrow \cos^2\theta = \frac{4}{8}, \frac{2}{8} = \frac{1}{2}, \frac{1}{4} \Rightarrow \cos\theta = \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{2} \Rightarrow \theta = \frac{\pi}{4}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}, \frac{5\pi}{4}, \frac{5\pi}{3}, \frac{7\pi}{4}$

17. $\cos 6\theta + \cos 4\theta + \cos 2\theta + 1 = 0 \Rightarrow 2\cos 4\theta\cos 2\theta + (1 + \cos 4\theta) = 0$

$\Rightarrow 2\cos 4\theta\cos 2\theta + 2\cos^2 2\theta = 0 \Rightarrow 2\cos 2\theta(\cos 4\theta + \cos 2\theta) = 0$

$\Rightarrow \cos 2\theta(2\cos 3\theta\cos\theta) = 0 \Rightarrow \theta = (2n+1)\frac{\pi}{2}, (2n+1)\frac{\pi}{4}, (2n+1)\frac{\pi}{6}$

18. $\tan\theta + \tan 2\theta + \tan 3\theta - \tan\theta\tan 3\theta\tan 2\theta = 0$ ...(1)

$\Rightarrow \tan(\theta + 2\theta + 3\theta) = 0 \Rightarrow \tan 6\theta = 0 \Rightarrow 6\theta = n\pi \Rightarrow \theta = \frac{n\pi}{6}$

19. $\cos x . \sin y = 1 \Rightarrow$ Either $\cos x = 1$ and $\sin y = 1$

or $\cos x = -1$ and $\sin y = -1 \Rightarrow (x,y) = \left(0, \frac{\pi}{2}\right), \left(0, \frac{5\pi}{2}\right), \left(2\pi, \frac{\pi}{2}\right), \left(2\pi, \frac{5\pi}{2}\right)$

or $(x, y) = \left(\pi, \frac{3\pi}{2}\right), \left(3\pi, \frac{3\pi}{2}\right)$

$\therefore$ Number of pairs = 6

20. $\sin 3\theta = 4\sin\theta\sin 2\theta\sin 4\theta \Rightarrow \sin 3\theta = (2\sin\theta)(2\sin 2\theta\sin 4\theta)$

$\Rightarrow 3\sin\theta - 4\sin^3\theta = 2\sin\theta(\cos 2\theta - \cos 6\theta) \Rightarrow 3 - 4\sin^2\theta = 2(\cos 2\theta - \cos 6\theta)$ or $\sin\theta = 0$

$\Rightarrow 3 - 2(1 - \cos 2\theta) = 2\cos 2\theta - 2\cos 6\theta$ or $\sin\theta = 0$

$\Rightarrow 1 = -2\cos 6\theta \Rightarrow \cos 6\theta = \frac{-1}{2}$ or $\sin\theta = 0$

$\therefore \sin\theta = 0$ or $\cos 6\theta = \frac{-1}{2}$

$\Rightarrow \theta = n\pi$ or $\theta = \frac{2n\pi \pm \left(\frac{2\pi}{3}\right)}{6} = \frac{n\pi}{3} \mp \frac{\pi}{9}$

$\Rightarrow \theta = 0, \pi, \frac{\pi}{9}, \frac{\pi}{3} \mp \frac{\pi}{9}, \frac{2\pi}{3} \mp \frac{\pi}{9}, \pi - \frac{\pi}{9}$

So eight solutions.

21. $\cos 6x\,(1 + \tan^2 x) = 1 - \tan^2 x$ or $\cos 6x = \dfrac{1-\tan^2 x}{1+\tan^2 x}$

or $\cos 6x = \cos 2x$ or $6x = 2n\pi \pm 2x \Rightarrow x = \dfrac{n\pi}{2}, \dfrac{n\pi}{4}$

$\Rightarrow x = 0, \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \pi, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}, 2\pi$ (At, $x = \dfrac{\pi}{2}, \dfrac{3\pi}{2}$, tan x does not exist)

22. $\tan\theta + \sin\phi = \dfrac{3}{2}$ ...(1)

As $\tan^2\theta + \cos^2\phi = \dfrac{7}{4} \Rightarrow \left(\dfrac{3}{2} - \sin\phi\right)^2 + \cos^2\phi = \dfrac{7}{4} \Rightarrow \dfrac{9}{4} + \sin^2\phi - 3\sin\phi + \cos^2\phi = \dfrac{7}{4}$

$\Rightarrow \dfrac{3}{2} = 3\sin\phi \Rightarrow \sin\phi = \dfrac{1}{2} \Rightarrow \phi = n\pi + (-1)^n \dfrac{\pi}{6}$

from (1) ((1)), $\tan\theta = \dfrac{3}{2} - \sin\phi = \dfrac{3}{2} - \dfrac{1}{2} = 1 \Rightarrow \theta = n\pi + \dfrac{\pi}{4}$.

23. Let $\sin x(\sin x + \cos x) = n \Rightarrow \sin^2 x + \sin x \cos x = n \Rightarrow 1 - \cos 2x + \sin 2x = 2n$

$\Rightarrow \sin 2x - \cos 2x = 2n - 1 \Rightarrow -\sqrt{2} \le 2n - 1 \le \sqrt{2} \Rightarrow \dfrac{1-\sqrt{2}}{2} \le n \le \dfrac{1+\sqrt{2}}{2} \Rightarrow n = 0, 1$

So number of integral values of n = 2

24. $\cot x - 2\sin 2x = 1 \Rightarrow \dfrac{1}{\tan x} - \dfrac{4\tan x}{1+\tan^2 x} - 1 \Rightarrow 1 + t^2 - 4t^2 = t + t^3$ where $t = \tan x$

$\Rightarrow t^3 + 3t^2 + t - 1 = 0 \Rightarrow (t+1)(t^2 + 2t - 1) = 0 \Rightarrow t = -1, -1 \pm \sqrt{2} = -1, -1-\sqrt{2}, -1+\sqrt{2}$

$\Rightarrow \tan x = \tan\left(-\dfrac{\pi}{4}\right), -\tan\left(\dfrac{\pi}{2} - \dfrac{\pi}{8}\right), \tan\dfrac{\pi}{8} \Rightarrow x = n\pi - \dfrac{\pi}{4}, n\pi + \dfrac{\pi}{8}, n\pi + \dfrac{\pi}{8} - \dfrac{\pi}{2}$

$\Rightarrow x = n\pi - \dfrac{\pi}{4}, \dfrac{2n\pi}{2} + \dfrac{\pi}{8}, (2n-1)\dfrac{\pi}{2} + \dfrac{\pi}{8} = n\pi - \dfrac{\pi}{4}, \dfrac{n\pi}{2} + \dfrac{\pi}{8}$.

25. $\sin\theta + 2\sin 2\theta + 3\sin 3\theta + 4\sin 4\theta = 10 \Rightarrow \sin\theta = \sin 2\theta = \sin 3\theta = \sin 4\theta = 1$ is $(0, \pi)$ which is not possible

26. RHS = $3x^2 + 2x + 3$; Minimum value = $\dfrac{4(3)(3) - 4}{4(3)} = \dfrac{8}{3} > 2$

whereas LHS ≤ 2 no solution.

27. $\sin x \cos x - 3\cos x + 4\sin x - 12 - 1 > 0 \Rightarrow (\sin x - 3)(\cos x + 4) - 1 > 0 \Rightarrow$ (–ve) (+ve) $-1 > 0$
(which is not posible) $\Rightarrow x \in \phi$

PART - III

1. $\dfrac{1+\tan 11^\circ}{1-\tan 11^\circ} = \tan(45^\circ + 11^\circ) = \tan 56^\circ$

2. $\sin t + \cos t = \dfrac{1}{5} \Rightarrow \dfrac{2\tan\dfrac{t}{2} + 1 - \tan^2\dfrac{t}{2}}{1+\tan^2\dfrac{t}{2}} = \dfrac{1}{5} \Rightarrow 10\tan\dfrac{t}{2} + 5 - 5\tan^2\dfrac{t}{2} = 1 + \tan^2\dfrac{t}{2}$

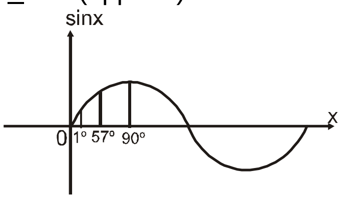
$\Rightarrow 6\tan^2\dfrac{t}{2} - 10\tan\dfrac{t}{2} - 4 = 0 \Rightarrow 3\tan^2\dfrac{t}{2} - 6\tan\dfrac{t}{2} + \tan\dfrac{t}{2} - 2 = 0$

$\Rightarrow 3\tan\dfrac{t}{2}\left(\tan\dfrac{t}{2} - 2\right) + 1\left(\tan\dfrac{t}{2} - 2\right) = 0 \Rightarrow \tan\dfrac{t}{2} = 2, \tan\dfrac{t}{2} = -\dfrac{1}{3}$

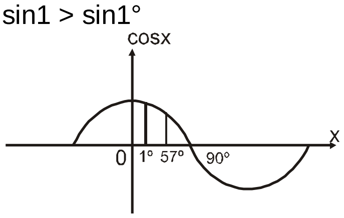
3. $\dfrac{\sin x + \cos x}{\cos^3 x}$ = tanx $\sec^2$x + $\sec^2$x = $\sec^2$x (1 + tanx) = (1 + $\tan^2$x) (1 + tanx)

4. 1 = [(sec A – tan A) (sec B – tan B) (sec C – tan C)]^2
$\Rightarrow$ (sec A – tan A) (sec B – tan B) (sec C – tan C) = ± 1
So each side is equal to ± 1

5. 1 radian $\simeq$ 57º (approx.)



$\therefore$ sin1 > sin1°



$\therefore$ cos1° > cos1

6. sinx + sin y = a .....(1) cosx + cos y = b .....(2)

$$\frac{2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)} = \frac{a}{b} \Rightarrow \tan\left(\frac{x+y}{2}\right) = \frac{a}{b}$$

$\Rightarrow$ $\sin\left(\frac{x+y}{2}\right) = \dfrac{a}{\sqrt{a^2+b^2}}$, $\cos\left(\frac{x+y}{2}\right) = \dfrac{b}{\sqrt{a^2+b^2}}$

$\Rightarrow$ $\sin(x+y) = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x+y}{2}\right) = \dfrac{2ab}{a^2+b^2}$

Now for $\tan\left(\frac{x-y}{2}\right)$ $\left(\tan\left(\frac{x-y}{2}\right)\right) \Rightarrow (1)^2 + (2)^2 \Rightarrow 1 + 1 + 2\cos(x-y) = a^2 + b^2$

$\cos(x-y) = \dfrac{a^2+b^2-2}{2}$

$\because$ $\tan^2\left(\frac{x-y}{2}\right) = \dfrac{1-\cos(x-y)}{1+\cos(x-y)}$

$\Rightarrow$ $\tan^2\left(\frac{x-y}{2}\right) = \dfrac{1-\left(\frac{a^2+b^2-2}{2}\right)}{1+\frac{a^2+b^2-2}{2}}$ $\Rightarrow$ $\tan\left(\frac{x-y}{2}\right) = \pm\sqrt{\dfrac{4-a^2-b^2}{a^2+b^2}}$

7. $\cos(A-B) = \frac{3}{5}$ & tan A tan B = 2

$\cos A\cos B + \sin A\sin B = \frac{3}{5} \Rightarrow (1 + \tan A\tan B)\cos A\cos B = \frac{3}{5}$

$\Rightarrow (1+2) \times \cos A\cos B = \frac{3}{5} \Rightarrow \cos A\cos B = \frac{1}{5}$ $\therefore \sin A\sin B = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$

$\cos(A+B) = \cos A\cos B - \sin A\sin B = \frac{1}{5} - \frac{2}{5} = \frac{-1}{5}$.

8. $P_n - P_{n-2} = \cos^n\theta + \sin^n\theta - \cos^{n-2}\theta - \sin^{n-2}\theta = \cos^{n-2}\theta(\cos^2\theta - 1) + \sin^{n-2}\theta(\sin^2\theta - 1)$

$= \cos^{n-2}\theta(-\sin^2\theta) + \sin^{n-2}\theta(-\cos^2\theta) = (-\sin^2\theta\cos^2\theta)\{\cos^{n-4}\theta + \sin^{n-4}\theta\} = (-\sin^2\theta\cos^2\theta)P_{n-4}$

put $n = 4 \Rightarrow P_4 - P_2 = (-\sin^2\theta\cos^2\theta)P_0 \Rightarrow P_4 = P_2 - 2\sin^2\theta\cos^2\theta = 1 - 2\sin^2\theta\cos^2\theta$

similarly we can prove the other result also.

9. $\tan^2\alpha + 2\tan\alpha \,.\, \tan 2\beta = \tan^2\beta + 2\tan\beta \,.\, \tan 2\alpha$

$\Rightarrow\quad (\tan^2\alpha - \tan^2\beta) + 4\tan\alpha\tan\beta\left(\frac{1}{1-\tan^2\beta} - \frac{1}{1-\tan^2\alpha}\right) = 0$

$\Rightarrow\quad (\tan^2\alpha - \tan^2\beta) + 4\tan\alpha\tan\beta\ \frac{(\tan^2\beta - \tan^2\alpha)}{(1-\tan^2\alpha)(1-\tan^2\beta)} = 0$

$\Rightarrow\quad (\tan^2\alpha - \tan^2\beta)\left\{1 - \frac{4\tan\alpha\tan\beta}{(1-\tan^2\alpha)(1-\tan^2\beta)}\right\} = 0$

$\Rightarrow\quad (\tan^2\alpha - \tan^2\beta)(1 - \tan 2\alpha \,.\, \tan 2\beta) = 0$

$\Rightarrow\quad \tan^2\alpha = \tan^2\beta$ or $\tan 2\alpha \,.\, \tan 2\beta = 1$

L.H.S.$= \tan^2\alpha + 2\tan\alpha\ \frac{1}{\tan 2\alpha}. = \tan^2\alpha + \frac{2\tan\alpha}{2\tan\alpha}.\ (1 - \tan^2\alpha) = 1$

R.H.S. $= \tan^2\beta + 2\tan\beta.\ \frac{1}{\tan 2\beta} = \tan^2\beta + \frac{2\tan\beta}{2\tan\beta}.\ (1 - \tan^2\beta) = 1$

10. $h = \sqrt{\{\cos 2\alpha + \cos 2\beta + 2\cos(\alpha+\beta)\}^2 + \{\sin 2\alpha + \sin 2\beta + 2\sin(\alpha+\beta)\}^2}$

$\Rightarrow\quad h = [4\cos^2(\alpha+\beta)(\cos(\alpha-\beta)+1)^2 + 4\sin^2(\alpha+\beta)(\cos(\alpha-\beta)+1)^2]^{1/2}$

$\Rightarrow\quad h = [4\{\cos(\alpha-\beta)+1\}^2\{\cos^2(\alpha+\beta) + \sin^2(\alpha+\beta)\}]^{1/2}$

$\Rightarrow\quad h = 2(1 + \cos(\alpha-\beta)) \quad\Rightarrow\quad h = 2 \times 2\cos^2\left(\frac{\alpha-\beta}{2}\right)$

$\Rightarrow\quad h = 4\cos^2\left(\frac{\alpha-\beta}{2}\right)$

11. $0 < \theta < \pi/2 \Rightarrow \tan 3\theta = \tan(2\theta + \theta) = \frac{\tan 2\theta + \tan\theta}{1 - \tan 2\theta\ \tan\theta}$

$\Rightarrow \tan 3\theta - \tan\theta\tan 2\theta\tan 3\theta = \tan 2\theta + \tan\theta$

$\Rightarrow \tan 3\theta - \tan 2\theta - \tan\theta = \tan\theta\tan 2\theta\tan 3\theta$

$\Rightarrow \tan 3\theta + \tan 3\theta = \tan\theta.\ \tan 2\theta\tan 3\theta$

($\because$ given that $\tan\theta + \tan 2\theta + \tan 3\theta = 0$)

$\Rightarrow \tan 3\theta(2 - \tan\theta\tan 2\theta) = 0 \Rightarrow \tan 3\theta = 0$ or $\tan\theta\tan 2\theta = 2$.

12. Let $\tan\frac{\alpha}{2} = t$

$\Rightarrow\quad (a+2)\,2t + (2a-1)(1-t^2) = (2a+1)(t^2+1)$

$\Rightarrow\quad 2a\,t + 4t + 2a - 2a\,t^2 - 1 + t^2 = 2a + 1 + 2a\,t^2 + t^2$

$\Rightarrow\quad 4a\,t^2 - 2t(2+a) + 2 = 0 \quad\Rightarrow\quad 2a\,t^2 - 2t - a\,t + 1 = 0$

$\Rightarrow\quad 2t(at-1) - 1(at-1) = 0 \quad\Rightarrow\quad t = 1/2,\ t = 1/a$

$\Rightarrow\quad \tan\alpha = \frac{2\tan\alpha/2}{1-\tan^2\alpha/2} \quad\Rightarrow\quad \tan\alpha = \frac{2\times 1/2}{1-1/4} = 4/3$

or $\quad \tan\alpha = \frac{2/a}{1-1/a^2} = \frac{2a}{a^2-1}$

13. $y = a\cos^2 x + 2b\sin x\cos x + c\sin^2 x$ & $\tan x = \frac{2b}{a-c}$

$z = a\sin^2 x - 2b\sin x\cos x + c\cos^2 x \quad \Rightarrow \quad y + z = a + c$

and $y - z = (a - c)\left(\cos^2 x - \sin^2 x\right) + 4b\sin x\cos x$

$= (a - c)\cos 2x + 2b\sin 2x \qquad (\because\ 2b = (a - c)\tan x)$

$= (a - c)[\cos 2x + \tan x.\sin 2x] = (a - c)\left[2\cos 2x + \dfrac{\sin x}{\cos x}\sin 2x\right] = \dfrac{(a-c)\ \cos(2x - x)}{\cos x} = (a - c).$

14. $\left\{\dfrac{2\cos\left(\frac{A+B}{2}\right)\ .\ \cos\left(\frac{A-B}{2}\right)}{2\cos\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right\}^n + \left\{\dfrac{2\sin\left(\frac{A+B}{2}\right)\ .\ \cos\left(\frac{A-B}{2}\right)}{-2\sin\left(\frac{A+B}{2}\right).\sin\left(\frac{A-B}{2}\right)}\right\}^n$

$= \cot^n\left(\dfrac{A-B}{2}\right) + (-1)^n\cot^n\left(\dfrac{A-B}{2}\right) = \begin{cases} 0\ ;\ n \in \text{odd} \\ 2\cot^n\left(\dfrac{A-B}{2}\right);\ n \in \text{even}\end{cases}$

15. $\sin^6 x + \cos^6 x = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)(\sin^4 x + \cos^4 x - \sin^2 x\cos^2 x) = a^2$

$\Rightarrow \quad (\sin^2 x + \cos^2 x)^2 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow \quad 1 - 3\sin^2 x\cos^2 x = a^2$

$\Rightarrow \quad 1 - \dfrac{3}{4}\sin^2 2x = a^2$

$\Rightarrow \quad \dfrac{4\ (1-a^2)}{3} = \sin^2 2x$

$\Rightarrow \quad 0 \le \dfrac{4}{3}(1 - a^2) \le 1$

$1 - a^2 \ge 0 \qquad$ and $\qquad 4 - 4a^2 \le 3$

$a^2 \le 1 \qquad$ and $\qquad \dfrac{1}{4} \le a^2$

$-1 \le a \le 1 \qquad$ and $\qquad a \ge \dfrac{1}{2}$ or $a \le -\dfrac{1}{2}$

$a \in \left[-1, -\dfrac{1}{2}\right] \cup \left[\dfrac{1}{2},\ 1\right]$

16. $\sin(x - y) = \dfrac{1}{2}$ and $\cos(x + y) = \dfrac{1}{2}$

$\Rightarrow x - y = \dfrac{\pi}{6}, \dfrac{5\pi}{6}$ and $x + y = \dfrac{\pi}{3}, \dfrac{5\pi}{3}$

Adding $2x = \dfrac{\pi}{2}$ or $\dfrac{7\pi}{6}$ or $\dfrac{11\pi}{6}$

$\Rightarrow x = \dfrac{\pi}{4}$ or $\dfrac{7\pi}{12}$ or $\dfrac{11\pi}{12}$ when $x = \dfrac{\pi}{4}, y = \dfrac{\pi}{12}$

when $x = \dfrac{7\pi}{12}$ no value of y is possible.

when $x = \dfrac{11\pi}{12}, y = \dfrac{3\pi}{4}$.

17. $\Rightarrow \quad 2(\sec^2\alpha - \text{cosec}^2\alpha) + (\text{cosec}^2\alpha + \sec^2\alpha)(\text{cosec}^2\alpha - \sec^2\alpha) = \dfrac{15}{4}$

$\Rightarrow$ $(\text{cosec}^2\alpha - \sec^2\alpha)\,[\text{cosec}^2\alpha + \sec^2\alpha - 2] = \frac{15}{4}$

$\Rightarrow$ $4(\cot^2\alpha - \tan^2\alpha)(\cot^2\alpha + \tan^2\alpha) = 15$ $\Rightarrow$ $4(\cot^4\alpha - \tan^4\alpha) = 15$

$\Rightarrow$ $4(1 - \tan^8\alpha) = 15\tan^4\alpha$ $\Rightarrow$ $4\tan^8\alpha + 15\tan^4\alpha - 4 = 0$

$\Rightarrow$ $4\tan^8\alpha + 16\tan^4\alpha - \tan^4\alpha - 4 = 0$ $\Rightarrow$ $(4\tan^4\alpha - 1)(\tan^4\alpha + 4) = 0$

$\Rightarrow$ $\tan^4\alpha = \frac{1}{4}$ or $\tan^4\alpha = -4$ (not possible) $\Rightarrow$ $\tan^2\alpha = \pm\frac{1}{2}$

$\Rightarrow$ $\tan^2\alpha = +\frac{1}{2}\left(\because\ \tan^2\alpha \neq -\frac{1}{2}\right)$ $\Rightarrow$ $\tan\alpha = \pm\frac{1}{\sqrt{2}}$

18. $3\sin\beta = \sin(2\alpha + \beta)$, (Given) $\Rightarrow \tan(\alpha + \beta) - 2\tan\alpha$

$= \tan(\alpha + \beta) - \tan\alpha - \tan\alpha = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} - \frac{\sin\alpha}{\cos\alpha} - \tan\alpha$

$= \frac{\sin(\alpha+\beta-\alpha)}{\cos\alpha\ \cos(\alpha+\beta)} - \tan\alpha = \frac{\sin\beta}{\cos(\alpha+\beta)\ \cos\alpha} - \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\beta - \sin\alpha\cos(\alpha+\beta)}{\cos(\alpha+\beta)\ \cos\alpha}$

$= \frac{2\sin\beta - [\sin(2\alpha+\beta) - \sin\beta]}{2\cos(\alpha+\beta)\ \cos\alpha} = \frac{-[\sin(2\alpha+\beta) - 3\sin\beta]}{2\cos(\alpha+\beta)\cos\alpha} = 0$

19. $\frac{\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} = \pi \Rightarrow \frac{\alpha}{2} + \frac{\beta}{2} = \pi - \frac{\gamma}{2} \Rightarrow \tan\left(\frac{\alpha}{2} + \frac{\beta}{2}\right) = \tan\left(\pi - \frac{\gamma}{2}\right)$

$\Rightarrow \frac{\tan\frac{\alpha}{2} + \tan\frac{\beta}{2}}{1 - \tan\frac{\alpha}{2}\tan\frac{\beta}{2}} = -\tan\frac{\gamma}{2} \Rightarrow \tan\frac{\alpha}{2} + \tan\frac{\beta}{2} + \tan\frac{\gamma}{2} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}\tan\frac{\gamma}{2}$

20. $\cos^2x + \cos^2y + \cos^2z - 2\cos x\cos y\cos z$ (Given $x + y = z$)
$= 1 + \cos(x + y)\cos(x - y) + \cos^2z - 2\cos x\cos y\cos z$
$= 1 + \cos z\,[\cos(x - y) + \cos(x + y)] - 2\cos x\cos y\cos z$
$= 1 + \cos z \,.\, 2\cos x\cos y - 2\cos x\cos y\cos z = 1 = \cos(x + y - z)$

21. $\tan A + \tan B + \tan C - \tan A\tan B\tan C = 0$

$\Rightarrow$ $\sin A\cos B\cos C + \sin B\cos A\cos C + \sin C\cos A\cos B - \sin A\sin B\sin C = 0$

$\Rightarrow$ $\sin(A + B + C) = 0$

$\Rightarrow$ $A + B + C = n\pi,\ n \in I$

22. Let $y = 2\sin t$ $\Rightarrow$ $y = \frac{1 - 2x + 5x^2}{3x^2 - 2x - 1}$

$(3y - 5)x^2 - 2x(y - 1) - (y + 1) = 0$

$x \in R - \left\{1, -\frac{1}{3}\right\}$

$\therefore$ $D \geq 0$ $\Rightarrow$ $y^2 - y - 1 \geq 0$

$\therefore$ $y \geq \frac{1+\sqrt{5}}{2}$ or $y \leq \frac{1-\sqrt{5}}{2}$

$\Rightarrow$ $\sin t \geq \frac{1+\sqrt{5}}{4}$ or $\sin t \leq \frac{1-\sqrt{5}}{4}$

$\therefore$ range of t is $\left[-\frac{\pi}{2}, -\frac{\pi}{10}\right] \cup \left[\frac{3\pi}{10}, \frac{\pi}{2}\right]$

23. $2\sin2x = \sin x + \sin3x \Rightarrow 2\sin2x = 2\sin2x\cos x \Rightarrow \sin2x = 0$ or $\cos x = 1$

$\Rightarrow$ 2x = nπ or x = 2mπ $\Rightarrow x = \frac{n\pi}{2}$, 2mπ. options (A), (B), (C), (D) are all a part of $x = \frac{n\pi}{2}$.

24. sin x + sin 2x + sin 3x = 0 $\Rightarrow$ 2 sin 2x cos x + sin 2x = 0 $\Rightarrow$ sin 2x = 0 or $\cos x = \frac{-1}{2}$.

25. cos4x cos8x – cos5x cos9x = 0 $\Rightarrow$ 2cos4x cos8x = 2cos5x cos 9x $\Rightarrow$ cos12x + cos4x = cos14x + cos4x

$\Rightarrow$ 14x = 2nπ ± (12x) $\Rightarrow$ 2x = 2nπ or 26x = 2nπ $\Rightarrow$ x = nπ or $\frac{n\pi}{13}$

$\therefore$ sinx = 0 or sin13x = 0

26. Let E = sin x – $\cos^2$ x – 1 $\Rightarrow$ E = sin x – 1 + $\sin^2$ x – 1 = $\sin^2$ x + sin x – 2

$= \left(\sin x + \frac{1}{2}\right)^2 - \frac{9}{4}$ assumes least value when sin x = $-\frac{1}{2}$ $\Rightarrow$ x = nπ + $(-1)^n \left(-\frac{\pi}{6}\right)$.

27. $(X\cos\theta + Y\sin\theta)^2 + 4(X\cos\theta + Y\sin\theta)(X\sin\theta - Y\cos\theta) + (X\sin\theta - Y\cos\theta)^2 = aX^2 + bY^2$

$\Rightarrow$ (1 + 2 sin2θ) X^2 + (1–2 sin2θ) Y^2 – (cos2θ) XY = $aX^2 + bY^2$

$\Rightarrow$ cos2θ = 0 $\Rightarrow$ θ = 45° $\Rightarrow$ a = 3, b = – 1

28. $\sin(\pi x^2) = \sin(\pi(x^2 + 2x))$ $\Rightarrow$ $\pi x^2 = n\pi + (-1)^n(x^2 + 2x)$

Case- I n is even $\Rightarrow$ n = 2m then $x^2 = 2m + x^2 + 2x$ $\Rightarrow$ x = – m

This gives positive root as 1, 2, 3, ... corresponding to m = – 1, –2, – 3, ...

Case-II n is odd $\Rightarrow$ n = 2m+1 $\Rightarrow$ $x^2 = (2m+1) - (x^2+2x) \Rightarrow x = \frac{-1 \pm \sqrt{3+4m}}{2}$

This gives poistive roots $\frac{-1+\sqrt{3}}{2}$, $\frac{-1+\sqrt{7}}{2}$, $\frac{-1+\sqrt{11}}{2}$ .....

$\Rightarrow$ positive roots in increasing sequence are

$\frac{-1+\sqrt{3}}{2}, \frac{-1+\sqrt{7}}{2}, 1, \frac{-1+\sqrt{11}}{2}, \frac{-1+\sqrt{15}}{2}, \frac{-1+\sqrt{19}}{2}, \frac{-1+\sqrt{23}}{2}, 2, ...$

$\Rightarrow$ 8 term of sequence is 2

29. cosx. cos6x = –1

$\Rightarrow$ Either cosx = 1 and cos6x –1 or cosx = –1 and cos6x = 1

$\Rightarrow$ x = 2nπ and cos6x = –1 or x = (2n + 1) π and cos6x = 1

If x = 2nπ then cos6x cannot be –1

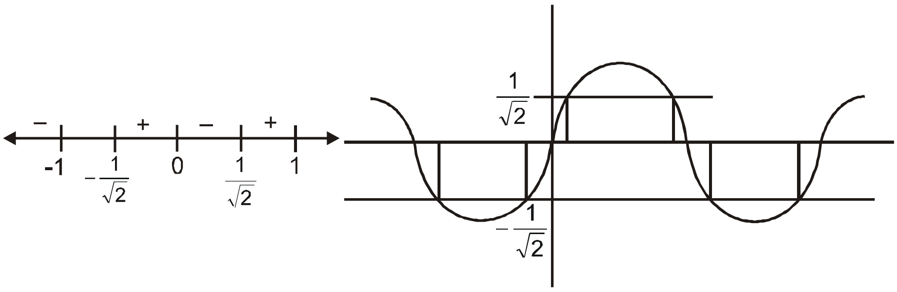
However if x = (2n +1)π then cos6x = 1

x = (2n + 1) π

$\Rightarrow$ x = (2n – 1) π is also as above.

30. **(i)** $\sin 3x < \sin x \Rightarrow 3 \sin x - 4 \sin^3 x - \sin x < 0 \Rightarrow 2 \sin x (1 - 2 \sin^2 x) < 0$

$\Rightarrow \sin x \left(\sin x + \frac{1}{\sqrt{2}}\right)\left(\sin x - \frac{1}{\sqrt{2}}\right) > 0$



$\Rightarrow \quad \sin \in \left[-1, \frac{-1}{\sqrt{2}}\right) \cup \left(\frac{-1}{\sqrt{2}}, 0\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right) \Rightarrow x \in \left(\frac{-\pi}{4}, 0\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right)$

$\therefore$ General solution $x \in \left(2n\pi - \frac{\pi}{4}, 2n\pi\right) \cup \left(2n\pi + \frac{\pi}{4}, 2n\pi + \frac{3\pi}{4}\right) \cup \left(2n\pi + \pi, 2n\pi + \frac{5\pi}{4}\right)$

(ii) $\tan^2 x - \tan x - \sqrt{3} \tan x + \sqrt{3} < 0$

$\tan x(\tan x - 1) - \sqrt{3} (\tan x - 1) < 0 \Rightarrow (\tan x - 1)(\tan x - \sqrt{3}) < 0$

$\Rightarrow \quad \tan x \in (1, \sqrt{3}) \Rightarrow x \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$\therefore$ general solution is $x \in \left(n\pi + \frac{\pi}{4}, n\pi + \frac{\pi}{3}\right)$, $n \in I$

(iii) Clearly $5 - 2 \sin x > 0$

case-I : If $6 \sin x - 1 \le 0 \Rightarrow -1 \le \sin x \le \frac{1}{6}$ then inequality is true

case-II : If $\frac{1}{6} < \sin x < 1$ then we have $5 - 2 \sin x \ge 36 \sin^2 x - 12 \sin x + 1$

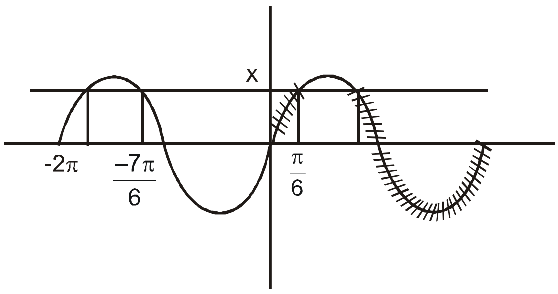
$36 \sin^2 x - 10 \sin x - 4 \le 0$

$\Rightarrow 18 \sin^2 x - 5 \sin x - 2 \le 0$

$18 \sin^2 x - 9 \sin x + 4 \sin x - 2 \le 0$

$\Rightarrow 9 \sin x (2 \sin x - 1) + 2 (2 \sin x - 1) \le 0$

$\Rightarrow (2 \sin x - 1)(9 \sin x + 2) \le 0 \Rightarrow \sin x \in \left[-\frac{2}{9}, \frac{1}{2}\right]$



$\therefore \sin x \in \left(\frac{1}{6}, \frac{1}{2}\right]$

$\therefore \sin x \in \left[-1, \frac{1}{2}\right]$

31. $2\sin\frac{x}{2} . \cos^2x + \sin^2x = 2\sin\frac{x}{2} . \sin^2x + \cos^2x \Rightarrow 2\sin\frac{x}{2}(\cos^2x - \sin^2x) = \cos^2x - \sin^2x$

$\Rightarrow \cos2x = 0$ or $2\sin\frac{x}{2} = 1$

$\Rightarrow \sin2x = 1$ or -1 and $\cos x = 1 - 2\sin^2\frac{x}{2} = 1 - 2\left(\frac{1}{4}\right) = \frac{1}{2}$

$\cos2x = 2\cos^2 x - 1 = 2\times\frac{1}{4} - 1 = -\frac{1}{2}$

32. $\cos 15x = \sin 5x \Rightarrow \cos 15x = \cos\left(\frac{\pi}{2} - 5x\right)$ or $\cos\left(\frac{3\pi}{2} + 5x\right)$

$15x = 2n\pi \pm \left(\frac{\pi}{2} - 5x\right)$ or $15x = 2n\pi \pm \left(\frac{3\pi}{2} + 5x\right)$

$\Rightarrow x = \frac{n\pi}{10} + \frac{\pi}{40}, n \in I, \quad x = \frac{n\pi}{5} + \frac{3\pi}{20}, n \in I$

and $x = \frac{n\pi}{5} - \frac{\pi}{20}, n \in I$ and $x = \frac{n\pi}{10} - \frac{3\pi}{40}, n \in I$

33. $5\sin^2 x + \sqrt{3}\sin x\cos x + 6\cos^2x = 5$

Case-I $\cos x = 0 \Rightarrow 5 + 0 + 0 = 5$

$\therefore x = n\pi + \frac{\pi}{2}$

Case -II $\cos x \neq 0$

$\therefore 5\tan^2x + \sqrt{3}\tan x + 6 = 5(1 + \tan^2x) \Rightarrow \tan x = -\frac{1}{\sqrt{3}}$

34. $\sin^2 x + 2\sin x\cos x - 3\cos^2 x = 0$

case-I : $\cos x \neq 0$

$\therefore \tan^2 x + 2\tan x - 3 = 0$

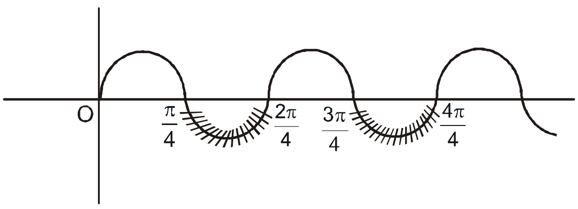
$\Rightarrow \tan x = 3, 1 \Rightarrow x = n\pi + \tan^{-1}(-3), n\pi + \frac{\pi}{4}$

case-II : $\cos x = 0 \Rightarrow 1 + 0 - 0 = 0$ not true.

35. $\sin^3 x\cos x > \cos^3 x\sin x$

$\Rightarrow \sin x\cos x\left(\cos^2 x - \sin^2 x\right) < 0$

$\Rightarrow \frac{\sin 2x}{2} . \cos 2x < 0 \Rightarrow \sin 4x < 0$



$\therefore x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{4}, \pi\right)$

36. $4\sin^4 x + (1 - \sin^2 x)^2 = 1 \Rightarrow \quad 5\sin^4 x - 2\sin^2 x = 0$

$\sin^2 x\,(5\sin^2 x - 2) = 0 \quad \Rightarrow \quad \sin^2 x = 0;\ \ \sin^2 x = \dfrac{2}{5}$

$\Rightarrow \quad x = n\pi\ ;\ n \in I \quad$ or $\quad \cos 2x = 1 - 2\sin^2 x = 1 - \dfrac{4}{5}$

$\therefore \quad \cos 2x = \dfrac{1}{5} = \cos\alpha$

$\therefore \quad 2x = 2n\pi \pm \alpha$

$\therefore \quad x = n\pi \pm \dfrac{1}{2}\cos^{-1}\left(\dfrac{1}{5}\right)\ ;\ n \in I$

37. $\because \quad 2\sin 2x\cos x + \sin 2x = 2\cos 2x\cos x + \cos 2x$

$\sin 2x\,(1 + 2\cos x) = \cos 2x\,(2\cos x + 1)$

$\Rightarrow \quad (2\cos x + 1)(\sin 2x - \cos 2x) = 0$

$\Rightarrow \quad \cos x = -\dfrac{1}{2} \quad$ or $\quad \tan 2x = 1$

$\Rightarrow \quad x = 2n\pi \pm \dfrac{2\pi}{3},\ n \in I \quad$ or $\quad x = \dfrac{n\pi}{2} + \dfrac{\pi}{8},\ n \in I$

PART - IV

1. $\tan A\tan B\tan C = \tan A + \tan B + \tan C = \dfrac{p}{q}$

2. $\tan A\tan B + \tan B\tan C + \tan C\tan A = \dfrac{\sin A\ \ \sin B\ \ \cos C + \cos A\ \ \sin B\ \ \sin C + \cos B\ \ \sin C\ \ \sin A}{\cos A\ \ \cos B\ \ \cos C}$

$= \dfrac{\sin B.\sin(A + C) + \cos B\ \ \sin C\ \ \sin A}{q} = \dfrac{1 - \cos^2 B + \cos B\ \ \sin C\ \ \sin A}{q}$

$= \dfrac{1 + \cos B\ \ (\sin C\ \ \sin A - \cos B)}{q} = \dfrac{1 + \cos B\ \ \cos C\ \ \cos A}{q} = \dfrac{1 + q}{q}$

3. $= (\tan A + \tan B + \tan C)\ \{(\tan A + \tan B + \tan C)^2 - 3(\tan A\tan B + \tan B\tan C + \tan C\tan A)\}$

$= \dfrac{p}{q}\left[\left(\dfrac{p}{q}\right)^2 - 3\left(\dfrac{1 + q}{q}\right)\right]$

$\therefore \quad \tan^3 A + \tan^3 B + \tan^3 C = \dfrac{3p}{q} + \dfrac{p}{q^3}\,(p^2 - 3q - 3q^2) = \dfrac{3pq^2 + p^3 - 3pq - 3pq^2}{q^3} = \dfrac{p^3 - 3pq}{q^3}$

4. $4\sin^3 x + 2\sin^2 x - 2\sin x - 1 = (2\sin x + 1)(2\sin^2 x - 1) = 0$

$\therefore \quad \sin x = -\dfrac{1}{2},\ \pm\dfrac{1}{\sqrt{2}} \quad \therefore$ there are 6 solutions.

5. $3 = \cos 4x + \dfrac{10\tan x}{1 + \tan^2 x} = \cos 4x + 5\sin 2x$

i.e $\quad 3 = 1 - 2\sin^2 2x + 5\sin 2x$

i.e $\quad \sin 2x = \dfrac{1}{2}$

$\therefore \quad 2x = \dfrac{\pi}{6},\ \dfrac{5\pi}{6}$

Thus there are two solutions.

6. (i) when $\tan x \geq 0$, then the equation becomes $\tan x = \tan x + \frac{1}{\cos x}$ i.e $\frac{1}{\cos x} = 0$ (not possible)

(ii) when $\tan x < 0$, then the equation becomes $-\tan x = \tan x + \frac{1}{\cos x}$ i.e $\sin x = -\frac{1}{2}$

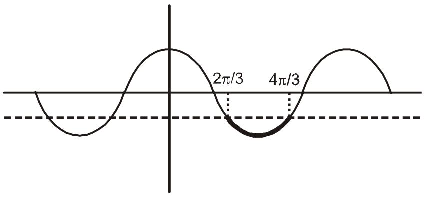
$\therefore$ $x = \frac{11\pi}{6}$ is the only solution.

7. $\sin^6 x + \cos^6 x < \frac{7}{16}$ $\Rightarrow 1 - 3\sin^2 x \cos^2 x < \frac{7}{16}$ $\Rightarrow \sin^2 x \cos^2 x > \frac{3}{16}$ $\Rightarrow \sin^2 2x > \frac{3}{4}$

$\Rightarrow \frac{1-\cos 4x}{2} > \frac{3}{4}$ $\Rightarrow 1 - \cos 4x > \frac{3}{2} \Rightarrow \cos 4x < -\frac{1}{2}$ $\Rightarrow$ Principal is value $4x \in \left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

$\Rightarrow$ General value is $4x \in \left(2n\pi + \frac{2\pi}{3}, \quad 2n\pi + \frac{4\pi}{3}\right)$

$\Rightarrow$ $x \in \left(\frac{n\pi}{2} + \frac{\pi}{6}, \quad \frac{n\pi}{2} + \frac{\pi}{3}\right)$, $n \in I$

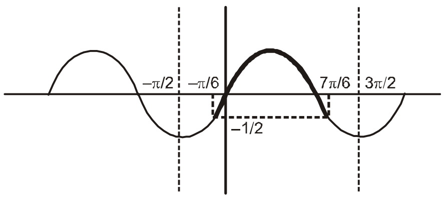


8. $\cos 2x + 5\cos x + 3 \geq 0$ $\Rightarrow 2\cos^2 x + 5\cos x + 2 \geq 0 \Rightarrow (\cos x + 2)(2\cos x + 1) \geq 0$

$\Rightarrow 2\cos x + 1 \geq 0$ $(\because \cos x + 2 > 0)$

$\Rightarrow \cos x \geq -\frac{1}{2}$ $\Rightarrow x \in \left[-\frac{2\pi}{3}, \frac{2\pi}{3}\right]$

9. $2\sin^2\left(x + \frac{\pi}{4}\right) + \sqrt{3}\cos 2x \geq 0$



$\Rightarrow$ $1 - \cos\left(2x + \frac{\pi}{2}\right) + \sqrt{3}\cos 2x \geq 0$

$\Rightarrow$ $\sqrt{3}\cos 2x + \sin 2x \geq -1$

$\Rightarrow$ $\frac{\sqrt{3}}{2}\cos 2x + \frac{1}{2}\sin 2x \geq -\frac{1}{2}$

$\Rightarrow$ $\sin\left(2x + \frac{\pi}{3}\right) \geq -\frac{1}{2}$

$\Rightarrow$ $2x + \frac{\pi}{3} \in \left[2n\pi - \frac{\pi}{6}, \quad 2n\pi + \frac{7\pi}{6}\right]$

$\Rightarrow$ $2x \in \left[2n\pi - \frac{\pi}{2}, \quad 2n\pi + \frac{5\pi}{6}\right]$

$\Rightarrow$ $x \in \left[n\pi - \frac{\pi}{4}, \quad n\pi + \frac{5\pi}{12}\right]$

$\Rightarrow$ $x \in \left[-\pi, \frac{-7\pi}{12}\right] \cup \left[-\frac{\pi}{4}, \frac{5\pi}{12}\right] \cup \left[\frac{3\pi}{4}, \pi\right]$ in $[-\pi, \pi]$

EXERCISE # 3

PART - I

1. $f(\theta) = \dfrac{1}{\sin^2\theta + 3\sin\theta\cos\theta + 5\cos^2\theta} = \dfrac{1}{\dfrac{1-\cos 2\theta}{2} + \dfrac{3}{2}\sin 2\theta + \dfrac{5(1+\cos 2\theta)}{2}} = \dfrac{2}{6 + 3\sin 2\theta + 4\cos 2\theta}$

$\therefore \quad f(\theta)_{max} = \dfrac{2}{6-5} = 2$

2. $\dfrac{1}{\sin\frac{\pi}{n}} - \dfrac{1}{\sin\frac{3\pi}{n}} = \dfrac{1}{\sin\frac{2\pi}{n}}$; $\dfrac{2\cos\frac{2\pi}{n}\sin\frac{\pi}{n}}{\sin\frac{\pi}{n}\sin\frac{3\pi}{n}} = \dfrac{1}{\sin\frac{2\pi}{n}}$; $\sin\dfrac{4\pi}{n} = \sin\dfrac{3\pi}{n}$

$\dfrac{4\pi}{n} = (-1)^k\,\dfrac{3\pi}{n} + k\pi,\ k \in I$

If $k = 2m \Rightarrow \dfrac{\pi}{n} = 2m\pi \quad \dfrac{1}{n} = 2m$, not possible

If $k = 2m + 1 \Rightarrow \dfrac{7\pi}{n} = (2m+1)\pi \Rightarrow n = 7, m = 0$

3. $\tan\theta = \cot 5\theta$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = \dfrac{\cos 5\theta}{\sin 5\theta} \Rightarrow \cos 6\theta = 0 \Rightarrow 6\theta = (2n+1)\dfrac{\pi}{2} \Rightarrow \theta = (2n+1)\dfrac{\pi}{2}$; $n \in I$

$\Rightarrow \quad \theta = -\dfrac{5\pi}{12}, -\dfrac{\pi}{4}, -\dfrac{\pi}{12}, \dfrac{\pi}{12}, \dfrac{\pi}{4}, \dfrac{5\pi}{12}$ ....(1)

$\sin 2\theta = \cos 4\theta$

$\Rightarrow \quad \sin 2\theta = 1 - 2\sin^2 2\theta \Rightarrow 2\sin^2 2\theta + \sin 2\theta - 1 = 0 \Rightarrow \sin 2\theta = -1, \dfrac{1}{2}$

$\Rightarrow \quad 2\theta = (4m-1)\dfrac{\pi}{2},\ p\pi + (-1)^p\dfrac{\pi}{6}$

$\Rightarrow \quad \theta = (4m-1)\dfrac{\pi}{4},\ \dfrac{p\pi}{2} + (-1)^p\dfrac{\pi}{12}$; $m, p \in I$

$\Rightarrow \quad \theta = -\dfrac{\pi}{4}, \dfrac{\pi}{12}, \dfrac{5\pi}{12}$ ....(2)

From (1) & (2) $\theta \in \left\{-\dfrac{\pi}{4}, \dfrac{\pi}{12}, \dfrac{5\pi}{12}\right\}$

Number of solution is 3.

4. $P = \{\theta : \sin\theta - \cos\theta = \sqrt{2}\cos\theta\}$

$\sin\theta = (\sqrt{2}+1)\cos\theta \Rightarrow \tan\theta = \sqrt{2} + 1 \Rightarrow \theta = n\pi + \dfrac{3\pi}{8}$; $n \in I$

$Q = \{\theta : \sin\theta + \cos\theta = \sqrt{2}\sin\theta\}$

$\therefore \quad \cos\theta = (\sqrt{2} - 1)\sin\theta$

$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{2}-1} = \sqrt{2} + 1$

$\Rightarrow \quad \theta = n\pi + \dfrac{3\pi}{8}$; $n \in I$

$\therefore \quad P = Q$

5.* As $\tan(2\pi - \theta) > 0$, $-1 < \sin\theta < -\frac{\sqrt{3}}{2}$, $\theta \in [0, 2\pi] \Rightarrow \frac{3\pi}{2} < \theta < \frac{5\pi}{3}$

Now $2\cos\theta(1 - \sin\phi) = \sin^2\theta(\tan\theta/2 + \cot\theta/2)\cos\phi - 1$

$\Rightarrow 2\cos\theta(1 - \sin\phi) = 2\sin\theta\cos\phi - 1$

$\Rightarrow 2\cos\theta + 1 = 2\sin(\theta + \phi)$

As $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow 2\cos\theta + 1 \in (1, 2) \Rightarrow 1 < 2\sin(\theta + \phi) < 2 \Rightarrow <\sin(\theta + \phi) < 1$

As $\theta + \phi \in [0, 4\pi] \Rightarrow \theta + \phi \in \left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$ or $\theta + \phi \in \left(\frac{13\pi}{6}, \frac{17\pi}{6}\right)$

$\Rightarrow \frac{\pi}{6} - \theta < \phi < \frac{5\pi}{6} - \theta$ or $\frac{13\pi}{6} - \theta < \phi < \frac{17\pi}{6} - \theta \Rightarrow \phi \in \left(-\frac{3\pi}{2}, \frac{-2\pi}{3}\right) \cup \left(\frac{2\pi}{3}, \frac{7\pi}{6}\right)$

$\therefore$ correct option is (A, C, D)

6. $\sin x + 2\sin 2x - \sin 3x = 3$. $\sin x(1 + 2\cos x - 3 + 4\sin^2 x) = 3$.

$(4\sin^2 x + 2\cos x - 2) = \frac{3}{\sin x}$

$2 - 4\cos^2 x + 2\cos x = \frac{3}{\sin x}$

$\frac{9}{4} - \left(2\cos x - \frac{1}{2}\right)^2 = \frac{3}{\sin x}$.

L.H.S. $\le \frac{9}{4}$ R.H.S. ≥ 3.

No solution.

7. $\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - \frac{1}{2}\sin^2 2x + 1 - \frac{3}{4}\sin^2 2x = 2$

$\Rightarrow \cos^2 2x = \sin^2 2x \Rightarrow \tan^2 2x = 1$

Now $2x \in [0, 4\pi] \Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}$

so number of solution = 8

8. $$\sum_{k=1}^{13} \frac{\sin\left[\left(\frac{\pi}{4} + \frac{k\pi}{6}\right) - \left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right]}{\sin\frac{\pi}{6}\left(\sin\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\sin\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right)\right)} = 2\sum_{k=1}^{13}\left(\cot\left(\frac{\pi}{4} + (k-1)\frac{\pi}{6}\right) - \cot\left(\frac{\pi}{4} + \frac{k\pi}{6}\right)\right)$$

$= 2\left(\cot\frac{\pi}{4} - \cot\left(\frac{\pi}{4} + \frac{13\pi}{6}\right)\right) = 2\left(1 - \cot\left(\frac{29\pi}{12}\right)\right) = 2\left(1 - \cot\left(\frac{5\pi}{12}\right)\right) = 2(1 - (2 - \sqrt{3})) = 2(-1 + \sqrt{3})$

$= 2(\sqrt{3} - 1)$

9. $\sqrt{3}$ secx + cosecx + 2 (tanx – cotx) = 0 $\Rightarrow$ $\sqrt{3}$ sinx + cosx + 2 ($\sin^2$x – $\cos^2$x) = 0

$\sqrt{3}$ sinx + cosx – 2cos2x = 0 $\Rightarrow$ sin $\left(x+\frac{\pi}{3}\right)$ = cos2x

cos (π/3 – x) = cos2x $\Rightarrow$ 2x = 2nπ ± (π/3 – x) $x = \frac{2n\pi}{3}+\frac{\pi}{9}$ or $x = 2n\pi - \frac{\pi}{3}$.

– 100º – 60º + 20º +140º = 0

10. $\cos\alpha = \left(\frac{1-a}{1+a}\right)$; $a = \tan^2\frac{\alpha}{2}$

$\cos\beta = \left(\frac{1-b}{1+b}\right)$; $b = \tan^2\frac{\beta}{2}$

$$2\left(\left(\frac{1-b}{1+b}\right)-\left(\frac{1-a}{1+a}\right)\right) + \left(\left(\frac{1-a}{1+a}\right)\left(\frac{1-b}{1+b}\right)\right)=1$$

$\Rightarrow$ 2((1 – b)(1 + a) – (1 – a)(1 + b)) + (1 – a)(1 – b) = (1 + a)(1 + b)

$\Rightarrow$ 2(1 + a – b – ab – (1 + b – a – ab)) + 1 – a – b + ab = 1 + a + b + ab

$\Rightarrow$ 4(a – b) = 2(a + b)

$\Rightarrow$ 2a – 2b = a + b

$\Rightarrow$ a = 3b

$\tan^2\frac{\alpha}{2} = 3\tan^2\frac{\beta}{2}$

$\tan\frac{\alpha}{2} = \pm\sqrt{3}\tan\left(\frac{\beta}{2}\right)$

11. $\sqrt{3}$ a cos x + 2b sin x = c

$x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\sqrt{3}\,a\left(\frac{1-t^2}{1+t^2}\right) + 2b\left(\frac{2t}{1+t^2}\right)$ = c, where $t = \tan\frac{x}{2}$

$\sqrt{3}\,a(1 - t^2) + 4bt = c\,(1 + t^2)$

$t^2(c + \sqrt{3}\,a) - 4bt + c - \sqrt{3}\,a = 0$

$\frac{\alpha+\beta}{2} = \frac{\pi}{6}$

$\tan\left(\frac{\alpha+\beta}{2}\right) = \frac{1}{\sqrt{3}}$

$\Rightarrow \frac{t_1+t_2}{1-t_1t_2}=\frac{1}{\sqrt{3}}$

$\Rightarrow \frac{4b}{c+\sqrt{3}a-c+\sqrt{3}a} = \frac{1}{\sqrt{3}}$ $\frac{b}{a} = \frac{1}{2}$

12. $f(n) = \dfrac{\sum_{k=0}^{n} \sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n} 2\sin^2\left(\frac{k+1}{n+2}\right)\pi} = \dfrac{\sum_{k=0}^{n}\left(\cos\frac{\pi}{n+2} - \cos\left(\frac{2k+3}{n+2}\right)\pi\right)}{\sum_{k=0}^{n} 2\sin^2\left(\frac{k+1}{n+2}\right)\pi}$

$$= \frac{(n+1)\cos\frac{\pi}{(n+2)} - \dfrac{\cos\left(\frac{n+3}{n+2}\right)\pi \sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\frac{\pi}{n+2}}}{(n+1) - \dfrac{\cos\pi . \sin\left(\frac{n+1}{n+2}\right)\pi}{\sin\frac{\pi}{n+2}}}$$

$$= \frac{(n+1)\cos\frac{\pi}{(n+2)} + \cos\left(\frac{n+3}{n+2}\right)\pi}{(n+1)+1} = \frac{(n+1)\cos\left(\frac{\pi}{n+2}\right) + \cos\left(\frac{\pi}{n+2}\right)}{n+2} = \cos\left(\frac{\pi}{n+2}\right)$$

(A) $f(4) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ correct

(B) $\alpha = \tan(\cos^{-1} f(6)) = \tan|\cos^{-1}(\cos\frac{\pi}{8})| = \tan\frac{\pi}{8}$

$\tan\frac{\pi}{4} = \dfrac{2\tan\frac{\pi}{8}}{1-\tan^2\frac{\pi}{8}} \Rightarrow 1 = \dfrac{2\alpha}{1-\alpha^2} \Rightarrow \alpha^2 + 2\alpha - 1 = 0$ correct

(C) $\sin(7\cos^{-1} f(5)) = \sin(7\cos^{-1}(\cos\frac{\pi}{7})) = \sin\pi = 0$ correct

(A), (C), (D) correct

(D) $\lim_{n\to\infty} f(n) = \cos\left(\frac{\pi}{n+2}\right) = 1$ Incorrect

(13 to 14)

$f(x) = 0 \Rightarrow \sin(\pi \cos x) = 0$

$\Rightarrow \pi \cos x = n\pi \Rightarrow \cos x = n \Rightarrow \cos x = -1, 0, 1 \Rightarrow X = \{n\pi, (2n+1)\frac{\pi}{2}\} = \{n\frac{\pi}{2}, n \in I\}$

$f'(x) = 0 \Rightarrow \cos(\pi \cos x)(-\pi \sin x) = 0 \Rightarrow \pi \cos x = (2n+1)\frac{\pi}{2}$ or $x = n\pi$

$\Rightarrow \cos x = n + \frac{1}{2}$ or $x = n\pi \Rightarrow \cos x = \pm\frac{1}{2}$ or $x = n\pi \Rightarrow Y = \left\{2n\pi \pm \frac{\pi}{3}, 2n\pi \pm \frac{2\pi}{3}, n\pi, n \in I\right\}$

$= \left\{.......-\frac{2\pi}{3}, -\frac{\pi}{3}, 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, ...\right\}$ which is an arithmetic progression

$g(x) = 0 \Rightarrow \cos(2\pi \sin x) = 0 \Rightarrow 2\pi \sin x = (2n+1)\frac{\pi}{2}$

$\Rightarrow \sin x = \frac{2n+1}{4} = \pm\frac{1}{4}, \pm\frac{3}{4} \Rightarrow Z = \left\{n\pi \pm \sin^{-1}\frac{1}{4}, n\pi \pm \sin^{-1}\frac{3}{4}, n \in I\right\}$

$g'(x) = 0 \Rightarrow -\sin(2\pi \sin x)(2\pi \cos x) = 0 \Rightarrow 2\pi \sin x = n\pi$ or $x = (2n+1)\frac{\pi}{2}$

$\Rightarrow \sin x = \frac{n}{2} = 0, \pm\frac{1}{2}, \pm 1$ or $x = (2n+1)\frac{\pi}{2} \Rightarrow W = \left\{n\pi, (2n+1)\frac{\pi}{2}, n\pi \pm \frac{\pi}{6}, n \in I\right\}$

PART - II

1. $\tan 2\alpha = \tan((\alpha+\beta)+(\alpha-\beta)) = \dfrac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)} = \dfrac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}} = \dfrac{(9+5)4}{48-15} = \dfrac{14\times4}{33} = \dfrac{56}{33}$

Hence correct option is (1)

2. $A = \sin^2x + \cos^4x = \sin^2x + (1-\sin^2x)^2 = \sin^4x - \sin^2x + 1 = \left(\sin^2x - \dfrac{1}{2}\right)^2 + \dfrac{3}{4} = \dfrac{3}{4} \le A \le 1$

3. $3\sin P + 4\cos Q = 6$ ...(i)
$4\sin Q + 3\cos P = 1$ ...(ii)

Squaring and adding (i) & (ii) we get $\sin(P+Q) = \dfrac{1}{2}$

$\Rightarrow \quad P + Q = \dfrac{\pi}{6}$ or $\dfrac{5\pi}{6} \quad \Rightarrow \quad R = \dfrac{5\pi}{6}$ or $\dfrac{\pi}{6}$

If $R = \dfrac{5\pi}{6}$ then $0 < P, Q < \dfrac{\pi}{6}$

$\Rightarrow \quad \cos Q < 1$ and $\sin P < \dfrac{1}{2} \Rightarrow \quad 3\sin P + 4\cos Q < \dfrac{11}{2}$

So $R = \dfrac{\pi}{6}$

4. Given expression

$$= \frac{\sin A}{\cos A} \times \frac{\sin A}{\sin A - \cos A} + \frac{\cos A}{\sin A} \times \frac{\cos A}{\cos A - \sin A}$$

$$= \frac{1}{\sin A - \cos A}\left\{\frac{\sin^3 A - \cos^3 A}{\cos A \ \sin A}\right\} = \frac{\sin^2 A + \sin A\cos A + \cos^2 A}{\sin A \ \cos A} = 1 + \sec A \operatorname{cosec} A$$

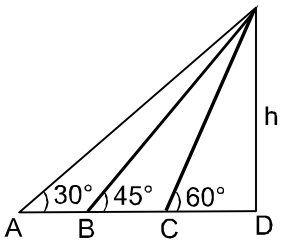
5. $f_k(x) = \dfrac{1}{k}(\sin^k x + \cos^k x)$

$f_4 - f_6 = \dfrac{1}{4}(\sin^4x + \cos^4x) - \dfrac{1}{6}(\sin^6x + \cos^6x) = \dfrac{1}{4}(1 - 2\sin^2x\cos^2x) - \dfrac{1}{6}(1 - 3\sin^2x\cos^2x)$

$\dfrac{1}{4} - \dfrac{1}{6} = \dfrac{1}{12}$

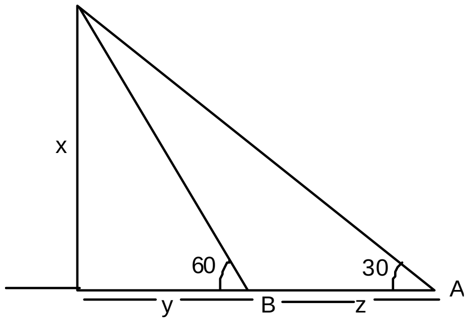
6. $\tan 30° \dfrac{h}{AD} \quad \Rightarrow \quad AD = h\sqrt{3}$

$BD = h$; $CD = \dfrac{h}{\sqrt{3}}$



$\dfrac{AB}{BC} = \dfrac{AD - BD}{BD - CD} = \dfrac{\sqrt{3}-1}{1-\frac{1}{\sqrt{3}}} = \dfrac{3-\sqrt{3}}{\sqrt{3}-1} = \sqrt{3}$

7.



$\tan 30^o = \frac{x}{y+z} = \frac{1}{\sqrt{3}} \Rightarrow \sqrt{3}\,x = y + z \quad \Rightarrow \quad \tan 60^o = \frac{x}{y} = \sqrt{3} \Rightarrow x = \sqrt{3}\,y = y + z$

$3y = y + z \quad \Rightarrow \quad 2y = z$

for 2y distance time = 10 min. so for y dist time = 5 min.

8. $0 \le x < 2\pi$

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$(\cos x + \cos 4x) + (\cos 2x + \cos 3x) = 0$

$2\cos\frac{5x}{2}\cos\frac{3x}{2} + 2\cos\frac{5x}{2}\cos\frac{x}{2} = 0$

$2\cos\frac{5x}{2}\left[2\cos x\cos\frac{x}{2}\right] = 0$

$\cos\frac{5x}{2} = 0 \quad$ or $\quad \cos x = 0$ or $\cos\frac{x}{2} = 0$

$x = \frac{(2n+1)\pi}{5}$ or $x = (2n+1)\frac{x}{2}$ or $x = (2n+1)\pi$

$x = \left\{\frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$ Number of solution is 7

9. $5(\tan^2 x - \cos^2 x) = 2\cos 2x + 9$

$5\left(\tan^2 x - \frac{1}{1+\tan^2 x}\right) = 2\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right) + 9$

$5(\tan^4 x + \tan^2 x - 1) = 2 - 2\tan^2 x + 9 + 9\tan^2 x$

$5\tan^4 x - 2\tan^2 x - 16 = 0$

$5\tan^4 x - 10\tan^2 x + 8\tan^2 x - 16 = 0$

$5\tan^2 x(\tan^2 x - 2) + 8(\tan^2 x - 2) = 0$

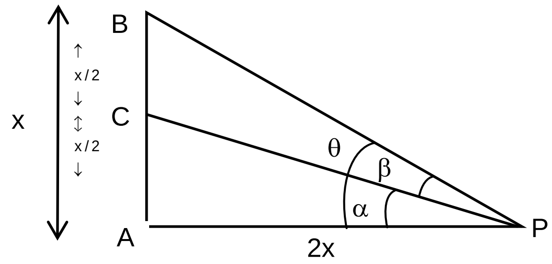
$(5\tan^2 x + 8)(\tan^2 x - 2) = 0$

$\tan^2 x = 2$

$\cos 2x = \frac{1-2}{1+2} = -\frac{1}{3}$

$\cos 4x = 2\cos^2 2x - 1 = -\frac{7}{9}$

10.



$\tan\theta = \frac{1}{2}$, $\tan\alpha = \frac{1}{4}$, $\tan\beta = y \Rightarrow \tan\theta = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\frac{1}{2} = \frac{\frac{1}{4} + y}{1 - \frac{y}{4}} \Rightarrow \frac{1}{2} = \frac{1+4y}{4-y}$ $4 - y = 2 + 8y$ $\frac{2}{9} = y$

11. $8\cos x\left(\left(\cos^2\frac{\pi}{6} - \sin^2 x\right) - \frac{1}{2}\right) = 1 \Rightarrow 8\cos x\left(\left(\frac{3}{4} - \sin^2 x\right) - \frac{1}{2}\right) = 1$

$6\cos x - 8\cos x(\sin^2 x) - 4\cos x = 1$

$6\cos x - 8\cos x(1 - \cos^2 x) - 4\cos x - 1 = 0$

$8\cos^3 x - 6\cos x - 1 = 0$

$2(4\cos^3 x - 3\cos x) = 1$

$\cos 3x = \frac{1}{2}$

$3x = 2n\pi \pm \frac{\pi}{3}$

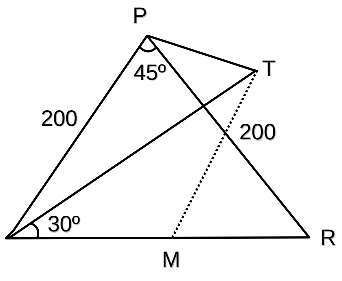
$x = (6n \pm 1)\frac{\pi}{9}$

$n = 0,\ x = \frac{\pi}{9}$

$n = 1,\ x = \frac{7\pi}{9}, \frac{5\pi}{9}$

$s = \frac{13\pi}{9}$

$k = \frac{13}{9}$



12.

Let height of the tower is TM = h and QM = MR = x

$PM = \sqrt{40000 - x^2} \Rightarrow \tan 45^\circ = \frac{TM}{PM} = \frac{h}{\sqrt{40000} - x^2} \Rightarrow h^2 = 40000 - x^2$

$\Rightarrow h^2 + x^2 = 40000$ ....(i)

$\tan 30^\circ = \frac{TM}{QM} \Rightarrow x = \sqrt{3}\,h.$ ....(ii)

by (i) and (2) $4h^2 = 40000 \Rightarrow h = 100.$ m

13. $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4} \quad \Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

Let $\cos^2 2\theta = t \Rightarrow t^2 - t + \frac{1}{4} = 0$

$\Rightarrow \left(t - \frac{1}{2}\right)^2 = 0 \Rightarrow t = \frac{1}{2} \Rightarrow \cos^2 2\theta = \frac{1}{2}$

$\Rightarrow 2\cos^2 2\theta - 1 = 0 \Rightarrow \cos 4\theta = 0 \Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$

$\Rightarrow \theta = (2n + 1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$

sum of values of θ is $\frac{\pi}{2}$

14. $\cos\frac{\pi}{2^2} . \cos\frac{\pi}{2^3} \ldots. \cos\frac{\pi}{2^{10}} . \sin\frac{\pi}{2^{10}} = \frac{\sin\left(2^9 . \frac{\pi}{2^{10}}\right)}{2^9 . \sin\frac{\pi}{2^{10}}} . \sin\frac{\pi}{2^{10}} = \frac{1}{2^9}$

15. AM $\geq$ GM

$\frac{\sin^4\alpha + 4\cos^4\beta + 1 + 1}{4} \geq \left(4\sin^4\alpha\cos^4\beta\right)^{\frac{1}{4}}$

so AM = GM $\Rightarrow \sin^4\alpha = 4\cos^4\beta = 1$

$\sin^4\alpha = 1 \Rightarrow \alpha = \frac{\pi}{2} \quad \Rightarrow \cos\beta = \frac{1}{\sqrt{2}} \quad \Rightarrow \beta = \frac{\pi}{4}$

hence $-2\sin\alpha \sin\beta = -2 \times 1 \times \frac{1}{\sqrt{2}} = -\sqrt{2}$

16. $\underbrace{1 + \sin^4 x}_{\geq 1} = \underbrace{\cos^2 3x}_{\leq 1}$

Hence for equality to hold $\sin^4 x = 0$ & $\cos^2 3x = 1$

$\sin^4 x = 0 \Rightarrow x = -2\pi, -\pi, 0, \pi, 2\pi$

All of which satisfy $\cos^2 3x = 1 \Rightarrow$ 5 solutions.

17. $(\sin 30°)(\sin 10°)(\sin(60°-10°))(\sin(60°+10°)) = \frac{1}{2}\left(\frac{1}{4}\sin 30°\right) = \frac{1}{16}$

18. $2^{\sqrt{(\sin x - 1)^2 + 4}} \leq 4^{\sin^2 y}$

$2\sin^2 y \geq \sqrt{(\sin x - 1)^2 + 4}$

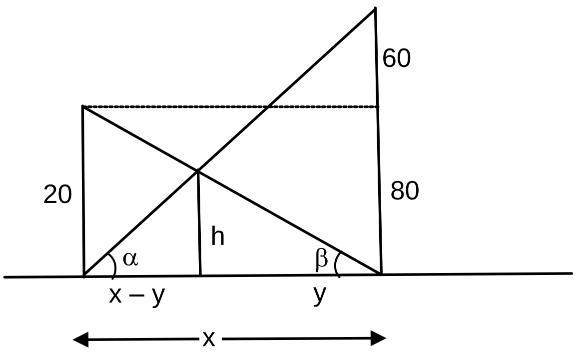
$\because\ 2\sin^2 y \in [0, 2]$

$\sqrt{(\sin x - 1)^2 + 4} \in [2,\ 2\sqrt{2}\,]$

Hence $2\sin^2 y = \sqrt{(\sin x - 1)^2 + 4}$,

for $|\sin y| = 1$ and $\sin x = 1 \Rightarrow |\sin y| = \sin x$

19. Height of two tower are 20 m c 80 m …..



$\frac{h}{y} = \tan\beta \Rightarrow \frac{h}{y} = \frac{20}{x}$, $\frac{h}{x-y} = \frac{80}{x} \Rightarrow \frac{hx}{20} = y$, $\frac{hx}{80} = x - y$

$\frac{hx}{20} + \frac{hx}{80} = x \Rightarrow \frac{h}{20} + \frac{h}{80} = x \Rightarrow \frac{h}{20} + \frac{h}{80} = 1$

$5h = 80 \Rightarrow h = 16$

20. $(k+1)\tan^2 x - \sqrt{2}\lambda\ \tan x + (k-1) = 0$

$\tan\alpha + \tan\beta = \frac{\sqrt{2}\lambda}{k+1} \Rightarrow \tan\alpha\tan\beta = \frac{k-1}{k+1} \Rightarrow \tan(\alpha+\beta) = \frac{\frac{\sqrt{2}\lambda}{k+1}}{1-\frac{k-1}{k+1}} = \frac{\sqrt{2}\lambda}{2} = \frac{\lambda}{\sqrt{2}}$

$\tan^2(\alpha+\beta) = \frac{\lambda^2}{2} = 50 \Rightarrow \lambda = 10$

21. $\cos^3\frac{\pi}{8}\left[4\cos^3\frac{\pi}{8} - 3\cos\frac{\pi}{8}\right] + \sin^3\frac{\pi}{8}\left[3\sin\frac{\pi}{8} - 4\sin^3\frac{\pi}{8}\right] = 4\cos^6\frac{\pi}{8} - 4\sin^6\frac{\pi}{8} - 3\cos^4\frac{\pi}{8} + 3\sin^4\frac{\pi}{8}$

$= 4\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]\left[\left(\sin^4\frac{\pi}{8} + \cos^4\frac{\pi}{8} + \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right)\right] - 3\left[\left(\cos^2\frac{\pi}{8} - \sin^2\frac{\pi}{8}\right)\right]$

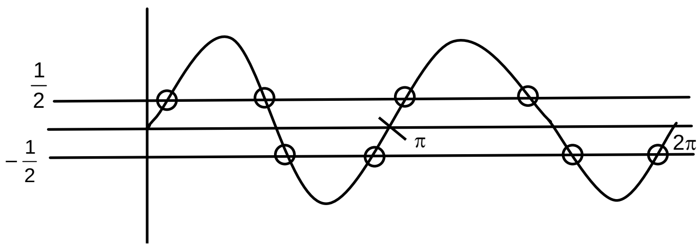
$= \cos\frac{\pi}{4}\left[4\left(1 - \sin^2\frac{\pi}{8}\cos^2\frac{\pi}{8}\right) - 3\right] = \frac{1}{\sqrt{2}}\left[1 - \frac{1}{2}\right] = \frac{1}{2\sqrt{2}}$

22. $\log_{1/2}|\sin x| = 2 - \log_{1/2}|\cos x|$

$\log_{1/2}|\sin x \cos x| = 2$

$|\sin x \cos x| = \frac{1}{4}$

$\sin 2x = \pm\frac{1}{2}$



Number of solution = 8.

ALP Solutions

1. **(i)** L.H.S. = $\sec^4 A\ (1 + \sin^2 A)\ (1 - \sin^2 A) - 2\tan^2 A$

$= \sec^2 A + \sec^2 A \sin^2 A - 2\tan^2 A$

$= 1 + \tan^2 A + \tan^2 A - 2\tan^2 A$

$= 1$ = R.H.S.

(ii) LHS = $\dfrac{\cot^2\theta\ \ (\sec\theta - 1)\ \ (1-\sin\theta)\ \ (1+\sec\theta)}{(1+\sin\theta)(1-\sin\theta)\ \ (1+\sec\theta)}$

$= \dfrac{\cot^2\theta(\sec^2\theta - 1)}{\cos^2\theta}\ .\ \dfrac{(1-\sin\theta)}{(1+\sec\theta)} = \sec^2\theta . \dfrac{(1-\sin\theta)}{(1+\sec\theta)}$ = R.H.S.

2. The given expression is equal to $\sqrt{\sin^4 x + 4(1-\sin^2 x)} - \sqrt{\cos^4 x + 4(1-\cos^2 x)}$

$= \sqrt{(2-\sin^2 x)^2} - \sqrt{(2-\cos^2 x)^2} = (2 - \sin^2 x) - (2 - \cos^2 x) = \cos^2 x - \sin^2 x = \cos 2x$

3. Two given equation as

$\sin a - 8 \sin d = 4 \sin c - 7\sin b,$

$\cos a - 8 \cos d = 4 \cos c - 7 \cos b.$

By squaring the above two equalities and adding them, we obtain

$1 + 64 - 16(\cos a \cos d + \sin a \sin d) = 16 + 49 - 56 (\cos b \cos c + \sin b \sin c)$, and the conclusion follows from the addition formulas.

4. We have $\cos 3x = 4\cos^3 x - 3\cos x$, so $4\cos^2 x - 3 = \dfrac{\cos 3x}{\cos x}$ for all $x \neq (2k + 1).\ \dfrac{\pi}{2}$, $k \in Z$.

Thus $(4\cos^2 9^o - 3)\ (4\cos^2 27^o - 3) = \dfrac{\cos 27^o}{\cos 9^o}\ .\ \dfrac{\cos 81^o}{\cos 27^o} = \dfrac{\cos 81^o}{\cos 9^o} = \tan 9^o$

5. $\cos(\alpha + \beta) = \dfrac{4}{5}$, $\sin(\alpha - \beta) = \dfrac{5}{13}$, $\left(\begin{matrix} \because & 0 < \alpha,\ \ \beta < \pi/4 \\ \Rightarrow & 0 < \alpha + \beta < \pi/2 \end{matrix}\right)$

$\Rightarrow$ $\tan 2\alpha = \tan \{(\alpha + \beta) + (\alpha - \beta)\}$

$= \dfrac{\tan(\alpha+\beta) + \tan(\alpha-\beta)}{1 - \tan(\alpha+\beta)\tan(\alpha-\beta)}$

here $\tan(\alpha + \beta) = 3/4$, $\tan(\alpha - \beta) = 5/12$

$\Rightarrow$ $\tan 2\alpha = \dfrac{\frac{3}{4} + \frac{5}{12}}{1 - \frac{3}{4} \times \frac{5}{12}} = \dfrac{56}{48-15} \Rightarrow \tan 2\alpha = \dfrac{56}{33}$ Ans.

6. $a \tan\theta + b \sec\theta = c$

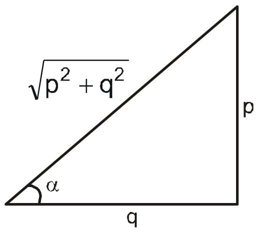
$\Rightarrow (b\sec\theta)^2 = (c - a\tan\theta)^2 \Rightarrow b^2\ (1 + \tan^2\theta) = c^2 + a^2\tan^2\theta - 2ac\tan\theta$

$\Rightarrow \tan^2\theta\ (b^2 - a^2) + 2ac\tan\theta + (b^2 - c^2) = 0$

$\Rightarrow \tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\ \ \tan\beta} = \dfrac{\frac{-2ac}{b^2 - a^2}}{1 - \frac{b^2 - c^2}{b^2 - a^2}} = \dfrac{2ac}{a^2 - c^2}$.

7. $\tan\alpha = \dfrac{p}{q}$

$$\text{LHS} = \frac{1}{2}(p \text{ cosec } 2\beta - q \sec 2\beta) \times \frac{\sqrt{p^2+q^2}}{\sqrt{p^2+q^2}} = \frac{1}{2}\left\{\frac{p}{\sqrt{p^2+q^2}}\text{cosec}\,2\beta - \frac{q}{\sqrt{p^2+q^2}}\sec 2\beta\right\} \times \sqrt{p^2+q^2}$$



$$\sin\alpha = \frac{p}{\sqrt{p^2+q^2}}, \ \cos\alpha = \frac{q}{\sqrt{p^2+q^2}} \quad = \frac{1}{2}\left(\frac{\sin\alpha\cos 2\beta - \cos\alpha \sin 2\beta}{\sin 2\beta \cos 2\beta}\right) \times \sqrt{p^2+q^2}$$

$$= \frac{\sin(\alpha - 2\beta)}{\sin 4\beta} \times \sqrt{p^2+q^2} \quad = \frac{\sin 4\beta}{\sin 4\beta} \times \sqrt{p^2+q^2}$$

$(\because \quad \alpha = 6\beta)$

8. If $a = \sin(\theta + \alpha)$, $b = \sin(\theta + \beta)$

$\therefore \quad 2ab = 2\sin(\theta + \alpha)\sin(\theta + \beta)$

$2ab = \cos(\alpha - \beta) - \cos(2\theta + \alpha + \beta)$

Multiply both sides by $2\cos(\alpha - \beta)$

$\Rightarrow \quad 4ab\cos(\alpha - \beta) = 2\cos^2(\alpha - \beta) - 2\cos(2\theta + \alpha + \beta).\cos(\alpha - \beta)$

$= 1 + \cos 2(\alpha - \beta) - \cos 2(\theta + \alpha) - \cos 2(\theta + \beta)$

$\Rightarrow \quad \cos 2(\alpha - \beta) - 4ab\cos(\alpha - \beta) = \cos 2(\theta + \alpha) + \cos 2(\theta + \beta) - 1$

$= 1 - 2\sin^2(\theta + \alpha) + 1 - 2\sin^2(\theta + \beta) - 1 = 1 - 2a^2 - 2b^2$

9. **(i)** $\cot 7\dfrac{1^\circ}{2} = \tan 82\dfrac{1^\circ}{2} = \dfrac{\cos 7\frac{1}{2}^\circ}{\sin 7\frac{1}{2}^\circ} = \dfrac{2\cos^2 7\frac{1}{2}^\circ}{\sin 15^\circ} = \dfrac{1+\cos 15^\circ}{\sin 15^\circ}$

$$= \frac{1+\cos(45^\circ - 30^\circ)}{\sin(45^\circ - 30^\circ)} = \frac{1+\frac{\sqrt{3}+1}{2\sqrt{2}}}{\frac{\sqrt{3}-1}{2\sqrt{2}}} = \frac{2\sqrt{2}+\sqrt{3}+1}{\sqrt{3}-1}$$

$$= \frac{(2\sqrt{2}+\sqrt{3}+1)(\sqrt{3}+1)}{3-1} = \sqrt{2}+\sqrt{3}+2+\sqrt{6} = \sqrt{2}+\sqrt{3}+\sqrt{4}+\sqrt{6} = (\sqrt{2}+\sqrt{3})(\sqrt{2}+1)$$

(ii) $\tan 142\frac{1}{2}^{o} = -\cot 52\frac{1}{2}^{o} = \frac{-1}{\tan 52\frac{1}{2}^{o}} = \frac{-1}{\tan\left(45+7\frac{1}{2}^{o}\right)}$

$= -\frac{1-\tan 7\frac{1}{2}^{o}}{1+\tan 7\frac{1}{2}^{o}} = -\frac{\cos 7\frac{1}{2}^{o} - \sin 7\frac{1}{2}^{o}}{\cos 7\frac{1}{2}^{o} + \sin 7\frac{1}{2}^{o}}$

$= -\frac{\left(\cos 7\frac{1}{2}^{o} - \sin 7\frac{1}{2}^{o}\right)^2}{\cos 15°} = -\frac{1-\sin 15°}{\cos 15°} = -\left(\frac{1-\frac{\sqrt{3}-1}{2\sqrt{2}}}{\frac{\sqrt{3}+1}{2\sqrt{2}}}\right) = -\frac{(2\sqrt{2}-\sqrt{3}+1)(\sqrt{3}-1)}{2}$

$= -\frac{[2\sqrt{2}\ \ (\sqrt{3}-1)-(\sqrt{3}-1)^2]}{2} = -\frac{[2\sqrt{2}\ \ (\sqrt{3}-1)-(4-2\sqrt{3})]}{2}$

$= -[\sqrt{2}(\sqrt{3}-1)-(2-\sqrt{3})] = -\sqrt{6}+\sqrt{2}+2-\sqrt{3} = 2+\sqrt{2}-\sqrt{3}-\sqrt{6}$

10. L.H.S. $\sin 2\beta = \frac{2\tan\beta}{1+\tan^2\beta} = \frac{\frac{2(\tan\alpha+\tan\gamma)}{(1+\tan\alpha\tan\gamma)}}{1+\left(\frac{\tan\alpha+\tan\gamma}{1+\tan\alpha\tan\gamma}\right)^2}$

$= \frac{2(\tan\alpha+\tan\gamma)(1+\tan\alpha\tan\gamma)}{(1+\tan\alpha\tan\gamma)^2+(\tan\alpha+\tan\gamma)^2} = \frac{2(\sin(\alpha+\gamma)\cos(\alpha-\gamma))}{\cos^2(\alpha-\gamma)+\sin^2(\alpha+\gamma)}$

$= \frac{\sin 2\alpha + \sin 2\gamma}{1+\sin 2\alpha\ .\ \sin 2\gamma}$ = RHS

$\because$ $\sin^2A - \sin^2B = \sin(A+B).\sin(A-B)$

11. $a\cos 2\theta + b\sin 2\theta = c$

$\Rightarrow \frac{a(1-t^2)}{1+t^2} + \frac{b(2t)}{1+t^2} = c$

where $t = \tan\theta$

$\Rightarrow (c+a)t^2 - 2bt + (c-a) = 0 \Rightarrow t_1 + t_2\ \frac{2b}{c+a} = t_1t_2 = \frac{c-a}{c+a}$

$\therefore\ \cos^2\alpha + \cos^2\beta = \frac{1+\cos 2\alpha + 1 + \cos 2\beta}{2} = 1 + \frac{1}{2}[\cos 2\alpha + \cos 2\beta] = 1 + \frac{1}{2}\left[\frac{1-t_1^2}{1+t_1^2}+\frac{1-t_2^2}{1+t_2^2}\right]$

simplifying and using values for t_1, t_2 we get $\cos^2\alpha + \cos^2\beta = 1 + \frac{ac}{a^2+b^2} = \frac{a^2+b^2+ac}{a^2+b^2}$

12. $4\sin 27° = (5+\sqrt{5})^{1/2} - (3-\sqrt{5})^{1/2}$

$\because$ $16\sin^2 27 = 8(1-\cos 54°) = 8\left(1-\frac{\sqrt{10-2\sqrt{5}}}{4}\right) = 2\left(4-\sqrt{10-2\sqrt{5}}\right) = 8-2\sqrt{10-2\sqrt{5}}$

$= (5+\sqrt{5}) + (3-\sqrt{5}) - 2\sqrt{(5+\sqrt{5})\ \ (3-\sqrt{5})} = \left\{\sqrt{5+\sqrt{5}}-\sqrt{3-\sqrt{5}}\right\}^2$

$\Rightarrow$ $4\sin 27° = \left(\sqrt{5+\sqrt{5}}\right)-\left(\sqrt{3-\sqrt{5}}\right)$ Ans.

13. Let x = tan A, y = tan B, z = tan C

$\because$ $\tan(A + B + C) = \dfrac{\tan A + \tan B + \tan C - \tan A \tan B \tan C}{1 - \tan A \tan B - \tan B \tan C - \tan C \tan A}$

$\because$ xy + yz + zx = 1

$\Rightarrow$ 1 – tan A tan B – tan B tan C – tan C tan A = 0

$\Rightarrow$ tan (A + B + C) = does not exist.

$\Rightarrow$ $A + B + C = (2n + 1)\dfrac{\pi}{2}$, $n \in I$

So 2A + 2B + 2C = (2n + 1)π

$\Rightarrow$ tan (2A + 2B + 2C) = 0

$\Rightarrow$ tan 2A + tan 2B + tan 2C = tan 2A tan 2B tan 2C

$\Rightarrow$ $\dfrac{2\tan A}{1-\tan^2 A} + \dfrac{2\tan B}{1-\tan^2 B} + \dfrac{2\tan C}{1-\tan^2 C} = \dfrac{8\tan A \tan B \tan C}{(1-\tan^2 A)\ (1-\tan^2 B)(1-\tan^2 C)}$

$\Rightarrow$ $\dfrac{x}{1-x^2} + \dfrac{y}{1-y^2} + \dfrac{z}{1-z^2} = \dfrac{4xyz}{(1-x^2)\ (1-y^2)\ (1-z^2)}$

14. **(a)** $\because$ $\sin^2 3a - \sin^2 a$ = (sin 3a + sin a)(sin 3a – sin a)

= (2sin 2a cos a)(2sin a cos 2a)

= (2sin 2a cos 2a)(2sin a cos a)

= sin 4a sin 2a = sin 2a sin 3a,

as desired. The last identity is evident by noting that 4a + 3a = π (and so sin 3a = sin 4a)

(b) cosec a = cosec 2a + cosec 4a

$\Rightarrow$ sin 2a sin 4a = sin a(sin 2a + sin 4a)

$\because$ R.H.S. = sin a(sin 2a + sin 4a)

= sin a (2 sin 3a cos a) = sin 2a sin 3a = sin 2a sin 4a = L.H.S.

(c) $\because \cos a - \cos 2a + \cos 3a = -(\cos 6a + \cos 2a + \cos 4a) = -\dfrac{\cos\left(\dfrac{2a+6a}{2}\right)\sin\left(\dfrac{3(2a)}{2}\right)}{\sin\left(\dfrac{2a}{2}\right)}$ $(\because 7a = \pi)$

$= -\dfrac{\cos 4a \sin 3a}{\sin a} = \dfrac{2\sin 3a \cos 3a}{2\sin a} = \dfrac{1}{2}$

(d) Because 3a + 4a = π, it follows that sin 3a = sin 4a.

$\because$ $\sin a \neq 0$ as $a = \dfrac{\pi}{7}$

$\Rightarrow$ $\sin a\,(3 - 4\sin^2 a)$ = 2 sin 2a cos 2a = 4 sin a cosa cos 2a,

$\Rightarrow$ $3 - 4(1 - \cos^2 a) = 4\cos a\,(2\cos^2 a - 1)$. It follows that

$\Rightarrow$ $8\cos^3 a - 4\cos^2 a - 4\cos a + 1 = 0$ ....(i)

From equation (i), we can say that cos a is a root of $8x^3 - 4x^2 - 4x + 1 = 0$

(e), **(f)**, **(g)** & **(h)** Because 3a + 4a = π, it follows that tan 3a + tan 4a = 0.

$\Rightarrow \quad \dfrac{\tan a + \tan 2a}{1 - \tan a \tan 2a} + \dfrac{2\tan 2a}{1 - \tan^2 2a} = 0$

or $\quad \tan a + 3\tan 2a - 3\tan a \tan^2 2a - \tan^3 2a = 0$

Let $\quad \tan a = x$. Then $\tan 2a = \dfrac{2\tan a}{1 - \tan^2 a} = \dfrac{2x}{1 - x^2}$. Hence

$x + \dfrac{6x}{1 - x^2} - \dfrac{12x^3}{(1 - x^2)^2} - \dfrac{8x^3}{(1 - x^2)^3} = 0$ or $(1 - x^2)^3 + 6(1 - x^2)^2 - 12x^2(1 - x^2) - 8x^2 = 0$

Expanding the left-hand side of the above equation gives

$x^6 - 21x^4 + 35x^2 - 7 = 0$ ....(i)

Thus tan a is a root of the above equation. Note that $6a + 8a = 2\pi$ and $9a + 12a = 3\pi$, and so $\tan[3(2a)] + \tan[4(2a)] = 0$ and $\tan[3(3a)] + \tan[4(3a)] = 0$. Hence tan 2a and tan 3a are also the roots of equation (i).

putting $x^2 = t$ in (i), we get $t^3 - 21t^2 + 35t - 7 = 0$. ....(ii)

Therefore $\tan^2 ka$, k = 1,2,3 are the distinct roots of the cubic equation (ii)

$\therefore \quad \tan^2 a + \tan^2 2a + \tan^2 3a = 21$

$\tan^2 a \tan^2 2a + \tan^2 2a \tan^2 3a + \tan^2 a \tan^2 3a = 35$

$\tan^2 a \,.\, \tan^2 2a \,.\, \tan^2 3a = 7 \Rightarrow \tan a \tan 2a \tan 3a = \sqrt{7}$

$\because \quad \cot^2 a + \cot^2 2a + \cot^2 3a = \dfrac{\tan^2 a \tan^2 2a + \tan^2 2a \tan^2 3a + \tan^2 3a \tan^2 a}{\tan^2 a \tan^2 2a \tan^2 3a} = \dfrac{35}{7} = 5$

15. L.H.S. $= 2\sin\left(\dfrac{A+B}{2.2}\right)\cos\left(\dfrac{A-B}{2.2}\right) + \cos\left(\dfrac{\pi}{2} - \dfrac{C}{2}\right)$

$= 2\sin\left(\dfrac{\pi - C}{4}\right)\cos\left(\dfrac{A-B}{4}\right) + 1 - 2\sin^2\left(\dfrac{\pi}{4} - \dfrac{C}{4}\right)$

$= 2\sin\left(\dfrac{\pi - C}{4}\right)\left[\cos\left(\dfrac{A-B}{4}\right) - \sin\left(\dfrac{\pi}{4} - \dfrac{C}{4}\right)\right] + 1$

$= 1 + 2\sin\left(\dfrac{\pi - C}{4}\right)\left[\cos\left(\dfrac{A-B}{4}\right) - \cos\left(\dfrac{\pi}{4} + \dfrac{C}{4}\right)\right]$

$= 1 + 2\sin\left(\dfrac{\pi - C}{4}\right)\left[2\sin\left(\dfrac{A+C+\pi-B}{4.2}\right)\sin\left(\dfrac{\pi+C+B-A}{4.2}\right)\right]$

$= 1 + 2\sin\left(\dfrac{\pi - C}{4}\right)\left[2\sin\dfrac{2(\pi - B)}{8}\sin\dfrac{2(\pi - A)}{8}\right] = 1 + 4\sin\left(\dfrac{\pi - A}{4}\right)\sin\left(\dfrac{\pi - B}{4}\right)\sin\left(\dfrac{\pi - C}{4}\right)$ = R.H.S.

16. Let P denotes the desired product, and let Q = sin a sin 2a sin 3a.......sin 999a.

Then 2^{999} PQ = (2 sin a cos a)(2 sin 2a cos 2a).........(2 sin 999a cos 999a)

= sin 2a sin 4a .....sin 1998a

= (sin 2a sin 4a ....sin 998a) [–sin(2π – 1000a)]. [–sin(2π – 1002a)]........[–sin(2π – 1998a)]

= sin 2a sin 4a ....sin 998a sin 999a sin 997a ....sin a = Q.

It is easy to see that $Q \neq 0$. Hence the desired product is $P = \dfrac{1}{2^{999}}$.

17. We have to prove that

$2\sin 2^\circ + 4\sin 4^\circ + ... + 178\sin 178^\circ = 90\cot 1^\circ$

which is equal to

$2 \sin 2^o . \sin 1^o + 2(2\sin 4^o.\sin 1^o) + ... + 89\,(2\sin 178^o . \sin 1^o) = 90 \cos 1^o$

We know that

$2 \sin 2k^o \sin 1^o = \cos(2k - 1)^o - \cos(2k + 1)^o$

We have $2 \sin 2^o. \sin 1^o + 2\,(2\sin 4^o.\sin 1^o) + 3\,(2 \sin 6^o \sin 1^o) + 4(2 \sin 8^o \sin 1^o) + ... + 89$ $(2\sin 178^o. \sin 1^o) = (\cos 1^o - \cos 3^o) + 2(\cos 3^o - \cos 5^o) + ... + 89(\cos 177^o - \cos 179^o)$

$= \cos 1^o + \cos 3^o + \cos 5^o + \cos 177^o + 89 \cos 1^o$

$= \cos 1^o + 89 \cos 1^o + (\cos 3^o + \cos 5^o + ... + \cos 177^o) = 90 \cos 1^o + 0 = 90 \cos 1^o$

18. $\tan 2\theta = \tan \frac{2}{\theta}$

$2\theta = n\pi + \frac{2}{\theta}$

$2\theta - \frac{2}{\theta} - n\pi = 0$

$2\theta^2 - n\pi\theta - 2 = 0$

$\theta = \frac{n\pi \pm \sqrt{n^2\pi^2 + 16}}{4}$, $n \in I$

19. We have

$5 \sin x \cos y = 1$and $4 \tan x = \tan y$

$\Rightarrow$ $5 \sin x \cos y = 1$ and $4 \sin x \cos y = \sin y \cos x$

$\Rightarrow$ $\sin x \cos y = \frac{1}{5}$ and $\cos x \sin y = \frac{4}{5}$

$\Rightarrow$ $\sin x \cos y + \cos x \sin y = 1$ and $\sin x \cos y - \cos x \sin y = -\frac{3}{5}$

$\Rightarrow$ $\sin (x + y) = \sin \frac{\pi}{2}$ and $\sin (x - y) = \sin\left\{\sin^{-1}\left(-\frac{3}{5}\right)\right\}$

$\Rightarrow$ $x + y = n\pi + (-1)^n \frac{\pi}{2} n \in Z$ and $x - y = m\pi + (-1)^m \sin^{-1}\left(-\frac{3}{5}\right)$; $m \in Z$

$\Rightarrow$ $x = (m + n) \frac{\pi}{2} + (-1)^n \frac{\pi}{4} + (-1)^m \frac{1}{2} \sin^{-1}\left(-\frac{3}{5}\right)$; $m, n \in Z$

and $y = (n - m) \frac{\pi}{2} + (-1)^n \frac{\pi}{4} + (-1)^{m+1} \frac{1}{2} \sin^{-1}\left(-\frac{3}{5}\right)$; $m, n \in Z$

20. $\frac{\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right)\left(1 + \sin\frac{x}{2}\cos\frac{x}{2}\right)}{2\left(1 + \sin\frac{x}{2}\cos\frac{x}{2}\right)} = \frac{\cos x}{3}$ or $\sin \frac{x}{2} - \cos\frac{x}{2} = \frac{2}{3}\cos x$

or $1 - \sin x = \frac{4}{9}\cos^2 x$ or $\frac{4}{9}\sin^2 x - \sin x + 1 - \frac{4}{9} = 0$

or $4 \sin^2 x - 9 \sin x + 5 = 0$ or $(4 \sin x - 5)(\sin x - 1) = 0$

or $\sin x = 1$ $\Rightarrow$ $x = 2n\pi + \frac{\pi}{2}$

21. We have,

$x + y = \frac{2\pi}{3}$ and $\cos x + \cos y = \frac{3}{2}$

Now, $\cos x + \cos y = \frac{3}{2}$ $\Rightarrow$ $2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$

$\Rightarrow$ $\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ [using : $x + y = \frac{2\pi}{3}$]

Clearly, $\cos\left(\frac{x-y}{2}\right) = \frac{3}{2}$ is not possible.

Hence the given system of equations has no solution.

22. Let $4^{\sin x} = u$ and $3^{1/\cos y} = v$. Then, the given system of equations reduces to

$u + v = 11$ ..... (i)

$5u^2 - 2v = 2$ ..... (ii)

Eliminating v between (i) and (ii), we get $5u^2 - 22 + 2u = 2$

$\Rightarrow$ $5u^2 - 22 + 2u = 2$ $\Rightarrow$ $(5u + 12)(u - 2) = 0$

$\Rightarrow$ $u - 2 = 0$ [$\because u = 4^{\sin x} > 0$ for all x $\therefore 5u + 12 > 0$]

$\Rightarrow$ $u = 2$ $\Rightarrow$ $4^{\sin x} = 2$

$\Rightarrow$ $2^{2\sin x} = 2^1$ $\Rightarrow$ $2\sin x = 1$

$\Rightarrow$ $\sin x = \frac{1}{2}$ $\Rightarrow$ $\sin x = \frac{1}{2}$

$\Rightarrow$ $\sin x = \sin\frac{\pi}{6}$ $\Rightarrow$ $x = n\pi + (-1)^n\frac{\pi}{6}$; $n \in Z$

putting the value of u in (i), we get v = 9

$\Rightarrow$ $3^{1/\cos y} = 3^2$ $\Rightarrow$ $\frac{1}{\cos y} = 2$

$\Rightarrow$ $\cos y = \frac{1}{2}$ $\Rightarrow$ $\cos y = \cos\frac{\pi}{3}$

$\Rightarrow$ $y = 2m\pi \pm \frac{\pi}{3}$; $n \in Z$

Hence, $x = n\pi + (-1)^n\frac{\pi}{6}$ and $y = 2m\pi \pm \frac{\pi}{3}$, where $m, n \in Z$

23. $\cos\theta + \sin\theta = \cos 2\theta + \sin 2\theta$

or $\cos\theta - \cos 2\theta = \sin 2\theta - \sin\theta$

or $2\sin\frac{3\theta}{2}\sin\frac{\theta}{2} = 2\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$

$\Rightarrow$ $\sin\frac{\theta}{2} = 0$ or $\sin\frac{3\theta}{2} = \cos\frac{3\theta}{2}$

$\Rightarrow$ $\theta = 2n\pi$ or $\theta = \frac{2n\pi}{3} + \frac{\pi}{6}$

24. $8\sin^2 x\cos x = \sqrt{3}\sin x + \cos x.$ $\Rightarrow 4(2\sin x\cos x)\sin x = \sqrt{3}\sin x + \cos x.$

$\Rightarrow$ $2(2\sin 2x\sin x) = \sqrt{3}\sin x + \cos x.$ $\Rightarrow 2\cos x - 2\cos 3x = \sqrt{3}\sin x + \cos x.$

$\Rightarrow$ $\cos x - \sqrt{3}\sin x = 2\cos 3x.$ $\Rightarrow \cos 3x = \cos\left(x + \frac{\pi}{3}\right) \Rightarrow 3x = 2n\pi \pm \left(x + \frac{\pi}{3}\right), n \in I$

(i) by taking positive sign $x = n\pi + \frac{\pi}{6}$, $n \in I$

(ii) by taking negative sign $x = \frac{n\pi}{2} - \frac{\pi}{12}$, $n \in I$

25. $\because$ $\sin^3 x\cos 3x + \cos^2 x\sin 3x + \frac{3}{8} = 0$

$\Rightarrow$ $\sin^3 x(3\cos^3 x - 3\cos x) + \cos^3 x(3\sin x - 4\sin^3 x) + \frac{3}{8} = 0$

$\Rightarrow$ $3 \sin x \cos x(\cos^2 x - \sin^2 x) + \frac{3}{8} = 0$

$\Rightarrow$ $8(\sin x \cos x) \cos 2x + 1 = 0$

$\Rightarrow$ $2 \sin 4x = -1$

$\Rightarrow$ $\sin 4x = -\frac{1}{2}$

$\therefore$ $x = \frac{n\pi}{4} + (-1)^{n+1} \cdot \frac{\pi}{24}$; $n \in I$

26. $\because$ $\sqrt{3} \sin x = 2(\cos x + \cos^2 x)$

Squaring both sides, we get

$3 \sin^2 x = 4(\cos^2 x + \cos^4 x + 2 \cos^3 x)$

$\Rightarrow$ $3(1 - \cos^2 x) = 4 \cos^2 x + 4 \cos^4 x + 8 \cos^3 x.$

$\Rightarrow$ $4 \cos^4 x + 8 \cos^3 x + 7 \cos^2 x - 3 = 0$

$\Rightarrow$ $(\cos x + 1)(2 \cos x - 1)(2 \cos^2 x + 3 \cos x + 3) = 0$

$\Rightarrow$ $\cos x = -1$ $\Rightarrow$ $x = (2n + 1)\pi : n \in I$

or $\cos x = \frac{1}{2}$ $\Rightarrow$ $x = 2n\pi + \frac{\pi}{3}, n \in I$

27. $\because$ $\sin^4 x + \cos^4 x - 2\sin^2 x + \frac{3}{4} \sin^2 2x = 0$

$\Rightarrow$ $(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x - 2 \sin^2 x + \frac{3}{4} . 4 \sin^2 x . \cos^2 x = 0$

$\Rightarrow$ $1 - 2 \sin^2 x + \sin^2 x \cos^2 x = 0$

$\Rightarrow$ $\sin^4 x + \sin^2 x - 1 = 0$

$\Rightarrow$ $\sin^2 x = \frac{\sqrt{5} - 1}{2}$

$\therefore$ $\cos 2x = 2 - \sqrt{5}$

$\Rightarrow$ $x = n\pi \pm \frac{1}{2} \cos^{-1}(2 - \sqrt{5}), n \in I$

28. $\sqrt{13 - 18\tan x} = 6 \tan x - 3$ ....(1)

$\Rightarrow 13 - 18 \tan x = 36 \tan^2 x + 9 - 36 \tan x$

$\Rightarrow \tan x = \frac{2}{3}, -\frac{1}{6}$

Put in (1) $\Rightarrow \tan x = \frac{2}{3}$ is correct

$\Rightarrow x = n\pi + \tan^{-1}\frac{2}{3} = n\pi + \alpha = \alpha, \pi + \alpha, -\pi + \alpha, -2\pi + \alpha$ in $(-2\pi, 2\pi)$

29. $3 - 2\cos\theta - 4 \sin\theta - \cos 2\theta + \sin 2\theta = 0$

$\Rightarrow 3 - 2\cos\theta - 4 \sin\theta - 2\cos^2\theta + 1 + 2\sin\theta \cos\theta = 0$

$\Rightarrow 4 - 2\cos\theta - 4 \sin\theta - 2\cos^2\theta + 2\sin\theta \cos\theta = 0$

$\Rightarrow 2 - 2\cos\theta - 4 \sin\theta + 2\sin^2\theta + 2\sin\theta \cos\theta = 0$

$\Rightarrow 2 - 2(\cos\theta + \sin\theta) - 2\sin\theta + 2\sin\theta (\sin\theta + \cos\theta) = 0$

$\Rightarrow$ (sin θ + cos θ) (– 2 + 2sinθ) + 2 – 2sinθ = 0 $\Rightarrow$ (sin θ + cos θ –1) (– 2 + 2sinθ) = 0

$\Rightarrow$ sin θ = 1 or sinθ + cosθ = 1 $\Rightarrow \theta = 2n\pi + \frac{\pi}{2}$ or $\cos\left(\theta - \frac{\pi}{4}\right) = \cos\frac{\pi}{4}$

$\Rightarrow \theta = 2n\pi + \frac{\pi}{2}$ or $\theta - \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4} \Rightarrow \theta = 2n\pi + \frac{\pi}{2}, 2n\pi$

30. $\sin^2 4x + \cos^2 x = 2\sin 4x \cdot \cos^4 x$ $\Rightarrow \sin^2 4x - 2\sin 4x\cos^4 x + \cos^2 x = 0$

$\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x - \cos^8 x = 0$ $\Rightarrow (\sin 4x - \cos^4 x)^2 + \cos^2 x\,(1 - \cos^6 x) = 0$

$\Rightarrow \sin 4x - \cos^4 x = 0$ ....(1)

and $\cos^2 x\,(1 - \cos^6 x) = 0$ ....(2)

From (2) $\cos^2 x = 0, 1$

Case-I $\cos^2 x = 0$ $\Rightarrow x = n\pi \pm \frac{\pi}{2}$ $\Rightarrow 4x = 4n\pi \pm 2\pi$

$\therefore$ sin 4x = 0

$\Rightarrow$ equation (1) is also true

Case-II $\cos^2 x = 1$ $\Rightarrow \sin^2 x = 0$

$\Rightarrow x = n\pi$ equation (1) becomes

0 – 1 = 0 false solution is $x = n\pi \pm \frac{\pi}{2}$

31. We have cos 5A = cos(3A + 2A) = cos 3A cos 2A – sin 3A sin 2A

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)(2\sin A\cos A)$

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3 - 4\sin^2 A)(2\sin^2 A\cos A)$

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - \{3 - 4(1 - \cos^2 A)\}\{2(1 - \cos^2 A)\cos A\}$

$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - 2\cos A(1 - \cos^2 A)(4\cos^2 A - 1)$

$= (8\cos^5 A - 10\cos^3 A + 3\cos A) - 2\cos A(5\cos^2 A - 4\cos^4 A - 1) = 16\cos^5 A - 20\cos^3 A + 5\cos A$ = RHS

32. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta = \cos\theta(4\cos^2\theta - 3) = \frac{1}{2}\left(a + \frac{1}{a}\right)\left\{4 \times \frac{1}{4}\left(a + \frac{1}{a}\right)^2 - 3\right\}$

$= \frac{1}{2}\left(a + \frac{1}{a}\right)\left\{a^2 + \frac{1}{a^2} - 1\right\} = \frac{1}{2}\left(a^3 + \frac{1}{a^3}\right)$

$\cos\theta = \frac{1}{2}\left(a + \frac{1}{a}\right) \Rightarrow a = 1$ or – 1

$\cos 3\theta = \frac{1}{2}\left(a^k + \frac{1}{a^k}\right) \Rightarrow a^k = 1$ or – 1

so k must be odd integer.

k = 1, 3, 5,........, 49.

33. $\left(\sin 2\theta + \sqrt{3}\cos 2\theta\right)^2 - 5 = \cos\left(\frac{\pi}{6} - 2\theta\right) \Rightarrow 4\left(\frac{1}{2}\sin 2\theta + \frac{\sqrt{3}}{2}\cos 2\theta\right)^2 - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0$

$\Rightarrow 4\cos^2\left(\frac{\pi}{6} - 2\theta\right) - \cos\left(\frac{\pi}{6} - 2\theta\right) - 5 = 0 \Rightarrow \cos\left(\frac{\pi}{6} - 2\theta\right) = \frac{5}{4}, -1 \Rightarrow \cos\left(\frac{\pi}{6} - 2\theta\right) = -1 = \cos\pi$

$\Rightarrow \frac{\pi}{6} - 2\theta = 2n\pi \pm \pi \Rightarrow 2\theta = \frac{\pi}{6} - 2n\pi \mp \pi \Rightarrow \theta = \frac{2n\pi}{2} + \frac{\pi}{12} \pm \frac{\pi}{2} \Rightarrow \theta = \frac{7\pi}{12}, \frac{19\pi}{12}$.